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SELECTION OF SURFACE THERMOMETERS FOR MEASURING HEAT FLUX

By: John W. Kurzrock

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FOREWORD

This report contains the analytical and experimental results obtained during the investigation of heat flux measurement by using surface thermometers applicable to the Wave Superheater Hypersonic Tunnel.

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ABSTRACT

Classical solid conduction theory is applied to a composite semi-infinite slab for the constant surface heat flux case to determine the operating limits of surface thermometers. It is shown that two dimensionless parameters σ and θ_F specify the operating range of surface thermometers. A surface thermometer is selected on the basis of these dimensionless parameters, the heat flux range, the testing time, and the output sensitivity. Experimental results of thin and thick film surface thermometers are compared with solid conduction theory to indicate the effect of thermal contact resistance between the film and the mounting material, and to verify the theoretical film thickness. The results indicate that thick film thermometers (calorimeters) can measure heat flux one to two orders of magnitude higher than thin film thermometers for the same time interval. The thin film thermometers are useful for measuring lower heat flux where high sensitivity is required.

A comparison of experimental heat flux results using thin and thick film thermometers indicated that the thin film data was 15 to 40% below the thick film data. This difference was postulated to be the use of thin film thermometers with thicknesses of 1μ instead of the required thickness of $.1\mu$ or less.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

NOMENCLATURE

T	=	temperature rise above initial temperature = $t - t_i$
T_∞	=	surface temperature rise of semi-infinite slab
$\bar{T}$	=	average temperature of film = $\frac{1}{\delta}\int_0^\delta T\,dx = \bar{t} - t_i$
q_0	=	constant heat flux at surface
q	=	instantaneous heat flux
δ	=	film thickness
τ	=	time
x	=	distance normal to film measured from surface of mounting material
k	=	thermal conductivity
ρ	=	mass density
c	=	specific heat capacity
α	=	thermal diffusivity = $k/\rho c$
σ	=	$\sqrt{(k_M \rho_M c_M)/(k_F \rho_F c_F)}$
θ	=	Fourier modulus = $\frac{\alpha\tau}{\delta^2}$
p	=	transformed variable
U	=	Laplace transform of $T(\tau)$
$erfc\,X$	=	complementary error function = $1 - erf\,X$
$erf\,X$	=	$\frac{2}{\sqrt{\pi}}\int_0^X e^{-\lambda^2}d\lambda$
$i\,erfc\,X$	=	complementary integral error function = $\int_X^\infty erfc\,\lambda\,d\lambda$
	=	$\frac{1}{\sqrt{\pi}}e^{-X^2} - X\,erfc\,X$

NOMENCLATURE (Cont.)

A	=	heat transfer surface area
m	=	mass
R	=	film resistance
I	=	film current

Subscripts

F	refers to film material
M	refers to mounting material
I	refers to interface

I. INTRODUCTION

Surface thermometers have received wide recognition within the last decade.[1,2] These instruments have become an indispensable tool for obtaining surface temperature and heat flux (rate of heat transfer per unit area) measurements under transient heating conditions in intermittent and continuous flow facilities. The magnitude of the heat flux may vary from typical values of 1 to 10^5 Btu/ft^2-sec where the measurement must be obtained within typical times varying from microseconds to seconds.

Surface thermometers have been further subdivided into "thin film" and "thick film" (calorimeter) thermometers. The purpose of this report is to investigate the criteria for the selection of thin and thick film thermometers utilizing solid conduction theory and to formulate some parameters which will specify the operating range of the surface thermometer. It is hoped that this report will clarify some of the nebulous numbers that are used to specify the selection of thin and thick film surface thermometers as well as substantiate others. Researchers should be able to quickly select surface thermometers or assess the accuracy of their measurements using thin and thick film thermometers by evaluating the magnitude of the parameters θ_F and σ. Finally, the range of these parameters was modified to take into account the effect of thermal contact between the film and the mounting material by comparing the theoretical range with experimental data.

II. SURFACE THERMOMETER RANGE DICTATED BY COMPOSITE SEMI-INFINITE SLAB CONDUCTION THEORY

Surface thermometers consist of a film (usually metallic) mounted on a semi-infinite backing material. The heat conduction problems can be formulated by applying one-dimensional heat conduction theory to a composite semi-infinite slab. The constant surface heat flux case will be considered here since the heat transfer source is usually at a high temperature and the film temperature rise does not exceed 500°F during the time which usable data is obtained.

The formal heat conduction solution was obtained in Appendix I. The temperature T_I at the interface was normalized with respect to the surface temperature of a semi-infinite slab with a constant heat flux at its surface, $T_\infty = 2q_0\sqrt{\tau/\pi k_M \rho_M c_M}$. The temperature ratio T_I/T_∞ was found to be

$$\frac{T_I}{T_\infty} = \frac{2\sigma}{1+\sigma}\sqrt{\pi}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n ierfc\left(\frac{2n+1}{2\sqrt{\theta_F}}\right) \tag{1}$$

The heat flux at the interface q_I was found by differentiating the expression for the temperature distribution with respect to the normal to the surface and solving for $x = 0$. The resulting heat flux ratio was

$$\frac{q_I}{q_0} = \frac{2\sigma}{1+\sigma}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n erfc\left(\frac{2n+1}{2\sqrt{\theta_F}}\right) \tag{2}$$

Equations (1) and (2) were solved on an IBM 704 computer. The results were presented in graphical form in Figures 1 and 2 and in tabular form in Tables I and II. The results illustrate the importance of the Fourier modulus, $\theta_F = \alpha_F\tau/\delta^2$, where α_F is the thermal diffusivity of the film of thickness δ, exposed to heat transfer for the time τ. Other authors[1, 2, 3] prefer to use the ratio of the film thickness δ, to the thermal diffusion length $\sqrt{\alpha_F\tau}$ which is the square root of the inverse of the Fourier modulus.

Figure 2 indicates that the heat flux transferred through the interface becomes negligible when the Fourier modulus approaches zero. This means that the heat is being stored in the film and a negligible amount of heat is being conducted into the mounting material. As the Fourier modulus approaches infinity, the heat flux entering the film surface is transmitted through the interface to the mounting material with a negligible amount of heat being stored in the film. In reality, the film is recording the surface temperature of the mounting material as shown in Figure 1. These two limits, small and large values of the Fourier modulus, define the appropriate operating range of transient surface thermometers.

The other important parameters that must be given consideration is sigma, $\sigma = \sqrt{(k\rho c)_M/(k\rho c)_F}$. The thermal properties of various materials are summarized in Table III. The range of sigma varies between .03 and .10 for a metallic film mounted to an insulative backing material.

A. Thin Film Surface Thermometers

Surface thermometers that are used to measure the surface temperature of the mounting material are called thin film thermometers. The reference to thin film is readily apparent when it is remembered that the Fourier modulus must be of the order of 10^5 so that the film has a negligible heat capacity. To achieve such large values of the Fourier modulus the metallic film must be made extremely thin to function as a thin film thermometer or be used for long testing times.

The surface temperature of the mounting material with zero film thickness was shown to be[4]

$$T_{\infty} = 2q_0\sqrt{\frac{\tau}{\pi k_M \rho_M c_M}} \tag{3}$$

or it can be deduced from the temperature expression for the composite semi-infinite slab derived in Appendix I by considering the film to be of the same material as the semi-infinite mounting slab and let the film thickness δ approach zero. Figure 1 indicates that the thin film surface thermometer does indeed approach the surface temperature of the semi-infinite mounting material for large values of the Fourier modulus. In fact, Figures 1 and 2 indicate as the asymptotic values of the surface temperature or heat flux are approached the product of the Fourier modulus of the film and the square of sigma is a constant for a given value of temperature ratio or heat flux ratio. For example $\theta_F = 100/\sigma^2$ or $\tau = 100(\rho c)_F^2 \delta^2/(k\rho c)_M$ for $q_x/q_o = .943$.

When the testing time is much greater than the time for the thin film thermometer to approach the asymptotic value of the surface temperature of the mounting material, the heat flux q_o can be calculated from Equation (3) using the measured temperature-time history and knowing the thermal properties of the mounting material.

The successful use of thin film thermometers is dependent upon the precision with which the properties of the backing materials are known. The techniques that are presently used in determining the thermal properties of the mounting materials are described in References 1, 2, 5, 6 and 7. The surface temperature rise of the thin film is usually restricted to 500°F[1] because of variation of the thermal properties of the mounting material with temperature. The variation of the thermal properties with temperature for pyrex, quartz, and plate glass was considered in References 7 and 8.

The general case of determining the heat flux which is a function of time was discussed in References 1, 2, 6, 9 and 10, but their solutions still require that the lag of the surface thermometer is negligible, that is, the surface temperature of the thin film has approached the asymptotic surface temperature of the mounting material.

Thin film thermometers are generally used as resistance thermometers. Resistance thermometers measure the change in film resistance with temperature by measuring the change in output voltage for a constant value of current. The small thicknesses of thin film thermometers produce resistances of the order of ohms and only require milliamp currents. The output sensitivity of a thin film thermometer might be 1.17 mv/°F[6]. The thin film thermometer also gains an order of magnitude increase in sensitivity due to the measurement of the surface temperature of an insulative material instead of a metallic material. The thin film temperature has also been measured by infrared techniques (bolometers)[11] and thermocouple techniques.[9,12]

Thin film thermometers that are used in shock tubes and hypersonic shock tunnels are made by painting a thin layer of Hanovia Liquid Bright Platinum solution on pyrex, quartz, or glass mounting material and curing at high temperatures.[1,2,6,7,9,10,13,14] The solution contains platinum and gold (silver) compounds of a resinous character in volative oils and other solvents. The resulting thickness of the film is quoted by the manufacturer to be $.1\mu$ while measurements indicate $.3$ to 4.5μ.[6,13,14] The resulting thin films are actually an alloy of platinum and small amount of gold (silver). The alloy would be expected to have a slightly lower density and a much lower thermal conductivity than pure platinum. Figures 1 and 2 indicate that a platinum alloy should not influence the response of a thin film thermometer if the product $(\rho c)_F$ does not change since the heat flux ratio (temperature ratio) becomes independent of the film thermal conductivity as its asymptotic value is approached.

B. Thick Film Thermometers

Surface thermometers that measure the average temperature of the film with negligible conduction to the mounting material are called thick film thermometers. When the film makes perfect thermal contact with the mounting material the value of the Fourier modulus must never exceed .25 for negligible heat conduction through the interface, (Figure 2 for $\sigma = .10$). If poor thermal contact exists at the interface, the thick film thermometer can be used at larger values of the Fourier modulus. If the mounting material is considered to be air at standard conditions the heat flux through the interface would be reduced by two orders of magnitude. An upper limit on the Fourier modulus for thick film thermometers with poor thermal contact might be 100. To keep the values of the Fourier modulus small the films must be made thicker, hence the name thick film thermometer.

The difficulty in using thick film thermometers is the ability to record the variation of the average film temperature with time. The temperature distribution in the thick film will reveal the value of the Fourier modulus of the film at which the temperature gradient through the film becomes negligible.

The temperature distribution in a thick film for negligible heat conduction to the mounting material can be developed from Equation (15) in Appendix I by assuming that the thermal conductivity of the mounting material is zero since $q_I = -k_M \frac{\partial T}{\partial M}\Big|_M$ or found on page 112 of Reference 4. The value of sigma would be zero for $k_M = 0$ and Equation (15) would become

$$T_F = 2q_o\sqrt{\frac{\tau}{k_F \rho_F c_F}} \sum_{n=0}^{\infty} \left\{ i\,erfc\left[\frac{(2n+1)+\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] + i\,erfc\left[\frac{(2n+1)-\frac{x}{\delta}}{2\sqrt{\theta_F}}\right]\right\} \tag{4}$$

for the temperature distribution in an insulated slab. Equation (4) was solved on an IBM 704 for values of $\frac{x}{\delta}$ = 0, -.25, -.50, -.75 and -1.0.* The results were plotted in Figure 3 and tabulated in Table IV. The temperature gradient between the surface of the film and the backside of the film converges very rapidly with increase in Fourier modulus (Figure 3). The temperature gradient through the film was obtained from Equation (4) and plotted in Figure 4. The temperature gradient is less than 3% for a Fourier modulus greater than 10 and t_i/t_I greater than .4 or for a Fourier modulus greater than 1.0 and t_i/t_I greater than .95.

The average temperature of an insulated film can be found by integrating Equation (4)

$$\bar{T}_F = \frac{1}{\delta}\int_{-\delta}^{0} T_F\,dx = \frac{q_o \alpha_F \tau}{\delta k_F} = \frac{q_o \tau}{\rho_F c_F \delta} = \frac{q_o \tau A}{m_F c_F} \tag{5}$$

and the change of the average temperature of the thick film with time will be

$$\frac{\partial \bar{T}_F}{\partial \tau} = \frac{q_o \alpha_F}{\delta k_F} = \frac{q_o}{\rho_F c_F \delta} = \frac{q_o A}{m_F c_F} \tag{6}$$

*The negative values of the ratio x/δ are consistent with the geometry as specified in the sketch in Appendix I.

The change of the film temperature with time can be shown to be (Appendix II)

$$\frac{\partial T_F}{\partial \tau} = \frac{q_o}{\sqrt{\pi \tau k_F \rho_F c_F}} \sum_{n=0}^{\infty} \left\{ e^{-\left[\frac{(2n+1)+\frac{x}{\delta}}{2\sqrt{\theta_F}}\right]^2} + e^{-\left[\frac{(2n+1)-\frac{x}{\delta}}{2\sqrt{\theta_F}}\right]^2} \right. \tag{7}$$

for a constant heat flux. Equation (7) was normalized with respect to Equation (6) and was plotted in Figure 5 for x/δ = -1 and 0. The rate of change of the thick film surface temperature is 1.17 and 1.00 of the average temperature rate of change for θ_F = .25 and 1.0, respectively. The rate of change of the interface temperature is .83 and .995 of the average temperature rate of change for θ_F = .25 and 1.0, respectively.

The limitations of achieving negligible temperature gradients and negligible heat conduction to the mounting material dictates the range of the Fourier modulus that the thick film thermometers should be used. When both of these limitations are satisfied the thick film thermometer can be called a calorimeter, and the instantaneous heat flux can be calculated from the expression

$$q = \frac{d}{d\tau}\left[\frac{m_F c_F}{A} \bar{T}_F\right] = \frac{d}{d\tau}\left[\rho_F c_F \delta \bar{T}_F\right] \tag{8}$$

Thick films with good thermal contact at the interface must be restricted to values of the Fourier modulus of .4 to 1.0 for $\sigma <$ .10, Figures 2 and 5. Thick films with poor thermal contact at the interface should be restricted to Fourier modulus values of .4 to 100.

Several methods have been used to measure the average temperature of the thick film thermometer. Thick film thermometers used as resistance thermometers tend to suffer a decrease in output sensitivity compared to thin film resistance thermometers. The decrease in resistance due to the increase in thickness of the film has to be offset by an increase in current. However, the joule heating, I^2R, limits the current increase. The thick film also suffers an order of magnitude decrease in sensitivity since it measures the temperature rise of a metallic material instead of an insulative material.

Thick film thermometers that use chromel/alumel thermocouples to sense the temperature rise of the film would have an output sensitivity of .022 mv/°F.

Other temperature measurement techniques that have been applied to thick film surface thermometers with varying degrees of sensitivities are summarized in Reference 1. Some of these techniques are: replacement of a thick film with a thermistor; the replacement of the thick film with a pyroelectric material; and the use of the thick film temperature rise to vary the reluctance of a magnetic circuit.[15]

III. EXPERIMENTAL RESULTS USING SURFACE THERMOMETERS

The previous conduction theory for transient heating provided an estimate of the approximate range of the Fourier modulus for perfect thermal contact at the interface of thin and thick film surface thermometers. In practice a perfect thermal contact never exists and the ordinate of the curves in Figures 1 and 2 would be displaced downward a finite amount. Poor thermal contact is desirable for thick film thermometers since it extends the range of the Fourier modulus. On the other hand, thin film thermometers are hampered by poor thermal contact since they would not approach the asymptotic surface temperature of the mounting material as quickly as expected. Examination of experimental results from thin and thick film thermometers used at various laboratories will reveal the effect of contact resistance and material properties on the Fourier modulus range.

Thick film resistance thermometers used in shock tubes and hypersonic shock tunnels were made of chemically pure platinum foil bonded to pyrex.[3,13,16] Stagnation heat flux measurements in a shock tube[3,13] were made with thick film resistance thermometers with a film thickness of 33μ. The thick films exhibited a constant slope during testing time of 10 to 100 microseconds or a Fourier modulus of .2 to 2 for stagnation point heat flux of 1300 to 35,000 Btu/ft^2-sec. In another application[16] a 126μ platinum film used as a resistance thermometer was bonded to pyrex to measure the throat heat transfer in a 2-inch diameter nozzle utilizing the Cornell Aeronautical Laboratory 48-Inch Hypersonic Shock Tunnel. The oscillograms indicated a constant slope during the time interval of .3 to 1.3 milliseconds or a Fourier modulus of .5 to 2 for a heat flux of 1900 to 2400 Btu/ft^2-sec, Figure 6.

Reference 17 reports the measurement of heat flux in a hotshot tunnel with thick film (calorimeter) thermometers made of .0002 to .020 inch thick copper. Chromel/constantan thermocouples are used to sense the thick film temperature. Heat flux in the range of 2 to 1000 Btu/ft^2-sec has been measured for 5 to 80 milliseconds testing time. The approximate range of the Fourier modulus is 1 to 100 since the copper films are insulated from the nylon mounting bushings by an air space.

Thick film surface thermometers are being used in the Cornell Aeronautical Laboratory Wave Superheater Hypersonic Tunnel. The calorimeters are made from 1/8 to 1/4 inch diameter electrolytic tough pitch copper with film thickness of 1/32 to 1/8 of an inch. The calorimeter "buttons" were mounted in a fiberglass cloth which used a melamine resin. Forty gauge chromel/alumel thermocouples are used to measure the temperature of the copper button. The mass, the heat transfer surface area, and the thickness of the copper buttons were determined by measurement and the heat flux was then computed from Equation (8) using the slope of the temperature-time curve recorded on an oscillograph.

Heat flux in the range of 50 to 3000 Btu/ft^2-sec has been measured in the Wave Superheater Tunnel. Constant slope data was obtained during the time interval of .03 to 1.2 seconds for the 1/32 to 1/8 inch gages indicating a Fourier modulus of .4 to 70. The testing time was 3 to 6 seconds.

A typical temperature-time trace is reproduced in Figure 7. Two copper calorimeters were mounted flush with the flat face of 3/4-inch diameter rod 120° apart to compare the heat flux results when the range of the Fourier modulus is varied. The 1/32 inch thick button indicated a heat flux of 66.7 Btu/ft^2-sec for a Fourier modulus of 11 to 51. The 1/8 inch thick button indicated a heat flux of 70.0 Btu/ft^2-sec for a Fourier modulus of .4 to 11. It should be mentioned that the lower limits of the Fourier modulus can only be approximate since the model is injected into the air flow and first passes through a boundary layer with a stagnation pressure higher than the tunnel stagnation pressure.

The experimental data for thick film thermometers indicate that they can be operated over a Fourier modulus range of .2 to 2.0 when used as a resistance thermometer with negligible heat lost to the backing material. When the thick film temperature is sensed by thermocouples attached to the back of the film the Fourier modulus can range from .4 to 100 if poor thermal contact exists between the film and mounting material (air space). The experimental results for thick film thermometers clearly indicate that the rate of heat transfer through the interface has been reduced one to two orders of magnitude by an air space or poor thermal contact when compared with the results in Figure 2 for perfect thermal contact. The experimental data for thick films using thermocouples as film temperature sensors verify the theoretical heat conduction solutions (Figures 4 and 5) that even though a temperature gradient exists through the thick film the temperature-time slope can be considered to be constant for a Fourier modulus greater than .40.

References 9, 10, 13 and 14 report the use of thin film thermometers to measure stagnation heat flux in shock tubes and shock tunnels. The thin films were all made of Hanovia Liquid Bright Platinum. Henshall has compared the measured stagnation heat flux results of his Laboratory[10] utilizing thin film thermometers with other investigators.[9,13] The thin film results are 15 to 40% lower than the theory of Fay and Riddell[19] while the thick film results[13] agree with the theory of Fay and Riddell. Hartunian[7] indicates that the thin film heat flux results in References 9 and 10 would require a correction up to 20 to 40% due to the change of the thermal properties of the mounting material with temperature for heat flux values greater than 1000 Btu/ft^2-sec. The thin film experimental results at lower values of the heat flux cannot be accounted for by the change in thermal properties of the mounting material. The thin film data was taken over a time interval of 20 to 500 microseconds indicating that q_x/q_o was .96 to .99 for a film .1μ thick or that q_x/q_o was .69 to .93 for a film 1μ thick, Figure 2. The thickness of the thin film thermometers used in these experiments seems to be open to question. It appears that the thin films were too thick and could not monitor the actual surface temperature of the mounting material. More direction should be given to the measurement of the properties of the thin films used and in particular their thicknesses. The thermal contact between the mounting material was assumed to be perfect. Every effort must be made to insure good thermal contact between the thin film and the mounting material. If good thermal contact cannot be attained, the usefulness of the thin film thermometer for measuring surface temperature and thus heat flux is seriously limited.

IV. SELECTION OF THIN AND THICK FILM THERMOMETERS

The selection of surface thermometers for a specific application can be best illustrated by two examples which are representative of hypersonic testing facilities. First, consider the problem of determining the heat flux in a hypersonic shock tunnel which has a testing time of 1 millisecond. Since the testing time is short, a surface thermometer with a fast response and good sensitivity will be required. These considerations would justify the use of resistance thermometry for measuring the film temperature of the surface thermometers. A thin and thick film surface thermometer made of platinum and used as a resistive element will indicate representative values of heat flux that are obtainable for testing times of 1 millisecond.

Representative surface temperature rises* are .5 to 400°F and 50 to 400°F for thin and thick film resistance thermometers, respectively. The mounting material will be pyrex and good thermal contact at the interface between the platinum film and the pyrex would be expected. The value of sigma would be .1 in both cases. The required thickness of the films can be calculated from the Fourier modulus ranges suggested in the previous discussions. That is, the thickness of the thick film thermometer can be calculated from $\theta_F = 2$ and the testing time of 1 millisecond. The time at which the data should exhibit constant slope can be obtained from the value of the thickness and $\theta_F = .2$. The thickness of the thin film thermometer can be calculated from the Fourier modulus and an assumed time which is less than the testing time so that the thin film operates during a time interval where the lag of the thin film is negligible. The value of the Fourier modulus used depends upon the accuracy desired. A value of $\theta_F = 10^5$ will result in a lag of approximately 3% between the thin film temperature and the surface temperature of the mounting material. The range of the constant heat flux that can be measured can be calculated from Equation (3) for the thin film and Equation (8) for the thick film. The results are tabulated in Table V.

*The lower limit of the temperature rise is dependent upon the sensitivity of the thermometer while the upper limit of the temperature rise is dependent on the variation of the thermal properties of the film.

TABLE V

Selection of Thin and Thick Film Surface Thermometers for the Case of a One Millisecond Test in a Hypersonic Shock Tunnel

Quantity	Thin Film	Thick Film
Film material	platinum	platinum
Temp. measurement technique	resistance thermometer	resistance thermometer
Assumed temperature rise (°F)	.5 to 400	50 to 400
Initial time for taking data (millisec)	.04 (assumed)	.10
θ_F at initial time for data	10^5	.2
θ_F at τ = 1 millisecond	2.5×10^6	2 (assumed)
Film thickness δ (microns)	.10	110
Constant heat flux range (Btu/ft^2-sec)	1 to 800	785 to 6,300

The results indicate that the thin and thick film thermometers complement each other. The thin film thermometer should be used to measure the lower range of heat flux values while the thick film thermometer is useful for the higher range of heat flux values.

As a second example consider the measurement of heat flux in a hypersonic tunnel facility where longer testing times are available. The heat flux data is to be obtained during a 1 second time interval. A thin film thermometer made of platinum and used as a resistance thermometer will be compared with a thick film thermometer made of copper and using a thermocouple to measure its temperature. The thin film thermometer would be mounted on pyrex while the thick film thermometer will be mounted on plastic or nylon with an air space behind the copper film. Good thermal contact at the interface would be expected for the thin film while the thick film would have a very poor contact. The calculations for both thermometers will follow those outlined in the first example with the exception that the Fourier modulus for a thick film thermometer in poor thermal contact might be 50. The results are summarized in Table VI.

TABLE VI

Selection of Thin and Thick Film Surface Thermometers for the Case of a One-Second Time Interval in a Hypersonic Shock Tunnel

Quantity	Thin Film	Thick Film
Film material	platinum	copper
Temp. measurement technique	resistance thermometer	thermocouple
Assumed temperature rise (°F)	.5 to 400	50 to 400
Initial time for taking data (sec)	.008 (assumed)	.008
θ_F at initial time	10^5 (assumed)	.40
θ_F at 1 second	1.25×10^7	50 (assumed)
Film thickness δ (microns)	1.4	1500
Constant heat flux range (Btu/ft^2-sec)	.03 to 24	11 to 87

The two examples indicate the effect of the time interval on the heat flux range. An increase of the time interval causes a reduction in the magnitude of the heat flux that can be measured assuming an upper limit on the temperature rise of the film. The measurement of heat flux with magnitudes of thousands (units of Btu/ft^2-sec) must be done during time intervals of 10^{-2} second or less with thick film thermometers. For the same time interval the thick film thermometer can measure heat flux that has a magnitude one to two times that of the thin film thermometer.

V. CONCLUDING REMARKS

Classical one-dimensional heat conduction theory was applied to a finite slab in perfect thermal contact with a semi-infinite slab for the constant heat flux case under transient heating conditions. The rate of heat transferred from the finite slab to the semi-infinite slab was shown to be a function of the Fourier modulus θ_F and the thermal properties σ, Figure 2. These two parameters defined the applicable range of thick and thin film surface thermometers.

The thin film surface thermometer records the instantaneous surface temperature of the mounting material from which the instantaneous heat flux can be calculated. The rate of heat transferred to a thin film thermometer can be used to measure heat flux when the Fourier modulus $\theta_F \geqq 10^5$ with negligible error ($q_I/q_o \geq .97$) for a $\sigma = .10$. A typical thin film made of platinum would satisfy these conditions after 40 microseconds if it had a thickness of $.1\mu$. The experimental data from shock tubes indicate that thin films are being used for testing times from 20 to 500 microseconds with film thicknesses greater than $.1\mu$ which can result in lower calculated heat flux values than actually exist.

The thick film surface thermometer stores the heat in the film with negligible heat conduction to the mounting material. The instantaneous temperature is recorded and the instantaneous heat flux is determined from the slope of the temperature-time curve. Experimental data indicate that thick film platinum resistance thermometers that have been used in shock tubes and tunnels can be used over a Fourier modulus range of .2 to 2.0 since perfect thermal contact does not exist between the film and the mounting material. Thick film copper thermometers that use thermocouples attached to the back surface of the film can be used over a Fourier modulus range of .4 to 100 if very poor thermal contact exists between the film and the mounting material.

The successful design of a highly accurate surface thermometer entails the integration of the conditions required by solid conduction theory with the

available testing time, limitations of film temperature rise because of variations in thermal properties and radiation, the magnitude of the heat flux being measured, and the sensitivity of the temperature measuring technique that is being used. Over-all accuracies of thin and thick film thermometers are probably ±5% to ±15% if the designer is careful in his surface thermometer selection.

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REFERENCES

1. Vidal, R. J.:
   "Transient Surface Temperature Measurements," CAL Report No. 114 (March 1962)

2. Hall, J. G. and Hertzberg, A.:
   "Recent Advances in Transient Surface Temperature Thermometry," Jet Propulsion, v 28, no. 11, pp 719-723 (November 1958)

3. Rose, P.H.:
   "Development of the Calorimeter Heat Transfer Gage for Use in Shock Tubes," AVCO Research Report 17 (February 1958)

4. Carslaw, H.S. and Jaeger, J.E.:
   "Conduction of Heat in Solids," Clarendon Press, Oxford (1959) Second ed, II, p 75

5. Skinner, G. T.:
   "A New Method of Calibrating Thin Film Gauge Backing Materials," CAL Report No. 105 (June 1962)

6. Vidal, R. J.:
   "Model Instrumentation Techniques for Heat Transfer and Force Measurements in a Hypersonic Shock Tunnel," CAL Report No. AD-917-A-1 (February 1956) WADC TN 56-315

7. Hartunian, R.A. and Varwig, R.L.:
   "On Thin Film Heat Transfer Measurements in Shock Tubes and Shock Tunnels," Physics of Fluids, v 5, no. 2, pp 169-174 (February 1962)

8. Somers, L.M.:
   "The Variation of $(k\rho c)^{1/2}$ with Temperature in "Pyrex," CAL Report No. 106 (July 1961)

9. Sabol, A.P.:
   "Measurements in a Shock Tube of Heat Transfer Rates at the Stagnation Point of a One-Inch Diameter Sphere for Real Gas Temperatures up to 7900°R," NACA TN 4354 (August 1958)

10. Henshall, B.D.:
    "Stagnation-Point Heat Transfer Rate Measurements in the Unexpanded Flow of the NPL Hypersonic Shock Tunnel," Aeronautical Research Council Technical Report C.P. No. 468 (1960)

11. Camac, M. and Feinberg, R.:
"High Speed Infrared Bolometer," AVCO Research Report 120 (March 1962)

12. Bendersky, D.:
"A Thermocouple for Measuring Transient Temperatures," Mechanical Engineering, v 75, no 2, pp 117-121 (February 1953)

13. Rose, P.H. and Stark, W.I.:
"Stagnation Point Heat Transfer Measurements in Dissociated Air," AVCO Research Report 3 (April 1957)

14. Vidal, R.J., Wittliff, C.E., Bartlett, G.E. and Logan, J.G.:
"Investigation of Stagnation Point Heat Transfer in the CAL Hypersonic Shock Tunnel," CAL Report No. AA-966-A-1 (November 1955)

15. Andrews, J.C. and Morgan, C.C.:
"Morgandyne Heat-Transfer Transducer and a Flame-Torch Calibration Technique for Hypervelocity Wind Tunnels," Arnold Engineering Development Center, AEDC TR 60-1 (February 1960)

16. Private communication with L. Bogdan and E. Czeck, Cornell Aeronautical Laboratory (July 1962)

17. Ledford, R.L.:
"A Device for Measuring Heat Transfer Rates in Arc-Discharge Hypervelocity Wind Tunnels," Arnold Engineering Development Center, AEDC TDR-62-64 (May 1962)

18. Carpenter, J.E.:
"Wave Superheater Tunnel," CAL Report No. AD-1345-W-7, (1 July 1961 to January 1962)

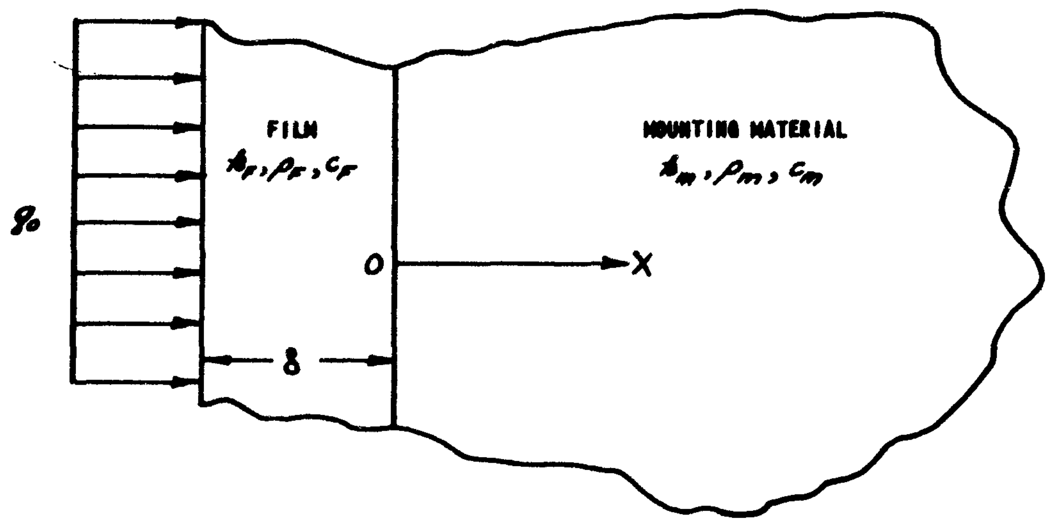
19. Fay, J.A. and Ridell, F.R.:
"Theory of Stagnation Point Heat Transfer in Dissociated Air," Journal of the Aeronautical Sciences, v 25, no 2, pp 73-85, (February 1958)

20. Scott, E.J.:
"Transform Calculus," Harper and Brothers, New York (1955) p 322

APPENDIX I

ONE-DIMENSIONAL HEAT CONDUCTION IN A COMPOSITE SEMI-INFINITE SLAB FOR THE CASE OF A CONSTANT SURFACE HEAT FLUX AND PERFECT THERMAL CONTACT BETWEEN THE SLABS

The transient one-dimensional heat conduction problem for a film in perfect thermal contact with a semi-infinite mounting material is illustrated in the following sketch.



COMPOSITE SEMI-INFINITE SLAB GEOMETRY

The Fourier heat conduction equation is

$$\frac{\partial T}{\partial \tau} = \alpha \frac{\partial^2 T}{\partial x^2} \qquad -\delta \le x \le \infty \qquad \tau > 0 \tag{I-1}$$

with the boundary conditions

$$T = 0 \qquad x \to \infty \qquad \tau \geq 0 \tag{I-2}$$

$$T_F = T_M = T_I \qquad x = 0 \qquad \tau \geq 0 \tag{I-3}$$

$$k_F \frac{\partial T}{\partial x}\Big)_F = k_M \frac{\partial T}{\partial x}\Big)_M \qquad x = 0 \qquad \tau \geq 0 \tag{I-4}$$

$$\frac{\partial T}{\partial x}\Big)_F = \frac{-q_0}{k_F} \qquad x = \delta \qquad \tau \geq 0 \tag{I-5}$$

The Laplace transform of Equation (I-1) with respect to time τ is

$$PU - T(\tau = 0) = \alpha \frac{d^2U}{dx^2} \qquad -\delta \leq x \leq \infty \qquad \tau > 0 \tag{I-6}$$

where T (τ = 0) is zero since the temperature T is defined as the temperature rise above the initial temperature. The boundary conditions now become

$$U_M = 0 \qquad x \to \infty \qquad \tau \geq 0 \tag{I-7}$$

$$U_F = U_M \qquad x = 0 \qquad \tau \geq 0 \tag{I-8}$$

$$k_F \frac{\partial U}{\partial x}\Big)_F = k_M \frac{\partial U}{\partial x}\Big)_M \qquad x = 0 \qquad \tau > 0 \tag{I-9}$$

$$\frac{\partial U}{\partial x}\Big)_F = -\frac{q_0}{k_F P} \qquad x = -\delta \qquad \tau > 0 \tag{I-10}$$

The general solution of Equation (I-6) is

$$U_F = C_1 \cosh x\sqrt{P/\alpha_F} + C_2 \sinh x\sqrt{P/\alpha_F} \tag{I-11}$$

$$\frac{\partial U}{\partial x}\Big)_F = \sqrt{P/\alpha_F}\left(C_1 \sinh x\sqrt{P/\alpha_F} + C_2 \cosh x\sqrt{P/\alpha_F}\right) \tag{I-12}$$

for $\delta \leq x \leq 0$ and

$$U_M = C_3\, e^{x\sqrt{P/\alpha_M}} + C_4\, e^{-x\sqrt{P/\alpha_M}} \tag{I-13}$$

$$\left.\frac{\partial U}{\partial x}\right)_M = \sqrt{P/\alpha_M}\left(C_3\, e^{x\sqrt{P/\alpha_M}} - C_4\, e^{-x\sqrt{P/\alpha_M}}\right) \tag{I-14}$$

for $x \geq 0$. To satisfy boundary conditions (I-7) C_3 must be zero. To satisfy boundary condition (I-8) C_1 must equal C_4. To satisfy boundary condition (I-9)

$$\frac{C_2}{C_4} = \frac{C_2}{C_1} = -\frac{k_M}{k_F}\sqrt{\frac{\alpha_F}{\alpha_M}} = -\sqrt{\frac{k_M \rho_M c_M}{k_F \rho_F c_F}} = -\sigma$$

The application of the final boundary condition (I-10) results in

$$C_4 = C_1 = \frac{q/A)_0 \sqrt{\alpha_F}}{k_F P^{3/2}\left(\sinh \delta\sqrt{P/\alpha_F} + \sigma \cosh \delta\sqrt{P/\alpha_F}\right)}$$

The particular solution for U_F is

$$U_F = \frac{q_0\sqrt{\alpha_F}\left[\cosh x\sqrt{P/\alpha_F} - \sigma \sinh x\sqrt{P/\alpha_F}\right]}{k_F P^{3/2}\left[\sinh \delta\sqrt{P/\alpha_F} + \sigma \cosh \delta\sqrt{P/\alpha_F}\right]}$$

$$U_F = \frac{q_0\sqrt{\alpha_F}\left[e^{-\sqrt{P/\alpha_F}(\delta+x)} + \left(\frac{1-\sigma}{1+\sigma}\right) e^{-\sqrt{P/\alpha_F}(\delta-x)}\right]}{k_F P^{3/2}\left[1-\left(\frac{1-\sigma}{1+\sigma}\right) e^{-2\delta\sqrt{P/\alpha_F}}\right]}$$

The denominator can be expanded in an infinite series, $\frac{1}{1-x} = \sum_{n=0}^{\infty} = X^n$ so that U_F becomes

$$U_F = \frac{q_0\sqrt{\alpha_F}}{k_F P^{3/2}}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n \left\{e^{-\sqrt{P/\alpha_F}\,[(2n+1)\delta+x]} + \frac{1-\sigma}{1+\sigma}\, e^{-\sqrt{P/\alpha_F}\,[(2n+1)\delta-x]}\right\}$$

Similarly U_M becomes

$$U_M = \frac{q_0\sqrt{\alpha_F}\, e^{-x\sqrt{P/\alpha_M}}}{k_F P^{3/2}(\sinh\delta\sqrt{P/\alpha_F} + \sigma\cosh\delta\sqrt{P/\alpha_F})}$$

$$U_M = \frac{2q_0\sqrt{\alpha_F}}{(1+\sigma)k_F P^{3/2}}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n e^{-\sqrt{P/\alpha_F}\left[(2n+1)\delta + \sqrt{\frac{\alpha_F}{\alpha_M}}x\right]}$$

The inverse Laplace transform of U_F and U_M is[20]

$$T_F = \frac{2q_0\sqrt{\alpha_F\tau}}{k_F}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n\left\{ierfc\left[\frac{(2n+1)+x/\delta}{2\sqrt{\theta_F}}\right]+\left(\frac{1-\sigma}{1+\sigma}\right)ierfc\left[\frac{(2n+1)-x/\delta}{2\sqrt{\theta_F}}\right]\right\} \quad \text{(I-15)}$$

$$T_M = \frac{4q_0\sqrt{\alpha_F\tau}}{(1+\sigma)k_F}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n ierfc\left[\frac{(2n+1)+\sqrt{\frac{\alpha_F}{\alpha_M}}\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] \quad \text{(I-16)}$$

where θ_F is the Fourier modulus $\alpha_F\tau/\delta^2$.

The temperature T_I at the interface between the film and the semi-infinite slab can be found from Equations (I-15) or (I-16) for $x = 0$.

$$T_I = \frac{4q_0\sqrt{\alpha_F\tau}}{(1+\sigma)k_F}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n ierfc\left[\frac{2n+1}{2\sqrt{\theta_F}}\right] \quad \text{(I-17)}$$

The interface temperature T_I was normalized with respect to the surface temperature of a semi-infinite slab with a constant heat flux at its surface, $T_\infty = 2q_0\sqrt{\alpha_M\tau/\pi}/k_M$. The temperature ratio becomes

$$\frac{T_I}{T_\infty} = \frac{2\sigma}{1+\sigma}\sqrt{\pi}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n ierfc\frac{2n+1}{2\sqrt{\theta_F}} \quad \text{(I-18)}$$

The heat flux at any location in the mounting material can be determined by differentiating the expression for the temperature distribution in the mounting material with respect to x .

$$q = -k_M \left.\frac{\partial T}{\partial x}\right)_M$$

$$= -\frac{4\dot{q}_0 \sqrt{a_F \tau}\, k_M}{k_F(1+\sigma)} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma}\right)^n \frac{\partial}{\partial x}\left\{ i\,erfc\left[\frac{2n+1+\sqrt{\frac{\alpha_F}{\alpha_M}}\frac{x}{\delta}}{2\sqrt{\theta_F}}\right]\right. \tag{I-19}$$

$$q = \frac{2\sigma q_0}{1+\sigma}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n erfc\left[\frac{(2n+1)+\sqrt{\frac{\alpha_F}{\alpha_M}}\frac{x}{\delta}}{2\sqrt{\theta_F}}\right]$$

The heat flux at the interface, $x = 0$ becomes

$$\frac{q_I}{q_0} = \frac{2\sigma}{1+\sigma}\sum_{n=0}^{\infty}\left(\frac{1-\sigma}{1+\sigma}\right)^n erfc\left(\frac{2n+1}{2\sqrt{\theta_F}}\right) \tag{I-20}$$

APPENDIX II

TEMPERATURE DISTRIBUTION RATE OF CHANGE OF TEMPERATURE FOR AN INSULATED SLAB (THICK FILM)

The temperature distribution in an insulated slab can be found from Equation (I-15) of Appendix I by setting $\sigma = 0$ ($k_M = 0$) since the boundary condition of an insulated slab is that $q_x = -k \left.\frac{\partial T}{\partial x}\right)_{x=0} = 0$. The expression for temperature distribution becomes

$$T_F = \frac{2q_o\sqrt{\alpha_F \tau}}{k_F} \sum_{n=0}^{\infty} \left\{ i\,erfc\left[\frac{(2n+1)+\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] + i\,erfc\left[\frac{(2n+1)-\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] \right\} \tag{II-1}$$

Equation (II-1) can be normalized with respect to the surface temperature that would occur if it were a semi-infinite slab

$$\frac{T_F}{\frac{2q_o\sqrt{\alpha_F \tau/\pi}}{k_F}} = \frac{T_F}{T_\infty} = \sqrt{\pi} \sum_{n=0}^{\infty} \left\{ i\,erfc\left[\frac{(2n+1)+\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] + i\,erfc\left[\frac{(2n+1)-\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] \right\} \tag{II-2}$$

The change of the film temperature with time will be

$$\frac{\partial T_F}{\partial \tau} = \frac{q_o}{k_F}\sqrt{\frac{\alpha_F}{\tau}} \sum_{n=0}^{\infty} \left\{ i\,erfc\left[\frac{(2n+1)+\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] + i\,erfc\left[\frac{(2n+1)-\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] \right\}$$

$$+ \frac{2q_o\sqrt{\alpha_F \tau}}{k_F} \frac{\partial}{\partial \tau} \left\{ \sum_{n=0}^{\infty} \left\{ i\,erfc\left[\frac{(2n+1)+\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] + i\,erfc\left[\frac{(2n+1)-\frac{x}{\delta}}{2\sqrt{\theta_F}}\right] \right\} \right\}$$

The partial derivative of the complementary integral error function can be evaluated in the following manner:

$$\frac{\partial}{\partial\tau}\left[ierfc\left(\frac{C}{\sqrt{\tau}}\right)\right] = \frac{\partial}{\partial\tau}\left[\frac{1}{\sqrt{\pi}}e^{-\frac{C^2}{\tau}} - \frac{C}{\sqrt{\tau}}\left(1-erf\left(\frac{C}{\sqrt{\tau}}\right)\right)\right]$$

$$= \frac{C^2}{\sqrt{\pi}\,\tau^2}e^{-\frac{C^2}{\tau}} + \frac{1}{2}\frac{C}{\tau^{3/2}}\left[1-erf\left(\frac{C}{\sqrt{\tau}}\right)\right]$$

$$-\frac{C}{\sqrt{\tau}}\left[0+\frac{C}{\sqrt{\pi}\,\tau^{3/2}}\,e^{-\left(\frac{C}{\sqrt{\tau}}\right)^2}\right]$$

$$= \frac{C^2}{\sqrt{\pi}\,\tau^2}\,e^{-\left(\frac{C^2}{\tau}\right)} + \frac{1}{2}\frac{C}{\tau^{3/2}}\,erfc\left(\frac{C}{\sqrt{\tau}}\right) - \frac{C^2}{\sqrt{\pi}\,\tau^2}\,e^{-\left(\frac{C}{\sqrt{\tau}}\right)^2}$$

$$= \frac{1}{2}\frac{C}{\tau^{3/2}}\,erfc\left(\frac{C}{\sqrt{\tau}}\right)$$

The rate of change of the film temperature can now be written as

$$\frac{\partial T_F}{\partial\tau} = \frac{q_0}{k_F}\sqrt{\frac{\alpha_F}{\tau}}\sum_{n=0}^{\infty}\left\{\frac{1}{\sqrt{\pi}}e^{-\left[\frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}\right]^2} - \frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}erfc\left(\frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}\right)\right.$$

$$+\frac{1}{\sqrt{\pi}}e^{-\left[\frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}\right]^2} - \frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}erfc\left(\frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}\right)$$

$$\left.+\frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}erfc\left(\frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}\right) + \frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}erfc\left(\frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}\right)\right\}$$

$$\frac{\partial T_F}{\partial\tau} = \frac{q_0}{\sqrt{\pi k_F \rho_F C_F \tau}}\sum_{n=0}^{\infty}\left\{e^{-\left[\frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}\right]^2} + e^{-\left[\frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}\right]^2}\right\} \qquad \text{(II-3)}$$

The average temperature of the insulated film can be found by integrating Equation (I-1)

$$\bar{T}_F = \frac{1}{\delta}\int_{-\delta}^{0} T_F \,\delta x = \frac{q_o \tau}{\rho_F C_F \delta} \tag{II-4}$$

and the rate of change of the average film temperature will be

$$\frac{\partial \bar{T}_F}{\partial \tau} = \frac{q_o}{\rho_F C_F \delta} \tag{II-5}$$

The ratio of the rate of change of the film temperature with respect to the rate of change of the average film temperature is

$$\frac{\partial T_F/\partial \tau}{\partial \bar{T}_F/\partial \tau} = \frac{1}{\sqrt{\pi \theta_F}} \sum_{n=0}^{\infty} \left\{ e^{-\left[\frac{(2n+1)+(x/\delta)}{2\sqrt{\theta_F}}\right]^2} + e^{-\left[\frac{(2n+1)-(x/\delta)}{2\sqrt{\theta_F}}\right]^2} \right\} \tag{II-6}$$

TABLE I

TEMPERATURE AT INTERFACE OF A COMPOSITE SEMI-INFINITE SLAB FOR A CONSTANT SURFACE HEAT FLUX AND PERFECT THERMAL CONTACT AT THE INTERFACE

θ_F	T_I/T_∞						
	σ = 0.005	σ = 0.01	σ = 0.03	σ = 0.10	σ = 0.50	σ = 1.0	σ = 2.0
1.0000E-02*	0.	0.	0.	0.	0.	0.	0.
0.4000E-01	0.12564941E-05	0.25005479E-05	0.73559808E-05	0.22959577E-04	0.84185114E-04	0.12627767E-03	0.16837022E-03
0.6250E-01	0.17257659E-04	0.34344450E-04	0.10103270E-03	0.31534450E-03	0.11562631E-02	0.17343947E-02	0.23125263E-02
0.9000E-01	0.77173053E-04	0.15358201E-03	0.45179953E-03	0.14101622E-02	0.51705946E-02	0.77558921E-02	0.10341189E-01
0.1225E-00	0.20051719E-03	0.39904907E-03	0.11739016E-02	0.36639959E-02	0.13434651E-01	0.20151978E-01	0.26869304E-01
0.1600E-00	0.38598205E-03	0.76814251E-03	0.22596813E-02	0.70529442E-02	0.25860781E-01	0.38791173E-01	0.51721563E-01
0.2500E-00	0.88636111E-03	0.17639454E-02	0.51890709E-02	0.16196078E-01	0.59383997E-01	0.89074323E-01	0.11876353E-00
0.3600E-00	0.14633296E-02	0.29121462E-02	0.85665162E-02	0.26735077E-01	0.97987799E-01	0.14693961E-00	0.19586335E-00
0.4900E-00	0.20466298E-02	0.40728281E-02	0.11979304E-01	0.37370615E-01	0.13674743E-00	0.20483433E-00	0.27273004E-00
0.6400E 00	0.26104249E-02	0.51943964E-02	0.15273625E-01	0.47602250E-01	0.17353532E-00	0.25926599E-00	0.34430607E-00
0.8100E 00	0.31512293E-02	0.62697364E-02	0.18426509E-01	0.57337876E-01	0.20772324E-00	0.30899698E-00	0.40855236E-00
0.1000E 01	0.36727320E-02	0.73060703E-02	0.21457838E-01	0.66626107E-01	0.23930869E-00	0.35385468E-00	0.46503249E-00
0.4000E 01	0.84228996E-02	0.16707765E-01	0.48528931E-01	0.14544445E-00	0.45679855E-00	0.61874365E 00	0.74789596E 00
0.9000E 01	0.12894774E-01	0.25491117E-01	0.73074520E-01	0.21035463E-00	0.57909376E 00	0.73224095E 00	0.83849858E 00
0.1600E 02	0.17272820E-01	0.34026004E-01	0.96246073E-01	0.26639187E-00	0.65739293E 00	0.79402767E 00	0.88143397E 00
0.2500E 02	0.21594231E-01	0.42388526E-01	0.11831950E-00	0.31549969E-00	0.71159741E 00	0.83273793E 00	0.90638667E 00
0.3600E 02	0.25871618E-01	0.50605642E-01	0.13942061E-00	0.35891188E-00	0.75122884E 00	0.85923191E 00	0.92267987E 00
0.4900E 02	0.30110567E-01	0.58690222E-01	0.15962981E-00	0.39754254E-00	0.78140988E 00	0.87849380E 00	0.93414927E 00
0.6400E 02	0.34314017E-01	0.66649806E-01	0.17900854E-00	0.43210975E-00	0.80513120E 00	0.89312531E 00	0.94265900E 00
0.8100E 02	0.38483737E-01	0.74489546E-01	0.19760831E-00	0.46319361E-00	0.82425035E 00	0.90461515E 00	0.94922275E 00
0.1000E 03	0.42620911E-01	0.82213385E-01	0.21547436E-00	0.49127042E-00	0.83997927E 00	0.91387624E 00	0.95443919E 00
0.4000E 03	0.82333681E-01	0.15365614E-00	0.36138345E-00	0.66955139E 00	0.91573439E 00	0.95631358E 00	0.97753188E 00
0.9000E 03	0.11927744E-00	0.21592599E-00	0.46469941E-00	0.75722056E 00	0.94285899E 00	0.97073686E 00	0.98509068E 00
0.1600E 04	0.15371870E-00	0.27056575E-00	0.54093529E 00	0.80864988E 00	0.95678118E 00	0.97800057E 00	0.98884406E 00
0.2500E 04	0.18588264E-00	0.31879520E-00	0.59910814E 00	0.84228047E 00	0.96525037E 00	0.98237544E 00	0.99108772E 00
0.3600E 04	0.21596825E-00	0.36160061E-00	0.64474600E 00	0.86593135E 00	0.97094488E 00	0.98529898E 00	0.99258002E 00
0.4900E 04	0.24415336E-00	0.39978646E-00	0.68138841E 00	0.88344894E 00	0.97503615E 00	0.98739063E 00	0.99364429E 00
0.6400E 04	0.27059732E-00	0.43401429E-00	0.71138820E 00	0.89693572E 00	0.97811761E 00	0.98896123E 00	0.99444152E 00
0.8100E 04	0.29544307E-00	0.46483148E-00	0.73636074E 00	0.90763499E 00	0.98052197E 00	0.99018390E 00	0.99506105E 00
0.1000E 05	0.31881993E-00	0.49269421E-00	0.75744672E 00	0.91632757E 00	0.98245034E 00	0.99116272E 00	0.99555636E 00
0.4000E 05	0.49270438E-00	0.67004913E 00	0.86599178E 00	0.95693050E 00	0.99118137E 00	0.99557512E 00	0.99778132E 00
0.9000E 05	0.59915576E 00	0.75746490E 00	0.90766172E 00	0.97101117E 00	0.99411118E 00	0.99704868E 00	0.99852157E 00
0.1600E 06	0.66991755E 00	0.80879186E 00	0.92960509E 00	0.97815479E 00	0.99557971E 00	0.99778599E 00	0.99889145E 00
0.2500E 06	0.71987464E 00	0.84237055E 00	0.94313312E 00	0.98247404E 00	0.99646200E 00	0.99822854E 00	0.99911328E 00
0.3600E 06	0.75681023E 00	0.86599108E 00	0.95230383E 00	0.98536734E 00	0.99705071E 00	0.99852366E 00	0.99926115E 00
0.4900E 06	0.78513107E 00	0.88348903E 00	0.95892937E 00	0.98744076E 00	0.99747141E 00	0.99873446E 00	0.99936672E 00
0.6400E 06	0.80748928E 00	0.89696211E 00	0.96393938E 00	0.98899950E 00	0.99778709E 00	0.99889260E 00	0.99944591E 00
0.8100E 06	0.82556532E 00	0.90765126E 00	0.96786048E 00	0.99021398E 00	0.99803269E 00	0.99901561E 00	0.99950750E 00
0.1000E 07	0.84046908E 00	0.91633621E 00	0.97101268E 00	0.99118708E 00	0.99822921E 00	0.99911402E 00	0.99955677E 00
0.4000E 07	0.91258331E 00	0.95690843E 00	0.98536698E 00	0.99558088E 00	0.99911410E 00	0.99955694E 00	0.99977841E 00
0.9000E 07	0.93850961E 00	0.97098020E 00	0.99021325E 00	0.99705107E 00	0.99940928E 00	0.99970461E 00	0.99985228E 00
0.1600E 08	0.95183377E 00	0.97811969E 00	0.99264793E 00	0.99778711E 00	0.99955691E 00	0.99977845E 00	0.99988919E 00

*INDICATES POWER OF 10 THAT NUMBER IS TO BE MULTIPLIED BY; e.q. 1.000 - 02 = 1.000 x 10^{-2}

TABLE II

HEAT FLUX AT INTERFACE OF A COMPOSITE SEMI-INFINITE SLAB FOR A CONSTANT SURFACE HEAT FLUX AND PERFECT THERMAL CONTACT AT THE INTERFACE

θ_r	q_I/q_o						
	σ = 0.005	σ = 0.01	σ = 0.03	σ = 0.10	σ = 0.50	σ = 1.0	σ = 2.0
1.0000E-02*	0.	0.	0.	0.	0.	0.	0.
0.4000E-01	0.40513328E-05	0.80625534E-05	0.23717997E-04	0.74028900E-04	0.27143930E-03	0.40715895E-03	0.54287860E-03
0.6250E-01	0.46542112E-04	0.92623412E-04	0.27247470E-03	0.85045133E-03	0.31183215E-02	0.46774823E-02	0.62366430E-02
0.9000E-01	0.18330478E-03	0.36479467E-03	0.10731338E-02	0.33494783E-02	0.12281420E-01	0.18422130E-01	0.24562841E-01
0.1225E-00	0.43136328E-03	0.85845564E-03	0.25253598E-02	0.78821836E-02	0.28901339E-01	0.43352010E-01	0.57802679E-01
0.1600E-00	0.76716512E-03	0.15267345E-02	0.44912674E-02	0.14018195E-01	0.51400014E-01	0.77099984E-01	0.10279992E-00
0.2500E-00	0.15653829E-02	0.31152626E-02	0.91642664E-02	0.28603119E-01	0.10487090E-00	0.15729894E-00	0.20972202E-00
0.3600E-00	0.23780680E-02	0.47325113E-02	0.13920942E-01	0.43441065E-01	0.15915229E-00	0.23859273E-00	0.31794268E-00
0.4900E-00	0.31327377E-02	0.62339818E-02	0.18333330E-01	0.57167339E-01	0.20882418E-00	0.31242234E-00	0.41547798E-00
0.6400E 00	0.38278511E-02	0.76162365E-02	0.22387062E-01	0.69694428E-01	0.25295356E-00	0.37675934E-00	0.49878738E-00
0.8100E 00	0.44814087E-02	0.89148203E-02	0.26183520E-01	0.81307043E-01	0.29213926E-00	0.43205865E-00	0.56790330E 00
0.1000E 01	0.51090340E-02	0.10160714E-01	0.29812580E-01	0.92273746E-01	0.32722928E-00	0.47950031E-00	0.62432954E 00
0.4000E 01	0.10941857E-01	0.21680637E-01	0.62704735E-01	0.18538303E-00	0.55268904E 00	0.72367334E 00	0.84731004E 00
0.9000E 01	0.16541488E-01	0.32640731E-01	0.92914852E-01	0.26170021E-00	0.66942217E 00	0.81366357E 00	0.90274310E 00
0.1600E 02	0.22051658E-01	0.43331577E-01	0.12140211E-00	0.32651754E-00	0.73986381E 00	0.85968389E 00	0.92814940E 00
0.2500E 02	0.27497963E-01	0.53807406E-01	0.14840969E-00	0.38224831E-00	0.78643237E 00	0.88753734E 00	0.94290978E 00
0.3600E 02	0.32889039E-01	0.64088441E-01	0.17407222E-00	0.43056969E-00	0.81926655E 00	0.90618570E 00	0.95259812E 00
0.4900E 02	0.38228846E-01	0.74185457E-01	0.19848999E-00	0.47276036E-00	0.84355572E 00	0.91953868E 00	0.95945892E 00
0.6400E 02	0.43519600E-01	0.84105650E-01	0.22174720E-00	0.50982787E 00	0.86220059E 00	0.92956819E 00	0.96457642E 00
0.8100E 02	0.48762762E-01	0.93854634E-01	0.24391811E-00	0.54258014E 00	0.87693827E 00	0.93737695E 00	0.96854293E 00
0.1000E 03	0.53959399E-01	0.10343715E-00	0.26507000E-00	0.57167236E 00	0.88886651E 00	0.94362838E 00	0.97170826E 00
0.4000E 03	0.10351653E-00	0.19092526E-00	0.43204942E-00	0.74442007E 00	0.94380172E 00	0.97179658E 00	0.98588516E 00
0.9000E 03	0.14904551E-00	0.26536383E-00	0.54338837E 00	0.82093054E 00	0.96245327E 00	0.98119555E 00	0.99059378E 00
0.1600E 04	0.19096657E-00	0.32918515E-00	0.62141411E 00	0.86296865E 00	0.97181838E 00	0.98589610E 00	0.99294639E 00
0.2500E 04	0.22964222E-00	0.38428878E-00	0.67838319E 00	0.88927811E 00	0.97744676E 00	0.98871671E 00	0.99435758E 00
0.3600E 04	0.26539125E-00	0.43217868E-00	0.72141851E 00	0.90721301E 00	0.98120195E 00	0.99059713E 00	0.99529806E 00
0.4900E 04	0.29849563E-00	0.47405615E-00	0.75486567E 00	0.92019327E 00	0.98388553E 00	0.99194032E 00	0.99596979E 00
0.6400E 04	0.32920499E-00	0.51088820E 00	0.78149012E 00	0.93001018E 00	0.98589876E 00	0.99294782E 00	0.99647376E 00
0.8100E 04	0.35774187E-00	0.54345893E 00	0.80311739E 00	0.93768890E 00	0.98746487E 00	0.99373132E 00	0.99686553E 00
0.1000E 05	0.38430384E-00	0.57240835E 00	0.82099228E 00	0.94385645E 00	0.98871797E 00	0.99435820E 00	0.99717902E 00
0.4000E 05	0.57241278E 00	0.74460053E 00	0.90722194E 00	0.97182509E 00	0.99435829E 00	0.99717914E 00	0.99858965E 00
0.9000E 05	0.67835916E 00	0.82099561E 00	0.93769079E 00	0.98120373E 00	0.99623872E 00	0.99811938E 00	0.99905970E 00
0.1600E 06	0.74431733E 00	0.86299566E 00	0.95314428E 00	0.98589932E 00	0.99717906E 00	0.99858959E 00	0.99929480E 00
0.2500E 06	0.78851656E 00	0.88928758E 00	0.96246898E 00	0.98871807E 00	0.99774321E 00	0.99887161E 00	0.99943575E 00
0.3600E 06	0.81986903E 00	0.90721232E 00	0.96870280E 00	0.99059776E 00	0.99811938E 00	0.99905979E 00	0.99952996E 00
0.4900E 06	0.84312686E 00	0.92018571E 00	0.97316283E 00	0.99194059E 00	0.99838807E 00	0.99919417E 00	0.99959716E 00
0.6400E 06	0.86100402E 00	0.92999768E 00	0.97651093E 00	0.99294773E 00	0.99858943E 00	0.99929473E 00	0.99964729E 00
0.8100E 06	0.87514430E 00	0.93767256E 00	0.97911694E 00	0.99373122E 00	0.99874615E 00	0.99937310E 00	0.99968653E 00
0.1000E 07	0.88659243E 00	0.94383717E 00	0.98120264E 00	0.99435802E 00	0.99887150E 00	0.99943577E 00	0.99971786E 00
0.4000E 07	0.93932807E 00	0.97179209E 00	0.99059678E 00	0.99717870E 00	0.99943575E 00	0.99971797E 00	0.99985905E 00
0.9000E 07	0.95722966E 00	0.98116604E 00	0.99373013E 00	0.99811896E 00	0.99962391E 00	0.99981213E 00	0.99990629E 00
0.1600E 08	0.96621738E 00	0.98585930E 00	0.99529707E 00	0.99858912E 00	0.99971788E 00	0.99985909E 00	0.99992966E 00

*INDICATES POWER OF 10 THAT NUMBER IS TO BE MULTIPLIED BY- e.q. 1.000 - 02 = 1.000 x 10^{-2}

TABLE III

HANDBOOK VALUES OF THERMAL PROPERTIES OF SOME COMMON MATERIALS

MATERIAL	THERMAL CONDUCTIVITY k Btu/hr-ft-°F	DENSITY ρ lb/ft^3	SPECIFIC HEAT c Btu/lb-°F	ρc $\frac{Btu}{ft^3\text{-}°F}$	$k\rho c$ $\frac{Btu^2}{sec\text{-}ft^4\text{-}°F^2}$	THERMAL DIFFUSIVITY α ft^2/sec
SILVER	242.000	657	0.0559	36.75	2.470	18.340×10^{-4}
GOLD	172.000	1206	0.0312	37.63	1.80	12.710×10^{-4}
COPPER	223.000	559	0.0915	51.15	3.170	12.090×10^{-4}
ALUMINUM	118.000	169	0.2140	36.17	1.190	10.180×10^{-4}
NICKEL	52.000	556	0.1065	59.21	0.855	2.450×10^{-4}
PLATINUM	41.100	1338	0.0324	43.35	0.495	2.630×10^{-4}
STEEL (1% C)	25.000	487	0.1130	55.03	0.382	1.260×10^{-4}
SAPPHIRE	15.700	248	0.1800	44.64	0.195	0.977×10^{-4}
CARBON	8.700	98	0.1600	15.68	0.379	1.540×10^{-4}
QUARTZ (CLEAR FUSED)	0.836	137	0.1760	24.15	0.560×10^{-2}	0.961×10^{-5}
PYREX (7740)	0.654	139	0.1850	25.70	0.466×10^{-2}	0.695×10^{-5}
GLASS (SODA-LIME)	0.416	154	0.1810	27.85	0.322×10^{-2}	0.415×10^{-5}
MELAMINE GLASS CLOTH	0.290	118	0.3000	35.40	0.285×10^{-2}	0.228×10^{-5}

TABLE IV

TEMPERATURE DISTRIBUTION IN AN INSULATED SLAB (THICK FILM) FOR A CONSTANT SURFACE HEAT FLUX

θ_F	T_F/T_∞				
	x/δ = -1	x/δ = -0.75	x/δ = -0.50	x/δ = -0.25	x/δ = 0
1.000000E-02*	0.99999999E 00	0.38791174E-01	0.12627769E-03	0.25207201E-07	0.
0.400000E-01	0.99999999E 00	0.25926600E-00	0.38791199E-01	0.31127211E-02	0.25255539E-03
0.625000E-01	0.99999999E 00	0.35385486E-00	0.89079347E-01	0.15409298E-01	0.34687899E-02
0.900000E-01	0.10000011E 01	0.43024351E-00	0.14706589E-00	0.39948552E-01	0.15511785E-01
0.122500E-00	0.10000275E 01	0.49201565E-00	0.20569487E-00	0.75591635E-01	0.40303959E-01
0.160000E-00	0.10002525E 01	0.54291015E 00	0.26237871E-00	0.11991301E-00	0.77582398E-01
0.250000E-00	0.10034687E 01	0.62466942E 00	0.36926416E-00	0.22463039E-00	0.17815869E-00
0.360000E-00	0.10155129E 01	0.69492021E 00	0.47019207E-00	0.33777937E-00	0.29413178E-00
0.490000E-00	0.10403315E 01	0.76398399E 00	0.56760729E 00	0.45035689E-00	0.41138974E-00
0.640000E 00	0.10778349E 01	0.83562654E 00	0.66282316E 00	0.55925588E 00	0.52475743E 00
0.810000E 00	0.11257701E 01	0.91038693E 00	0.75657478E 00	0.66430502E 00	0.63355239E 00
0.100000E 01	0.11816274E 01	0.98776795E 00	0.84929980E 00	0.76622094E 00	0.73852833E 00
0.400000E 01	0.19201557E 01	0.18232255E 01	0.17539889E 01	0.17124468E 01	0.16985995E 01
0.900000E 01	0.27571469E 01	0.26925269E 01	0.26463692E 01	0.26186746E 01	0.26094428E 01
0.160000E 02	0.36187550E 01	0.35702905E 01	0.35356721E 01	0.35149013E 01	0.35079776E 01
0.250000E 02	0.44902106E 01	0.44514391E 01	0.44237444E 01	0.44071277E 01	0.44015887E 01
0.360000E 02	0.53665894E 01	0.53342798E 01	0.53112012E 01	0.52973540E 01	0.52927381E 01
0.490000E 02	0.62457818E 01	0.62180882E 01	0.61983060E 01	0.61864372E 01	0.61824808E 01
0.640000E 02	0.71267321E 01	0.71025002E 01	0.70851915E 01	0.70748061E 01	0.70713440E 01
0.810000E 02	0.80088550E 01	0.79873153E 01	0.79719295E 01	0.79626983E 01	0.79596204E 01
0.100000E 03	0.88917983E 01	0.88724127E 01	0.88585660E 01	0.88502576E 01	0.88474876E 01
0.400000E 03	0.17739283E 02	0.17729590E 02	0.17722667E 02	0.17718512E 02	0.17717127E 02
0.900000E 03	0.26596615E 02	0.26590154E 02	0.26585539E 02	0.26582769E 02	0.26581846E 02
0.160000E 04	0.35456395E 02	0.35451551E 02	0.35448088E 02	0.35446011E 02	0.35445318E 02
0.250000E 04	0.44317170E 02	0.44313297E 02	0.44310526E 02	0.44308864E 02	0.44308309E 02
0.360000E 04	0.53178440E 02	0.53175207E 02	0.53172901E 02	0.53171518E 02	0.53171057E 02
0.490000E 04	0.62039954E 02	0.62037182E 02	0.62035204E 02	0.62034021E 02	0.62033626E 02
0.640000E 04	0.70901658E 02	0.70899241E 02	0.70897510E 02	0.70896471E 02	0.70896122E 02
0.810000E 04	0.79763492E 02	0.79761343E 02	0.79759800E 02	0.79758878E 02	0.79758570E 02
0.100000E 05	0.88625409E 02	0.88623476E 02	0.88622095E 02	0.88621261E 02	0.88620985E 02

*INDICATES POWER OF 10 THAT NUMBER IS TO BE MULTIPLIED BY; e.g. 1.000 - 02 = 1.000×10^{-2}

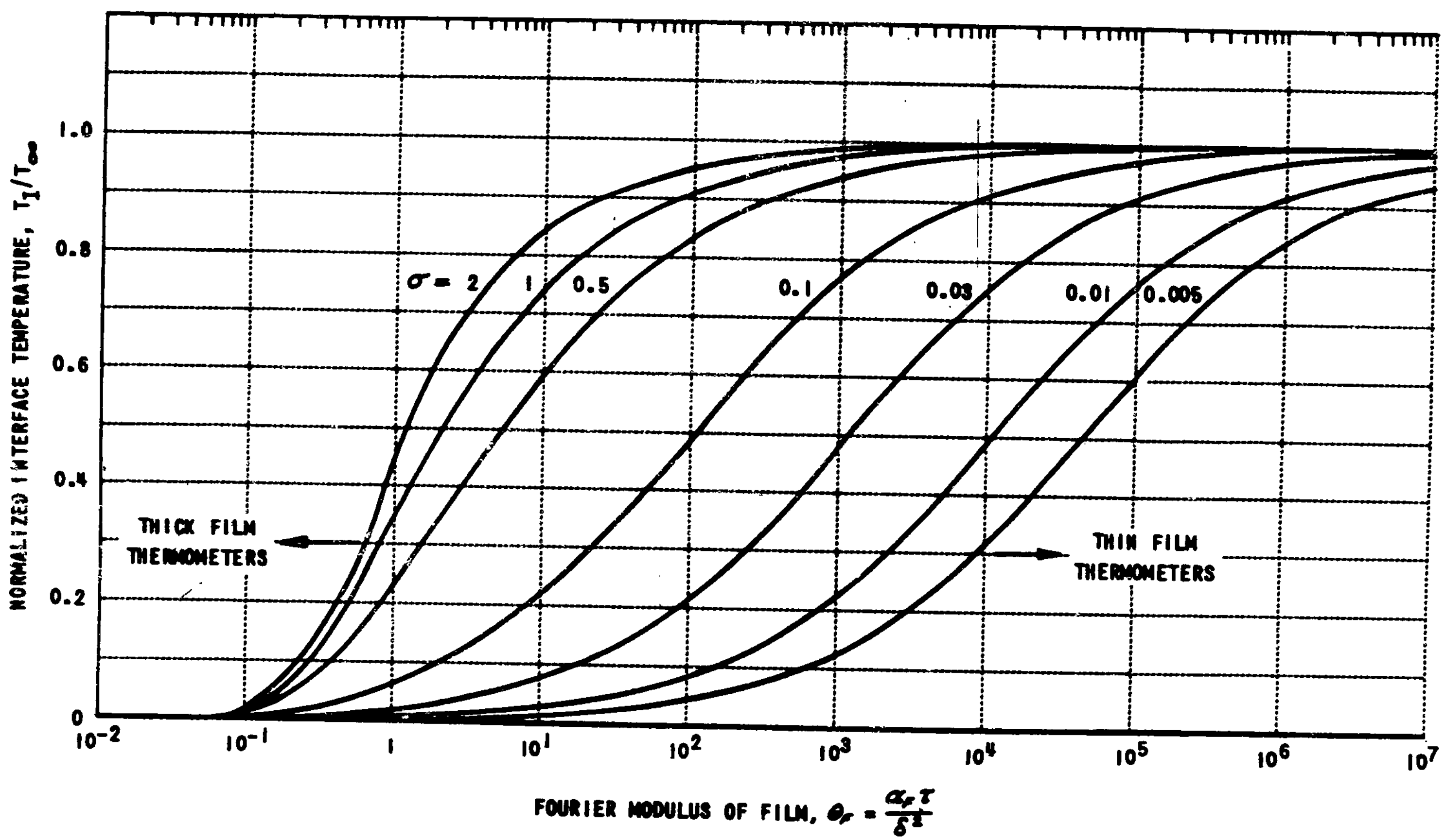


Figure 1 TEMPERATURE AT INTERFACE OF A COMPOSITE SEMI-INFINITE SLAB FOR A CONSTANT SURFACE HEAT FLUX AND PERFECT THERMAL CONTACT AT THE INTERFACE

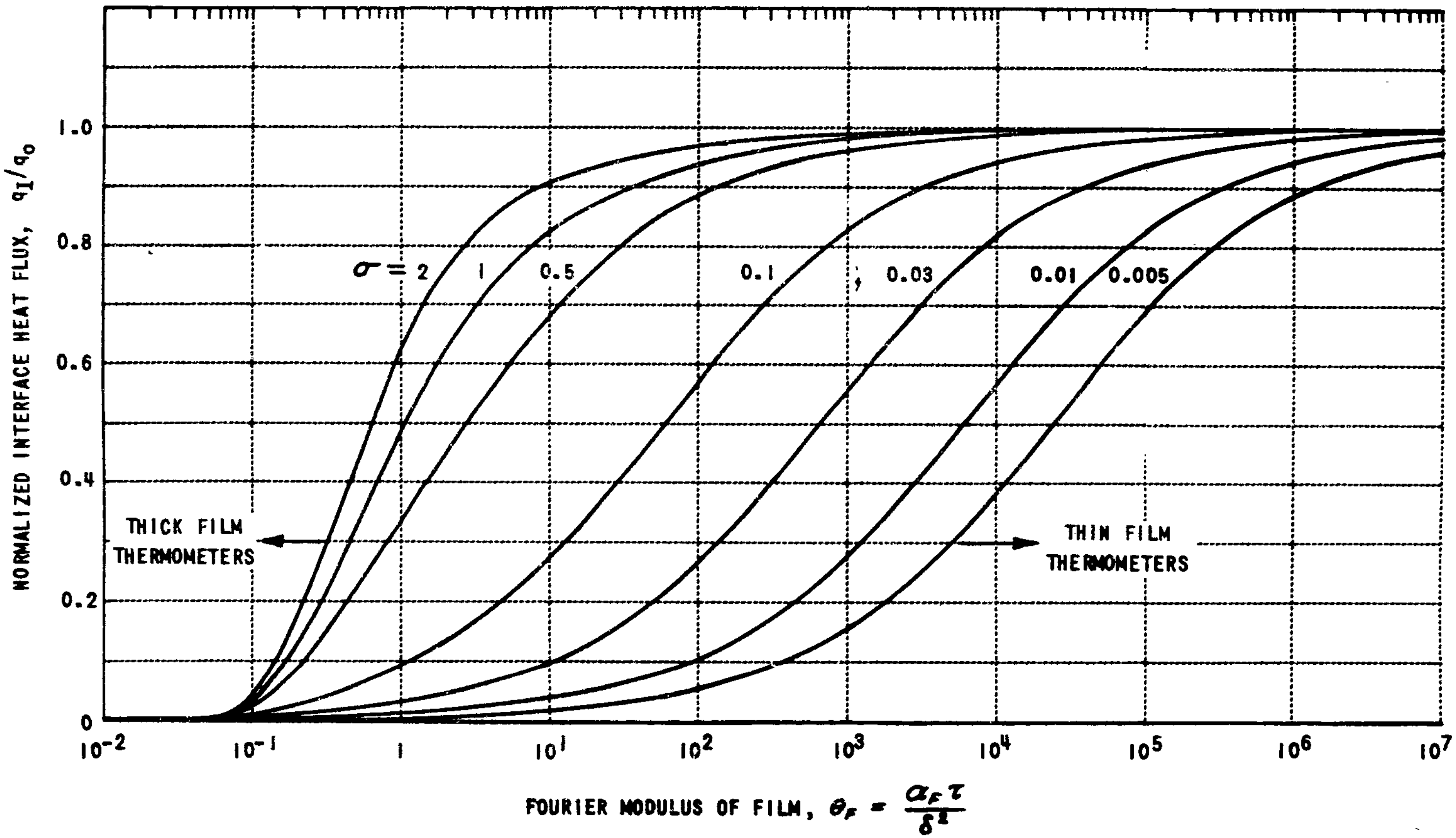


Figure 2 HEAT FLUX AT INTERFACE OF A COMPOSITE SEMI-INFINITE SLAB FOR A CONSTANT SURFACE HEAT FLUX AND PERFECT THERMAL CONTACT AT THE INTERFACE

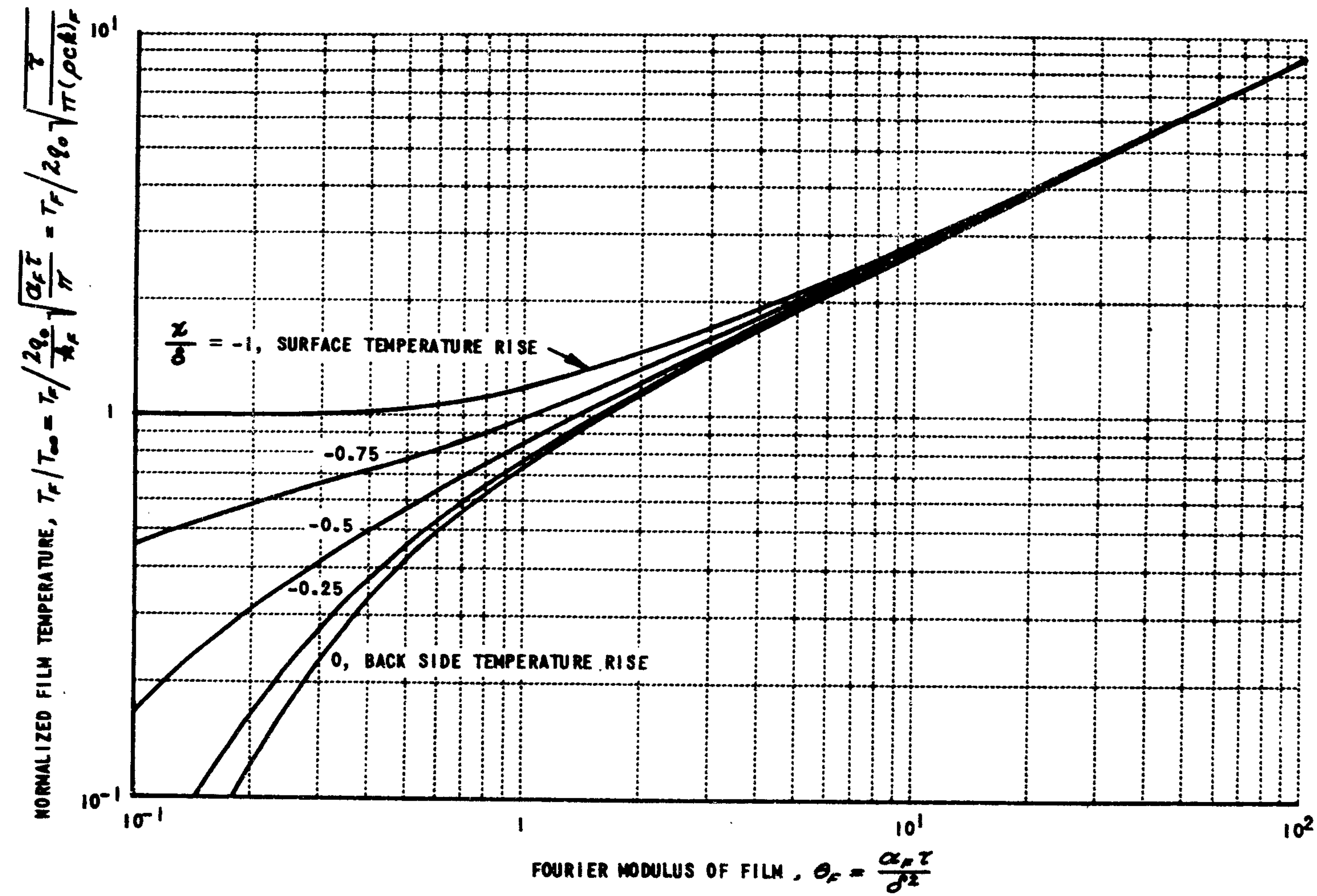


Figure 3 TEMPERATURE DISTRIBUTION IN AN INSULATED SLAB (THICK FILM) FOR A CONSTANT SURFACE HEAT FLUX

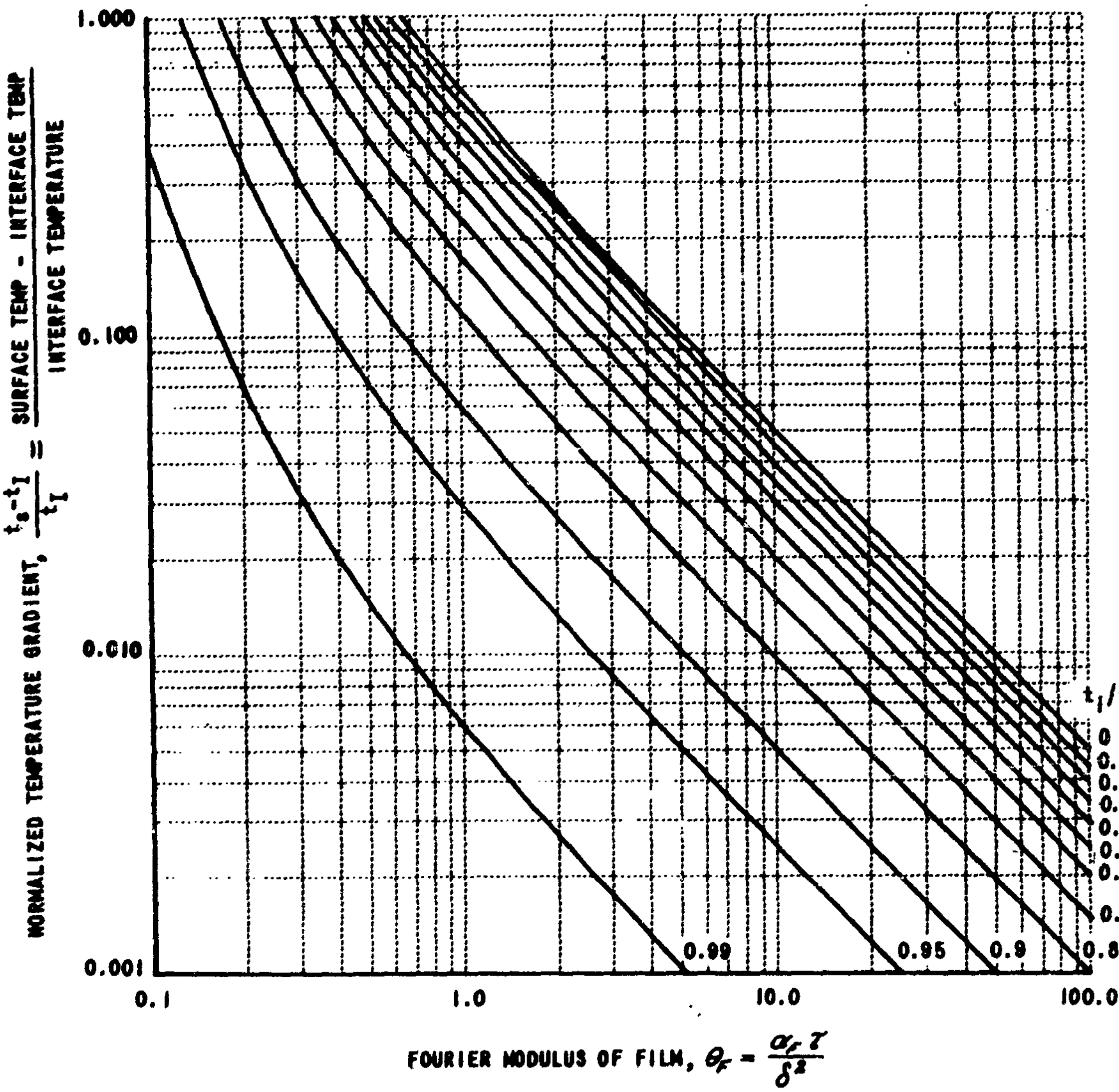


Figure 4 TEMPERATURE GRADIENT IN AN INSULATED SLAB FOR A CONSTANT SURFACE HEAT FLUX

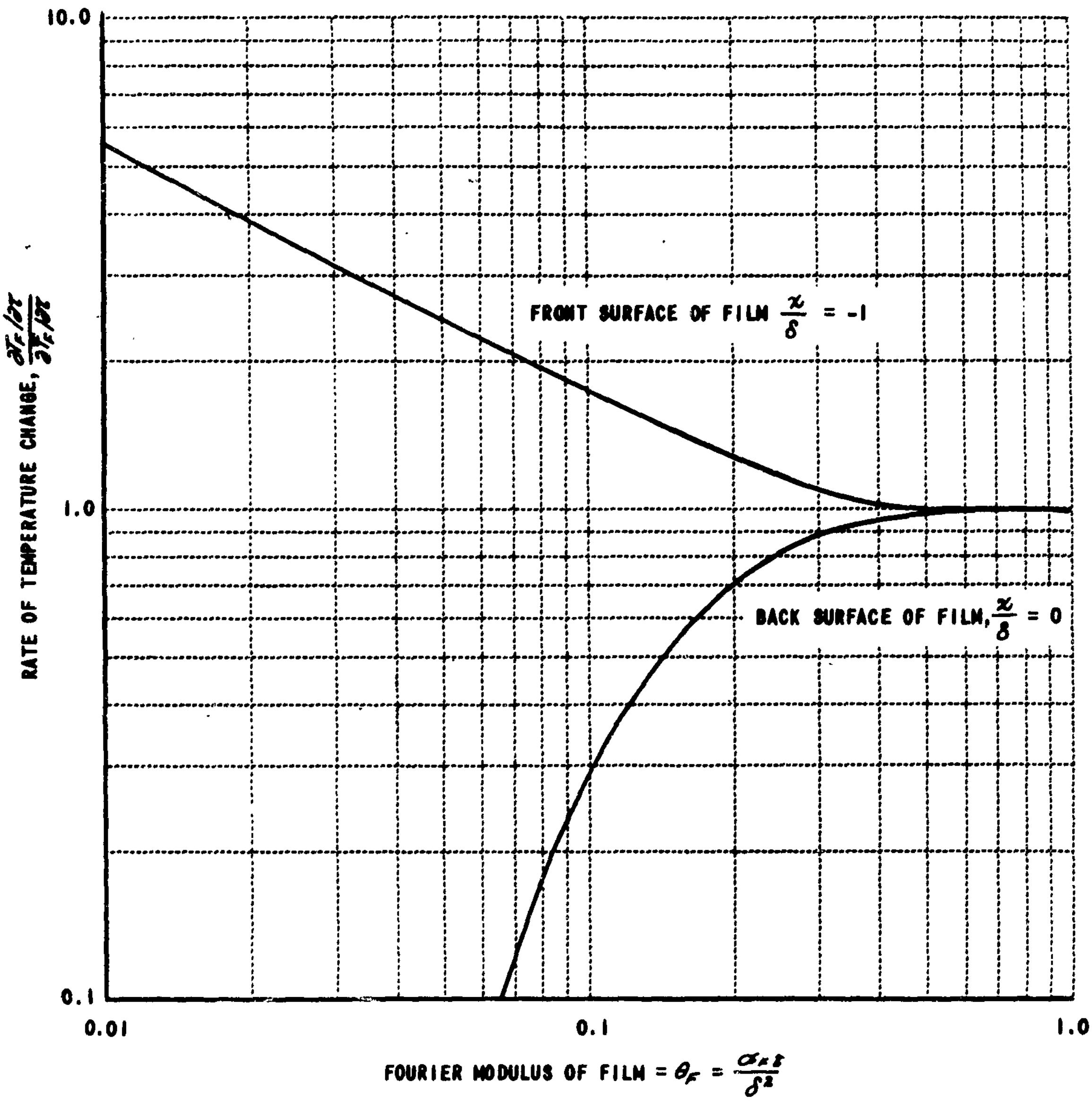
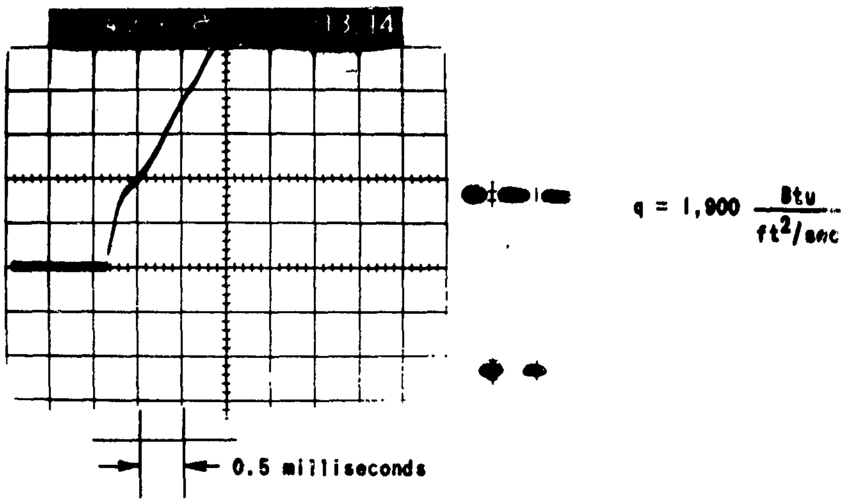


Figure 5 RATE OF CHANGE OF TEMPERATURE OF AN INSULATED SLAB FOR A CONSTANT SURFACE HEAT FLUX



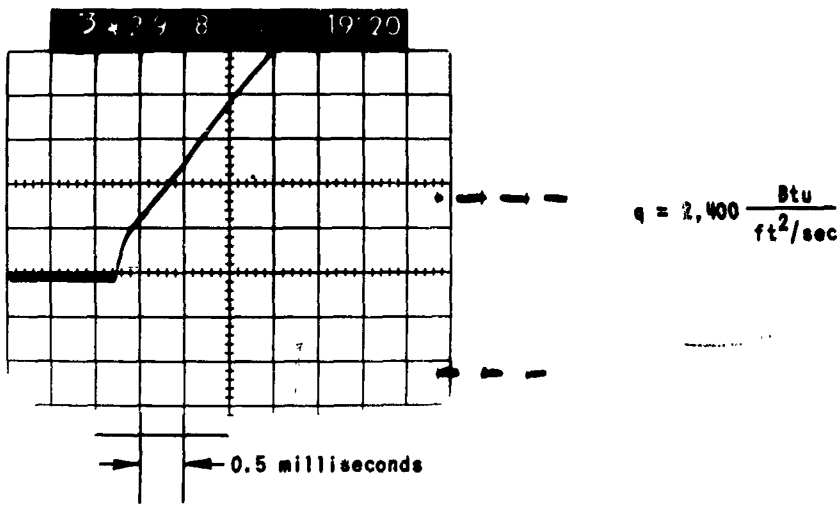


Figure 6 OSCILLOGRAM TRACE OF A PLATINUM THICK FILM RESISTANCE THERMOMETER (REFERENCE 16)