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THE OPTIMIZATION OF SATELLITE RECONNAISSANCE BY THE APPLICATION OF DYNAMIC PROGRAMMING TECHNIQUES

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-168

April, 1963

DIRECTORATE OF ANALYSIS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts

(Prepared under Contract Number AF33(600)-39852,
Project 600 by J. A. Hunt of The MITRE Corporation,
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ABSTRACT

This paper points out the application of dynamic programming techniques to the optimization of a satellite reconnaissance system with a constrained "picture-taking" capability. The technique is illustrated by an example and a brief description of a possible implementation procedure.

I. INTRODUCTION

A satellite reconnaissance system will probably have a restricted intelligence gathering capability. This restriction may be due to limitations on the data which can be transmitted to a readout station during a pass or the film available in the satellite. In any case, there is a bound on the "picture-taking" capability of the satellite.

Generally this "picture-taking" capability will be a parameter in the design of the vehicle and the over-all command and control network. We may select this parameter by deciding on the maximum (or the expected) number of targets the satellite will pass over between readouts. Generally, an increase in "picture-taking" capability will decrease the resolution of a system with fixed readout capability or increase the cost of the system by demanding a greater readout capability (i.e., larger bandwidth or more readout stations).

As some targets will be cloud covered, a design based on reserving "pictures" for all targets would be wasting some "pictures" at the sacrifice of either resolution or cost. Hence, it may be reasonable to consider designing the system so that the number of "pictures" is less than the total number of targets.

However, this design creates an additional problem; "What targets does the satellite photograph?" When the satellite passes over a target it observes the conditions over this target and has a probabilistic notion about cloud conditions over the future targets. Hence, the satellite must make decisions sequentially. This paper points out the applicability of dynamic programming to the optimization of this problem. In other words, a dynamic programming solution will provide a criterion by which the satellite can decide optimally when to photograph a particular target.

A particular case which can be solved intuitively is when photographic conditions are binary (cloud, no cloud) and the targets are equal valued. Then the satellite should always take pictures under "no-cloud" conditions, for this is the optimum policy.

However, atmospheric phenomena may be such that picture quality can have several possible values or a continum of values. Also the target pictures may not be of equal importance. Clearly, the satellite would need a more sophisticated method of deciding whether to take a picture of the rather hazy target which is approaching.

This paper presents a dynamic programming formulation for this latter problem, and applies the model to a small example.

II. FORMULATION OF THE PROBLEM

Let the state of the reconnaissance vehicle be given by the two dimensional representation $S(i,j)$ where

i = number of targets remaining until a readout station becomes available

j = number of remaining pictures which the reconnaissance vehicle can take

The initial state of the vehicle is given by $S(n, m)$. Hence,

n = total number of targets

m = total number of pictures which the reconnaissance vehicle can take

In accordance with the discussion of Section I, we let $n > m$.

It may be more important to get pictures of some targets then others. Hence, we consider the value of target pictures to be given by

$$t(1),\ t(2)\ \ldots\ t(n)$$

where

$$0 \leq t(i) \leq 1 \qquad \text{for } 1 \leq i \leq n.$$

These values are pre-assigned and do not change while the reconaissance vehicle passes between readout stations. We designate the targets and pictures as follows:

If "i" targets remain, then the i'th target is the target approaching and the first target is the target immediately before the readout station.

Picture quality is affected by a number of generally uncontrollable factors -- atmospheric conditions, sunlight, etc. We can model this affect by calling the picture quality of the i'th target a random variable, q_i, $(0 \le q_i \le 1)$ and define $p(q_i = x) = p_i(x)$ as the probability density function for picture quality of the i'th target.

In our example we will consider these random variables to be independent and identically distributed. We can define the picture value v_i of the i'th target:

-If a picture is not taken of the i'th target, $v_i = 0$

-If a picture is taken of this target, $v_i = t_i q_i$.

The decision facing the reconnaissance vehicle at i'th target can be expressed as follows:

> Given the present state is S(i,j) and the picture value of i'th target from direct measurement is v_i, should the i'th target be photographed?

Let us define a decision criterion d(i,j) for state S(i,j) such that

-If $v_i \ge d(i,j)$ then a picture will be taken of the i'th target

-If $v_i < d(i,j)$ then a picture will not be taken.

A listing of d(i,j) for all i and j allows decisions to be made for any state S(i,j) on the basis of the picture value v(i) of the target. Such a listing is called a policy.

For example, the following listing of d(i,j) represents a policy

$$d(i,j) = 0 \qquad \text{for all } i \text{ and } j.$$

This means that the first m targets would be photographed because their picture value $v_i \geq 0$ by definition. However, as some of these targets may be clouded over ($v_i = 0$) it is unlikely that this policy would be optimum.

The dynamic programming problem is to determine the optimum policy, that is, determine the values of $d(i,j)$ for all i and j that will maximize the total picture value for the n targets (or m pictures).

The procedures of dynamic programming are inspired by the Principle of Optimality. "An optimum policy has the property that, whatever the initial state and the initial decision, the remaining decisions must constitute an optimum policy with regard to the state resulting from the first decision."

To apply this principle, we must define $V(i,j)$ as the expected total value of the remaining j pictures when the optimum policy is followed from the present state, $S(i,j)$.

We can then write the basic dynamic programming equation.

$$V(i,j) = \max_{d(i,j)} \left\{ pr(i,j) \left[E(v_i) + V(i-1, j-1)\right] + \left[1-pr(i,j)\right] \left[V(i-1,j)\right] \right\} \quad (1)$$

where $pr(i,j)$ is the probability of photographing a target at state $S(i,j)$ or $pr(i,j) = \text{probability} \left[v_i \geq d(i,j)\right]$

where $E(v_i)$ is the expected picture value of the i'th target under the condition that a picture is taken of this target.

Briefly, this relation, Eq. 1, means that $V(i,j)$ is maximized for any state $S(i,j)$ with respect to the decision to be made immediately, e.g., $d(i,j)$. Hence, a reverse solution is obtained from $V(0,0)$ through successive values of i and j to $V(n,m)$.

Now applying Eq. 1 to this particular problem

$$V(i,j) = \max_{d(i,j)} \left\{ \int_{d(i,j)}^{1} t_i x \, p_i(x) \, dx + V(i-1, j-1) \int_{d(i,j)}^{1} p_i(x) \, dx \right.$$

$$\left. + V(i-1,j) \int_{0}^{d(i,j)} p_i(x) \, dx \right\} \qquad (2)$$

Taking partial derivative

$$\frac{\partial V(i,j)}{\partial d(i,j)} = -t_i d(i,j) \, p_i[d(i,j)] - V(i-1,j-1) \, p_i[d(i,j)]$$

$$+ V(i-1,j) \, p_i[d(i,j)]$$

and setting it equal to zero

$$d(i,j) = \frac{V(i-1,j) - V(i-1,j-1)}{t_i} \qquad (3)$$

This is a rather intuitive result; it says we take a picture of the i'th target if the value of this target is greater than the difference between expected value of remaining i-1 targets when picture is taken and when picture is not taken.

Now substitute Eq. 3 into Eq. 2,

$$V(i,j) = \int_{d(i,j)}^{1} t_i x \, p_i(x) \, dx + V(i-1,j-1) \int_{d(i,j)}^{1} p_i(x) \, dx$$

$$+ V(i-1,j) \int_{0}^{d(i,j)} p_i(x) \, dx \qquad (4)$$

By noting the initial conditions

(a) $V(i,0) = 0$ for all i, and

(b) $V(1,1) = v_1 = t_1 \int_0^1 x p_1(x)\, dx$

we can solve Eq. 4 repeatedly for increasing i and j. Also using Eq. 3, we can determine the optimum policy, d(i,j).

This completes the dynamic programming solution. An example in the following section will illustrate the procedure.

III. EXAMPLE

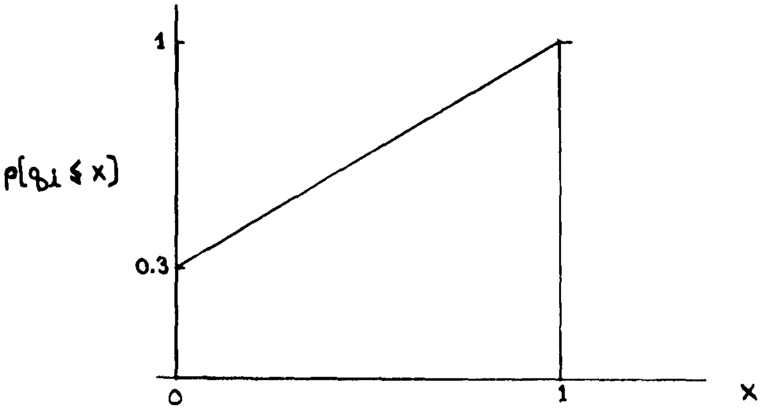
Assume all targets are equal valued

$$t_i = 1 \qquad 1 \le i \le n$$

Assume the random variables for picture quality, q_i, are mutually independent and identically distributed random variables from the cumulative distribution below.

$$\text{probability } (q_i \le x) = 0.3 + 0.7\,x$$

This distribution is both discrete and continuous which demands the use of distribution functions rather than density functions.



This distribution corresponds to a situation with a discrete probability $p(q_i = 0) = 0.3$ for cloud cover ($q_i = 0$) and a continuous uniform probability for other improved values of picture quality.

Let us assume there are five targets and three pictures, $m = 5$, $n = 3$.

The V(i,j) have been computed from Eq. 4 and are given in the following matrix.

j i	V(i,j)			
	0	1	2	3
0	0	--	--	--
1	0	0.35	--	--
2	0	0.50	0.70	--
3	-	0.59	0.93	1.05
4	-	--	1.07	1.32
5	-	--	--	1.52

The omitted elements correspond to states which the reconnaissance vehicle can not attain.

From the V(i,j) matrix the d(i,j) matrix can be computed by using Eq. 3.

j i	d(i,j)			
	0	1	2	3
0	-	-	-	-
1	-	0	-	-
2	-	.35	0	-
3	-	.50	.20	0
4	-	-	.34	.12
5	-	-	-	.25

To illustrate these computations, consider a specific state is three targets and two pictures remaining, S(3,2); a picture should be taken of the third last target if its picture quality is greater than 0.20, ($q_3 \geq 0.2$) (e.g., d(3,2) = 0.20). The remaining expected picture quality (before condition of third last target is observed) is given by V(3,2) = 0.93.

Now consider an alternate policy of photographing the first three targets without observing the photographic conditions which is the ex-

$$V(5,3) = 3(0.35) = 1.05$$

pected value of the reconnaissance mission in terms of picture quality. This compares unfavorably with the V(5,3) = 1.52 for the optimum policy from the dynamic programming solution.

Let us consider another policy of photographing the first three targets without cloud cover ($v_i \neq 0$).

prob (number of clear targets = 0) = 0.0024
prob (number of clear targets = 1) = 0.028
prob (number of clear targets = 2) = 0.13
prob (number of clear targets = 3) = 0.31
prob (number of clear targets = 4) = 0.36
prob (number of clear targets = 5) = 0.17
1.00

$\therefore$ Expected value of reconnaissance mission

$$= (0.0024)(0) + (0.03)(0.5) + (0.13)(1.00) + (0.84)(1.50)$$

$$= 1.41$$

For obvious reasons, this policy is better than the previous one, however, dynamic programming yields an even greater expected value.

The difference which depends on the shape of the picture value probability distribution is not very great in this case. For other distributions it would be greater or less. Some general study of the effect of this distribution could be performed.

IV. CONCLUSION

The purpose of this paper was to apply dynamic programming to a satellite reconnaissance problem involving sequential decisions concerning when observations should be made. The paper formulates the dynamic programming model, and illustrates its use for a particular example. The example indicates that the dynamic programming technique provides substantial improvement in reconnaissance performance over less sophisticated decision methods.

In this example, the distribution function for picture quality was selected quite arbitrarily. In a design study, this distribution would be obtained from actual observations. The example does indicate that it may be profitable to place infrared or albido measurement devices in the satellite so that picture quality can be determined before film is exposed.

In order to implement the dynamic programming technique, the reconnaissance system would need a ground capability to calculate the decision matrix, d(i,j), from the statistics of meterorological conditions. This matrix would be fed into the satellite before passage over targets. The satellite would compare measurements with the matrix to arrive at reconnaissance decisions.

A final caveat is worth noting, the motivation for this study and preceeding discussion has been the assumption that the picture taking capability will be a design or operational constraint. If this constraint is not important in practice, then much of this discussion is inappropriate.

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