63-3-3

BRL

MEMORANDUM REPORT NO. 1450
JANUARY 1963

METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL

Arthur D. Groves

Department of the Army Project No. 503-05-005

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

MEMORANDUM REPORT NO. 1450

JANUARY 1963

METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL

Arthur D. Groves

Weapons Systems Laboratory

Department of the Army Project No. 503-05-005

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

MEMORANDUM REPORT NO. 1450

ADGroves/djt
Aberdeen Proving Ground, Maryland
January 1963

METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL

ABSTRACT

An extensive and detailed analysis is made of the factors which contribute to the accuracy of a spotting rifle controlled antitank weapon. The manner in which each source of error contributes to the total error is discussed, and two methods of computing quasi-combat first-round hit probability are presented. The entire method is illustrated by a complete determination of the hitting potential of the 106mm recoilless rifle (M40) using the .50 caliber spotter (M8) under an assumed quasi-combat environment.

TABLE OF CONTENTS

INTRODUCTION

Important among the current and proposed anti-tank weapons are those which acquire target range data through the use of a separate, small-caliber spotting rifle. This type of weapon system is used by firing spotting rounds, with adjustment between rounds, until one hits the target, and then, without further adjustment, firing the first major caliber round. It is clearly desirable for the spotting round and the major caliber round to be ballistically matched, that is, to fly exactly the same trajectories under both standard and non-standard conditions. However, if past experience is any indication, it is impossible to achieve this goal. The two rounds generally are not matched. They do not fly the same trajectories under standard conditions, and they are not equally sensitive to variations from standard. For example, the two weapons might be adjusted so that their centers of impact are at the same point when no wind is blowing, but, given a constant cross wind, one type of round might be blown further off course than the other, so that they no longer have the same center of impact.

In this report a mathematical model is presented which allows for the computation of quasi-combat hit probability for the first major-caliber round fired after achieving a hit with a spotting round. The resulting probabilities are quasi-combat, that is, representative of the true combat performance of the weapon to the best degree possible, since all of the variable quantities considered as affecting the hit probability are assumed to vary in magnitude as they would from round to round or from occasion to occasion in a true combat situation.

The interactions of the sources of error are simplified to the degree that they are assumed to be independent of each other. For example, the horizontal error introduced as a result of occasion-to-occasion variation in weapon cant is assumed to be independent of the vertical error introduced due to the lot-to-lot variation in muzzle velocity. In truth, these are not independent, since, as will be shown, the error due to cant depends on the angles of elevation of the two weapons when firing, and these in turn depend on the particular muzzle velocity of the lot of ammunition being used. Dependences such as this are second order in their effect on hit probability, and are best ignored in a method which lends itself to hand computation, as does the one presented here.

Certain ballistic data regarding both the spotting round and major caliber round are required, as well as estimates of the standard deviations of the variation in the identified parameters. Specifically, for standard conditions, the angle of elevation and the time of flight as a function of target range are required, as well as the sensitivities of the two types of ammunition to changes in range wind, ballistic coefficient, muzzle velocity, and air density. These sensitivities, presented in the form of unit differential effects (e.g., change in impact point resulting from a 1-fps change in muzzle velocity), as well as the angle of elevation and time of flight data are best obtained by the computation of the pertinent trajectories under both standard and nonstandard conditions. These trajectory computations are normally performed in the Ballistic Research Laboratories by the Artillery and Missile Ballistics Branch of the Computing Laboratory.

Two methods of computing hit probability will be considered. The first assumes that the spotting-round impact is, from occasion to occasion, distributed normally over the target, while the second assumes that it is distributed uniformly over the target.

It is also assumed that the weapon system will be re-zeroed whenever a new lot of either major caliber or spotting ammunition is obtained.

The methods presented will be illustrated by a determination of the hit probabilities for the standard BAT system (Rifle, Recoilless, 106mm, M 40, firing shell, HEAT, M344A1, using Spotting Rifle, Caliber .50 inch, M8, firing Spotter Tracer Bullet M48A1).

NOMENCLATURE AND TERMINOLOGY

As will become apparent to the reader, the number of quantities involved in a detailed discussion of the sources of errors inherent in the overall accuracy of a system of the type under consideration is so large as to cause considerable difficulty in selecting symbols to represent them. The following general concepts have been used: Whenever possible, a symbol suggestive of the quantity represented has been used. For example, V indicates velocity, t indicates time, and N indicates number. Also wherever possible, standard notations are used. For example, Δ indicates a change in some quantity. Since two weapons are involved, (major caliber and spotter) and many quantities apply in type to both weapons but in different magnitude to each, capital letters (either Greek or English) are used when the quantity is to apply to the major caliber weapon or ammunition, and lower case letters (either Greek or English) are used when the quantity is to apply to the spotting weapon or ammunition. Errors in the vertical direction or quantities which result in vertical errors will carry the subscript y, while those associated with horizontal errors will carry the subscript x.

A complete listing of all symbols, their units, and their definitions is located at the end of the report.

TYPES OF ERRORS

In a weapon system of the type under consideration, the sources of delivery error fall generally into three categories. These are

(1) Fixed Bias Errors

(2) Variable Bias Errors

(3) Random Errors

The fixed bias errors are the trajectory mismatch errors computed under standard conditions. The magnitudes of the fixed bias errors depend on the range to the target when firing for effect and also the range at which the system has been zeroed, as well as the ballistic differences between the major caliber and spotting weapons. (Zeroing simply means the adjustment of the spotting rifle relative to the major caliber weapon so as to cause the centers of impact of groups of rounds fired from each weapon to coincide at some range, called the zeroing range.) In the vertical plane this bias results from differences in muzzle velocity, ballistic coefficient and drag between the spotting round and the major caliber round. In the horizontal plane, it results from the difference in drift between the two rounds. The fixed bias errors can be thought of as those not varying from firing occasion to firing occasion or from round to round on a given firing occasion.

The variable bias errors are those which vary from firing occasion to firing occasion but remain fixed from round to round on any given firing occasion. That is to say, they are the errors introduced by the particular nonstandard conditions prevalent on a given occasion, which generally vary from occasion to occasion. These include variations in atmospheric conditions as well as occasion-to-occasion variation in firing conditions such as the cant of the weapon and the error introduced during the process of zeroing at the last time the weapon was zeroed.

The random errors are those which vary from round to round on a given occasion. These are composed primarily of round-to-round differences in ammunition performance, but also include the effect of cross wind and range wind gustiness and round-to-round aiming error.

These sources of error will be discussed in a somewhat different order from that in which they were introduced. The first to be discussed will be the random errors, and then the variable bias errors introduced during the zeroing process will be discussed. Then the remainder of the variable bias errors will be defined, and finally the fixed bias errors will be introduced. Then all of the errors will be combined to determine the hit probability on a target.

Random Errors

Horizontal

Jump. There is a round-to-round variation in the angle at which the projectile (either major caliber or spotter) departs the launcher relative to the intended angle of the trajectory. This variation is called jump variation. It is composed partly of an angular difference between the projectile axis and the launcher axis at the instant of the projectile's departure from the launcher, and partly due to the motion of the launcher induced by the forces associated with the expanding propellant gases and moving projectile. Since jump is an angular error, it is expressed in mils. The standard deviation of horizontal jump for the spotting weapon will be denoted σ_{j_x}, while that for the major caliber weapon will be denoted σ_{J_x}.

Cant Variation. Cant error is the error in placing a weapon in firing position so that its elevating trunions are level. Such an error causes an elevated launcher to point at a horizontally measured angle different from that intended. For a shoulder-fired system this cant error may vary from round to round and so is properly discussed under random error. (For a tripod-mounted system, the cant error will not vary from round to round on a given occasion, but will vary from occasion to occasion, and hence will be discussed under variable biases.) The standard deviation in round-to-round variation in cant may be different when zeroing than when firing for effect. Let $\sigma_{c'}$ and $\sigma_{C'}$ be the standard deviations of round-to-round cant variation when zeroing and when firing for effect, respectively. Then, when zeroing at range R_z, $\sigma_{x_{c'A}}(R_z)$ and $\sigma_{x_{c'a}}(R_z)$, the standard deviations of horizontal random error due to round-to-round cant variation for the major caliber and spotting rounds respectively are given by

$$\sigma_{x_{c'A}}(R_z) = 1018.59 \tan \phi(R_z) \sin \sigma_{c'} \quad \text{mils.}$$

and
$$\sigma_{x_{c'a}}(R_z) = 1018.59 \tan \phi(R_z) \sin \sigma_{c'} \quad \text{mils.}$$

Then, when firing for effect at a target at range R, $\sigma_{x_{C'A}}(R,R_z)$ and $\sigma_{x_{C'a}}(R)$, the standard deviations of random error due to cant variation for the major caliber and spotter respectively are given by

$$\sigma_{x_{C'A}}(R,R_z) = 1018.59 \tan \left[\phi(R) + \phi(R_z) - \phi(R_z)\right] \sin \sigma_{C'}, \text{ mils}$$

and
$$\sigma_{x_{C'a}}(R) = 1018.59 \tan \phi(R) \sin \sigma_{C'}, \text{ mils,}$$

where $\phi(R) + \phi(R_z) - \phi(R_z)$ is the angle of elevation of the major caliber weapon when firing for effect at range R after zeroing at range R_z.

Crosswind Gustiness. Crosswind gustiness is the round-to-round variation in crosswind on a given occasion. Its effect on a projectile in flight is a random error. If σ_{WG_x} is the standard deviation of crosswind gustiness, then $\sigma_{x_{WG}}(r)$ and $\sigma_{x_{wg}}(r)$, the standard deviation of horizontal impact error at range r yards for the major caliber round and spotter respectively, caused by the variation in wind gustiness, are

$$\sigma_{x_{WG}}(r) = \frac{1018.59}{3r} \sigma_{WG_x} \left\{T(r) - \frac{3r}{V \cos \phi(r)}\right\} \quad \text{mils}$$

and
$$\sigma_{x_{wg}}(r) = \frac{1018.59}{3r} \sigma_{WG_x} \left\{t(r) - \frac{3r}{v \cos \phi(r)}\right\} \quad \text{mils,}$$

where $T(r)$ and $t(r)$ are the times of flight to range r for the major caliber round and the spotter, respectively; V and v are the muzzle velocities for the major caliber and spotter rounds respectively, and $\Phi(r)$ and $\phi(r)$ are the angles of launcher elevation above bore sight required to fire the major caliber and spotter respectively at range r. Note that the quantity in braces is simply the difference between the projectile's times of flight in air and in vacuum. The factor $\frac{1018.59}{3r}$ provides the conversion from deflection in feet at the target to a corresponding angular deflection subtended at the launcher.

Aiming Error. In general there will be a round-to-round variation in aiming due to the gunner's inability to hold the sight recticle on the desired point. If the system is shoulder fired, this error may be large; if the system is tripod mounted, it may be negligible. When zeroing, it may be smaller than when firing for effect. The standard deviation of this aiming error when zeroing will be denoted σ_{l_x} mils. When firing for effect the standard deviation of aim error will be denoted σ_{L_x} mils.

Total Random Error. The total horizontal random error is the combination of the errors in jump, cant, that due to crosswind gustiness, and aiming. If R_z is the zeroing range, then $\sigma_{x_{RA}}(R_z)$ and $\sigma_{x_{Ra}}(R_z)$, the total standard deviations of random error when zeroing for the major caliber and spotter respectively are given by

$$\sigma_{x_{RA}}(R_z) = \sqrt{\sigma_{J_x}^2 + \left[\sigma_{x_{c'A}}(R_z)\right]^2 + \left[\sigma_{x_{WG}}(R_z)\right]^2 + \sigma_{l_x}^2} \text{ mils,}$$

and

$$\sigma_{x_{Ra}}(R_z) = \sqrt{\sigma_{j_x}^2 + \left[\sigma_{x_{c'a}}(R_z)\right]^2 + \left[\sigma_{x_{wg}}(R_z)\right]^2 + \sigma_{l_x}^2} \text{ mils.}$$

When firing for effect at range R after zeroing at range R_z, $\sigma_{x_{RA}}(R,R_z)$

and $\sigma_{x_{Ra}}(R)$, the total standard deviations of horizontal random error for the major caliber and spotter respectively are given by

$$\sigma_{x_{RA}}(R,R_z) = \sqrt{\sigma_{J_x}^2 + \left[\sigma_{x_{C'A}}(R,R_z)\right]^2 + \left[\sigma_{x_{WG}}(R)\right]^2 + \sigma_{L_x}^2} \text{ mils,}$$

and

$$\sigma_{x_{Ra}}(R) = \sqrt{\sigma_{j_x}^2 + \left[\sigma_{x_{C'a}}(R)\right]^2 + \left[\sigma_{x_{wg}}(R)\right]^2 + \sigma_{L_x}^2} \text{ mils.}$$

Vertical

Jump. There is a round-to-round error in vertical jump similar to that in the horizontal direction. Its standard deviation will be denoted σ_{j_y} for the spotting weapon and σ_{J_y} for the major caliber weapon.

Range Wind Gustiness. The round-to-round variation in range wind on a given occasion influences the vertical coordinate of impact just as was the case with crosswind gustiness and horizontal impact error. If σ_{WG_y} is the standard deviation in range wind gustiness, and $\left(\frac{\Delta y}{\Delta W}\right)_r$ and $\left(\frac{\delta y}{\delta w}\right)_r$ give the vertical change in impact at range r corresponding to a 1 foot per second change in range wind for the major caliber weapon and spotting weapon respectively, then $\sigma_{y_{WG}}(r)$ and $\sigma_{y_{wg}}(r)$, the standard deviations of impact error at range r due to range wind gustiness for the major caliber weapon and the spotting weapon respectively are given by

$$\sigma_{y_{WG}}(r) = \sigma_{WG_y} \cdot \left(\frac{\Delta y}{\Delta W}\right)_r \text{ mils and}$$

$$\sigma_{y_{wg}}(r) = \sigma_{WG_y} \cdot \left(\frac{\delta y}{\delta w}\right)_r \text{ mils.}$$

The quantities $\left(\frac{\Delta y}{\Delta W}\right)_r$ and $\left(\frac{\delta y}{\delta w}\right)_r$ are termed "unit differential effects" giving the effect of range wind on the vertical coordinate of impact.

Ballistic Coefficient Variation. The round-to-round variation in ballistic coefficient causes a variation in the vertical coordinate of impact. If σ_B and σ_b are the standard deviations of ballistic coefficient variation for the major caliber ammunition and spotting ammunition respectively, and $\left(\frac{\Delta y}{\Delta B}\right)_r$ and $\left(\frac{\delta y}{\delta b}\right)_r$ are the unit differential effects giving the change in vertical impact coordinate resulting from a 1% change in ballistic coefficient for the major caliber weapon and spotting weapon respectively, then $\sigma_{y_B}(r)$ and $\sigma_{y_b}(r)$, the standard deviations in vertical impact error due to round-to-round ballistic coefficient variation for the major caliber weapon and spotting weapon respectively, are given by

$$\sigma_{y_B}(r) = \sigma_B \cdot \left(\frac{\Delta y}{\Delta B}\right)_r \text{ mils and}$$

$$\sigma_{y_b}(r) = \sigma_b \cdot \left(\frac{\delta y}{\delta b}\right)_r \text{ mils.}$$

Within-Lot Muzzle Velocity Variation. The next source of vertical random error is the variation in muzzle velocity among the rounds of a given lot of ammunition. Let σ_{VWL} and σ_{vwl} be the standard deviations of within-lot muzzle velocity variation for the major caliber ammunition and the spotting ammunition respectively. If $\left(\frac{\Delta y}{\Delta V}\right)_r$ and $\left(\frac{\delta y}{\delta v}\right)_r$ are the unit differential effects giving the vertical change in impact point at range r corresponding to a 1-foot-per-second change in muzzle velocity for the two kinds of ammunition, then the standard deviations in impact error at range r due to muzzle velocity variation [$\sigma_{y_{VWL}}(r)$ and $\sigma_{y_{vwl}}(r)$ for the major caliber ammunition and the spotting ammunition respectively] are given by

$$\sigma_{y_{VWL}}(r) = \sigma_{VWL} \cdot \left(\frac{\Delta y}{\Delta V}\right)_r \text{ mils}$$

and $$\sigma_{y_{vwl}}(r) = \sigma_{vwl} \cdot \left(\frac{\delta y}{\delta v}\right)_r \text{ mils.}$$

Aiming Error. There is generally a vertical round-to-round aiming error similar to the previously described horizontal error. The standard deviation of this error is denoted σ_{l_y} mils in the zeroing situation and σ_{L_y} mils when firing for effect.

Total Random Error. The total vertical random error is the combination of errors introduced by jump, range wind gustiness, ballistic coefficient variation, within-lot muzzle velocity variation, and aiming error. Hence, if R_z is the zeroing range, then $\sigma_{y_{RA}}(R_z)$ and $\sigma_{y_{Ra}}(R_z)$, the standard deviations of total vertical random error when zeroing for the major caliber and spotting rounds respectively, are given by

$$\sigma_{y_{RA}}(R_z) = \sqrt{\sigma_{J_y}^2 + \left[\sigma_{y_{WG}}(R_z)\right]^2 + \left[\sigma_{y_B}(R_z)\right]^2 + \left[\sigma_{y_{VWL}}(R_z)\right]^2 + \sigma_{l_y}^2} \text{ mils,}$$

and

$$\sigma_{y_{Ra}}(R_z) = \sqrt{\sigma_{j_y}^2 + \left[\sigma_{y_{wg}}(R_z)\right]^2 + \left[\sigma_{y_b}(R_z)\right]^2 + \left[\sigma_{y_{vwl}}(R_z)\right]^2 + \sigma_{l_y}^2} \text{ mils.}$$

When firing for effect at range R, the standard deviation of vertical random error for the major caliber weapon is given by

$$\sigma_{y_{RA}}(R) = \sqrt{\sigma_{J_y}^2 + \left[\sigma_{y_{WG}}(R)\right]^2 + \left[\sigma_{y_B}(R)\right]^2 + \left[\sigma_{y_{VWL}}(R)\right]^2 + \sigma_{L_y}^2} \text{ mils,}$$

while that for the spotter is given by

$$\sigma_{y_{Ra}}(R) = \sqrt{\sigma_{j_y}^2 + \left[\sigma_{y_{wg}}(R)\right]^2 + \left[\sigma_{y_b}(R)\right]^2 + \left[\sigma_{y_{vwl}}(R)\right]^2 + \sigma_{L_y}^2} \text{ mils.}$$

Variable Bias Errors

Zeroing. The process of zeroing a weapon system of this type consists of firing a group of rounds from the major caliber weapon and a group of rounds from the spotting weapon, estimating the separation of the centers of impact of these two groups, and adjusting the spotting weapon relative to the major caliber weapon so that these two centers of impact coincide. The range at which these firings take place is called the zeroing range and is denoted R_z. There are a number of factors which influence the magnitude of the error made in doing this. For example, the centers of impact of the two groups cannot be determined exactly from the small number of rounds fired in the group; the air temperature and hence the propellant temperature of the ammunition may not be standard, and since the two kinds of ammunition are affected differently by variation in propellant temperature, an error is introduced; or the mean muzzle velocities of the lots of ammunition being used for the zeroing may not be standard, leading to a zeroing error. Of interest is the standard deviation of error when firing for effect at range R caused by the error in zeroing at range R_z. It will be shown that all of the components of zeroing error, expressed in angular units (mils), are independent of R, (but of course dependent on R_z) except the error introduced by the non-standard mean velocities of the lots of ammunition used on a particular zeroing occasion. This component of error depends on both R and R_z. Within the framework of these general comments the zeroing error will be discussed in detail.

Horizontal

Location of Center of Impact. Suppose that in zeroing, N rounds of major caliber ammunition and n rounds of spotting ammunition are fired at range R_z. Since these firings are all made on a given occasion, the distributions of impacts for both weapons can be thought of as impacts selected at random from distributions having as standard deviations the previously discussed random errors. For each group of impacts, it is desired to estimate the mean (or center of impact) of the distribution from which the impacts were selected. Sampling theory indicates that the best estimate of the mean of the distribution is the mean of the sample. Hence the best the gunner can do is assume that the mean impact point as he observes it is really the center of impact of the population from which the impacts are drawn. Of course, the sample mean is not always the same as the mean of the population. Of interest here is to predict (in a probability sense) the error a gunner makes by assuming that the observed mean is the same as the center of impact of the population. Again looking at sampling theory, it is found that the distribution of sample means has a standard deviation given by $\frac{\sigma}{\sqrt{k}}$, where σ is the standard deviation of the population from which a sample of size k is examined. Hence, if $\sigma_{M_x}(R_z)$ and $\sigma_{m_x}(R_z)$ are the standard deviations of the error in assuming the means of the zeroing groups to be the means of the populations from which the groups are selected, for the major caliber and spotting weapons respectively, then

$$\sigma_{M_x}(R_z) = \sqrt{\frac{1}{N}} \cdot \sigma_{x_{RA}}(R_z) \text{ mils and}$$

$$\sigma_{m_x}(R_z) = \sqrt{\frac{1}{n}} \cdot \sigma_{x_{Ra}}(R_z) \text{ mils.}$$

Note that the random errors are computed at the specific range R_z at which the zeroing is taking place.

Observation of Center of Impact. In addition to the error in the actual location of the center of impact as described in the preceding paragraph, there is an error in estimating the mean of the shot group from the remote firing position. This error should be of the same magnitude for both the spotter and major caliber weapons, and its standard deviation be denoted σ_{o_x} and its units will be mils.

Cant Variation. Cant error is the error in placing a weapon in firing position so that its elevating trunnions are level. Such an error causes an elevated launcher to point at a horizontally-measured angle different from that intended. In general, if γ is the angle of cant and ϕ is the elevation angle of the launcher, the horizontal error ϵ introduced is given by

$$\epsilon = 1018.59 \tan \phi \sin \gamma \text{ mils.}$$

Since for a weapon of the type being considered here both launchers (major caliber and spotter) are rigidly fixed together, when one is canted, so is the other. Let $\Phi(R_z)$ and $\phi(R_z)$ be the angles of elevation of the major caliber launcher and spotting round launcher respectively required to fire them to zeroing range R_z. Then the horizontal error in the major caliber impact is given by

$$\epsilon_1 = 1018.59 \tan \Phi(R_z) \sin \gamma,$$ while that for the spotter is given by

$$\epsilon_2 = 1018.59 \tan \phi(R_z) \sin \gamma.$$

Since this weapon is employed by firing the spotter until it hits the target, only the difference between the errors ϵ_1 and ϵ_2 are of interest. This error is

$$\epsilon_1 - \epsilon_2 = 1018.59 \sin \gamma \left[\tan \Phi(R_z) - \tan \phi(R_z)\right].$$

If σ_c is the standard deviation of cant error measured from zeroing occasion to zeroing occasion, the corresponding standard deviation of zeroing error, σ_{x_c}, is given by

$$\sigma_{x_c}(R_z) = 1018.59 \sin \sigma_c \left[\tan \Phi(R_z) - \tan \phi(R_z)\right] \text{ mils.}$$

Crosswind Variation. The variation in average wind velocity from zeroing occasion to zeroing occasion introduces a zeroing error. In other words, if the system is zeroed on a windy day, it will not be correctly zeroed for firing on a subsequent calm day. Here again, it is the difference in impact points introduced by the fact that the spotter reacts differently to a wind than does the major caliber round.

If σ_{w_x} is the standard deviation of the variation of mean crosswind from zeroing occasion to zeroing occasion, then $\sigma_{x_w}(R_z)$, the standard deviation of zeroing error caused by this crosswind variation is given by

$$\sigma_{x_w}(R_z) = \frac{1018.59}{3R_z}\sigma_{w_x}\left| T(R_z) - t(R_z) - 3R_z \left\{ \frac{1}{V \cos \Phi(R_z)} - \frac{1}{v \cos \phi(R_z)} \right\} \right| \text{ mils}$$

Total Horizontal Zeroing Error. The total horizontal zeroing error is a combination of all of those errors listed. If the standard deviation of this error is denoted $\sigma_{x_z}(R_z)$, then

$$\sigma_{x_z}(R_z) = \sqrt{\left[\sigma_{M_x}(R_z)\right]^2 + \left[\sigma_{m_x}(R_z)\right]^2 + 2\,\sigma_{o_x}^2 + \left[\sigma_{x_c}(R_z)\right]^2 + \left[\sigma_{x_w}(R_z)\right]^2} \text{ mils.}$$

This can be interpreted as the standard deviation of error when firing for effect at range R due to the fact that the zeroing was accomplished at range R_z. Since $\sigma_{x_z}(R_z)$ is independent of R, it is a constant (in mils) at all target ranges, but changes only when zeroing range changes.

Vertical

Location of Center of Impact. As in the case of horizontal zeroing error, there is an error in assuming that the mean of the impacts of the zeroing group sample is the same as the center of impact of the population from which the sample is drawn. If $\sigma_{M_y}(R_z)$ and $\sigma_{m_y}(R_z)$ are the standard deviations of this error for the major caliber weapon and the spotting weapon, respectively, then

$$\sigma_{M_y}(R_z) = \sqrt{\frac{1}{N}} \cdot \sigma_{y_{RA}}(R_z) \text{ mils and}$$

$$\sigma_{m_y}(R_z) = \sqrt{\frac{1}{n}} \cdot \sigma_{y_{Ra}}(R_z) \text{ mils,}$$

where N and n are the number of major caliber and spotting rounds respectively, fired during zeroing, and $\sigma_{y_{RA}}(R_z)$ and $\sigma_{y_{Ra}}(R_z)$ are the standard deviations of random error for the two weapons at the zeroing range R_z.

Observation of Center of Impact. There is also a vertical error in estimating the mean of the sample shot group from a remote firing position. This error should be of the same magnitude for both the spotter and the major caliber weapons. The standard deviation of this error is denoted σ_{o_y} and its units are mils.

Temperature Variation. If the weapon system is zeroed on an occasion when nonstandard temperature prevails, an error will be introduced due to the fact that the spotting ammunition reacts differently from the major caliber ammunition as temperature varies. (A change in temperature results in a change in velocity which in turn results in a change in impact point.) Let $\frac{\Delta V}{\Delta F}$ and $\frac{\delta v}{\delta f}$ be the unit differential effects giving the change in muzzle velocity corresponding to a change of $1^{\circ}F$ in propellant temperature for the major caliber and spotting ammunition respectively, and let $\left(\frac{\Delta y}{\Delta V}\right)_{R_z}$ and $\left(\frac{\delta y}{\delta v}\right)_{R_z}$ be the unit differential effects giving the change in impact point resulting from a 1 foot per second change in velocity at range R_z for the major caliber and spotting ammunition respectively. Then the difference between the impact points of the two rounds at range R_z caused by a $1^{\circ}F$ change in propellant temperature is

$$\left[\left(\frac{\Delta V}{\Delta F}\right)\left(\frac{\Delta y}{\Delta V}\right)_{R_z} - \left(\frac{\delta v}{\delta f}\right)\left(\frac{\delta y}{\delta v}\right)_{R_z}\right].$$

If σ_f is the standard deviation of temperature variation from zeroing occasion to zeroing occasion, then $\sigma_{y_f}(R_z)$, the component of zeroing error due to this temperature variation is given by

$$\sigma_{y_f}(R_z) = \sigma_f \cdot \left| \left(\frac{\Delta v}{\Delta f}\right)\left(\frac{\Delta y}{\Delta v}\right)_{R_z} - \left(\frac{\delta v}{\delta f}\right)\left(\frac{\delta y}{\delta v}\right)_{R_z} \right| \text{ mils.}$$

Range Wind Variation. Corresponding to the horizontal zeroing error caused by the variation in mean crosswind from zeroing occasion to zeroing occasion, there is a vertical zeroing error caused by the variation in mean range wind from zeroing occasion to zeroing occasion. If $\left(\frac{\Delta y}{\Delta w}\right)_{R_z}$ and $\left(\frac{\delta y}{\delta w}\right)_{R_z}$ are the unit differential effects giving, at range R_z, the change in vertical impact point corresponding to a 1 foot per second change in range wind for the major caliber weapon and spotting weapon respectively, then the difference between the impact points of the two weapons caused by a 1 foot per second range wind change is $\left[\left(\frac{\Delta y}{\Delta w}\right)_{R_z} - \left(\frac{\delta y}{\delta w}\right)_{R_z}\right]$. If σ_{w_y} is the standard deviation in mean range wind variation from zeroing occasion to zeroing occasion, then $\sigma_{y_w}(R_z)$, the standard deviation in vertical zeroing error due to the variation in mean range wind from zeroing occasion to zeroing occasion is given by

$$\sigma_{y_w}(R_z) = \sigma_{w_y} \cdot \left| \left(\frac{\Delta y}{\Delta w}\right)_{R_z} - \left(\frac{\delta y}{\delta w}\right)_{R_z} \right| \text{ mils.}$$

Air Density Variation. If the weapon system is zeroed under conditions of nonstandard air density, an error will be introduced since the spotting round reacts differently from the major caliber round to changes in air density. If $\left(\frac{\Delta y}{\Delta D}\right)_{R_z}$ and $\left(\frac{\delta y}{\delta d}\right)_{R_z}$ are the unit differential effects at range R_z for the major caliber weapon and the spotting weapon respectively, giving the vertical change in impact point resulting from a 1% change in air density, then the difference in impact point resulting from the 1% change in air density is $\left[\left(\frac{\Delta y}{\Delta D}\right)_{R_z} - \left(\frac{\delta y}{\delta d}\right)_{R_z}\right]$. If σ_d is the standard deviation in the variation of air density from zeroing occasion to zeroing occasion, then $\sigma_{y_d}(R_z)$, the standard deviation of vertical zeroing error, due to variation in air density variation from zeroing occasion to zeroing occasion is given by

$$\sigma_{y_d}(R_z) = \sigma_d \cdot \left|\left(\frac{\Delta y}{\Delta D}\right)_{R_z} - \left(\frac{\delta y}{\delta d}\right)_{R_z}\right| \text{ mils.}$$

Lot-to-Lot Muzzle Velocity Variation. All of the components of zeroing error discussed heretofore produce errors which are constant (when expressed in mils) for all target ranges when firing for effect, the value of the constant depending on the range at which the system was zeroed. The lot-to-lot variation in muzzle velocity for both the spotter and major caliber weapon, however, produces errors which behave differently. Since it is assumed that the weapon system is re-zeroed whenever a new lot of either type of ammunition is acquired, it follows that the system will never be fired for effect with ammunition from a lot different from that used for zeroing. Hence the effect of lot-to-lot variation in muzzle velocity must be zero when firing for effect at the zeroing range. In order to determine the effect of nonstandard muzzle velocity for the spotting weapon, consider the following:

For standard muzzle velocity, let v be that velocity and $\phi(R_z)$ be the angle of elevation required to fire the spotter to range R_z. Let V and $\Phi(R_z)$ be the same quantities for the major caliber weapon. The process of zeroing is simply one of fixing the difference $\Phi(R_z) - \phi(R_z)$. Hence $\Phi(R_z) - \phi(R_z)$ will be the difference between the angles of elevation of the two weapons when subsequently firing for effect at any range. In particular, when firing for effect at range R, the angle of elevation of the spotter is $\phi(R)$,

and that for the major caliber weapon is $\phi(R) + \left[\Phi(R_z) - \phi(R_z)\right]$.
But in order for the major caliber weapon to hit the same point that the spotter has hit (nominally), it should be elevated to an angle of $\Phi(R)$. Hence the error in the angle of elevation of the major caliber weapon when firing for effect at range R after zeroing at range R_z is

$$\Phi(R) - \phi(R) - \left[\Phi(R_z) - \phi(R_z)\right] .$$

If it is assumed that a 1 mil change in angle of elevation results in a 1 mil change in impact point (which can safely be assumed for relatively flat trajectory weapons such as are under consideration here), then

$$\Phi(R) - \phi(R) - \left[\Phi(R_z) - \phi(R_z)\right]$$

also gives the vertical error present under standard conditions of spotting rifle muzzle velocity.

Now assume that the spotter has nonstandard muzzle velocity $v + \delta v$. In order to zero the system at range R_z, a new angle of elevation $\phi'(R_z)$ for the spotter will have to be used, and the fixed bias angle between the launchers on subsequent firings for effect will be $\Phi(R_z) - \phi'(R_z)$. Thus, when firing for effect at range R, the spotting weapon will be elevated to angle $\phi'(R)$, and the major caliber weapon to angle $\phi'(R) + \left[\Phi(R_z) - \phi'(R_z)\right]$. Now since the major caliber should be elevated to angle $\Phi(R)$, the error in angle of elevation and hence in vertical impact error (expressed in mils) under the nonstandard spotter velocity is $\Phi(R) - \phi'(R) - \left[\Phi(R_z) - \phi'(R_z)\right]$.
If the error present under the standard velocity condition is subtracted from this, then δy, the error due solely to the velocity error δv, is

$$\delta y = \phi(R) - \phi'(R) - \left[\phi(R_z) - \phi'(R_z)\right] .$$

But $\phi(R) - \phi'(R) = \delta v \cdot \left(\frac{\delta y}{\delta v}\right)_R$ and

$$\phi(R_z) - \phi'(R_z) = \delta v. \left(\frac{\delta y}{\delta v}\right)_{R_z} ,$$

where $\left(\frac{\delta y}{\delta v}\right)_r$ is the unit differential effect at range r giving the change in spotter impact caused by a 1 foot per second change in spotter muzzle velocity. Hence

$$\delta y = \delta v \cdot \left[\left(\frac{\delta y}{\delta v}\right)_R - \left(\frac{\delta y}{\delta v}\right)_{R_z}\right] .$$

If σ_{vll} is the standard deviation in lot-to-lot muzzle velocity variation for the spotting ammunition, then $\sigma_{y_{vll}}(R,R_z)$, the component of error caused by the lot-to-lot muzzle velocity variation for the spotting ammunition is given by

$$\sigma_{y_{vll}}(R,R_z) = \sigma_{vll} \cdot \left|\left(\frac{\delta y}{\delta v}\right)_R - \left(\frac{\delta y}{\delta v}\right)_{R_z}\right| \text{ mils.}$$

A very similar argument canbe followed in the case where the major caliber weapon is fired with nonstandard muzzle velocity, and a very similar result is obtained. If $\sigma_{y_{VLL}}(R,R_z)$ is the desired component of error due to lot-to-lot variation in the muzzle velocity of the major caliber ammunition, and σ_{VLL} is the standard deviation in this lot-to-lot variation, then

$$\sigma_{y_{VLL}}(R,R_z) = \sigma_{VLL} \cdot \left|\left(\frac{\Delta y}{\Delta V}\right)_{R_z} - \left(\frac{\Delta y}{\Delta V}\right)_R\right| \text{ mils, where } \left(\frac{\Delta y}{\Delta V}\right)_r \text{ is}$$

the unit differential effect giving the change in major caliber impact at range r caused by a 1 foot per second change in muzzle velocity for the major caliber weapon.

Total Vertical Zeroing Error. The total vertical zeroing error is a combination of all those listed. If the standard deviation of total vertical zeroing error is denoted $\sigma_{y_z}(R,R_z)$, then

$$\sigma_{y_z}(R,R_z) = \left\{\left[\sigma_{M_y}(R_z)\right]^2 + \left[\sigma_{m_y}(R_z)\right]^2 + 2\sigma_{o_y}^2 + \left[\sigma_{y_f}(R_z)\right]^2 + \left[\sigma_{y_w}(R_z)\right]^2 + \left[\sigma_{y_d}(R_z)\right]^2 + \left[\sigma_{y_{vll}}(R,R_z)\right]^2 + \left[\sigma_{y_{VLL}}(R,R_z)\right]^2\right\}^{1/2} \text{ mils.}$$

This should be interpreted as the standard deviation of error when firing for effect at range R due to the fact that the zeroing was accomplished at range R_z. Unlike the horizontal zeroing error, which, when expressed in mils, is independent of R, this vertical error is dependent on both R and R_z.

Other Variable Bias Errors. The remaining variable bias errors will be discussed in separate paragraphs depending on whether they are horizontal or vertical errors.

Horizontal

Cant Variation. The variation of weapon cant from occasion to occasion when firing for effect is a source of variable bias error. Since the major caliber and spotting weapon are rigidly fixed together at the time of zeroing with an angular separation depending on the zeroing range R_z, when subsequently fired for effect at range R, the spotter will be elevated to angle $\phi(R)$, and the major caliber to angle $\phi(R) + \left[\Phi(R_z) - \phi(R_z)\right]$, where $\Phi(R_z)-\phi(R_z)$ is the angular separation fixed at zeroing. Then, following the same development that was described in the section on the zeroing error introduced by variation in cant from zeroing occasion to zeroing occasion, $\sigma_{x_C}(R,R_z)$, the horizontal variable bias error introduced by the occasion-to-occasion cant variation when firing for effect is given by

$$\sigma_{x_C}(R,R_z) = 1018.59 \sin \sigma_C \left| \tan\left\{\phi(R) + \left[\Phi(R_z) - \phi(R_z)\right]\right\} - \tan \phi(R)\right|$$

mils, where σ_C is the standard deviation of occasion-to-occasion variation in cant when firing for effect. But, for any angles α and β,

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta} .$$

For small angles α and β, $\tan\alpha \tan\beta$ is very small, and negligible error is introduced by letting $\tan(\alpha-\beta) = \tan\alpha - \tan\beta$. Hence

$$\sigma_{x_C}(R, R_z) = 1018.59 \sin \sigma_C \left| \tan \left[\Phi(R_z) - \phi(R_z) \right] \right| \text{ mils,}$$

since all of the angles of elevation will be relatively small. Thus the effect of occasion-to-occasion cant variation depends only on the zeroing range.

Crosswind Variation. Let σ_{W_x} be the standard deviation of the variation in mean crosswind from occasion to occasion. Then, since both the major caliber and spotting projectile will be subjected to the same mean crosswind on a given occasion, only the difference between the errors of the two rounds is of interest. If $\sigma_{x_W}(R)$ is the standard deviation of variable bias error at range R due to occasion-to-occasion crosswind variation, then

$$\sigma_{x_W}(R) = \frac{1018.59}{3R} \sigma_{W_x} \left| T(R) - t(R) - 3R \left\{ \frac{1}{V \cos \Phi(R)} - \frac{1}{v \cos \phi(R)} \right\} \right|$$

mils, where $T(R)$, V and $\Phi(R)$ are the time of flight, muzzle velocity, and angle of elevation respectively for the major caliber weapon, and $t(R)$, v, and $\phi(R)$ are the same quantities for the spotting weapon.

Total Horizontal Variable Bias Error. The total horizontal variable bias error is the combination of the errors introduced during zeroing and the errors due to cant variation and mean crosswind variation from occasion to occasion. Thus if $\sigma_{x_{VB}}(R, R_z)$ is the standard deviation of total horizontal variable bias,

$$\sigma_{x_{VB}}(R,R_z) = \sqrt{\left[\sigma_{x_z}(R_z)\right]^2 + \left[\sigma_{x_C}(R,R_z)\right]^2 + \left[\sigma_{x_W}(R)\right]^2} \quad \text{mils.}$$

Vertical

Temperature Variation. The variation in temperature from occasion to occasion produces an occasion-to-occasion variation in velocity for both the spotting ammunition and the major caliber ammunition. Since the two have different sensitivity to temperature change, there will be a variable bias error introduced equal to the difference between the effects of temperature change on the impact position of the two kinds of rounds. If $\frac{\Delta V}{\Delta F}$ and $\frac{\delta v}{\delta f}$ are the unit differential effects giving the change in velocity resulting from a $1^{\circ}F$ change in temperature for the major caliber and spotting ammunition respectively, and $\left(\frac{\Delta y}{\Delta V}\right)_R$ and $\left(\frac{\delta y}{\delta v}\right)_R$ are the unit differential effects giving the change in impact point at range R resulting from a 1 foot per second change in velocity for the same weapons, respectively, then the change in impact at range R for the major caliber weapon due to a $1^{\circ}F$ change in temperature is $\left(\frac{\Delta V}{\Delta F}\right)\left(\frac{\Delta y}{\Delta V}\right)_R$ and for the spotting weapon is $\left(\frac{\delta v}{\delta f}\right)\left(\frac{\delta y}{\delta v}\right)_R$. Hence the variable bias error introduced as a result of a $1^{\circ}F$ temperature change is

$$\left[\left(\frac{\Delta V}{\Delta F}\right)\left(\frac{\Delta y}{\Delta V}\right)_R - \left(\frac{\delta v}{\delta f}\right)\left(\frac{\delta y}{\delta v}\right)_R\right].$$

If σ_F is the standard deviation of temperature variation from occasion to occasion when firing for effect, then $\sigma_{y_F}(R)$, the standard deviation of variable bias error due to occasion-to-occasion temperature variation is given by

$$\sigma_{y_F}(R) = \sigma_F \cdot \left|\left(\frac{\Delta V}{\Delta F}\right)\left(\frac{\Delta y}{\Delta V}\right)_R - \left(\frac{\delta v}{\delta f}\right)\left(\frac{\delta y}{\delta v}\right)_R\right| \quad \text{mils.}$$

Air Density Variation. The variation in air density from occasion to occasion when firing for effect causes a variable bias error since the major caliber ammunition and the spotting ammunition react differently to the same change in air density. If $\left(\frac{\Delta y}{\Delta D}\right)_R$ and $\left(\frac{\delta y}{\delta d}\right)_R$ are the unit differential effects giving the change in impact point at range R corresponding to a 1% change in air density, then the variable bias error resulting from a 1% change in air density is $\left[\left(\frac{\Delta y}{\Delta D}\right)_R - \left(\frac{\delta y}{\delta d}\right)_R\right]$. If σ_D is the standard deviation of occasion-to-occasion variation in air density, then $\sigma_{y_D}(R)$, the standard deviation of variable bias error due to air density variation is given by

$$\sigma_{y_D}(R) = \sigma_D \cdot \left|\left(\frac{\Delta y}{\Delta D}\right)_R - \left(\frac{\delta y}{\delta d}\right)_R\right| \quad \text{mils.}$$

Range Wind Variation. The final component of vertical variable bias error is the effect of occasion-to-occasion variation in mean range wind. Let σ_{W_y} be the standard deviation of the variation in mean range wind from occasion to occasion when firing for effect, and let $\left(\frac{\Delta y}{\Delta W}\right)_R$ and $\left(\frac{\delta y}{\delta w}\right)_R$ be the unit differential effects giving the change in impact point due to a 1 foot per second change in range wind for the major caliber weapon and spotting weapon, respectively. Then $\sigma_{y_W}(R)$, the standard deviation of variable error due to the wind variation is given by

$$\sigma_{y_W}(R) = \sigma_{W_y} \cdot \left|\left(\frac{\Delta y}{\Delta W}\right)_R - \left(\frac{\delta y}{\delta w}\right)_R\right| \quad \text{mils.}$$

Total Vertical Variable Bias Error. The total vertical variable bias error is the combination of the errors introduced during zeroing and those resulting from occasion-to-occasion variation in temperature, air density, and range wind. Hence, if $\sigma_{y_{VB}}(R,R_z)$ is the standard deviation of total variable bias when firing for effect at range R after zeroing at range R_z, then

$$\sigma_{y_{VB}}(R,R_z) = \sqrt{\left[\sigma_{y_z}(R,R_z)\right]^2 + \left[\sigma_{y_F}(R)\right]^2 + \left[\sigma_{y_D}(R)\right]^2 + \left[\sigma_{y_W}(R)\right]^2} \quad \text{mils.}$$

Fixed Bias Errors

Horizontal

Drift. The difference between the drift of the major caliber weapon and that of the spotting weapon is the source of a fixed bias error. Since the zeroing process removes all fixed bias errors at the zeroing range, the error due to drift difference must be zero when firing for effect at the zeroing range. Let $X_D(r)$ mils and $X_d(r)$ mils be the amount of drift at range r associated with the major caliber weapon and spotting weapon respectively. When zeroing at range R_z, the major caliber weapon will be fired at angle $-X_D(R_z)$ from line of sight, and the spotter will be fired at angle $-X_d(R_z)$ from the line of sight in order that the two projectiles will "drift" onto the zeroing target. Then $X_d(R_z) - X_D(R_z)$ will be the fixed angular separation of the two launchers on subsequent firings for effect. When firing for effect at range R, the spotting weapon will be aimed at angle $-X_d(R)$ from line of sight to target so that it will drift onto the target. At this time the major caliber weapon will be aimed at angle $-X_d(R) + \left[X_d(R_z) - X_D(R_z)\right]$ from line of sight. But at range R the amount of major caliber drift is $X_D(R)$. Therefore $\bar{X}_D(R,R_z)$, the fixed bias horizontal error due to drift difference is given by

$$\bar{X}_D(R,R_z) = \left[X_D(R) - X_d(R)\right] - \left[X_D(R_z) - X_d(R_z)\right] \quad \text{mils.}$$

Parallax. The distance by which the bore line of the spotting weapon and the bore line of the major caliber weapon are separated is called parallax, and is the source of a fixed bias error. If the spotting weapon is mounted with its bore line P_X inches to the right of the bore line of the major caliber weapon, as viewed by the gunner, then the error at range R when firing for effect caused by this separation is

$$P_X \left[\frac{R}{R_z} - 1 \right]$$ inches, where R_z is the range at which the

system is zeroed. (If the spotter is mounted to the left of the major caliber, P_X will be negative.) Expressed in mils at range R, this error, $\overline{X}_P(R,R_z)$ is given by

$$\overline{X}_P(R,R_z) = 28.29\ P_X \left[\frac{1}{R_z} - \frac{1}{R} \right] \text{ mils.}$$

Total Horizontal Fixed Bias. The total horizontal fixed bias error is the algebraic sum of the errors due to drift and parallax. Hence, if $\overline{X}(R,R_z)$ is the total error,

$$\overline{X}(R,R_z) = \overline{X}_D(R,R_z) + \overline{X}_P(R,R_z) \text{ mils.}$$

Vertical

Trajectory Mismatch. One component of vertical fixed bias error results from trajectory mismatch. Again this error must be zero at the zeroing range R_z since the zeroing process removes all fixed biases. During zeroing, the spotter is elevated to angle $\phi(R_z)$ and the major caliber weapon to angle $\Phi(R_z)$, so that the angle between them is $\Phi(R_z) - \phi(R_z)$ mils. The two weapons are then rigidly attached together, maintaining this angular separation. Subsequently, when firing for effect at range R, spotting rounds are fired until one hits the target, that is, until the spotter is elevated to angle $\phi(R)$. At this point the major caliber is fired at elevation $\phi(R) + \left[\Phi(R_z) - \phi(R_z) \right]$. But when firing at range R it should be elevated only to angle $\Phi(R)$. Hence the major caliber weapon is elevated $\phi(R) + \left[\Phi(R_z) - \phi(R_z)\right] - \Phi(R)$ above what is required. But it can be assumed, for relatively flat trajectory weapons of the type being considered that a 1 mil change in angle of elevation results in a 1 mil change in impact point. Thus $\overline{Y}_{TM}(R,R_z)$, the vertical fixed bias error due to trajectory mismatch, is given by

$$\overline{Y}_{TM}(R,R_z) = \left[\phi(R) - \Phi(R)\right] + \left[\Phi(R_z) - \phi(R_z)\right] \quad \text{mils.}$$

Parallax. If the spotting weapon is mounted above or below the major caliber weapon, there will be a vertical fixed bias error resulting from this parallax. Let P_Y inches be the vertical distance between the bore lines of the two weapons, with positive P_Y denoting the spotter being mounted above the major caliber. Then the vertical fixed bias error, $\overline{Y}_P(R,R_z)$, resulting from this parallax when firing for effect at range R after zeroing at range R_z is given by

$$\overline{Y}_P(R,R_z) = 28.29\, P_Y \left[\frac{1}{R_z} - \frac{1}{R}\right] \quad \text{mils.}$$

Total Vertical Fixed Bias. The total vertical fixed bias error, $\overline{Y}(R,R_z)$, is the algebraic sum of the errors due to trajectory mismatch and parallax. Thus

$$\overline{Y}(R,R_z) = \overline{Y}_{TM}(R,R_z) + \overline{Y}_P(R,R_z) \quad \text{mils.}$$

FIRST ROUND HIT PROBABILITY

The foregoing has been a discussion of all of the errors which can cause the first major-caliber round to miss the target after a spotting round has hit the target, except the error whose distribution describes the position of the spotting-round hit on the target. Two different assumptions will be made, and the associated first major-round hit probabilities will be given. The target will be rectangular of width W(R) mils and height H(R) mils. Of course the probability of hit will be expressed as a product of two probabilities, one being the horizontal hit probability and the other being the vertical hit probability.

It will first be assumed that the distribution of spotting-round hits from occasion to occasion is normal. Then since $\pm$ 3 standard deviations in a normal distribution encompass essentially the whole population, the standard deviation of the spotting impact is taken as $\frac{W(R)}{6}$ (or $\frac{H(R)}{6}$). Now since it is assumed that all of the previously discussed error distributions are normal, the resulting distribution of first major-caliber impacts will be normal. In the horizontal, the mean of the impact distribution measured from the target center as origin will be $\overline{X}(R,R_z)$, the total horizontal fixed bias, and the variance of the distribution will be the sum of the variances of horizontal random error, horizontal variable bias, and spotting impact error. Thus, if $p_x(R,R_z)$ is the horizontal hit probability at range R after having zeroed at range R_z, then

$$p_x(R,R_z) = \int_{\frac{-W(R)}{2}}^{\frac{W(R)}{2}} \frac{1}{\sqrt{2\pi}\,\sigma_x(R,R_z)}\, e^{-\frac{1}{2}\left[\frac{x-\overline{X}(R,R_z)}{\sigma_x(R,R_z)}\right]^2} dx,$$

where

$$\left[\sigma_x(R,R_z)\right]^2 = \left[\sigma_{x_{RA}}(R,R_z)\right]^2 + \left[\sigma_{x_{Ra}}(R)\right]^2 + \left[\sigma_{x_{VB}}(R,R_z)\right]^2 + \left[\frac{W(R)}{6}\right]^2 .$$

This probability can be rewritten

$$p_x(R,R_z) = \alpha\left(\frac{\frac{W(R)}{2} - \overline{X}(R,R_z)}{\sigma_x(R,R_z)}\right) - \alpha\left(\frac{-\frac{W(R)}{2} - \overline{X}(R,R_z)}{\sigma_x(R,R_z)}\right),$$

where $\alpha(x) = \int_{-\infty}^{X} \sqrt{\frac{1}{2\pi}}\, e^{-\frac{t^2}{2}}\, dt$, the cumulative normal distribution,

a table of which is included in Table 1.

The vertical hit probability is computed in exactly the same manner. Thus $p_y(R,R_z)$ the vertical hit probability for the first major-caliber round after achieving a spotting-round hit on a target at range R is

$$p_y(R,R_z) = \alpha\left(\frac{\frac{H(R)}{2} - \overline{Y}(R,R_z)}{\sigma_y(R,R_z)}\right) - \alpha\left(\frac{-\frac{H(R)}{2} - \overline{Y}(R,R_z)}{\sigma_y(R,R_z)}\right)$$

where

$$\left[\sigma_y(R,R_z)\right]^2 = \left[\sigma_{y_{RA}}(R)\right]^2 + \left[\sigma_{y_{Ra}}(R)\right]^2 + \left[\sigma_{y_{VB}}(R,R_z)\right]^2 + \left[\frac{H(R)}{6}\right]^2 .$$

The probability of hitting the target, $p_H(R,R_z)$, is then the product of the probabilities of horizontal and vertical hit. Hence

$$p_H(R,R_z) = p_x(R,R_z) \cdot p_y(R,R_z).$$

If it is assumed that the distribution of spotting round hits on the target is uniform random rather than normal, a different probability of hit will be obtained. In BRL Memorandum Report 636, "On Estimating Probabilities of Hitting for the Battalion Anti-Tank Weapon", it was shown that the horizontal first round hit probability,

$p'_x\ (R,R_z)$, is given by

$$p'_x(R,R_z) = \frac{1}{\lambda}\left\{\left[\beta-\lambda\right]\alpha(\beta-\lambda) + \left[\beta+\lambda\right]\alpha(\beta+\lambda) - 2\beta\alpha(\beta) + \theta(\beta+\lambda) + \theta(\beta-\lambda) - 2\theta(\beta)\right\}$$

where $\lambda = \dfrac{W(R)}{\sigma'_x(R,R_z)}$,

$$\beta = \frac{\overline{X}(R,R_z)}{\sigma'_x(R,R_z)} ,$$

$$\left[\sigma'_x(R,R_z)\right]^2 = \left[\sigma_{x_{RA}}(R,R_z)\right]^2 + \left[\sigma_{x_{Ra}}(R)\right]^2 + \left[\sigma_{x_{VB}}(R,R_z)\right]^2 ,$$

$$\alpha(X) = \int_{-\infty}^{X} \sqrt{\frac{1}{2\pi}}\, e^{-\frac{t^2}{2}}\, dt,$$

$$\text{and } \theta(X) = \sqrt{\frac{1}{2\pi}}\, e^{-\frac{X^2}{2}} .$$

The function $\theta(X)$ is the normal density function and is given in tabular form in Table 2.

Similarly $p'_y(R,R_z)$ is given by

$$p'_y(R,R_z) = \frac{1}{\lambda'}\left\{\left[\beta'-\lambda'\right]\alpha(\beta'-\lambda') + \left[\beta'+\lambda'\right]\alpha(\beta'+\lambda') - 2\beta'\alpha(\beta') + \theta(\beta'+\lambda') + \theta(B'-\lambda') - 2\theta(\beta')\right\}$$

where $\lambda' = \dfrac{H(R)}{\sigma'_y(R,R_z)}$,

$$\beta' = \frac{\overline{Y}(R,R_z)}{\sigma'_y(R,R_z)} ,$$

TABLE 1

CUMULATIVE NORMAL DISTRIBUTION

$$\alpha(X) = \int_{-\infty}^{X} \sqrt{\frac{1}{2\pi}}\, e^{-\frac{t^2}{2}}\, dt \qquad \alpha(-X) = 1-\alpha(X)$$

X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 2

NORMAL DENSITY FUNCTION

$$\theta(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}} \qquad \theta(-X) = \theta(X)$$

X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.3989	.3989	.3989	.3988	.3986	.3984	.3982	.3980	.3977	.3973
0.1	.3970	.3965	.3961	.3956	.3951	.3945	.3939	.3932	.3925	.3918
0.2	.3910	.3902	.3894	.3885	.3876	.3867	.3857	.3847	.3836	.3825
0.3	.3814	.3802	.3790	.3778	.3765	.3752	.3739	.3725	.3712	.3697
0.4	.3683	.3668	.3653	.3637	.3621	.3605	.3589	.3572	.3555	.3538
0.5	.3521	.3503	.3485	.3467	.3448	.3429	.3410	.3391	.3372	.3352
0.6	.3332	.3312	.3292	.3271	.3251	.3230	.3209	.3187	.3166	.3144
0.7	.3123	.3101	.3079	.3056	.3034	.3011	.2989	.2966	.2943	.2920
0.8	.2897	.2874	.2850	.2827	.2803	.2780	.2756	.2732	.2709	.2685
0.9	.2661	.2637	.2613	.2589	.2565	.2541	.2516	.2492	.2468	.2444
1.0	.2420	.2396	.2371	.2347	.2323	.2299	.2275	.2251	.2227	.2203
1.1	.2179	.2155	.2131	.2107	.2083	.2059	.2036	.2012	.1989	.1965
1.2	.1942	.1919	.1895	.1872	.1849	.1826	.1804	.1781	.1758	.1736
1.3	.1714	.1691	.1669	.1647	.1626	.1604	.1582	.1561	.1539	.1518
1.4	.1497	.1476	.1456	.1435	.1415	.1394	.1374	.1354	.1334	.1315
1.5	.1295	.1276	.1257	.1238	.1219	.1200	.1182	.1163	.1145	.1127
1.6	.1109	.1092	.1074	.1057	.1040	.1023	.1006	.0989	.0973	.0957
1.7	.0940	.0925	.0909	.0893	.0878	.0863	.0848	.0833	.0818	.0804
1.8	.0790	.0775	.0761	.0748	.0734	.0721	.0707	.0694	.0681	.0669
1.9	.0656	.0644	.0632	.0620	.0608	.0596	.0584	.0573	.0562	.0551
2.0	.0540	.0529	.0519	.0508	.0498	.0488	.0478	.0468	.0459	.0449
2.1	.0440	.0431	.0422	.0413	.0404	.0396	.0387	.0379	.0371	.0363
2.2	.0355	.0347	.0339	.0332	.0325	.0317	.0310	.0303	.0297	.0290
2.3	.0283	.0277	.0270	.0264	.0258	.0252	.0246	.0241	.0235	.0229
2.4	.0224	.0219	.0213	.0208	.0203	.0198	.0194	.0189	.0184	.0180
2.5	.0175	.0171	.0167	.0163	.0158	.0154	.0151	.0147	.0143	.0139
2.6	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110	.0107
2.7	.0104	.0101	.0099	.0096	.0093	.0091	.0088	.0086	.0084	.0081
2.8	.0079	.0077	.0075	.0073	.0071	.0069	.0067	.0065	.0063	.0061
2.9	.0060	.0058	.0056	.0055	.0053	.0051	.0050	.0048	.0047	.0046
3.0	.0044	.0043	.0042	.0040	.0039	.0038	.0037	.0036	.0035	.0034
3.1	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026	.0025	.0025
3.2	.0024	.0023	.0022	.0022	.0021	.0020	.0020	.0019	.0018	.0018
3.3	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014	.0013	.0013
3.4	.0012	.0012	.0012	.0011	.0011	.0010	.0010	.0010	.0009	.0009
3.5	.0009	.0008	.0008	.0008	.0008	.0007	.0007	.0007	.0007	.0006
3.6	.0006	.0006	.0006	.0005	.0005	.0005	.0005	.0005	.0005	.0004
3.7	.0004	.0004	.0004	.0004	.0004	.0004	.0003	.0003	.0003	.0003
3.8	.0003	.0003	.0003	.0003	.0003	.0002	.0002	.0002	.0002	.0002
3.9	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001
4.0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
4.1	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
4.2	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000	.0000	.0000

$$\left[\sigma_y'(R,R_z)\right]^2 = \left[\sigma_{y_{RA}}(R)\right]^2 + \left[\sigma_{y_{Ra}}(R)\right]^2 + \left[\sigma_{y_{VB}}(R,R_z)\right]^2,$$

and the functions $\alpha(x)$ and $\theta(x)$ are defined as before. Then the probability of hit, $p_H'(R,R_z)$ is given by

$$p_H'(R,R_z) = P_x'(R,R_z) \cdot P_y'(R,R_z).$$

EVALUATION OF 106MM RECOILLESS RIFLE SYSTEM (BAT)

As an example of the application of the methods presented herein, the BAT weapon system will be evaluated to determine the first round hit probability against the standard 7 1/2 foot by 7 1/2 foot target. As part of the investigation, the optimum zeroing range will be determined. Specifically the system being evaluated is the Rifle, Recoilless, 106mm, M40, firing Shell, HEAT, M344A1, using Spotting Rifle, Caliber .50 inch, M8, firing Spotter Tracer Bullet M48A1. Table 3 lists the nominal muzzle velocities, the angles of elevation, and the times of flight for the two rounds.

TABLE 3

FIRING DATA FOR M344A1 and M48A1

r	A M344A1 V = 1650 fps		B M48A1 V = 1760 fps	
Range in Yards	$\Phi(r)$ (mils)	T(r) (seconds)	$\Phi(r)$ (mils)	t(r) (seconds)
200	3.795	.377	3.389	.358
400	7.979	.783	7.252	.752
600	12.610	1.221	11.674	1.188
800	17.754	1.693	16.740	1.667
1000	23.488	2.205	22.535	2.191
1200	29.893	2.757	29.099	2.755
1400	36.992	3.341	36.387	3.344
1600	44.773	3.956	44.336	3.959
1800	53.224	4.598	52.901	4.596
2000	62.342	5.269	62.056	5.257

Tables 4 and 5 list the unit differential effects for the M344A1 and M48A1 respectively.

TABLE 4 UNIT DIFFERENTIAL EFFECTS - M344A1				
r Range in Yards	Range Wind $\left(\frac{\Delta y}{\Delta W}\right)_r$ mils/fps	Ballistic Coefficient $\left(\frac{\Delta y}{\Delta B}\right)_r$ mils/%	Muzzle Velocity $\left(\frac{\Delta y}{\Delta V}\right)_r$ mils/fps	Air Density $\left(\frac{\Delta y}{\Delta D}\right)_r$ mils/%
200	.000	.000	.005	.002
400	.001	.000	.010	.008
600	.002	.010	.016	.021
800	.004	.034	.022	.040
1000	.006	.059	.029	.068
1200	.011	.093	.037	.105
1400	.016	.132	.046	.149
1600	.024	.175	.054	.200
1800	.034	.225	.062	.256
2000	.046	.284	.070	.315
Propellant Temperature : $\frac{\Delta V}{\Delta F}$ = .265 fps/^{o}F				

TABLE 5 UNIT DIFFERENTIAL EFFECTS - M48A1				
r Range in Yards	Range Wind $\left(\frac{\delta y}{\delta w}\right)_r$ mils/fps	Ballistic Coefficient $\left(\frac{\delta y}{\delta b}\right)_r$ mils/%	Muzzle Velocity $\left(\frac{\delta y}{\delta v}\right)_r$ mils/fps	Air Density $\left(\frac{\delta y}{\delta d}\right)_r$ mils/%
200	.000	.000	.005	.002
400	.001	.002	.008	.008
600	.002	.017	.010	.032
800	.005	.028	.012	.054
1000	.010	.072	.020	.083
1200	.017	.108	.030	.121
1400	.024	.151	.038	.164
1600	.033	.196	.045	.212
1800	.046	.242	.051	.262
2000	.060	.290	.056	.310
Propellant Temperature: $\frac{\delta v}{\delta f}$ = 1.16 fps/^{o}F				

The specific values of the components of random error are listed in Table 6 for the two weapons

TABLE 6

COMPONENTS OF RANDOM ERROR

	M344A1	M48A1
Ballistic Coefficient	$\sigma_B = 1\%$	$\sigma_b = 1\%$
Within-Lot Muzzle Velocity Variation	σ_{VWL} = 7.82 fps	σ_{vwl} = 12 fps
Horizontal Jump	σ_{J_x} = .598 ₥	σ_{j_x} = .273 ₥
Vertical Jump	σ_{J_y} = .566 ₥	σ_{j_y} = .227 ₥
Cant Error	$\sigma_{c'} = \sigma_{C'} = 0$	
Range Wind Gustiness	σ_{WG_y} = 3.3 fps	
Aiming Error	$\sigma_{l_x} = \sigma_{l_y} = \sigma_{L_x} = \sigma_{L_y} = 0$	
Cross Wind Gustiness	σ_{WG_x} = 3.3 fps	

Table 7 summarizes the computation of the horizontal random errors, and Table 8 summarizes the computation of the vertical random errors in accordance with the formulas presented earlier in this report. Since all aiming errors are assumed to be zero for this tripod mounted system, the ranges r in Tables 7 and 8 can be interpreted as either zeroing range, R_z, or range to target when firing for effect, R, whichever applies at the time random error is being considered.

TABLE 7

COMPUTATION OF HORIZONTAL RANDOM ERROR

r = R or R_z (Yards)	M48A1			M344A1		
	Jump σ_{J_x}	Crosswind Gustiness $\sigma_{x_{wg}}(r)$	Total $\sigma_{x_{Ra}}(r)$	Jump σ_{J_x}	Crosswind Gustiness $\sigma_{x_{WG}}(r)$	Total $\sigma_{x_{RA}}(r)$
200	.273	.096	.28939	.598	.073	.60244
400	.273	.198	.33724	.598	.158	.61852
600	.273	.310	.41307	.598	.244	.64586
800	.273	.426	.50597	.598	.333	.68447
1000	.273	.548	.61224	.598	.432	.73772
1200	.273	.663	.71701	.598	.535	.80239
1400	.273	.769	.81602	.598	.637	.87371
1600	.273	.861	.90324	.598	.733	.94599
1800	.273	.950	.98845	.598	.822	1.01651
2000	.273	1.033	1.06847	.598	.911	1.08974

TABLE 8

COMPUTATION OF VERTICAL RANDOM ERROR

r = R or R_z (yards)	M48A1 Jump σ_{J_y}	Ballistic Coefficient σ_{y_B} (r)	Range Wind Gustiness $\sigma_{y_{WG}}$ (r)	Within Lot Velocity Var. $\sigma_{y_{VWL}}$	Total $\sigma_{y_{RA}}$ (r)
200	.227	.000	.000	.060	.23480
400	.227	.002	.003	.096	[illegible]
600	.227	.017	.007	.120	.25742
800	.227	.028	.016	.144	.27075
1000	.227	.072	.033	.240	.33971
1200	.227	.108	.056	.360	.44264
1400	.227	.151	.079	.456	.55713
1600	.227	.196	.109	.540	[illegible]
1800	.227	.242	.152	.612	[illegible]
2000	.227	.290	.198	.672	[illegible]

r = R or R_z (Yards)	M344A1 Jump σ_{J_y}	Ballistic Coefficient σ_{y_B} (r)	Range Wind Gustiness $\sigma_{y_{WG}}$ (r)	Within Lot Velocity Var. $\sigma_{y_{VWL}}$ (r)	Total $\sigma_{y_{RA}}$ (r)
200	.566	.000	.000	.059	[illegible]
400	.566	.000	.003	.078	[illegible]
600	.566	.010	.007	.125	[illegible]
800	.566	.034	.013	.172	[illegible]
1000	.566	.059	.020	.227	.61500
1200	.566	.093	.030	.289	[illegible]
1400	.566	.130	.053	.360	[illegible]
1600	.566	.175	.079	.422	.72109
1800	.566	.225	.110	.485	[illegible]
2000	.566	.284	.152	.547	[illegible]

These random errors are now used as a basis for the determination of the error introduced in zeroing. The values of the components of zeroing error are listed in Table 9.

TABLE 9

COMPONENTS OF ZEROING ERROR

	M48A1	M344A1
Number of Rounds Fired	$n = 5$	$N = 3$
Lot-to-Lot Muzzle Velocity Variation	$\sigma_{vll} = 13.7$ fps	$\sigma_{VLL} = 13.25$ fps
Cant Variation	$\sigma_c = 44$ ₥	
Temperature Variation	$\sigma_f = 16.1^\circ F$	
Air Density Variation	$\sigma_d = 6\%$	
Mean Crosswind Variation	$\sigma_{w_x} = 11$ fps	
Mean Range Wind Variation	$\sigma_{w_y} = 11$ fps	
Observation of Center of Impact	$\sigma_{o_x} = \sigma_{o_y} = .05$ ₥	

The horizontal variable bias error introduced during zeroing depends only on the zeroing range, while the vertical variable bias error introduced during zeroing depends both on the zeroing range and the range to the target when firing for effect. Table 10 summarizes the computation of the horizontal zeroing error, and Table 11 summarizes the computation of that portion of vertical zeroing error which depends only on zeroing range, i.e., all components except that due to lot-to-lot muzzle velocity variation, which will be computed subsequently.

TABLE 10

COMPUTATION OF HORIZONTAL ZEROING ERROR

R_z (Yards)	Location of Center of Impact		Observation of Center of Impact		Cant	Mean Cross Wind	Total Horizontal Zeroing Error
	M48A1 $\sigma_{m_x}(R_z)$	M344A1 $\sigma_{M_x}(R_z)$	M48A1 σ_{o_x}	M344A1 σ_{o_x}	$\sigma_{x_c}(R_z)$	$\sigma_{x_w}(R_z)$	$\sigma_{x_z}(R_z)$
200	.129	.348	.050	.050	.018	.077	.38600
400	.151	.357	.050	.050	.032	.132	.41677
600	.185	.373	.050	.050	.041	.220	.47795
800	.226	.395	.050	.050	.045	.308	.55587
1000	.274	.426	.050	.050	.042	.385	.64151
1200	.321	.463	.050	.050	.035	.429	.71251
1400	.365	.504	.050	.050	.027	.440	.76588
1600	.404	.546	.050	.050	.019	.429	.80668
1800	.442	.587	.050	.050	.014	.429	.85391
2000	.478	.629	.050	.050	.013	.407	.89160

TABLE 11

COMPUTATION OF VERTICAL ZEROING ERROR
(EXCEPT FOR EFFECT OF LOT-TO-LOT VELOCITY VARIATION)

R_z (Yards)	Location of Center of Impact		Observation of Center of Impact		Temperature $\sigma_{y_f}(R_z)$	Mean Range Wind $\sigma_{y_w}(R_z)$	Air Density $\sigma_{y_d}(R_z)$	Sub Total
	M48A1 $\sigma_{m_y}(R_z)$	M344A1 $\sigma_{M_y}(R_z)$	M48A1 σ_{o_y}	M344A1 σ_{o_y}				
200	.105	.328	.050	.050	.072	.000	.000	.35888
400	.110	.330	.050	.050	.108	.000	.000	.37119
600	.115	.335	.050	.050	.119	.000	.066	.38596
800	.121	.342	.050	.050	.130	.011	.084	.40085
1000	.152	.354	.050	.050	.250	.044	.090	.47535
1200	.198	.371	.050	.050	.402	.066	.096	.59751
1400	.240	.396	.050	.050	.514	.088	.090	.70672
1600	.281	.422	.050	.050	.610	.099	.072	.80569
1800	.319	.454	.050	.050	.689	.132	.036	.89795
2000	.354	.491	.050	.050	.747	.154	.030	.97674

Table 12 summarizes the components of vertical zeroing error resulting from lot-to-lot muzzle velocity variation. The first entry in each box gives the effect for the spotter, while the second entry gives it for the major caliber ammunition.

TABLE 12

COMPONENTS OF ZEROING ERROR DUE TO LOT-TO-LOT MUZZLE VELOCITY VARIATION

First Entry in Each Box - $\sigma_{y_{v11}}(R, R_z)$ {M48A1}

Second Entry in Each Box - $\sigma_{y_{VLL}}(R, R_z)$ {M344A1}

R (Yards)	R_z (Yards)									
	200	400	600	800	1000	1200	1400	1600	1800	2000
200	.000 .000	.041 .066	.068 .146	.096 .225	.206 .318	.342 .424	.452 .543	.548 .649	.630 .755	.699 .861
400	.041 .066	.000 .000	.027 .080	.055 .159	.164 .252	.301 .358	.411 .477	.507 .583	.589 .689	.658 .795
600	.068 .146	.027 .080	.000 .000	.027 .080	.137 .172	.274 .278	.384 .398	.480 .504	.562 .610	.630 .716
800	.096 .225	.055 .159	.027 .080	.000 .000	.110 .093	.247 .199	.356 .318	.452 .424	.534 .530	.603 .636
1000	.206 .318	.164 .252	.137 .172	.110 .093	.000 .000	.137 .106	.247 .225	.343 .331	.425 .437	.493 .543
1200	.342 .424	.301 .358	.274 .278	.247 .199	.137 .106	.000 .000	.110 .119	.206 .225	.288 .331	.356 .437
1400	.452 .543	.411 .477	.384 .398	.356 .318	.247 .225	.110 .119	.000 .000	.096 .106	.178 .212	.247 .318
1600	.548 .649	.507 .583	.480 .504	.452 .424	.343 .331	.206 .225	.096 .106	.000 .000	.082 .106	.151 .212
1800	.630 .755	.589 .689	.562 .610	.534 .530	.425 .437	.288 .331	.178 .212	.082 .106	.000 .000	.068 .106
2000	.699 .861	.658 .795	.630 .716	.603 .636	.493 .543	.356 .437	.247 .318	.151 .212	.068 .106	.000 .000

The data in Tables 11 and 12 are then combined to give the total vertical zeroing error. These are shown in Table 13.

TABLE 13

TOTAL VERTICAL ZEROING ERROR $\sigma_{y_z}(R, R_z)$

R (Yards)	R_z (Yards)									
	200	400	600	800	1000	1200	1400	1600	1800	2000
200	.35888	.37923	.41822	.46960	.60788	.80855	.99930	1.17074	1.33163	1.47782
400	.36719	.37119	.39509	.43473	.56246	.75880	.94652	1.11628	1.27591	1.42092
600	.39336	.38067	.38596	.40965	.52375	.71371	.89739	1.06468	1.22240	1.36513
800	.43432	.40754	.39509	.40085	.49670	.67648	.85283	1.01647	1.17148	1.31230
1000	.52188	.47768	.44420	.42595	.47535	.62211	.78172	.93613	1.08531	1.22144
1200	.65763	.59712	.54893	.51117	.50593	.59751	.72506	.86151	.99941	1.12771
1400	.79243	.73091	.67441	.62333	.58103	.61910	.70672	.81828	.93965	1.05648
1600	.92212	.85716	.79585	.73808	.67318	.67088	.72104	.80569	.90790	1.01082
1800	1.04677	.97950	.91483	.85249	.77301	.80593	.75900	.81676	.89795	.98483
2000	1.16564	1.09671	1.02884	.96374	.87399	.82142	.81338	.84669	.90674	.97674

Table 14 lists the components of variable bias error. These represent the standard deviation of the various quantities from occasion to occasion when firing for effect.

TABLE 14 COMPONENTS OF VARIABLE BIAS	
Cant Variation	$\sigma_C = 89$ ₥
Mean Crosswind Variation	$\sigma_{W_x} = 11$ fps
Mean Range Wind Variation	$\sigma_{W_y} = 11$ fps
Temperature Variation	$\sigma_F = 8.1^{\circ}F$
Air Density Variation	$\sigma_D = 2\%$

The total variable bias errors can now be computed. In the horizontal direction, this error includes the zeroing error and the errors due to cant variation and mean crosswind variation. The total horizontal variable bias error will be a function of both the zeroing range (R_z) and the range to the target when firing for effect, since the zeroing error and cant error depend only on zeroing range while the error due to crosswind variation depends only on range to the target. Table 15 shows the errors due to cant variation and those due to crosswind variation.

TABLE 15

VARIABLE BIAS ERROR DUE TO CANT VARIATION AND MEAN CROSSWIND VARIATION

Cant		Crosswind	
R_z (Yards)	$\sigma_{x_C}(R, R_z)$ (mils)	R (Yards)	$\sigma_{x_W}(R)$ (mils)
200	.036	200	.077
400	.065	400	.132
600	.083	600	.220
800	.090	800	.308
1000	.085	1000	.385
1200	.071	1200	.429
1400	.054	1400	.440
1600	.039	1600	.429
1800	.029	1800	.429
2000	.025	2000	.407

The data from Tables 10 and 15 can now be combined to give values for $\sigma_{x_{VB}}(R, R_z)$, the total horizontal variable bias error at range R after zeroing at range R_z. These values are given in Table 16.

TABLE 16

TOTAL HORIZONTAL VARIABLE BIAS ERROR $\sigma_{x_{VB}}(R, R_z)$

R Yards	R_z (Yards)									
	200	400	600	800	1000	1200	1400	1600	1800	2000
200	.395	.429	.491	.568	.652	.720	.772	.811	.858	.895
400	.410	.442	.503	.578	.660	.728	.779	.818	.865	.902
600	.446	.476	.533	.605	.683	.749	.799	.837	.882	.919
800	.495	.522	.575	.642	.717	.779	.827	.864	.908	.944
1000	.546	.571	.619	.682	.753	.813	.859	.895	.937	.971
1200	.578	.602	.648	.708	.776	.835	.880	.914	.956	.990
1400	.586	.610	.655	.715	.783	.840	.885	.920	.961	.995
1600	.578	.602	.648	.708	.776	.835	.880	.914	.956	.990
1800	.578	.602	.648	.708	.776	.835	.880	.914	.956	.990
2000	.562	.586	.633	.695	.764	.824	.869	.904	.946	.980

The total vertical variable bias error is a combination of the vertical zeroing error and the errors due to occasion-to-occasion variations in temperature, air density and mean range wind. These latter three standard deviations are shown as a function of target range in Table 17. These are then combined with the vertical zeroing error shown in Table 13 to give the total vertical variable bias error, $\sigma_{y_{VB}}(R, R_z)$, which is tabulated in Table 18.

TABLE 17

VARIABLE BIAS ERRORS DUE TO TEMPERATURE, AIR DENSITY, AND RANGE WIND

R (Yards)	Temperature $\sigma_{y_F}(R)$	Air Density $\sigma_{y_D}(R)$	Range Wind $\sigma_{y_W}(R)$
200	.036	.000	.000
400	.054	.000	.000
600	.060	.022	.000
800	.066	.028	.011
1000	.126	.030	.044
1200	.202	.032	.066
1400	.258	.030	.088
1600	.307	.024	.099
1800	.347	.012	.132
2000	.376	.010	.154

TABLE 18

TOTAL VERTICAL VARIABLE BIAS ERROR $\sigma_{y_{VB}}(R, R_z)$

R (Yards)	R_z (Yards)									
	200	400	600	800	1000	1200	1400	1600	1800	2000
200	.361	.381	.420	.471	.609	.809	1.000	1.171	1.332	1.478
400	.371	.375	.399	.438	.565	.761	.948	1.118	1.277	1.422
600	.399	.386	.391	.415	.528	.717	.900	1.067	1.224	1.367
800	.440	.414	.402	.407	.502	.680	.856	1.019	1.174	1.314
1000	.540	.497	.465	.428	.495	.637	.794	.946	1.094	1.229
1200	.692	.635	.589	.555	.550	.635	.756	.888	1.022	1.148
1400	.839	.781	.728	.681	.642	.677	.758	.863	.979	1.091
1600	.977	.916	.859	.806	.747	.745	.790	.868	.964	1.061
1800	1.111	1.048	.987	.930	.858	.887	.845	.897	.972	1.053
2000	1.234	1.170	1.106	1.046	.964	.916	.909	.939	.994	1.058

The horizontal fixed bias error results from the difference in drift between the spotter and major caliber weapons and the horizontal parallax. Since only a spinning projectile drifts, and the M344A1 is fin stabilized, its drift is zero. Also the horizontal parallax is zero since the spotting rifle is mounted directly above the major caliber rifle. The components of fixed bias errors are summarized in Table 19.

TABLE 19
COMPONENTS OF FIXED BIAS

Parallax	
Horizontal	$P_X = 0$
Vertical	$P_Y = 4.71$ inches
Drift	
M344A1	$X_D(r) = 0$
M48A1	$X_d(r) = .00033\ r$ ₥
Angles of Elevation	See Table 3

The total horizontal fixed bias $\overline{X}\ (R,\ R_z)$, which in this case is the same as $\overline{X}_D(R,\ R_z)$, the fixed bias due to drift difference, is shown in Table 20.

The computation of the vertical fixed bias errors are summarized in Table 21. Shown are the error due to trajectory mismatch, parallax, and the total error.

TABLE 20

TOTAL HORIZONTAL FIXED BIAS . $\bar{X}$ (R, R_z)

R (Yards)	R_z (Yards)									
	200	400	600	800	1000	1200	1400	1600	1800	2000
200	.000	.066	.132	.198	.264	.330	.396	.462	.528	.594
400	-.066	.000	.066	.132	.198	.264	.330	.396	.462	.528
600	-.132	-.066	.000	.066	.132	.198	.264	.330	.396	.462
800	-.198	-.132	-.066	.000	.066	.132	.198	.264	.330	.396
1000	-.264	-.198	-.132	-.066	.000	.066	.132	.198	.264	.330
1200	-.330	-.264	-.198	-.132	-.066	.000	.066	.132	.198	.264
1400	-.396	-.330	-.264	-.198	-.132	-.066	.000	.066	.132	.198
1600	-.462	-.396	-.330	-.264	-.198	-.132	-.066	.000	.066	.132
1800	-.528	-.462	-.396	-.330	-.264	-.198	-.132	-.066	.000	.066
2000	-.594	-.528	-.462	-.396	-.330	-.264	-.198	-.132	-.066	.000

TABLE 21

COMPUTATION OF VERTICAL FIXED BIAS ERROR

First Entry in Each Box - Error Due to Trajectory Mismatch $\overline{Y}_{TM}(R, R_z)$

Second Entry in Each Box - Error Due to Parallax $\overline{Y}_P(R, R_z)$

Third Entry in Each Box - Total Vertical Fixed Bias Error $\overline{Y}(R, R_z)$

R (Yards)	R_z (Yards)									
	200	400	600	800	1000	1200	1400	1600	1800	2000
200	.000 .000 .000	.321 -.333 -.012	.530 -.444 .086	.608 -.500 .108	.547 -.533 .014	.388 -.555 -.167	.199 -.571 -.372	.031 -.583 -.552	-.083 -.592 -.675	-.120 -.600 -.720
400	-.321 .333 .012	.000 .000 .000	.209 -.111 .098	.287 -.167 .120	.226 -.200 .026	.067 -.222 -.155	-.122 -.238 -.360	-.290 -.250 -.540	-.404 -.259 -.663	-.441 -.266 -.707
600	-.530 .444 -.086	-.209 .111 -.098	.000 .000 .000	.078 -.056 .022	.017 -.089 -.072	-.142 -.111 -.253	-.331 -.127 -.458	-.499 -.139 -.638	-.613 -.148 -.761	-.650 -.155 -.805
800	-.608 .500 -.108	-.287 .167 -.120	-.078 .056 -.022	.000 .000 .000	-.061 -.033 -.094	-.220 -.056 -.276	-.409 -.071 -.480	-.577 -.083 -.660	-.691 -.092 -.783	-.728 -.100 -.828
1000	-.547 .533 -.014	-.226 .200 -.026	-.017 .089 .072	.061 .033 .094	.000 .000 .000	-.159 -.022 -.181	-.348 -.038 -.386	-.516 -.050 -.566	-.630 -.059 -.689	-.667 -.067 -.734
1200	-.388 .555 .167	-.067 .222 .155	.142 .111 .253	.220 .056 .276	.159 .022 .181	.000 .000 .000	-.189 -.016 -.205	-.357 -.028 -.385	-.471 -.037 -.508	-.508 -.044 -.552
1400	-.199 .571 .372	.122 .238 .360	.331 .127 .458	.409 .071 .480	.348 .038 .386	.189 .016 .205	.000 .000 .000	-.168 -.012 -.180	-.282 -.021 -.303	-.319 -.029 -.348
1600	-.031 .583 .552	.290 .250 .540	.499 .139 .638	.577 .083 .660	.516 .050 .566	.357 .028 .385	.168 .012 .180	.000 .000 .000	-.114 -.009 -.123	-.151 -.017 -.168
1800	.083 .592 .675	.404 .259 .663	.613 .148 .761	.691 .092 .783	.630 .059 .689	.471 .037 .508	.282 .021 .303	.114 .009 .123	.000 .000 .000	-.037 -.007 -.044
2000	.120 .600 .720	.441 .266 .707	.650 .155 .805	.728 .100 .828	.667 .067 .734	.508 .044 .552	.319 .029 .348	.151 .017 .168	.037 .007 .044	.000 .000 .000

Before computations of hit probability can be carried out, the target dimensions must be expressed in mils. Since the target is a square 7 1/2 feet on a side,

$$W(R) = H(R) = \frac{7.5\ (1018.59)}{3R} = \frac{2546.475}{R} \text{ mils.}$$

Table 22 lists W(R) = H(R) as a function of R.

TABLE 22

TARGET DIMENSIONS IN MILS

R (Yards)	H(R) = W(R) (mils)
200	12.732
400	6.366
600	4.244
800	3.183
1000	2.546
1200	2.122
1400	1.819
1600	1.592
1800	1.415
2000	1.273

The random errors listed in Tables 7 and 8, the variable biases listed in Tables 16 and 18, the fixed biases listed in Tables 20 and 21, and the target dimensions in Table 22 are sufficient to compute the first round hit probabilities for both the situation where the spotting round impact is normally distributed and that where it is uniformly distributed. The results of these computations are shown in Figures 1 and 2 for normally distributed spotter and uniformly distributed spotter respectively. As can be seen, the probabilities depend strongly on the zeroing range, the target range, and on the assumption as to the distribution of spotter impact. It is not known which assumption more nearly describes the real situation. It is felt, however, that at short ranges, where the first round hit probability for the spotter is high, the assumption of normal distribution of spotter impact is more realistic. At long range, where the spotter hits the target only after a series of misses and corrections, the distribution of hits would probably be more nearly uniform. Thus if it desired to select a zeroing range which maximizes the range at which the hit probability is at least .75, this selection should probably be made from Figure 1, since the desired range is relatively small (between 500 and 800 yards) and hence the assumption of a normally distributed spotter impact is probably more realistic. Figure 1 indicates a hit probability of .75 or greater out to a range of slightly in excess of 800 yards if zeroed at 500 yards. It can also be seen that this zeroing range results in near-maximum hit probability at all target ranges. The maximum degradation occurs at target ranges beyond 1400 yards, where hit probabilities are unacceptably low regardless of zeroing range. If the system were zeroed so as to maximize hit probability at some long range, say 1800 yards, then the effective range (defined in terms of a .75 hit probability) drops to about 600 yards, an unacceptable loss. As a result, it is recommended that, as a matter of tactical policy, this system be zeroed at 500 yards. Figure 3 shows the first round hit probability as a function of target range for the system zeroed at 500 yards, under both assumptions of the distribution of spotting impact. It is felt that these curves bound the performance of the system when zeroed at 500 yards, with the true performance closer to the solid curve at short range, but approaching the dotted curve at long range.

An interesting phenomenon is apparent in Figures 1 and 2. Namely, that zeroing at a specific range is not the way to maximize the probability of hit at that range. For example, for a target at 100 yards, the optimum zeroing range (from Figure 1) is about 600 yards. The reason for this is as follows: The magnitudes of the fixed bias error and the variable bias error when firing at a target at range of 1000 yards depend on the zeroing range. The fixed bias error at 1000 yards range is minimized by zeroing at 1000 yards, but the variable bias errors at 1000 yards are minimized by zeroing at much shorter ranges (see Tables 16 and 18). Since both types of error influence hit probability, the interplay between the two determines the best zeroing range. The current doctrine used in training soldiers in the use of the BAT system is to zero at 1000 yards. This choice of zeroing range was not based on an analysis such as that presented here, but simply on a minimizing of the maximum fixed bias error over some span of ranges.

Arthur D. Groves

A. D. GROVES

FIGURE I

FIRST ROUND HIT PROBABILITY — 7.5' X 7.5' TARGET

106 mm RECOILLESS RIFLE FIRING M344 AI, USING .50 CALIBER SPOTTER FIRING M48 AI

NORMAL DISTRIBUTION OF SPOTTER IMPACTS

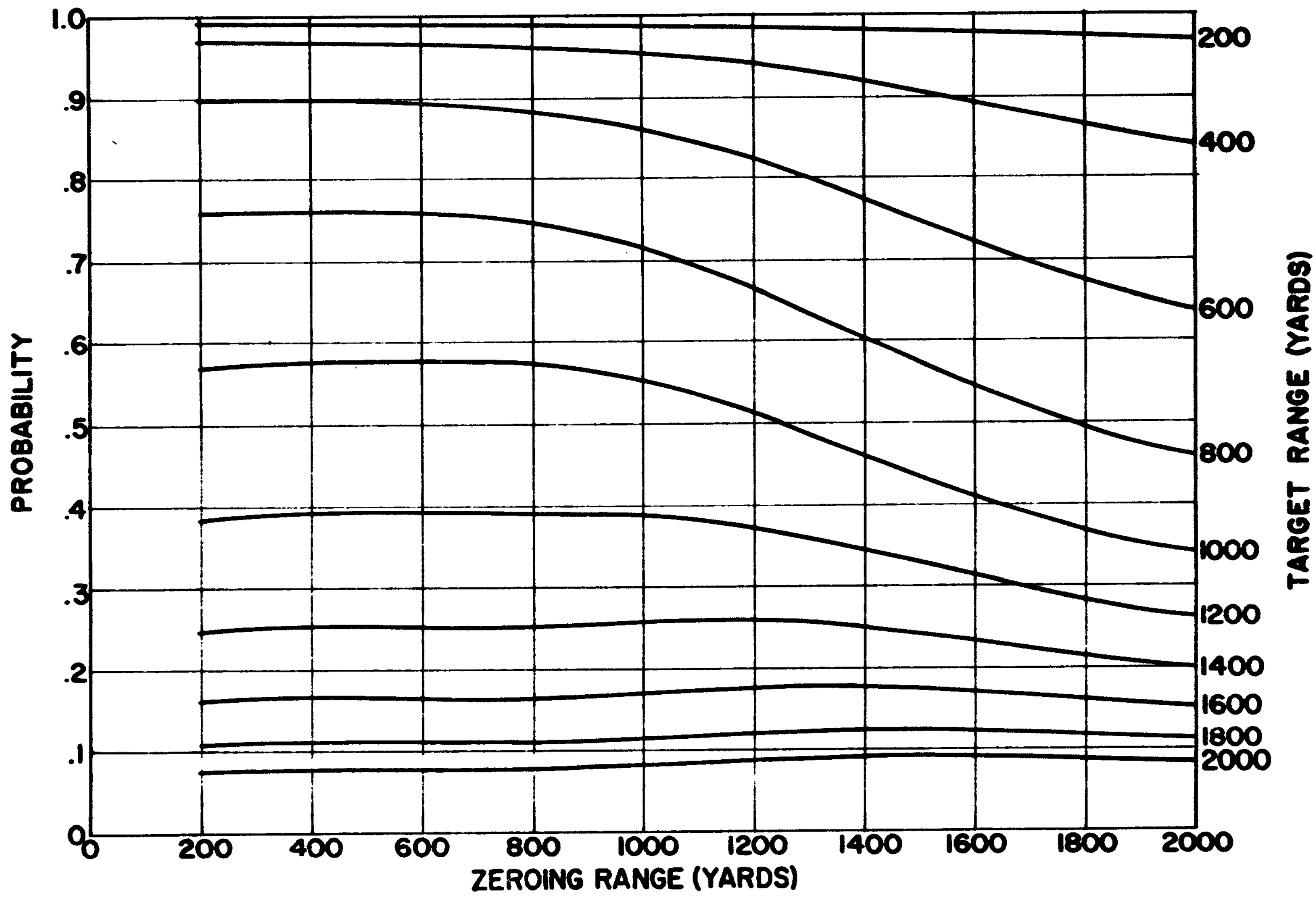


FIGURE 2

FIRST ROUND HIT PROBABILITY — 7.5' X 7.5' TARGET

106 mm RECOILLESS RIFLE FIRING M344A1, USING .50 CALIBER SPOTTER FIRING M48 A1

UNIFORM DISTRIBUTION OF SPOTTER IMPACTS

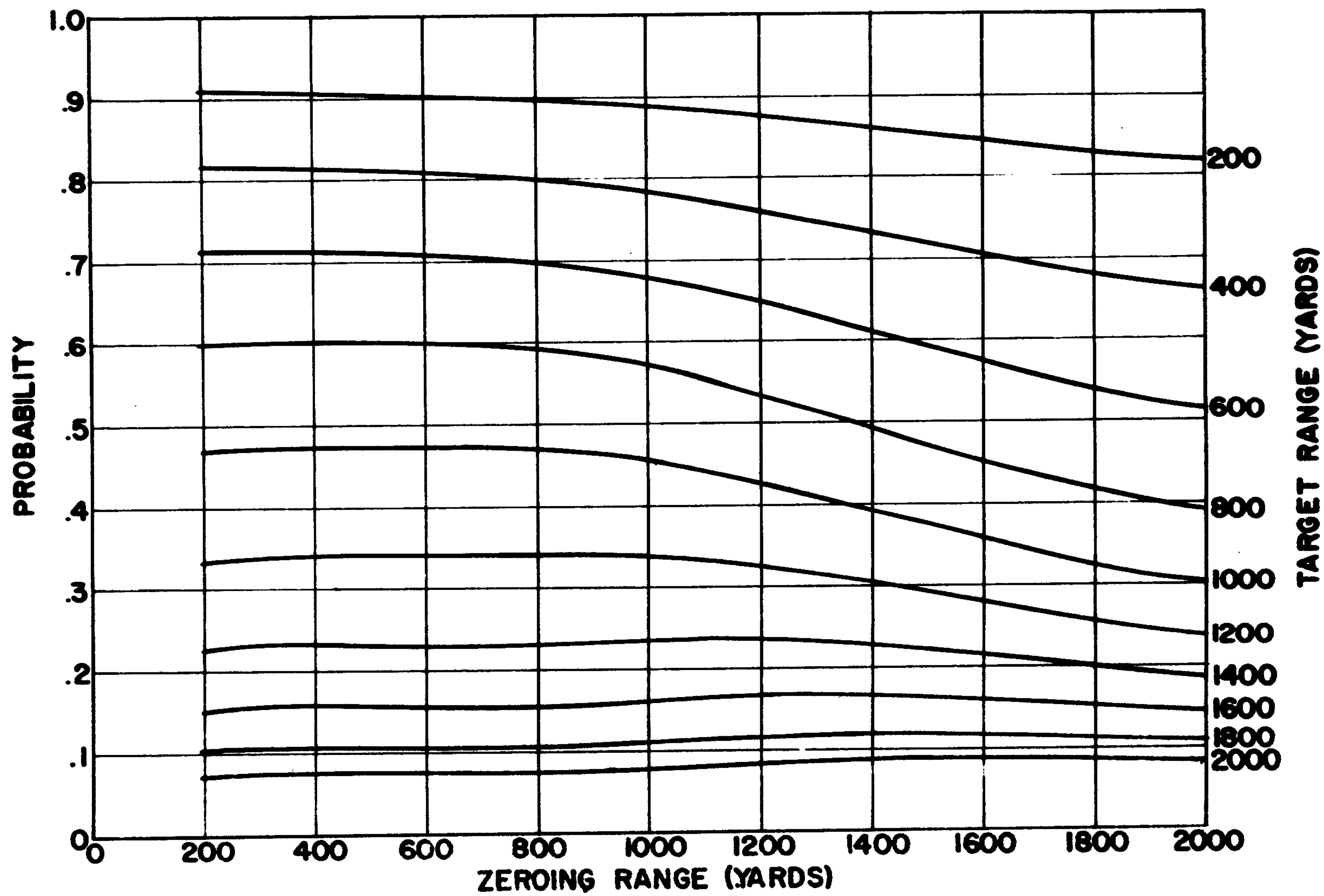
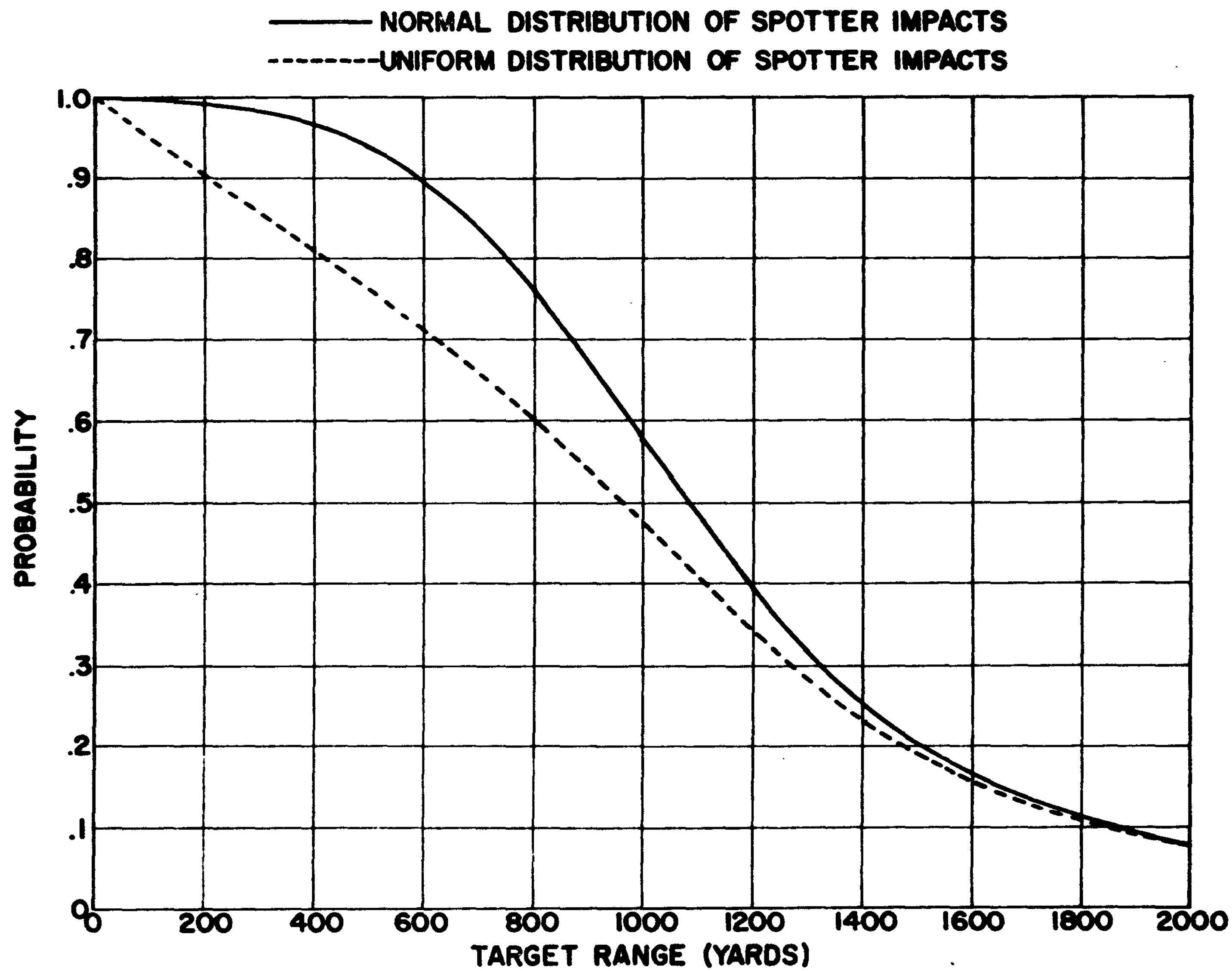


FIGURE 3

FIRST ROUND HIT PROBABILITY — 7.5' X 7.5' TARGET

106 mm RECOILLESS RIFLE FIRING M344A1, USING .50 CALIBER SPOTTER FIRING M48A1

SYSTEM ZEROED AT 500 YARDS



DEFINITION OF BASIC TERMS		
SYMBOL	**UNITS**	**DEFINITION**
A or a	-	Ammunition type - A denotes major Caliber; a denotes spotting ammunition.
N or n	-	Number of major caliber rounds (N) or spotting rounds (n) fired during zeroing.
r	Yards	Range - general; unspecified as to whether it is range to target when zeroing or when firing for effect.
R or R_z	Yards	Range - specific; range to target when firing for effect (R) or when zeroing (R_z).
V or v	feet/second	Nominal muzzle velocity of major caliber ammunition (V) or spotting ammunition (v).
$\Phi(r)$ or $\phi(r)$	Mils	Angle above boresight through which the weapon tube must be elevated when firing at range r for the major caliber weapon $\Phi(r)$ or the spotting weapon $\phi(r)$.
T(r) or t(r)	seconds	Time of flight to target at range r for major caliber weapon T(r) or spotting weapon t(r).
m̸	Mils	Unit of angular measure (1 m̸ = .05625 degrees). The mil is also used as a unit for expressing a linear dimension d at range r by expressing d in terms of the angle it subtends at the zero range point. In particular, a distance of d = 1 yard at a range of r=1018.59 yards, subtends an angle of 1 m̸. Thus to express a distance d in mils, multiply it by $\frac{1018.59}{r}$.

DEFINITION OF TERMS - RANDOM ERRORS

Symbol		Units	Definition
Horizontal	Vertical		
$\sigma_{x'_{cA}}(R_z)$ or $\sigma_{x'_{ca}}(R_z)$	-	Mils	Standard deviation of impact error in zeroing group at range R_z due to round to round cant variation for major caliber (A) or spotter (a).
$\sigma_{x'_{CA}}(R, R_z)$ or $\sigma_{x'_{Ca}}(R)$	-	Mils	Standard deviation of impact error when firing for effect at range R after zeroing at range R_z due to round to round cant variation for major caliber (A) or spotter (a).
$\sigma_{x_{WG}}(r)$ or $\sigma_{x_{wg}}(r)$	$\sigma_{y_{WG}}(r)$ or $\sigma_{y_{wg}}(r)$	Mils	Standard deviation in impact error at range r due to crosswind gustiness (x) or range wind gustiness (y) for the major caliber (WG) or spotter (wg)
-	$\sigma_{y_B}(r)$ or $\sigma_{y_b}(r)$	Mils	Standard deviation in impact error at range r due to round to round variation in ballistic coefficient for major caliber (B) or spotter (b).
-	$\sigma_{y_{VWL}}(r)$ or $\sigma_{y_{vwl}}(r)$	Mils	Standard deviation in impact error at range r due to within-lot muzzle velocity variation for major caliber (VWL) or spotter (vwl)
$\sigma_{x_{RA}}(R_z)$ or $\sigma_{x_{Ra}}(R_z)$	$\sigma_{y_{RA}}(R_z)$ or $\sigma_{y_{Ra}}(R_z)$	Mils	Standard deviation of total random error when zeroing at range R_z for major caliber (A) or spotter (a).
$\sigma_{x_{RA}}(R, R_z)$ or $\sigma_{x_{Ra}}(R)$	$\sigma_{y_{RA}}(R)$ or $\sigma_{y_{Ra}}(R)$	Mils	Standard deviation of total random error when firing for effect at range R after zeroing at range R_z for major caliber (A) or spotter (a).

DEFINITION OF TERMS - COMPONENTS OF RANDOM ERRORS

Symbol		Units	Definition
Horizontal	Vertical		
σ_{WG_x}	σ_{WG_y}	feet/ second	Standard deviation of crosswind gustiness (x) or rangewind gustiness (y).
-	σ_B or σ_b	Percent	Standard deviation in ballistic coefficient variation for major caliber round (B) or for spotter (b).
-	σ_{VWL} or σ_{vwl}	feet/ second	Standard deviation in within lot muzzle velocity variation for major caliber ammunition (VWL) or spotter (vwl).
σ_{j_x} or σ_{J_x}	σ_{j_y} or σ_{J_y}	Mils	Standard deviation of jump for spotter (j) or major caliber weapon (J).
σ_{l_x} or σ_{L_x}	σ_{l_y} or σ_{L_y}	Mils	Standard deviation of aiming error when zeroing (l) or when firing for effect (L).
$\sigma_{c'}$ or $\sigma_{C'}$	-	Mils	Standard deviation of round to round variation in cant when zeroing (c) or when firing for effect (C).

DEFINITION OF TERMS - VARIABLE BIASES

Symbol		Units	Definition
Horizontal	Vertical		
$\sigma_{x_c}(R_z)$ or $\sigma_{x_C}(R, R_z)$	-	Mils	Standard deviation of variable bias error due to occasion to occasion cant variation when zeroing (c) or when firing for effect (C).
$\sigma_{x_w}(R_z)$ or $\sigma_{x_W}(R)$	$\sigma_{y_w}(R_z)$ or $\sigma_{y_W}(R)$	Mils	Standard deviation of variable bias error due to mean crosswind variation (x) or mean range wind variation (y) when zeroing (w) or when firing for effect (W).
-	$\sigma_{y_f}(R_z)$ or $\sigma_{y_F}(R)$	Mils	Standard deviation of variable bias error due to occasion-to-occasion variation in temperature when zeroing (f) or when firing for effect (F).
-	$\sigma_{y_d}(R_z)$ or $\sigma_{y_D}(R)$	Mils	Standard deviation of variable bias error due to occasion-to-occasion variation in air density when zeroing (d) or when firing for effect (D).
-	$\sigma_{y_{vll}}(R, R_z)$ or $\sigma_{y_{VLL}}(R, R_z)$	Mils	Standard deviation of zeroing variable bias error due to lot-to-lot muzzle velocity variation of the spotting ammunition (vll) or the major caliber ammunition (VLL).
$\sigma_{x_z}(R_z)$	$\sigma_{y_z}(R, R_z)$	Mils	Total standard deviation of variable bias error when firing at range R introduced due to zeroing at range R_z.
$\sigma_{x_{VB}}(R, R_z)$	$\sigma_{y_{VB}}(R, R_z)$	Mils	Total standard deviation of variable bias error.

DEFINITION OF TERMS - COMPONENTS OF VARIABLE BIASES

Symbol: Horizontal	Symbol: Vertical	Units	Definition
$\sigma_{M_x}(R_z)$ or $\sigma_{m_x}(R_z)$	$\sigma_{M_y}(R_z)$ or $\sigma_{m_y}(R_z)$	Mils	Standard deviation of the error in assuming the mean of the zeroing shot group fired at range R_z to be equal to the center of impact of the population, for the major caliber weapon (M) or the spotter (m).
σ_{o_x}	σ_{o_y}	Mils	Standard deviation in estimating zeroing group mean from a remote firing position.
σ_c or σ_C	-	Mils	Standard deviation of cant variation from zeroing occasion to zeroing occasion (c) or from occasion-to-occasion when firing for effect.
σ_{W_x} or σ_{w_x}	σ_{W_y} or σ_{w_y}	feet/second	Standard deviation of mean crosswind (x) or mean range wind (y) variation from zeroing occasion to zeroing occasion (w) or from occasion-to-occasion when firing for effect (W).
-	σ_f or σ_F	^oF	Standard deviation of temperature variation from zeroing occasion to zeroing occasion (f) or from occasion to occasion when firing for effect (F).
-	σ_d or σ_D	%	Standard deviation of air density variation from zeroing occasion to zeroing occasion (d) or from occasion to occasion when firing for effect (D).
-	σ_{vll} or σ_{VLL}	feet/second	Standard deviation of lot-to-lot muzzle velocity variation for the spotting ammunition (vll) or the major caliber ammunition (VLL).

DEFINITION OF TERMS - FIXED BIAS ERRORS

Symbol		Units	Definition
Horizontal	Vertical		
$\overline{X}_D(R, R_z)$	-	Mils	Fixed bias error when firing at range R after zeroing at range R_z, resulting from the difference in drift between the spotting round and the major caliber round.
-	$\overline{Y}_{TM}(R, R_z)$	Mils	Fixed bias error when firing at range R after zeroing at range R_z, resulting from ballistic mismatch between the spotting round and the major caliber round.
$\overline{X}_P(R, R_z)$	$\overline{Y}_P(R, R_z)$	Mils	Fixed bias error when firing at range R after zeroing at range R_z, resulting from parallax between bore line of major caliber weapon and boreline of spotting weapon.
$\overline{X}(R, R_z)$	$\overline{Y}(R, R_z)$	Mils	Total fixed bias error when firing at range R after zeroing at range R_z.

DEFINITION OF TERMS - COMPONENTS OF FIXED BIAS ERRORS

Symbol		Units	Definition
Horizontal	Vertical		
$X_D(r)$ or $X_d(r)$	-	Mils	Drift of projectile at range r, for major caliber (D) or spotter (d).
-	$\Phi(r)$ or $\phi(r)$	Mils	Angle of elevation of major caliber launcher (Φ) or spotting round launcher (ϕ) when firing at range r.
P_X	P_Y	Inches	Parallax - the distance between the bore lines of the spotting weapon and the major caliber weapon.

DEFINITION OF TERMS - UNIT DIFFERENTIAL EFFECTS

Symbol	Units	Definition
$\left(\frac{\Delta y}{\Delta W}\right)_r$ or $\left(\frac{\delta y}{\delta w}\right)_r$	mils per foot/second	Vertical change in impact point at range r corresponding to a 1 foot/second change in range wind for major caliber weapon (Δ) or spotter weapon (δ).
$\left(\frac{\Delta y}{\Delta B}\right)_r$ or $\left(\frac{\delta v}{\delta b}\right)_r$	mils per percent	Vertical change in impact point at range r corresponding to a 1% change in ballistic coefficient for major caliber weapon (Δ) or spotting weapon (δ)
$\left(\frac{\Delta y}{\Delta V}\right)_r$ or $\left(\frac{\delta y}{\delta v}\right)_r$	mils per foot/second	Vertical change in impact point at range r corresponding to a 1 foot/second change in muzzle velocity for major caliber (Δ) or spotting weapon (δ)
$\left(\frac{\Delta y}{\Delta D}\right)_r$ or $\left(\frac{\delta y}{\delta d}\right)_r$	mils per %	Vertical change in impact point at range r corresponding to a 1% change in air density for the major caliber weapon (Δ) or the spotting weapon (δ).
$\frac{\Delta V}{\Delta F}$ or $\frac{\delta v}{\delta f}$	feet/second per ^{o}F	Change in muzzle velocity resulting from a $1^{o}F$ change in propellant temperature for the major caliber ammunition (Δ) or for the spotter (δ).

REFERENCES

1. D&PS/TS4-4020/25 "Engineering Test of Cartridge, Spotter-Tracer, Caliber .50, T189E3 for Use With The 106mm BAT Weapon System", Development and Proof Services, September 1958.
2. 1st Ind to Basic Ltr No. APG-474.1/278 (1956), Subj: Analysis of Spotting Rifle Ranging in the BAT Weapon System - TB3-1224" Ind prepared by BRL.
3. BRL Memorandum Report 636 "On Estimating Probabilities of Hitting for the Battalion Anti-Tank Weapon" Helen J. Coon and Frank E. Grubbs, 1953 (CONFIDENTIAL)
4. FA-R1380A "Fire Control Studies - Tank Gunnery Accuracy Evaluation" Frankford Arsenal 1958 (CONFIDENTIAL)
5. Surveillance Branch, WSL, BRL, Tests of 125 lots of Iowa Ordnance Plant Ammunition M344A1

DISTRIBUTION LIST

No. of Copies	Organization
10	Commander Armed Services Technical Information Agency ATTN: TIPCR Arlington Hall Station Arlington 12, Virginia
1	Director IDA/Weapons System Evaluation Group The Pentagon Washington 25, D. C.
3	Commanding General U. S. Army Materiel Command ATTN: AMCRD-RS-PE-Bal Research and Development Directorate Washington 25, D. C.
1	Commanding Officer Harry Diamond Laboratories ATTN: Technical Information Office Branch 012 Washington 25, D. C.
1	Research Analysis Corporation ATTN: Document Control Office 6935 Arlington Road Bethesda, Maryland Washington 14, D. C.
1	Commanding Officer U. S. Army Research Office (Durham) Box CM, Duke Station Durham, North Carolina
1	Commanding General Frankford Arsenal Philadelphia 37, Pennsylvania
1	Commanding General U. S. Army Mobility Command 28251 Van Dyke Avenue Centerline, Michigan

No. of Copies	Organization
1	Commanding Officer Picatinny Arsenal Dover, New Jersey
1	Commanding Officer Watervliet Arsenal Watervliet, New York
1	Commanding Officer Watertown Arsenal Watertown 72, Massachusetts
1	Commanding General U. S. Army Weapons Command Rock Island, Illinois
1	Commanding General Ammunition Command Joliet, Illinois
2	Commanding General U. S. Continental Army Command Fort Monroe, Virginia
1	Commanding General U. S. Army Combat Developments Command Fort Belvoir, Virginia
1	President U. S. Army Armor Board Fort Knox, Kentucky
2	President U. S. Army Infantry Board Fort Benning, Georgia
1	Commandant The Armored School Fort Knox, Kentucky
2	Commandant The Infantry School Fort Benning, Georgia

DISTRIBUTION LIST

No. of Copies	Organization
1	Commanding General U. S. Army Combat Development and Experimentation Center Fort Ord, California
1	Commanding Officer U. S. Army Infantry CD Agency Fort Benning, Georgia
1	Commanding Officer U. S. Army Armor CD Agency Fort Knox, Kentucky
1	Deputy Chief of Staff for Logistics Department of the Army Washington 25, D. C.
1	Deputy Chief of Staff for Military Operations Department of the Army Washington 25, D. C.
2	Chief, Research and Development Department of the Army Washington 25, D. C.

No. of Copies	Organization
1	Commander U. S. Naval Ordnance Test Station China Lake, California
1	Commander U. S. Naval Ordnance Laboratory White Oak Silver Spring 19, Maryland
1	Commandant U. S. Marine Corps ATTN: Ord. Section Washington 25, D. C.
10	The Scientific Information Officer Defence Research Staff British Embassy 3100 Massachusetts Avenue, N. W. Washington 8, D. C.
4	Defence Research Member Canadian Joint Staff 2450 Massachusetts Avenue, N. W. Washington 8, D. C.

AD ______ Accession No. ______ UNCLASSIFIED

Ballistic Research Laboratories, APG
METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL
A. D. Groves

Spotting rifle - Effectiveness
Antitank guns - Fire control

BRL Memorandum Report No. 1450 January 1963

DA Project No. 503-05-005
UNCLASSIFIED Report

An extensive and detailed analysis is made of the factors which contribute to the accuracy of a spotting rifle controlled antitank weapon. The manner in which each source of error contributes to the total error is discussed, and two methods of computing quasi-combat first-round hit probability are presented. The entire method is illustrated by a complete determination of the hitting potential of the 106mm recoilless rifle (M40) using the .50 caliber spotter (M8) under an assumed quasi-combat environment.

AD ______ Accession No. ______ UNCLASSIFIED

Ballistic Research Laboratories, APG
METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL
A. D. Groves

Spotting rifle - Effectiveness
Antitank guns - Fire control

BRL Memorandum Report No. 1450 January 1963

DA Project No. 503-05-005
UNCLASSIFIED Report

An extensive and detailed analysis is made of the factors which contribute to the accuracy of a spotting rifle controlled antitank weapon. The manner in which each source of error contributes to the total error is discussed, and two methods of computing quasi-combat first-round hit probability are presented. The entire method is illustrated by a complete determination of the hitting potential of the 106mm recoilless rifle (M40) using the .50 caliber spotter (M8) under an assumed quasi-combat environment.

AD ______ Accession No. ______ UNCLASSIFIED

Ballistic Research Laboratories, APG
METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL
A. D. Groves

Spotting rifle - Effectiveness
Antitank guns - Fire control

BRL Memorandum Report No. 1450 January 1963

DA Project No. 503-05-005
UNCLASSIFIED Report

An extensive and detailed analysis is made of the factors which contribute to the accuracy of a spotting rifle controlled antitank weapon. The manner in which each source of error contributes to the total error is discussed, and two methods of computing quasi-combat first-round hit probability are presented. The entire method is illustrated by a complete determination of the hitting potential of the 106mm recoilless rifle (M40) using the .50 caliber spotter (M8) under an assumed quasi-combat environment.

AD ______ Accession No. ______ UNCLASSIFIED

Ballistic Research Laboratories, APG
METHOD FOR COMPUTING THE FIRST-ROUND HIT PROBABILITY FOR AN ANTITANK WEAPON WITH SPOTTING RIFLE CONTROL
A. D. Groves

Spotting rifle - Effectiveness
Antitank guns - Fire control

BRL Memorandum Report No. 1450 January 1963

DA Project No. 503-05-005
UNCLASSIFIED Report

An extensive and detailed analysis is made of the factors which contribute to the accuracy of a spotting rifle controlled antitank weapon. The manner in which each source of error contributes to the total error is discussed, and two methods of computing quasi-combat first-round hit probability are presented. The entire method is illustrated by a complete determination of the hitting potential of the 106mm recoilless rifle (M40) using the .50 caliber spotter (M8) under an assumed quasi-combat environment.