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DETERMINATION OF THE STREAMLINES ON A SPHERE-CONE AT ANGLE OF ATTACK FROM THE MEASURED SURFACE PRESSURE DISTRIBUTION

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UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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DETERMINATION OF THE STREAMLINES ON A SPHERE-CONE AT ANGLE OF ATTACK FROM THE MEASURED SURFACE PRESSURE DISTRIBUTION

by

E. L. HARRIS

ABSTRACT: A method is given for computing the inviscid fluid streamlines on a sphere-cone at an angle of attack in supersonic flow from the measured surface pressure distribution. The boundary layer was assumed to be negligibly thin. The necessary equations are derived and put in a form suitable for programming on a digital computer.

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Determination of the Streamlines on a Sphere-Cone at Angle of Attack from the Measured Surface Pressure Distribution

This report presents a numerical method for calculating the inviscid surface streamline distribution over a sphere-cone at an angle of attack in a supersonic flow from the measured pressure distribution. The method can be easily adapted to any blunt-nosed body of revolution.

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By direction

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Figure 1 Sphere-Cone Geometry: (x,y,z) and (s,ψ,σ) Coordinate Systems

Figure 2 Sphere-Cone Geometry: (x_o,y_o,z_o) and (s_o,ψ_o,σ_o) Coordinate Systems, and Initial Circle

SYMBOLS

Roman

$A_{i/j}$	covariant derivative of the vector A_i
a_{ij}	metric tensor, equation (8)
b_{ij}	metric tensor, equation (40)
B_i	function of coordinates and surface streamline direction, equation (30)
$d\ell$	infinitesimal distance
h	enthalpy per unit mass, $h = U + p/\rho$
p	pressure
q	magnitude of the velocity vector
R	radius of spherical nose
s	distance along body from geometric stagnation point
S	entropy per unit mass
T	temperature
U	internal energy per unit mass
u^k	velocity vector
V_∞	free-stream velocity
W	function of coordinates, equation (50)
x^i	curvilinear coordinate system (s, ψ, σ)
X^i	curvilinear coordinate system (s_o, ψ_o, σ_o)
y^i	rectangular coordinate system (x, y, z)
Y^i	rectangular coordinate system (x_o, y_o, z_o)

Greek

α	angle of attack
β	cone semi-angle
Γ^i	initial unit tangent vector to the streamline

ϵ	semi-angle subtended by the initial circle at the sphere center
η	related to direction cosine of streamline, equation (47)
λ^i	unit tangent vector of surface streamline
ξ	related to direction cosine of streamline, equation (47)
ρ	density
σ	distance normal to body
τ	distance measured along streamline from aerodynamic stagnation point
ψ	roll angle of the body
ω	function of coordinates, equation (7)

Subscripts

i,j,k,l,m,n	covariant vector
c	value of variable at sphere-cone junction

Superscripts

i,j,k,l,m,n	contravariant vector
*	value of variable on initial circle

COORDINATE SYSTEMS

x,y,z	rectangular Cartesian, see figure 1
x_o, y_o, z_o	rectangular Cartesian, see figure 2
(s, ψ, σ)	curvilinear, see figure 1
(s_o, ψ_o, σ_o)	curvilinear, see figure 2

INTRODUCTION

In order to perform a boundary-layer calculation on any body, the velocity (both magnitude and direction) and the pressure just outside the boundary layer must be known. In other words, the streamlines at the boundary-layer edge must be known. In this report these streamlines shall be referred to as surface streamlines since the boundary-layer thickness is assumed to be very small. In a two-dimensional flow, the surface streamlines are sometimes known from symmetry. They are known, for example, on an axisymmetric body in a flow at zero angle of attack. In a general three-dimensional flow, however, they can only be obtained from an integration of the momentum equations. This report presents a method of finding these inviscid surface streamlines from an experimental surface pressure distribution on a sphere-cone in a supersonic flow at angle of attack. The following assumptions are made:

a. The aerodynamic stagnation point is known from symmetry considerations and is the most forward point of the sphere viewed from the oncoming flow (see fig. 1).

b. If a normal to the surface is drawn passing through this stagnation point, there is some small region near this point where the flow is axisymmetric if we regard this normal as the axis.

c. Viscosity and heat conduction are neglected and there are no shocks on the body surface. Thus, the flow over the surface is isentropic and the magnitude of the velocity depends only on the ratio of static pressure to the stagnation pressure behind the normal shock at the nose. It should be noted, although we do not consider it further here, that the streamlines in the boundary layer of a three-dimensional flow are not parallel to the surface streamlines. This results from the fact that the velocity of the fluid in the boundary layer is lower than the external velocity and, hence, the centrifugal forces may be quite different in the two cases.

A set of four simultaneous differential equations is derived from the inviscid momentum equation. The dependent variables are two coordinates on the body surface and the two direction cosines to the streamline. The independent variable is the distance measured along the streamline from the aerodynamic stagnation point from which all streamlines emanate. Initial conditions for the set of equations are obtained from assumption (b) which gives the fluid velocity and the streamline direction in a small region near the stagnation point. If the experimental pressure distribution is tabulated or analytical curve fits are provided so that at any point on the body the pressure gradient and the magnitude of the fluid velocity can be calculated, then the set of four differential equations is in a convenient form for numerical integration on a digital computer.

COORDINATE SYSTEMS

Four orthogonal coordinate systems are used. They are the (x,y,z), (s,ψ,σ), (x_o,y_o,z_o), and (s_o,ψ_o,σ_o) systems. Only the first two are used in the differential equations to be developed, while the last two are used in the specification of the initial conditions for these equations. In some of the equations to follow, the four coordinate systems will be designated by the symbols y^k, x^k, Y^k, X^k, respectively (k = 1, 2, 3). That is,

$$
\begin{aligned}
&y^1 = x \;; && y^2 = y \;; && y^3 = z \\
&x^1 = s && x^2 = \psi && x^3 = \sigma \\
&Y^1 = x_o && Y^2 = y_o && Y^3 = z_o \\
&X^1 = s_o && X^2 = \psi_o && X^3 = \sigma_o
\end{aligned}
\qquad (1)
$$

Tensor notation and the methods of tensor calculus will be used when it is convenient to do so. Reference (a) gives a good treatment of this subject. Letters which have the superscript (i,j,k,l,m,n) are contravariant vectors, while letters with (i....n) as subscripts are covariant vectors. The physical component of a vector is subscripted with a letter which identifies it with a coordinate axis in the (x,y,z), (s,ψ,σ), (x_o,y_o,z_o), or (s_o,ψ_o,σ_o) systems.

<u>The (x,y,z) and (s,ψ,σ) Coordinate Systems</u>. The geometry is shown in figure 1. The coordinates (x,y,z) are right-handed Cartesian with origin at the sphere center. z is in the downstream axial direction, y points down, and the free-stream velocity vector is in the y-z plane. The point of tangency of the sphere and cone is given by the angle β as shown.

Equation of sphere:

$$(x)^2 + (y)^2 + (z)^2 = R^2 \;, \quad z \leq -R \sin\beta \qquad (2)$$

Equation of cone:

$$(x)^2 + (y)^2 = \left[\tan\beta \cdot \left(z + \frac{R}{\sin\beta}\right)\right]^2 , \quad z \geq R \sin\beta \qquad (3)$$

The (s, ψ, σ) coordinate system is defined as follows. For a general point P as shown in figure 1, σ is the perpendicular distance to the body surface, s is distance measured from the geometric stagnation point A along the body in a meridian plane, and ψ is the angle the meridian plane makes with the y-z plane. The meridian plane contains the z-axis for all ψ ; $\psi = 0$ is the y-z plane, and $\psi = \pi/2$ is the x-z plane.

For a point P ahead of the point of tangency of the sphere cone, that is, in region I, we have

$$s = R \cos^{-1}\left[\frac{-z}{\sqrt{x^2+y^2+z^2}}\right]$$

$$\psi = \tan^{-1}(x/y) \tag{4}$$

$$\sigma = \sqrt{x^2+y^2+z^2} - R$$

and the inverse relations

$$x = (\sigma + R)\cdot \sin\frac{s}{R} \cdot \sin\psi$$

$$y = (\sigma + R)\cdot \sin\frac{s}{R} \cdot \cos\psi \tag{5}$$

$$z = -(\sigma + R)\cdot \cos\frac{s}{R}$$

In region II we have

$$s = s_c + z\cos\beta + \sqrt{x^2+y^2}\cdot\sin\beta$$

$$\psi = \tan^{-1}(x/y) \tag{6}$$

$$\sigma = \sqrt{x^2+y^2}\cdot\cos\beta - R - z\sin\beta$$

where $s_c = R\,(\pi/2 - \beta)$ and the inverse relations

$$x = \omega \cdot \sin\psi$$
$$y = \omega \cdot \cos\psi \tag{7}$$
$$z = (s-s_c)\cdot\cos\beta - (\sigma+R)\sin\beta$$

where $\omega = (s-s_c)\sin\beta + (\sigma + R)\cos\beta$.

The expressions for the metric (that is, definition of distance) in the two regions are

$$(d\ell)^2 = a_{ij}\, dx^i\, dx^j$$
$$= a_{11}\, ds^2 + a_{22}\, d\psi^2 + a_{33}\, d\sigma^2$$

since the coordinate system is orthogonal, where,

Region I: $$a_{11} = \left(\frac{\sigma+R}{R}\right)^2 \;;\; a_{22} = (\sigma+R)^2 \sin^2\frac{s}{R} \;;\; a_{33} = 1 \tag{8}$$

Region II: $$a_{11} = 1 \;;\; a_{22} = \omega^2 \;;\; a_{33} = 1$$

<u>The (x_o, y_o, z_o) and (s_o, ψ_o, σ_o) Coordinate Systems.</u> The geometry is shown in figure 2. The (x_o, y_o, z_o) system is formed by rotating the (x,y,z) system through an angle α about the x-axis. We obtain

$$x_o = x$$
$$y_o = y\cos\alpha + z\sin\alpha \tag{9}$$
$$z_o = -y\sin\alpha + z\cos\alpha$$

and the inverse relations

$$x = x_o$$
$$y = y_o\cos\alpha - z_o\sin\alpha \tag{10}$$
$$z = y_o\sin\alpha + z_o\cos\alpha$$

The (s_o, ψ_o, σ_o) system bears precisely the same relation to the (x_o, y_o, z_o) system as the (s, ψ, σ) system does to the (x,y,z). The reason for introducing the (s_o, ψ_o, σ_o) system is to specify the initial conditions for any streamline originating from a line s_o = constant, σ_o = 0 for values of ψ_o from 0 to π. We have

$$\begin{aligned} s_o &= R\cos^{-1}\left[\frac{-z_o}{\sqrt{x_o^2+y_o^2+z_o^2}}\right] \\ \psi_o &= \tan^{-1}(x_o/y_o) \\ \sigma_o &= \sqrt{x_o^2+y_o^2+z_o^2} - R \end{aligned} \tag{11}$$

and the inverse relations

$$\begin{aligned} x_o &= (\sigma_o+R)\cdot\sin\frac{s_o}{R}\cdot\sin\psi_o \\ y_o &= (\sigma_o+R)\cdot\sin\frac{s_o}{R}\cdot\cos\psi_o \\ z_o &= -(\sigma_o+R)\cdot\cos\frac{s_o}{R} \end{aligned} \tag{12}$$

Transformation from the (s,ψ,σ) to the $s_o,\psi_o,\sigma_o)$ System. It will be necessary later to have formulae relating the x^k and X^k systems directly. From equations (11), (9), and (5) we obtain

$$\begin{aligned} s_o &= R\cos^{-1}\left[\sin\frac{s}{R}\cdot\cos\psi\cdot\sin\alpha+\cos\frac{s}{R}\cdot\cos\alpha\right] \\ \psi_o &= \tan^{-1}\left[\frac{\sin\frac{s}{R}\cdot\sin\psi}{\sin\frac{s}{R}\cdot\cos\psi\cdot\cos\alpha-\cos\frac{s}{R}\cdot\sin\alpha}\right] \\ \sigma_o &= \sigma \end{aligned} \tag{13}$$

Combination of equations (12), (10), and (4) yields the inverse relations

$$\begin{aligned} s &= R\cos^{-1}\left[-\sin\frac{s_o}{R}\cdot\cos\psi_o\cdot\sin\alpha+\cos\frac{s_o}{R}\cdot\cos\alpha\right] \\ \psi &= \tan^{-1}\left[\frac{\sin\frac{s_o}{R}\cdot\sin\psi_o}{\sin\frac{s_o}{R}\cdot\cos\psi_o\cdot\cos\alpha+\cos\frac{s_o}{R}\cdot\sin\alpha}\right] \\ \sigma &= \sigma_o \end{aligned} \tag{14}$$

THE CONDITION OF CONSTANT ENTROPY ON THE BODY SURFACE

Since the thickness of the boundary layer is taken to be negligibly small, the surface streamlines as previously defined lie on the body surface. In assumption (c) of the Introduction, it was stated that the flow over the body surface was isentropic. This statement will now be given a rigorous basis, and a relation will be derived between the surface pressure gradient and velocity gradient.

As a particle moves along a streamline and if there are no discontinuities (shocks), then the entropy change between two neighboring positions of the particle is given by the usual thermodynamic relation

$$\begin{aligned} T\,dS &= dU + p\,d\left(\frac{1}{\rho}\right) \\ &= dh - \frac{dp}{\rho} \end{aligned} \tag{15}$$

If there is no heat conduction to the particle, then an energy balance requires

$$h + \frac{q^2}{2} = \text{constant} \tag{16a}$$

or

$$dh = -q \cdot dq \tag{16b}$$

Hence, from equations (15) and (16b)

$$T \cdot dS = -q \cdot dq - \frac{dp}{\rho} \tag{17}$$

Now the fluid dynamic inviscid momentum equation may be written

$$\operatorname{curl} \vec{q} \times \vec{q} + \frac{1}{2} \operatorname{grad} \vec{q}^{\,2} = -\frac{1}{\rho} \operatorname{grad} p \tag{18}$$

If we perform a scalar multiplication of each side of this equation by an infinitesimal displacement $\overrightarrow{dx}$ along the streamline, we obtain

$$\frac{1}{2}\, d(\vec{q}^{\,2}) = -\frac{1}{\rho}\, dp \qquad \text{along a streamline} \tag{19}$$

since the vector curl $\vec{q} \times \vec{q}$ is normal to the streamline. Combination of equations (17) and (19) gives

$$T \cdot dS = 0 \qquad \text{along a streamline} \tag{20}$$

Note that there are two assumptions contained in equations (19) and (20) - the streamline does not pass through a shock, and viscosity and heat conduction are negligible.

Equations (19) and (20) may be applied to the present problem of the surface streamlines on a sphere-cone as follows. Since all the surface streamlines emanate from the stagnation point of the sphere-cone, and since the entropy is constant on each streamline, therefore the complete surface of the body is one of constant entropy. Hence, equation (19) holds for any direction tangent to the body and may be written

$$q \frac{\partial q}{\partial x^{k}} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x^{k}} \tag{21}$$

Note that equation (21) is only valid on the body surface and in a direction tangent to the surface. Note also that equation (20) allows the use of the usual isentropic relations for the determination of the density, Mach number, and velocity from the surface pressure at any point on the body surface.

THE DIFFERENTIAL EQUATIONS FOR THE BODY STREAMLINES

Let λ^k be a unit vector along the streamline. Then

$$u^{k} = q \lambda^{k} \tag{22}$$

where q is the magnitude of the fluid velocity and u^k is the contravariant vector velocity. Let γ be distance measured along the streamline (take $\gamma = 0$ at the aerodynamic stagnation point). By definition

$$\frac{dx^{k}}{d\gamma} = \lambda^{k} \tag{23}$$

along the streamline where x^k (k = 1, 2, 3) refer to the coordinates s, ψ, σ, respectively. λ^i and λ_i have the property

$$\lambda^{1} = \lambda_{1}/a_{11} \; ; \; \lambda^{2} = \lambda_{2}/a_{22} \; ; \; \lambda^{3} = \lambda_{3}/a_{33} \tag{24}$$

The inviscid momentum equation is

$$u^{\kappa} u_{i/\kappa} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x^{i}} \tag{25}$$

where the symbol (/) indicates covariant differentiation (see ref. (a)). From equations (22) and (25)

$$g\lambda^{\kappa}(g\lambda_i)_{/\kappa} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x^{i}}$$

or

$$g\lambda^{\kappa}\left[g\lambda_{i/\kappa} + \lambda_i \frac{\partial g}{\partial x^{\kappa}}\right] = -\frac{1}{\rho}\frac{\partial p}{\partial x^{i}} \tag{26}$$

Now if A_i is any vector, then (ref. (a))

$$A_{i/j} = \frac{\partial A_i}{\partial x^{j}} - \left\{ {\ell \atop i\,j} \right\} A_{\ell} \tag{27}$$

where $\left\{ {\ell \atop i\,j} \right\}$ are the Christoffel symbols referred to the metric tensor a_{ij}. If we use equation (27) in equation (26), there results

$$g^2\lambda^{\kappa}\left[\frac{\partial \lambda_i}{\partial x^{\kappa}} - \left\{ {\ell \atop i\,\kappa} \right\}\lambda_{\ell}\right] = -\frac{1}{\rho}\frac{\partial p}{\partial x^{i}} - g\,\lambda_i\lambda^{\kappa}\frac{\partial g}{\partial x^{\kappa}} \;. \tag{28}$$

Substituting equation (21) in equation (28) and restricting our consideration to the surface streamlines gives

$$\frac{d\lambda_i}{d\tau} = \lambda^{\kappa}\lambda_{\ell}\left\{ {\ell \atop i\,\kappa} \right\} - \frac{1}{\rho g^2}\left[\frac{\partial p}{\partial x^{i}} - \lambda_i\lambda^{\kappa}\frac{\partial p}{\partial x^{\kappa}}\right] . \tag{29}$$

Equations (23) and (29) represent six equations for λ_1, λ_2, λ_3, s, ψ, σ. Only four are necessary since by definition $\lambda_3 = \sigma = 0$; that is, the streamline is on the body surface. We define

$$B_i = \lambda^{\kappa}\lambda_{\ell}\left\{ {\ell \atop i\,\kappa} \right\} \tag{30}$$

and write out the four equations. They are

$$\frac{ds}{d\gamma} = \lambda^1$$
$$\frac{d\psi}{d\gamma} = \lambda^2$$
$$\frac{d\lambda_1}{d\gamma} = B_1 - \frac{1}{\rho q^2}\left[\frac{\partial p}{\partial s} - \lambda_1\left\{\lambda^1\frac{\partial p}{\partial s} + \lambda^2\frac{\partial p}{\partial \psi}\right\}\right] \quad (31)$$
$$\frac{d\lambda_2}{d\gamma} = B_2 - \frac{1}{\rho q^2}\left[\frac{\partial p}{\partial \psi} - \lambda_2\left\{\lambda^1\frac{\partial p}{\partial s} + \lambda^2\frac{\partial p}{\partial \psi}\right\}\right].$$

B_1 and B_2 are functions of the coordinates only and are given below. ρ and q may be found from the pressure. Consequently, equations (31) along with equation (24) and an experimentally determined pressure distribution form a sufficient set of equations for the determination of the streamlines. That is, solution of equation (31) gives s and ψ as a function of γ along the streamline.

Calculation of B_1. Below is given a list of the Christoffel symbols in regions I and II, some of which are necessary for the calculation of B_1 and B_2. The formulae used may be found in reference (a), page 82.

In region I:

$$\left\{{1 \atop 11}\right\} = 0 \qquad \left\{{1 \atop 22}\right\} = -R\sin\frac{s}{R}\cdot\cos\frac{s}{R} \qquad \left\{{1 \atop 33}\right\} = 0$$

$$\left\{{1 \atop 12}\right\} = 0 \qquad \left\{{1 \atop 13}\right\} = \frac{1}{\sigma+R} \qquad \left\{{1 \atop 23}\right\} = 0$$

$$\left\{{2 \atop 11}\right\} = 0 \qquad \left\{{2 \atop 22}\right\} = 0 \qquad \left\{{2 \atop 33}\right\} = 0$$

$$\left\{{2 \atop 12}\right\} = \frac{1}{R}\cot\frac{s}{R} \qquad \left\{{2 \atop 13}\right\} = 0 \qquad \left\{{2 \atop 23}\right\} = \frac{1}{\sigma+R} \quad (32)$$

$$\left\{{3 \atop 11}\right\} = -\frac{\sigma+R}{R^2} \qquad \left\{{3 \atop 22}\right\} = -(\sigma+R)\sin^2\frac{s}{R} \qquad \left\{{3 \atop 33}\right\} = 0$$

$$\left\{{3 \atop 12}\right\} = 0 \qquad \left\{{3 \atop 13}\right\} = 0 \qquad \left\{{3 \atop 23}\right\} = 0$$

In region II:

$$
\begin{aligned}
&\left\{ {1 \atop 11} \right\} = 0 && \left\{ {1 \atop 22} \right\} = -\omega \sin\beta && \left\{ {1 \atop 33} \right\} = 0 \\
&\left\{ {1 \atop 12} \right\} = 0 && \left\{ {1 \atop 13} \right\} = 0 && \left\{ {1 \atop 23} \right\} = 0 \\
&\left\{ {2 \atop 11} \right\} = 0 && \left\{ {2 \atop 22} \right\} = 0 && \left\{ {2 \atop 33} \right\} = 0 \\
&\left\{ {2 \atop 12} \right\} = \frac{\sin\beta}{\omega} && \left\{ {2 \atop 13} \right\} = 0 && \left\{ {2 \atop 23} \right\} = \frac{\cos\beta}{\omega} \\
&\left\{ {3 \atop 11} \right\} = 0 && \left\{ {3 \atop 22} \right\} = -\omega \cos\beta && \left\{ {3 \atop 33} \right\} = 0 \\
&\left\{ {3 \atop 12} \right\} = 0 && \left\{ {3 \atop 13} \right\} = 0 && \left\{ {3 \atop 23} \right\} = 0
\end{aligned}
\tag{33}
$$

By expanding the expression on the right of equation (30) and using the fact that $\lambda_3 = \lambda^3 = 0$ we obtain the following simple expressions for B_1 and B_2.

In region I:

$$B_1 = \frac{\lambda^2 \lambda_2}{R} \cdot \cot \frac{s}{R}$$

$$B_2 = 0 \tag{34}$$

In region II:

$$B_1 = \lambda^2 \lambda_2 \, \frac{\sin\beta}{\omega}$$

$$B_2 = 0 \tag{35}$$

INITIAL CONDITIONS FOR THE STREAMLINE INTEGRATION

We construct an "initial circle" on the sphere as shown in figure 2 by cutting the sphere with a plane which is normal to the free-stream velocity. Let us call Γ^i the unit tangent vector to the streamline in the (s_o, ψ_o, σ_o) system on the initial circle. From assumption (b) of the Introduction

$$\Gamma_{s_o} = 1 \quad ; \quad \Gamma_{\psi_o} = 0 \quad ; \quad \Gamma_{\sigma_o} = 0 \tag{36}$$

Equation (36) gives the physical components of the unit vector Γ^i. According to the rule for transforming vectors from one coordinate system to another (ref. (a))

$$\lambda^i = \frac{\partial x^i}{\partial X^m} \cdot \Gamma^m \tag{37}$$

When equation (36) is used in equation (37)

$$\lambda^i = \frac{\partial x^i}{\partial X^1} \cdot \Gamma^1 \tag{38}$$

Now the metric in the X^k system is

$$\begin{aligned} (d\ell)^2 &= b_{ij} \cdot dX^i \cdot dX^j \\ &= b_{11}(dX^1)^2 + b_{22}(dX^2)^2 + b_{33}(dX^3)^2 \end{aligned} \tag{39}$$

where, by comparison with the expression for a_{ij} in equation (8)

$$b_{11} = \left(\frac{\sigma_o + R}{R}\right)^2 \; ; \; b_{22} = (\sigma_o + R)^2 \cdot \sin^2 \frac{s_o}{R} \; ; \; b_{33} = 1 \tag{40}$$

Hence, on the sphere where $\sigma_o = 0$, $b_{11} = 1$,

$$\Gamma_{s_o} = 1 = \Gamma^1 b_{11} = \Gamma_1 / b_{11}$$

Consequently, from equation (36)

$$\begin{aligned} &\Gamma^1 = \Gamma_1 = 1 \\ &\Gamma^2 = \Gamma_2 = \Gamma^3 = \Gamma_3 = 0 \end{aligned} \tag{41}$$

Hence, equation (38) gives for the initial values of λ^{i}

$$\lambda^1 = \frac{\partial s}{\partial s_0}$$
$$\lambda^2 = \frac{\partial \psi}{\partial s_0} \tag{42}$$

From equation (14)

$$\frac{\partial s}{\partial s_0} = \frac{\sin\frac{s_0}{R}\cdot\cos\alpha + \cos\frac{s_0}{R}\cdot\cos\psi_0\cdot\sin\alpha}{\sin\frac{s}{R}} \tag{43}$$

$$\frac{\partial \psi}{\partial s_0} = \frac{1}{R}\cdot\frac{\sin^2\psi\cdot\sin\alpha}{\sin^2\frac{s_0}{R}\cdot\sin\psi_0} \tag{44}$$

From equations (42), (43), (44), and (24), the initial values for λ_1 and λ_2 are

$$\lambda_1 = \frac{\sin\frac{s_0}{R}\cdot\cos\alpha + \cos\frac{s_0}{R}\cdot\cos\psi_0\cdot\sin\alpha}{\sin\frac{s}{R}} \tag{45}$$

$$\lambda_2 = \frac{R\,\sin^2\frac{s}{R}\cdot\sin^2\psi\cdot\sin\alpha}{\sin^2\frac{s_0}{R}\cdot\sin\psi_0} \tag{46}$$

To make clear as to how the above equations are used, the following rules are given for providing initial values of λ_1, λ_2, s, ψ.

a. Choose some small initial circle surrounding the aerodynamic stagnation point. This is equivalent to choosing some ϵ as shown in figure 2 (e.g., $\epsilon = \pi/20$ radians). This specifies some s_o/R since $\epsilon = s_o/R$. ψ is arbitrary.

b. Since the angle of attack, α, is given from the experimental conditions, equations (14) may be solved for the initial s and ψ.

c. Equations (45) and (46) may now be solved for the initial λ_1 and λ_2.

d. The initial value for γ is $\gamma = R\epsilon$.

MUSTER OF EQUATIONS

In order to get rid of all subscripts and superscripts define

$$\xi = \lambda_1 \qquad ; \qquad \eta = \lambda_2 \tag{47}$$

Then from equation (24)

In region I:

$$\begin{aligned} \lambda^1 &= \xi \\ \lambda^2 &= \frac{\eta}{\left(R \sin \frac{s}{R}\right)^2} \end{aligned} \tag{48}$$

In region II:

$$\begin{aligned} \lambda^1 &= \xi \\ \lambda^2 &= \frac{\eta}{W^2} \end{aligned} \tag{49}$$

where W is the value of ω with $\sigma = 0$. That is,

$$W = (s - s_c) \sin\beta + R \cos\beta \tag{50}$$

In region I, the differential equations (31) for the streamlines are

$$\begin{aligned} \frac{ds}{d\gamma} &= \xi \\ \frac{d\psi}{d\gamma} &= \frac{\eta}{\left(R \sin \frac{s}{R}\right)^2} \\ \frac{d\xi}{d\gamma} &= \frac{\eta^2 \cos \frac{s}{R}}{\left(R \sin \frac{s}{R}\right)^3} - \frac{1}{\rho q^2}\left[\frac{\partial p}{\partial s} - \xi\left\{\xi \frac{\partial p}{\partial s} + \frac{\eta}{\left(R \sin \frac{s}{R}\right)^2} \cdot \frac{\partial p}{\partial \psi}\right\}\right] \end{aligned} \tag{51a}$$

$$\frac{d\eta}{d\tau} = -\frac{1}{\rho q^2}\left[\frac{\partial p}{\partial \psi} - \eta\left\{\xi\frac{\partial p}{\partial s} + \frac{\eta}{(R\sin\frac{s}{R})^2}\cdot\frac{\partial p}{\partial \psi}\right\}\right] \tag{51a}$$

In region II:

$$\frac{ds}{d\tau} = \xi$$

$$\frac{d\psi}{d\tau} = \frac{\eta}{W^2}$$

$$\frac{d\xi}{d\tau} = \frac{\eta^2 \sin\beta}{W^3} - \frac{1}{\rho q^2}\left[\frac{\partial p}{\partial s} - \xi\left\{\xi\frac{\partial p}{\partial s} + \frac{\eta}{W^2}\frac{\partial p}{\partial \psi}\right\}\right] \tag{51b}$$

$$\frac{d\eta}{d\tau} = -\frac{1}{\rho q^2}\left[\frac{\partial p}{\partial \psi} - \eta\left\{\xi\frac{\partial p}{\partial s} + \frac{\eta}{W^2}\frac{\partial p}{\partial \psi}\right\}\right]$$

Let us use a superscript * for the initial conditions.

$$S^* = R\cos^{-1}\left[-\sin\epsilon\cdot\cos\psi_0\cdot\sin\alpha + \cos\epsilon\cdot\cos\alpha\right]$$

$$\psi^* = \tan^{-1}\left[\frac{\sin\epsilon\cdot\sin\psi_0}{\sin\epsilon\cdot\cos\psi_0\cdot\cos\alpha + \cos\epsilon\cdot\sin\alpha}\right]$$

$$\xi^* = \frac{\sin\epsilon\cdot\cos\alpha + \cos\epsilon\cdot\cos\psi_0\cdot\sin\alpha}{\sin S^*/R} \tag{52}$$

$$\eta^* = \frac{R\sin^2 S^*/R\cdot\sin^2\psi^*\cdot\sin\alpha}{\sin^2\epsilon\cdot\sin\psi_0}$$

Note that equation (52) contains ψ_0 as a parameter so that the streamline may be started at any point on the initial circle.

Solutions of equations (51a) and (51b), along with the initial conditions of equations (52), provide the streamline coordinates, s and ψ, as functions of γ, the distance along the streamline.

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(a) Sokolnikoff, I. S., "Tensor Calculus," John Wiley & Sons, 1951

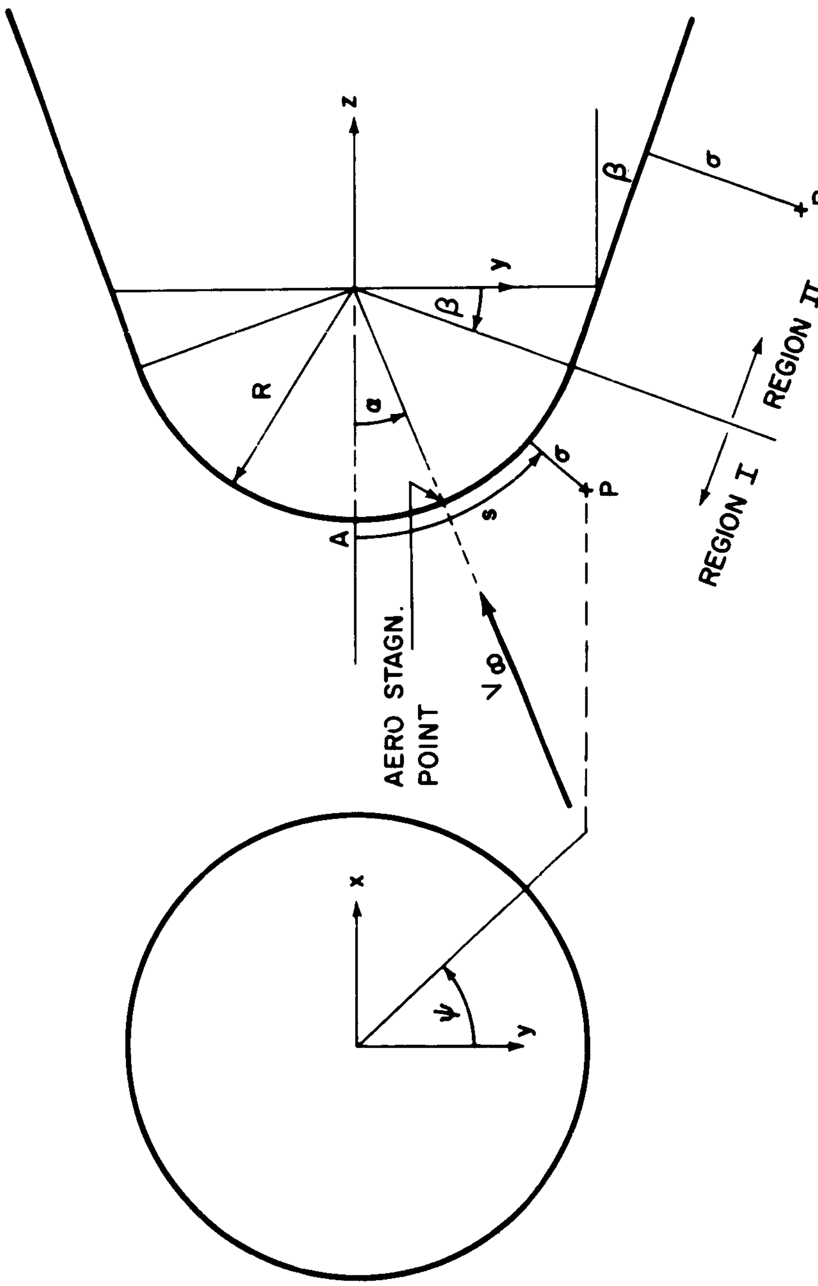

FIG. 1 SPHERE-CONE GEOMETRY: (x, y, z) AND (s, ψ, σ) COORDINATE SYSTEMS

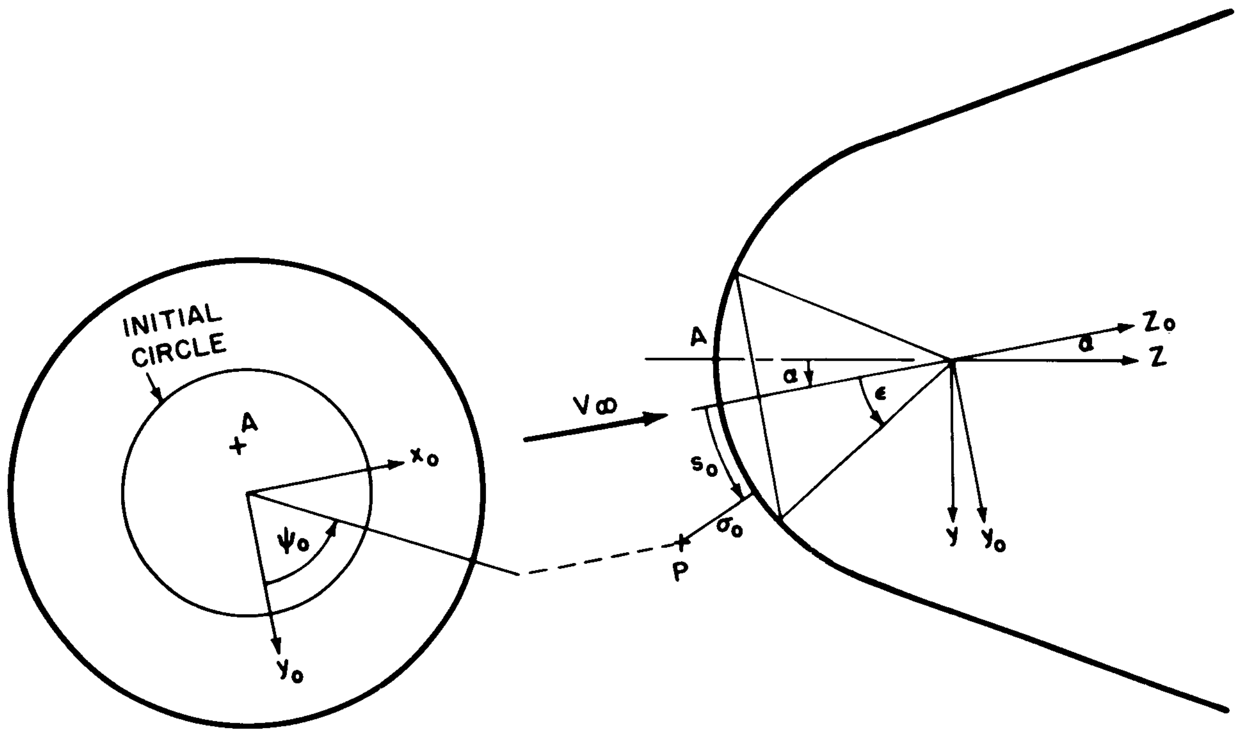

FIG. 2 SPHERE-CONE GEOMETRY: (x_o, y_o, z_o) AND (s_o, ψ_o, σ_o) COORDINATE SYSTEMS, AND INITIAL CIRCLE

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2. Bodies - Boundary layer
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Naval Ordnance Laboratory, White Oak, Md.
(NOL technical report 63-37)
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2. Bodies - Boundary layer
3. Bodies - Flow
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