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# TECHNICAL MEMORANDUM

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On Reliability Inference

by

E. L. Pugh

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ON RELIABILITY INFERENCE

by

E. L. Pugh

1. INTRODUCTION

An important aspect of any system is its reliability, that is, its probability of successful operation. However, such a probability,  $R$ , is never known exactly in practice but may only be approximated by an estimate,  $\hat{R}$ , based on either a subjective probability analysis or on an analysis of actual failure history of the system. The accuracy of the estimate  $\hat{R}$  would therefore appear to be at least as important a consideration as  $R$  itself, since it is  $\hat{R}$  and not  $R$  which is used in any systems analysis. This is especially true for a purchaser of a system as reflected in current military contract specifications. If the estimate  $\hat{R}$  is based on failure history, its accuracy can be precisely quantified in the form of "confidence"; this paper is concerned with such quantification under various assumptions regarding the failure law (including the case of no assumption). Moreover, for a given failure history, a "confidence relation" may be established which gives the confidence of the statement: "The reliability is at least  $R_0$ ", for every  $R_0$ ,  $0 \leq R_0 \leq 1$ . Thus, in addition to considering the best estimate  $\hat{R}$  and its confidence, the systems analyst may wish to compute a value  $R'_0$  such that the true reliability is at least  $R'_0$  with a very high confidence. The value  $R'_0$  could then be used as a lower bound on  $R$  for a maximum cost or risk study. The terms "reliability," "best estimate," and "confidence relation" will now be made mathematically precise.

Let  $F_c(t) = 1 - F(t)$  be the complementary distribution function of a non-negative random variable,  $T$ , known as the "time to failure" of a given system. Let  $t_m$  be a positive number known as the "mission time" of the system. Reliability,  $R$ , is defined as the probability that the system survives the mission time:

$$R = P\{T > t_m\} = F_c(t_m) \quad (1)$$

By a best point estimate for  $R$ , we shall mean a minimum-variance, unbiased estimate. By a confidence relation, depending on a statistic  $\Xi$ , we shall mean a real valued function,  $C_\xi(r)$ ,  $0 \leq r \leq 1$ , having the following property: for any observed value  $\xi$  of  $\Xi$ , and any  $r$ ,  $[r, 1]$  is a  $100 C_\xi(r)\%$  confidence interval for  $R$ . Thus, based on the observed value  $\xi$  of  $\Xi$ , one says for any  $r$ ,  $0 \leq r \leq 1$ : the confidence that  $R \geq r$  is  $C_\xi(r)$ . The function  $C_\xi(r)$  is derived from a "pivotal quantity," i.e., a statistic whose distribution is independent of  $R$ . However, it is of interest to note that in the cases studied:

$$C_\xi(r) = G(\xi; r) \quad (2)$$

where  $G(\xi; r)$  is the distribution function of  $\Xi$ , depending on the value  $r$  of the parameter  $R$ . Furthermore:

$$\varphi_\xi(r) = - \frac{\partial}{\partial r} G(\xi; r) \quad (3)$$

is the "fiducial" density function of the parameter  $R$  according to Fisher<sup>[2]</sup>. Hence,  $C_\xi(r)$  may be regarded as a complementary fiducial distribution of  $R$  in which the fiducial interval is equal to the confidence interval.

It appears that (2) will be true for a general parameter  $\theta$  whenever the solution,  $O(\xi)$ , of  $G(\xi; \theta) = 1 - \alpha$ , for fixed  $\alpha$ , is a 1-1 function of  $\xi$ . (See Lehmann<sup>[3]</sup>, p. 80, Corollary 3.) However, rather than appeal to this lemma, (2) will be demonstrated by direct evaluation for the particular cases studied. That is, the confidence relation will be derived from a pivotal quantity and then the distribution function  $G(\xi; r)$  will be derived directly and shown equal to  $C_{\xi}(r)$ .

The two inferences, a best estimate and a confidence relation, will now be evaluated under: (i) no assumption concerning  $F_c(t)$  and (ii) the assumption that  $F_c(t) = \exp(-t/\theta)^k$ ;  $\theta > 0$ ,  $k > 0$  (the Weibull distribution).

## 2. NO ASSUMPTION CONCERNING $F_c(t)$

Let  $T_1, \dots, T_n$  be  $n$  independent random variables corresponding to a random sample of size  $n$  from the distribution  $F_c(t)$ . Let  $S$  be the statistic defined as: "the number of sample values which exceed  $t_m$ ." Then since  $R = F_c(t_m) = P\{T_i > t_m\}$ , the well known binomial confidence relation for  $R$  may be derived from the observed values  $s$  of  $S$ :

$$C_s(r) \geq \sum_{j=0}^{s-1} \binom{n}{j} r^j (1-r)^{n-j} \quad (4)$$

and it is noted that this confidence relation is the distribution function of  $S$ ,  $G(s; r)$ , as per equation (2). The best estimate for  $R$  is simply the success ratio:

$$\bar{R} = S/n \quad (5)$$

It is of interest, in connection with subsequent developments of this paper, that the relation (4) can be derived by assuming  $F_c(t)$  continuous and considering  $F_c(T_v)$  where  $T_v$  here indicates the  $v^{\text{th}}$  order statistic. A confidence relation for the parameter  $R_{v^*} = F_c(t_{v^*})$ , where  $t_{v^*-1} \leq t_m \leq t_{v^*}$ , is given as the following incomplete beta function [10]:

$$\begin{aligned} C_{t_{v^*}}(r_{v^*}) &= \int_{r_{v^*}}^1 \frac{n!}{(v^*-1)!(n-v^*)!} x^{n-v^*} (1-x)^{v^*-1} dx \\ &= 1 - I_{r_{v^*}}^B(n+1-v^*, v^*) \end{aligned} \quad (6)$$

However, by choice of  $t_{v^*}$ ,  $R \geq R_{v^*}$  and hence a confidence relation for  $R$  is given as:

$$\begin{aligned} C_{t_{v^*}}(r) &\geq 1 - I_r^B(n+1-v^*, v^*) \\ &= 1 - \sum_{j=n+1-v^*}^n \binom{n}{j} r^j (1-r)^{n-j} \end{aligned} \quad (7)$$

But  $n+1-v^* = s$  and hence:

$$C_{t_{v^*}}(r) \geq \sum_{j=0}^{s-1} \binom{n}{j} r^j (1-r)^{n-j} \quad (8)$$

which is the binomial relation (4).

As an example of the use of this relation suppose that the following failure times, in hours, are recorded for a particular system:

20.2, 1.6, 3.5, 15.7, 62.1, 19.9

Further suppose that the mission time of this system is 2 hours. We have:  $n=6$ ,  $s=5$  and hence the best estimate of  $R$  is  $\bar{R} = 5/6 = .833$ . The confidence relation is:

$$\begin{aligned} C_s(r) &\geq 1 - I_r^B(s, n+1-s) \\ &= 1 - I_r^B(5, 2) \end{aligned} \quad (9)$$

In particular, the confidence that  $R \geq .833$  is  $1 - I_{.833}^B(5, 2) = 1 - .736 = .264$ . This is quite low and thus one would probably want to assert instead:  $R \geq .600$  with confidence  $1 - I_{.600}^B(5, 2) = 1 - .233 = .767$ . Note that one is virtually certain that  $R \geq .400$  since the confidence in this statement is  $1 - I_{.400}^B(5, 2) = .960$ . Thus, .400 could be used as a minimum reliability point for a maximum cost or maximum risk study.

In the above example, the best estimate and the confidence statements seem overly conservative. This is because a great deal of information contained in the sample, i.e., the magnitudes of the failure times, is not utilized. To utilize such information, some assumption must be made regarding the distribution of time to failure. This is done in the sections to follow where it is assumed that this distribution is Weibull.

### 3. THE WEIBULL ASSUMPTION

In the remainder of the paper it will be assumed that  $F_c(t)$  is Weibull:

$$F_c(t) = \exp(-t/\theta)^k; \theta, k > 0 \quad (10)$$



This distribution is well suited for reliability applications. The parameter  $k$  indicates the wear-out characteristics of the system involved. For  $k=1$  (exponential) there is no wear-out; for  $k > 1$ , wear-out occurs; and for  $k < 1$  there is negative wear-out, i.e., the longer a system remains in operation, the better its chances of surviving a subsequent time interval. These statements are made precise by considering the value of the conditional density function,  $f(t|T \geq \tau)$ , at  $t = \tau$ . This is known as the hazard,  $h(\tau)$ , and in the Weibull case is:

$$h(\tau) = (k/\theta^k) \tau^{k-1} \quad (11)$$

Thus the hazard is increasing, constant, or decreasing as  $k$  is respectively greater, equal, or less than one.

In what is to follow, it will be assumed that the value of  $k$  is known. Thus these results, to be useful, depend on empirical or theoretical evaluations of  $k$  for certain classes of systems. This has been done, for example, by Drenick<sup>[1]</sup> who has shown that  $k=1$  is a good assumption for highly complex systems.

#### 4. AN ORDER STATISTIC CONFIDENCE RELATION

Consider the pivotal quantity:  $\Lambda = (T_v/\theta)^k$  where  $T_v$  is the  $v^{\text{th}}$  order statistic ( $v$  arbitrary) from a random sample of size  $n$  taken from the Weibull distribution of equation (10). The distribution of  $\Lambda$  may be found to be:

$$\begin{aligned} H(\lambda) &= P\{\Lambda < \lambda\} \\ &= \int_0^1 \frac{n!}{(v-1)!(n-v)!} x^{n-v} (1-x)^{v-1} dx \\ &= 1 - I_{\exp(-\lambda)}^B(n+1-v, v) \end{aligned} \quad (12)$$

Hence:

$$\begin{aligned}
 H(\lambda) &= P\{T_v/\theta)^k < \lambda\} \\
 &= P\{\theta > T_v/\lambda^{1/k}\} \\
 &= P\{\exp(-t_m/\theta)^k > \exp[-\lambda(t_m/T_v)^k]\} \\
 &= P\{R > r\}
 \end{aligned} \tag{13}$$

where:

$$r = \exp[-\lambda(t_m/T_v)^k] \tag{14}$$

or:

$$\lambda = (-\ln r)(T_v/t_m)^k \tag{15}$$

and therefore:

$$P\{R > r\} = H(\lambda) = H[(-\ln r)(T_v/t_m)^k] \tag{16}$$

As a probability statement, this is interpreted in the following manner. For fixed  $\alpha$ , the random variable  $\hat{r}$  defined as the solution of:

$$1-\alpha = H[(-\ln r)(T_v/t_m)^k] \tag{17}$$

is such that:  $P_R\{R \geq \hat{r}\} = 1-\alpha$ , all  $R$ . Thus, to obtain the confidence relation  $C_{t_k}(\alpha)$ , an observed value  $t_v^k$  of the statistic  $T_v^k$  is substituted in (16):

$$\begin{aligned}
 C_{t_v^k}(r) &= H[(-\ln r)(t_v/t_m)^k] \\
 &= 1 - I_r^B(t_v/t_m)^k (n+1-\nu, \nu)
 \end{aligned}
 \tag{18}$$

Note that for  $k=0$  and  $\nu=\nu^*$ , this confidence relation reduces to the binomial one of equation (7). Also, to verify equation (2) in this context, note that the Weibull distribution written in terms of the value  $r$  of the parameter  $R = \exp(-t_m/\theta)^k$ , is:

$$F_c(t) = \exp(-t/\theta)^k = r(t/t_m)^k \tag{19}$$

Therefore the distribution of  $T_v^k$  may be found as:

$$G(t_v^k; r) = \int_0^{t_v^k} \frac{n! (1/t_m^k)}{(\nu-1)! (n-\nu)!} (-\ln r) r^{(x/t_m^k)^{\nu-1}} (1-r^{x/t_m^k})^{\nu-1} dx \tag{20}$$

which, if one lets  $y = r^{x/t_m^k}$ , becomes:

$$\begin{aligned}
 G(t_v^k; r) &= \int_{r(t_v/t_m)^k}^1 \frac{n!}{(\nu-1)! (n-\nu)!} y^{n-\nu} (1-y)^{\nu-1} dy \\
 &= 1 - I_r^B(t_v/t_m)^k (n+1-\nu, \nu)
 \end{aligned}
 \tag{21}$$

Therefore, in accordance with equation (2), we have:

$$G(t_v^k; r) = C_{t_v^k}(r) \tag{22}$$

As an example of the use of the relation (18), consider again the data of Section 2. Suppose that  $k = 1.2$  and we desire an inference based on the third order statistic:  $t_3 = 15.7$ . We have  $(t_3/t_m)^k = (15.7/2)^{1.2} = (7.85)^{1.2} = 11.85$  and therefore:

$$C_{27.4}(r) = 1 - I_r^B{}_{11.85}(4,3) \quad (23)$$

As a comparison with the earlier example, the confidence that  $R \geq .833$  is

$1 - I_{(.833)}^B{}_{11.85}(4,3) = 1 - I_{.115}^B(4,3) = 1 - .0022 = .9978$ . For the exponential case, i.e.,  $k=1$ , this confidence is  $1 - I_{(.833)}^B{}_{7.85}(4,3) = 1 - I_{.239}^B(4,3) = 1 - .0320 = .9680$ , and for the case  $k=.8$  we obtain:  $1 - I_{.387}^B(4,3) = 1 - .1617 = .8383$ . Note that these three confidence values are significantly higher than the value of .264 obtained in Section 2. This indicates the profound effect that a distributional assumption can have on the calculations.

##### 5. A SUFFICIENT CONFIDENCE RELATION

The Weibull density function (minus the derivative of (19)) may be written to indicate its membership in the exponential family<sup>[3]</sup>:

$$\begin{aligned} f(t) &= (-\ln r^{k/t_m})(t/t_m)^{k-1} r (t/t_m)^k \\ &= (-\ln r^{k/t_m})(t/t_m)^{k-1} \exp[(\ln r)(t/t_m)^k] \end{aligned} \quad (24)$$

Therefore the statistic:

$$\Psi = \frac{1}{n} \sum_{i=1}^n T_i^k \quad (25)$$

is sufficient for R. Consider the pivotal quantity:  $\Lambda = \bar{Y}/\theta^k$ . Its distribution may be found to be the following incomplete gamma function [5]:

$$\begin{aligned} H(\lambda) &= \int_0^{n\lambda} \frac{1}{(n-1)!} x^{n-1} e^{-x} dx \\ &= \Gamma^{\Gamma}(\lambda \sqrt{n}, n-1) \end{aligned} \quad (26)$$

Hence:

$$\begin{aligned} H(\lambda) &= P\{\bar{Y}/\theta^k < \lambda\} \\ &= P\{\theta > (\bar{Y}/\lambda)^{1/k}\} \\ &= P\{\exp(-t_m/\theta)^k > \exp(-t_m^k \lambda / \bar{Y})\} \\ &= P\{R > r\} \end{aligned} \quad (27)$$

where:

$$r = \exp(-t_m^k \lambda / \bar{Y}) \quad (28)$$

or:

$$\lambda = (-\ln r) (\bar{Y}/t_m^k) \quad (29)$$

and therefore:

$$P\{R > r\} = H[(-\ln r) (\bar{Y}/t_m^k)] = H(\lambda) \quad (30)$$

where this probability statement is interpreted as in equation (16). Thus, to

obtain the confidence relation  $C_\psi(r)$ , an observed value  $\psi$  of  $\Psi$  is substituted in (30):

$$\begin{aligned} C_\psi(r) &= H[(-\ln r)(\psi/t_m^k)] \\ &= \Gamma^{\Gamma} [(-\ln r)(\psi/t_m^k) \sqrt{n, n-1}] \end{aligned} \quad (31)$$

Finally, to verify equation (2), the distribution of  $\Psi$  may be found as:

$$G(\psi; r) = \int_0^\psi \frac{[(n/t_m^k)(-\ln r)]^n}{(n-1)!} x^{n-1} r^{nx/t_m^k} dx \quad (32)$$

Which, if one lets  $y = n(-\ln r)(x/t_m^k)$ , becomes:

$$\begin{aligned} G(\psi; r) &= \int_0^{n(-\ln r)(\psi/t_m^k)} \frac{1}{(n-1)!} y^{n-1} e^{-y} dy \\ &= \Gamma^{\Gamma} [(-\ln r)(\psi/t_m^k) \sqrt{n, n-1}] \end{aligned} \quad (33)$$

Therefore, again in accordance with (2):

$$G(\psi; r) = C_\psi(r) \quad (34)$$

As an example of the use of the relation (31), consider again the data of Section 2 and the value  $k = 1.2$ . We have  $\psi = (20.2^{1.2} + 1.6^{1.2} + 3.5^{1.2} + 15.7^{1.2} + 62.1^{1.2} + 19.9^{1.2})/6 = 41.45$  and therefore:

$$\begin{aligned}
 C_{41.45}(r) &= I^{\Gamma} [(-\ln r)(41.45/2^{1.2}) \sqrt{6}, 5] \\
 &= I^{\Gamma} [(-\ln r)(44.10), 5]
 \end{aligned}
 \tag{35}$$

The confidence that  $R \geq .833$  is  $I^{\Gamma} [(-\ln .833)(44.10), 5] = I^{\Gamma} [8.05, 5] = .9999$ . For the exponential case,  $\psi$  is the sample mean, 20.5, and the confidence is  $I^{\Gamma} [(-\ln .833)(20.5/2) \sqrt{6}, 5] = I^{\Gamma} [4.59, 5] = .9674$  and for the case  $k = .8$  we obtain  $I^{\Gamma} [(-\ln .833)(10.47/2^{.8}) \sqrt{6}, 5] = I^{\Gamma} [2.70, 5] = .6472$ . Comparing these values: .9999, .9674, and .6472 with the values obtained in the previous section: .9978, .9680, and .8383, it appears that  $C_{\psi}(r)$  is more sensitive to the distributional assumption (the value of  $k$ ) than is  $C_{t_k}(r)$ . This is because  $C_{\psi}(r)$ , being sufficient, utilizes all the information in the sample concerning  $R$ , whereas  $C_{t_k}(r)$  does not. Therefore  $C_{\psi}(r)$  should be used for the data considered. There do arise, however, situations in which  $C_{t_k}(r)$  is superior to  $C_{\psi}(r)$  and these are discussed in the next section.

## 6. THE USE OF THE CONFIDENCE RELATIONS

As pointed out above,  $C_{\psi}(r)$  is based on the sufficient statistic  $\bar{Y}$  whereas  $C_{t_k}(r)$  is based on  $T_v^k$  which is non-sufficient. Also, it follows from a corollary of Lehmann ([3], p. 80, Corollary 3) that the confidence bound,  $r$ , derived from  $C_{\psi}(r)$  is "uniformly most accurate." Therefore  $C_{\psi}(r)$  is mathematically superior to  $C_{t_k}(r)$ . However, in reliability estimation the statistic  $T_v^k$  has a great practical advantage over the statistic  $\bar{Y}$ , which in some cases makes  $C_{t_k}(r)$  the superior confidence relation. This advantage is the fact that to obtain  $T_v^k$ ,

only  $v$  systems need be destroyed but to obtain  $\bar{\Psi}$ , all systems must be destroyed, i.e., all failure times must be known. Thus, for example, in a life test in which  $n$  systems are placed on test simultaneously, the  $v^{\text{th}}$  observed failure time is the  $v^{\text{th}}$  order statistic. The test may therefore be discontinued after, say, one failure, and the relation  $C_{t_1^k}(r)$  used. To use  $C_{\psi}(r)$ , however, the test must be continued until all systems have failed.

This advantage may also be exploited in the analysis of field data such as the data used in the preceding examples. Suppose, for example, that in addition to the recorded failure times: 20.2, 1.6, 3.5, 15.7, 62.1, 19.9, it is known that 5 systems are still in operation and each has lasted longer than the longest recorded failure time, namely, 62.1 hours. The above failure times, when arranged in order, then represent the first six order statistics from a sample of size 11. It therefore appears that  $C_{t_v^k}(r)$ ,  $1 \leq v \leq 6$ , based on  $n = 11$  will be more accurate than  $C_{\psi}(r)$  based on  $n = 6$ . To illustrate the difference in values obtained, we have from Section 5 that for  $k = .8$  the confidence that  $R \geq .833$ , based on  $n = 6$  and  $\psi = 10.47$  is .6472. However, for  $k = .8$  the confidence that  $R \geq .833$ , based on  $n = 11$  and  $t_3 = 15.7$  is  $1 - I_{.387}^B(9,3) = 1 - .0045 = .9955$ .

#### 7. A BEST ESTIMATE FOR R

If  $T$  is a random variable having a Weibull distribution with parameters  $\theta$  and  $k$ , then  $T^k$  is a random variable having an exponential distribution with mean  $\theta^k$ . This fact has been used throughout this paper and it yields the solution of the problem of a best estimate for  $R$  as an immediate extension of



an earlier result derived by the author<sup>[6]</sup>. The best unbiased estimate for  $R$  is the conditional expectation,  $E\{\bar{R}|\Psi\}$  where  $\bar{R}$  is the success ratio of equation (5) and  $\Psi$  is the sufficient (and complete) statistic of equation (25). This estimate is:

$$R^* = E\{\bar{R}|\Psi\} = (1 - t_m^k/n\Psi)^{n-1} \quad (36)$$

where  $R^* = 0$  if  $\Psi \leq t_m^k/n$ . It is interesting to compare this estimate with the maximum likelihood estimate,  $\hat{R}$ :

$$\hat{R} = \exp(-t_m^k/\Psi) \quad (37)$$

It is noted that  $R^* \rightarrow \hat{R}$  as  $n$  becomes large.

Finally, it is of interest in connection with fiducial theory to compare (36) and (37) with two additional estimates of  $R$ . Recall that  $C_{t^k}(\nu)$  and  $C_\Psi(\nu)$  are equivalent to complementary fiducial distributions for  $R$ . We should therefore expect the means of these distributions (which are random variables) to be good point estimates for  $R$ . These are:

$$\hat{R}_1 = \int_0^1 C_{T^k}(\nu) dr = \frac{\prod_{i=0}^{\nu-1} (n-i)}{\prod_{i=0}^{\nu-1} [n-i+(t_m/T_\nu)^k]} \quad (38)$$

$$\hat{R}_2 = \int_0^1 C_\Psi(\nu) dr = \left[ \frac{n}{n+(t_m^k/\Psi)} \right]^\nu \quad (39)$$

Note that  $\hat{R}_2$  is sufficient (since it is a function of  $\Psi$ ) and since  $\hat{R}_2 \rightarrow \hat{R}$ , it

possesses the desirable asymptotic properties of  $\hat{R}$ . Also, it can be shown that  $\hat{R}_2$  has less bias than  $\hat{R}$ . Thus it appears that  $\hat{R}_2$  is superior to  $\hat{R}$ , though inferior to  $R^*$ , the best estimate. The estimate  $\hat{R}_1$ , on the other hand, is very poor since it is non-sufficient and non-consistent in addition to being biased.

The values of these four estimates, for  $k = 1$  and the data of Section 2, are given for comparison below.

$$\begin{aligned} R^* &= [1 - 2/(6)(20.5)]^5 \\ &= (.9837)^5 = .921 \end{aligned} \tag{40}$$

$$\begin{aligned} \hat{R} &= \exp(-2/20.5) \\ &= \exp(-.0975) = .907 \end{aligned} \tag{41}$$

$$\begin{aligned} \hat{R}_1 &= (6)(5)(4)/(6+2/15.7)(5+2/15.7)(4+2/15.7) \\ &= 120/129.8 = .925 \end{aligned} \tag{42}$$

$$\begin{aligned} \hat{R}_2 &= \left[ \frac{6}{6+(2/20.5)} \right]^6 \\ &= (.9845)^6 = .910 \end{aligned} \tag{43}$$

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such a probability (R), is never  
known exactly in practice but may  
only be approximated by an estimate  
( $\hat{R}$ ), based on either a subjective  
probability analysis or on an  
analysis of actual failure history  
of the system. States that if the  
estimate  $\hat{R}$  is based on failure  
history, its accuracy can be  
precisely quantified in the form  
of confidence; this paper is  
concerned with such quantification  
under various assumptions regarding  
the failure law (including the  
case of no assumption).

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