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PACIFIC MISSILE RANGE
POINT MUGU, CALIFORNIA

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AN APPROACH TO THE
SEQUENCING OF RANGE OPERATIONS

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SUMMARY

This report is one of a series which consider the problems of attempting to use a digital computer as an aid in scheduling operations at the Pacific Missile Range. A theoretical treatment of one portion of the mathematical problem of scheduling is presented herein. This portion of the problem is the sequencing or ordering of compound operations on a time scale. A compound operation is a set of individual operations which can be scheduled simultaneously.

The objective of the sequencing process is to arrange the operations such that the running time for the operations is minimized. Under certain assumptions, it is shown that the sequencing problem is the same as the classical traveling salesman problem.

The technique of dynamic programming is discussed as a means of finding an optimal or near-optimal sequence and the problem is formulated in that structure. As developed, the model can accommodate demands for resource time which vary as a function of the ordering of operations and can incorporate priority rules for operations which must be scheduled within a certain time period of the day.

INTRODUCTION

This report is one of a series which consider the problems of attempting to use a digital computer as an aid in scheduling operations at the Pacific Missile Range. Past work on OASIS (Operational Automatic Scheduling Information System) has assumed that a computer could be employed to suggest "optimum" schedules to the Range Scheduling Officer. The work of the Operations Research Group has been to consider *how* and to what extent the scheduling problem could be made amenable to solution by a computer.

This report is a theoretical treatment of one portion of the mathematical problem of scheduling. Other reports (references 1 and 2) also consider the mathematical aspects of scheduling. A further report is planned in which some of the practical aspects of computer-aided scheduling will be discussed in relation to the theoretical approaches previously described.

The present report is limited to determining optimal sequences of operations or compound operations. This latter terminology implies that reasonably efficient methods are available for grouping several operations which may be run simultaneously. A systematic procedure for doing this is presented in reference 1. Reference 1 also gives an alternative approach to the sequencing problem.

The problem under consideration is *not* the minute-by-minute rescheduling activity which takes place once operations have commenced and the effects of a dynamically changing situation begin to set in. Rather, the methodology of this report applies to the scheduling which takes place before operations begin.

The remainder of this report is divided into three major sections: (1) formulation of the sequencing problem as a traveling salesman problem, (2) outline of a dynamic programming algorithm for solution of the traveling salesman problem, and (3) discussion of the validity of the matrix model of the operations.

PROBLEM FORMULATION

For scheduling purposes, an operation can be visualized as in figure 1. The length of a horizontal bar represents the time for which a given resource must be allocated to a certain operation. The assumption is made that specific resources have already been assigned to support the operation. The starting and stopping times are relative to a somewhat arbitrary "zero" time associated with each operation plan. The earliest time the k^{th} resource is required, is indicated by a_k , measured from "zero," and the latest time it is required is indicated by b_k , also measured from "zero." The values assigned to a_k and b_k may be positive, negative, or zero. The sequencing problem is to determine an ordering of operations such that some measure of effectiveness is optimized. Particularly interesting measures can be treated by considering the minimum time between two compound operations. This is defined and symbolized in the following way,

t_{ij} = the minimum time between operations i and j where i is directly followed by j . (A strict ordering is assumed in that, for each value of k , a resource must be used and released by operation i before it can be used by operation j .)

From the definition it follows that

$$t_{ij} = \max_{k=1, 2, \dots, n} [b_{ik} - a_{jk}] \quad (1)$$

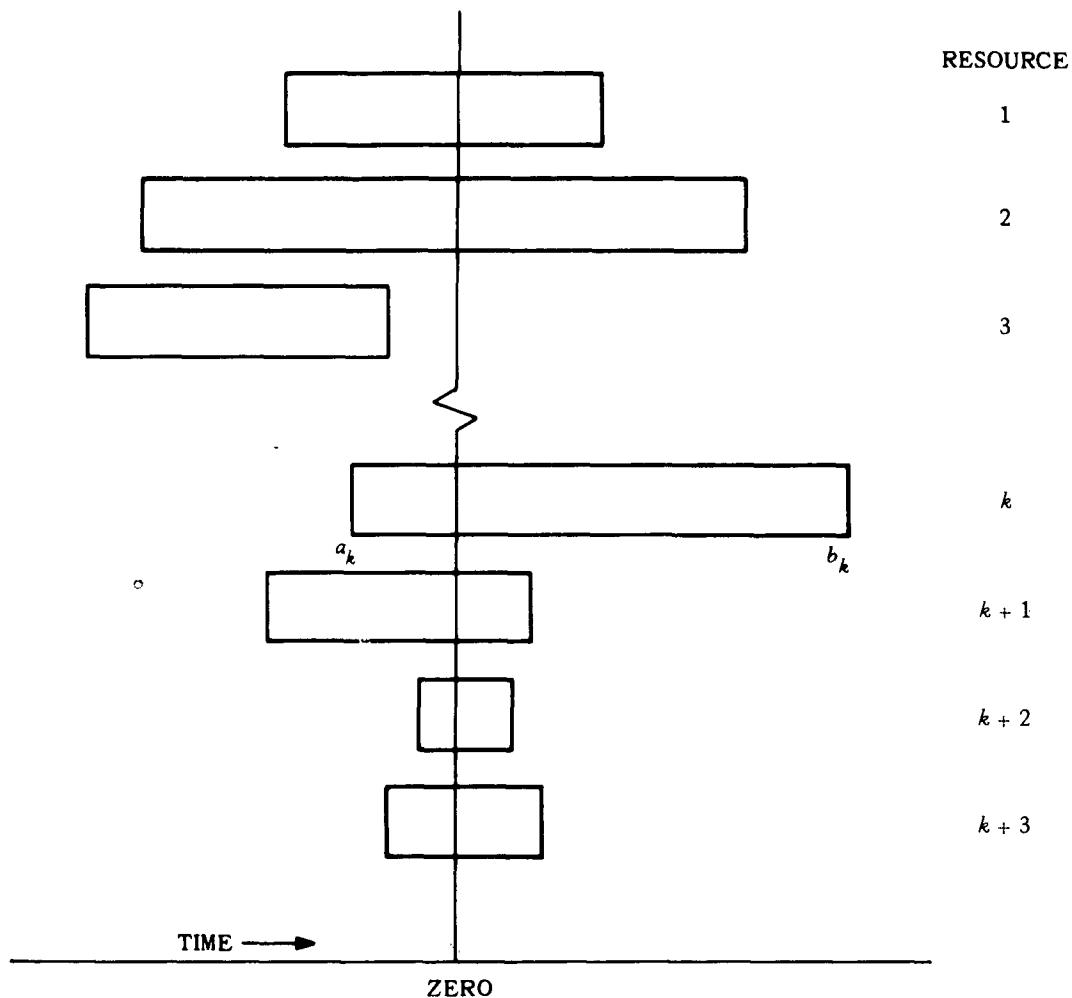


Figure 1. Plan for Operation i .

For an n -operations sequencing problem, one can represent all possible operation pairs by a square matrix $[t_{ij}]$ of size n where $t_{ii} = \infty$ to eliminate consideration of an operation following itself. The problem is to sequence n operations such that the total scheduled time is minimized. Formally, this means finding a subset of n matrix elements

$$t_{pq}, t_{qr}, t_{rs}, \dots, t_{uv}, t_{vp}$$

where p, q, r, s, \dots, u, v is some permutation of the integers $1, 2, \dots, n$ such that the sum of the elements of the subset is a minimum.

Because the ordering described is cyclic, n other permutations such as r, s, \dots, u, v, p, q also provide a minimum. Thus, although there are $n!$ permutations of the numbers $1, 2, \dots, n$, a given subset has $n-1$ "equivalent" subsets so that the optimum ordering may be found by searching among only $(n-1)!$ sequences.

Suitable measures of effectiveness can be introduced by using the concept of a pseudo-operation. For example, if one wished to find an optimal sequence of operations which would assume identical work shifts for all resources and yet provide for dissimilar daily warmup and shutdown times, a pseudo-operation could be specified in which the bars to the left of "zero" time would represent shutdown times and those to the right would represent warmup times. The objective would then be to find a sequence of operations such that the time from the beginning of warmup to the end of shutdown is minimized. From the subset of optimal permutations, the unique ordering desired would be that beginning with the pseudo-operation.

Another interesting measure would be the elapsed time between the first use of a resource on the first real operation to the last use of a resource on the last real operation. Such a measure can be handled by introducing a pseudo-operation which has infinitesimally thin bars centered on the "zero" line. This is the same as stating that every resource must be scheduled simultaneously and for a vanishingly small interval; i.e., both a_k and b_k approach zero for all k . Again, a unique solution is provided by choosing from the optimal subset that permutation which begins with the pseudo-operation.

Regardless of which measure of effectiveness is chosen, the problem is to choose a permutation of operations which minimizes the total scheduled time in some sense. Thus formulated, the operation sequencing problem is recognizable as the classical traveling salesman problem wherein it is desired to minimize the distance traveled in visiting n cities, each only once, and returning to the point of origin. (For a good review of the traveling salesman problem, see reference 3.) The magnitude of the problem can be seen by considering the solution of a 20-operation schedule by direct enumeration. At the rate of one sequence evaluation each microsecond, a computer would require over 77,000 years.

Solution of the traveling salesman problem by the technique of dynamic programming has recently been proposed by Bellman (references 4 and 5) and, independently, by Held and Karp (reference 6). As this approach exhibits considerable computational efficiency, it is outlined in the next section.

A DYNAMIC PROGRAMMING SOLUTION

Consider the problem of scheduling n real operations and a pseudo-operation called 0 . At a certain stage in an optimal schedule beginning (and ending) with 0 , the i^{th} operation has just begun and there remain k operations $j_1, j_2, j_3, \dots, j_k$ to be run before shutting down. Since the schedule is optimal, the total time from i through $j_1, j_2, j_3, \dots, j_k$ in some order to 0 is a minimum; otherwise the total schedule could not be optimal because the over-all time could be reduced by some other permutation of $j_1, j_2, j_3, \dots, j_k$.

This reasoning is in accordance with the principle of optimality as stated by Bellman: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (reference 7).

In order to express this verbal reasoning mathematically, define

$f(i; j_1, j_2, \dots, j_k)$ = the time (measured from the "zero" time of operation i to the "zero" time of operation 0) associated with an optimal schedule beginning with operation i and proceeding through operations j_1, j_2, \dots, j_k to 0 .

Application of the principle of optimality then gives

$$f(i; j_1, j_2, \dots, j_k) = \text{Min}_{1 \leq m \leq k} \{t_{ij_m} + f(j_m; j_1, j_2, \dots, j_{m-1}, j_{m+1}, \dots, j_k)\} \quad (2)$$

The recursive procedure indicated by equation (2) can be initiated through use of the known function

$$f(i; j) = t_{ij} + t_{j0} \quad (3)$$

Application of equation (2) then yields $f(i; j_1, j_2)$ in the following manner

$$f(i; j_1, j_2) = \text{Min}_{m=1, 2} \{t_{ij_m} + f(j_m; \bar{j}_m)\}$$

where $\bar{m} = 3 - m$

The iteration is continued until $f(0; j_1, j_2, \dots, j_n)$ is obtained. The sequence of functions $\{m_k(i)\}$ which results from the minimizing process of equation (2) will give the optimal ordering of operations.

The solution is carried out in two steps: first, n tables each of $f(i; j_1, j_2, \dots, j_k)$ and $m_k(i)$ are generated using equations (2) and (3); next, beginning with $m_n(i) = m_n(0)$, the optimum permutation is determined by a table look-up routine.

DISCUSSION

A model of the operation sequencing problem has been advanced which is analogous to the so-called traveling salesman problem. It is now important to consider the validity of the model with regard to the actual sequencing problem and as a part of the over-all scheduling process.

The fact that even a compound operation will very likely demand no time on some resources gives rise to an incongruity between reality and the model. In such a case, a_k and b_k are undefined and so the maximum operation (equation 1) cannot be carried out. One way of avoiding this inadmissible circumstance is to assume that the resource is required, but for an infinitesimally short time during the operation. Unfortunately, this subterfuge means that the chosen sequence cannot be guaranteed optimum because the position of the infinitesimally thin bar relative to the "zero" time must to some extent be arbitrary. As a result, there is the possibility that with the bar in some position other than that chosen, another permutation of the operations would be better. Present information about the nature of operations indicates, however, that the problem is not a serious one because the resulting deviation from optimality may well be negligible.

A combination of the immediately preceding constraint and the previous assumption of strict ordering means, in effect, that the elapsed time between two compound operations does not depend upon any other operations. The extent to which real operations exhibit this independence remains to be determined.*

*S. Zacks has recently generalized the approach to the sequencing problem discussed here by treating higher dependencies and applying the procedure to a scheduling problem (Stanford University, Applied Mathematics and Statistics Laboratories. On a Pseudo-Solution to a Scheduling Problem. Palo Alto, Calif., Technical Report No. 84, 14 Dec 1962.) As he notes however, increasing the generality rapidly increases the computational time.

The structure of the model offers a possible asset which has not yet been mentioned. For technical reasons, the resource time required for a given operation may depend upon the adjacent operations. As an example, the setup time required for an instrument may depend upon the kind of operation which came before. Obviously, this kind of variation may be incorporated into the operations matrix $[t_{ij}]$ if it is considered significant.

As yet, no restrictions have been imposed upon the time which an operation must be scheduled. Consider now the constraint that the scheduled "zero" time of a certain operation must come within a specified interval of time during the day. This restriction can be handled by introducing a priority rule which in effect says: if, during the recursive procedure indicated previously, the value of $f(i; j_1, j_2, \dots, j_k)$ shows that it is time for the special operation, then that operation takes precedence and it should be inserted into the ordering.

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