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# ORNELL AERONAUTICAL LABORATORY, INC.

AD NO 402 608

ANALYSIS OF PROBABILITY DISTRIBUTIONS  
AND POWER SPECTRA OF TIME HISTORIES  
OF MANEUVERING FLIGHT

Report No. AC-1428-F-1

Bureau of Naval Weapons Contract  
NO(s) 60-6114-c

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CORNELL AERONAUTICAL LABORATORY, INC.  
Buffalo, New York

Report No. AC-1428-F-1

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Bureau of Naval Weapons Contract

④ - NOa(s) 60-61142

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## ABSTRACT

Time histories of representative maneuvering flight were used to compute the power spectral densities of pertinent aircraft motions. Peak distributions were also examined. The analysis has shown that the mission and the type of aircraft are the two most important factors which influence the probability of occurrence of peak motions. The spectral analysis has shown that the predominant frequencies present in maneuver motions, and which appear to be closely related to peaks in the time histories, are not necessarily the natural frequencies of rigid airframe response such as Dutch roll and short period. A statistical model representative of maneuvering motion time histories is briefly described.

## FOREWORD

In 1960, the Loads and Dynamics Branch of the Airframe Design Division, Bureau of Naval Weapons, Department of the Navy, sponsored this analysis of maneuver time histories at CAL under contract number NOa(s) 60-6114-c. Mr. Edward J. Griffin of the Airframe Design Division was project administrator.

The author, Mr. Charles B. Notess of the Flight Research Department of the Cornell Aeronautical Laboratory, was project engineer in charge of this program. Work began in March 1960 and was completed in March 1961.

TABLE OF CONTENTS

LIST OF TABLES . . . . . vi  
LIST OF ILLUSTRATIONS . . . . . vi  
INTRODUCTION. . . . . 1  
    Background for Maneuver Load Considerations . . . . . 1  
    A Statistical Model for Maneuvering Motions. . . . . 2  
RESULTS OF THE INVESTIGATION . . . . . 5  
    The Probability Distributions of Maneuver Time Histories . . . . . 5  
    A Statistical Model for Representing Maneuver Time Histories. . . . . 9  
    Power Spectral Densities of Maneuvering Motions . . . . . 13  
CONCLUSIONS AND RECOMMENDATIONS . . . . . 21  
LIST OF SYMBOLS. . . . . 24  
REFERENCES . . . . . 25

## LIST OF TABLES

I	PEAKS PER HOUR FOR MANEUVERING FLIGHT . . . . .	26
II	DESCRIPTIVE INFORMATION FOR F-100A FLIGHT DATA. . . . .	27
III	DESCRIPTIVE INFORMATION FOR F11F, A4D, AND A3D FLIGHT DATA . . . . .	28

## LIST OF ILLUSTRATIONS

1	Cumulative Frequency of Occurrence of Normal Load Factor Peaks . . . . .	29
2	Cumulative Frequency of Occurrence of Normal Load Factor Peaks . . . . .	30
3	Cumulative Frequency of Occurrence of Pitch Acceleration Peaks. . . . .	31
4	Cumulative Frequency of Occurrence of Roll Acceleration Peaks. . . . .	32
5	Cumulative Frequency of Occurrence of Yaw Acceleration Peaks. . . . .	33
6	Cumulative Compilation of Probability Distribution . . . . .	34
7	Probability Distribution for F2H-2 Maneuvers . . . . .	35
8	Harmonic Analysis of Roll Rate - Flight 241, F-100 . . . . .	36
9	Illustration of the Significance of Knowing the Time Sequence of Data Points. . . . .	37
10	Probability Distribution of Fitted $n_z$ Pulses for A4D-1 . . . . .	38
11	Power Spectra of Maneuver Motions $\Delta n_z$ , Normal Acceleration . . . . .	39
12	Power Spectra of Maneuver Motions $\Delta n_z$ , Normal Acceleration . . . . .	40
13	Power Spectra of Maneuver Motions $\Delta n_z$ , Normal Acceleration . . . . .	41
14	Power Spectra of Maneuver Motions $\Delta n_z$ , Normal Acceleration . . . . .	42

LIST OF ILLUSTRATIONS  
(cont'd)

15	Power Spectra of Maneuver Motions $\dot{\phi}$ , Angular Acceleration . . . . .	43
16	Power Spectra of Maneuver Motions $\dot{\psi}$ , Angular Acceleration . . . . .	44
17	Power Spectra of Maneuver Motions $\dot{\theta}$ , Angular Acceleration . . . . .	45
18	Power Spectra of Maneuver Motions $\dot{\eta}$ , Angular Acceleration . . . . .	46
19	Power Spectra of Maneuver Motions $\dot{\xi}$ , Angular Acceleration . . . . .	47
20	Power Spectra of Maneuver Motions $\dot{\zeta}$ , Angular Acceleration . . . . .	48
21	Power Spectra of Maneuver Motions $\dot{\nu}$ , Angular Acceleration . . . . .	49
22	Power Spectra of Maneuver Motions $\dot{\rho}$ , Angular Acceleration . . . . .	50
23	Power Spectra of Maneuver Motions $\dot{\sigma}$ , Angular Acceleration . . . . .	51
24	Power Spectra of Maneuver Motions Angular Accelerations . .	52

## INTRODUCTION

### BACKGROUND FOR MANEUVER LOAD CONSIDERATIONS

One of the important considerations during the structural design of an aircraft is determination of the loads that may be experienced by the various components of the airframe during maneuvering flight. Recent structural designs have been based upon the ability to withstand several hypothetical maneuvers, some of which are quite unrealistic. The high performance requirements of modern aircraft require that anticipated maneuver histories be specified accurately in order to enable design within the bounds imposed by: 1) weight limitations, and 2) sufficient strength to perform the intended missions.

The maneuvers experienced by different aircraft vary considerably, depending upon such factors as the type of mission and the configuration of the aircraft. Because of the latter consideration, the aerodynamic, geometric, and mass characteristics of the particular aircraft concerned can be important factors which govern the nature of the maneuver history.

The designer must have a means for determining the expected maneuver history of his aircraft from a knowledge of the contemplated missions and the aircraft characteristics. At present, there is not sufficient data available which the designer can use to estimate maneuver histories. To provide such data, the maneuver histories of all classes of currently operational aircraft must be sampled. Records obtained in flight operations should be analyzed and the results presented in a manner which would relate maneuvering motions to the pertinent mission and aircraft characteristics. The statistical model, which will be the basis of such a presentation, must be designed to satisfy the necessary accuracy requirements for specifying expected motions.

The aircraft designer should be able to determine the probability of occurrence of the various peak motions which the aircraft may experience

during its active service life. From these motions, he will calculate the probability distributions of the peak loads on each component of the airframe. This information would be used to determine the frequency of occurrence of repeated load applications, of various magnitudes, for the purpose of estimating fatigue life.

The Bureau of Naval Weapons and the Wright Air Development Division are currently engaged in establishing a program which has as its purpose the collection of time histories of maneuver motions, analysis of this data, and reduction of the information to forms suitable for use in determining structural design criteria. A number of eight-channel recorders which will provide time histories (on magnetic tape) of airspeed, altitude, and of the aircraft motions in six degrees of freedom will be installed in a variety of operational service aircraft. The tape records will be reduced to statistical representations of maneuver motions by digital computation at a central Data Analysis Center.

#### A STATISTICAL MODEL FOR MANEUVERING MOTIONS

In recent years, considerable experience has been amassed concerning the development of statistical models which are representative of random processes. For example, the random time histories of atmospheric turbulence can be represented by a process having a Gaussian distribution for the magnitude of any component of gust velocity and having a power spectral density defined in many cases by two parameters, scale and intensity. This process is rather simple and therefore it is possible to generate, in the laboratory, typical time histories of gust velocity by taking the output of a Gaussian white noise generator and filtering it according to amplitude versus frequency characteristics obtained directly from the power spectral density. Gust time histories so generated cannot be distinguished from actual measurements of gust velocity.

Early attempts to represent the maneuvering motions of aircraft by a

statistical model proceeded along similar lines. Reference 1 showed that the probability distributions of time histories of maneuvering motions often approached the Gaussian distribution if a sufficient number of individual maneuvers were included in compilation of the probability distribution. To some extent, the central limit theorem was thought to support such a conclusion.

In addition to investigations of the probability distribution, the power spectral densities of normal acceleration for a number of fighter aircraft performing a variety of missions were also examined and are presented in Reference 1. The consistency of spectral shape obtained in these investigations led to attempts to represent the maneuver motions by a statistical model similar to that found satisfactory for atmospheric turbulence. On the basis of this model, techniques based upon knowledge of the spectral shape were employed to relate the frequency of occurrence of peaks in the motion time histories to the mean square value of the time history. These techniques were developed by Rice and are outlined in Reference 4. If their use were valid, it would mean that one need measure the mean square value of maneuver motion time histories and with a knowledge of the shape of the power spectral density, one could then estimate the number of peaks per second occurring in the time history.

At one time, the approach described above appeared quite promising. Therefore, it was decided to perform additional analysis of maneuver time histories for two reasons: 1) to check the probability distributions and determine if they were essentially Gaussian, and 2) to check the consistency of the shape of power spectral density curves. In this latter case, it was planned to determine to what extent the aerodynamic and mass characteristics of a particular aircraft can be used to predict deviations in the shape of the spectral curves not only for normal acceleration, but roll, yaw, and pitch rates also.

It is in the light of this background that the research at CAL under contract NOa(s) 60-6114-c was begun in the spring of 1960. This research

program, described in more detail in subsequent sections, had as its purpose investigation of the two factors mentioned above. This report summarizes the results of these investigations.

## RESULTS OF THE INVESTIGATION

The actual analysis performed under contract NOa(s) 60-6114-c can best be separated into three main categories. The first consisted of checking the probability distributions of the peak values and the amplitudes of maneuver time histories. The second included analysis of some statistical models which could be used to represent the important statistical characteristics of maneuver motions. The third included calculation of the power spectral densities of maneuver time histories. The analysis performed for each of these three categories will be described below.

### THE PROBABILITY DISTRIBUTIONS OF MANEUVER TIME HISTORIES

Two types of probability distribution were examined. They are: 1) the probability distribution of time history peaks, and 2) the probability distribution of amplitudes of the time histories of maneuver motions sampled at constant time increments. The peak distributions will be discussed first and then the amplitude distributions will be discussed.

#### Peak Distributions

A considerable amount of peak count data is presented in Reference 2. This data includes air to air, air to surface, and general flight mission categories. These mission categories are defined as follows:

1. "Air to air - includes attack or defensive tactics between aircraft and aerial gunnery practice with towed targets."
2. "Air to surface - includes attack tactics upon surface targets through dive-bombing, loft-bombing, strafing, etc."
3. "General - includes familiarization flights, instrument training, acrobatics, field-carrier landing practice, etc."

About 2000 flight hours, including data from thirteen representative types of aircraft, were analyzed. The data was presented in the forms of tables and curves relating the number of motion time history peaks per hour exceeding various values of angular acceleration and translational acceleration. The data was reworked and is shown for a combination of all three types of mission in Figures 1 through 5. The normal load factor data was presented in the form of a ratio; incremental normal load factor divided by the maximum permissible load factor (or limit load factor). To illustrate the shape of the frequency of occurrence curves, the data was normalized to pass through the point 10 peaks per hour exceeding a normal load factor ratio of .20. This data is plotted in Figure 2.

#### A. Normal Load Factor

The curves on Figures 1 and 2 show that there is a distinct trend for the early AD type aircraft to perform a proportionately high number of high "g" maneuvers with sharp decreases in the number of peaks occurring only above a  $\frac{\Delta n_z}{\Delta n_{z\_LIMIT}}$  of 0.5. The twin-engine patrol bombers (S2F, P2V) and the A3D twin-engine attack bomber are at the other extreme of performing the majority of maneuvers at values of load factor ratio under 0.4.

We can summarize these two extremes of characteristic peak distribution by saying that the AD's maneuver at considerably higher load factor ratios than do the bombers. The restriction of the maximum limit load factor causes the pilot to truncate the curve so that generally the magnitudes do not exceed a load factor ratio of unity. However, occasionally (the S2F in Figure 2) a load factor ratio of unity is exceeded. If we group the 13 aircraft into four categories, as the shapes of the distribution curves vary from severely maneuvering truncated AD-5 type curves toward the other extreme of light maneuvers (the majority of which are at low load factor ratios), the result would be approximately as follows:

<u>Predominant Characteristics</u>	<u>Aircraft Type</u>
1. Strongly convex	AD's
2. Less convex	Early jets
3. Linear	Recent jets
4. Concave	Bombers (twin-engine)

This tendency tends to indicate that the type of aircraft influences the nature of load factor peak occurrences. However, another important factor is involved. This factor is the type of mission. A comparison of the relative characteristics of the distribution of normal load factor peak ratios for any given aircraft type indicates that the "general" type of mission generates considerably more peaks with small load factor ratios than the other mission types. Thus we might say that the type of mission also influences the nature of load factor peak occurrences. In particular, the AD-5 and the AD-4B data contain a much smaller proportion of general type missions than the other aircraft. More data will have to be analyzed before one can say which factor is the governing one, aircraft type or mission type

#### B. Pitch Acceleration

An interesting fact evident in Figure 3 is that the relative shape of the curves is somewhat similar to that of Figure 2, but also appears to be correlated to some extent, with wing aspect ratio and sweep angle or lift curve slope. This seems reasonable if one realizes that delta wing aircraft must fly at considerable angles of attack to generate large lift coefficients. The case is similar for swept wing aircraft. Thus, Figure 3 shows that as one considers aircraft which have wing planforms such that the lift curve slope decreases, the aircraft utilize larger pitch accelerations. The basic mission, however, also affects the curves similarly to the case for normal load factor. This fact is shown by the extremely low values of pitch acceleration which predominate for the S2F and the P2V as compared to the AD's. More data would be necessary to enable one to specify which factor affects pitch acceleration peaks the greatest amount, and how the two factors interact.

### C. Roll Acceleration

The information on roll acceleration is based upon considerably less flight data than is the case for normal load factor. The number of peak occurrences per hour which exceed various magnitudes of roll acceleration are shown in Figure 4. The curves were not normalized in this case, as the general shape of the curves is so similar for all the fighter aircraft. The twin engine bombers have a steeper slope than do the fighters. A characteristic of equal importance to curve shape is the relative order of magnitude of the peaks. This is shown in Figure 4. The curves indicate that the aileron effectiveness divided by the large roll inertias, resulting from engines mounted outboard on the wings, provides considerably smaller roll accelerations than is the case for single engine fighters. The low roll inertias of swept and delta wing aircraft contribute towards larger roll accelerations.

### D. Yaw Acceleration

Very little data was available on yaw acceleration. However, Figure 5 shows that the AD-5 and 6 have as much variation as the other two aircraft, which implies that for the AD's, the type of mission and pilot characteristics produce variations comparable to the variations between the A4D and F2H which might be due to mission, pilot, and also aircraft variations.

### E. Summary of Peak Distribution Analysis

The foregoing brief analysis of the frequency of occurrence of peak values which exceed various magnitudes of motion indicates that the type of mission and the particular type of aircraft configuration do influence the peak distributions. Analysis of additional data will be required before it is possible to quantitatively relate these influences to specific causes. It does appear that additional data and analysis will be quite fruitful in leading ultimately to quantitative definition of the variation in peak distributions caused by types of mission and types of aircraft.

### Amplitude Distributions

Early in the program, probability distributions of time histories of normal acceleration and elevator deflection were determined from NACA data. A plot was made of the distributions based upon consideration of one individual maneuver, then two maneuvers, then three maneuvers, etc. As the number of maneuvers included in the determination of a probability distribution increased, the shape of the distribution approached that of a truncated Gaussian or "normal" distribution as defined in Reference 1. This similarity became noticeable when about 16 maneuvers, comprising about 6 different types of maneuvers from loops to Immelmans, to Cuban eights, were included in determination of the distribution. Figure 6 shows the progressive compilation of a probability distribution obtained by adding one maneuver at a time. The maneuvers included several rolls, loops, Immelmans, Split-S's, and a Cuban eight. The respective maneuvers are identified with the letters R, L, I, S, and C, respectively. Figure 7 shows the cumulative probability distribution plotted on arithmetic probability paper. It is evident that the pull-up data does not follow a normal distribution which would appear as a straight line on such paper. The push-over data does not contain a sufficient number of class intervals to indicate any non-normality. Results similar to those shown for the F2H-2 were obtained for an F-86A for elevator deflection and normal acceleration. Similar tendencies are shown also in Figure 11 of Reference 1. The important questions which became evident were: 1) how many maneuvers were contained in the average flight of an aircraft? and 2) were they enough to enable use of a Gaussian distribution?

### A STATISTICAL MODEL FOR REPRESENTING MANEUVER TIME HISTORIES

Further thought given to the random process from other points of view raised another very important question. It is true that, as mentioned previously, a typical time history of atmospheric turbulence can be generated by filtering the output of a Gaussian white noise generator according to frequency characteristics indicated by the power spectrum. However, if one tries a similar approach for generating a typical maneuver time history, the result is

not at all typical. The question is, Why? The answer must be that the statistical process was not adequately represented by a Gaussian model similar to that used for the case of turbulence.

Perhaps the most important deficiency of the Gaussian model described above is based upon the assumption that the relative phasing of the sinusoids which are contained in the time history is random. Inspection of time histories of individual maneuvers indicates that clearly this is not so for individual maneuvers. For example, a typical dive and pull-up maneuver will produce a time history of normal load factor which will look like a sinusoid of one predominant frequency with other frequencies definitely correlated in phase. The roll rate time histories often appear like a square wave which also indicates phase correlation.

In order to check the relative phasing of the sinusoids of various frequencies, the roll rate time history for F-100 Flight 241 was harmonically analyzed. The relative phase angle plotted versus frequency is shown in Figure 8. It is evident that the relative phase angles do not vary randomly, but are essentially a function of frequency.

A second important deficiency of the model is that the central limit theorem requires, in cases such as these, that the individual non-Gaussian, independent processes (individual maneuvers) which combine to form a Gaussian process, must each act simultaneously with all the others during the entire duration of the sampling period. Such is truly not the case for maneuvering flight. Each maneuver is performed separately in time. All 16 maneuvers including loops, Immelmans, and Cuban eights would have to be performed simultaneously for the central limit theorem to apply in this situation. Hence, it would appear that a different statistical model is required to represent maneuvering flight. It should be mentioned that if one were not concerned with the time order of data points used to construct a probability distribution, but were only interested in the number of times the time history fell within certain class

intervals such as 3.1 g - 4 g, 4.1 g - 5 g, 5.1 g - 6 g, etc., then the central limit theorem would be applicable. However, in our case, we are concerned with load cycles and thus, the time ordering of these numbers is important. Figure 9 illustrates the importance of load cycle information. If the two hypothetical time histories shown in Figure 9 were sampled at a time interval of .5 seconds, the data for the dashed curve would indicate 13 data points greater than 3 g, while the data for the solid curve would indicate two data points greater than 3 g. For fatigue considerations, it is important to know that the two data points represent two distinct load cycles or load applications while the 13 data points represent only one distinct load application.

Some time was spent in trying to construct a statistical model which would more adequately represent the maneuver process than does the Gaussian model outlined in the previous section. A classical process described in Chapters 14 and 15 of John Aseltine's text on transform methods, Reference 3, seemed to be better suited for the case on hand. This process is that of the random phase square wave or random pulse. This process is defined by two variables. One variable is the magnitude or amplitude of the pulse, which has a Gaussian probability distribution, and the other is the duration of the pulse, which has a Poisson probability distribution.

The power spectral density for such a bivariate model is derived in Reference 3 and has the form

$$\bar{\Phi}(\omega) = 2\bar{a}^2\beta / (\beta^2 + \omega^2)$$

where  $\bar{a}^2$  is the variance of the Gaussian distribution of amplitude,

$\omega$  is the frequency in radians per second, and

$\beta$  is equal to the inverse of the average pulse duration. It will be noted that this form is identical to that which is obtained from the Gaussian model used to represent atmospheric turbulence where  $\bar{a}^2$  is equal to the intensity of the turbulence and  $\beta$  is a function of the scale of the turbulence. Several variations of this basic bivariate model were examined in an attempt to

obtain a model which would adequately represent the process.

One might remark at this point that maneuver time histories do not look like rectangular pulses of random duration. This is quite true, but the time history of a rectangular pulse signal can very simply be changed to a more realistic maneuver time history by passing it through a low pass filter which would round off the square corners. This low pass filter might also contain a resonance at the natural frequencies of the aircraft motions, and thus further improve the representation.

The Poisson distribution for maneuver duration can be represented as follows: the probability that the elapsed time between two successive events (or the maneuver duration) lies between  $T_0$  and  $T_0 + dt$  equals  $\beta e^{-\beta T} dT$ . To check the suitability of this form for representing maneuvers, a time history of normal acceleration for an A4D-1 was approximated by a series of rectangular pulses. This representation was done manually on the oscillograph record and therefore is very approximate; however, it was thought that this approximate representation would provide a useful first indication of the probability distribution of maneuver duration. Figure 10 shows the number of pulses having durations within class intervals of width 2 seconds. The solid curve is plotted for intervals of 0.1-2, 2.1 - 4, 4.1 - 6 seconds, etc., while the dashed curve is plotted for intervals of 1.1 - 3, 3.1 - 5, 5.1 - 7 seconds, etc. It will be noticed that a definite peak occurs for pulse durations of 9, 20 and 34 seconds. The 34 second pulses are probably the result of long maneuvers such as pull-ups, while the 9 second pulses might be associated with lateral motions, such as rolls and turns. The mean interval length is 13 seconds.

A factor which tends to support this type of a model is the number of peaks per hour of maneuvering (light time). This factor indicates that one load factor peak per maneuver, or perhaps two roll acceleration peaks per maneuver are very common. If this is the case for average missions, it would indicate that each pulse can be considered as representing an individual maneuver such as a pull-up which generates one peak. Hence, the average pulse duration

should be related to average peaks per hour of maneuver flight time,  $T_m$ . The proportion ( $T_m/T$ ) usually varies from 0 to 50%; however, the majority of values is under 20%. The factor  $T_m/T$ , to a large extent, is independent of airplane characteristics and will depend primarily on the mission and the distance from the airfields used in a flight to the target or combat areas. Figure 1 shows the extreme variation in peaks per hour caused by variations in  $T_m/T$ . The F8U-1 had a maximum rate of peaks per hour of 43.9. If one assumes that  $T_m/T$  was 20% for the F8U-1, the peaks per hour of maneuvering flight time would be about 220, or roughly one peak every 16 seconds. Thus, an average maneuver or pulse duration of 16 seconds might represent the F8U-1 data. 16 seconds, as an over-all average, is of the same order as the 13 seconds mentioned above for the A4D-1.

The AD-5 had a maximum rate of 110 peaks per hour for the case of roll acceleration. Table I shows the average number of peaks per hour found for the four classes of aircraft which are contained in Reference 2. It is evident that when normalized on the basis of one load factor peak per hour, the roll acceleration has a considerably greater peak occurrence, especially so for the AD aircraft. This is to be expected since a turn will contribute peaks to roll acceleration upon entry and exit while the normal load factor will normally have one peak in a turn. It is difficult to draw any further conclusions from this data because there is no way of determining the type of maneuvers contained in the roll, pitch and yaw data and in the load factor data, which is based on considerably more flight hours.

The exploration of statistical models was not carried to completion as it would have been beyond the scope of this program.

#### POWER SPECTRAL DENSITIES OF MANEUVERING MOTIONS

Under the present contract, the power spectral densities of normal acceleration and of the angular accelerations about the three axes, roll, pitch, and yaw, were calculated from time history data. The types of mission

included air to air, air to ground, and air combat maneuvers with an F-100A aircraft; high altitude maneuvers with an F11F-1; and high altitude instrumentation checkout maneuvers (pull-ups, rolls, fast rocks, fishtails, and bank reversals) with an A4D-2N. In addition, the power spectrum of normal acceleration for high altitude maneuvers with an A3D-2 was calculated. Additional information which describes the flight conditions appears in Tables II and III. The F-100A data was obtained from NASA (Langley Research Center) in the form of punched IBM cards. The remainder of the data was obtained in the form of oscillograph records from NAMC, Philadelphia. The F-100 punched card data contained time histories of roll and yaw rates, normal acceleration, pitch angular acceleration, airspeed, Mach number, altitude, weight, measured horizontal tail load, and other information. The time histories were presented at a time interval of 0.2 seconds. The punched card data contained information which was transcribed only during periods when the oscillograph traces were varying, thus indicating maneuvering flight. Periods of non-maneuvering flight, which amounted to about 77% of the total flight time, were not transcribed.

#### Linear Interpolation

Examination of the F-100 time histories which were punched into the cards indicated that the normal acceleration and the angular rate time histories often were not completely returned to steady state (level flight) values during the interval which was covered by the transcription. For this reason, it was assumed that the time histories would return slowly to their steady state value or to the value at the start of the next maneuver which was transcribed. To represent this slow trend, the maneuvering portions of the time histories (those portions which were transcribed onto the cards) were assumed to be joined together with straight line segments. An IBM program was set up to perform this linear interpolation. This interpolation was not necessary for angular acceleration time histories, since these variables inherently have little low frequency content and were always close to their steady state value of zero.

When the oscillograph records were transcribed, care was taken to

start and end the transcription when all traces were very close to steady state, or at least well below any reasonable threshold. The sampling interval was 0.33 seconds. The transcribed oscillograph data included roll, pitch, and yaw angular acceleration time histories, which as mentioned above, contain little low frequency content and also included normal acceleration at or near the cg of the aircraft. To be consistent, the normal acceleration time histories were linearly interpolated between maneuvers in a manner similar to that used for the F-100 data.

#### Power Spectral Computations

The power spectral density was computed for each individual maneuver by standard methods described generally in Reference 4, and specifically in Reference 5. Since the spectra of angular accelerations in maneuvering flight is essentially constant over a large part of the frequency range, no prewhitening was used in the computation of the spectra. For computation of the spectra of incremental normal acceleration and angular rates of roll and yaw, the linear interpolation was applied to the time histories. The interpolated time histories were then passed twice through sharp high pass  $\sigma$  filters (described in Reference 5) with  $\sigma = .06$ . This operation essentially forced the time histories to start at zero for each individual maneuver. If the originally transcribed time history did not return to zero at the end of the transcribed maneuver duration, the linear interpolation represented an extension of the time history beyond this time. If this was the case, the extremely sharp high pass filter would bring the time history to zero in several seconds after entering the interpolated extension of the maneuver time history. The spectral calculations were based upon time history lengths which terminated as soon as the high pass filter brought the time history to a threshold value near zero. The threshold was chosen to be consistent with transcription accuracies which produced time history data accurate to .01 inches. The attenuation of low frequency energy resulting from use of the  $\sigma$  filters was compensated for by properly increasing the amplitude of the computed power spectral density values at the low frequencies.

Another important point which should be mentioned here is that no corrections were made to the plotted spectra to remove the effects of folding, which caused the slight leveling off of the spectra evident near the folding frequency.

Figures 11 through 23 contain the power spectra computed from the maneuver time histories. The spectra of angular accelerations were normalized by dividing by  $\sigma^2$ , the mean square value of the time history.  $\sigma^2$  is based upon integration of the spectrum of each flight over the range of spectrum points plotted on the graphs. For example, if the first plotted point is at  $f = .025$  cps, the integration is assumed to cover frequencies from .0125 cps on upward. Because of folding considerations, the contribution to the mean square from power or energy contained in frequency bands above the folding frequency will be included automatically by the integration of power spectrum up to the folding frequency.

The spectra for normal acceleration were handled similarly, except that the normalizing operation was performed upon the values of frequency times  $\Delta n_z$  spectrum, rather than the  $\Delta n_z$  spectrum alone. Thus, the mean square does not represent mean square normal acceleration. For this reason, the symbol  $\Sigma^2$  was used. The curve of  $f \times \overline{\Phi \Delta n_z}$  decreases less with increasing frequency and thus changes in curve shape caused by differing maneuver or aircraft configurations are more readily apparent in the plots of  $f \times \overline{\Phi \Delta n_z}$  as presented than would be the case of plots of  $\overline{\Phi \Delta n_z}$ . Since the primary purpose of normalization in this case was to eliminate the effects of maneuver severity and to illustrate changes in spectral curve shape, the basis for normalization is not critical.

### Spectrum Results

The first reported calculations of power spectral densities of angular accelerations for maneuvering flight were presented in Reference 6. The spectra are shown in Figure 24. They are essentially flat over the frequency range from .05 to .70 cps. The curves drop off rapidly at frequencies above the Dutch

roll (apparently .6 cps) and the short period (apparently .8 cps). Spectra of the control surface deflection rates shown in Reference 6 decrease with frequency at values from .05 down to .01 cps, and therefore, the angular acceleration spectra similarly decrease with decreasing frequency below .05 cps.

Generally speaking, the maneuver data presented for the F-100 pitch and yaw accelerations do not indicate a consistent drop-off at very low frequencies similar to that shown for one flight of the F-84G in Figure 24. To some extent, this is a result of the fact that very few of the maneuvers were long enough to supply accurate spectral data at these low frequencies. The majority of maneuvers were of 100 second duration or less. This means that only one cycle of frequencies near .01 to .02 cps would be included in the computation of the spectrum for an individual maneuver. In this range of very low frequencies, any errors resulting from linear interpolation would also be most noticeable. For the special case of the spectra determined directly from angular acceleration time histories, the low energy of frequency content under .05 cps is highly sensitive to transcription errors in extracting the information from the oscillograph records. Hence, the actual shape of spectral curves in the very low frequency regions below  $f = .05$  cps is highly suspect. Only in the case of a few runs which were considerably longer than 100 seconds in duration can one expect reasonable accuracy to frequencies as low as .02 cps. Such long runs were plotted separately as Runs 240L and 237L. The time durations of these runs are given in Tables II and III.

These facts raise the question: If individual maneuvers are of the order of 50 seconds duration, what is the significance of power spectral density at frequencies near .01 cps? The answer to this question is that in such instances, the spectrum contains information dependent upon the spacing between maneuvers. If the power spectrum is computed on the basis of one continuous time history representing several independent and separated (time-wise) maneuvers, the very low frequency spectral density points define the average "dead" time separating maneuvers. If maneuvers are treated individually and then the spectra are

averaged (a weighted average based on maneuver duration), the low frequency information represents nothing except error accumulation and thus should be ignored. The latter approach was used in calculations reported herein.

The general conclusions which can be drawn from the computed power spectra can be briefly outlined as follows:

1. The energy content decreases sharply at frequencies greater than the short period natural frequency for cases of longitudinal motion. This is evident from examination of  $\dot{q}$  and  $\Delta n_z$  spectra.
2. The energy content decreases sharply at frequencies greater than the Dutch roll for many cases of lateral-directional motion, but for some cases, energy may persist at frequencies higher than the Dutch roll due to energy inputs at these frequencies (see A4DS, Figures 17 and 23). This characteristic occurs primarily in roll response and to a lesser degree in yawing motions. This is evident from examination of the roll and yaw spectra.
3. In roll response, the maximum energy frequently occurs at frequencies considerably lower than the Dutch roll. (The Dutch roll is clearly indicated for the F-100 in Figures 21 and 22.) This frequency appears to be governed by the time to bank 90° or the roll mode time constant. Considerable energy at this frequency is often associated with the presence of peaks in the time history or roll rate.
4. Somewhat similarly as Case 3 above, the normal acceleration response often has its predominant energy content at frequencies below .05 cps. This is governed by the duration of pull-up maneuvers. It is this energy peak which most often is associated

with peaks in the time history.

An analysis of the spectral curve shapes is best summarized as follows. The specific dynamic characteristics of an aircraft do influence the shape of power spectral curves for maneuvering motions; however, a brief examination of time histories seems to indicate that the energy which contributes to the peak loads or motions is often at frequencies other than the aircraft natural frequencies. Hence, the peak load occurrence rate is not highly sensitive to the specific dynamic characteristics of an aircraft. The number of peak loads per hour is primarily dependent upon the type of mission and varies considerably with individual maneuvers within any given mission. In other words, it appears that the variability of spectral curve shapes and peak counts is sufficiently great to mask the effect of particular dynamic characteristics of an aircraft. What is the cause of this variability? Can it be due to the maneuver character only or does each pilot contribute toward the variability? It is desirable to explore further this variability of spectral shape and to determine the contributions to this variability resulting from different pilots flying the same aircraft on the same mission (interpilot variability) and resulting from the same pilot flying the same aircraft and mission at different times or on different days (intrapilot variability).

#### Significance of Predominant Frequencies for Load Calculations

The expressions which can be used to compute approximate values for the loads acting on the various components of an airframe are given on page 17 of Reference 7. These equations relate the angular and translational accelerations and angular velocities to the component loads. A number of cross products such as:  $qr$ ,  $r^2$ ,  $p^2$ , and  $pq$  are involved. If the exact form of motion is known, these cross coupling terms can be taken into account. However, if no information is available as to the type of motion, it is difficult, if not impossible, to specify whether a peak in  $r^2$  occurs at the same time as a peak in  $pq$ . If the type of motion is known to be a Dutch roll oscillation, the cross coupling can be specified from information such as the moments and cross products of inertia

and the roll to yaw ratio of the aircraft at that particular range of dynamic pressure and Mach number. If the motion is of a type different from the Dutch roll, it may be possible to similarly specify the required cross coupling.

The type of motion in which an aircraft is involved is closely related in many instances to the basic frequency content of the motion. To the extent that this is true, knowledge of the basic frequencies which contribute to various peaks in the motion time histories will be useful in enabling one to estimate cross coupling effects in the computation of component loads.

## CONCLUSIONS AND RECOMMENDATIONS

A number of aspects of the general maneuver loads problem have been examined. The characteristics of time histories of maneuver motions which are most important for the specification of maneuver load design criteria are the probability distributions of peak accelerations and velocities, and the predominant frequencies present in these peaks. The rate of occurrence of peaks is a primary factor in determination of fatigue life. The frequency content of peaks is directly related to the type of motions which produce the peaks and thus is important for converting the magnitudes of peak motions to magnitudes of peak loads on the horizontal tail or on other airframe components.

The analysis has shown that the type of mission (e. g. , air to air, air to surface, etc.) and the type of aircraft (delta wing fighter, twin-engine bomber, etc.) are the two most important factors which influence the peak probability distributions. The spectral analysis has shown that the predominant frequencies present in maneuver motions, and which appear to be closely related to peaks in the time histories, are not necessarily the natural frequencies of rigid airframe response such as Dutch roll and short period. They appear, in the case of normal load factor and of roll acceleration, to be related to the inherent character of how pilots perform such common maneuvers as pull-ups, turns, and rolls.

A statistical model of maneuvering flight was examined and appeared quite promising in its ability to represent the important characteristics of the time histories for maneuver motions.

An important outcome of continued research along the lines indicated above may be that the character of important time history peaks can be generalized in such a manner that their frequency of occurrence can be sampled and analyzed by methods somewhat simpler than those which seem practical at the present time. The simplification implied here is related to two aspects

of the large scale sampling program. One aspect is determination of the criteria and means for locating peaks in time histories of maneuvering motions and the second is determination of the type of motions involved in the peaks which are essential for accurate peak load estimation.

The direction of continued research along these lines would seem to be to obtain more peak count data and to analyze it by separation into categories of mission type and aircraft type, and then to examine how one can relate these categories as a function of aircraft characteristics. For continued research into the frequency content of peaks, it is desirable to analyze the causes of variability in spectral shape and also to analyze the maneuver spectra for aircraft different from those types thus far examined.

Investigation of the frequencies which contribute to the peak motions and peak loads should be based upon the following approach. The recorded time histories would be passed through band pass filters which have central frequencies in the vicinity of the short period, the Dutch roll and other frequencies of the order of .1 and .05 cps where the power spectrum was found to contain maximum energy concentrations. The filtered time histories would then be compared to the original. If a significant peak in the original unfiltered time history coincided (occurred at the same point in time) with a significant peak in one of the filtered time histories, it could be concluded that the particular peak was the result of the motions near the central frequency of the particular band pass filter. Such information is very useful for developing simple statistical models which will accurately represent peak occurrences. Such information also is extremely useful to the consideration of load cycle type of information needed for analysis of structural fatigue accumulation. A third use for such information is that it indicates much about the type of motion which caused the peak. If a natural rigid body mode of response (e.g., Dutch roll mode) is the frequency which contributes to certain peak motions, the computation of component loads such as tail load can be made accurately since the particular aerodynamic cross coupling and inertia coupling relating

roll to yaw can be estimated with reasonable accuracy for any particular type of aircraft in the Dutch roll mode. Thus, the equations relating component load to aircraft motion can be utilized in their more exact form rather than in an approximate form required by unavailability of coupling information.

## LIST OF SYMBOLS

$\Delta n_z$	normal acceleration, plus for a pull-up, "g" units
$p$	angular rate of roll, rad/sec
$q$	angular rate of pitch, rad/sec
$r$	angular rate of yaw, rad/sec
$\dot{p}$	angular acceleration in roll, rad/sec <sup>2</sup>
$\dot{q}$	angular acceleration in pitch, rad/sec <sup>2</sup>
$\dot{r}$	angular acceleration in yaw, rad/sec <sup>2</sup>
$\Phi_x$	power spectral density of variable $x$ , units of $(x)^2/\text{cps}$
$\sigma^2$	mean square value of $x$ , $= \int \Phi_x df$
$\Sigma^2$	$= \int f \times \Phi df$
$T_m$	time duration of maneuvering motions
$T$	total time duration of flight record, including maneuvering and straight and level flight

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3. Aseltine, J.A.: Transform Method in Linear System Analysis. McGraw-Hill Electrical and Electronic Engineering Series, 1958, Chapter 14.
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5. Notess, C.B.: Analysis of Turbulence Data Measured in Flights at Altitudes up to 1600 Feet Above Three Different Types of Terrain. CAL Report No. TE-1215-F-1. April 1959.
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TABLE I  
PEAKS PER HOUR FOR MANEUVERING FLIGHT  
FROM REFERENCE 2

<u>Aircraft Type</u>	<u>Average Number of Peaks per Hour</u>			
1. AD's	6.8	72.7	26.8	11.0
2. Early Jets	10.9	22.1	33.5	17.8
3. Recent Jets	34.7	46.2	13.4	-
4. Bombers	11.2	15.3	9.0	-

<u>Aircraft Type</u>	<u>Normalized Peaks per Hour</u>			
1. AD's	1	10.7	4.0	1.6
2. Early Jets	1	2.0	3.1	1.7
3. Recent Jets	1	1.3	.38	
4. Bombers	1	1.4	.81	

NOTE:

1. includes AD-4B, AD-4N, AD-5 and AD-6.
2. includes F9F-8, A4D-1, and F2H-4.
3. includes F11F-1, F3H-2, and F8U-1.
4. includes P2V-7, S2F-1, and A3D-2.

**TABLE II**  
**DESCRIPTIVE INFORMATION FOR F-100A FLIGHT DATA**

Flight No.	Individual Maneuver No.	Duration seconds	Pilot Comments	
241	1	50	Air Combat Maneuvers:	
	2	60	40,000 feet simulated attacks - supersonic	
	3	45		
	4	55	35,000 feet simulated defensive breaks at M = .9, 1.1, and 1.5	
	5	25		
	6	25		
240	2	70	Air to Air Gunnery:	
	4	115	Simulated passes on other airplanes, firing at tow targets	
	5	90		
	6	95	Pilot tracking fair (no Yaw Damper)	
	7	30	Last two passes were supersonic. Tracked target, but didn't fire.	
	8	10	Target speed 165 knots IND.	
	9	55		
	240L	1	210	
	237A	3	60	Air to Ground Gunnery and Bombing:
4		50	1 spacer pass on air to ground range	
5		60	4 low angle or skip bomb runs	
6		50	6 low angle strafing passes	
7		50	4 rocketry passes	
8		90	4 high angle bomb passes	
237B		9	40	2 loops on return to base
		10	50	
	11	40		
	12	50		
	13	50		
237L	1	327.4		
	2	252.4		

TABLE III  
 DESCRIPTIVE INFORMATION FOR F11F, A4D, AND A3D  
 FLIGHT DATA

<u>Aircraft, Flt. No.</u>	<u>Individual Maneuver No.</u>	<u>Duration seconds</u>	<u>Remarks on Maneuvers</u>	
F11F	1	100	Long 3"g" pull-ups	
	3	130	Sharp turns	
	4	80	Throttle change	
	5	60	One severe roll oscillation	
	6	190	Maneuver in rough air	
	7	70	Maneuver in rough air	
	8	100	Maneuver in rough air	
	9	140	Turn	
	A4D-L	1	73.7	Slow rock
2		55.0	Low "g" roller coaster	
5		77.3	High "g" pull-up	
6		107.3	High "g" pull-up	
7		90.7	45° - 60° turn	
9		112.0	Throttle change	
10		73.0	Two turns	
11		115.7	Split-S and turn	
12		53.5	360° turn	
13		96.3	360° turn	
14		30.7	360° roll	
15		30.0	90° bank reversal	
A4D-S		3	31.7	Fast rock
		4	60.3	Fishtails
A3D-2		1	600	General Maneuvering

FIGURE 1  
 CUMULATIVE FREQUENCY OF OCCURRENCE  
 OF NORMAL LOAD FACTOR PEAKS

Data from Reference 2

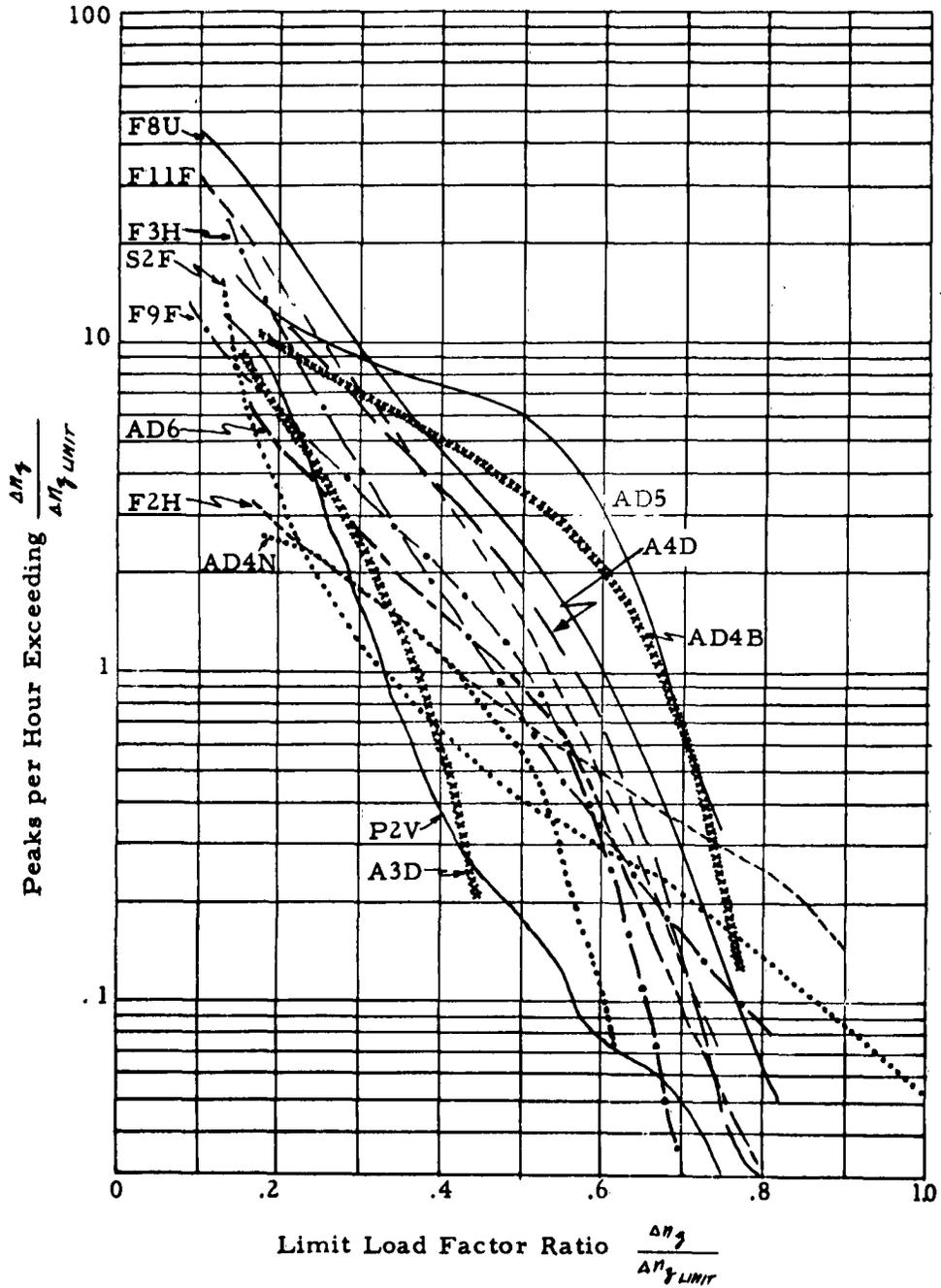


FIGURE 2  
 CUMULATIVE FREQUENCY OF OCCURRENCE  
 OF NORMAL LOAD FACTOR PEAKS

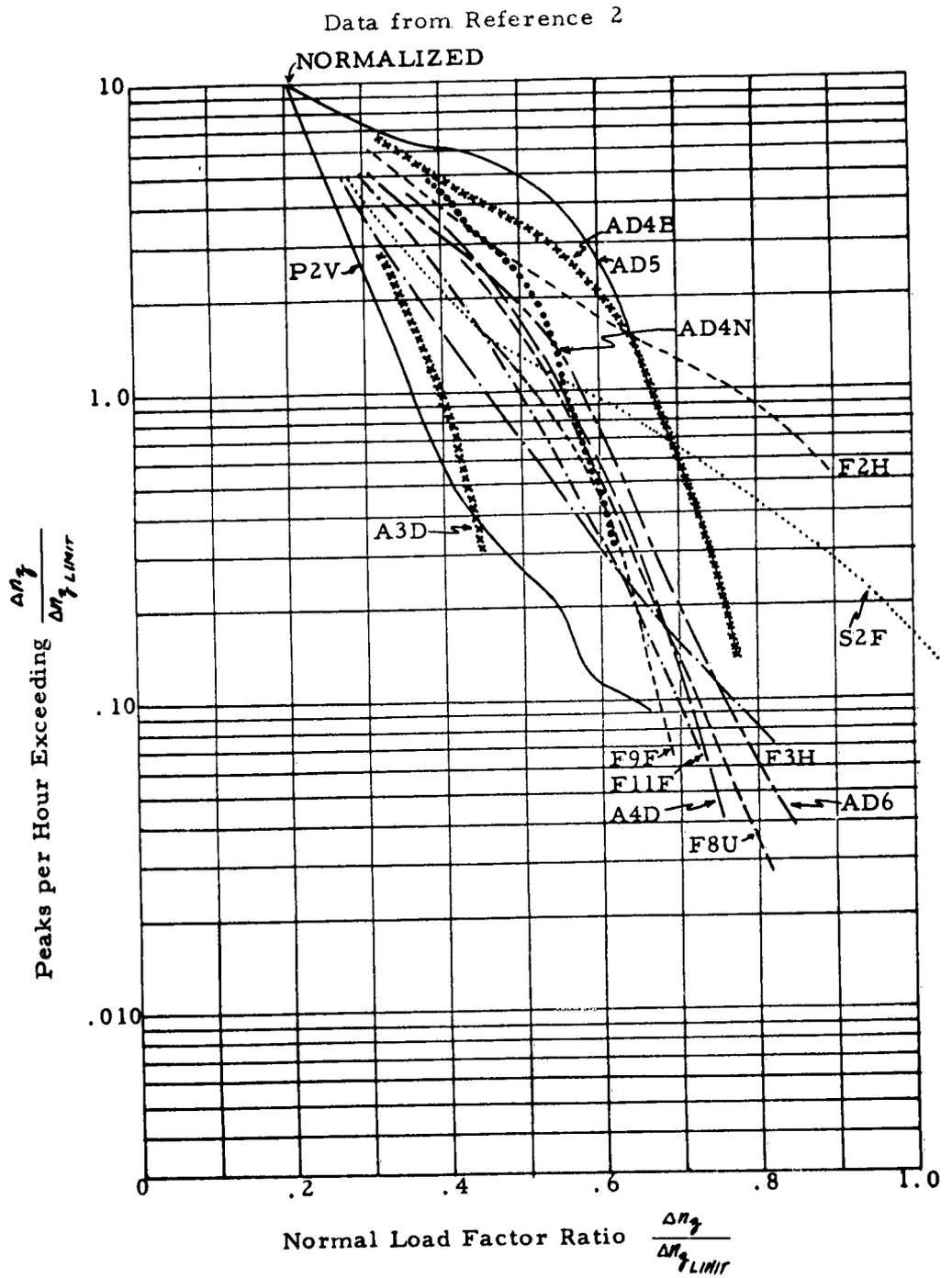


FIGURE 3  
 CUMULATIVE FREQUENCY OF OCCURRENCE  
 OF PITCH ACCELERATION PEAKS

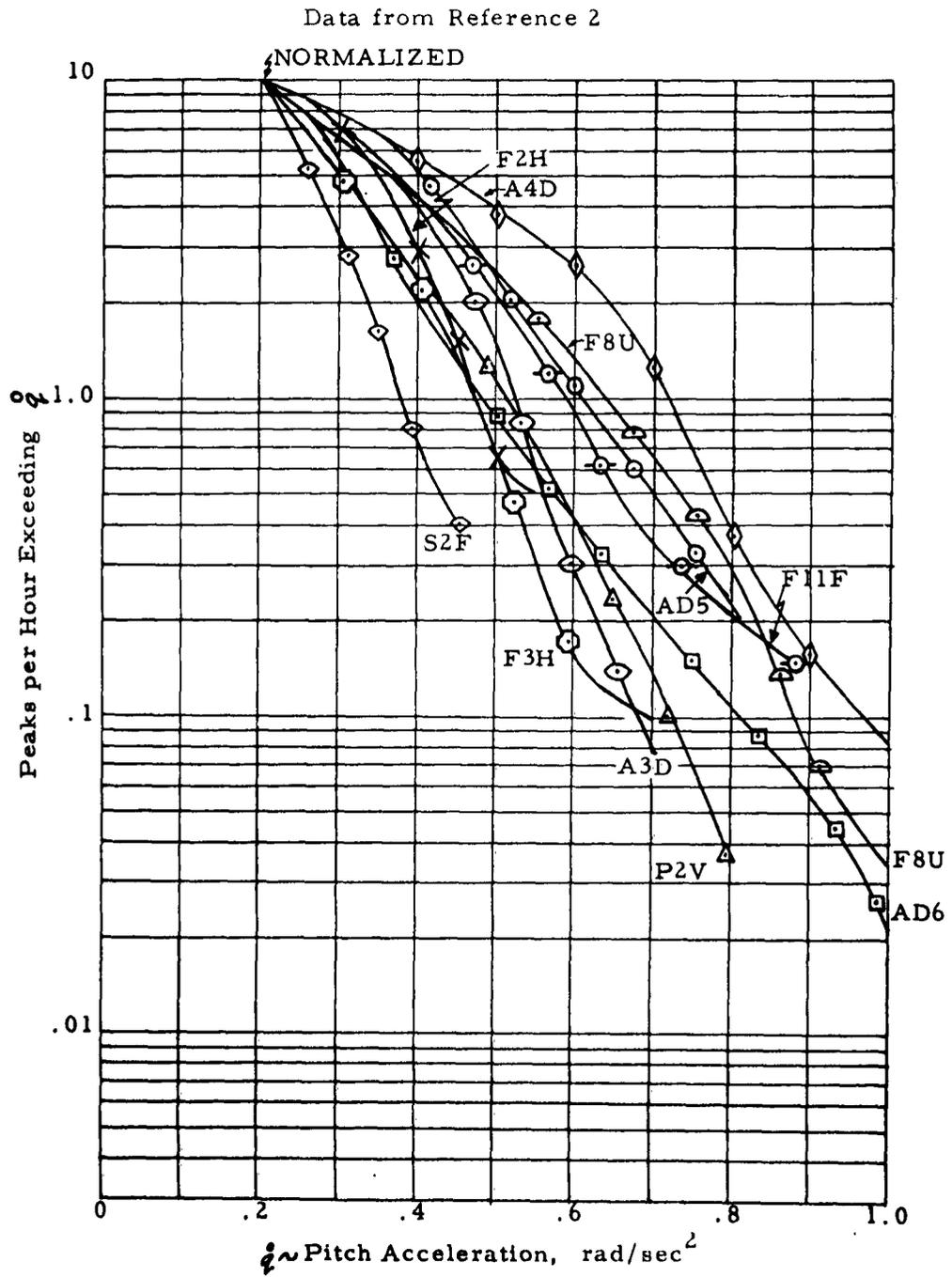


FIGURE 4  
 CUMULATIVE FREQUENCY OF OCCURRENCE  
 OF ROLL ACCELERATION PEAKS

Data from Reference 2

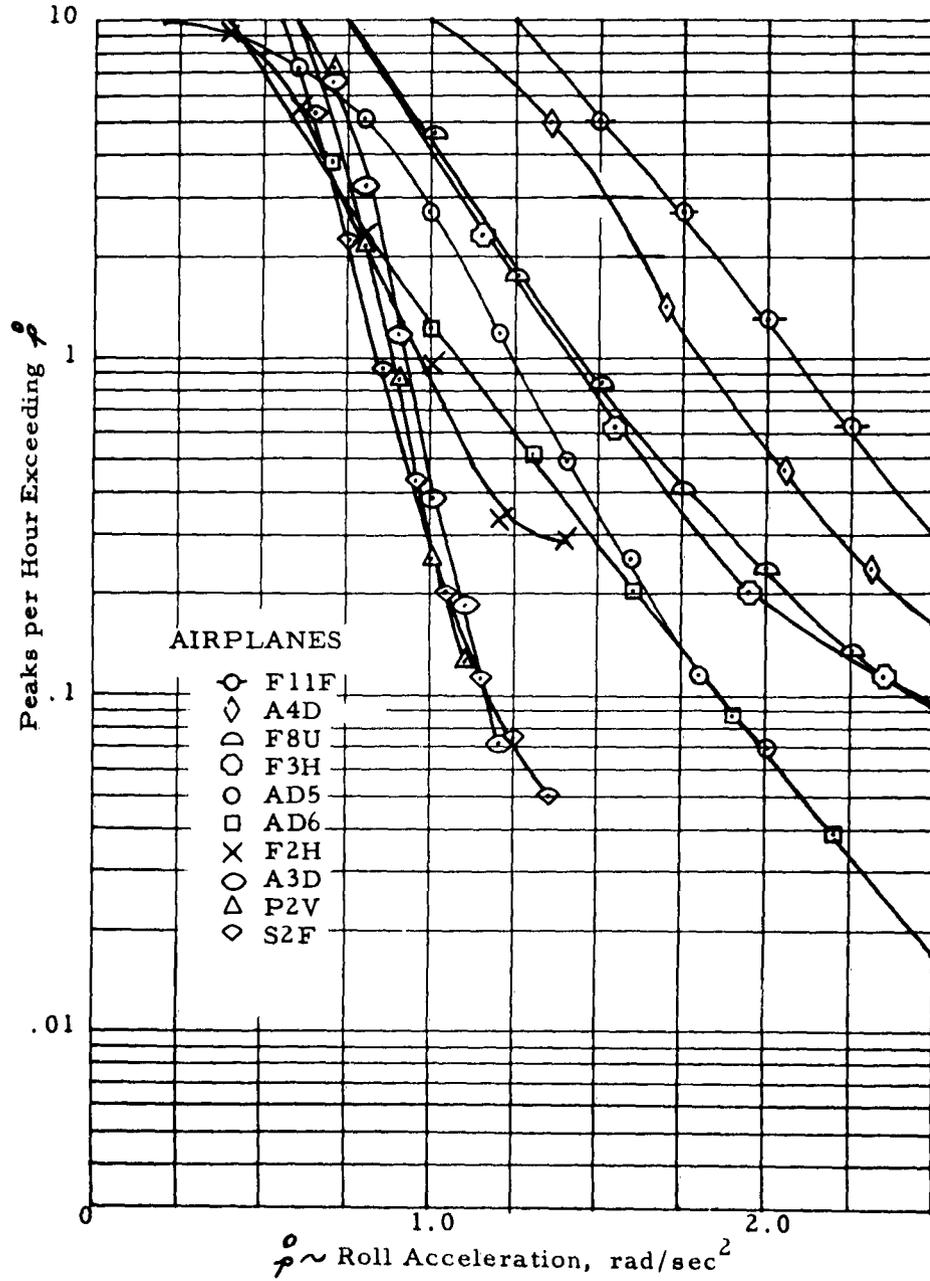


FIGURE 5  
 CUMULATIVE FREQUENCY OF OCCURRENCE  
 OF YAW ACCELERATION PEAKS

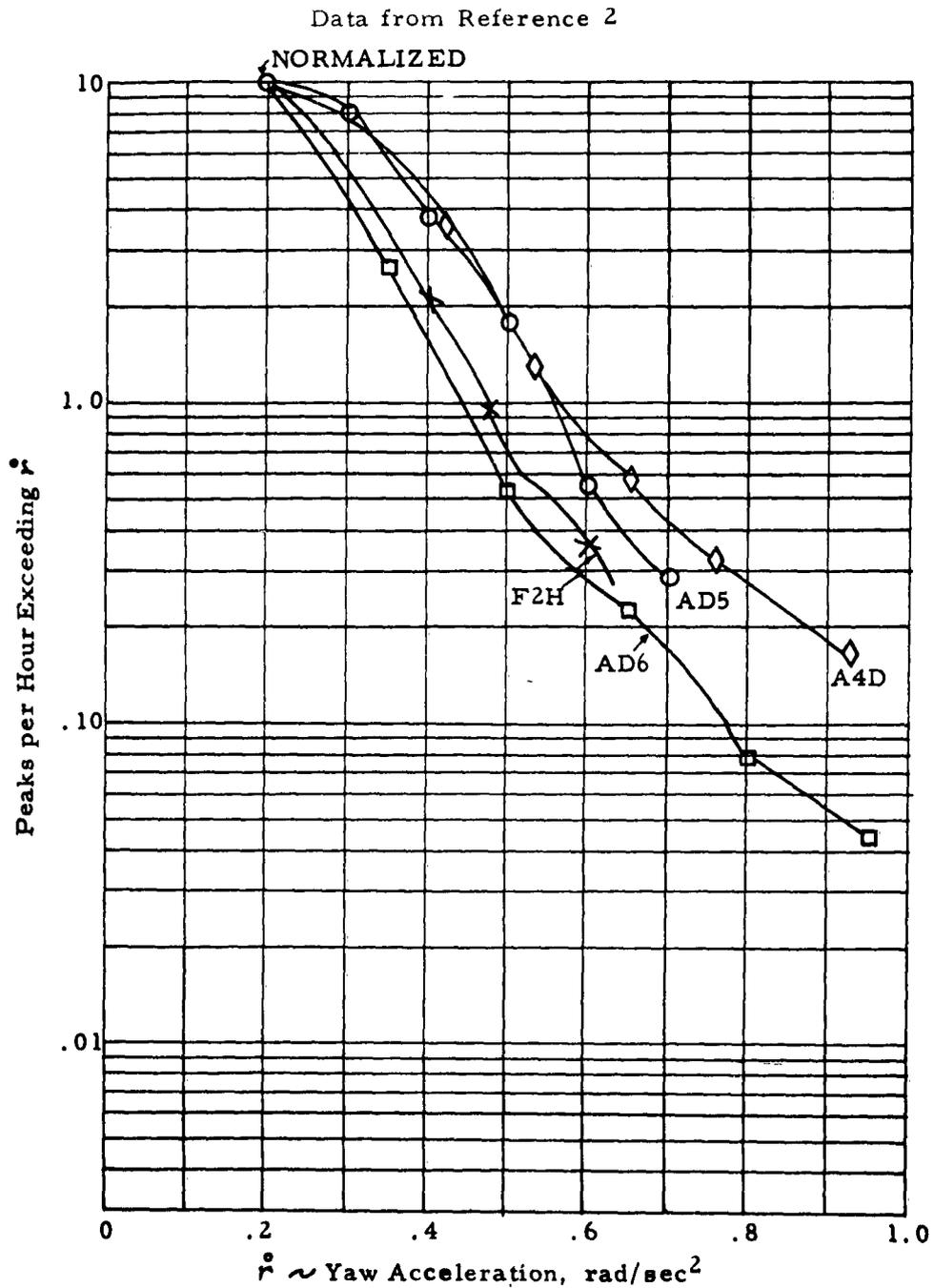
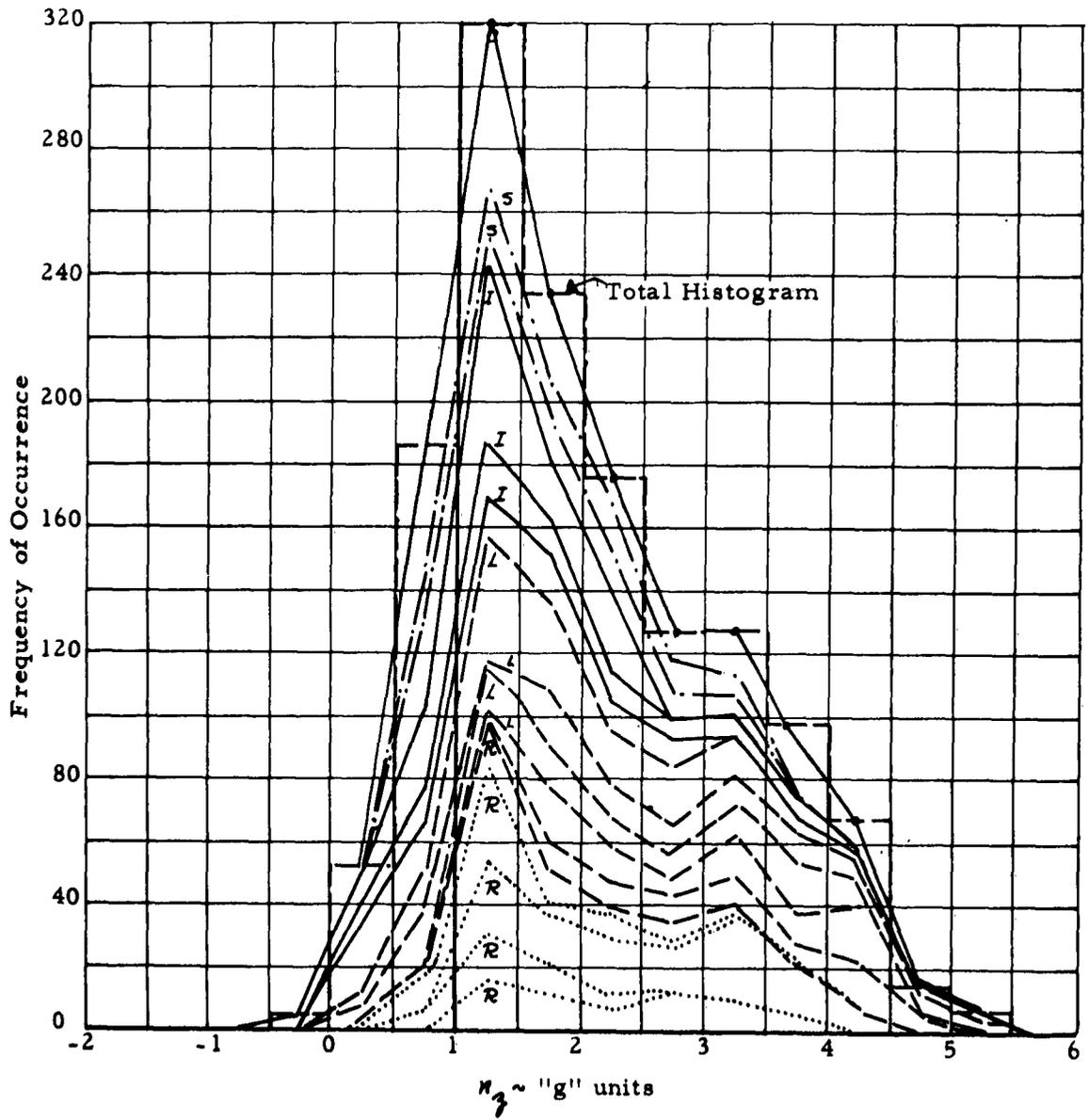


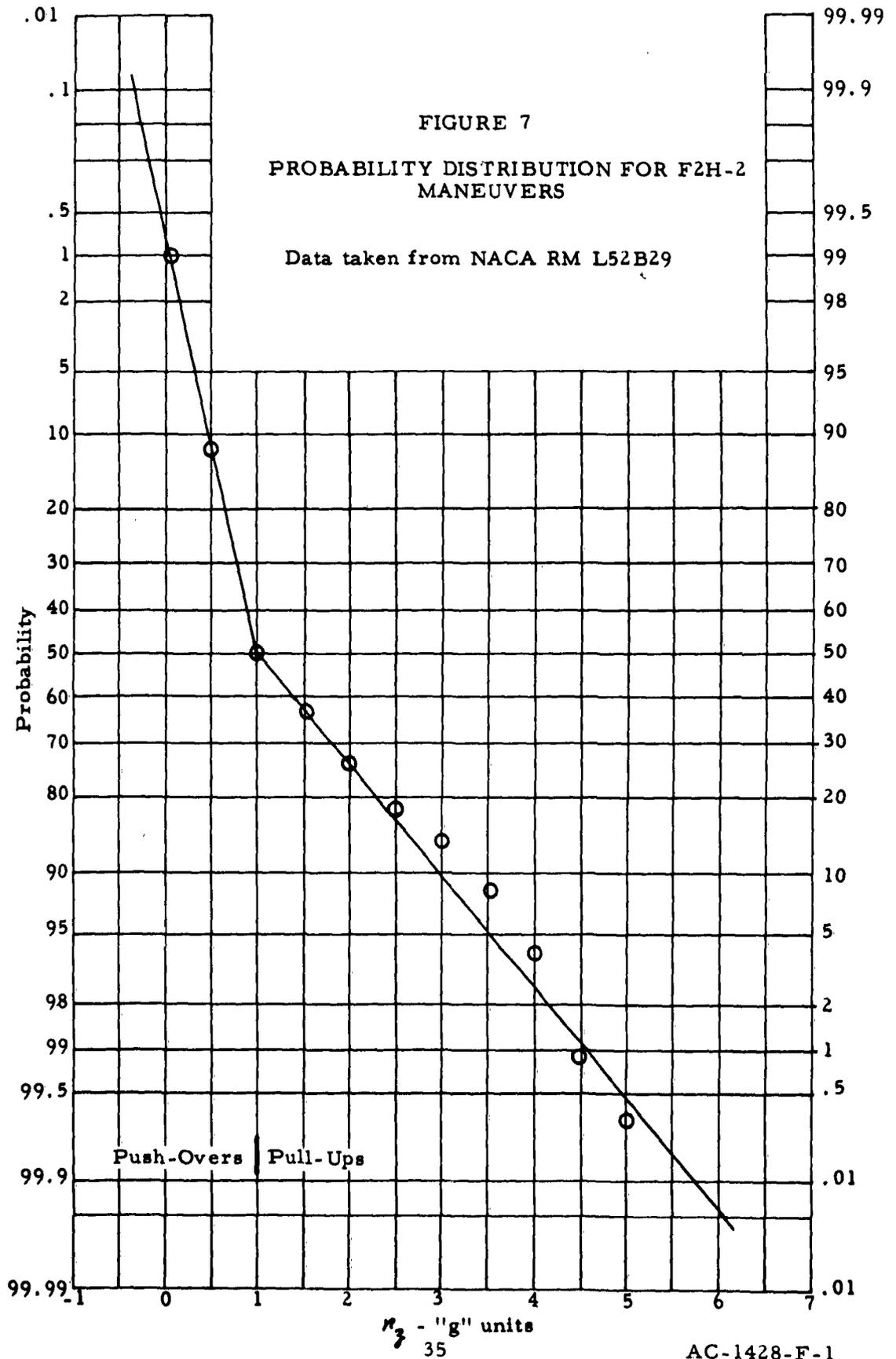
FIGURE 6

CUMULATIVE COMPILATION OF PROBABILITY DISTRIBUTION

$n_z$  - Normal Acceleration for an F2H-2  
Time Histories Obtained from NACA RM L52B29

Sampling Interval  $\Delta t = .5$  Sec





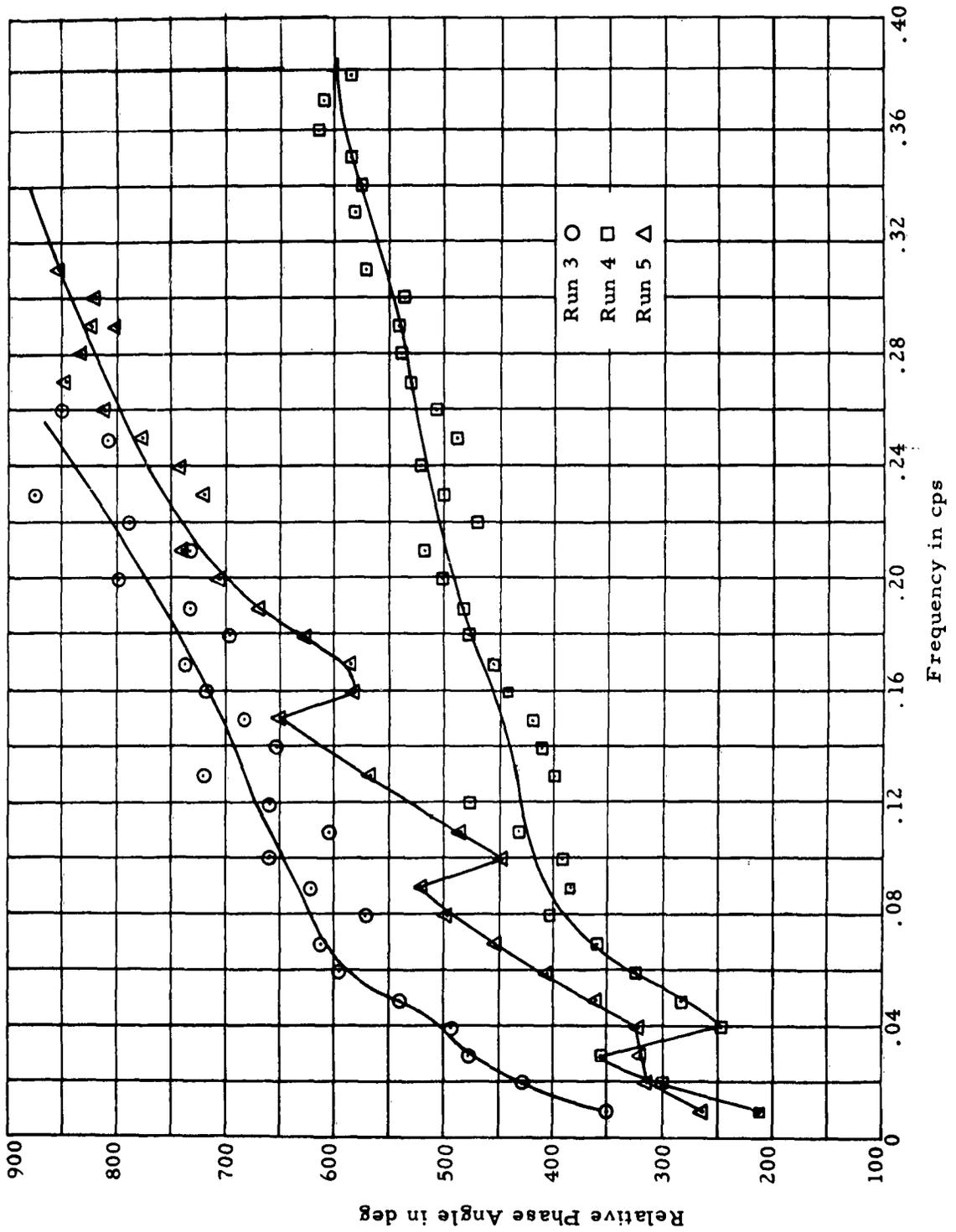


FIGURE 8 HARMONIC ANALYSIS OF ROLL RATE - FLIGHT 241, F-100

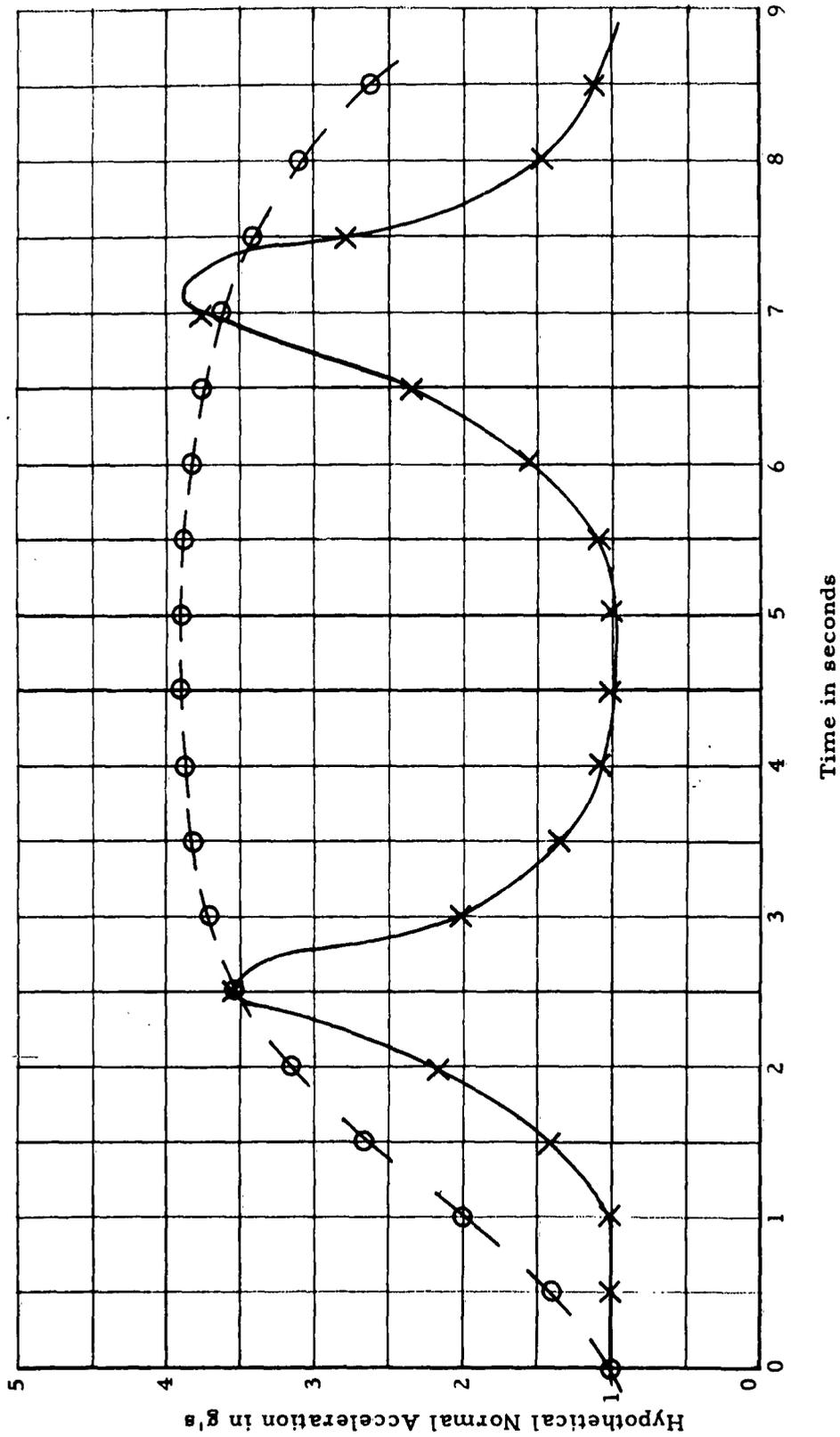
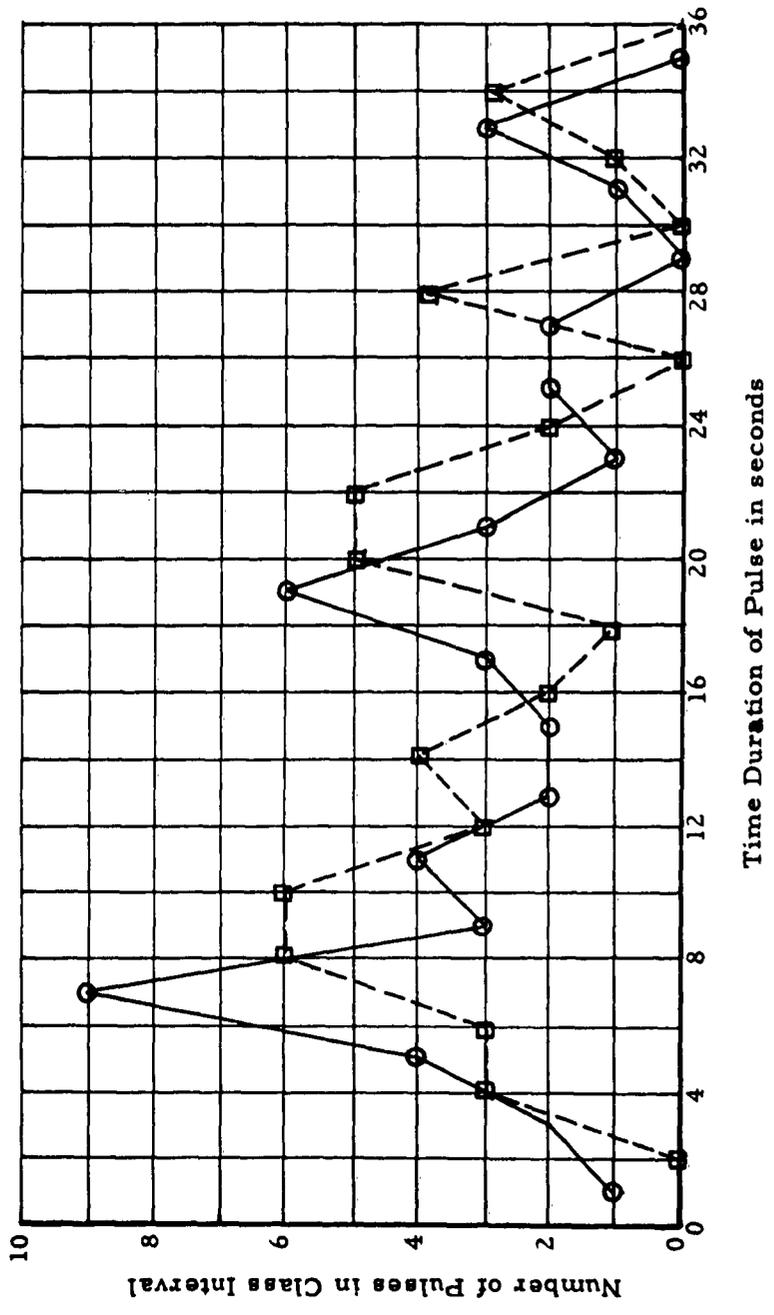
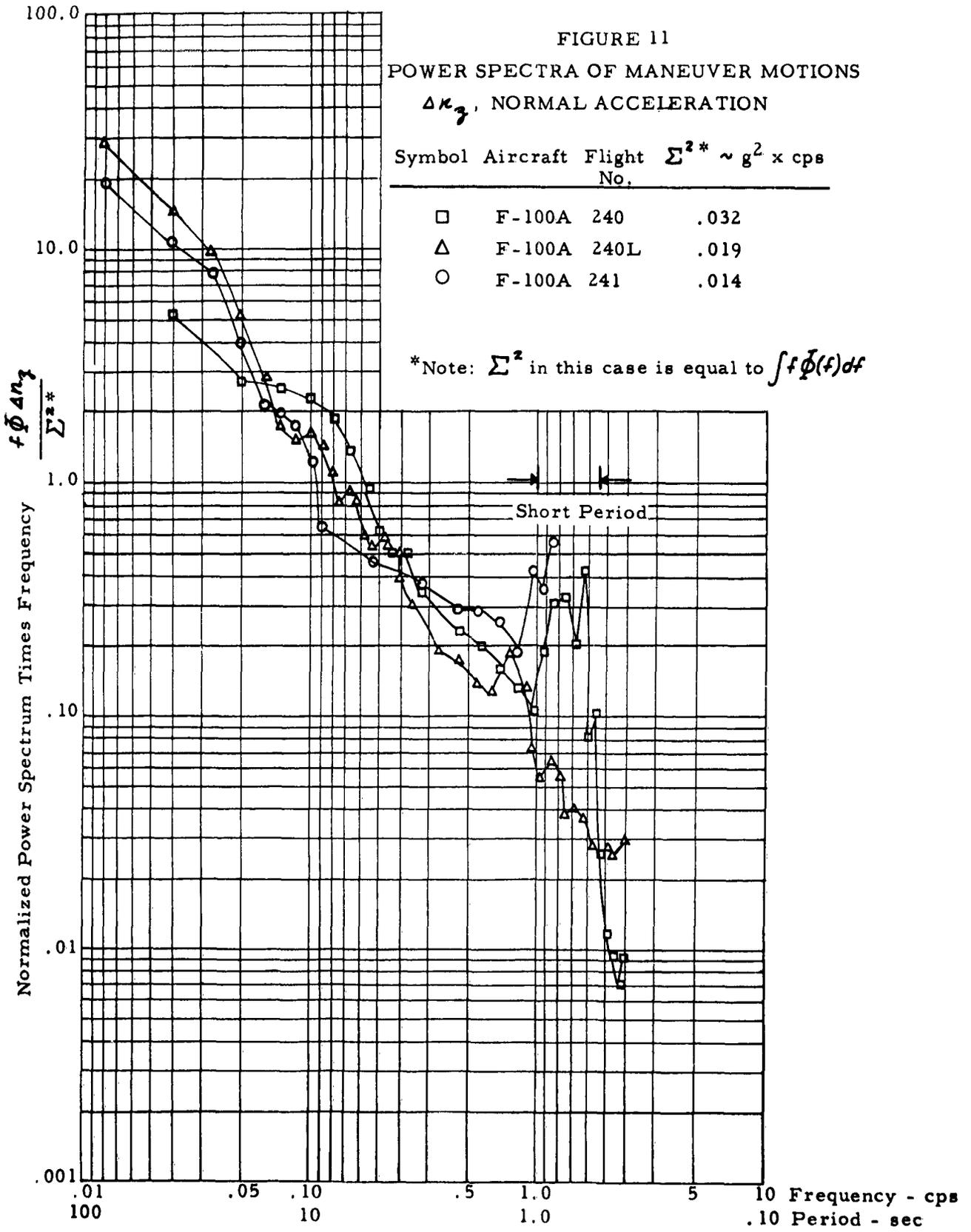


FIGURE 9 ILLUSTRATION OF THE SIGNIFICANCE OF KNOWING THE TIME SEQUENCE OF DATA POINTS

FIGURE 10  
 PROBABILITY DISTRIBUTION OF FITTED  $n_2$  PULSES FOR A4D-1  
 48 Major Peaks Roughly Fitted with Rectangular Pulses  
 ----- Class Interval 0.1 - 2, 2.1 - 4, 4.1 - 6  
 - - - - - Class Interval 1.1 - 3, 3.1 - 5, 5.1 - 7





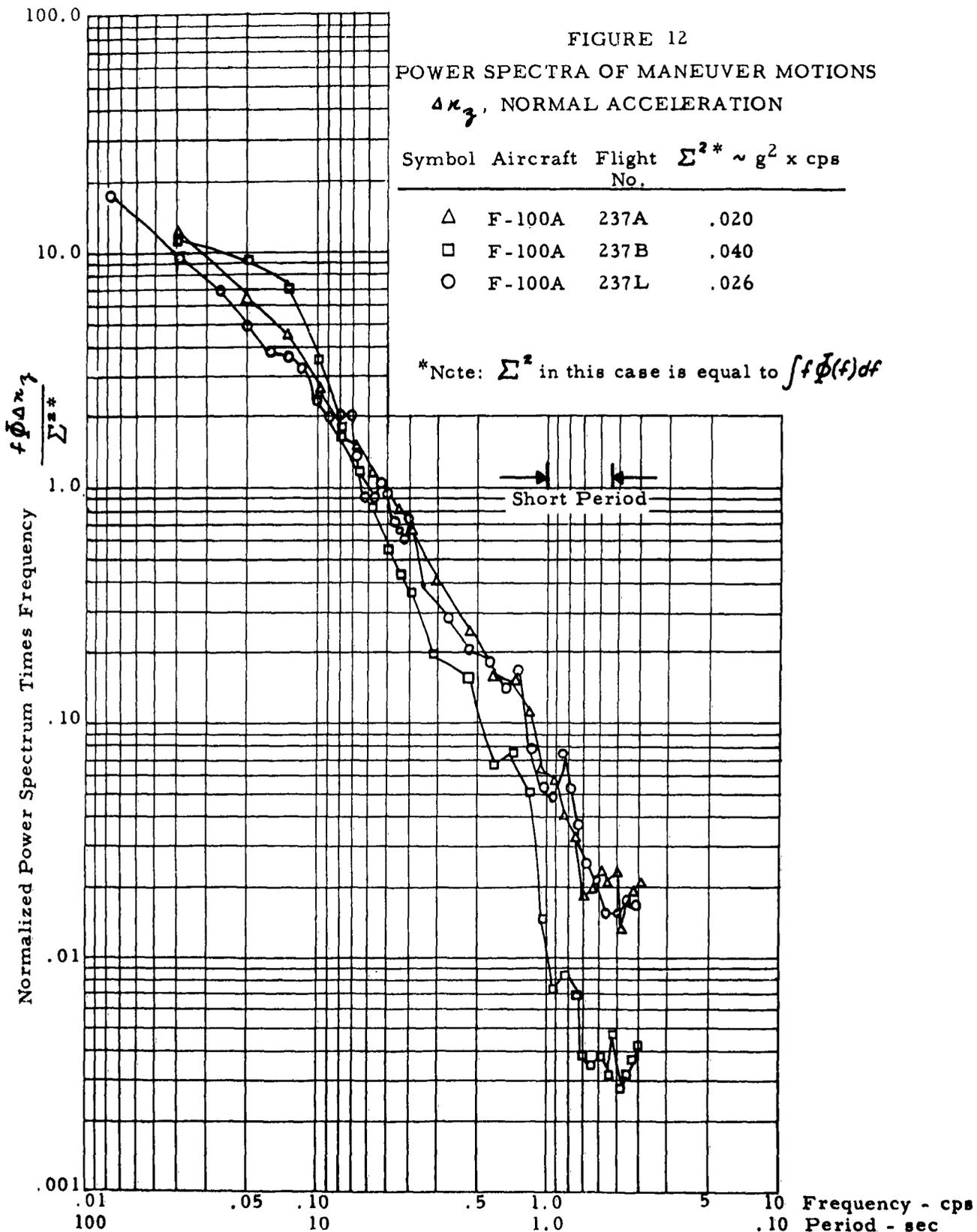


FIGURE 13  
 POWER SPECTRA OF MANEUVER MOTIONS  
 $\Delta \kappa_z$ , NORMAL ACCELERATION

Symbol	Aircraft	Flight No.	$\Sigma^2 \sim g^2 \times \text{cps}$
$\Delta$	A4D	Run 1, 2, 5-7, 9-15	.011
$\circ$		Run 3, 4	.0036
$\square$	F11F		.0044

\*Note:  $\Sigma^2$  in this case is equal to  $\int f \phi(f) df$

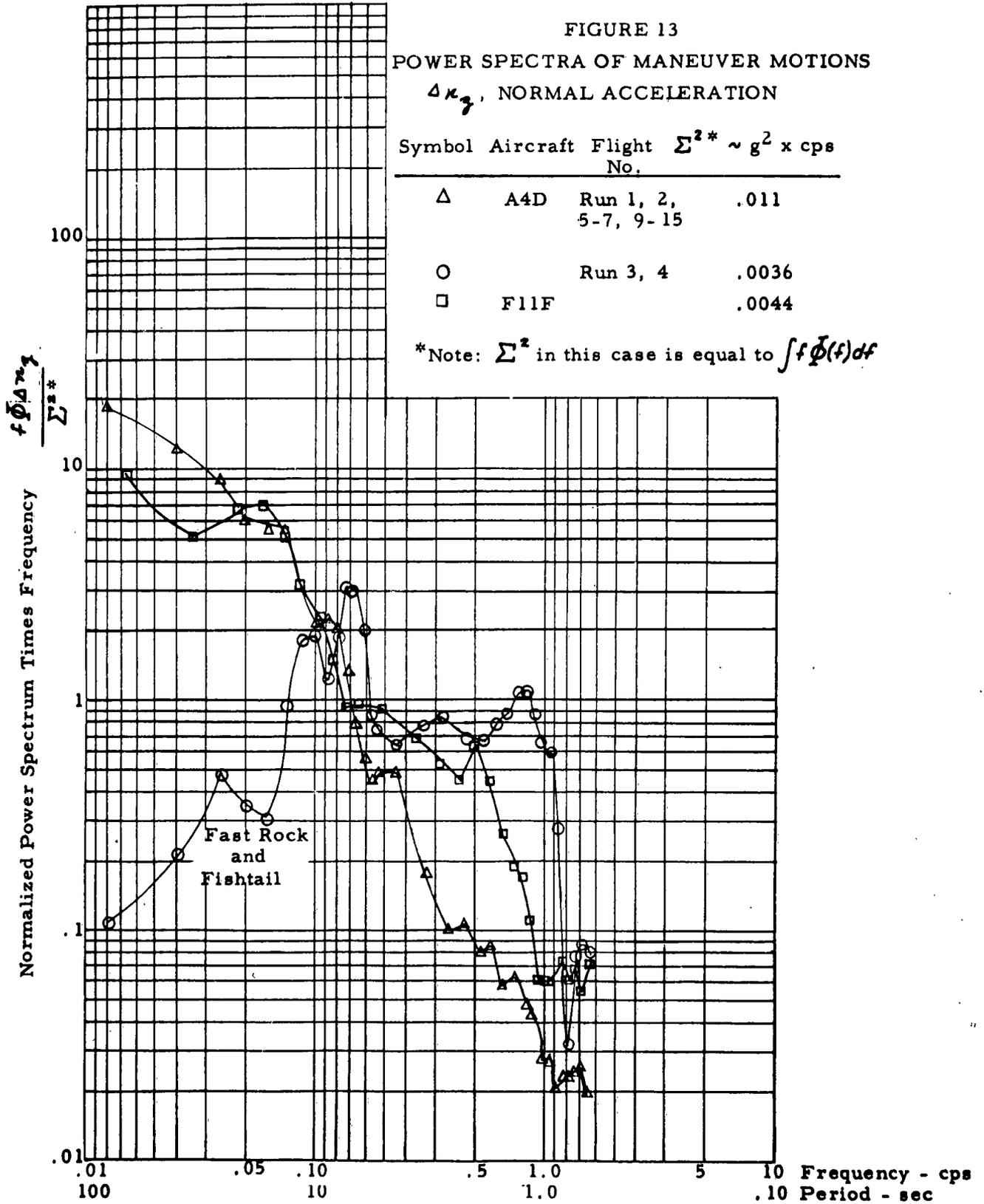


FIGURE 14  
 POWER SPECTRA OF MANEUVER MOTIONS  
 $\Delta \kappa_3$ , NORMAL ACCELERATION

Symbol Aircraft Flight  $\Sigma^{2*} \sim g^2 \times \text{cps}$   
 No.

X A3D-2 .0031

$T_m$  = Record Length - 600 seconds of maneuvers

\*Note:  $\Sigma^2$  in this case is equal to  $\int f \Phi(f) df$

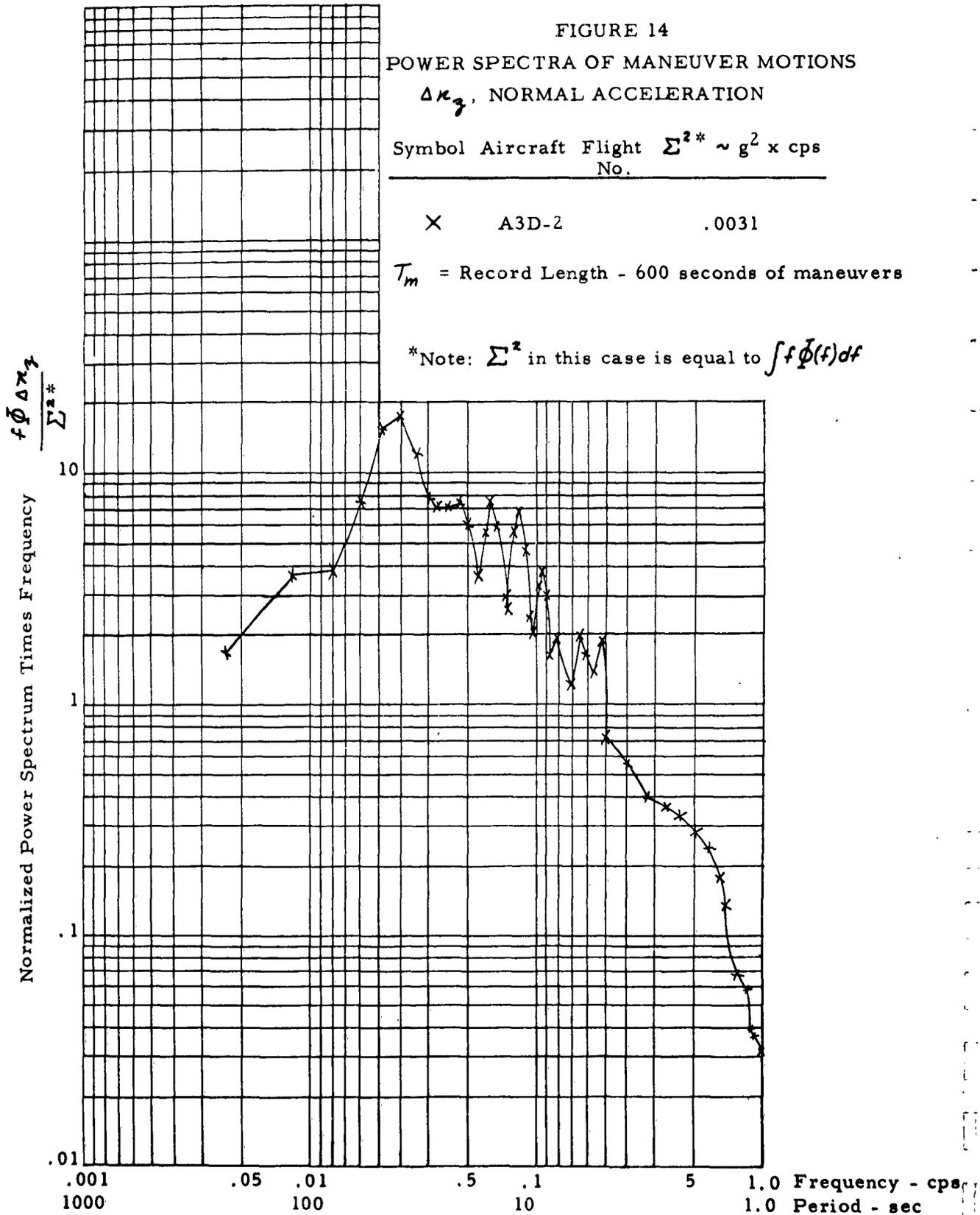


FIGURE 15

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{\phi}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
□	F-100A	240	.080
△	F-100A	240L	.017
○	F-100A	241	.032

\* Note:  $\sigma^2$  in this case is equal to  $\int \dot{\phi}_p^2 df$

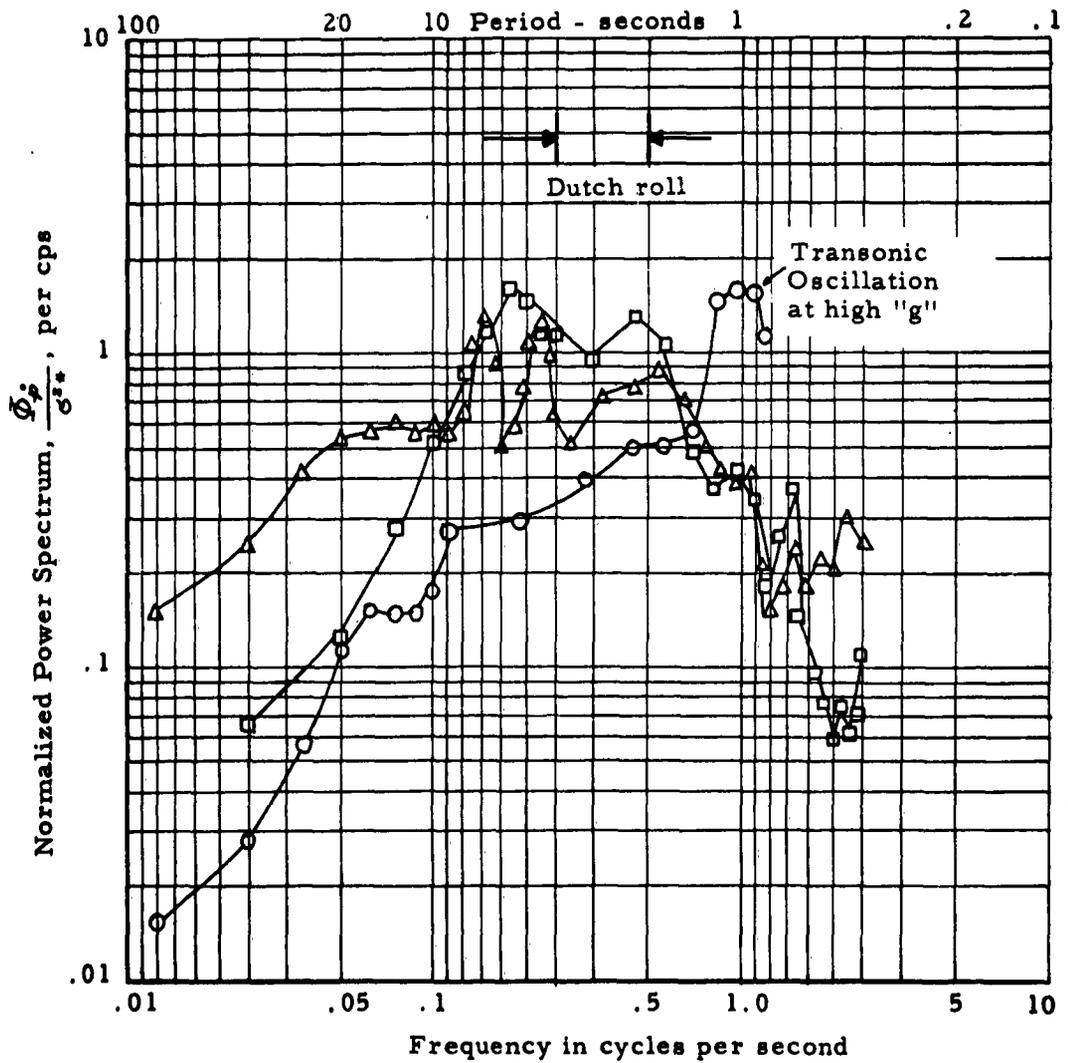


FIGURE 16

POWER SPECTRA OF MANEUVER MOTIONS

$\ddot{\phi}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
Δ	F-100A	237A	.012
□	F-100A	237B	.019
○	F-100A	237L	.038

\* Note:  $\sigma^2$  in this case is equal to  $\int \ddot{\phi}^2 df$

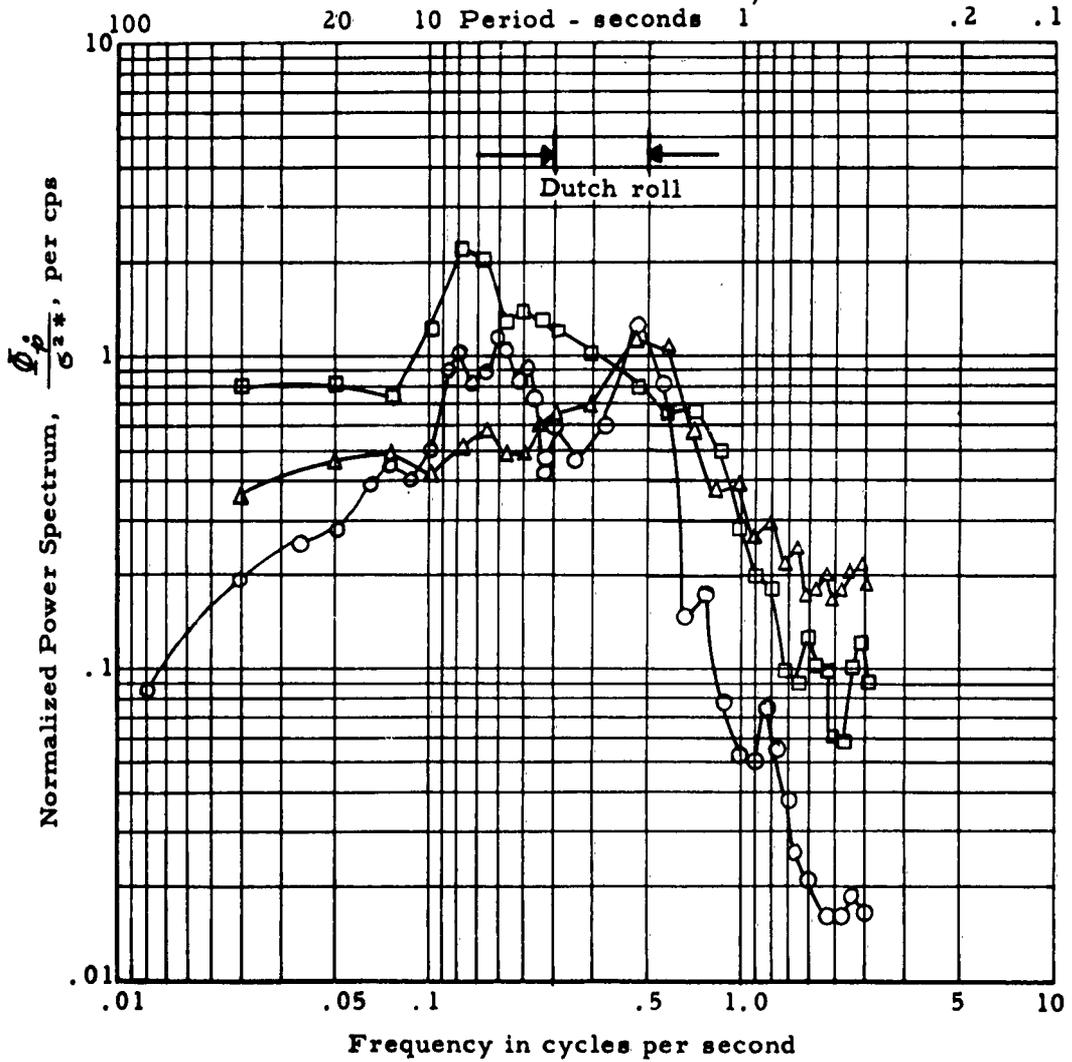


FIGURE 17

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{\phi}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
△	F11F		.039
□	A4DL	Runs 1, 2, 5-7, 9-15	.069
○	A4DS	Runs 3, 4	1.79

\* Note:  $\sigma^2$  in this case is equal to  $\int \dot{\phi}^2 df$

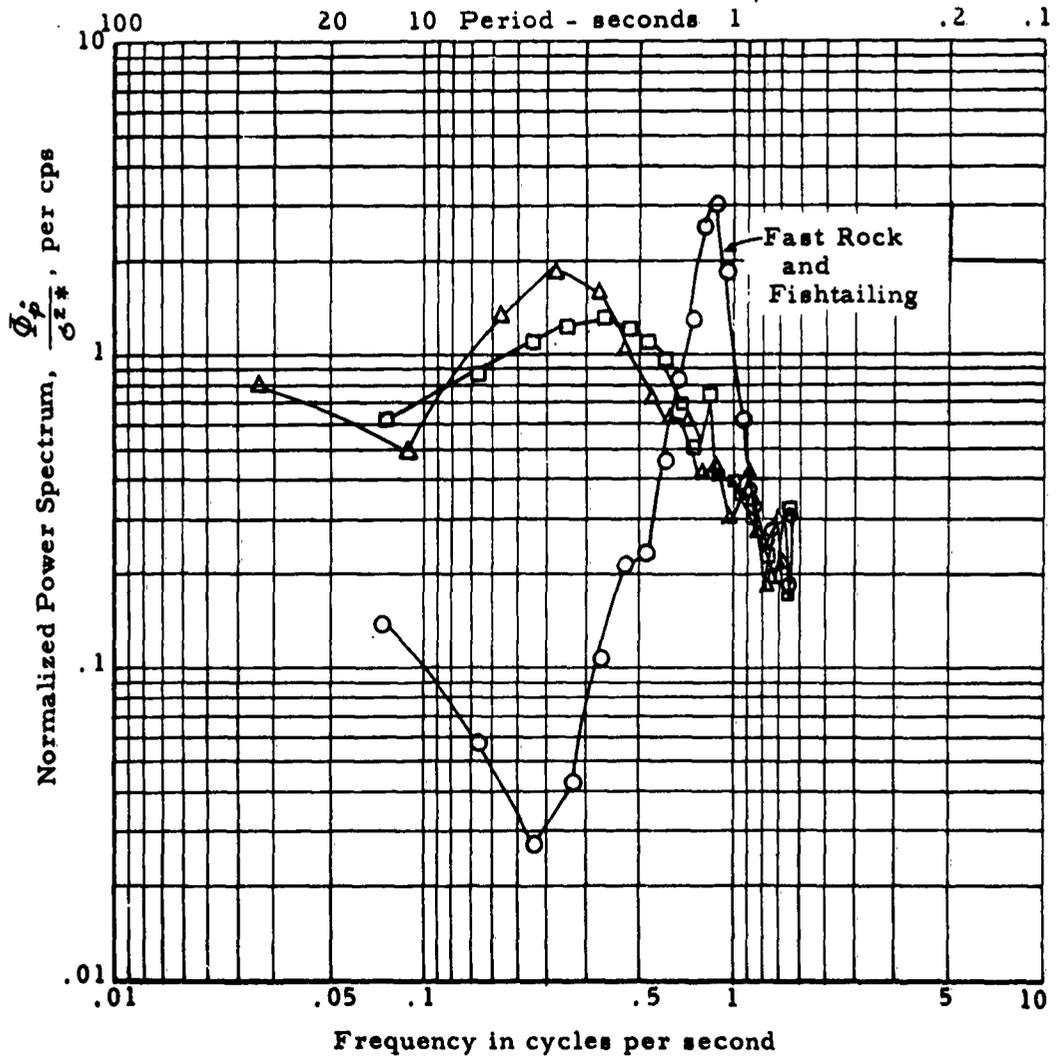


FIGURE 18

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{q}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
□	F-100A	240	.014
△	F-100A	240L	.0010
○	F-100A	241	.019

\* Note:  $\sigma^2$  in this case is equal to  $\int \Phi_{\dot{q}} df$

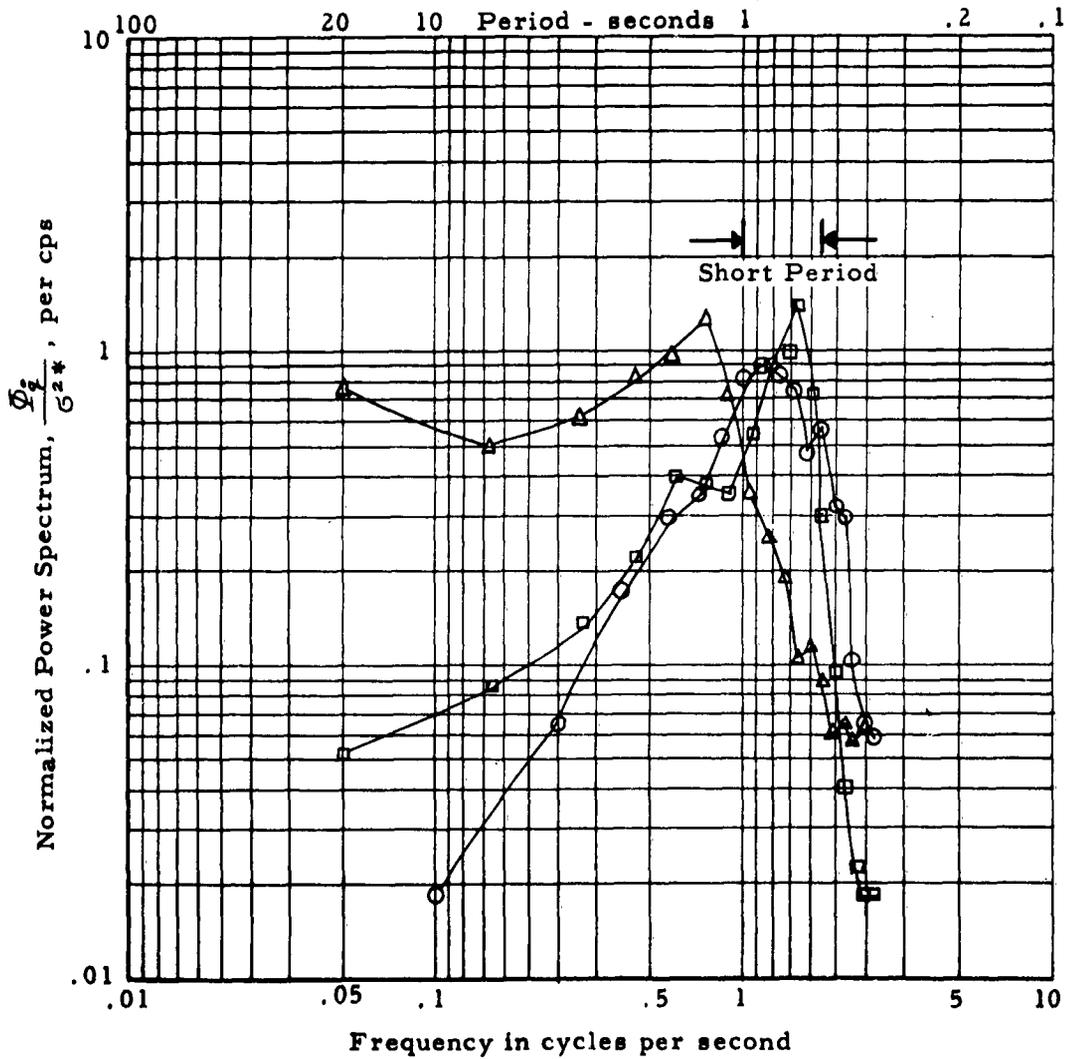


FIGURE 19

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{\phi}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
△	F-100A	237A	.0018
□	F-100A	237B	.0026
○	F-100A	237L	.0041

\* Note:  $\sigma^2$  in this case is equal to  $\int \dot{\phi}_i^2 dt$

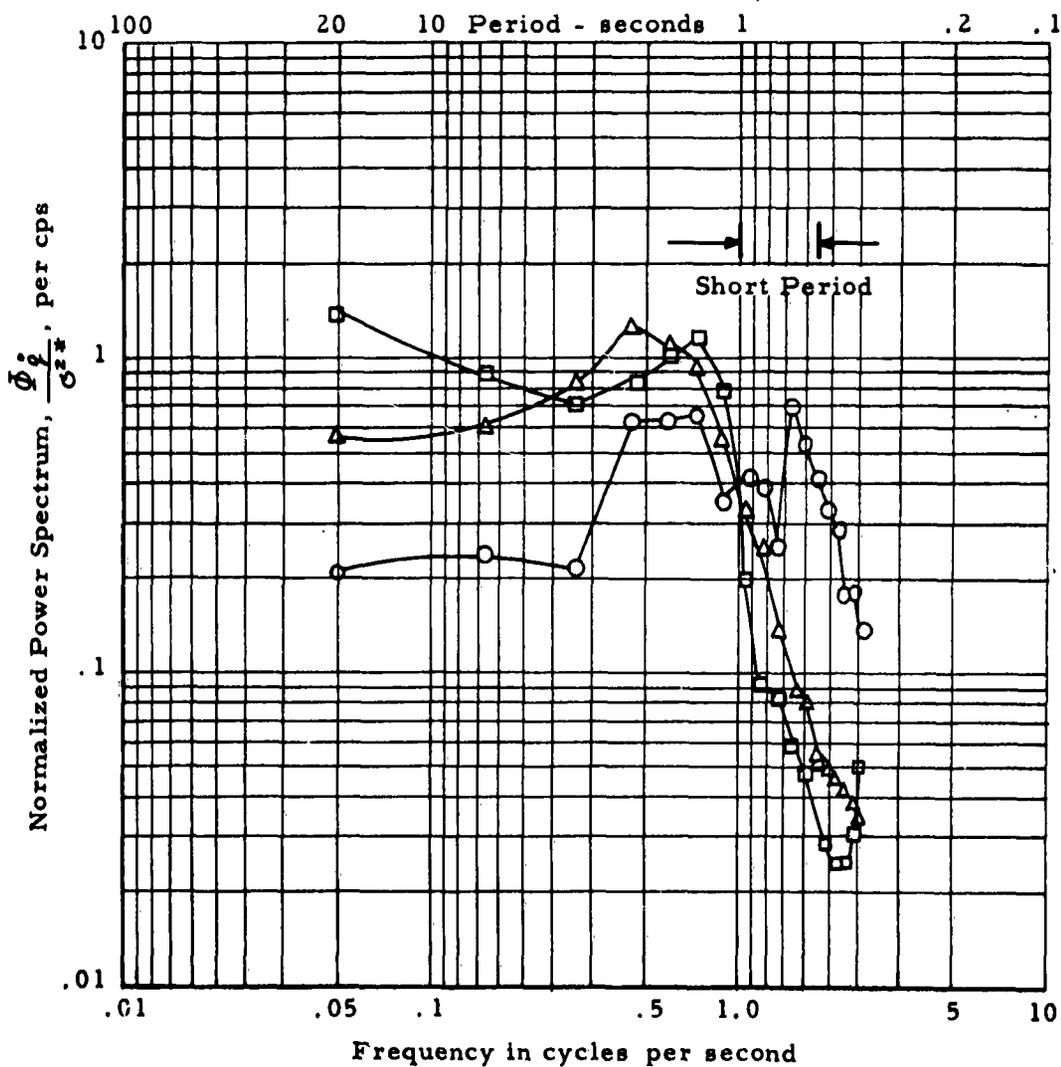


FIGURE 20

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{q}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
△	F11F		.0033
□	A4DL	Runs 1, 2, 5-7, 9-15	.0021
○	A4DS	Runs 3, 4	.0090

\* Note:  $\sigma^2$  in this case is equal to  $\int \dot{\Phi}_{\dot{q}} dt$

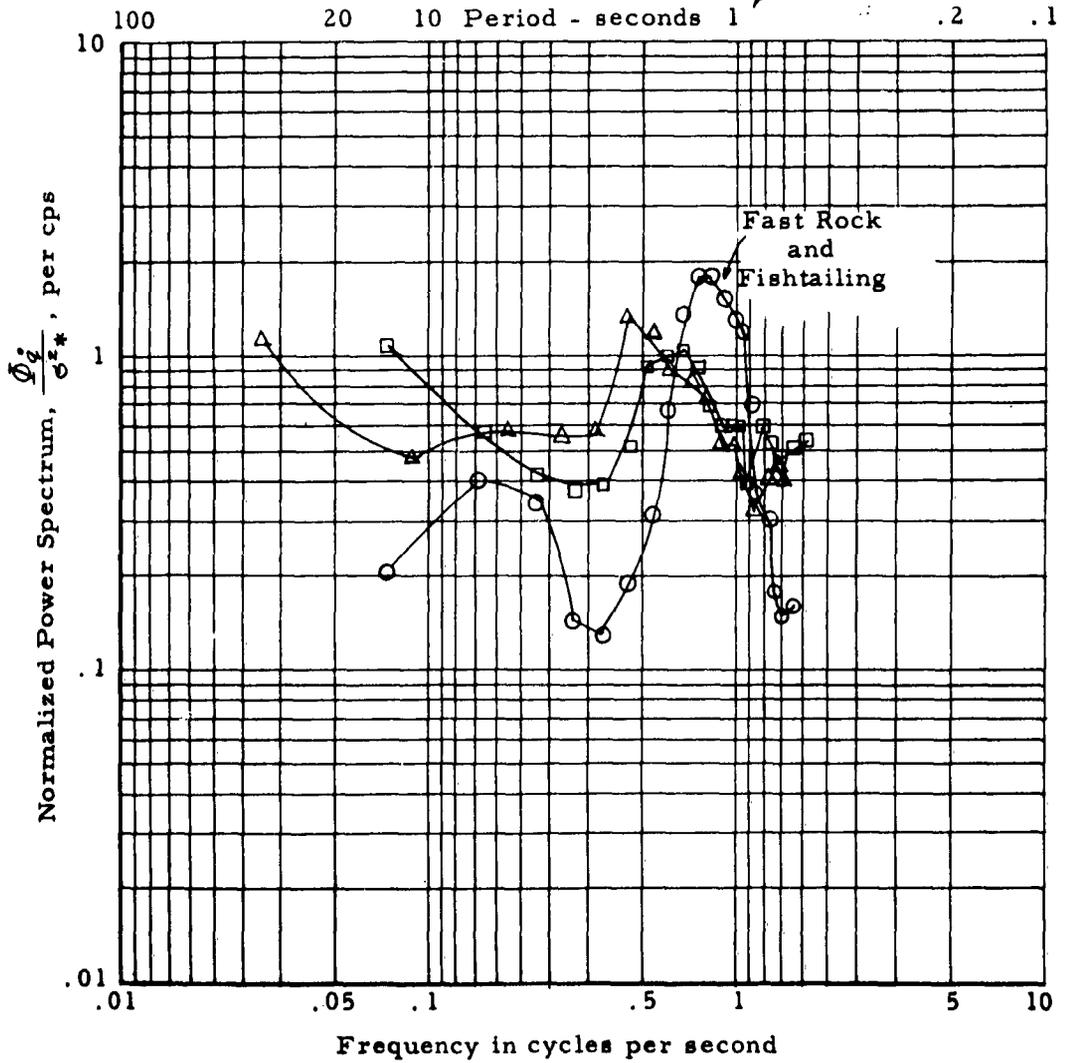


FIGURE 21

POWER SPECTRA OF MANEUVER MOTIONS

$\ddot{\psi}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2 * (\text{rad/sec}^2)^2$
□	F-100A	240	.00090
△	F-100A	240L	.00047
○	F-100A	241	.00026

\* Note:  $\sigma^2$  in this case is equal to  $\int \ddot{\psi}_s^2 dt$

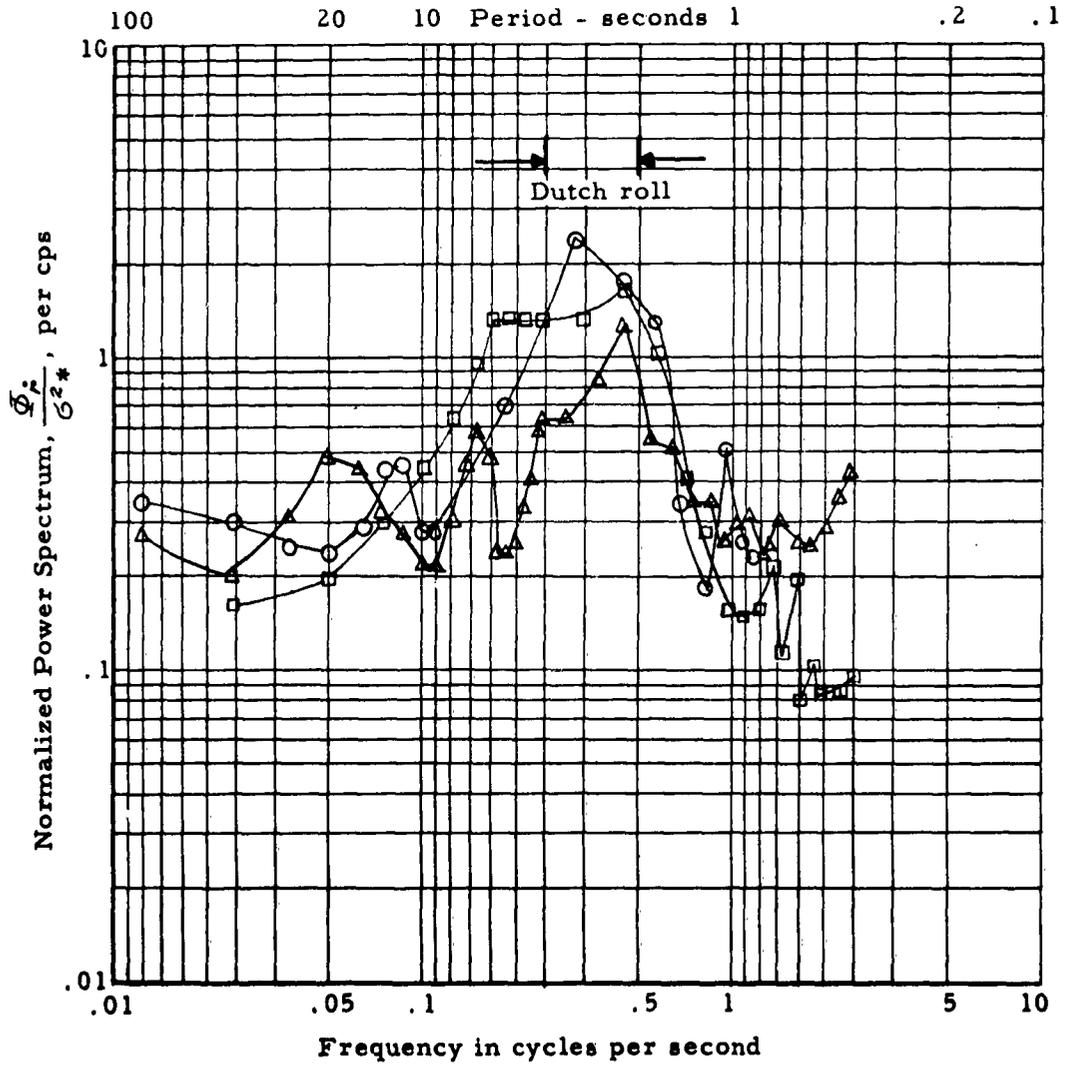


FIGURE 22

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{\phi}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2 * (\text{rad/sec}^2)^2$
$\Delta$	F-100A	237A	.00045
$\square$	F-100A	237B	.00047
$\circ$	F-100A	237L	.00093

\* Note:  $\sigma^2$  in this case is equal to  $\int \dot{\phi}^2 df$

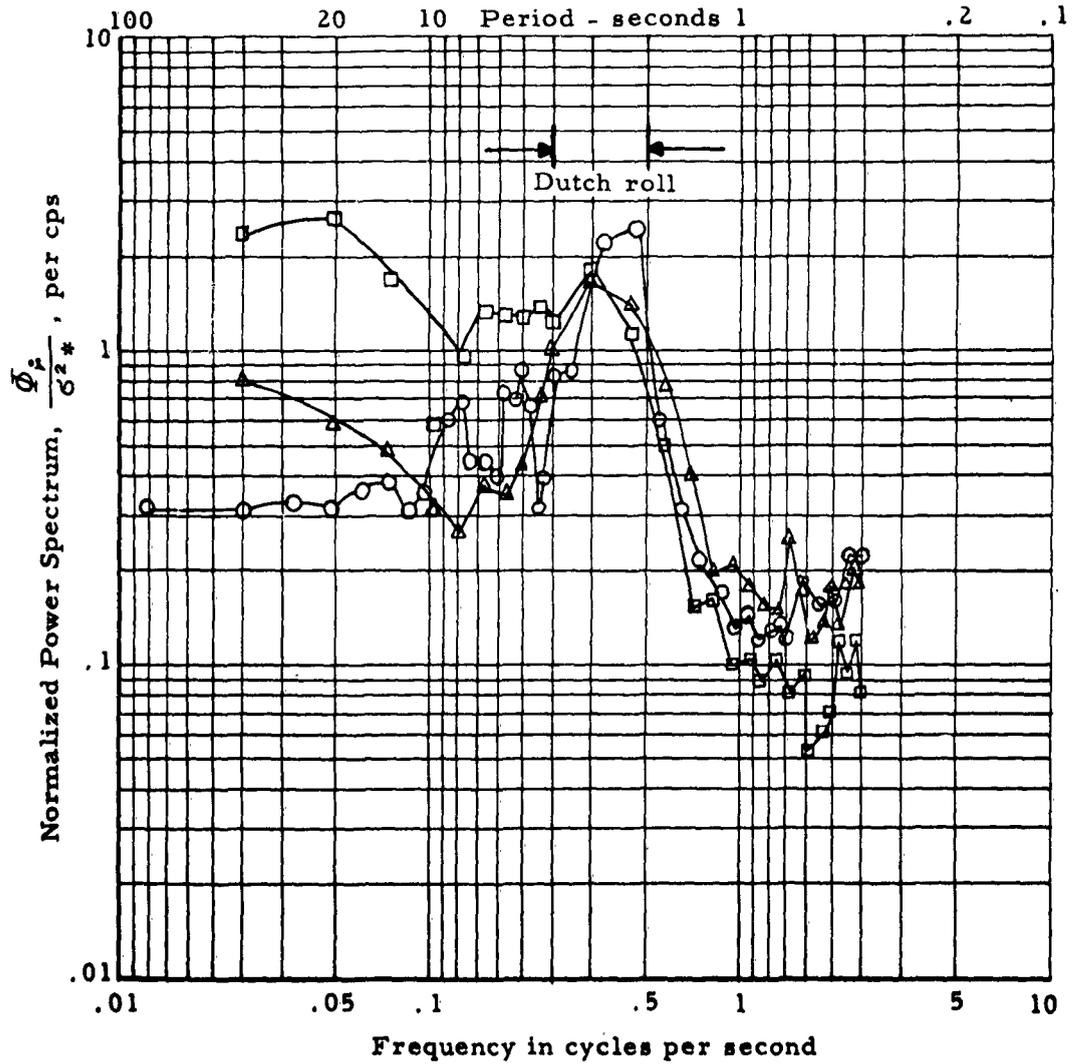


FIGURE 23

POWER SPECTRA OF MANEUVER MOTIONS

$\dot{\rho}$ , ANGULAR ACCELERATION

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
Δ	F11F		.00044
□	A4DL	Runs 1, 2 5-7, 9-15	.0014
○	A4DS		.063

\* Note:  $\sigma^2$  in this case is equal to  $\int \dot{\rho}^2 df$

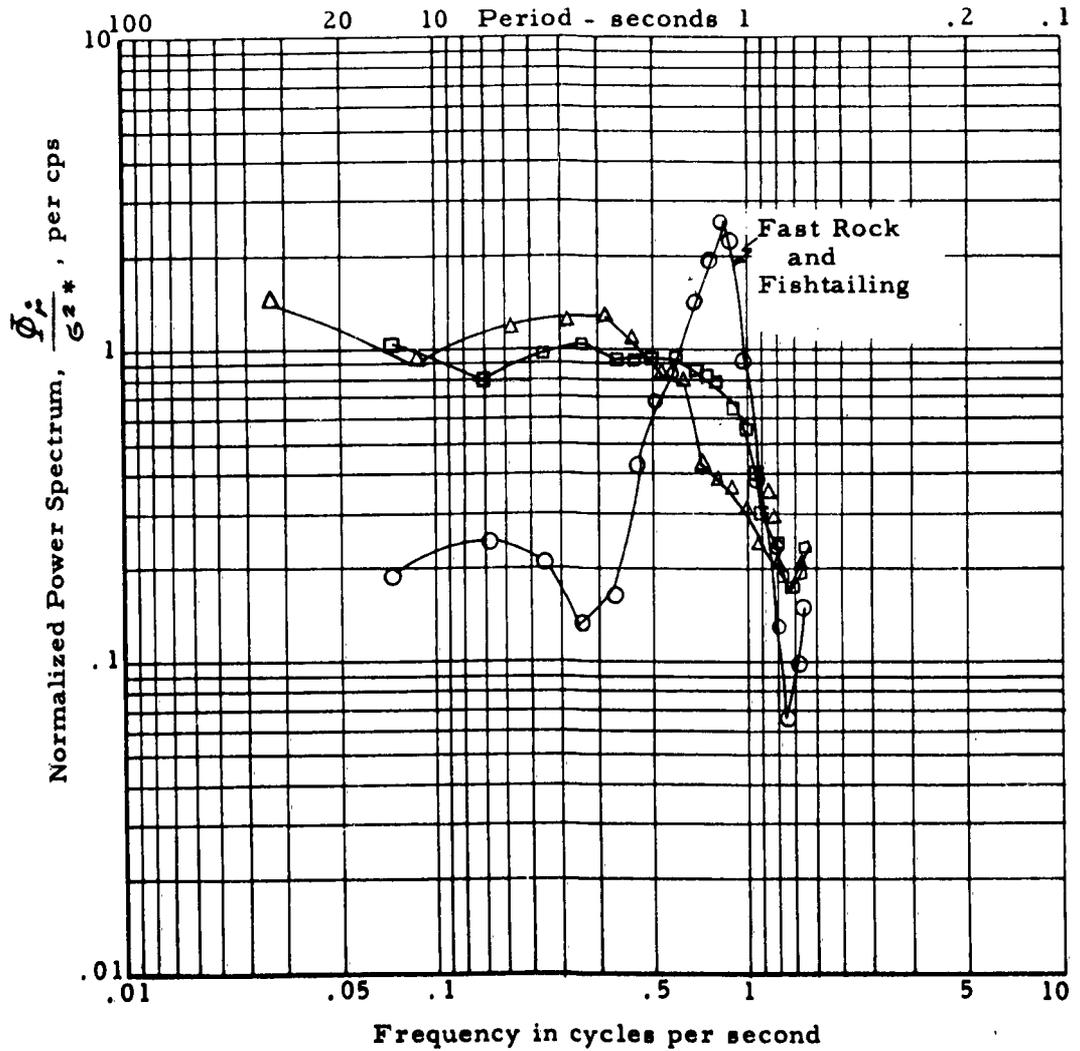


FIGURE 24

POWER SPECTRA OF MANEUVER MOTIONS  
ANGULAR ACCELERATIONS

Symbol	Aircraft	Flight	$\sigma^2$ * (rad/sec <sup>2</sup> ) <sup>2</sup>
—	F-84G	2	.0341
- - -		1	.00324
.....		3	.0016

\* Note:  $\sigma^2$  in this case is equal to

