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TRANSLATION

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STATIONARY FLOW OF A CONDUCTING MEDIUM IN THE PRESENCE OF A MAGNETIC FIELD

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1. Rakhmatullin has derived a closed system of hydrodynamic equations for the motion of monophase media [1]. Using this system makes it possible to study the behavior of nonconducting liquid or gaseous mixtures.

Also of interest is the problem of the motion, for instance, of a two-phase medium where one of the phases is a conductor. A mixture of this type may evidently be described by the system of equations indicated above, provided that an additional term of the electromagnetic derivation is included in the equation of motion for the conducting medium and if the system is completed with the Maxwell equations. Study of such a system makes it possible to examine, for example, how the nonconducting fluid influences the motion of the conducting one.

Limiting ourselves to two incompressible media, we may write

$$
\left.
\begin{aligned}
&\operatorname{div} v = 0\\
&\rho \frac{d\vec{v}}{dt} = -f\nabla p + K\left(\vec{v}_n - \vec{v}\right) + \mu\left[\vec{j}\vec{H}\right] + \eta f \Delta \vec{v}\\
&\operatorname{div} v_n = 0\\
&\rho_n \frac{d\vec{v}_n}{dt} = -f_n \nabla p + K\left(\vec{v} - \vec{v}_n\right) + \eta_n f_n \Delta \vec{v}_n\\
&f + f_n = 1
\end{aligned}
\right\} \qquad (1)
$$

Here the first two equations refer to the conducting fluid and the second two to the nonconducting one (corresponding quantities indicated by the subscript n);

$\vec{v}$, ρ - velocity and average density of the medium;

η - viscosity;

K - interaction factor which in the case of incompressible media may be considered constant;

$\vec{j}$, $\vec{H}$, p - density of the electrical current, magnetic field strength and pressure common for a two-component mixture respectively;

μ - permeability of the medium; and

$$f = \frac{\rho}{\rho_l}, \quad f_n = \frac{\rho_n}{\rho_{nl}},$$

where ρ_1 and ρ_{n1} are specific densities.

The Maxwell equations must be added to system (1).

2. As an example of the application of the system of equations (1) let us consider the problem of a stationary one-dimensional flow of a conducting fluid in mixture with a nonconducting one between a pair of parallel planes in the presence of a transverse external magnetic field. Let the motion of the media be in the direction of the x-axis, the external uniform magnetic field $\vec{H}_0$ be directed along z, and the boundary planes be at $z = \pm l$.

As is known, flow spreads the lines of force of the magnetic field perpendicular to it. Therefore, together with the transverse component H_0 there appears also a component h_x parallel to the motion. The latter is a function of $\underline{z}$ since the component of velocity in the $\underline{x}$ direction depends on $\underline{z}$. Let us consider an uncomplicated solution of the problem where in the oy direction there is applied an external uniform electrical field $E_y = E_0$.

In the stationary case $(\frac{\partial}{\partial t} = 0)$, taking into account the geometry of the problem we obtain the following system for determining

$$\left.\begin{aligned} &v_x = v(z),\ \ v_{nx} = v_n(z),\ \ h_x = h(z): \\ &f\eta \frac{d^2v}{dz^2} + \mu j_y H_0 - f\frac{\partial p}{\partial x} + K(v_n - v) = 0 \\ &f_n \eta_n \frac{d^2 v_n}{dz^2} - f_n \frac{\partial p}{\partial x} + K(v - v_n) = 0 \\ &\frac{dh}{dz} = 4\pi j_y,\ j_y = \sigma(E_0 - \mu H_0 v) \end{aligned}\right\} \tag{2}$$

The last two expressions are a Maxwell equation and a relationship for the density of the electrical current, while σ is the conductivity of the media. The pressure gradient $\frac{\partial p}{\partial x}$ = const.

Taking advantage of the expression for j_y we may determine $\underline{v}$ and v_n from the first two equations of system (2). The problem is solved for the boundary conditions v = v = 0 for $z = \pm a$. The solution takes the form

$$v = -\frac{1}{\eta}\frac{\partial p}{dx}\left(a\frac{\text{ch}\, z/r_1}{\text{ch}\, l/r_1} + b\,\frac{\text{ch}\, z/r_2}{\text{ch}\, l/r_2} + \frac{l^2}{M^2}\right) + u. \tag{3*}$$

$$v_n = -\frac{1}{\eta_n}\frac{\partial p}{dx}\left(a_n\frac{\text{ch}\, z/r_1}{\text{ch}\, l/r_1} + b_n\,\frac{\text{ch}\, z/r_2}{\text{ch}\, l/r_2} + \frac{l^2}{M_n^2} + \frac{l^2}{L_n^2}\right) + u_n, \tag{4}$$

where $\underline{u}$ and u_n are the velocities specified by the presence of an electrical field E_0 perpendicular to H_0

$$u = \frac{E_0}{\mu H_0}\left(\alpha\frac{\text{ch}\, z/r_1}{\text{ch}\, l/r_1} + \beta\,\frac{\text{ch}\, z/r_2}{\text{ch}\, l/r_2} + 1\right), \tag{5}$$

$$u_n = \frac{E_0}{\mu H_0}\left(\alpha_n\frac{\text{ch}\, z/r_1}{\text{ch}\, l/r_1} + \beta_n\,\frac{\text{ch}\, z/r_2}{\text{ch}\, l/r_2} + 1\right). \tag{6}$$

*ch = cosh

The following definitions enter into expressions (3)-(6):

$$a = \delta\left[\left(\frac{l}{r_2}\right)^2 \frac{1}{M^2} - 1\right], \quad a_n = \delta\left[\left(\frac{l}{r_2}\right)^2\left(\frac{1}{M_n^2} + \frac{1}{L_n^2}\right) - 1\right], \tag{7}$$

$$b = -\delta\left[\left(\frac{l}{r_1}\right)^2 \frac{1}{M^2} - 1\right], \quad b_n = -\delta\left[\left(\frac{l}{r_1}\right)^2\left(\frac{1}{M_n^2} + \frac{1}{L_n^2}\right) - 1\right], \tag{8}$$

$$\alpha = \delta\left(\frac{1}{r_2^2} - \frac{M^2}{f}\frac{1}{l^2}\right), \quad \beta = -\delta\left(\frac{1}{r_1^2} - \frac{M^2}{f}\frac{1}{l^2}\right), \tag{9}$$

$$\alpha_n = \delta/r_2^2, \quad \beta_n = -\delta/r_1^2, \quad \delta = \frac{1}{\sqrt{s^2 - 4q}}, \tag{10}$$

$$r_1 = \sqrt{\frac{2}{s + \sqrt{s^2 - 4q}}}, \quad r_2 = \sqrt{\frac{2}{s - \sqrt{s^2 - 4q}}}, \tag{11}$$

$$= 1/l^2\left(L^2 + L_n^2 + \frac{1}{f}M^2\right), \quad q = 1/l^4 \frac{M^2 L_n^2}{f}, \tag{12}$$

$$L = l\left(\frac{K}{f\eta}\right)^{1/2}, \quad L_n = l\left(\frac{K}{f_n \eta_n}\right)^{1/2},$$

$$M = \mu H_0 l\left(\frac{\sigma}{\eta}\right)^{1/2}, \quad M_n = \mu H_0 l\left(\frac{\sigma}{\eta_n}\right)^{1/2}$$

which are dimensionless combinations (M being the Hartman number).

In order to determine the magnetic field h we have the equation

$$\frac{d^2h}{dz} = -4\pi\sigma H_0 \frac{dv}{dz}. \tag{13}$$

Substituting here the value of v from Eq. (3) and integrating when th boundary conditions are h = 0 and z = $\pm l$ (these conditions follow from the fact that the components of the magnetic field $\vec{H}$ are continuously tangent to the boundary) we find

$$h = A\frac{\operatorname{sh} z/r_1 - z/l \operatorname{sh} l/r_1}{\operatorname{ch} l/r_1} + B\frac{\operatorname{sh} z/r_2 - z/l \operatorname{sh} l/r_2}{\operatorname{ch} l/r_2}, \tag{14}$$

where

$$\left.\begin{aligned} A &= 4\pi\sigma r_1\left(\frac{\mu H_0}{\eta} a \frac{\partial p}{\partial x} - \alpha E_0\right) \\ B &= 4\pi\sigma r_2\left(\frac{\mu H_0}{\eta} b \frac{\partial p}{\partial x} - \beta E_0\right) \end{aligned}\right\}. \tag{15}$$

The average value of the magnetic field $\underline{h}$ for $-l < z < l$ is equal to zero. In the region $sl^2 \ll 1$ for $E_0 = 0$ it is proportional to H_0 and does not depend on the interaction factor or the properties of the nonconducting fluid:

$$h = -\frac{2\pi}{3}\frac{\sigma\mu H_0}{\eta}\frac{\partial p}{\partial x}z(z^2 - l^2). \tag{16}$$

3. Expressions (3)-(4) give the distribution of velocity over the transverse cross section, while (14) and (16) determine the distribution of the field. The quantities r_1 and r_2 which enter here are real since, as is easily seen from (11) and (12),

$$s^2 - 4q \geq 0.$$

Equality occurs when $H_0 = K = 0$.

In the limiting case of the absence of a magnetic and electrical field, the dependence of $\underline{v}$ and v_n on $\underline{z}$ must be the same as if both media were nonconducting. Passing to the limit indicated in the expressions (3) and (4), we obtain the same result as did Fayzullayev [2] for the case of the motion of a two-phase nonconducting medium. When $K \to 0$ and $f = 1$, formula (3) becomes the well-known expression for the case of flow of a conducting fluid in a magnetic field (see, for example, Kauling [3]).

The presence of both a transverse magnetic field and a nonconducting medium shows up in the appearance of an additional resistance to the motion of the conducting fluid. With the other given parameters an increase in the magnetic field H_0 or the coefficient of friction K leads to a decrease in the velocities $\underline{v}$ and v_n. The magnetic field in altering the motion of the conducting medium by that very fact affects the nonconducting medium because of friction.

From Fayzullayev's work [2] it follows that of two media moving together, the one with the lowest viscosity has the greater velocity. In the presence of a magnetic field this cannot be said since it directly slows only the conducting component of the mixture. Whether the inequality $v_n > v$ or the reverse takes place here depends also on the magnetic field. Here it can only be said that both $\eta_n > \eta$ and $\eta_n < \eta$ satisfy the inequality

$$v_n - v \leqslant -\frac{f_n}{K}\frac{\partial p}{\partial x} \tag{17}$$

(the equals sign refers to the case $\eta = \eta_n = 0$). In a region where the viscosity is slight and the field not too great it may be expected that $v > v_n$.

The velocity v_n also depends on the electric field. The presence of the latter leads to the appearance of an additional velocity u_n, it being expected that $u_n < u$. Not only viscosity and induction braking but also dynamic friction between the components of the mixture now has an effect on the velocity profile.

In the case where $l \ll r_1$ ($sl^2 \ll 1$) the influence of the magnetic field and of friction between the fluids becomes insignificant in comparison with the internal friction in each component (η, η_u) and the expressions for velocity take the usual parabolic form

$$v = \frac{1}{2\eta}\left(\frac{\sigma E_0 H_0}{f} - \frac{\partial p}{\partial x}\right)(l^2 - z^2), \tag{18}$$

$$v_n = -\frac{1}{2\eta_n}\frac{\partial p}{\partial x}(l^2 - z^2). \tag{19}$$

When in Eq. (2) it is possible to neglect the term with viscosity (low viscosity, large magnetic field) we obtain (in the absence of an $F_\jmath$ field)

$$v_n = -\frac{l^2}{\eta_n}\frac{\partial p}{\partial x}\left(\frac{1}{M_n^2} + \frac{1}{L_n^2}\right)\left(1 - \frac{\operatorname{ch} z/r}{\operatorname{ch} l/r}\right), \tag{20}$$

$$v - v_n = -\frac{1}{L^2 + \frac{M^2}{f}}\frac{\partial p}{\partial x}\left[\frac{l^2}{\eta} - \frac{1}{K}\left(L^2 + \frac{f_n}{f}M^2\right)\left(1 - \frac{\operatorname{ch} z/r}{\operatorname{ch} l/r}\right)\right]. \tag{21}$$

where

$$r = l\sqrt{\frac{f_n}{M_n^2} + \frac{1}{L_n^2}}.$$

The basic influence here is magnetic "viscosity" which impedes motion perpendicular to the lines of force.

Expression (20) and (21) for $l \ll r$ takes the form

$$v_n = -\frac{1}{2\eta_n}\frac{\partial p}{\partial x}\frac{M_n^2 + L_n^2}{M_n^2 + f_n L_n^2}(l^2 - z^2). \tag{22}$$

$$v - v_n = -\frac{1}{2\eta_n}\frac{\partial p}{\partial x}\left[\frac{2fl^2}{L_n^2 + M_n^2} - \frac{M_n^2\left(L_n^2 + M_n^2\right)}{\left(f_n L_n^2 + M_n^2\right)}(l^2 - z^2)\right]. \tag{23}$$

If, moreover, it is possible in Eq. (2) to neglect the term with viscosity η_n then we will obtain expressions independent of <u>z</u>

$$v = -\frac{1}{\sigma\mu^2 H_0^2}\frac{\partial p}{\partial x} + \frac{E_0}{\mu H_0}, \tag{24}$$

$$v_n - v = -\frac{f_n}{K}\frac{\partial p}{\partial x}. \tag{25}$$

As can be seen from (24) the velocity <u>v</u> decreases inversely as the square of the field and does not depend on the interaction factor. For very high values of K the velocities become the same for both components. It should be noted that expressions (24) and (25) may actually be satisfied only at some distance from the walls where viscosity can have no significant influence. However, in the thin layer near the wall its influence always shows up.

In problems associated with flow rate of liquids (or gases) it is important to know the average velocity. From (3) and (4) we have

$$\bar{v} = -\frac{1}{l}\frac{\partial p}{\partial x}\left(ar_1 \operatorname{th} l/r_1 + br_2 \operatorname{th} l/r_2 + \frac{l}{\sigma H_0^2}\right) + \bar{u}, \tag{26}$$

$$\bar{v}_n = -\frac{1}{l}\frac{\partial p}{\partial x}\left(a_n r_1 \operatorname{th} l/r_1 + b_n r_2 \operatorname{th} l/r_2 + \frac{l}{\sigma H_0^2} + \frac{lf_n}{K}\right) + \bar{u}_n. \tag{27}$$

For the region where $\frac{l}{r^2}$ is of the order of several units and greater formulas (26) and (27) may be approximated by the following expressions (we will not write the terms proportional to E_0)

$$v = -\frac{1}{\eta}\frac{\partial p}{\partial x}\left\{ Q\left[\left(\frac{L}{M}\right)^2 + \left(\frac{L_n}{M}\right)^2 + \frac{1}{f'_n}\frac{L_n}{M} + \frac{f_n}{f}\right] + \frac{l^2}{M^2}\right\}, \qquad (28)$$

$$v_n - v = -\frac{1}{\eta_n}\frac{\partial p}{\partial x}\left[Q\left(\frac{1}{f}\frac{M^2}{L_n^2} + \frac{1}{f'_n}\frac{M}{L_n} + \frac{1}{f}\frac{\eta_n}{\eta}\right) + \frac{l^2}{L_n^2}\right], \qquad (29)$$

where

$$Q = \frac{1}{l}\left[\sqrt{\frac{s - \sqrt{s^2 - 4q}}{2q\,(s^2 - 4q)}} - \sqrt{\frac{s + \sqrt{s^2 - 4q}}{2q\,(s^2 - 4q)}}\right].$$

Our results refer to the case of motion between a pair of parallel flat walls. However, they may be extended for use with experiments with tubes having rectangular cross section if one of the sides of the rectangle is much longer than the other and the magnetic field is perpendicular to the long side. It is possible to measure the average velocities $\underline{v}$ and v_n experimentally. Knowledge of the latter makes it possible to determine for example K if the viscosities η and η_n as well as H_0, f, and f_n are known.

REFERENCES

1. Kh. A. Rakhmatullin, Prikladnaya matematika i mekhanika , 1956, t. 20, vyp. 2.

2. D. G. Faysullayev, Izv. AN UzSSR , seriya tekhn. nauk, 1958, No. 3.

3. T. Kauling, Nagnitnaya gidrodinamika, M., IL, 1959.

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