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# TRANSLATION

ACOUSTIC CONDUCTIVITY OF A  
RIGID BURNING SURFACE

By

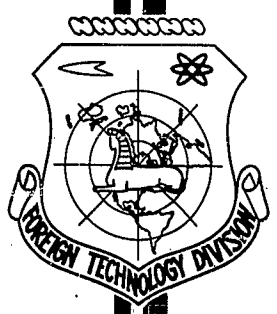
S. S. Novikov and Yu. S. Ryazantsev

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## UNEDITED ROUGH DRAFT TRANSLATION

ACOUSTIC CONDUCTIVITY OF A RIGID BURNING SURFACE

BY: S. S. Novikov and Yu. S. Ryazantsev

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Acoustic Conductivity of a Rigid Burning Surface

By  
S. S. Novikov and Yu. S., Ryazantsev

Acoustic properties of a shock front, such as the front of a flame in gas, were investigated theoretically in (Bibl. 1 thru 4). It was demonstrated in (Bibl. 4) that taking into account the flow of mass through the flame front and reaction of the flame front to a change in thermodynamic parameters of gas in acoustic wave may cause a substantial change in the magnitude of acoustic conductivity of this surface, determined by the kind of dependence of the flame propagation velocity on thermodynamic parameters. Similar examination can be performed also in the case of burning of condensed systems. In addition, whereas the applicability of such an examination in case of the flame in gas was restricted particularly by the fact that even a laminar flame is not planar because of instability (Bibl. 5), but has a cellular structure (Bibl. 6, 7), the concept of the plane front of burning is fully justified in case of the burning of condensed systems. The problem is of interest because the magnitude of acoustic conductivity determines the boundary condition on burning surface in existing theories of resonance burning (Bibl. 8, 9).

Subsequent examination permits to draw the following conclusions:

1. If the steady-state law of burning of the kind  $U = ap^{\nu}$  ( $\nu < 1$ ) is fulfilled under non-steady conditions, the rigid burning surface is acoustically stable.
2. Taking into account the appearance of entropic wave during the interaction of acoustic wave with burning surface changes substantially the region of acoustic stability.
3. It is possible to intensify a weak pressure wave during the interaction with a rigid burning surface only if the nonsteady-state law of burning satisfies a certain condition (23) (see below).

On the plane front of burning, regarding which one assumes that it coincides with the surface of condensed phase (Fig. 1), are fulfilled the mass, impulse and energy conservation laws which, in the coordinate system connected with condensed phase and constituting simultaneously the laboratory system, are written in the following form

$$\begin{aligned} \rho_1 U &= \rho_2 (u_2 + U) \\ p_1 + \rho_1 U^2 &= p_2 + \rho_2 (u_2 + U)^2 \quad (1) \\ w_1 + \frac{U^2}{2} &= w_2 + \frac{(u_2 + U)^2}{2} \end{aligned}$$

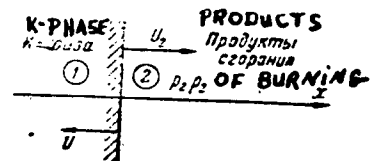


Fig. 1

Here  $p, \rho, w$  are the pressure, density and enthalpy;  $U$  is the absolute value of linear burning rate;  $u_2$  is the rate of outflow of gases, products of burning.

If the quantities  $p_2, \rho_2, u_2$  experience a weak disturbance  $\delta p_2, \delta \rho_2, \delta u_2$ , e. g., in a weak pressure wave ( $\delta p_2 \ll p_2$ ) coming in from the direction of combustion products, then these disturbances on the front of burning are connected by relations that are obtained through varying the equations of system (1). Discarding the terms which are quadratic in relation to variations, we obtain from (1) the system of equations for variations

$$\begin{aligned} \rho_1 \delta U + U \delta \rho_1 &= \rho_2 (\delta u_2 + \delta U) + \frac{\rho_1}{\rho_2} U \delta \rho_2 \\ \delta p_1 + U^2 \delta \rho_1 &= \delta p_2 + 2 \rho_2 U \delta u_2 + \frac{\rho_1^2}{\rho_2^2} U^2 \delta \rho_2 \quad (2) \\ \delta w_1 + U \delta U &= \delta w_2 + \frac{\rho_1}{\rho_2} U (\delta u_2 + \delta U) \end{aligned}$$

Assuming that the condensed phase is non-compressible, i. e.,  $\rho_1 = \text{const}$ , and discarding small members on the order of  $U^2/c_2^2$  (where  $c_2$  is the speed of sound), which is justified by the fact that  $U/c_2 \ll 1$  is always fulfilled, we obtain instead of (2)

$$\begin{aligned}
\rho_1 \delta U &= \rho_2 (\delta u_2 + \delta U) + \frac{p_1}{\rho_2} U \delta \rho_2 \\
\delta p_1 &= \delta p_2 + 2\rho_1 U \delta u_2 \\
\delta w_1 + U \delta U &= \delta w_2 + \frac{p_1}{\rho_2} U (\delta u_2 + \delta U)
\end{aligned} \tag{3}$$

Disturbances in combustion products are composed of incident and reflected pressure waves and entropic wave

$$\begin{aligned}
\delta p_2 &= \delta p_2^- + \delta p_2^+, & \delta \rho_2 &= \delta \rho_2^- + \delta \rho_2^+ + \delta \rho_2^\circ \\
\delta u_2 &= \delta u_2^- + \delta u_2^+, & \delta w_2 &= \delta w_2^- + \delta w_2^+ + \delta w_2^\circ
\end{aligned} \tag{4}$$

Apart from that, variations of parameters on waves are interconnected by equalities:

incident pressure wave

$$\delta u_2^- = -\frac{\delta p_2^-}{\rho_2 c_2}, \quad \delta \rho_2^- = \frac{\delta p_2^-}{c_2^2}, \quad \delta w_2^- = \frac{\delta p_2^-}{\rho_2} \tag{5}$$

reflected pressure wave

$$\delta u_2^+ = \frac{\delta p_2^+}{\rho_2 c_2}, \quad \delta \rho_2^+ = \frac{\delta p_2^+}{c_2^2}, \quad \delta w_2^+ = \frac{\delta p_2^+}{\rho_2} \tag{6}$$

entropic pressure wave

$$\delta u_2^\circ = 0; \quad \delta \rho_2^\circ = 0, \quad \delta w_2^\circ = -\frac{c_2^2 \delta \rho_2^\circ}{\rho_2 (\gamma_2 - 1)} \tag{7}$$

It is assumed, here and further on, that combustion products are the ideal gas. We shall consider the function  $U = U(p_2 T_2)$  as being known. This will permit to express  $\delta U$  through  $\delta p_2$  and  $\delta \rho_2^\circ$

$$\delta U = A \delta p_2 + B \delta \rho_2^\circ \tag{8}$$

Here

$$A = \left( \frac{\partial U}{\partial p_2} \right)_{T_2} + \frac{\gamma_2 - 1}{\gamma_2} \frac{T_2}{p_2} \left( \frac{\partial U}{\partial T_2} \right)_{p_2}, \quad B = -\frac{c_2^2}{\rho_2 c_{p_2} (\gamma_2 - 1)} \left( \frac{\partial U}{\partial T_2} \right)_{p_2} \tag{9}$$

From equations (3) thru (9) one can obtain the expression for acoustic conductivity

which is equal to normal component of acoustic velocity to acoustic pressure.

With the above-indicated accuracy relative to  $U/c_2$ , the non-dimensional acoustic conductivity of the rigid burning surface equals

$$\zeta = -\frac{\partial u_2}{\partial p_2} \rho_2 c_2 = -(\rho_1 - \rho_2) c_2 A - (\gamma_2 - 1)(\rho_1 - \rho_2)^2 \frac{B}{\rho_1 c_2} + \left[ \gamma_2 - (\gamma_2 - 1) \frac{\rho_2}{\rho_1} \right] \frac{\rho_1 U}{\rho_2 c_2} - (\gamma_2 - 1)(\rho_1 - \rho_2)^3 \frac{ABU}{\rho_2 c_2} - (\gamma_2 - 1)^2 (\rho_1 - \rho_2)^4 \frac{B}{\rho_1 \rho_2 c_2^3} \frac{U}{c_2} \quad (10)$$

In (Bibl. 4) was obtained an approximate formula for  $\zeta$ , containing only the first term, which corresponds to the case of large  $A$ .

If the dependence of burning rate on temperature of combustion products is weak, i. e.,  $\left| \frac{\partial U}{\partial T_2} \right|_{p_2} \approx 0$ , then formula (10) becomes considerably simplified

$$\zeta = -(\rho_1 - \rho_2) c_2 A + \left[ \gamma_2 - (\gamma_2 - 1) \frac{\rho_2}{\rho_1} \right] \frac{\rho_1 U}{\rho_2 c_2} \quad (11)$$

The following law of burning is valid for a wide class of condensed systems

$$U = a p_2^\nu \quad (12)$$

In this case

$$\frac{\partial U}{\partial T_2} = 0, \quad B = 0, \quad A = \frac{\partial U}{\partial p_2} = \gamma_2 \nu \frac{U}{p_2 c_2} \quad (a)$$

Then

$$\zeta = \left[ -\left(1 - \frac{\rho_2}{\rho_1}\right) \gamma_2 \nu + \gamma_2 - (\gamma_2 - 1) \frac{\rho_2}{\rho_1} \right] \frac{\rho_1 U}{\rho_2 c_2} \quad (13)$$

The possibility of intensification, i. e., increasing the amplitude of incident wave on reflection, is determined by the sign of real component of the acoustic conductivity. Intensification takes place if  $\text{Re} \zeta < 0$ . In the given case, quantity  $\zeta$  is real and  $\zeta < 0$  corresponds to intensification. This condition is reduced to



$$N > \frac{\gamma_2 - (\gamma_2 - 1) p_1 / p_2}{\gamma_2 (1 - p_2 / p_1)} \approx 1 + \frac{1}{\gamma_2} \frac{p_2}{p_1} \quad (14)$$

In the first approximation on  $e_2/e_1$ , condition (14) coincides with the condition of burning stability of condensed systems. Since the inequality  $v < 1$  is fulfilled for all normally burning secondary explosives and, consequently,  $\zeta > 0$ , one can state that the fulfillment of the steady-state law of burning under non-steady conditions would guarantee wave attenuation on reflection, i. e., acoustic stability.

Let us note that the form of disturbances was not concretely defined here, same as in (Bibl. 4), and our conclusion is valid for weak waves of any form, including harmonic waves. Of course, one can assume that the law of burning may differ noticeably from the steady-state law for rapid pressure changes, e. g., for periodic changes with a high frequency. The effect of unsteadiness on the process of interaction of shock waves with flame in gas was taken into account in (Bibl. 11, 12). The unsteadiness of burning process on interaction of weak harmonic waves with burning surface was acknowledged in theories of resonance burning, developed in (Bibl. 8, 9). In these papers, the unsteadiness is taken into account by introducing into the steady-state law of burning the combustion-process delay time relative to instantaneous values of thermodynamic parameters.

One of the possible methods for introduction of delay time is the method used in theory of non-equilibrium processes (Bibl. 13), according to which the speed of approximation of the non-equilibrium burning rate to its equilibrium value is proportional to the difference between instantaneous and equilibrium values, i. e., satisfies equation

$$\frac{dU}{dt} = \frac{1}{\tau} (U_0 - U) \quad (15)$$

where  $\tau$  is the relaxation time;  $U_0$  is the steady-state rate of burning;  $U$  is the instantaneous rate of burning.

This method was employed in (Bibl. 8). Assuming that all the disturbances depend on time according to the law  $e^{i\omega t}$ , and using equations (3) thru (9) and (15), one can easily demonstrate that the presence of relaxation time of delay will be taken into account if

$$\frac{A}{1+i\omega\tau}, \quad \frac{B}{1+i\omega\tau} \quad (b)$$

is substituted, instead of A and B, in the expression for acoustic conductivity.

For the same assumptions under which formula (13) was obtained we shall have now

$$\zeta = \left[ -\left(1 - \frac{\rho_2}{\rho_1}\right) \frac{\gamma_2 v}{1+i\omega\tau} + \gamma_2 - (\gamma_2 - 1) \frac{\rho_2}{\rho_1} \right] \frac{\rho_1 U}{\rho_2 c_2} \quad (16)$$

In this case, the condition of intensification will have the form

$$\operatorname{Re} \zeta = \left[ -\left(1 - \frac{\rho_2}{\rho_1}\right) \frac{\gamma_2 v}{1+\omega^2\tau^2} + \gamma_2 - (\gamma_2 - 1) \frac{\rho_2}{\rho_1} \right] \frac{\rho_1 U}{\rho_2 c_2} < 0 \quad (17)$$

One can see that the presence of relaxation time of delay increases the acoustic stability of the system.

Another method of obtaining nonsteady-state law of burning by modifying the well-known steady-state law of burning through introduction of delay time was formulated first for the case of burning in liquid fuel rocket engines in (Bibl. 14) and applied to burning of condensed systems in (Bibl. 9). It is assumed in this method that the process of burning progresses in two stages, viz., gasification and combustion proper, separated by the time interval equal to the induction time  $\tau$ , so that

$$\dot{m}_b(t) = \left(1 - \frac{d\tau}{dt}\right) \dot{m}_i(t - \tau) \quad (\dot{m}_i = a p^n(t)) \quad (18)$$

Here  $\dot{m}_1$  is the mass rate of gasification;  $\dot{m}_2$  is the mass rate of formation of combustion products, and induction time  $\tau$  is determined from equation

$$\int_{-\tau}^{\tau} p^m(t') dt' = \text{const} \quad (19)$$

where  $m$  is a certain constant on the order of one.

If the pressure disturbance and, at the same time, disturbances in all thermodynamic parameters and in outflow rate of combustion products depend on time as  $e^{i\omega t}$ , then it is possible to obtain from equations (3) through (9), (18) and (19) the expression for acoustic conductivity in form

$$\zeta = \left\{ (\gamma_2 - 1) \left( 1 - \frac{p_2}{p_1} \right) + 1 - \gamma_2 [m + (v - m) e^{-i\omega\tau}] \right\} \frac{u_2}{c_2} \quad (20)$$

Hence the condition of intensification

$$m + (v - m) \cos \omega\tau > 1 - \left( 1 - \frac{1}{\gamma_2} \right) \frac{p_2}{p_1} \quad (21)$$

It can be seen from (21) that intensification is possible with corresponding  $\omega$  and  $\tau$ , even at  $v < 1$ . One can show that, in this case, a single pressure peak attenuates, although the periodic wave may intensify.

It should be noted that intensification criteria (17) and (21), obtained on the basis of nonsteady-state laws of burning, postulated in (Bibl. 8, 9), differ from criteria ensuing directly from expressions for acoustic activity, presented in these papers. These discrepancies are caused by the fact that the authors of (Bibl. 8, 9) ignored the appearance of entropic wave on interaction of a weak wave with burning surface, whereas taking the latter into consideration leads to a substantial change in expression for acoustic conductivity and, consequently, to a change in the criterion of acoustic stability.

The methods of delay time introduction, considered above, are arbitrary to a certain degree, due to the absence of experimental data confirming that or other initial hypothesis. At the same time, attempts at the direct numerical calculation of magnitude of the acoustic conductivity of a burning rigid surface, based on analysis of a concrete model of burning (Bibl. 15), fail to give a clear mechanism of intensification on reflection. Therefore, further searches for nonsteady-state law of burning are

are justified.

Based on laws of mass, impulse and energy conservation on the front of burning, as well as on the condition of acoustic stability, the general requirement as to such a law can be formulated. Indeed, in the case of disturbances depending on time as  $e^{i\omega t}$ , assuming for the sake of simplicity that the rate of burning depends on pressure only, one can connect instantaneous values the combustion product outflow rate and the pressure by formula

$$\frac{\delta(\rho_2 u_2)}{\rho_2 u_2} = D \frac{\delta p_2}{p_2} \quad (22)$$

where D is a certain complex number determined by the law of burning. Finding a corresponding expression by means of (3) through (8) and (22), we establish that the following inequality must be satisfied, if the incident wave is intensified on reflection:

$$\operatorname{Re} D > 1 - \left(1 - \frac{1}{\gamma_2}\right) \frac{\rho_2}{\rho_1} \quad (23)$$

It is pointed out in (Bibl. 16) that the compressibility of condensed phase must be taken into account in studying the properties of burning surface. Conclusions here obtained can be easily generalized on the case of compressible condensed phase.

The authors are grateful to A. D. Margolin for his critique in which he demonstrated that condition (14) can be obtained also within the framework of combustion theory formulated by Ya. B. Zeldovich and A. F. Belyayev (Bibl. 10, 17).

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