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TRANSLATION

LAMINAR BOUNDARY LAYER ON A POROUS PLATE WITH CHEMICAL REACTIONS AT THE SURFACE

BY

V. S. Avduyevskiy and Ye. I. Obroskova

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

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LAMINAR BOUNDARY LAYER ON A POROUS PLATE WITH CHEMICAL REACTIONS AT THE SURFACE

V. S. Avduyevskiy and Ye. I. Obroskova

The efficiency of transpiration cooling strongly depends on the physical properties of the coolant used. The results of the calculations of a laminar boundary layer on a porous surface can be presented within a known accuracy as engineering formulas, from which it follows that a decrease in the heat-transfer coefficient and friction coefficient is greater the smaller the molecular weight of the gas fed through the surface.

If the gas which is being delivered enters into chemical reactions, the molecular weight and physical properties of the mixture can substantially change. In this study we investigated a laminar boundary layer on a porous plate in the presence of chemical reactions. We will give the results of numerical calculations of boundary-layer equations for the case of burning of carbon and hydrogen at the surface with binary diffusion inside the boundary layer, and we will suggest approximating formulas.

Designations

x, y — coordinates

u, v — velocity components on the axes

ρ — density

p — pressure

T — temperature

J — enthalpy

c_p — specific heat at constant pressure

C — concentration

μ — viscosity coefficient

λ — coefficient of heat conductivity

D — diffusion coefficient

P — Prandtl number

τ — friction stress

q — heat flow

Q — diffusion flow

W — velocity of chemical reaction

G — gas flow through surface

h_x — heat of formation

Δh — thermal effect of reaction

h — total enthalpy

α — heat-transfer coefficient

C_f — friction coefficient

R — Reynolds number

M — Mach number

M_+ — molecular weight

P_g — Prandtl diffusion number

$$P = \frac{\mu c_p}{\lambda}, \qquad L = \frac{P}{P_g}, \qquad J = \int_0^T c_p dT, \qquad P_g = \frac{\mu}{\rho D}$$

The indexes mean: w — conditions at the wall; 0 — conditions without injection at the same wall temperature; ∞ — conditions outside the boundary layer: 1 — gas delivered through wall (fuel); 2 — gas outside boundary layer (oxidant): paired indexes w2, 12 denote relationship of corresponding magnitudes.

1. The equations of a boundary layer in a reacting mixture without consideration of thermal diffusion can be presented in the form:

equation of motion:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} \tag{1.1}$$

equation of continuity for the i-th component of mixture:

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = -\frac{\partial}{\partial y} K_i - W_i \qquad \left(C_i = \frac{\rho_i}{\rho}\right) \tag{1.2}$$

equation of continuity for entire mixture:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad \left(\sum_1^n C_i = 0, \quad \sum_1^n W_i = 0\right) \tag{1.3}$$

equation of energy:

$$\rho u \frac{\partial J}{\partial x} + \rho v \frac{\partial J}{\partial y} = -\frac{\partial q}{\partial y} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + u \frac{\partial p}{\partial x} + H_x \tag{1.4}$$

Boundary conditions

$$\begin{aligned} &u = 0, \quad v = v_w, \quad J = J_w, \quad C_i = C_{iw} \quad \text{when } y = 0 \\ &u = u_\infty, \quad J = J_\infty, \quad C_i = C_{i\infty} \quad \text{when } y = \infty \end{aligned} \tag{1.5}$$

Here u, v are velocity components on the x-, y-axes (Fig. 1), ρ is the density of mixture, p is static pressure, μ is viscosity coefficient, C_i is the weight concentration, W_i is the rate of consumption of i-th component per unit volume, n is the total number of components.

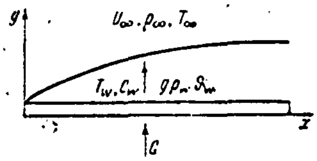

Fig. 1.

Enthalpy of the mixture is defined as

$$J = \sum_{i=1}^n J_i C_i, \qquad J_i = \int_0^T c_{pi}\, dT \tag{1.6}$$

Here H_x is heat liberated as a result of chemical reactions. For diffusion flow K_i we have

$$K_i = \sum_{j \neq i} \frac{M_i}{M^2} \rho D_{ij}^* \frac{\partial}{\partial y} (C_j M_+), \qquad \left(\frac{1}{M} = \sum \frac{C_i}{M_i}, C_i = \frac{s_i M_i}{s M} \right) \tag{1.7}$$

Here M_i is molecular weight of i-th component, D_{ij}^* is the multicomponent of the coefficient of diffusion for a multicomponent mixture, s_i is the number of molecules of the i-th component in unit volume.

The heat flow owing to diffusion and heat conductivity equals

$$q = -\lambda \frac{\partial T}{\partial y} + \sum_{i=1}^{n} K_i J_i \tag{1.8}$$

The rates of consumption of components W_i are related by stochiometric relations expressing the conditions of conservation of mass of each element

$$\sum_{i=1}^{n} W_i \nu_{ij} = 0 \qquad \left(\sum_{i=1}^{n} W_i = 0 \right) \qquad (j = 1, \ldots, m) \tag{1.9}$$

where ν_{ij} is the weight fraction of element *j* in the composition of component *i*.

If the number of possible chemical reactions equals *l*, and the number of elements equals *m*, condition n - m = l is fulfilled.

If we multiply Eq. (1.2) by ν_{ij} and sum up, we obtain (m - 1) of the diffusion equations which do not contain the consumption rate of components in the right-hand part.

$$\rho u \frac{\partial}{\partial x} \sum_{i=1}^{n} C_i \nu_{ij} + \rho v \frac{\partial}{\partial y} \sum_{i=1}^{n} C_i \nu_{ij} = -\frac{\partial}{\partial y} \sum_{i=1}^{n} K_i \nu_{ij} \tag{1.10}$$

Equations (1.10) are valid for any assumptions concerning the nature of the chemical reations.

In addition to these equations, for calculation in the general case we must use linearly independent equations of systems (1.2) based on the number of reactions *l*. In the case of chemical equilibrium inside the boundary layer, system (1.9) is supplemented by a system containing $\underline{l}$ algebraic equations of equilibrium. In certain cases we can assume that chemical reactions proceed in very thin fronts.

In this study it was assumed that the burning front coincides with the plate surface.

2. The values ν_w, J_w, C_w are interrelated by the conditions of equilibrium of matter and heat at the surface with the chemical reactions taking place on it taken into consideration.

The condition of the conservation of mass for the entire mixture yields the following expression for the flow of gas through the wall:

$$G = g\rho_w v_w \tag{2.1}$$

The condition of continuity for each component has the form

$$GC_i^\circ = g\rho_w v_w C_{iw} + K_{iw} + W_{iw} \tag{2.2}$$

Using (1.9) we obtain the condition of stochiometry

$$G \sum_{i=1}^{n} C_i^\circ \nu_{ij} = g\rho_w v_w \sum_{i=1}^{n} C_{iw}\nu_{ij} + \sum_{i=1}^{n} K_{iw}\nu_{ij} \quad (i = 1, \ldots, m-1) \tag{2.3}$$

Here C_i^o is the weight fraction of the i-th component in a gas fed through the wall.

Equations (2.1) and (2.3) yield $\underline{m}$ conditions of mass equilibrium. In addition to this, the parameters at the wall are associated by equations (2.2) based on the number of reactions $\underline{l}$.

In a number of cases we can assign the values of the component concentrations, for example, assuming that the chemical equilibrium in the reactions under consideration is shifted to some side. Here the number of unknown concentrations equals n - 1 = m and Eqs. (2.1) and (2.3) close the system of boundary conditions of mass equilibrium.

Let us consider the condition of heat equilibrium when $y = 0$. We will denote by h the total enthalpy of the gas including the energy of chemical formation h_x

$$h = J + h_x = \sum_{i=1}^{n} C_i h_i$$

Then, considering the energy flows into and out of the circuit (Fig. 1), we obtain that the total heat flow including the energy transported by diffusion equals

$$q_\Sigma = \lambda \frac{\partial T}{\partial y} - \sum_{i=1}^{n} K_i h_i = G\,(h_w - h^\circ) + q_- + q_r \tag{2.4}$$

Here h_w is the total enthalpy of gas at the wall, h^o is the total enthalpy of gas delivered through the wall at the initial temperature, q_- is the heat forcefully being withdrawn from the wall area under consideration or the heat absorbed by the wall in unsteady heating, q_r is the radiant heat flow.

Taking into account Eqs. (1.8), (2.1) and (2.2), Eq. (2.4) can be reduced to the form

$$\lambda \frac{\partial T}{\partial y} + \sum_{i=1}^{n} W_i h_{xi} = G\Delta J + q_- + q_r \tag{2.5}$$

The magnitude $W_1h_{1+} + \dots + W_nh_{n+}$ determines the heat effect of all the actions occurring at the wall, ΔJ determines the heat consumed for heating the coolant in the wall with consideration of phase changes. It follows from Eq. (2.5) that the heat transmitted from the reacting gas to the wall is composed of the heat transmitted as a result of conduction and heat liberated during chemical reactions at the wall. The diffusion portion of the heat flow (1.8) compensates the change of gas enthalpy on escape from the wall

$$\sum_{i=1}^{n} K_{iw} J_{iw} = g\rho_w v_w \Sigma\,(C_i^\circ - C_{iw})\,J_{iw} \tag{2.6}$$

3. To investigate the effect of chemical reactions at the surface on heat and mass transfer, we calculated the boundary layer on a flat porous plate. The gas (fuel) being fed through the surface chemically reacts with the gas diffusing from the outer flow (oxidant). It was assumed that the concentration of fuel at the wall is equal to zero which can be, if the reaction is shifted to one side and the gas flow through the surface does not exceed a certain limiting value. The gas fed through the surface can also be a mixture of a fuel with a neutral gas, for which we took a gas that is a product of the reaction. Inside the boundary layer reactions do not occur and there is a binary diffusion between the reaction products and the oxidant.

Equations (1.1-1.4) reduced to the form

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{3.1}$$

$$\rho u \frac{\partial J}{\partial x} + \rho v \frac{\partial J}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \rho D \frac{\partial C}{\partial y} \frac{\partial}{\partial y}\left(J^{(1)} - J^{(2)}\right) \tag{3.2}$$

$$\rho u \frac{\partial C}{\partial x} + \rho v \frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial C}{\partial y}\right) \tag{3.3}$$

Here C is the concentration of combustion products, $J^{(1)}$ and $J^{(2)}$ are the enthalpies of the combustion product and oxidant, D is the coefficient of binary diffusion.

Boundary conditions (1.4) with consideration of (2.3) and (2.6) acquire the form

$$\begin{gathered} u = u_\infty, \quad C = C_\infty, \quad J = J_\infty \ \text{when } y = \infty \\ u = 0, \quad C = C_w, \quad \rho_w v_w\left(1 - C_w + k\frac{1-\nu}{\nu}\right) + \rho_w D\left(\frac{\partial C}{\partial y}\right)_w = 0 \\ J = J_w \ \text{when } y = 0 \end{gathered} \tag{3.4}$$

The condition $J = J_w$ when $y = 0$ can be replaced by the condition of heat equilibrium (2.6)

$$\lambda \frac{\partial T}{\partial y} + g\rho_w v_w \Delta h_x k = g\rho_w v_w (\Delta J) + q_- + q_r \tag{3.5}$$

Here ν is the weight fraction of the element of which the fuel consists in the reaction product; k is the weight fraction of the fuel in the mixture delivered through the wall. The values $k = 0$ or $\nu = 1$ correspond to the limiting cases of the supply of one of the gases in the mixture and the absence of a chemical reaction; Δh_x is the thermal effect of the reaction referred to a unit weight of fuel; ΔJ is the change of enthalpy of the gas supply inside the wall.

We will introduce the dimensionless variables and parameters

$$\eta = \sqrt{\frac{U_\infty}{\nu_\infty x}} \int \frac{\rho}{\rho_\infty} dy, \quad f' = \frac{U}{U_\infty}, \quad P_\infty = \frac{\mu_\infty c_{p\infty}}{\lambda_\infty}, \quad P_{g\infty} = \frac{\mu_\infty}{\rho_\infty D_\infty}$$

We will denote in terms of $c_p^{(1.2)}$ the difference of specific heats of the combustion products and the oxidant.

Then for the case $J_w = \text{const}$, $C_w = \text{const}$ and with delivery of the coolant according to the law $\rho_w v_w \sim x^{0.5}$, system (3.1)-(3.3) can be transformed to a system of ordinary differential equations

$$-\frac{1}{2} f f'' = (\mu\rho f'')' \tag{3.6}$$

$$-\frac{1}{2} c_p f T' = (\lambda\rho T')\frac{1}{P_\infty} + \rho^2 D C' T' \frac{c_p^{(1.2)}}{P_{g\infty}} + \mu\rho f'' (\varkappa - 1) M_\infty^2 \tag{3.7}$$

$$-\frac{1}{2} f C' = \frac{1}{P_{g\infty}} (\rho^2 D C')' \tag{3.8}$$

Here all magnitudes without indexes are reduced to a dimensionless form and the gas parameters outside the boundary layer are taken as scales.

The boundary conditions are transformed to the form:

when $\eta = 0$

$$T = T_w, \qquad C = C_w, \qquad f' = 0$$

$$f_w = 2\rho_w{}^2 D C_w{}' \frac{1}{P_{g\infty}} \frac{\nu}{(1 - C_w)\nu + (1 - \nu)k}$$

or

$$(\lambda\rho T')_w + f_w \Delta h_x k \frac{1-\nu}{\nu} = f_w \Delta J + q_- + q_r \tag{3.9}$$

when $\eta = \infty$

$$T = 1, \qquad C = 1, \qquad f' = 1$$

4. System of equations (3.4)-(3.6) was solved by the method of successive approximations on the "Strela" computer.

As a fuel we examined hydrogen H_2 or carbon C, as an oxidant, oxygen.

It was assumed that during burning one of the following reactions takes place

$$H_2 + \frac{1}{2}O_2 = H_2O \qquad \left(\nu = \frac{1}{9},\ k = 1\right) \tag{4.1}$$

$$C + O_2 = CO_2 \qquad \left(\nu = \frac{3}{11},\ k = 0.25\right) \tag{4.2}$$

$$C + \frac{1}{2}O_2 = CO \qquad \left(\nu = \frac{3}{7},\ k = 0.25\right) \tag{4.3}$$

Calculations were carried out for values of M_∞ equal to 1.5, 6, 10, 15, 20 and T_∞ = 300°C. The following conditions were used:

a) wall temperature was taken as $T_W/T_\infty = 0.5, 5.0$;

b) wall temperature was determined from the condition of thermal equilibrium when $q_- = q_r = 0$; initial temperature of the gas being supplied was equal to 300°.

The physical parameters μ, D, λ (dimensionless values) were presented in the form of the product of two functions: one depending on temperature and the other depending on viscosity [1, 2]. The first function is the same for all transfer coefficients

$$f_1(T) = \frac{1+S}{T+S} T^{1.5} \tag{4.4}$$

For S we took values 1.25, 0.57, 0.506 for reactions (4.1), (4.2), (4.3) respectively.

We used the formulas of Hirschfelder to determine the dependence of μ and λ on the concentration [3]. The limiting value of $\mu\rho$ when $C_w = 1$ and $T_w = T_\infty$ were equal to 0.219, 1.006, and 0.83 for reactions (4.1), (4.2) and (4.3) respectively. The values of $\lambda\rho$ for these cases were equal to 0.251, 0.862, and 0.95. The number P_∞ is equal to 0.72, the values of $P_{g\infty}$ equal 0.49, 0.925, and 0.695.

5. As a result of the calculation we obtained the values of friction stresses, specific heat and diffusion flows as a function of the injection parameter.

Figures 2, 4, 5 show the dependence of magnitudes

$$\tau_w^\circ = \frac{\tau_w}{\tau_{w0}} = \frac{\rho_w\mu_w}{(\rho_w\mu_w)_0} \frac{f_w''}{(f_w'')}, \qquad q_w^\circ = \frac{q_w}{q_{w0}} = \frac{\rho_w\lambda_w}{(\rho_w\lambda_w)_0} \frac{T_w'}{(T_w')_0}$$
$$Q_w^\circ = \frac{Q_w}{Q_{w0}} = \frac{\rho_w^2}{(\rho_w^2)_0} \left(\frac{C_w'}{C_w}\right) \left(\frac{C_{w0}}{C_{w0}'}\right) \tag{5.1}$$

on the dimensionless flow of the coolant

$$G^\circ = G \Big/ \left(\frac{\alpha}{c_p}\right)_0 \tag{5.2}$$

Here the index "0" means that the corresponding magnitudes were taken at G = 0 and at the same values of the wall temperature and flow parameters outside the boundary layer; the prime mark denotes a derivative with respect to η

The condition

$$\left(\frac{\alpha}{c_p}\right)_0 = \frac{q_{w0}}{J_e - J_w} = \frac{\tau_{w0}}{u_\infty} P_\infty^{-2/3}$$

is fulfilled with sufficient accuracy for an impermeable surface.

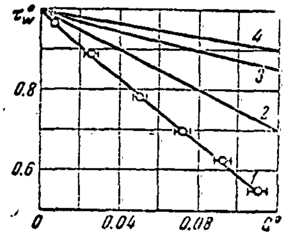

Fig. 2.

The dimensionless flow is associated with the parameter f_w by the relationship

$$G^\circ = \frac{G_1}{(\alpha/c_p)_0} = \frac{G}{\tau_{w0}} u_\infty P_\infty^{1/3} = \frac{f_w P_\infty^{2/3}}{\left(C_{f\infty} \sqrt{R_\infty}\right)_0} \tag{5.3}$$

As we see from Figs. 2, 4, 5, the dependence on the Mach number and the ratio of the temperatures of the wall and flow is very weakly demonstrated with such treatments, which enables us to use the calculation results in a wide range of variation of these parameters. The corresponding dependences for the case of injection of various gases without burning have the same properties.

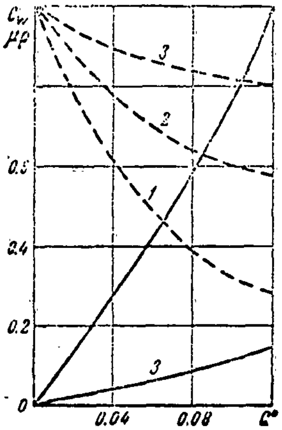

Fig. 3.

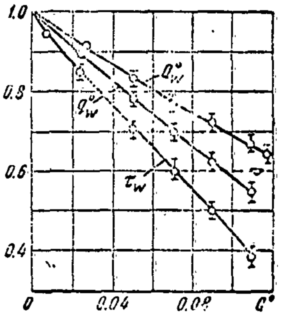

Fig. 4.

Figure 2 shows the relative change of the friction coefficient from the dimensionless flux. Curve 1 corresponds to the burning of hydrogen (reaction 4.1), curves 2, 3, and 4, plotted from the data of other studies [1, 2] correspond to the injection of hydrogen without burning, injection of water vapors, and air respectively.

As we see, in the case of the burning of hydrogen the effect of a decrease in the friction coefficient is more appreciable.

Figure 3 shows, for the same cases, the curves of the change in values of $\mu\rho$ (dashed line) and concentrations at the wall depending on the gas flow at a constant wall temperature. As is seen, in the case of burning the concentration of water vapors rapidly increased which leads to a significant decrease of $\mu\rho$. In cases 2 and 3 the decrease of $\mu\rho$ is weaker, in case 4 (with delivery of air) the value of $\mu\rho$ does not depend on the injection intensity.

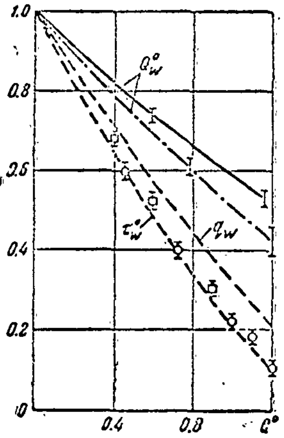

Fig. 5.

We can also conclude from an examination of Figs. 2 and 3 that a decrease of the friction coefficient in burning of hydrogen mainly occurs not because of the difference of the vertical component of velocity from zero but due to changes in the physical properties of the gas near the wall.

Figure 4 shows curves of the relative change of τ_ω, q_w, and Q_w

as a function of G^0. In all cases the scattering of the calculated points upon a change of the Mach number from 1.5 to 20 and T_w/T_∞ from 0.5 to 5 does not exceed ±5%. It is interesting to note also that the decrease of q_w^0 is more appreciable and the decrease of Q_w^0 less appreciable than the decrease of friction. This can be explained by the additional effect of the numbers P and P_g and by the change in the heat capacity of the mixture. The calculation results of the burning of carbon with the formation of CO_2 are shown in Fig. 5. The value of $\mu\rho$ and the Prandtl number in this case weakly depend on the concentration. Owing to this the calculated point of the changes of τ_ω and q_w differ little from the corresponding curves for the injection of air into air (dashed curves). The change of the diffusion flow Q_{wo} as a function of the dimensionless flux is appreciably weaker.

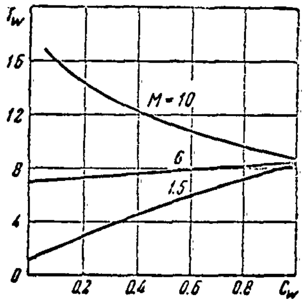

Fig. 6.

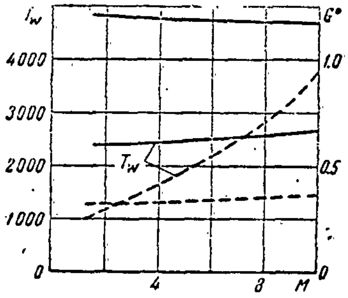

Fig. 7.

Figure 5 also shows the curve of the change of Q_w^0 for burning of carbon with the formation of CO (dot-dash line). The surface temperature was established from the condition of heat equilibrium (for $\varepsilon = 0$, $q_{-} = 0$, $T_\infty = 300^\circ$ and an initial temperature of carbon also equal to 300°) and was different for different flows of the coolant and different values of M (Fig. 6).

Figure 7 shows the curves of the change in surface temperature and dimensionless flow upon burning in oxygen and in a mixture similar to air in composition (dashed line, $C_w = 0.32$). At small values of M the temperature for the case of burning in oxygen is higher owing to the greater flow of carbon (G^o) and the correspondingly greater release of heat in the chemical reactions. Here the role of convective heating is small because of the pronounced decrease of q_w, and the surface temperature slightly changes with an increase of M. In the case of burning in air the value of G^o is small, the amount of heat released in the chemical reactions is also small, and the surface temperature rises together with an increase of M.

6. Let us examine the possible methods of approximation and generalization of the calculation results in order to derive engineering formulas and to extend of the obtained results to the case of burning and injection of gases with other laws of the change in physical properties and to other flow conditions.

As we see from Figs. 2, 4, 5, owing to the introduction of the injection parameter G^o we are able to take into account the effect of a change in the Mach number and the temperature ratio T_w/T_∞. This is possible because the dependences of the change in the friction and heat-transfer coefficient on parameter f_w under different flow conditions are laid out along a line close to parallel straight lines at low injection temperatures.

A parallel transfer of all curves to one curve corresponding to the case of the flow of an incompressible fluid is performed when plotting curves of the relative change in the magnitude as a function of the parameter G^o.

This property can be used when plotting engineering formulas.

We will assume that in all cases we can write

$$\frac{\tau_w}{\tau_{w0}'} = 1 - \beta \frac{G}{\tau_{w0}'/u_\infty} P_\infty^{2/3} \qquad (6.1)$$

Here τ_{wo}' is the value of friction stress on an impermeable surface with a change in the physical properties of the gas the same as that as in the case under consideration. Coefficient β is a constant and can be taken from the solution for an incompressible boundary layer with constant physical properties ($\beta \approx 0.85$).

The value of τ_{wo}' depends on the ratio of the product $\mu\rho$ at the wall and outside the layer. The best results are yielded by the formula

$$\tau_{w0}' = \tau_{w0}\left(\frac{M_w}{M_2}\frac{\mu_w}{\mu_2}\right)^{0.3} = \tau_{w0}\left(M_{w2}\mu_{w2}\right)^{0.3} \tag{6.2}$$

Here M_w and M_2 are the molecular weights of the gas near the wall and outside the layer, μ_w and μ_2 are the corresponding coefficients of viscosity at identical temperature (the variation of temperature across the flow was already taken into account in formula (6.1)).

Using such an approach, we obtained the following approximate formulas:

$$\begin{aligned}\frac{\tau_w}{\tau_{w0}} &= K_\tau\left(1-0.85\frac{G^\circ}{K_\tau}\right)\\ \frac{q_w}{q_{w0}} &= K_\alpha\left(1-0.85\frac{G^\circ}{K_\alpha}P_\infty^{2/3}\right)\\ \frac{Q_w}{Q_{w0}} &= K_c\left(1-0.85\frac{G^\circ}{K_c}P_{g\infty}^{2/3}\right)\end{aligned} \tag{6.3}$$

where

$$K_\tau = (\mu_{w2}M_{w2})^{0.3}, \qquad K_\alpha = \left(\frac{\mu_{w2}M_{w2}}{P_{w2}}\right)^{0.3} c_{pw2}^{-0.13} = (\lambda_{w2}M_{w2})^{0.3} c_{pw2}^{-0.43}$$
$$K_c = (M_{w2})^{0.6} \tag{6.4}$$

The factors $P_\infty^{2/3}$ and $P_{g\infty}^{2/3}$ were introduced into the formulas for concordance of the angular coefficient since

$$q_{w0} \sim \tau_{w0}P_\infty^{-2/3}, \qquad Q_{w0} \sim \tau_{w0}P_{g\infty}^{-2/3}$$

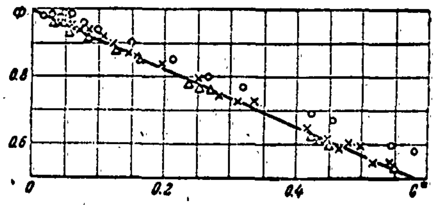

Fig. 8.

When used in formulas (6.3) and (6.4) it is necessary to know the physical properties of the gas at the wall.

For a binary mixture

$$\begin{aligned} \lambda_{w2} &= 1 + C_w(\lambda_{12} - 1) \\ c_{pw2} &= 1 + C_w(c_{p12} - 1) \\ M_{w2} &= \frac{1}{1 + C_w(M_{21} - 1)} \end{aligned} \qquad (6.5)$$

Here the index 2 corresponds to the gas in the outside flow, index 1 corresponds to the gas which is an admixture; C_W denotes the concentration of the admixture.

To check the accuracy of formulas (6.3), in Fig. 8 we have plotted on the ordinate

$$\Phi_\tau = \frac{\tau_w}{\tau_0}\frac{1}{K_\tau}, \qquad \Phi_\alpha = \frac{q_w}{q_{w0}}\frac{1}{K_\alpha}, \qquad \Phi_c = \frac{Q_w}{Q_{w0}}\frac{1}{K_c}$$

and on the abscissa correspondingly

$$G_\tau{}^* = \frac{G^\circ}{K_\tau}, \qquad G_\alpha{}^* = \frac{G^\circ P_\infty{}^{2/3}}{K_\alpha}, \qquad G_c{}^* = \frac{G^\circ P_{g\infty}{}^{2/3}}{K_c}$$

As we see the calculated points for τ_W^o (triangles) and q_W^o (crosses) within an accuracy of ±5% are plotted on a single curve with an angular coefficient equal to 0.85. The scattering of points for Q_W^o (circles) is somewhat greater, moreover, all points lie slightly higher than the generalized curve. However, the error does not exceed 10%, thus the accuracy of the formulas can be considered satisfactory.

7. Formulas (6.3) and (6.4) together with the conditions of heat and mass equilibrium can be used for calculating the amount of required coolant in transpiration cooling from a given surface temperature or for calculating the burning velocity or sublimation of the surface.

From condition (3.9) we derive a formula for determining the gas flow relative to the concentration of combustion products at the surface

$$G = \frac{[1 + C_w (M_{12} - 1)]^{0.6}}{kl + 1 + C_w (0.85 P_\infty^{2/3} - 1)} L_\infty^{2/3} C_w \left(\frac{\alpha_0}{c_p} \right) \qquad (7.1)$$

Here G is the total quantity of matter borne away from the wall with the gas, k is the fuel fraction, $l = (1 - \nu)/\nu$ is the stochiometric ratio of the oxidant and fuel, M_1 is the molecular weight of the combustion product, M_2 is the molecular weight of the gas in the outside flow.

The value $C_W = 1$ corresponds to complete oxidation in pure oxygen. The value $C_W < 1$ can approximately correspond to burning in an oxygen-hydrogen mixture.

To determine the surface temperature we must use the condition of heat equilibrium (3.5) which can be reduced to the form

$$G = \frac{K_\alpha q_0 - \sigma \varepsilon T_w^4}{\Delta J - \Delta h_x + 0.85 (J_e - J_w) P_\infty^{2/3}} \qquad (7.2)$$

The value K_α is calculated by formula (6.4). Formulas (7.1) and (7.2) can be used for determining the required flow of coolant at a given wall temperature.

8. Formulas (6.3) and (6.4) can be used only with conditions of equilibrium (3.5) since they contain unknown values of C_W. However, the calculated dependences are close to straight lines. Using this condition, we obtain simple approximate formulas

$$\frac{\tau_w}{\tau_{w0}} = 1 - 0.85G^\circ \left\{1 + (M_{21})^{0.6} \frac{kl + 0.85P_\infty^{2/3}}{0.85L_\infty^{2/3}} [1 - (M_{12}\mu_{12})^{0.3}\right\} \quad (8.1)$$

$$\frac{q_w}{q_{w0}} = 1 - 0.85G^\circ \left\{P_\infty^{2/3} + (M_{21})^{0.6} \frac{kl + 0.85P_\infty^{2/3}}{0.85L_\infty^{2/3}} [1 - (M_{12}\lambda_{12})^{0.3} c_{p12}^{-0.43}]\right\} \quad (8.2)$$

$$\frac{Q_w}{Q_{w0}} = 1 - 0.85G^\circ P_{g\infty}^{2/3} \left\{1 + [(M_{21})^{0.6} - 1] \frac{kl + 0.85P_\infty^{2/3}}{0.85P_\infty^{2/3}}\right\} \quad (8.3)$$

Here the index 1 pertains to a gas which is a combustion product, index 2 pertains to the gas in the outside flow.

In the case of mass transfer through a porous surface without burning we obtain the following formulas for the region of small and average values of G°

$$\begin{aligned} \frac{\tau_w}{\tau_{w0}} &= 1 - 0.85G^\circ (M_{21}\mu_{21})^{0.3} \\ \frac{q_w}{q_{w0}} &= 1 - 0.85G^\circ (M_{21}\lambda_{21})^{0.3} c_{p12}^{-0.43} P_\infty^{2/3} \\ \frac{Q_w}{Q_{w0}} &= 1 - 0.85G^\circ M_{21}^{0.6} P_{g\infty}^{2/3} \end{aligned} \quad (8.4)$$

The formulas proposed for this case in Gross' study [2], in which only the ratio of molecular weights is taken into account, in our opinion do not sufficiently account for the effect of all factors.

In conclusion we point out that the magnitude G° used here is related with the magnitude of the so-called effective enthalpy J_{ef} of the surface material by the simple relation

$$J_{ef} = \frac{q_0}{G} = \frac{J_e - J_w}{G^\circ}.$$

We easily see that in the case of burning, this magnitude can assume negative values.

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