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CURRENT TRANSITION GAS-METAL

III. STABILITY OF GLOW AND ARC DISCHARGES

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CONTENTS.

LIST OF SYMBOLS.

This list contains only those symbols which are used consistently throughout the whole report. Symbols which have only special meanings for certain sections will be appropriately defined in the text.

$\vec{B}$	magnetic field
D	diffusion coefficient
$\vec{E}$	electric field
e	unit charge
f	distribution function
h	Planck's constant
I	saturation current
$\overleftrightarrow{I}$	identity tensor
$J_{o,1}$	Bessel functions
$N_{o,1}$	Neumann functions
$\vec{j}$	electric current density
k	Boltzmann's constant
m	mass of the charge carriers
n	particle density
$\overleftrightarrow{P}$	pressure tensor
p	scalar pressure
R	radius of the discharge vessel
$\vec{r}$	space vector
r	radial distance
T	temperature
t	time
U_i	ionization potential
U	electric potential
$\vec{v}_d$	drift velocity
α	ionization coefficient per particle and per sec.

α^* ionization coefficient per particle and per cm

γ second Townsend coefficient

$\varkappa$ specific heat capacity

λ_- mean free path of the electrons

μ mobility

ρ_m mass density

σ recombination coefficient

ω_k eigenvalue

$\vec{\Gamma}$ particle current density

I. INTRODUCTION.

The object of gaseous electronics is a system which consists of various particle components. Some or all of these components may be singly or multiply charged ions. It is assumed that there is no coordination of the same neighbouring particles for times essentially longer than the lifetime of the system in a microstate of the Γ -space.

Obviously there is an enormous variety of systems which fulfil these conditions. The quantities of these systems depend strongly on the external parameters and the boundary conditions which are imposed by the experimental setup. Nevertheless the principal properties of the problem of stability can be discussed in general form.

Let us introduce a set of parameters X_ν which we will call the EXTERNAL PARAMETERS. In this group we will take all the quantities which define the experimental setup and which can be disposed of by external manipulation. Such parameters are for example the geometry of the device, the material and temperature of the walls, the potential applied, the external magnetic field etc. Let us further introduce a set of variables Y_ν which we call the INTERNAL VARIABLES. These internal variables cannot be disposed of from outside but for a given set of external parameters X_ν they are determined by the interior mechanism of the system. Of course the number of these variables introduced must be sufficient to allow a complete description of the mechanism within the range of accuracy required. Examples for such internal variables are e.g. the particle number densities n, the particle velocities v, the temperatures of the particle components T, the local electric field E, etc.

The time and local dependences of the internal variables are governed by a system of differential equations prescribed by the relevant physical laws. The external parameters enter these equations and determine the boundary conditions.

For the STATIONARY STATES this general system of equations simplifies due to the relation $\partial/\partial t = 0$. In the following we call such stationary states "S-MODES".

In general the possibility cannot be excluded that we have more than one s-mode for one given set of external parameters. There are two reasons. First, our system with the boundary conditions can have more than one solution. Secondly, one of the boundary conditions may be rendered ineffective due to the statistical fluctuations always present in a real physical system.

As an example we may refer to the Schottky solutions for different currents belonging to the external parameters of the longitudinal field, tube radius, gas kind and pressure. Interesting examples are also provided by Kenty's diffuse and filamentary discharge (Fig.15) and by the various arc spot modes (Fig. 16).

Let us form a multi-dimensional space of the parameters X_ν. Then let us introduce a function $U(X_\nu)$ which can take only integer values $(U=0, 1, 2. \ldots)$. The value of this function determines the number of possible s-modes for the set of external parameters.

Independent of the value of the U function it is quite possible that the experiment will not produce any s-mode. The reason would be that all modes are unstable. That means they are unable to sustain themselves in the stationary state against small perturbations, which always occur in any kind of a physical system. Here is the point where the problem of stability comes into consideration and we see immediately that it is of excellent importance.

II. THE CONCEPT OF STABILITY.

We call a s-mode stable if small deviations from the Y_ν state are damped. We call it unstable if the small deviations from the state grow in amplitude in such a way that the system progressively departs from the initial state and never returns to it.

This general definition is not sufficient for detailed discussions because the following questions are not taken into account.

1. The problem of stability can depend on the amplitude of the disturbance. A system may be stable against small deviations but unstable for slightly larger deviations.

2. The problem of stability can depend on the kind of disturbance considered. A certain s-mode may be stable with respect to disturbances of a certain kind but unstable with respect to different disturbances.

3. If a particular s-mode is stable in one part of the X_ν -space and unstable in the other part then there must be a region of the parameters X_ν in which it is neither stable nor unstable but neutral. The situation in this "neutral state" shows two different forms depending on whether or not the imaginary part of the coefficient determining the time dependence is zero. If the imaginary part is zero then in the neutral state the system passes from an aperiodically growing to an aperiodically damped state. If the imaginary part is finite then the system passes from an oscillatory growing to an oscillatory decaying state.

We base our discussions on the following definitions.

<u>ABSOLUTE STABILITY:</u> A given system in a certain stationary state shows absolute stability if it returns to the stationary state after any kind and any magnitude of parameter disturbance.

RELATIVE STABILITY: A given system in a certain stationary state shows relative stability if it returns to this stationary state after any kind of infinitesimal parameter disturbance.

TYPE-A-STABILITY: A given system in a certain stationary state shows relative or absolute stability of type A if it returns to this stationary state after an infinitesimal respectively arbitrary disturbance of parameter A.

MARGINAL INSTABILITY: If all initial states are classified as stable or unstable according to the criteria stated above then in the Γ -space the locus which separates the two classes of states defines the state of marginal or neutral stability.

SECONDARY FLOW INSTABILITY: If in the state of marginal instability the imaginary part of the coefficient determining the time dependence is zero then we say that the principle of the exchange of stabilities is valid or that we have an instability of secondary flow. In this case the restoring forces are to small to sustain the s-mode.

OVERSTABILITY: If at the onset of the instability the imaginary part of the coefficient determining the time dependence is finite then the oscillatory motion prevails and we have a state of overstability. In this case the restoring forces are so large that they overshoot the stationary state of the s-mode and cause the increasing oscillations.

III. MODE ANALYSIS OF THE STABILITY PROBLEM.

In this investigation we will not treat the problem of "absolute stability".

The theory of relative stability has the decisive advantage to allow linearization of the equations.

Our preceding definition of "relative stability" suggests the following way to investigate the stability question:

We consider small perturbations of the internal variables and study the time dependence of these deviations. Here two difficulties occur. The one is that our system is stable only if it is stable with respect to all possible initial increments. The second is that the time dependence not only changes the amplitude of a given initial disturbance but also its form. It is possible that with the change in the form an initially growing amplitude might gradually die out again. Such a state we would not call unstable.

To overcome the difficulties described above we consider the eigensolutions of our perturbation (index (1)) which are characterized by the time dependence

$$F^{(1)}(\vec{r},t) = F^{(1)}(\vec{r})\, e^{\omega t} \tag{1}$$

Introducing equation (1) into our system we find a set of equations which defines together with the boundary conditions an infinite number of eigensolutions and eigenvalues ω . These eigenfunctions provide a complete orthogonal system of functions. Any initial disturbance may be developed after these functions. Consequently the question of stability is reduced to the question whether anyone of the eigenvalues ω has a positive real part.

Therewith our task is to determine the eigenvalues.

Only in special cases the eigenfunctions can be determined in closed form and we have directly a dispersion relation

$$W(\omega) = 0 \tag{2}$$

defining the eigenvalues ω .

In general normal mode analysis is applied. In this case the eigenfunction is developed into a series of appropriate orthonormal functions. (normal modes).

(3)
$$F^{(1)}(\vec{r}) = \sum_k a_k f_k(\vec{r})$$

The terms of the differential equations can still depend on the coordinates via the imperturbed stationary solution. Therefore we encounter products of the form

(4)
$$g^{(0)}(\vec{r}) \cdot \sum_k a_k f_k(\vec{r})$$

Now we use for this whole term a mode representation in the form

(5)
$$g^{(0)}(\vec{r}) \cdot \sum_k a_k f_k(\vec{r}) = \sum_j b_j f_j(\vec{r})$$

where the coefficients b_j are related to the coefficients a_k by

(6)
$$b_j = \sum_k \left[\int g^{(0)}(\vec{r}) f_k(\vec{r}) f_j(\vec{r}) w(\vec{r}) d\tau \right] a_k$$

We introduce this into our equations and collect all terms belonging to the same normal mode. Since our relations must be fulfilled for any value of the coordinates, the sum over the coefficients of each mode must be identical zero for all equations. This provides an infinite set of homogeneous equations. We have as many equations as coefficients a_k and therefore a solution exists only if the secular determinant of the coefficients disappears. This condition leads to the characteristic dispersion equation for the eigenvalues ω

Introducing these eigenvalues into the equations for a_k allows to calculate these coefficients and with that the eigenfunctions.

If we have already the eigenvalues ω from equation (2) or from the secular determinent then we can decide the question of stability by studying the sign of the real parts of ω

However if we are merely interested in stability we do not really have to solve the dispersion relations. The question whether all the roots lie in the negative Gaussian plane can be answered by the following criteria.

Following Nyquist let us assume that our general dispersion relation (2) describes a conformal mapping of the ω -plane in the W-plane. Then from Cauchy's theorem we have

$$\int_{\mathcal{L}} \frac{W'(\omega)}{W(\omega)}\, d\omega = 2\pi i N \tag{7}$$

Where N is the number of zeros of W within the contour of the integral. Using the substitution $W = R. \exp\ (i\phi)$ it follows then

$$\Delta\phi = 2\pi i N \tag{8}$$

Consequently the question of stability may be decided by plotting in the W-plane the curve corresponding to the integral contour in the ω -plane (See Fig. 1). From the number of encirclings of this curve we may conclude to the number of roots lying within the contour of our integral in the ω -plane. This provides us with the criterion for stability.

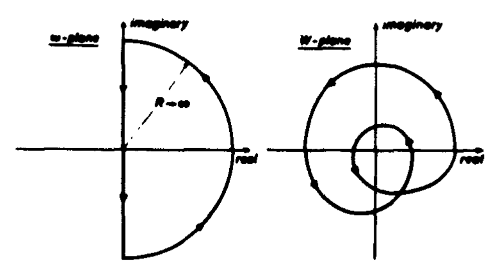

Fig. 1: Schematical sketch of the Nyquist mapping.

In the case of the normal mode analysis the dispersion relation is in general represented by an infinite power series.

It can happen that this series reduces to a polynomial or can be represented as a product of polynomials. Then we have simpler criteria to decide the question of stability.

A necessary condition for all the roots having negative real parts -that means a necessary condition for stability - is that all the coefficients of the polynomial have the same sign.

If the polynomial considered is of the quadratic type then the criterion of equal sign of all coefficients is also sufficient. For higher order polynomials the sufficient criteria are more complicated.

The best known necessary and sufficient criteria to decide whether all roots have negative real parts are those of Routh and Hurwitz.

According to the criterion of Hurwitz we have stability provided that all elements c and all "Hauptabschnittdeterminanten" D of the following matrix

$$(9) \qquad \left\{ \begin{matrix} c_1 & c_0 & 0 & 0 & \cdots & 0 \\ c_3 & c_2 & c_1 & c_0 & \cdots & 0 \\ c_5 & c_4 & c_3 & c_2 & \cdots & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & 0 & & c_n \end{matrix} \right\}$$

have a positive sign. The elements of the matrix correspond to the coefficients of the secular equation. The "Hauptabschnittsdeterminenten" are

$$(10) \quad D_1 = c_1, \quad D_2 = \begin{vmatrix} c_1 & c_0 \\ c_3 & c_2 \end{vmatrix}, \quad D_3 = \begin{vmatrix} c_1 & c_0 & 0 \\ c_3 & c_2 & c_1 \\ c_5 & c_4 & c_3 \end{vmatrix}, \ \ldots \ D_n = c_n D_{n-1}$$

The Routh criterion can be understood from the following scheme.

$$
\begin{array}{llll|l}
c_n & c_{n-2} & c_{n-4} & \cdots & \\
c_{n-1} & c_{n-3} & c_{n-5} & \cdots & \cdot(-c_n)/c_{n-1} \\
a_{n-2} & a_{n-4} & a_{n-6} & \cdots & \cdot(-c_{n-1})/a_{n-2} \\
b_{n-3} & b_{n-5} & b_{n-7} & \cdots & \cdot(-a_{n-2})/b_{n-3} \\
\vdots & \vdots & \vdots & \cdots & \vdots
\end{array}
\qquad (11)
$$

In this scheme the third equation is produced from the first two ones by multiplying the second with the factor indicated on the right hand side and adding it to the first equation. The result is given in the third equation where we have shifted every coefficient by one place to the left hand side. The fourth equation is produced in a similar way by the multiplication of the third equation and addition to the second one etc. If all the roots are supposed to have negative real parts, then all the numbers of the first column must be positive. That means proceeding from one equation to the next we may stop our calculation at the very moment if zero or a negative quantity appears, because then we know that the problem under consideration exhibits instability.

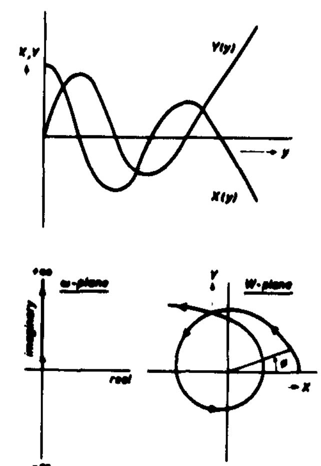

Fig. 2: Schematical sketch for the "Ortskurvenkriterium".

The "ORTSKURVENKRITERIUM" considers the mapping of the complex ω -plane into the W-plane as prescribed by the dispersion relation $W(\omega)$. The imaginary axis of the ω -plane ($\omega = iy$) produces a certain curve in the W-plane (See Fig. 2).

$$W(\omega) = c_0 + c_1\omega + c_2\omega^2 + \cdots + c_n\omega^n$$

(12) $$= c_0 - c_2 y + c_4 y^4 - + \cdots + i\,(c_1 y - c_3 y^3 + c_5 y^5 - + \ldots)$$

$$= X(y) + i\,Y(y)$$

$$= R\,e^{i\phi}$$

If we follow the imaginary axis of the ω -plane from $-\infty$ to $+\infty$ then again a certain path in the W-plane the socalled "Ortskurve" is travelled trough. It is sufficient to consider only the part $0 \leq y < +\infty$ since we have the relations X(- y) = X (y) and Y (-y) = - Y(y).

The criterion reads now: Equation (12) has roots with negative real parts only if the "Ortskurve" in the W-plane surrounds the origin in such away that the angle ϕ of the pointer in the W-plane covers the range $\phi = 0 \ldots\ldots n\pi/2$ if y goes from zero to infinity.

If there are roots with positive real parts and the angle covered may be written in the form

(13) $$\Delta\phi = (q-p)\frac{\pi}{2} = l\,\frac{\pi}{2}$$

then the number of roots with positive real part is given by

(14) $$p = \frac{n-l}{2}$$

and number of roots with negative real part is given by

(15) $$q = \frac{n+l}{2}$$

To evaluate the "Ortskurvenkriterium" it is not really necessary to plot the path in the W-plane. It is already sufficient to show that the following is correct:

"<u>LÜCKENKRITERIUM</u>": The equation $W(\omega) = 0$ has roots with negative real parts only if the equations

$$X(s) = 0; \quad \frac{1}{y} Y = \overline{Y}(s) = 0 \tag{16}$$

with $s = y^2$ have positive roots s_j respectively $\bar{s}_j$ only,if the zeros follow the sequence

$$s_1 < \bar{s}_1 < s_2 < \bar{s}_2 < s_3 < \bar{s}_3 \ldots \tag{17}$$

It is not even necessary to solve for these roots since it is sufficient to show that for $c_n > 0$ the equation

$$\overline{Y}(s) = 0 \tag{18}$$

has positive real parts only

$$0 < \bar{s}_1 < \bar{s}_2 < \bar{s}_3 \ldots \tag{19}$$

and that the values of the function $X_j = X(\bar{s}_j)$ changes its sign from one to the next root.

$$X_1 < 0, \; X_2 > 0, \; X_3 < 0, \ldots \tag{2o}$$

The normal mode analysis as described above has been frequently used. For a specific problem it is a powerful instrument. It allows to decide not only the question of stability but also implies the determination of the eigenfunctions and the growth of the instability respectively the decay time.

The second procedure to tackle the problem of stability restricts itself from the outset to the calculation of the stability criterion. Therefore the mathematical handling is simpler. The approach we refer to is the METHOD OF VARIATIONAL PRINCIPLE.

IV. VARIATIONAL PRINCIPLE ANALYSIS OF THE STABILITY PROBLEM.

There are essentially two difficulties inherent in the normal mode analysis. After choosing an appropriate set of orthogonal modes to expand the eigenfunctions the one problem arises from the calculation of the coefficients of the expansions (6). These coefficients include difficult integral expressions which cannot be evaluated in general form. The other problem is caused by the formidable difficulties in the evaluation of the secular equation.

A simplification can be expected by renouncing the specific knowledge of the frequencies and growth rates and by simply restricting our question to the criterion for the onset of instabilities.

This is the basic idea of the variational approach to the stability problem.

The principal features of the calculations are quite similar to those of the normal mode analysis. We again consider small deviations from an s-mode. This allows us to linearize the relations and we have a set of differential equations for the first order perturbation where the coefficients may depend on the qualities of the stationary state solution.

As in the case of the normal mode analysis the eigenfunctions of this first order problem are characterized by the relation (1).

We reduce our system of differential equations to one equation

(21) $$\mathcal{L}(\psi) = 0$$

for the dependent variable Ψ. We develop Ψ after the eigen-functions Ψ_k in the form

$$\Psi = \sum_k a_k \Psi_k = \sum_k a_k \psi_k e^{\omega_k t} \tag{22}$$

The applicability of the variational method depends on the character of the operator $\mathcal{L}$. Therefore specification of this operator is required for further discussion. As an example we treat here the case, where equation (21) reduces to

$$\mathcal{L}_r (\psi_k) = \omega_k \psi_k \tag{23}$$

where the operator $\mathcal{L}_r$ does not contain the time anymore. Introducing equation (22) into equation (21) and using equation (23) we find

$$\sum_k a_k \mathcal{L}_r (\psi_k) = \sum_k a_k \omega_k \psi_k \tag{24}$$

We now develop the adjoint function $\tilde{\Psi}$ after the adjoint eigenfunctions $\tilde{\Psi}_k$.

$$\tilde{\Psi} = \sum_k \tilde{a}_k \tilde{\Psi}_k = \sum_k \tilde{a}_k \tilde{\psi}_k e^{\tilde{\omega}_k t} \tag{25}$$

Multiplication of equ. (24) with $\tilde{\Psi}$ and the appropriate weight function w and integration produces

$$\int \sum_{j,k} \tilde{a}_j \tilde{\psi}_j a_k \mathcal{L}_r (\psi_k) w(\vec{r}) d\tau = \int \sum_{j,k} \tilde{a}_j a_k \omega_k \tilde{\psi}_j \psi_k w(\vec{r}) d\tau \tag{26}$$

If the operator $\mathcal{L}_r$ is self-adjoint it follows

$$\int \sum_{j,k} \tilde{a}_j \tilde{\psi}_j a_k \mathcal{L}_r (\psi_k) w(\vec{r}) d\tau = \sum_k \tilde{a}_k a_k \omega_k \tag{27}$$

Remembering that the a_k are first order quantities we aim to represent the right hand side of equ. (27) as a variation of a physical quantity Q. If this is possible we arrive at

(28) $$\sum_k \tilde{a}_k a_k \omega_k = \delta Q$$

Since all values of $\tilde{a}_k a_k$ are positive, the left hand side can be negative only if at least one of the eigenvalues has a negative real part. In this simple example the imaginary parts must all be zero, since the physical quantity Q is real.

Therefore if all the eigenvalues ω_k are real and positive, the right hand side of the equation (28) is a positive definite form of the deviations from the stationary state. Since it is sufficient if one value becomes negative we can conclude the following stability criterion:

The system under consideration is stable then and only then if the physical quantity Q is a positive definite form with respect to all possible deviations from the stationary state.

V. STABILITY CONSIDERATIONS IN PLASMA PHYSICS.

The methods prescribed in the preceding paragraphs were originally developed in connection with the stability problems in hydrodynamics.

During the recent years they have been widely applied also to problems of plasma physics. The applications may be grouped according to the underlying physical model.

The M H D approximation is based on the essential assumptions of skalar pressure, an adiabatic invariant

$$\frac{d}{dt}\left(p \rho_m^{-\gamma_r}\right) = 0 \tag{29}$$

and infinite conductivity.

For this model the gross instabilities of a cylindrical plasma column have been calculated [53]. Also the stability of a plasma in contact with a magnetic wall has been studied. Here normal mode analysis produces similar to the Rayleigh Taylor instabilities of hydrodynamics, gravitational, flute and ripple instabilities

The variational analysis has been applied to magnetohydrodynamics by Lundquist [57] and later by Bernstein, Frieman, Kruskal and Kulsrud [9]. They showed that for a magnetohydrodynamic device the function Q can be identified with the potential energy. That means a magnetohydrodynamic system is in equilibrium if the variation of the potential energy is a positive definite quadratic form, - provided that certain conditions are satisfied.

Chew, Goldberger and Low [13] have treated a different approximation in which the pressure is not scalar and there are two adiabatic invariants for the pressure parallel and perpendicular to the magnetic field. In the C. G. L. approximation the stability of a plasma may be also decided by an energy variation [9].

Generalizations of the energy principles of the M.H. and the C. G. L. approximation have been given by Kruskal and Oberman [54] and Rosenbluth and Rostocker [72]. The energy change calculated by these authors is bounded below by the M.H. energy change and above by the C.G.L. energy change.

In a recent publication Rosenbluth and Fuerth [71] study the problem of micro instabilities in a system of finite resistivity.

For the general phenomenon of micro instabilities there is no general physical model but various special cases have been treated [71, 73].

For our investigation of the stability qualities of the arc and the glow discharge the results quoted above are not applicable, because the underlying conditions are not met.

Since the equations describing the system are characteristic and decisive for the stability consideration one would like to define our problem by stating the equations and the approximations.

Unfortunately in the case of the gas discharges there is no unique set of equations describing the whole arc and glow region. Rather we have in each of these discharge types a number of model zones governed by different relations.

As a consequence we will expect that the various model regions of the discharges can exhibit different stability qualities. Our discussion must be specified with respect to each model region.

Since all stability calculations require the knowledge of the s-modes we at first discuss the possible s-modes. This presents at the same time the equations and approximations of our considerations.

VI. BASIC EQUATIONS, BOUNDARY CONDITIONS, AND APPROXIMATIONS.

Most of our investigations will be based on the following equations.

From the Boltzmann transport equation we derive by multiplication with m, mv, $mv^2/2$ and integration over the momentum space the following transport equations

$$\frac{\partial}{\partial t} n + \vec{\nabla} \cdot (n \vec{v}_d) = \alpha_t \tag{30}$$

$$\frac{\partial}{\partial t} \vec{\Gamma} + \frac{e}{m} \vec{B} \times \vec{\Gamma} + \sum_i \nu_i (\vec{\Gamma} - \vec{\Gamma}_i) = \frac{e}{m} n \vec{E} - \vec{\nabla} \cdot (\frac{\overleftrightarrow{P}}{m}) \tag{31}$$

$$\frac{\partial}{\partial t} (n v^2) + \vec{\nabla} (\overline{n v^2 \vec{v}}) - \frac{2ne}{m} \vec{E} \cdot \vec{v}_d =$$

$$\sum_i \frac{2 \nu_i n}{m_i + m} (m_i \overline{v_i^2} - m v^2) - \sum_{i,x} \frac{2 U_{ix} e n}{m} (\nu_{ix} - \bar{\nu}_{ix}) - S \tag{32}$$

where the approximations

$$\overleftrightarrow{P} = n k T \overleftrightarrow{I}$$

$$\int [v'^2 - v^2] f_i (\vec{v}_i) f(\vec{v}) c_i \sum_x \varrho_{ix} d\Omega \, dv_i^3 dv^3 = -\frac{2me}{m} \sum_x (\nu_{ix} - \bar{\nu}_{ix}) U_{ix} \tag{33}$$

have been used.

α_t denotes the net particle production per second and unit volume. The collision frequencies

$$\nu = n \int \varrho c \, d\Omega \tag{34}$$

are related to the mobilities μ and the electron-ion interaction parameters η according to

$$\nu_n = \frac{m}{e} \mu \;\; ; \;\; \nu_{-+} = \frac{e^2 n}{m_-} \eta \;\; ; \;\; \nu_{+-} = \frac{e^2 n}{m_+} \eta \tag{35}$$

A THE GLOW APPROXIMATION.

Characteristic for the glow approximation is the following: In the particle conservation law for the charged particle components the destruction is dominated by diffusion and mobility losses and not by volume recombination. In other words: The fact, that each volume element is an open system with respect to the charged particle balance is decisive and characteristic for the glow approximation.

Characteristic for the glow model is also the boundary condition at the wall. We have at the sheath edge for the particle number density the condition

(38) $$(D_+ \, grad \, n)_w = (n \, \bar{v}_+)_w \cdot \gamma_n$$

where γ_n is an uncertainty factor of magnitude one, accounting for the influence of the geometry and transition region [16a]. For very small values of the mean free path equation (38) simplifies to

(39) $$(n)_w = 0$$

Apart from these characteristic features one in general also assumes weak ionization

(36) $$n_n \, k \, T_n = p = const$$

quasineutrality

(37) $$n_- \approx n_+ = n$$

and the validity of the energy balances

$$T_+ = T_n \tag{40}$$

and

$$T_- = c\,E\,\lambda_- \tag{41}$$

with C being a constant typical for the gas.

B THE ARC APPROXIMATION.

Characteristic for the arc approximation is the following: In the particle conservation law for the charged particle components the destruction is dominated by volume recombination and not by diffusion and mobility losses. In other words: The fact, that each volume element is a closed system with respect to the charged particle balance is decisive and characteristic for the arc approximation.

Characteristic for the arc model is also the boundary condition at the wall

$$(\Gamma_\perp)_w = 0 \tag{42}$$

From these conditions the local validity of the Saha equation

$$n = \frac{2p^{1/2}(kT)^{1/4}(2\pi m_-)^{3/4}}{h^{3/2}} \exp\left(-\frac{e U_i}{2kT}\right) \tag{43}$$

follows.

Apart from these characteristic conditions one in general also assumes

(44) $$T_+ = T_- = T_n = T$$

and quasineutrality.

C <u>LIMITATION OF GLOW AND ARC APPROXIMATION.</u>

The glow and the arc approximation described in the preceding chapters are complementary. We therefore need only one limitation.

According to the definitions this limitation is given by

(45) $$div\left(D_{eff}\ grad\ n\right) = \bar{\sigma}\, n^2 n_n$$

where ($\bar{\sigma}$) is the recombination coefficient.

D INEFFECTIVE BOUNDARY CONDITIONS.

The above definitions of the glow and the arc model are based on the assumption that the boundary conditions (38) and (42) are the effective ones.

Physically these boundary conditions formulate the requirement of particle current continuity and energy current continuity respectively, at the sheath edge. There may be physical influences not included in our description above which take an effect on these continuity requirements. These effects may either be due to additional phenomena like external fields, convection etc. or they may be due to the normal statistical fluctuations present in any kind of real discharge.

If these influences become stronger than the terms included in the formulation of our boundary conditions (38) and (42) then they render these conditions ineffective.

The behaviour of our discharge is different depending on whether the ineffectiveness is caused by an other external influence or by statistical fluctuations.

In the first case the additional phenomenon determines the selection of the observed mode and we have a different effective boundary condition.

In the second case the statistical fluctuations which outrule the boundary conditions allow transitions between all those modes which are possible without the wall boundary condition. In this case the observed mode is determined by a stability criterion.

VII STATIONARY MODES OF THE GLOW- AND ARC DISCHARGE.

Without a claim to completeness we underlay our investigations the following stationary modes of the arc- and glow discharge.

A) NORMAL GLOW COLUMN (SCHOTTKY-COLUMN
B) THERMAL GLOW COLUMN
C) GLOW COLUMN WITH NEGATIVE IONS
D) WALL DETACHED GLOW COLUMN
E) WALL STABILIZED ARC COLUMN
F) WALL DETACHED ARC COLUMN
G) CATHODIC PART OF THE ARC DISCHARGE
H) ANODIC PART OF THE ARC DISCHARGE
I) CATHODIC PART OF THE GLOW DISCHARGE
J) ANODIC PART OF THE GLOW DISCHARGE

A NORMAL GLOW COLUMN (SCHOTTKY COLUMN).

The normal glow column is governed by the following mechanism:

Electrons and ions move in the axial direction under the influence of a longitudinal electric field. At the same time they move towards the wall under the influence of a concentration gradient and a radial ambipolar field. Particle production is due to collisions of electrons with neutral particles. Particle loss is due to wall recombination only. Volume recombination is neglected. The temperature of the electrons is defined by the energy gain in the electric field on the one hand and the energy loss by collisions with neutral particles on the other hand. The temperature of the ions is determined by the energy gain from the electric field and the energy loss due to collisions with the neutral particles. There is no radiative loss. Therefore the total energy gained from the electric field is carried to the walls by heat conduction of the neutral particles. The energy input is so small that the neutral particle temperature can be considered constant across the discharge. Then - for weak ionization - also the electron- and ion temperature are constant. The longitudinal electric field adjusts itself in such a way that the number of particles produced in the discharge covers the particle loss to the walls.

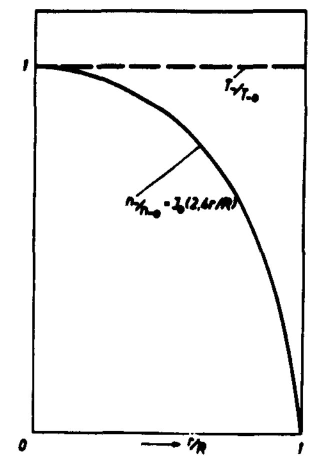

Fig. 3: Schematical sketch of the density and temperature distribution in the normal glow column.

Under these assumptions the glow approximation (VI A) is applicable throughout the whole discharge volume.

The longitudinal electric field is constant due to

$$rot\ \vec{E} = 0 \tag{46}$$

In equations (31) we use the conditions of stationarity $\partial / \partial t = 0$, weak ionization $p = n_n k T_n$ and the relation

$$D_{\pm} = \mu_{\pm} P_{\pm} / ne = \mu_{\pm} \cdot k T_{\pm} / e \tag{47}$$

With the condition of congruence

$$\Gamma_{r-} = \Gamma_{r+} \tag{48}$$

we eliminate all azimuthal components and the electric field and arrive at

(49) $$\Gamma_{r+} = \Gamma_{r-} = -D_{eff} \frac{\partial}{\partial r} n$$

with

(50) $$D_{eff} = \frac{D_+\mu_- + D_-\mu_+}{\mu_- + \mu_+} \; \frac{1}{1+\mu_+\mu_- B^2} = D_{am} \frac{1}{1+\mu_+\mu_- B^2}$$

Introducing this into the particle conservation law we have the equation

(51) $$\frac{1}{r}\frac{d}{dr}\left(r D_{eff} \frac{dn}{dr}\right) + \alpha n = 0$$

which together with the boundary condition (39) defines an eigenvalue problem for the electron density and the eigenvalue α . Since the particle density n can only assume finite positive values the well known solution of this eigenvalue problem is

(52) $$n(r) = n_o \, J_o\left(\left[\alpha/D_{eff}\right]^{\frac{1}{2}} \cdot r\right)$$

with

(53) $$\alpha = (2{,}4/R)^2 D_{eff}$$

Since the production coefficient α depends on the electron temperature according to the relation

(54) $$\alpha(T_-) = A(T_-) \exp\left(-\frac{eU_i}{kT_-}\right)$$

and the electron temperature is determined by the electric field according to equation (41) the eigenvalue (53) defines the eigenvalue of the electric field. Obviously the gradient E_z is independent of the current within the frame of the Schottky theory.

Schottky's theory underlies the assumption that the charged carriers are produced by a single ionizing collision. In rare gases where the formation of metastable atoms is common cumulative ionization may occur. One expects these processes to increase with current. The cumulative ionization causes a field decrease with increasing pressure. Spenke and Fabrikant [22, 81] included cumulative ionization by adding nonlinear terms in the equation (51). In an exact treatment the additional particle components, like e. g. the metastables, should be treated with a separate particle balance equation (see e. g. Chapter VII C).

Schottky's theory also does not account for volume recombination. This effect becomes important with increasing particle densities. Particularly in the presence of electronegative gases or molecular gases volume recombination may be of remarkable influence.

B THERMAL GLOW COLUMN.

One of the characteristic approximations for the Schottky theory of the normal glow column is the assumption of constant electron, ion and neutral gas temperature across the discharge.

In this chapter we will give an approximate calculation of the column in which the thermal effects cannot be considered as a small perturbation of the normal glow theory.

For the thermal glow column the whole model concept of the normal glow column holds with one exception: The energy input is so large that a variation of the neutral gas temperature is caused which cannot be considered as a small perturbation. As a consequence, electron and ion temperature show a radial variation.

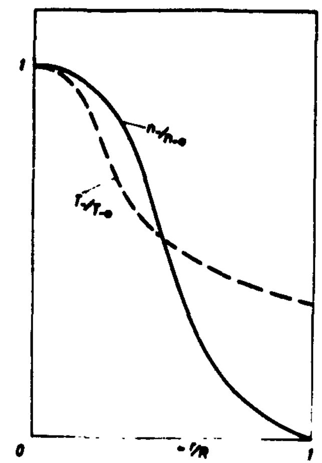

Fig. 4: Schematical sketch of the density and temperature distribution in the thermal glow column.

Therefore the following equations apply.

The particle conservation law for the electrons

(55) $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right)+\frac{\alpha}{D_{eff}}\,n=0$$

Under the assumption of weak ionization the particle balance for the neutral component simplifies to

(56) $$n_n k T_n = p = const$$

The energy balance of the electrons reads

(57) $$T_- = c\,E\,\lambda_-$$

and the energy balance of the neutral particle component is

(58) $$\frac{\lambda}{r}\frac{d}{dr}\left(r\frac{dT_n}{dr}\right)+e(\mu_+ + \mu_-)E^2 n = 0$$

The coefficient of the volume particle production per sec. will be represented in the form

$$\alpha = A \exp\left(-\frac{eU_i}{kT_-}\right) \tag{59}$$

The experiments provide the information

$$\frac{\alpha^*}{p} = A^* \exp\left(-\frac{B \cdot p}{E}\right) \tag{60}$$

Here it should be remembered that the characteristic parameters are actually not α^*/p and E/p but α^*/n_n and E/n_n where n_n is the neutral particle density. Since all experiments are carried out with constant neutral particle temperature, p reflects the dependence of n_n. In our calculations however the temperature of the neutral gas can cause a variation in the neutral gas density. Therefore we should apply the relation

$$\alpha = v_d A^* p \frac{T_{nw}}{T_n} \exp\left\{-\frac{Bp T_{nw}}{E T_n}\right\} = A \exp\left\{-\frac{eU_i}{kT_-}\right\} \tag{61}$$

The experimental knowledge (60) not only provides the coefficient A but also defines the coefficient c of equation (57) according to

$$T_- = \frac{eU_i}{k} \frac{1}{Bp} \frac{T_n}{T_{nw}} E = c \lambda_- E \tag{62}$$

Now we introduce equations (56) and (57) into (58) and have the following relation for the electron temperature

$$\frac{\lambda'}{r} \frac{d}{dr}\left(r \frac{dT_-}{dr}\right) + e(\mu_+ + \mu_-) E^2 n = 0 \tag{63}$$

With the abbreviations

$$\lambda' = \frac{\lambda k B \cdot p \cdot T_{nw}}{e U_i E} \tag{64}$$

Equations (55) and (63) are two simultaneous differential equations for the electron temperature and the electron density in the discharge.

In addition we have the boundary conditions

$$\frac{dn}{dr} = 0 , \quad \frac{dT}{dr} = 0 \qquad \text{for } r = 0$$

$$n = 0 , \quad T_n = T_W \qquad \text{for } r = R \tag{65}$$

This is an eigenvalue problem with the eigenvalue E .

We base our approximation on the fact that the production coefficient due to the strong dependence on the electron temperature shows a shape similar to that of a step function. Consequently we represent α by the step function shown in Fig. 5. The three quantities α_o , α_a , ρ , characterize the step function. α_o is the production coefficient corresponding to the electron temperature in the axis. α_a is the production coefficient corresponding to the wall temperature of the electrons. The position of the step (ρ) is derived from the condition $\alpha_o / \alpha(\rho) = e$ to be

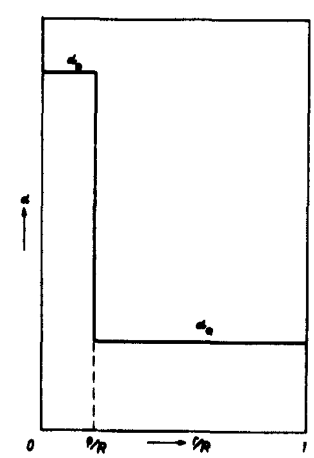

Fig. 5: Step approximation of the production coefficient.

$$J_o\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}} \rho\right) = 1 - \frac{kT_e^2}{eU_i} \frac{\lambda'}{e(\mu_+ + \mu_-)E^2 n_o} \frac{\alpha_o}{D_{eff}} \tag{66}$$

With this step function for α it is not difficult to find the density and temperature distributions in the interior and in the outer part independently. The result is

$$n_i = n_o \, J_o\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}} r\right) \tag{67}$$

$$T_i = T_{i1} \, J_o\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}} r\right) + T_{i2} \tag{68}$$

$$n_a = n_{a1} \, J_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} r\right) + n_{a2} \, N_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} r\right) \tag{69}$$

$$T_a = T' J_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} r\right) + T'' N_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} r\right) + T_{a1} + T_{a2} \ln r \tag{70}$$

In these equations the boundary conditions at $r = 0$ are taken into account but not those at $r = R$.

$$n_i = n_a \;, \quad \frac{dn_i}{dr} = \frac{dn_a}{dr} \;; \quad T_i = T_a \;, \quad \frac{dT_i}{dr} = \frac{dT_a}{dr} \tag{71}$$

These conditions secure finite values of the particle current densities and finite values of the sources at $r = \rho$.

The application of the conditions (71) at $r = \rho$ and the conditions (65) at $r = R$ to the equations (67) - (70) results in

$$n_o J_o\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}} \rho\right) = n_{a1} J_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} \rho\right) + n_{a2} N_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} \rho\right) \tag{72}$$

$$(73)\quad \left(\frac{\alpha_i}{D_{eff}}\right)^{\frac{1}{2}} n_o J_1\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}}\rho\right) = \left(\frac{\alpha_a}{D_{eff}}\right)^{\frac{1}{2}}\left\{ n_{a1} J_1\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}\rho\right) + n_{a2} N_1\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}\rho\right)\right\}$$

$$(74)\quad T_{i1} J_o\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}}\rho\right) + T_{i2} = T' J_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}\rho\right) + T'' N_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}\rho\right) + T_{a1} + T_{a2} \ln r$$

$$(75)\quad \left(\frac{\alpha_o}{D_{eff}}\right)^{\frac{1}{2}} T_{i1} J_1\left(\left[\alpha_o/D_{eff}\right]^{\frac{1}{2}}\rho\right) = \left(\frac{\alpha_a}{D_{eff}}\right)^{\frac{1}{2}}\left\{ T' J_1\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}\rho\right) + T'' N_1\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}\rho\right)\right\} - T_{a2}\frac{1}{\rho}$$

$$(76)\quad n_{a1} J_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} R\right) + n_{a2} N_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}} R\right) = 0$$

$$(77)\quad T_{-W} = T_{a1} + T_{a2} \ln R$$

Clearly a general solution of this system of equations is impracticable. We therefore restrict ourselves to the case of small values of ρ using the condition

$$(78)\quad 1 \gg \left(\alpha_o/D_{eff}\right)^{\frac{1}{2}}\rho \gg \left(\alpha_a/D_{eff}\right)^{\frac{1}{2}}\rho$$

This is not really a new restriction since it is included in our assumption of a strong radial dependence of the neutral gas temperature as may be seen from our equation (66).

With equation (78) it is possible to develop all special functions occuring in equations (72) - (77) and to neglect higher order terms. With equation (66) and equations (72) - (77) we have seven simultaneous equations for the quantities n_{a1}, n_{a2}, T_{a1}, T_{a2}, T_o, ρ and α_a. The quantities n_{a1}, n_{a2}, T_{a1}, T_{a2}, T_ρ and ρ define uniquely the electron temperature and density distributions in the discharge. The quantities T_o, ρ and α_a are also of interest with respect to our application in the instability calculation of chapter (VIII).

With respect to the application in Chapter (VIII) we concentrate on the calculation of the latter quantities.

We use the equations (66), (74) and (76) develop equation (66) and introduce it into equation (76). This results in

$$J_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}R\right)\Big/N_o\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}R\right) = \frac{\pi\,\lambda'}{e(\mu_+ + \mu_-)D_{eff}\,n_o}\,\frac{k\,T_o^2}{eU_i}\,\frac{\alpha_a}{E^2} \qquad (79)$$

which together with

$$T_o = \frac{e(\mu_+ + \mu_-)D_{eff}\,n_o}{\lambda'}\,\frac{E^2}{\alpha_a} + T_{-w} \qquad (80)$$

defines T_ρ and α_a. These equations contain apart from T_o and α_a the quantity E which we express by α_a according to equation (60). We see from equation (60) that the quantity E varies very slowly with α_a and therefore we use the approximation

$$\alpha_a = E_s\,A^*\mu_-\,p\,\exp\left(-Bp/E\right) \qquad (81)$$

where E_s is the longitudinal electric field of the corresponding normal glow column.

Then the two equations for T_o and α_a read

$$T_o = \frac{e(\mu_+ + \mu_-)D_{eff}\,n_o}{\lambda'}\,\frac{E_s^2}{\alpha_a} + T_s \qquad (82)$$

and

$$J_0\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}R\right)/N_0\left(\left[\alpha_a/D_{eff}\right]^{\frac{1}{2}}R\right) = \tag{83}$$
$$\frac{\pi\lambda'}{e(\mu_+ + \mu_-)D_{eff}n_0}\;\frac{kT_0^2}{eU_i}\;\frac{\mu_- A^* p}{E_s}\;\exp\left\{-\frac{eU_i}{kT_0}\right\}$$

T_s is the electron temperature in the normal glow column.

To calculate T_o and the electric field E, respectively α_a, we would have to introduce experimental data.

However without detailed calculation the following general conclusions can already be drawn.

Remembering that the right hand side of equation (83) can assume positive values only and that we should include the transition to the normal glow theory the inequality

$$0{,}9 \leq (\alpha_a/D_{eff})^{\frac{1}{2}}R \leq 2{,}4 \tag{84}$$

must hold independent of the choice of the experimental parameters. At least this is true within the frame of our approximations. It follows that the longitudinal electric field component E differs only little from the Schottky value E_s. With increasing particle number density in the center there is a slight decrease in E.

With increasing discharge current the particle number density in the axis increases slightly more than proportional to the current.

Equation (82) shows that the axial electron temperature T_o increases with the particle number density in the axis and with the total discharge current.

In judgement of our results let us always remember that we must exclude small discharge currents due to our initial assumptions. Also very large discharge currents are excluded since in this region the glow model breaks down.

C GLOW COLUMN WITH NEGATIVE IONS.

In this chapter we consider the influence of negative ions on the normal glow column. Subject of this investigation is therefore the model described in chapter (VII A) extended by the addition of a negative ion component.

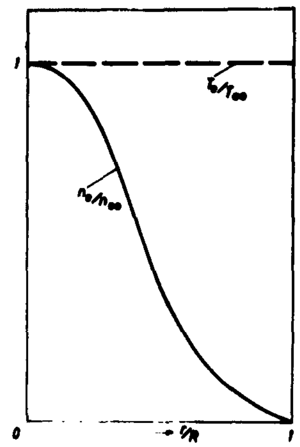

Fig. 6: Schematical sketch of the density and temperature distribution in the glow column with negative ions.

Consequently we have the following system of equations describing our problem:

The particle conservation laws

$$\vec{\nabla} \cdot \vec{\Gamma_i} = \Delta_i \tag{85}$$

the momentum balances

$$\vec{\Gamma_i} = - D_i \vec{\nabla} n_i + \frac{e_i}{|e_i|} \mu_i n_i \vec{E} \tag{86}$$

the assumption of quasineutrality

$$\sum_{i=1}^{3} e_i n_i = 0 \tag{87}$$

the condition of congruence

$$\sum_{i=1}^{3} e_i \Gamma_{ir} = 0 \tag{88}$$

the neutral particle balance under the assumption of constant pressure, weak ionization and small energy input

(89) $$n_n = const$$

and the boundary conditions

(90) $$\frac{dn_i}{dr} = 0 \quad for \ r = 0 \ ; \quad n_i = 0 \quad for \quad r = R$$

Introducing equation (86) into equation (88) gives the radial electric field

(91) $$E_r = \frac{\sum_k e_k D_k \nabla_r n_k}{\sum_k |e_k| \mu_k n_k}$$

and with that the radial particle current density

(92) $$\Gamma_{ir} = - D_i \nabla_r n_i + \frac{\mu_i n_i \sum_k e_k D_k \nabla_r n_k}{\sum_k |e_k| \mu_k n_k}$$

In the above equations the indices can assume the values (e), (+) and (-) referring to electrons, positive and negative ions respectively. We stress particularly that in this chapter - and only in this chapter - the index (-) refers to the negative ions and not to the electrons.

For a three-component system consisting of electrons, singly charged positive and negative ions the following processes contribute to the net-production terms Δ_i :

Ionization of neutral particles by electron collision

$$\Delta_a = (\alpha^*/p)(p/n_n)\, v_{ed}\; n_n n_e \tag{93}$$

Attachment of electrons to neutral particles

$$\Delta_b = (\beta^*/p)(p/n_n)\, v_{ed}\; n_n n_e \tag{94}$$

Detachment of electrons from negative ions by electron collision

$$\Delta_c = \delta\, v_{e\,th}\; n_- n_e \tag{95}$$

Recombination of negative ions with positive ions

$$\Delta_d = \sigma\, 2^{\frac{1}{2}}\, v_{\pm th}\; n_+ n_- \tag{96}$$

From the equations (85), (87), (92) and (93) - (96) we derive the following simultaneous differential equations for the electron density n_e and the negative ion density n_- .

$$\frac{1}{r}\frac{d}{dr} r\left[-\frac{n_e(\mu_e D + \mu D_e) + 2\mu D_e n_-}{n_e(\mu_e + \mu) + 2\mu n_-}\,\frac{dn_e}{dr}\right] = n_e\left[\left(\frac{\alpha^*}{p} - \frac{\beta^*}{p}\right) v_{ed}\, p - \delta v_{eth}\, n_-\right] \tag{97}$$

$$\frac{1}{r}\frac{d}{dr} r\left[-D\frac{dn_-}{dr} + \frac{(D_e + D)\mu n_-}{n_e(\mu_e + \mu) + 2\mu n_-}\,\frac{dn_e}{dr}\right] = n_e\left[\frac{\beta^*}{p}\, p\, v_{ed} - (\delta v_{eth} + \sigma 2^{\frac{1}{2}} v_{\pm th}) n_-\right] - \sigma 2^{\frac{1}{2}} v_{\pm th}\, n_-^2 \tag{98}$$

with the boundary conditions (90).

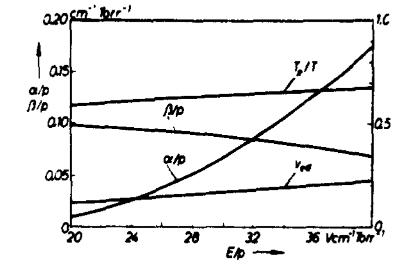

Fig. 7: Numerical values of the experimental parameters. The right ordinate gives 10^{-2} T_e/T_n resp. 10^{-8} v_{ed} (cm/s)

EVALUATION.

We do not know of an electro-negative gas which can be described by a three-component system and whose experimental data are sufficiently known. Therefore we have to apply our calculations to a ficticious gas. In this model gas we replace the various ion components by one positive and one negative ion component with average parameters. We will also choose the numerical values close to those known for oxygen.

For the evaluation we introduce the following abbreviations

(99)

$$X = T/R \;,\quad Y = n_e/n_{eo} \;,\quad Z = n_-/n_{eo}$$

$$A = [(p\cdot R)^2 v_{ed}]/[p\,D_{am}] \approx [(p\cdot R)^2 v_{ed}]/[p\,D(T_e/T)]$$

and find

$$(100)\quad \frac{1}{x}\frac{d}{dx}\,x\,\frac{y+2z}{y+2(\mu/\mu_e)z}\,\frac{dy}{dz} = -Ay\left[\frac{\alpha'}{p} - \frac{\beta'}{p} + \delta\,\frac{v_{eth}}{v_{ed}}\,z\,\frac{n_{eo}}{n_n}\,\frac{n_n}{p}\right]$$

and

$$(101)\quad \frac{1}{x}\frac{d}{dx}\,x\left[-\frac{T}{T_e}\frac{dz}{dx} + \frac{z}{y+2(\mu/\mu_e)z}\,\frac{dy}{dx}\right] =$$

$$Ay\left[\frac{\beta^*}{p} - \left(\delta\,\frac{v_{eth}}{v_{ed}} + \sigma\,2^{\frac{1}{2}}\,\frac{v_{\xi th}}{v_{ed}}\right)z\,\frac{n_{eo}}{n_n}\,\frac{n_n}{p}\right] - A\sigma\,2^{\frac{1}{2}}\,\frac{v_{\xi th}}{v_{ed}}\,z^2\,\frac{n_{eo}}{n_n}\,\frac{n_n}{p}$$

with the boundary conditions

$$\frac{dy}{dx} = 0, \quad \frac{dz}{dx} = 0 \quad \text{for } x = 0$$
$$y = 0, \quad z = 0 \quad \text{for } x = 1 \tag{102}$$

In Fig. 7 the parameters entering equations (100, 101) are shown as a function of E/p.

The numerical values used for the coefficients are

$$\delta = 10^{-14}\,[cm^2] \qquad \mu/\mu_e = 10^{-2}$$
$$\sigma = 10^{-12}\,[cm^2] \qquad T_+ = T_- = T_n = 400\,[°K] \tag{103}$$

The system of differential equations (100, 101) was solved by an appropriate approximation procedure with the help of a digital computer. The results are shown in Figs. 8-11.

DISCUSSION.

The parameters of our calculation are n_{eo}/n_n and E/p.

The solution of the eigenvalue problem produces the product of the total current with the pressure, Ip, and the product of the discharge radius with the pressure, Rp. The corresponding relations are shown in the Figs. 8 and 9.

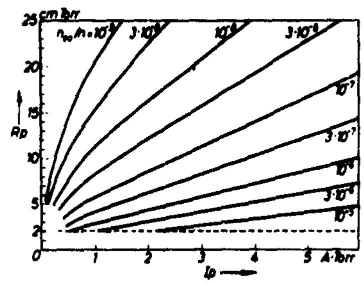

Fig. 8: Relative electron density n_{eo}/n_n as a function of the experimental parameters Rp and pI. n_n is the neutral gas density.

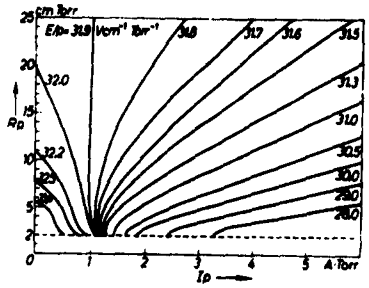

Fig. 9: The reduced electric field E/p as a function of the experimental parameters Rp and pI.

Together with the eigenvalues Ip, Rp the evaluation of the problem produces a 2-manifoldness of radial electron density distributions. We normalize these distributions using $y = n_e/n_{eo}$ and $x = r/R$. We further introduce the relative halfwidth h/R as a characteristic parameter. Then we find that curves with the same constriction parameter h/R show practically the same radial dependence. Consequently our 2-manifoldness is reduced to the 1-manifoldness shown in Fig. 10.

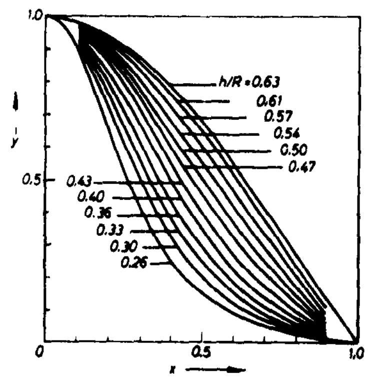

Fig. 10: Normalized electron density distributions. h/R is the constriction parameter.

The electron density distributions may therefore be characterized by the constriction parameter h/R only. In Fig. 10 we have given this typical parameter as a function of the experimental data Rp, Ip.

The range of the experimental parameters Rp, Ip is limited by the assumptions underlying our model. Since in a collision dominated plasma the mean free path of the carriers has to be much smaller than the dimensions of the vessel it follows that we have a lower limit for the product Rp, which is indicated in the dotted line in the Figs. 8, 9, 11. An upper limit for the product Ip is required by our assumption of constant temperature. This upper limit is found from a careful consideration of the energy balance and is approximately identical with the largest value shown in Figs. 8, 9, 11.

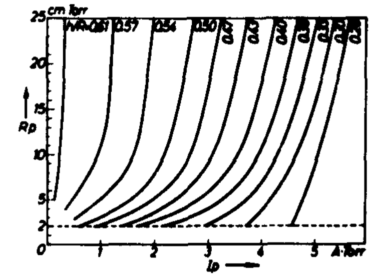

Fig. 11: Constriction parameter h/R as a function of the experimental parameters Rp and pI.

From the Figs. 8-11 the following conclusions may be drawn:

A negative ion component can cause remarkable constriction of the positive column.

For a constant value of Rp the constriction increases with the current as shown in Fig. 11. The constriction is more pronounced in the range of small Rp values and practically negligible for large Rp values. For constant current the constriction increases with pressure.

The characteristic (E-I) of the column can be found from horizontal intersections in Fig. 9. It shows a negative slope. This effect increases with decreasing discharge radius.

We stress particularly the fact that the constriction effect calculated above is nonthermal. By this we mean that within the range of our experimental parameters the assumption of constant particle component temperatures across the whole discharge is valid. If we would proceed to larger values of Ip the neutral gas temperature develops remarkable radial dependence and with that also the electron temperature becomes a function of the radial coordinate. By the same token the production coefficient increases in the center and decreases towards the wall. These effects may cause an additional constriction. We described the influence of the thermal effects in the preceding chapter (VII B).

It is interesting to note that already Güntherschulze, Seeliger and Sommermeyer[34,79] expected as one cause of the constriction process the presence of negative ions and Güntherschulze[34] formulated in this connection the concept of the "ion shield effect".

D THE WALL DETACHED GLOW COLUMN.

The preceding calculations of the glow column are distinguished by the underlying physical model. They have in common the asumption that at the wall the Schottky boundary condition is effective.

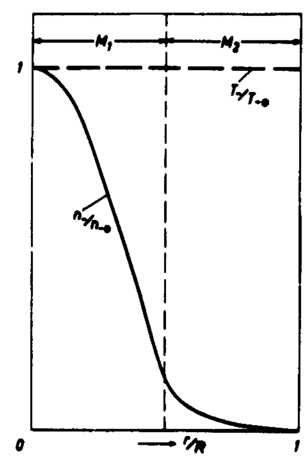

Fig. 12: Schematical sketch of the density and the temperature distribution in the wall detached glow column. M_1 model zone of the normal glow approximation. M_2 model zone of the glow approximation with volume recombination effects.

We discussed already in chapter (VI) that this boundary condition may be rendered ineffective. For instance there can be a competitive selecting mechanism - like for instance gas convection - which renders the simple particle balance (38) invalid. Or even the statistical fluctuations of the system may be sufficient to outrule Schottky's condition if the particle current density - that means the gradient of the particle number density - has become sufficiently small in the neighbourhood of the wall. In the first case the solution is selected by the new dominating mechanism which may be taken into account by a new effective boundary condition. In the second case the solution must be determined from stability considerations.

If the boundary condition (38) is not effective any more but the glow approximation is still applicable then we call the system a wall detached glow column.

Let us consider a discharge volume in which the glow approximation is applicable and let us increase the diameter of the discharge vessel. Then - keeping the total current constant - the particle number density and the importance of the divergence term will decrease and we will reach a limit where volume recombination and electron attachment become important. This effect will first come into play in the outer regions of the discharge. It is not our intention to present here a calculation of these volume effects. Fortunately we can refer the reader to the preceding chapter describing the glow column with negative ions. Here particle attachment and volume recombination is an important phenomenon. The numerical evaluation of the equations of this chapter showed that with the increasing contribution of these volume effects the gradient of the particle density decreases rapidly near the wall. This is just the effect which renders the Schottky condition invalid and causes wall detachment.

E WALL STABILIZED ARC COLUMN.

In the preceding investigations we applied the "glow approximation".

The case of the wall stabilized arc column is characterized by the fact that the "arc approximation" described in chapter (VI) is applicable throughout the whole cylindrical discharge volume.

Equation (32) provides three relations for the energy balance of electrons, ions and neutrals.

Assuming stationarity, quasineutrality and equal temperatures $T_+ = T_- = T_n$ the first term on the left hand side and on the right hand side of equation (32) vanishes. Multiplication by $^m/2$ and addition results in the relation

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda\frac{dT}{dr}\right) + j E_z = -\frac{1}{r}\frac{d}{dr}\left(r U_i j_r\right) + S \tag{104}$$

Here the extensive quantities without index describe quantities summed over all particle components. For instance the quantity S represents the radiation of the neutral particles, the electrons and the ions. is the total heat conduction coefficient of all components. The first term on the right hand side arises from the ambipolar diffusion of ionization energy. Diffusion of excitation energy is neglected.

We have further from equation (31) the momentum balances

$$\vec{j}_- = e\mu_- n\vec{E} + eD_-\vec{\nabla}n \tag{105}$$

$$\vec{j}_+ = e\mu_+ n\vec{E} - eD_+\vec{\nabla}n \tag{106}$$

The neutral particle momentum balance and conservation law will be replaced by the relation

$$n_n k T_n = p = const. \tag{107}$$

due to the assumption of constant pressure and weak ionization.

We further have the two particle conservation laws for electrons and ions. However with our assumption of quasineutrality and the basic concept of our arc approximation we may use instead of the particle conservation law the charge conservation law

$$\vec{\nabla}\left(\vec{j}_{+}+\vec{j}_{-}\right)=0 \tag{108}$$

and the S a h a equation

$$n=\frac{2p^{\frac{1}{2}}(kT)^{\frac{5}{4}}(2\pi m_{-})^{\frac{3}{4}}}{h^{\frac{3}{2}}}\exp\left(-\frac{eU_i}{2kT}\right) \tag{109}$$

Further M a x w e l l ` s equations provide the relation

$$\vec{E}=-\vec{\nabla}U \tag{110}$$

The equation

$$\vec{\nabla}\cdot\vec{E}=4\pi e\left(n_{+}-n_{-}\right) \tag{111}$$

defines the deviations from quasineutrality and is only of interest for higher approximations.

Altogether we have six unknown quantities

$$U,\ \vec{E},\ \vec{j}_{-},\ \vec{j}_{+},\ n_{-},\ T \tag{112}$$

and the six equations (104-106) and (108-110), assuming that the radiation term S in the energy balance is expressed by the variables n and T. The form of this relation depends on the state of the plasma, in particular on the question whether the plasma is opaque or not. Since we are considering a cylinder symmetrical problem it follows from (1o8)

$$j_{-r} + j_{+r} = \frac{C}{r} \tag{113}$$

where the constant C must be identical zero since the sum of the radial current densities should be finite in the axis. By elimination of the electric field the momentum balances (105, 106) provide for the radial particle current density the relation

$$j_{-r} = e\, D_{am} \frac{dn}{dr} \tag{114}$$

and for the longitudinal components

$$j_{-z} = e \mu_- \, n \, E_z \quad , \quad j_{+z} = e \mu_+ \, n \, E_z \tag{115}$$

Introducing the radial current density and the longitudinal current density into the equation (104) and remembering that the longitudinal component of the electric field E_z is constant due to Maxwell's equations, we arrive at

$$\frac{1}{r} \frac{d}{dr} \left(r \lambda \frac{dT}{dr} \right) + e \left(\mu_+ + \mu_- \right) n \, E_z^2 = - \frac{1}{r} \frac{d}{dr} \left(r e U_i \, D_{am} \frac{dn}{dr} \right) + S \tag{116}$$

This equation (116), the Saha equation (109) and the function S (n, T) expressing the radiative losses, determine together with the boundary conditions the temperature distribution in the wall stabilized thermal column.

As boundary conditions we have due to symmetrical reasons and according to equation (44)

$$\frac{dT}{dr} = 0 \;\; \text{for} \;\; r = 0 \quad ; \quad T = T_W \;\; \text{for} \;\; r = R \tag{117}$$

The equation (116) is still very complicated. A simplification can be reached by neglecting the influence of the ambipolar diffusion of ionization energy in comparison to the normal heat conduction.

Then we have the Elenbaas Heller differential equation

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda\frac{dT}{dr}\right)+e\left(\mu_{+}+\mu_{-}\right)n\,E_z^2 = S \tag{118}$$

The formulation of the radiative losses requires further discussion.

Under the assumption of a transparent channel we would in principal have to sum over all emissions of all lines and the continuum. This is a too difficult problem. The usual procedure therefore replaces all levels by one effective energy and writes the emission in the form

$$S(T) = (A/T)\exp\left\{-\frac{eU_a}{kT}\right\} \tag{119}$$

where A is proportional to the pressure and the average transition probability.

With increasing energy input the contribution of the continuous radiation becomes more important. Consequently the average energy level in our formula (119) rises.

If one increases the energy input even further one reaches the region in which the channel is still transparent but the contribution of the discrete spectrum is negligible in comparison to the contribution of the continuous spectrum. In this region we can describe the radiative loss by

$$S(T) \approx 4\pi\,\varepsilon_{\nu}\,\nu_g \tag{120}$$

with

$$(121) \qquad \varepsilon_{\nu} \approx 6{,}36 \cdot 10^{-47} Z^{2} \frac{n^{2}}{(kT)^{\frac{1}{2}}}$$

and $h \cdot \nu_g$ characterizing the energy interval near the ionization boundary through which the energy levels may be considered "smeared out".

Depending on the situation we introduce either (12o) or (119) into (118). Then we have together with the boundary conditions (117) an eigenvalue problem which provides the temperature distribution and the corresponding eigenvalues of the longitudinal electric field E_z.

One can reduce this eigenvalue problem to an initial value problem by using the coordinate transformation

$$(122) \qquad \vartheta = \frac{kT}{eU_i} \quad ; \quad y = \frac{C_1}{2}\left(\frac{r}{R}\right)^{2}$$

which leads to an equation of the form

$$(123) \qquad \frac{d}{dy}\left(y \frac{d\vartheta}{dy}\right) = \frac{C_2}{C_1} f_2(\vartheta) - f_1(\vartheta)$$

with the boundary conditions

$$(124) \qquad (C_1 y)^{\frac{1}{2}} \frac{d\vartheta}{dy} = 0 \quad \text{for} \quad y = 0 \; ; \quad y_R = \frac{C_1}{4}$$

Here the symbols C_1, C_2, f_1 and f_2 have the following meaning

$$(125) \quad C_1 = E_z^{2} R^{2} p^{\frac{1}{2}} \frac{2e\mu_{-}(2\pi m)^{\frac{1}{2}} k}{\lambda h^{\frac{1}{2}} (eU_i)^{\frac{1}{2}}} \; , \quad C_2 = p R^{2} \frac{Ak}{\lambda (eU_i)^{2}}$$

$$(126) \quad f_1(\vartheta) = \vartheta^{\frac{1}{2}} \exp\left\{-1/2\vartheta\right\} \; , \quad f_2(\vartheta) = \vartheta^{-1} \exp\left\{-U_a/U_i\vartheta\right\}$$

Now, assuming an initial value of ϑ_0 and choosing the ratio C_2 / C_1 we can integrate down to the value $\vartheta = 0$ and find a corresponding value of y_R. This value defines the value of the constant C_1 through equation (124). Since we have already chosen the ratio of C_2 / C_1 both constants C_1 and C_2 are defined. From these two relations we calculate with equation (125) the experimental parameters R_p and E_z. The current can be calculated from the temperature distribution using the Saha equation and the mobilities for the electrons and ions.

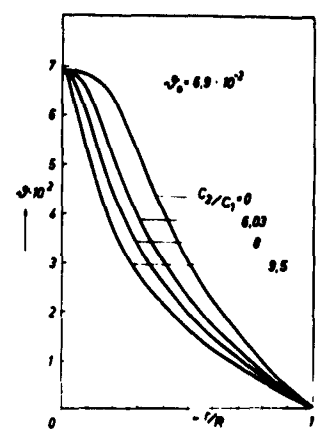

Fig. 13: Typical temperature distributions for the wall stabilized arc [77]. The meaning of the symbols is explained in the text.

The considerations sketched above have been carried through by Weizel and Schmitz[100] and Koch, Lesemann, Walther[50]. Some of the typical results are shown in Fig. (13).

Moreover it is possible to define from equation (123) similarity laws and draw general conclusions with respect to the dependence of the discharge qualities on the experimental parameters. Within the frame of our investigations here we cannot consider these details.

One more word about the radiative loss. If the channel of the wall-stabilized arc column is not transparent then the theoretical description of the radiation becomes extremely complicated due to the absorption and reemission processes. We do not know of any theoretical treatment of this problem.

F WALL DETACHED ARC COLUMN.

In all modes treated so far either the "glow model" or the "arc model" was applicable throughout the whole discharge volume. Only a very small region near the walls - order of magnitude of the mean free path - was excluded. The qualities of the "sheath" were only of interest for the boundary conditions.

The wall detached arc column mode - which we investigate here - is characterized by the fact that both - the arc and the glow model - are of importance.

To understand under what conditions this wall detached arc mode occurs let us consider the following picture.

We chose a value of the axial temperature T_o and the eigenvalue E_z. Since per definition the arc model holds near the axis we can find the temperature distribution by integration of equation (118) starting with the boundary value $dT/dr = o$ and $T = T_o$ at $r = o$. The corresponding density is given by the Saha equation (109).

This integration may be carried out down to a limiting value $r = r_c$ at which our arc approximation breaks down. The value of r_c can be found from the condition (132).

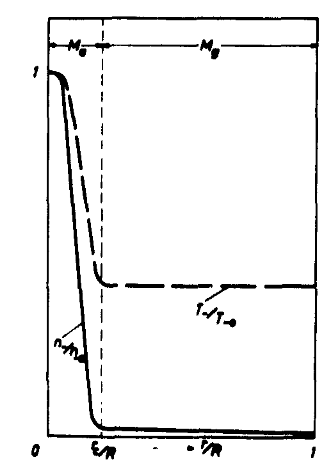

Fig. 14: Schematical sketch of the density and the temperature distribution in the wall detached arc column. r_c seperates the model region of the arc approximation (M_a) from that of the glow approximation (M_g).

We now discuss the possible solutions which can exist for the summed eigenvalue E_z and the axial temperature T_o.

If $R < r_c$ then the arc approximation holds throughout the whole discharge vessel. We have a wall stabilized arc column. A solution exists if the temperature of the wall T_w is identical with the temperature calculated at the distance $r = R$. $T_w = T(R)$.

If the discharge radius R is larger than r_c our arc model breaks down at r_c and a different model must be used in the outer part of the plasma column.

This is the glow approximation which is complimentary to the arc approximation.

Naturally in the environment of $r = r_c$ there is a transition region in which none of the two models is applicable. The general difficulties of such transition regions have been discussed in detail in FTR 1. In accordance with the general use we replace here also the transition region by appropriate boundary conditions. To secure finite current densities and finite sources we have the necessary conditions

$$n_a = n_g \, , \quad \frac{dn_a}{dr} = \frac{dn_g}{dr} \; ; \quad T_a = T_g \, , \quad \frac{dT_a}{dr} = \frac{dT_g}{dr} \tag{127}$$

at $r = r_c$.

The connection of the arc approximation with the glow approximation via equation (127) defines together with the boundary condition (39) one and only one value for the discharge diameter 2R in the range $R > r_c$. Our conclusion is therefore:

If $R < r_c$ holds we have a solution with T_o and E_z for any value of R assuming that we can still choose T_w. Should however T_w be prescribed then there is only one or no possible solution.

If $R > r_c$ then the boundary condition (39) allows only one value of R for the given values T_o and E_z. However this latter statement should be carefully used remembering the discussion of chapter (VI c). There it was stated that for small gradients dn/dr condition (39) may be rendered ineffective. Other influences may become effective and determine R or - if statistical fluctuations prevail - all values of R are possible and stability criteria select the observed mode.

Under these aspects it is of particular interest to see if dn/dr decreases for the wall detached arc column and if the solution for the thermal nucleus is only little influenced by the position of the wall.

The preceding general considerations are formulated in the following calculation:

For the calculation of the thermal nucleus we use a simplified form of equation (118). We neglect the radiative losses. The temperature dependence of the production term via the Saha equation suggests - as in the case of the normal glow column - the representation by a step function. The position of the step is characterized by r_1. In the range $o < r < r_1$ the production term is constant. In the range $r_1 < r < r_c$ the production term is zero.

These are serious simplifications but they do not affect the general conclusions we want to draw.

The quantity r_1 is defined by

$$K = \frac{2p^{1/2}(kT_o)^{1/4}(2\pi m_-)^{3/4}}{h^{3/2}}$$

(128)
$$\frac{\Delta T}{T_o} = \frac{e E^2(\mu_+ + \mu_-) K r_1^2}{\lambda' \cdot 4 \cdot T_o} \exp\left\{-\frac{eU_i}{2kT_o}\right\} = \alpha \ll 1;$$

For reasons of finite energy current density and finite energy sources the steadiness of the temperature and the temperature gradient dT/dr must be required. The the temperature distribution

$$T = T_0 \left\{ 1 - \alpha \left(\frac{r}{r_1} \right)^2 \right\} \quad \text{for} \quad r \leq r_1$$
$$T = T_0 \left\{ 1 - \alpha \left(1 + 2 \ln \frac{r}{r_1} \right) \right\} \quad \text{for} \quad r \geq r_1 \tag{129}$$

follows.

The Saha equation determines the particle density

$$n = K \exp\left\{ - \frac{eU_i}{2kT_0} \left[1 + \xi \left(\frac{r}{r_1} \right)^2 \right] \right\} \quad \text{for} \quad r \leq r_1$$

$$n = K \exp\left\{ - \frac{eU_i}{2kT_0} \left[1 + \xi \left(1 + 2 \ln \frac{r}{r_1} \right) \right] \right\} \quad \text{for} \quad r \geq r_1 \tag{130}$$

From our criterion for the validity of the arc approach (45) the critical density

$$n_c = \frac{1}{\bar{\sigma} n_n^2} \tag{131}$$

defines the radius r_c to be

$$r_c = r_1 \exp\left\{ \frac{1}{2\xi} \left[\frac{2kT_0}{eU_i} \ln \left(K \bar{\sigma} n_n^2 \right) - 1 - \xi \right] \right\} \tag{132}$$

In the region $r > r_c$ where the glow approximation applies we remember that the specific particle production is small in comparison to the net production in the arc center of our discharge. Consequently we neglect this contribution. Then we have, using condition (127) in the region $r > r_c$, the solution

$$n = n_c \left\{ 1 - \frac{eU_i}{kT_o} \xi \ln \frac{r}{r_1} \right\} \tag{133}$$

Condition (39) determines the value of the discharge radius R .

$$R = r_1 \exp \left\{ \frac{1}{2\xi} \left[\frac{2kT_o}{eU_i} \left\langle \ln \left(K\bar{\sigma} n_n^2 \right) + 1 \right\rangle - (1+\xi) \right] \right\} \tag{134}$$

Two quantities are of particular interest. The one is the derivative dR/dT_o given by

$$\frac{dR}{dT_o} = R \frac{1}{2\xi} \frac{2k}{eU_i} \left\langle \ln \left(K\bar{\sigma} n_n^2 \right) + 1 \right\rangle \tag{135}$$

and the other the derivative $(dn/dr)_R$ given by

$$\left. \frac{dn}{dr} \right|_{r=R} = - \frac{n_c \cdot \xi}{R} \frac{eU_i}{kT_o} \tag{136}$$

From equations (135) and (136) certain interesting conclusions can be drawn.

Let us first assume that there are no effects like convection, volume recombination or external fields which could render the Schottky condition invalid. In this case the wall is the stabilizing influence and the discharge is definitely symmetrical to the axis of our discharge vessel. Equation (135) shows that under these conditions the axial temperature T_o and with that the profile of the thermal nucleus is only weak dependent on the diameter 2R of our discharge vessel. Since the thermal nucleus determines the observed phenomenon, this phenomenon is practically independent of the size of the discharge vessel.

Then from equation (136) we can see that the gradient $(dn/dr)_R$ is small. Since in many discharges we have additional influences like volume recombination, convection etc., we can expect that these effects render the Schottky condition (39) invalid. Therefore the thermal nucleus of the discharge is not forced any more to occupy the center of our discharge vessel but under the influence of other effects may show a different geometrical shape.

Even if the latter case occurs the conclusion that the axial temperature and the temperature profile is little influenced by the parameter R will still hold in a good approximation.

In summarizing:

Our calculations show the existence of the "wall detached arc mode". If the boundary condition (39) at the wall governs the solution we find that the discharge body is axially symmetric and that the maximum temperature T_o is rather independent of the diameter of the discharge vessel 2R.

We further recognize that for the wall detached arc mode with increasing R the boundary condition (39) may be outruled by other influences like convection, external fields, volume recombination etc. In this latter case the geometry of the wall detached column is not defined anymore by the presence of the wall but by the qualities of the stabilizing influence. If the stabilizing influence shows assymmetric features then the geometry of the wall detached arc column will be assymmetric. In this latter case too the axial temperature and the thermal profile of the nucleus of the wall detached arc column should be practically independent of the effective diameter of the stabilizing influence.

It is often quite difficult to relate experimental observations to theoretical modes. Experimental and theoretical knowledge may be incomplete. Also there are frequently non-typical cases.

For the wall detached arc column however there are obviously corresponding observations.

Seeliger [78] described for the first time the continous change from the wall stabilized glow to the wall detached glow and the discontinous change to a mode which he called "Schlauchentladung" and which might correspond to our wall detached arc column. Similar observations with careful experimental study have been carried out by Edels, Gambling and Price [19, 26, 27, 28, 68]. They find that the highly contracted - our wall detached arc column - appéars at a certain characteristic gas temperature.

The most recent results follow from the very interesting investigation of Kenty [45, 46, 47]. He finds that under certain conditions his diffuse "normal Schottky mode" changes discontinously to a very narrow "filamentary discharge". This filamentary discharge shows all the qualities which we have described here for the wall detached arc column.

(A temperature differencc for electrons and ions is not principally excluded in our arc approximation or for our wall detached arc column).

Fig. 15: 100 mA discharge and 3 mA diffuse discharge at 60 mm Xe (45).

The investigations of Kenty prove that the thermal nucleus of the wall detached discharge may be displaced within the container without difficulties by very small forces. The position, the length and the form of the discharge is a very uncertain one. Kenty has shown that convective forces are able to elongate the discharge column extensively and even to push it into small extensions of the discharge vessel. Also he observes that the discharge column forms a little arch at the side of the cathode due to the upward convection of the gas.

G CATHODIC PART OF THE ARC DISCHARGE.

In the calculations of the s-modes of the column presented in the preceding chapters the governing boundary condition defined the symmetry of the model regions. For the cylindrical column therefore all model zones showed cylinder symmetry.

Only in the case of the wall detached arc-mode we pointed out that if the stabilizing influence of the wall is outruled, the geometrical shape of the thermal nucleus of the wall detached arc column is not defined by the geometry of the wall and may show various forms. Which of the possible geometrical forms we will observe depends on the presence of another stabilizing influence like e.g. external fields, convection etc. or on the stability qualities of the s-mode. In this respect we refer to the investigations in chapter (VI C) and (X).

In more general terms we can say that in the case of the wall detached arc column the discharge volume does not prescribe the interface separating the two model regions of the "arc approximation" and the "glow approximation". We rather have a whole continuum of stationary modes of different geometrical form. Which of them is realized in a particular experimental setup depends on influences not included in the calculations of chapter (VII F). Either we have strong additional stabilizing factors or the observed mode is selected by stability principles.

We are confronted with exactly the same problem in studying the possible s-modes in front of the electrodes of an arc discharge. Here the existence of more than one model region with the uncertainty of the interface is not the exception - as it was in the case of the wall detached arc column - but rather the normal form.

Approaching the electrode, the conditions change from those presented in the undisturbed column into those of the inertia limited region immediately in front of the electrode. Consequently we have in the electrode region always several model regions seperated by interfaces.

The geometrical form of these interfaces are in principal again defined by the boundary conditions. However we have now two competitive conditions, one from the electrodes and the other from the walls. In certain regions they counteract each other and reduce the stabilizing effect so that statistical fluctuations can render them invalid. This leaves the geometrical form of the model undefined and allows the existence of a whole manifoldness of s-modes.

In an earlier treatment of the electrode components of the arc discharge FTR I[1] we have distinguished three model regions in front of the cathode.

The first region was the "contraction region" which extends from the undisturbed column down to the beginning of the inertia limited region in front of the cathode. Within the contraction region the arc approximation holds. It is similar to the nucleus of the wall detached arc column. As the name suggests the contraction region is not of cylindrical shape but describes the area in which the diameter (2R) of the column constricts to the value ($2R_e$) in front of the electrode. The geometrical shape of this contraction region as well as the parameter of the end contraction R_e are not prescribed by the boundary conditions due to the arguments stated above.

The second model region is the inertia limited zone in front of the cathode in which the charged carriers move according to the laws of the free fall.

Finally the third model region is the whole rest of the discharge volume between the interface of the contraction and inertia limited region on the one hand and the walls and the electrode surface on the other hand.

In the preceding report [1] we have discussed in detail the qualities of the contraction region and the inertia limited zone. This can be done without specifying the laws within the third regions. The details of the calculations are not of interest here. But there is one essential result which relates to our present consideration.

We stated above that we expect in front of the electrode a whole manifoldness of s-modes corresponding to the possible geometrical shape of the interface between the contraction region and the external region. Since the conditions immediately in front of the cathode are not determined and influenced by the cylindrical walls a circular onset might in principal be just as possible as a quadratic or an elliptic onset. Moreover we could imagine that a stationary mode exists with more than one circular cathode spot or even with cathode spots of different values of end contraction meaning that the cathode spot area might be characterized by a set of end contraction parameters $1/R_{e\nu}$. Furthermore the shape of the interface connecting the column with the cathodic onset could show various geometrical forms.

Our investigation in FTR I pointed out that this latter influence of the geometrical form of the contraction region is of little importance for the stationarity condition in the cathodic and anodic area of an arc discharge.

The situation is however quite different with respect to the pattern of the end contraction. Unfortunately the calculations are so extremely involved that we could follow up only the case of a spherical single type cathode onset. Therefore we have only one parameter of end contraction $1/R_e$. We investigated the possibility of stationary modes within the range $1 < R/R_e < \infty$. We found that depending on the experimental parameters stationary modes may be expected in all ranges of the parameter $1/R_e$ and we distinguished three different modes according to the physical mechanism governing the current continuity in the inertia limited region.

There is a thermal spot mode. In general the thermal mode occurs in the range $R/R_e \approx 1$. Under these conditions the energy transport from the arc to the electrode is large. It is based on heat conduction, ion bombardment and radiation. The electrode is on high temperature. The gas temperature differs only little from the electrode temperature. The process governing the condition of current continuity in the inertia limited region is the thermal electron emission from the cathode surface. Due to the high temperature and the moderate current densities this process is able to provide the necessary particle current component. For the same reason the influence of field emission or more exactly temperature field emission (T-F emission FIR I) is only of little importance. Also the ion component, flow from the gas to the cathode surface is small due to the small gas temperature.

The second mode is the "combined spot mode (II)". This mode occurs in the range of medium values of end contraction. It is characterized by the fact that the combined action of electrons emitted from the cathode as well as ions coming from the gas governs the current transport in the inertia limited region. The electrons may be deliberated from the cathode by the temperature effect or by the influence of the electric field. Also it is quite possible that the process of individual field emission is of importance. Due to the increase of the contraction parameter R/R_e the gas temperature in front of the electrode can be much higher than the temperature at the electrode surface and results in a markable ion component flowing from the gas to the cathode surface.

The third mode is the "field mode (III)". This mode occurs with extreme end contractions. The electrode temperature is small. Due to the high current densities a strong electric field at the cathode surface exists. The process of field emission - in particular the I-F emission - is operating and provides large electron emission currents. The gas temperature in front of the electrode may be high but the ion current component is nevertheless small since the contact area is so extremely small.

It is necessary to distinguish two different types (IIIa) and (IIIb) of the "field mode".

In the case IIIa the field at the cathode surface is produced by ions which originate at least partly from thermal means. With other words the ion saturation current flowing from the hot gas region in front of the cathode to the cathode surface provides an essential part of the ion current necessary to build up the cathodic field. Such a process is according to the calculations in FTR I definitely bound to the condition that the radius of end contraction is much larger than the mean free path of the ions λ_+ .

For the mode IIIb the thermal ion component flowing from the gas to the cathode surface is altogether negligible. In this case the ions necessary to produce the electric field at the cathode must be created by the electrons coming from the cathode. Unfortunately with the small cathode drop and the corresponding small ionization ability of the electrons the number of ions produced by a single electron is very small.

In fact we require - due to the energy balance - multiple ionization processes to produce an ion. Consequently the ratio of the electron current to the ion current density j_- / j_+ must be a very large one if the discharge is to operate in the IIIb mode. This field mode IIIb is what was originally understood by the field emission process at the cathode of an arc and was already criticized by Compton. One can see from Fig. 2 of FTR I that it requires current densities of extreme values (order of magnitude 10^9 amp/cm^2) or the presence of processes favouring electron emission.

We emphasize that the various modes are possible depending on the experimental parameter. The essential result of our calculation in FTR I was however that for a fixed set of experimental parameters the number of possible stationary modes - characterized by our contraction parameter R/R_e - was quite restricted. The solution of the stationary equations of our problem did not allow all possible values of end contraction R/R_e, but only a certain set which could be determined by the so-called "EXISTENCE DIAGRAM". Such a diagram defines the possible stationary solutions from the intersection of two curves representing the ion saturation current and the ion defect current. The points of intersection are called POINTS OF EXISTENCE (E-POINTS) and determine those values of the parameter R/R_e which correspond to the possible stationary modes. In general such an existence diagram can have up to four intersection points. But there may be also one or even no intersection points within the range of the stationary modes I, II, and III.

This result means that we do not have - as one might expect - a continous manifoldness of s-modes for the cathode spot onset. The result also explains the phenomenon of constriction of the arc in front of the cathode. It explains moreover the existence of certain critical pressure and current values (p_c, I_c) in very satisfactory agreement with experimental observations.

In conclusion then we state that in front of the cathode of an arc discharge we have in general a finite number of possible s-modes. For the case of a circular single type onset we have four or less than four of such modes. Whether other multiple spot modes or modes of different geometrical shape may exist in front of the cathode hitherto has not been investigated. But there is no doubt that other modes of that type exist. To prove this we show in Fig. 16 pictures of several modes of high pressure mercury arc discharge which exist for exactly the same values of external parameters and were observed to change from one into the other without any change in the external parameters.

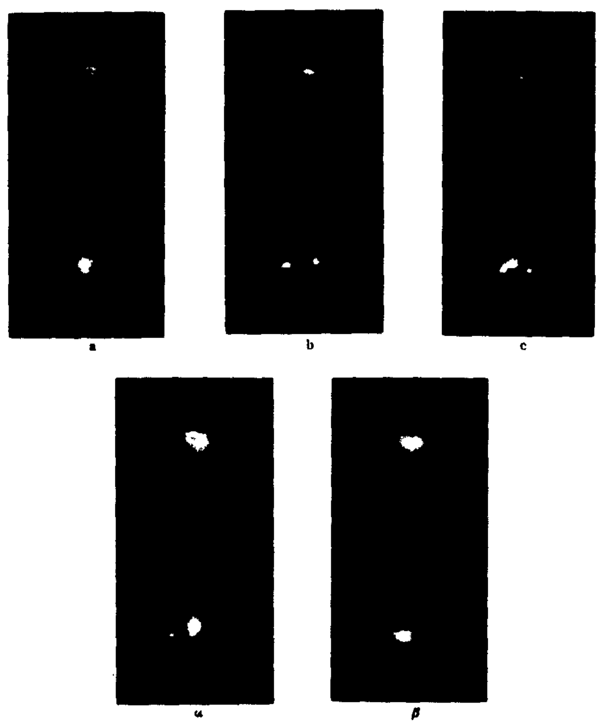

Fig. 16: Examples for the different spot modes.

H ANODIC PART OF THE ARC DISCHARGE.

It is one of the satisfactory features of our theoretical picture about the electrodic parts of an arc discharge that it does n o t underlie different concepts for the cathodic and the anodic region.

The basic mechanism which uses only the well known classical physical laws applies as well to the anodic region as to the cathodic region. The difference observed in front of the anode in comparison to the cathode can be caused only by quantitative differences in the parameters entering the calculations. There are essentially two of such features.

One is the lack of an anodic emission process corresponding to the emission from the cathode. (We restrict here from the special case of the B e c k -arc where we have an ion emission component).

The second feature is the difference in the mass of the particle component providing the saturation current flowing from the gas to the electrode. In front of the cathode this component is carried by the ions. In front of the anode by the electrons. For corresponding conditions in the end contraction region the saturation currents flowing will show the ratio

$$I_-/I_+ = (m_+/m_-)^{\frac{1}{2}} \tag{137}$$

It is possible to explain the principal differences between the cathodic and anodic parts of the arc with the help of this simple relation. We refer to the difference in the end contraction at the two electrodes and the so-called micro-spot mode.

Concerning the details of the calculations for the anode onset we refer to FTR I. The basic result is again - as in front of the cathode - a limited number of s-modes. For the single circular type onset we have four or less than four possible modes. Whether for different onset pattern of multiple structure and noncircular shape additional s-modes exist has never been investigated theoretically but as in the case of the cathode onset there is experimental evidence for this fact. (FTR I).

I CATHODIC PART OF THE GLOW DISCHARGE.

The "glow approximation" and the "arc approximation" described in chapter VI was used to distinguish the various s-modes of the arc and glow column. They are also characteristic for the distinction of the electrodic parts of the glow and arc.

The electrodic parts of the arc are described in their main body - the contraction region - by the assumptions underlying the arc approximation. There is a direct resemblance to the theoretical description of the wall detached column. The electrodic parts of the glow on the other hand are governed by the approximations characteristic for the "glow approach". At least this is true as far as the particle conservation is concerned. With respect to the energy balance there are certain differences.

We use two experimental parameters to characterize the cathodic part of the glow discharge. One is the extension l and the other the total voltage drop U_c. We also assume that we can treat our problem one-dimensionally. This latter assumption does not exclude a radial limitation of our discharge area but assumes only that the radial influences are negligibly small.

Our interest is here focussed on the question what stationary modes may exist for a given set of experimental parameters U_c, l.

First of all we have to distinguish between two principly different kinds of s-modes depending on whether the electric field through the distance 1 is approximately constant or shows a distorted shape due to the presence of space charges.

The first case occurs in the range of small current densities. Here we have only one model region. The requirement of particle conservation has the simple form

$$\frac{dj_-}{dz} = -\alpha^* |j_-| \tag{138}$$

Here α^* is the production coefficient per cm and no volume recombination is assumed.

The integration of this equation gives the current density of the electrons at the anodic end as a function of the electron current density at the cathode surface in the form

$$j_-(z) = j_-(o) \exp\left\{\int_0^z \alpha^*(\zeta)\, d\zeta\right\} \tag{139}$$

Due to the current continuity the ion current must be given by the formula

$$j_+(z) - j_+(o) = j_-(o)\left[1 - \exp\left\{\int_0^z \alpha^*(\zeta)\, d\zeta\right\}\right] \tag{140}$$

If we assume that no ions enter the anodic end and use the cathodic boundary condition that each ion at the cathode surface deliberates electrons according to the relation

$$j_-(o) = \gamma\, j_+(o) \tag{141}$$

then the well known stationarity condition of the Townsend discharge results

$$1 = \gamma \left[\exp \left\{ \int_0^d \alpha^*(\zeta)\, d\zeta \right\} - 1 \right] \tag{142}$$

This is one possible s-mode for our cathodic region characterized by U_c, l. In fact it does not describe only one mode but a whole series of modes since the parameter j does not occur in the stationarity condition. Therefore for a discharge with a given total current the radial extension is still a variable parameter. The condition is that the current density does not exceed the limit prescribed by the space charge effects.

If this last condition is not met then we approach a completely different type of cathodic mode. This new type of mode is characterized by the fact that the Poisson equation causes a strong change in the field distribution. To describe this s-mode we must distinguish three different model regions.

The first one is the so-called cathode fall region (CFR). In this area we find practically the whole cathode fall U_c. The electrons move from the cathode through the cathode fall region producing in a rather strong field additional electrons and ions. At the end of the CFR the electrons - at least a large portion of them - is in a "runaway state"[16b]. We stress that for the existence of the glow cathode mechanism described here the production of electron-runaways is decisive. The ions produced in the CFR move towards the cathode, bombard the cathode and liberate electrons. But not only the ions produced in the CFR reach the cathode. There is also an ion component entering the CFR from the neighbouring model zone. The so-called negative glow region (NGR). On the other hand electrons coming from the CFR enter the negative glow.

The second model zone is the negative glow. Here the electrons from the cathode fall have beam character and enter the negative glow region with a certain initial energy. In passing through the gas they produce secondary electrons and ions which leave the negative glow by ambipolar diffusion. Within the negative glow area we have hardly any electric field. Also the space charge is negligible. As we know from experiments there are three energy groups in the negative glow. The primary electrons which have the properties of a beam. The secondary electrons with an average energy of order or magnitude 5-10 eV. These are the particles produced by the primaries. Finally the dominating "ultimate electrons" which have low energy and show practically a Maxwellian velocity distribution.

The third model region is the Faraday dark space. Here the production of particles is practically zero. The runaways from the CFR have been broken down in the negative glow. Only secondary particles diffuse through the cathodic end into the Faraday dark space. There is no carrier production and no carrier destruction in the Faraday dark space. All particles diffuse in the axial and radial directions and are destroyed by wall recombination. There is little electric field.

To find the stationary modes possible within the frame of these model regions we construct as in the case of the arc an existence diagram plotting the extension l on the abscissa and the voltage U_c on the ordinate. (See Fig. 17).

The stationary states are governed by two relations (FTRII). One is the Poisson equation integrated under the assumption $n_+ \approx n_-$. It gives

$$E = -\left(\frac{3}{2}\,\frac{4\pi p^{\frac{1}{2}} j d}{q}\right)^{\frac{2}{3}}\left[\left(1-\frac{z}{d}\right)-\frac{\gamma}{d(1-\gamma)}\int_z^d \exp\left\{\int_0^\eta \alpha^*(\zeta)\,d\zeta\right\}d\eta\right]^{\frac{2}{3}} \tag{143}$$

where d is the extension of the CFR.

The other is the stationarity law which we find from a calculation similar to that for the Townsend discharge (142) but now limited to the extension of the CFR. The only difference to the Townsend calculation is that we now take into account the possibility of an ion component flowing from the negative glow to the CFR. This provides the boundary condition

$$j_+(d) = \delta j_-(d) \tag{144}$$

Under these circumstances we have the stationarity condition

$$1 = \gamma\left[(1+\delta)\exp\left\{\int_0^d \alpha^*(\zeta)\,d\zeta\right\}-1\right] \tag{145}$$

Let us draw attention to the fact that our existence diagram here has quite a similarity to the existence diagram in arc theory. Our stationarity condition (145) describes nothing else than the current continuity in our glow discharge. Just as the existence diagram in the case of the arc was based on the current continuity requirement in the inertia limited zone.

Fig. 17 shows schematically A-curves and B-curves. The A-curves correspond to the stationarity condition (145). The parameter for the curves is the value d/l. The thin lines correspond to the Poisson equation after eliminating the quantity d with the help of the stationarity condition (145). The resulting law is independent of l and consequently is represented in our $U_c - l$ diagram by lines parallel to the abscissa. The parameter of the B-curves is the current density j.

Any point of intersection of a B-curve with an A-curve is a possible mode (point of existence) for the values U_c and l. The parameters d/l, j are defined by these intersection points. That means for a given total current J with j the radial existence of the discharge is defined.

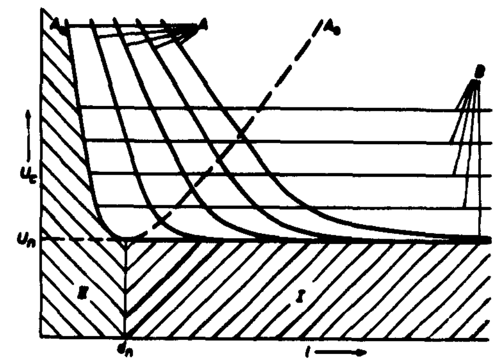

Fig. 17: E-diagram of the glow discharge.

Our existence diagram is quite revealing. It not only defines the parameter values of j and d. It also shows that there are areas in which no modes of the present type exist. In the Fig. 17 we have indicated these areas by I and II. The existence of I explains the phenomenon of a certain minimum current density which is required for the cathodic part of the glow discharge. It also explains the existence of a certain minimum voltage, which is well known. The existence of the region II corresponds to the phenomenon of the obstructed glow discharge. It shows that below a certain extension d_n of the cathodic discharge parts the minimum voltage increases with decreasing value of l. This too has been observed.

Finally we have indicated in our existence diagram Paschen's law which corresponds to the Townsend mode described above. (A_o). Our existence diagram gives the following results. There is a certain area of the parameter values U_c - l in which no stationary modes of the glow discharge type (CFR, NGR, FDR) exist. Then there is an area of U_c - l in which we have always a stationary mode but only with a definite value of the parameter d (extension of the CFR) and a definite value of the current density j (radial extension of the discharge for a given value of the total current, J). In the same region there exists for any value of U_c respectively l also an additional mode which has only one model region and an undistorted electric field - the T o w n s e n d mode.

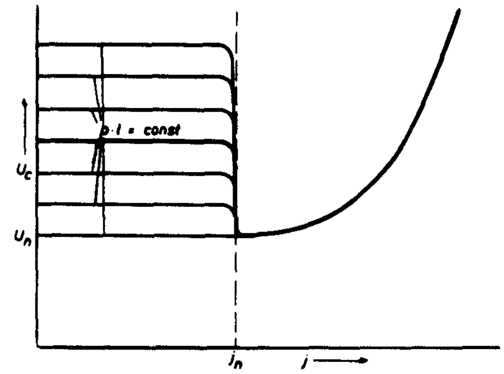

Fig. 18: Voltage- current density characteristic for different values p^{l}, including the Townsend, normal and abnormal glow regions.

So far we have discussed the question of stationary modes using U_c - l as parameters. It may be desirable to consider the same situation using U_c and J. To find these results we may first take from Fig. 17 the cathodic voltage drop as a function of j. This is demostrated in Fig. 18. As this figure shows for $j > j_n$ we have an increase in U_c with the current density which corresponds to the anomalous glow discharge. For current densities $j < j_n$ we are in the region of the T o w n s e n d discharge where the space charge is neglected and consequently the voltage is independent of the current density. Then however in this region there is an influence of the parameter l due to the stationarity condition and consequently a splitup of the

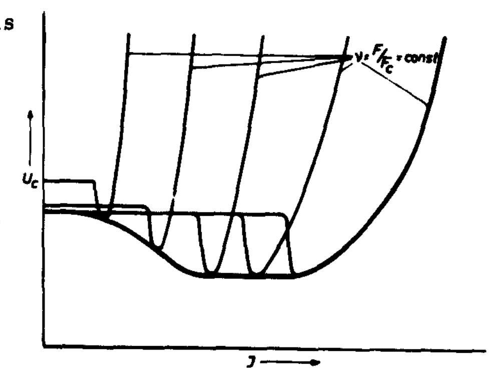

Fig. 19: Voltage- current characteristic for different cathode coverage, including Townsend, normal and abnormal glow regions.

characteristic in several branches. The continous transition from Townsend to glow discharge shown in Fig. 18 is due to space charge effects and cannot be taken from the calculations presented above. Again we have the problem of a transition region.

Now from Fig. 18 we may derive the desired characteristic U - J. We introduce the relative cathodic area covered by a discharge cross section $\nu = F/F_c$ (F_c = cathode area) as a parameter and draw the corresponding characteristics as shown in Fig. 19 . The parameter $\nu \leq 1$ has a lower limit determined mainly by the influence of the radial effects, which is indicated in Fig. 19 by the upward shift of the corresponding $(U - J)_\nu$ characteristics. Obviously there is no unique relation between U and J in Fig. 19.

We may therefore conclude:

There exists a whole area U_c and J where we have no stationary solution of the Townsend or glow discharge type. Then there is an area in which we have at least two stationary modes of the glow discharge type which are distinguished by the parameter ν describing the coverage of the cathode surface. Finally there is a certain number of combinations U_c - J where we have not only solutions of the glow discharge type but also solutions of the Townsend type.

J ANODIC PART OF THE GLOW DISCHARGE.

The situation of the anodic parts of the glow discharge is quite different from that of the cathodic parts.

The cathodic part of the glow discharge is of vital importance. The anodic part is not. There is a substantial theoretical understanding of the cathodic part and we can describe various possible modes on the basis of one typical model, the "glow model". There is hardly any quantitative theory for the anodic part and there is also no typical anode model.

However the existence of various s-modes has been observed though they could not be treated theoretically. We restrict ourselves therefore here to give a qualitative picture of these observations.

If we first consider a plane anode placed into the discharge and being in the negative glow then we have no detectable light phenomenon and the anode drop is negative. This effect has been termed "the negative anode fall".

When the anode is gradually moved away from the negative glow into the Faraday dark space and further into the positive column then a luminous layer forms in front of the anode, - the so-called anode glow. At the same time the potential increases to positive values with respect to the plasma potential. These observations are quite understandable on the simple basis that the current of the charge carriers diffusing to the anode surface must be identical with the total current flowing in the discharge. Within the negative glow the electron current component is too large and therefore a retarding field is necessary. In the positive column however the electron diffusion current is not sufficient and therefore an accelerating electric field is required which at the same time produces the light phenomenon of the anode glow. The increase in the positive anode drop is limited when ions are produced within the anode fall region so that the positive space charge can break down the development. This simple type of anode mode can be described roughly by an analysis which is similar to that applied to the CFR. We refer the reader to FTR II.

Modes different from the simple picture given above may be observed in those cases where the current density in front of the anode is increased due to a special geometry or limited surface of the anode. Also if a gas load is present one may observe the so-called "pearls and bubbles" which present a different anode s-mode. Fig. 2o shows some examples of this type of mode. The axial extension of these spots is much larger than the mean free path of the electrons which characterize the extension of the anode glow. The anode spots occur preferentially at corners and edges where the inhomogenity of the field causes an increase in the current density. Depending on the experimental setup

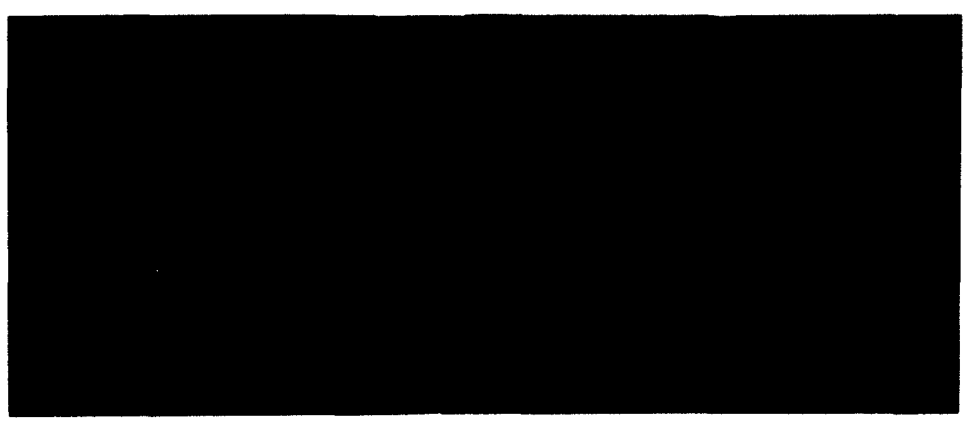

Fig. 20: "Pearls and bubbles" at the anode [illegible].

there may be a large number of anode spots which may be arranged in a regular geometric pattern. (Fig. 20a). If the pressure is decreased the pearls develop into opaque bubbles (Fig. 20b).

The model underlying the phenomenon of pearls and bubbles is not yet fully understood. Weizel and Hornberg claim that such a pearl has an extension one order of magnitude larger than that of the normal anode fall region. Therefore the concepts of mobility and diffusion should be appropriate for describing the situation in the pearls and bubbles. Weizel and Hornberg used these concepts in the Poisson equations and at the same time applied the stationarity condition similar to the calculation for the CFR. In agreement with the geometrical shape they applied a spherical polar coordinate system and came to the conclusion that the pearls may be described as a plasma with a positive space charge. The plasma is an extensive one which produces many more carriers than are required for the current continuity, into the column. The region of the pearls immediately in front of the anode is described in linear form and essentially based on the concept of ambipolar diffusion towards the anode. From these considerations Weizel and Hornberg found the potential distribution. Again we refer for details to FTR II.

For our present application the essential conclusion is:

In front of the anode - as in front of the cathode - a limited number of s-modes are possible. The normal planar form of the anode fall region can be described in terms similar to those of the CFR. In addition "pearl and bubble modes" have been experimentally observed and according to Weizel and Hornberg may be described as a plasma with a positive space charge and an electrodic sheath which is governed by the laws of ambipolar diffusion. The normal one-dimensional AFR-s-mode and the pearl and bubble-s-mode can exist in the same range of experimental parameters.

VIII THERMAL INSTABILITY OF THE GLOW COLUMN.

INTRODUCTION.

Kadomtsev and Nedospasov [42] were the first to tackle the problem of the instability of a collision dominated plasma column with a superimposed external longitudinal magnetic field. In subsequent papers Johnson and Jerde, Holter and others [40, 41, 37] treated the problem in more detail and with more mathematical rigour. Simon and Krall [51] considered the Kadomtsev type instability of a low pressure discharge.

The result of these calculations shows that with increasing magnetic field the m = 1-mode becomes unstable if the magnetic field surpasses a certain critical value B_c.

The physical picture behind this instability is rather simple. A density disturbance of the stationary state produces an increase in the particle production on the one hand and an increase in the particle loss on the other hand. If the increase in the particle production overcomes the increase in the particle loss and the two effects do not change the initial shape of the disturbance then we have an unstable eigensolution of the problem.

One of the basic assumptions of all the calculations quoted above is that together with the density disturbance no temperature disturbance occurs.

Under these circumstances the m=o-mode is stable, independent of the magnetic field. The onset of the instability is determined by the m=1-mode.

However there are experiments which show an instability of the m=o-mode with increasing current and increasing pressure. As an example we refer to the experiments of Kenty[47] and Hermann[97]

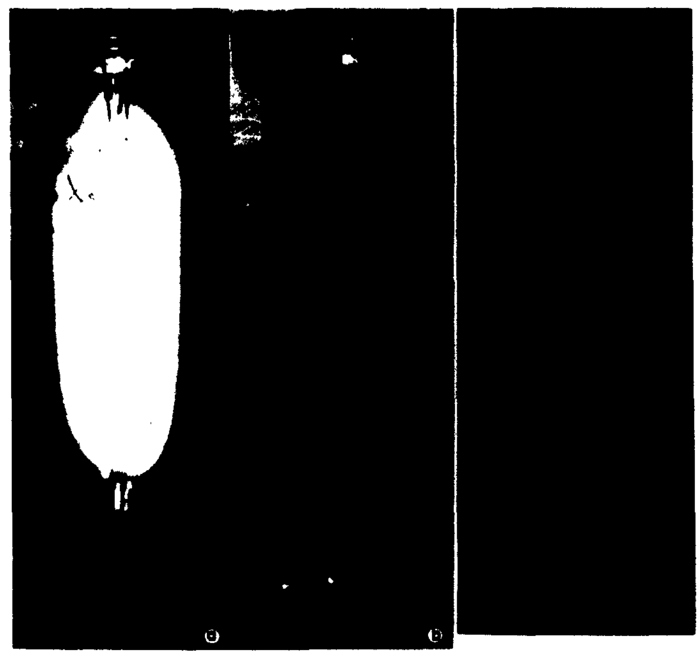

Fig. 21: Examples for column modes before and after thermal instability.

Due to the neglection of a temperature disturbance the calculations of the Kadomtsev type cannot be applied to these observations.

A temperature disturbance may cause instability even in the absence of a magnetic field. It is the aim of this chapter to investigate this "THERMAL INSTABILITY" of a cylindrical collision dominated plasma column with and without a longitudinal magnetic field.

SYSTEM.

Subject of this investigation is again an infinite cylindrical plasma column of radius R. The column is homogeneous in the z-direction.

It contains gas of one kind. The theory of a column with several neutral particle components follows exactly the same lines and requires only additional summation. The gas is partially ionized and we have electrons and one kind of singly charged ions. The concept of quasineutrality holds. A longitudinal magnetic field of strength B can be superimposed.

CONCEPT.

Within the range of the normal glow column (Schottky column) we expect no temperature instability. As we will see the temperature instability depends decisively on the coupling between density disturbance and temperature disturbance. In the Schottky concept the temperature is assumed independent of the density distribution and there is consequently no coupling between density disturbance and temperature disturbance.

The situation is different in the case of the thermal glow column described in chapter VII B. Here per definition the energy balance is of importance and causes a coupling of the temperature distribution with the density distribution. Consequently a density disturbance by necessity produces a temperature disturbance. The temperature disturbance results in a change of the particle production coefficient. In this way an additional influence on the particle production term via the electron temperature can be expected and may cause instability even if there is no instability of the Kadomtsev type.

For instance: A spontaneous increase in the electron density near the axis of the discharge results in a decrease of the neutral particle density which in turn produces an increase in the electron temperature and with that an additional particle production. This mechanism can be the root of an instability.

BASIC EQUATIONS.

The equations governing the thermal glow column can be taken from chapter VII B. They are represented in the law of particle conservation of electrons

$$\frac{\partial n}{\partial t} - \frac{D_{eff}}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right) - \alpha \cdot n = 0 \qquad (146)$$

and the equation for the electron temperature

$$\varkappa' \frac{\partial T}{\partial t} - \frac{\lambda'}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) - e\left(\mu_+ + \mu_-\right)E^2 n = 0 \qquad (147)$$

with the particle production per second

$$\alpha = A \exp\left\{-\frac{eU_i}{kT}\right\} \qquad (148)$$

and the coefficients

$$\lambda' = \frac{\lambda k B p T_{nw}}{e U_i E} \qquad (149)$$

and

$$\varkappa' = \frac{\varkappa k B p T_{nw}}{e U_i E} \qquad (150)$$

The quantities B and A^* are known from experiment (see equ. 60).

Equations (146) and (147) are two simultaneous differential equations for the variables T_- and n_- .

These equations together with the boundary conditions in the axis

(151) $$\frac{dn}{dr} = 0 \, , \quad \frac{dT}{dr} = 0 \quad for \; r = 0$$

and at the wall

(152) $$n = 0 \; , \quad T = 0 \quad for \; r = R$$

define the radial distributions of temperature and particle density as well as the eigenvalue of the longitudinal electrid field E_z.

SOLUTION OF THE EQUATIONS.

We want to treat the stability problem by the method of normal modes which is based on a first order perturbation theory of the steady state solution of our system. We therefore use the development

(153) $$n = n^{(0)} + n^{(1)} \, , \quad T = T^{(0)} + T^{(1)} \, , \quad \alpha = \alpha^{(0)} + \alpha^{(1)}$$

where the index (o) refers to the steady state. The index (1) designates the disturbance. Introducing equation (153) into equations (146) and (147) and remembering that the quantities with the index zero satisfy the equations (55) and (63) we arrive at a set of relations for the first order contributions

(154) $$\frac{\partial n^{(1)}}{\partial t} - \frac{D_{ef}}{r} \frac{d}{dr} \left(r \frac{dn^{(1)}}{dr} \right) - \alpha^{(0)} n^{(1)} - \alpha^{(1)} n^{(0)} = 0$$

and

(155) $$\varkappa' \frac{\partial T^{(1)}}{\partial t} - \frac{\lambda'}{r} \frac{d}{dr} \left(r \frac{dT^{(1)}}{dr} \right) - e \, (\mu_+ + \mu_-) \, E^2 n^{(1)} = 0$$

where all higher order terms are neglected.

We look for the eigensolutions of our perturbed problem by introducing the relations

$$n^{(1)}(\vec{r},t) = n_\mu(\vec{r})\,e^{\omega_\mu t}\,, \quad T^{(1)}(\vec{r},t) = T_\mu(\vec{r})\,e^{\omega_\mu t} \tag{156}$$

which gives

$$\left\{\omega_\mu n_\mu - \frac{D_{eff}}{r}\frac{d}{dr}\left(r\frac{dn_\mu}{dr}\right) + \alpha^{(0)} n_\mu\right\} e^{\omega_\mu t} - \alpha^{(1)} n^{(0)} = 0 \tag{157}$$

and

$$\omega_\mu \varkappa' T_\mu - \frac{\lambda'}{r}\frac{d}{dr}\left(r\frac{dT_\mu}{dr}\right) - e(\mu_+ + \mu_-)E^2 n_\mu = 0 \tag{158}$$

Since the equations are dominated by the operator

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) = \Delta_r \tag{159}$$

we develop our eigensolutions in a set of Bessel functions according to

$$n_\mu(\vec{r}) = \sum_\nu n_{\mu,\nu}\, J_0(\beta_\nu r/R)$$

$$T_\mu(\vec{r}) = \sum_\nu T_{\mu,\nu}\, J_0(\beta_\nu r/R) \tag{160}$$

where the β_ν are the zeros of the zero order Bessel function J_0.

From

$$\alpha^{(1)} = \frac{d\alpha}{dT}\Big|^{(0)} T^{(1)} = (\alpha')^{(0)} T^{(1)} \tag{161}$$

we find the relation

$$\alpha^{(1)} = (\alpha')^{(0)} e^{\omega_\mu t} \sum_\nu T_{\mu,\nu}\, J_0(\beta_\nu r/R) \tag{162}$$

If we introduce these developments (160) and (162) into equations (157) and (158) a system of simple uncoupled linear equations for the coefficients $n_{\mu,\nu}$ and $T_{\mu,\nu}$ would result if the coefficients of the equations (157) and (158) were all constants.

Now in our present problem the coefficients $D, \mu, \varkappa'$ and λ' are indeed considered constant. However the terms $\alpha^{(1)} n^{(0)}$ and $\alpha^{(0)} n^{(1)}$ show a strong radial dependence which is defined by the solutions of the zero order problem given in chapter VII B. This dependence cannot be neglected and consequently we have to use for these terms new developments of the form

$$\alpha^{(1)} n^{(0)} = e^{\omega_\mu t} \sum_\nu a_{\mu,\nu} J_0(\beta_\nu r/R)$$
$$\alpha^{(0)} n_\mu = \sum_\nu b_{\mu,\nu} J_0(\beta_\nu r/R) \qquad (163)$$

The relation between the corresponding coefficients and $n_{\mu,\nu}$ and $T_{\mu,\nu}$ are

$$a_{\mu,s} = \frac{2}{R^2[J_1(\beta_s)]^2} \times \sum_\nu T_{\mu,\nu} \int_0^R (\alpha')^{(0)} n^{(0)} J_0(\beta_\nu r/R) J_0(\beta_s r/R) r dr \qquad (164)$$

$$b_{\mu,s} = \frac{2}{R^2[J_1(\beta_s)]^2} \sum_\nu n_{\mu,\nu} \int_0^R \alpha^{(0)} J_0(\beta_\nu r/R) J_0(\beta_s r/R) r dr \qquad (165)$$

Now we introduce equations (160), (162), (164) and (165) into equations (157) and (158) and collect all terms with the same value $J_0(\beta_\nu r/R)$. Then the coefficients of these terms must all be zero due to the fact that the relations (157) and (158) hold for any value of r in the range $0 \leq r \leq R$. This produces an infinite system of coupled linear equations for the coefficients $n_{\mu,\nu}$ and $T_{\mu,\nu}$. Since we have as many homogeneous equations as we have variables, the determinant of the system must vanish. This secular determinant is

$$\left|\begin{array}{ccc|cc} \omega+\frac{D_{eff}\beta_0^2}{R^2}-C_0B_{00}\,; & -C_0B_{10}\,; & \cdots & \cdots\; C_0A_{s0} & \cdots \\ -C_1B_{01}\,; & \omega+\frac{D_{eff}\beta_1^2}{R^2}-C_1B_{11}\,; & \cdots & \cdots\; C_1A_{s1} & \cdots \\ \vdots & & & & \\ \hline -e(\mu_+ + \mu_-)E^2\,; & 0 & \cdots & \varkappa'\omega+\frac{\lambda'\beta_0^2}{R^2}\,; & 0\;\cdots \\ 0 & ;\,-e(\mu_+ + \mu_-)E^2\,; & 0\;\cdots & 0 & ;\,\varkappa'\omega+\frac{\lambda'\beta_1^2}{R^2}\,;\,0\cdots \\ \vdots & & & \vdots & \end{array}\right| \qquad (166)$$

$$B_{\nu s}=\int_0^R \alpha^{(0)} J_0(\beta_\nu r/R)\,J_0(\beta_s r/R)\,r\,dr \qquad C_\nu=\frac{2}{R^2[J_1(\beta_\nu)]^2}$$

$$A_{\nu s}=\int_0^R (\alpha')^{(0)} u^{(0)} J_0(\beta_\nu r/R)\,J_0(\beta_s r/R)\,r\,dr$$

The secular determinant defines the eigenvalues of our system. Introducing one of these eigenvalues in the equations allows to calculate the coefficients of the corresponding eigensolution.

Our stability problem is therewith reduced to the solution of the secular determinant.

There are two reasons why this solution is extremely difficult. One is that we are dealing with an infinite determinant, the other that the coefficients occuring in this determinant are complicated integral expressions. Approximations will therefore be unavoidable.

We suggest to simplify the problem by omitting the terms $\nu > s$ in the relations (164, 165). This seems possible since the numerical value of the integral expressions decreases rapidly with increasing index number $\nu > s$. This is particularly true for small values of s. It is favourable that our investigation is just interested in these eigensolutions with a small number of modes (small s) which decide the onset of the instability.

Under these circumstances the determinant (166) after suitable rearrangement can be written as a product of determinants in the form

$$\Xi = \prod_{\nu} \begin{vmatrix} \omega + \frac{D_{eff}\beta_\nu^2}{R^2} - C_\nu B_{\nu\nu}, & C_\nu A_{\nu\nu} \\ -e(\mu_+ + \mu_-)E^2 & , \varkappa'\omega + \frac{\lambda'\beta_\nu^2}{R^2} \end{vmatrix} \tag{167}$$

Each of these factors provides a quadratic equation for two of the possible eigenvalues. From chapter III we know that a necessary and sufficient condition for stability is that the coefficients in this relation have the same sign.

This provides for the zero order mode the following instability criterion

$$\bar{j}\, E > \frac{\lambda' D_{eff}\, \beta_0^2}{R^2\, \overline{(\alpha')^{(0)}}} \left[1 - \frac{\overline{\alpha^{(0)}}}{D_{eff}} \left(\frac{R}{\beta_0}\right)^2\right] \tag{168}$$

The higher order modes become unstable if the condition

$$\bar{j}\, E > \frac{\lambda' D_{eff}\, \beta_\nu^2}{R^2\, \overline{(\alpha')^{(0)}}} \left[1 - \frac{\overline{\alpha^{(0)}}}{D_{eff}} \left(\frac{R}{\beta_\nu}\right)^2\right] \tag{169}$$

is fulfilled.

Here we have used the abbreviations

$$\overline{\alpha^{(0)}} = \frac{2}{R^2[J_1(\beta_\nu)]^2} \int_0^R \alpha^{(0)} \left[J_0(\beta_\nu r/R)\right]^2 r\,dr$$

$$\overline{(\alpha')^{(0)}} \cdot \overline{n^{(0)}} = \overline{(\alpha')^{(0)} n^{(0)}} = \frac{2}{R^2[J_1(\beta_\nu)]^2} \int_0^R (\alpha')^{(0)} n^{(0)} \left[J_0(\beta_\nu r/R)\right]^2 r\,dr \tag{170}$$

and

$$\overline{n^{(0)}} = \frac{2}{R^2} \int_0^R n^{(0)} r \, dr \tag{171}$$

An estimation of the quantities entering our criterion shows that the onset of the instability is determined by the zero order mode.

Since our calculations do not exclude the case of the normal glow column, let us investigate whether our criterion (168) supports our expectation that the normal glow column should be always stable.

In the case of S c h o t t k y's theory α is constant through the discharge volume. Consequently we find $\overline{\alpha_0^{(0)}} = \alpha^{(0)}$. Using the eigenvalue $\alpha^{(0)}/D_{eff} = (2,4/R)^2$ from chapter VII A we arrive at

$$\bar{j} E > 0 \tag{172}$$

and should conclude - contrary to our expectations - that the normal glow column is always unstable. This conclusion is of course incorrect. Since in the normal S c h o t t k y theory the first term in equation (168) is small of higher order we must include in our stability consideration higher order contributions to the stationary eigenvalues

Let us therefore study higher approximations of the stationary solution by using the development

$$n^{(0)} = n^{(00)} + n^{(01)} \; ; \quad T^{(0)} = T^{(00)} + T^{(01)} \tag{173}$$

Introducing this into equations (55, 63) we find

$$(174)\quad \frac{D_{eff}}{r}\frac{d}{dr}\left(r\,\frac{dn^{(01)}}{dr}\right) + \alpha^{(00)}n^{(01)} + \varepsilon^{(1)}n^{(00)} + \alpha^{(01)}n^{(00)} = 0$$

and

$$(175)\quad \frac{\lambda'}{r}\frac{d}{dr}\left(r\,\frac{dT^{(01)}}{dr}\right) + e\,(\mu_+ + \mu_-)\,E^2 n^{(00)} = 0$$

Here $\alpha^{(00)}$ is the zero order of the eigenvalue $(\alpha^{(00)}) = (2{,}4/R)^2 D_{eff}$ $\varepsilon^{(1)}$ is the correction term of the eigenvalue. $\alpha^{(01)}$ is the perturbing function which can be found from equation (30) using the relation

$$(176)\quad \alpha^{(01)} = (\alpha')^{(0)}T^{(01)}$$

$$= (\alpha')^{(0)}\,\frac{e(\mu_+ + \mu_-)E^2R^2}{(2{,}4)^2}\,n_0^{(00)}\,J_0(\beta_0 r/R)$$

From well known perturbation theory it follows therefore for the first order correction of the eigenvalue $\alpha^{(00)}$

$$(177)\quad \varepsilon^{(1)} = -\frac{\int_0^R \alpha^{(01)}[n^{(00)}]^2 r\,dr}{\int_0^R [n^{(00)}]^2 r\,dr} = \frac{e(\mu_+ + \mu_-)E^2R^2n_0^{(00)}}{(2{,}4)^2}\,\frac{\int_0^R (\alpha')^{(0)}[J_0(\beta_0 r/R)]^3 r\,dr}{\int_0^R [J_0(\beta_0 r/R)]^2 r\,dr}$$

Introducing $\alpha^{(00)} + \varepsilon^{(1)}$ in our criterion (23) gives after some simple rearrangements the condition for instability $1 > 1$. Clearly this condition cannot be fulfilled and as expected the S c h o t t k y column is always stable. If we carry the perturbation theory on to the second order this fact is emphasized.

The situation is quite different for the thermal glow column. We refer to the results calculated in chapter VII B. Introducing the eigenvalue found in equation (81) our criterion determining the stability reads now

$$(178)\quad \bar{j}\,E > \lambda'\,\frac{T_{nw}}{T_s}\,\frac{kT_0^2}{eU_i}\left(\frac{\beta_0}{R}\right)^2\left[\exp\left\{\frac{eU_i}{k}\left(\frac{1}{T_0} - \frac{1}{T_s}\right)\right\} - 1\right]$$

and we see that instability of the thermally constricted Schottky column can well occur.

IX INSTABILITY OF THE CATHODIC PART OF THE GLOW DISCHARGE.

In the following we discuss a special type of instability of the cathodic part of the glow discharge: The " $\gamma - \delta$ instability".

Let us recall the phenomenology and the mechanism of the cathodic part of the glow discharge.

The cathodic part of the glow discharge is that region which is influenced by the cathode. If we carry out an infinitesimal displacement of the cathode keeping the current constant then all regions where we find changes in the discharge parameters belong to the cathodic part.

The so defined cathodic part of the glow discharge is subdivided in a number of phenomenological zones and a number of model regions. The two subdivisions are not identical.

The phenomenological regions:

1) The Aston dark space is in general small and may be absent with high cathode falls. It lies immediately in front of the cathode and is characterized by the fact that the electrons deliberated from the cathode do not have sufficient energy to cause notable excitation.

2) The first cathode layer is a small layer of faint light which is diffuse in the direction towards the anode. This is a zone in which the electrons have just reached energies corresponding to the

first excitation levels of the gas. Its decreasing visibility towards the anode is due to the acceleration of the electrons and the corresponding decrease in their excitation probability.

3) Hittorf's or Crook's dark space appears dark in contrast to the neighbouring regions of the first cathode layer and the negative glow. When the electrons enter the Hittorf or Crook dark space they have already energies above the excitation energies of the gas and accummulate further energy in the electric field. Due to this influence they finally surpass the state of drift motion and become runaways [16b]. The beam character of the electrons in the cathode fall region has been proved by a number of experiments. One of the most obvious proofs is provided by an obstacle in the Crook dark space. Under these circumstances the part of the negative glow behind the obstacle disappears and also all the corresponding cathode parts in the area of the obstacle are destroyed.

4) The negative glow is the most luminous region of the whole cathode zone with a sharp boundary on the cathode side. This boundary is called the glow edge. There is a continous decrease in brightness towards the anode. The geometry of the negative glow is dependent only on the geometrical form of the cathode and independent of position and form of the anode. This is due to the fact that the electron beams are practically always perpendicular to the cathode surface.

5) Faraday dark space is a region of low luminosity which is limited by the anode itself or by the head of the positive column. The length of the Faraday dark space increases with the radius of the discharge vessel and is in general larger than the extension of the negative glow. In the Faraday dark space the charged particles diffuse towards the anode and towards the wall. There is virtually no electric field and no particle production in this region.

The model regions describing the situation in the cathodic part of the glow discharge are the cathode fall region (CFR), the negative glow region (NGR) and the Faraday dark space region (FDR). We have already discussed the properties of these model regions in our description of the s-mode of the cathodic part of the glow discharge in chapter VII I .

It is impracticable to treat the complicated problem of instability of the cathode part of the glow taking into account all the details and extensions given in FTR II. Rather we will have to restrict ourselves here from the very beginning to tne most simple description of the cathodic part of the glow discharge, which is possible without affecting the principal features of the instability criterion.

It is sufficient to consider the CFR. The conditions in the negative glow enter these calculations via the boundary condition

$$j_+(d) = \delta\, j_-(d) \tag{179}$$

A complete theory would have to calculate the coefficient δ from the theory of the NGR. This was not achieved in the case of the stationary solution and is of course even less possible in the nonstationary case. However this lack of knowledge will not affect our following calculations since we allow quite generally for a variation of δ with the electron current density $j_-(d)$.

We write down the nonstationary equations governing the CFR in the usual one-dimensional approximation. We have the particle conservation laws

$$\frac{\partial n_-}{\partial t} - \frac{\partial (j_-/e)}{\partial z} - \alpha^* \left| j_-/e \right| = 0 \tag{180}$$

and

(181) $$\frac{\partial n_+}{\partial t} + \frac{\partial (j_+/e)}{\partial z} - \alpha^* |j_-/e| = 0$$

where j_-, j_+ describes the current density and n_-, n_+ the average particle number density for the electrons and ions respectively.

The coefficient α^* designates the production of particles per cm and electrons in the CFR. This is in general a very complicated quantity since the electron in the CFR have partly runaway character. Their velocity depends on their origin in the CFR and consequently their velocity distribution is upto now unknown. The problem has been discussed in FTR II. In general α^* must be considered as a function of the electric field $E(z)$ and the time t

(182) $$\alpha^* = \alpha^* (E, t)$$

However we will study a variation of our system with a characteristic time large in comparison to the relaxation time of the electron motion and production. Under these circumstances the motion of the electrons and their production is defined by the momentary electric field $E(z)$ in the CFR and we have

(183) $$\alpha^* = \alpha^*(E)$$

and for the momentum balances

(184) $$j_- = -\varkappa_- (E)\, n_-$$

and

(185) $$j_+ = -\varkappa_+ (E)\, n_+$$

where the coefficients $\varkappa_-$ and $\varkappa_+$ are both positive quantities. The geometry of our coordinate system is shown in Fig. 22

For the ions it may be safe to assume mobility limited motion. For the electrons this assumption is rather doubtful. Fortunately we do not need to specify the quantities $\varkappa_-$, $\varkappa_+$.

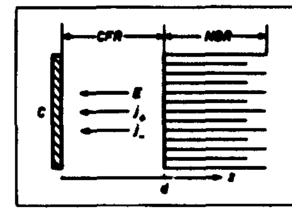

Fig. 22: Coordinate system and symbols used in the cathodic part of the glow.

We have further the Poisson equation

$$\frac{dE}{dz} = 4\pi e\,(n_+ - n_-) \tag{186}$$

Equations (180, 181) and (183-186) define the situation in the CFR together with the boundary conditions at the cathode surface and at the anode surface.

At the NGR boundary we have the condition (179). We recall that the coefficient δ in general must be considered depending on the current $j_-(d)$. Also at the NGR boundary we assume for the electric field

$$E(d) = 0 \tag{187}$$

At the cathode surface the electrons deliberated from the cathode are due to ion bombardment of the cathode surface. Therefore we have the condition

$$j_-(0) = \gamma\, j_+(0) \tag{188}$$

where the coefficient γ - for the same reason as δ - may be still a function of the current density $j_+(0)$.

We suggest to study the $\gamma - \delta$ instability defined by the following concept.

We have a given stationary solution of the model regions of the cathodic part of the glow discharge as described in FTR II. It is assumed that radial boundary effects are negligible, so that the whole situation can be treated one-dimensional. Now we superimpose a momentary disturbance of the parameters describing the cathodic part. This disturbance is limited to a small fraction of the whole cathode area. The lateral extension of this disturbance is subject to two counteracting conditions. On the one hand the lateral extension is still large enough to allow the one-dimensional description. On the other hand the lateral extension of the channel has to be small enough so that most of the field lines originating from the space charge caused by the disturbance leave the discharge channel in lateral direction and do therefore not effect the coefficients $\varkappa$ and α^*. There is a wide range of disturbances for which these conditions are fulfilled simultaneously.

To the so defined model we apply perturbation theory and have

$$n = n^{(o)} + n^{(1)} \qquad E^{(1)} = 0 \;, \quad \varkappa^{(1)} = 0$$

(189)

$$j = j^{(o)} + j^{(1)} \qquad \alpha^{*(1)} = 0$$

Here the index (o) characterizes the stationary solution, the index (1) the nonstationary disturbance.

Recalling our assumptions introduced above the first order components of the current densities are defined by the equations

(190)
$$\frac{\partial (j_-^{(1)}/\varkappa_-)}{\partial t} + \frac{\partial (j_-^{(1)}/e)}{\partial z} + \alpha^* \left| j_-^{(1)}/e \right| = 0$$

and

$$\frac{\partial(j_+^{(1)}/\varkappa_+)}{\partial t} - \frac{\partial(j_+^{(1)}/e)}{\partial z} + \alpha^* |j_+^{(1)}/e| = 0 \tag{191}$$

The boundary conditions (179) and (188) read in the first order approximation, - remembering the dependence of δ on j_(d) and γ on j_+ (o)

$$j_+^{(1)}(d) = \bar{\delta}\, j_-^{(1)}(d) \; , \quad \bar{\delta} = \delta^{(0)} + \frac{d\delta}{dj_-(d)}\Big|^{(0)} \cdot j_-^{(0)}(d) \tag{192}$$

and

$$j_-^{(1)}(o) = \bar{\gamma}\, j_+^{(1)}(o) \; , \quad \bar{\gamma} = \gamma^{(0)} + \frac{d\gamma}{dj_+(d)}\Big|^{(0)} j_+^{(0)}(o) \tag{193}$$

We look for the eigensolutions of this problem by introducing

$$j^{(1)}(z,t) = j^{(\lambda)}(z)\, e^{\lambda t} \tag{194}$$

into equations (190) and (191). The result is

$$\frac{dj_-^{(\lambda)}}{dz} + \left(\alpha^* Sg + \frac{\lambda e}{\varkappa_-}\right) j_-^{(\lambda)} = 0 \tag{195}$$

and

$$-\frac{dj_+^{(\lambda)}}{dz} + \frac{\lambda e}{\varkappa_+} j_+^{(\lambda)} = \alpha^* Sg\, j_-^{(\lambda)} \tag{196}$$

In these equations the symbol Sg is identical with the sign of the current $j_-^{(\lambda)}$. Equation (195) may be at once integrated resulting in

$$j_-^{(\lambda)} = j_-^{(\lambda)}(o) \exp\left\{ -\int_0^z \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_-}\right) d\zeta \right\} \tag{197}$$

where we have now introduced a new production coefficient of the form

(198)
$$\bar{\alpha} = \alpha^{*}\,\delta g$$

This fact should be carefully observed in case $j_-^{(\lambda)}$ goes through zero.

The method of the variation of the constant gives for the second equation

(199)
$$j_+^{(\lambda)} = \exp\left\{\int_0^z \frac{\lambda e}{\varkappa_+}\,d\zeta\right\} \times \left[j_+^{(\lambda)}(0) + j_-^{(\lambda)}(0)\int_0^z \bar{\alpha}\,\exp\left\{-\int_0^\zeta \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_-} + \frac{\lambda e}{\varkappa_+}\right) d\eta\right\} d\zeta\right]$$

Applying the boundary conditions (193) and (192) we find

(200)
$$j_+^{(\lambda)}(0) = j_-^{(\lambda)}(0)/\bar{\gamma}$$

and

(201)
$$\bar{\delta}\,\exp\left\{-\int_0^d \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_-}\right) dz\right\} = \exp\left\{\int_0^d \frac{\lambda e}{\varkappa_+}\,dz\right\}\left[\frac{1}{\bar{\gamma}} + \int_0^d \bar{\alpha}\cdot\exp\left\{-\int_0^z \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_-} + \frac{\lambda e}{\varkappa_+}\right) d\zeta\right\} dz\right]$$

Since $\varkappa_-$ is very much larger than $\varkappa_+$ we may reduce this equation to

(202)
$$\bar{\delta}\,\exp\left\{-\int_0^d \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_+}\right) dz\right\} = \left[\frac{1}{\bar{\gamma}} + \int_0^d \bar{\alpha}\,\exp\left\{\int_0^z \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_+}\right) d\zeta\right\} dz\right]$$

We further introduce the new variable

$$u = \int_0^z \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_+}\right) d\zeta \; ; \qquad u\,(d) = \omega \tag{203}$$

which gives

$$\bar{\delta} \exp(-\omega) = \frac{1}{\bar{\gamma}} + \int_0^\omega \frac{\bar{\alpha} \exp(-u)}{\bar{\alpha} + \lambda e/\varkappa_+} \, du \tag{204}$$

Within the frame of our approximations it is quite safe here to consider the term $\bar{\alpha}/(\bar{\alpha} + \lambda e/\varkappa_+)$ in the integral (204) as constant in comparison to the exponential. We replace it by the average value

$$\int_0^d \bar{\alpha}\, dz \Big/ \int_0^d \left(\bar{\alpha} + \frac{\lambda e}{\varkappa_+}\right) dz = \int_0^d \bar{\alpha}\, dz/\omega \tag{205}$$

and arrive at

$$\left\{ \bar{\delta} + \int_0^d \bar{\alpha}\, dz/\omega \right\} \exp(-\omega) = \frac{1}{\bar{\gamma}} + \int_0^d \bar{\alpha}\, dz/\omega \tag{206}$$

Equation (206) is the dispersion relation defining the eigenvalues of our instability problem.

This equation provides a whole set of complex numbers ω_ν which are related to the eigenvalues λ_ν by

$$\lambda_\nu = \left(\omega_\nu - \int_0^d \bar{\alpha}\, dz\right) \Big/ \int_0^d \frac{e}{\varkappa_+}\, dz \tag{207}$$

Our cathodic part of the glow discharge can be considered stable then and only then if the real parts of the complex numbers λ_ν have all negative values. With other words the relation must hold

$$\omega_\nu^r < \int_0^d \bar{\alpha}\, dz \tag{208}$$

Let us now - following Nyquist - consider the function

$$W = \left\{ \bar{\delta} + \int_0^d \bar{\alpha}\, dz/\omega \right\} \exp(-\omega) - \frac{1}{\bar{\gamma}} - \int_0^d \bar{\alpha}\, dz/\omega \tag{209}$$

which describes a transformation from the ω - complex plane into the W-complex plane.

We decide on the question of stability by using the relations (7) and (8) of chapter III. According to equation (209) we have to choose our integral contour for equation (7) as shown in Fig. 23. (remember that $\bar{\alpha}$ is a negative quantity due to the sign $j_-^{(\lambda)}$).

To see under what circumstances the path in the W-plane encircles the origin let us construct the W number vector by adding it up from the various contributions of relation (209).

There is first the constant contribution $-1/\bar{\gamma}$. (See Fig. 23).

The second contribution

$$\varrho_1 = \int_0^d \bar{\alpha}\, dz/\omega = \int_0^d \bar{\alpha}\, dz/(x+iy) \tag{210}$$

is in comparison to the exponential contribution only little varying. Its absolute value is also small in comparison to the exponential contribution. It is given in Fig. 23 by the vector ϱ_1. Finally there is the third contribution

$$\left\{ \bar{\delta} + \int_0^d \bar{\alpha}\, dz/\omega \right\} \exp(-\omega) = \rho_r \tag{211}$$

consisting of two parts which are represented in Fig. 23 by the vectors ρ_2 and ρ_3 . ρ_r rotates around the point $-1/\bar{\gamma} + \bar{\rho}_1$ in the W-plane with a frequency defined by the imaginary part of ω .

A detailed discussion of the vector diagram of Fig. 23 indicates that the largest value of the real part of W occurs for

$$y = o \quad and \quad x = - \int_0^d |\bar{\alpha}|\, dz \tag{212}$$

Therefore encircling of the W-origin can be expected only if for the values of equation (212) W has a positive real part ($W^r > o$). It follows

$$(1 + \bar{\delta}) \exp\left\{ \int_0^d |\bar{\alpha}|\, dz \right\} < 1 + \frac{1}{\bar{\gamma}} \tag{213}$$

for the stability of our system.

Let us remember that the stationarity condition has the form

$$(1 + \delta) \exp\left\{ \int_0^d |\bar{\alpha}|\, dz \right\} = 1 + \frac{1}{\gamma} \tag{214}$$

Then we recognize that $\gamma - \delta$ instability will occur if the relation

$$\frac{\bar{\gamma}\,(\gamma + 1)(\bar{\delta} + 1)}{\gamma\,(\bar{\gamma} + 1)(\delta + 1)} > 1 \tag{215}$$

holds.

If either $\bar{\delta}$ or $\bar{\gamma}$ or both of these quantities are larger than δ or γ , then instability will occur. But instability can even occur if one of these quantities is sufficiently larger than the undisturbed value and the other one is small but in effect not overcoming the influence of the first quantity.

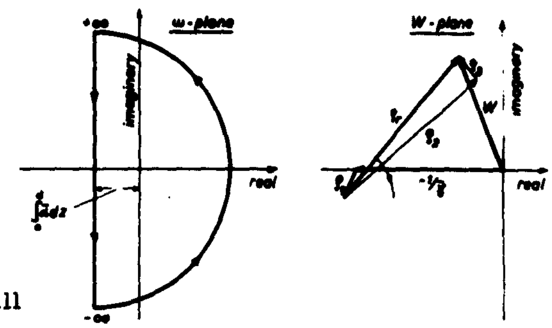

Fig. 23: Nyquist mapping for the dispersion relation of the γ-δ instability.

These results are quite visible. If an increase in the current density of the electrons results in an ion component from the negative glow larger than that necessary for the stationary state we will expect an avalanche process causing instability. The same holds if an increase in the ion current density produces an electron cathode emission too large for the stationary state.

It should be noted particularly that a spontaneous increase in the current density of the CFR itself does n o t produce instability. The instability occurs only if the current density increase goes along with a change in the coefficients γ and δ !

It is very difficult to relate these results to experimental observations. There are quite a number of experiments (e. g. 91, 92, 93, 58, 18, 19, 25, 27, 28, 95, 96, 8, 10, 38) which show cathodic glow instability and point directly to the importance of small size surface impurities. The experiments would allow an explanation with our γ - δ instability but they do not exclude other possibilities.

X STABILITY PRINCIPLES IN GASEOUS ELECTRONICS.

In chapter III we sketched a general method to derive variational principles for stability. This method requires a certain set of equations which describes the whole phenomenon under consideration. As an example we refer to the calculation of the magneto hydrodynamic principle [9]

The application of this general method to problems of gaseous electronics confronts us with the difficulty that there is in gaseous electronics no unique set of differential equations describing the whole field. Rather in any gas discharge we have a certain number of model regions and each of them is governed by different sets of equations. In contrast to the situation in magneto dydrodynamics we can therefore not expect to find o n e principle governing all gas discharges. In the best case we can hope to have different principles for different discharges and discharge parts.

The derivation of such variational principles is a very involved problem as can be seen for instance from the investigation [9] . It is therefore beyond the scope of this investigation.

We will however discuss here two important problems. The one is the question question whether the well known "p r i n c i p l e o f m i n i m u m e n t r o p y p r o d u c t i o n" of irreversible thermdynamics may be used to decide on the stability in gaseous electronics. The second is the discussion of "S t e e n b e c k`s m i n i m u m p r i n c i p l e" which has been widely applied in gaseous electronics to decide on stable modes.

A THE PRINCIPLE OF MINIMUM ENTROPY PRODUCTION.

The "principle of minimum entropy production" can be applied if the following conditions are met:

1.) Gibb's fundamental equation

$$TdS = dU + PdV - \sum_i a_i \, dM_i \tag{216}$$

must hold.

2.) The linear phenomenological relations between the thermodynamical fluxes and forces should be valid.

3.) The Onsager Casimir reciprocity relations should be applicable.

4.) The phenomenological coefficients should be time-independent.

5.) The system should be in mechanical equilibrium or at least in a stationary mechanical state according to

$$\frac{\partial \vec{v}}{\partial t} = 0 \quad , \quad \frac{\partial \overleftrightarrow{p}}{\partial t} = 0 \tag{217}$$

6.) At all parts of the surface of the system one of the two following conditions must hold. Either the temperature and the chemical potentials should be time-independent or the temperature should be time-independent and the normal component of all mass current should disappear.

These conditions imply that each component of the system is close to equilibrium. The principle of minimum entropy production states:

The stationary states of a system subject to the above conditions are characterized by a minimum of entropy production. These states are also stable

If we write down the equations of irreversible thermodynamics for a plasma satisfying the above conditions and apply the principle of minimum entropy production then the Euler equations of the variational problem are identical with the transport equations. Since we are on the other hand close to equilibrium the Saha equation is valid. The application of the energy transport equation and the Saha equation is the basic scheme of the "arc approximation" described in chapter VI B.

Therefore an important conclusion can be drawn from the principle of minimum entropy production:

Any discharge part which can be described by the arc approximation and has boundary conditions which meet the conditions (6) and (7) is always a stable one.

On the other hand, any discharge part which is not governed by the laws of the arc approximation cannot be subject to the application of the principle of minimum entropy production.

Therefore the wall stabilized arc column treated in chapter VII E is always a stable mode. By the same token all other modes described in chapter VII cannot be subject to the application of the principle of minimum entropy production.

However it is still possible to apply the principle to that part of a discharge mode described by the arc approximation - provided that the interface limiting this region is not varied.

With this restriction the thermal nucleus of the wall detached arc column shows the same stability as the wall stabilized arc.

If we allow for variations of the interface between the two model regions then no conclusion on the stability can be drawn from the application of the minimum entropy production principle since all the calculations carried out for this principle underlie the basic assumption that we have boundary conditions fixed in space. The latter problem of the stability of the wall detached arc column with respect to variations of the diameter of the thermal nucleus leads us over to the problem of the Steenbeck minimum principle.

B Steenbeck`s MINIMUM PRINCIPLE.

Following an investigation of Compton and Morse, Steenbeck has claimed the following postulate:

In a real gas discharge of constant total discharge current the energy production and with that the voltage applied to the electrodes is a minimum.

If one identifies a "real discharge" with a discharge which is stable then Steenbeck`s minimum principle is a variational principle for stability.

Steenbeck `s minimum principle could be varified in a number of experiments, e.g. 20, 83, 99 . The theoretical discussion has investigated whether Steenbeck`s empirical postulate can be justified.

There are essentially two questions.

1) Is Steenbeck`s minimum principle a true variational principle for stability?

2) If it is a true variational principle what are the conditions for the application of this principle?

Weizel and Rompe [70] investigated the question whether in the range of the arc approximation the Elenbaas-Heller equation is Euler's equation of Steenbeck's minimum principle. They come to the conclusion that in general this is not the case.

Peters [65] starts from the minimum entropy production principle of irreversible thermodynamics and tries to deduce Steenbeck's postulate from this principle. He finds Steenbeck's principle. However Krause and Steenbeck [52] pointed out that these calculations include the inaccurate restriction to variations into stationary states.

According to Krause and Steenbeck such variations into stationary states are possible if one uses the "canal model". Therefore for this canal model Steenbeck's minimum principle follows from the principle of minimum entropy production and for this special case is therefore manifested as a true variational principle.

In consideration of the canal model other investigators [20] came to the conclusion that Steenbeck's principle is a mean to determine the additional parameter of the discharge diameter introduced by the canal model.

The most refined discussion of Steenbeck's principle has been given recently by Steenbeck himself [84]. Steenbeck states that Elenbaas-Heller's equation is only an approximation. A more exact description including additional effects would increase the number of degrees of freedom. The corresponding boundary conditions for these variables may be ineffective. Then Steenbeck says there is not only space for the application of a minimum principle but there is even the necessity of the application of such a principle to select the actual discharge mode.

Our interpretation of the Steenbeck principle may be described as follows:

Let us consider quite generally any kind of a discharge volume described by a certain set of differential equations. Let us assume further that we have a sufficient number of effective boundary conditions for all variables occuring in these equations.

Then there is one and only one stationary solution for these differential equations with the given boundary conditions. If there is at all a stable solution it must be this stationary solution.

Now any physical system is subject to statistical fluctuations. If the influence of one or more of the boundary conditions is decreased then we reach a point where the influence of these boundary conditions comes below the level of the average statistical fluctuations. In this case the corresponding boundary condition becomes ineffective (see chapter VI C).

Due to the lack of this boundary condition we do not have only one solution but a whole manifoldness of stationary solutions of our system. In a physical experiment we will observe that one of all these stationary solutions which is stable. Under these circumstances we therefore always need a stability criterion which selects the solution observed in the experiment. The kind of this criterion depends on the physical situation.

With respect to S t e e n b e c k`s minimum principle we now consider a discharge consisting of an arc model region and a glow model region separated by an interface (e. g. wall detached arc column, cathodic part of the arc discharge).

As was explained in the chapters VI C, VII F, VII G the boundary conditions for this arrangement may become ineffective if the walls are removed too far. Then the shape of the discharge is not uniquely determined anymore. In our model representation this is reflected in the variability of the position of the interface. For the case of the wall detached arc column the position of the interface is described by the diameter of the arc nucleus.

It is our opinion that S t e e n b e c k's minimum principle selects that mode which is stable with respect to disturbances in the position of the interface. With respect to all other disturbances the thermal nucleus is stable according to the principle of minimum entropy production. Therefore - for our special arrangement of an arc model region and a glow model region - S t e e n b e c k's minimum principle is a sufficient criterion.

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