63-3-2

AFCRL - 63 - 79

MONITORING AGENCY DOCUMENT No.
ASTIA DOCUMENT No.

402133

MODES WITH COMPLEX PROPAGATION CONSTANT IN RECTANGULAR GUIDES LOADED WITH TRANSVERSELY MAGNETIZED LOSSLESS FERRITE

by

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ROMA - ITALY

Technical Note No 5

Contract No. AF 61 (052) - 101

20th June 1962

« The research reported in this document has been sponsored by the Cambridge Research Laboratories, OAR through the European Office, Aerospace Research, United States Air Force. »

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Summary.

The complete modal spectrum in the complex plane of the propagation constant for a rectangular guide partially filled with transversely magnetized lossless ferrite has been investigated and some numerical calculations have been carried out.

In a previous work[1] we have investigated the modal spectrum for a rectangular guide with perfectly conducting walls partially filled with transversely magnetized lossless ferrite (Figure 1).

Our discussion was however limited to real values of the propagation constant k_x.

It is the purpose of this work to investigate the modal spectrum in the whole complex k_x plane. Our discussion will be limited to zero order modes ($k_z = 0$).

The characteristic equation to be investigated has been known for some time[2], and it is the following .

$$\left(\frac{\mu_2}{\mu_1} k_x \operatorname{sen} k_{y2} b_f + k_{y2} \cos k_{y2} b_f\right) \operatorname{sen} k_{y0} b_0 + \tag{1}$$
$$+ k_{y0} \frac{\mu_1^2 - \mu_2^2}{\mu_1} \operatorname{sen} k_{y2} b_f \cos k_{y0} b_0 = 0 \ .$$

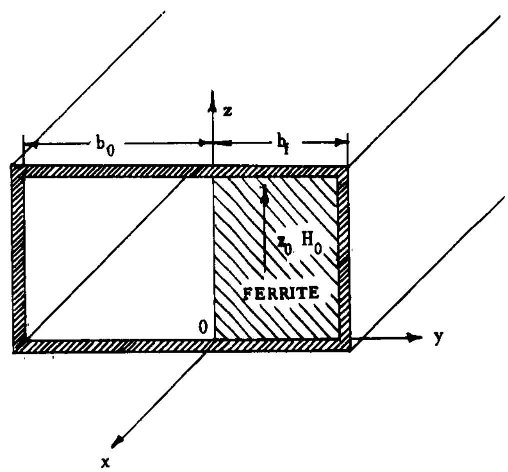

Fig. 1 - The rectangular guide partially filled with ferrite.

In equation (1) dependence of the form $\exp[j(\omega t - k_x x)]$ has been assumed, the propagation constants k_x, k_{y0} and k_{y2} are measured by assuming as unit $\omega\sqrt{\mu_0 \epsilon_0}$, and .

$$k_{y0} = \sqrt{1 - k_x^2} \quad ; \quad k_{y2} = \sqrt{t_2^2 - k_x^2} \quad ; \quad t_2^2 = \epsilon \frac{\mu_1^2 - \mu_2^2}{\mu_1} \ ,$$

where ϵ is the relative dielectric constant of the ferrite and μ_1 and μ_2 are the components of the ferrite tensor relative permeability :

$$\mu = \begin{vmatrix} \mu_1 & j\mu_2 & 0 \\ -j\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

(1) - G. Barzilai and G. Gerosa, «Modes in Rectangular Guides Partially Filled with Transversely Magnetized Ferrite» IRE Trans. on Antennas and Propagation, vol. AP-7, Special Supplement, pp. S471-S474, December 1959. For details: Istituto Elettrotecnico dell'Università di Roma - Techn. Note No. 1, Contract No. AF 61(052)-101, June 3, 1959.

(2) - M.L. Kales, H.N. Chait e N.G. Sakiotis, «A Nonreciprocal Microwave Component» - J. Appl. Phys., vol. 24, pp. 816-817, June 1953 - errata: p. 1528; December 1953.

For a lossless ferrite μ_1 and μ_2 are real and are given by the following expressions :

$$\mu_1 = 1 + \frac{\rho}{1 - \tau^2} \quad ; \quad \mu_2 = \frac{\tau\rho}{1 - \tau^2} \quad ,$$

where :

$$\rho = \frac{M_0}{\mu_0 H_0} \quad ; \quad \tau = \frac{\omega}{\omega_0} \quad ; \quad \omega_0 = -\gamma' H_0 \quad ;$$

M_0 is the saturation magnetization of the ferrite, H_0 is the applied d. c. magnetic field, γ is the gyromagnetic ratio for the electron.

To our knowledge discussions of equation (1) have been rather limited and the only numerical results available for lossless structures refer to real values of the propagation constant.

In what follows we shall consider only lossless structures. It is easy to verify that if k_x is a solution of (1), k_x^* (the asterisk indicates the complex conjugate) is also a solution.

To simplify our analysis we shall assume $b_f = b_0$. For this case it is easy to carry out an asymptotic analysis by assuming :

(2) $$|k_x| >> 1 \quad \text{and} \quad |k_x| >> |t_2^2|$$

and in addition :

$$\mu_1 + \mu_2 \neq 0 \quad \text{and} \quad \mu_1 - \mu_2 + 1 \neq 0 \, .$$

From (1) we obtain the following asymptotic solutions ($k_x = k_x^R + j\, k_x^J$) :

(3) $$\begin{cases} k_x^R = 0 \\ k_x^J = \dfrac{n\pi}{b_f} \end{cases}$$

and

(4) $$\begin{cases} k_x^R = \dfrac{1}{b_f} \tanh^{-1} \dfrac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} \\ k_x^J = \dfrac{n\pi}{b_f} \end{cases}$$

provided that :

$$0 < \frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} < 1$$

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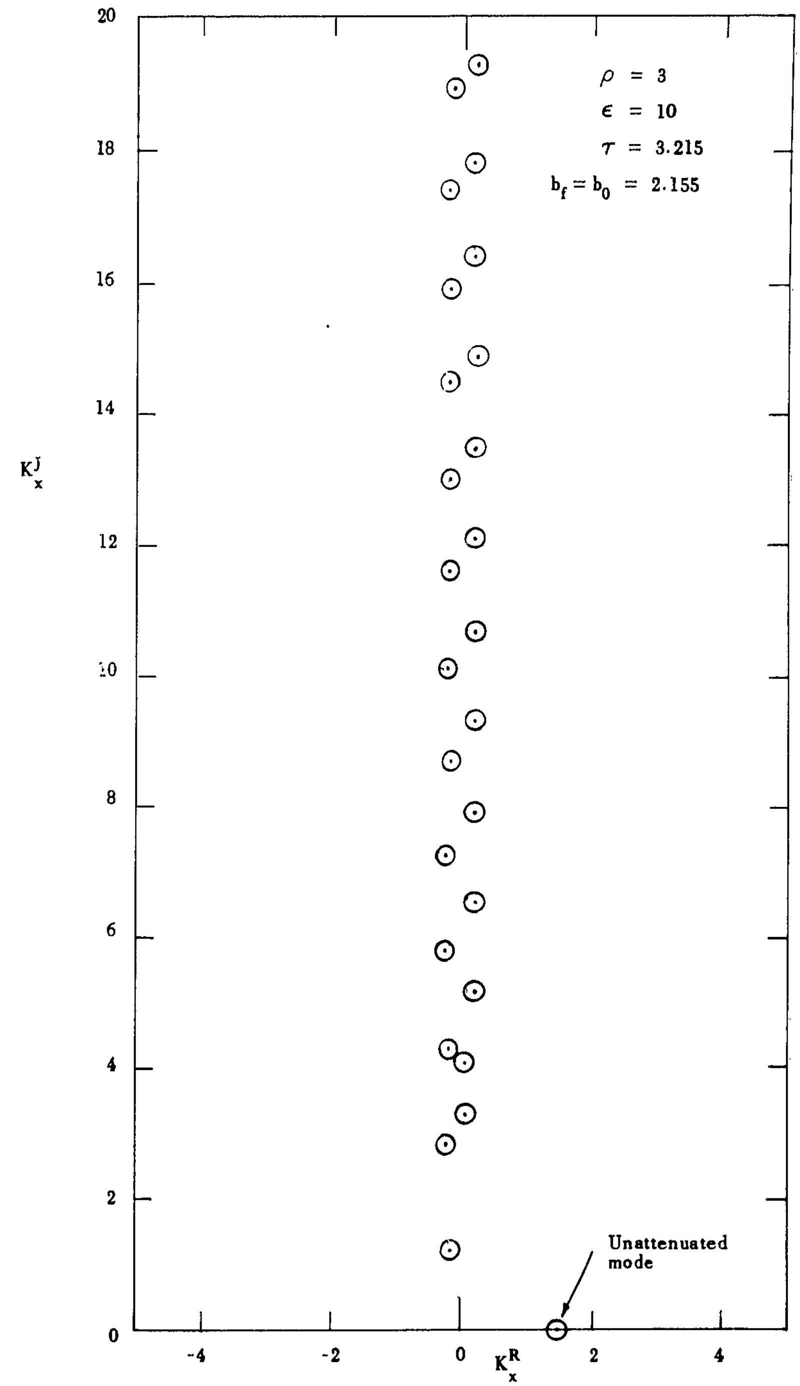

Fig. 2 - Solutions of equation (1) in the complex plane of the propagation constant $k_x = k_x^R + j k_x^J$. The numerical results refer to the numerical values of ρ, ϵ, τ, b_f and b_0 indicated in the figure. These values may be taken to correspond to ·

$M_0 = 0.3$ Wb/m^2 · $H_0 = 10^6/4\pi$ A/m ; $f = \omega/2\pi = 9000$ Mc/s ; $(b_f + b_0)/\omega\sqrt{\mu_0 \epsilon_0} = 0.9"$.

or :

$$
(5) \qquad \begin{cases} k^R = \dfrac{1}{b_f} \coth^{-1} \dfrac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} \\[2ex] k_x^J = \dfrac{(n + 1/2)\,\pi}{b_f} \end{cases}
$$

if :

$$
\frac{\mu_2^2 - \mu_1^2 - \mu_1}{\mu_2} > 1 \ .
$$

In (3), (4) and (5) n is any integral number sufficient to fullfill (2).

We have considered a numerical case for which we have solved equation (1) without any asymptotic approximation. The values of the parameters are indicated in Fig. 2, in which the solutions of equation (1) are recorded only in the $k_x^J \geq 0$ plane, since the solutions are symmetric with respect to the real axis.

The solution of equation (1) have been obtained by separating real and immaginary parts and setting them separately equal to zero. In the plane k_x^R , k_x^J , each of these two equations is represented by a curve with many brances. From the intersections of these two curves we obtain the solutions sought.

We have investigated the following zone of the k_x complex plane :

$$
-20 \leq k_x^R \leq 20 \qquad ; \qquad 0 \leq k_x^J \leq 20 \ .
$$

This investigation has required about twenty five hours of a medium speed electronic digital computer.

With the numerical values assumed in Fig. 2, the asymptotic solutions are given by (3) and (4) .

From the numerical results and the asymptotic analysis it appears that the structure considered possesses only one zero order unattenuated propagating mode and an infinite number of attenuated propagating modes, lying in a narrow strip including the immaginary axis.

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