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Some Problems in Phenomenological Fracture Mechanics

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MARCH 1963

Directorate of Materials and Processes
Aeronautical Systems Division
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

Project No. 7351, Task No. 735106

(Prepared under Contract No. AF 33(616) 6112
by Columbia University, New York, N. Y.

A. M. Freudenthal, M. P. Bieniek, authors)

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FOREWORD

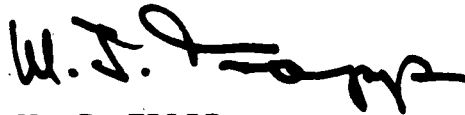
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This report covers work conducted from March 1961 to March 1962.

ABSTRACT

This report deals with certain problems of propagation of microcracks (Griffith cracks) and fracture cracks. The phenomenological approach based on the concept of the continuum and the methods of mechanics of solids is used. An energy criterion for equilibrium of cracks in inelastic solids is formulated in the form which reduces to Griffith's criterion in the case of brittle solids and, with certain simplifying assumptions, results in Orowan's criterion for elastic-plastic solids. Another criterion is derived from the analysis of the stresses at the edge of a crack. This stress criterion is also extended over time-dependent crack resistance by relating it to certain characteristics of the inelastic deformation.

This technical documentary report has been reviewed and is approved.



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1. INTRODUCTION

Fracture Mechanics deals with the following two phenomena:

(1) The development and propagation of cracks in stressed and deformed solid bodies; questions arise concerning the conditions that cause a crack to develop and propagate or a propagating crack to be stopped, and the relations between the mechanical properties of a body and its behavior with respect to crack initiation and propagation.

(2) The well-known discrepancy between the theoretical cohesive strength of the atomic lattice and the actual (technical) fracture strength of solids.

In order to explain the latter phenomenon, A. A. Griffith¹ assumed that in all solids certain cracks exist which at their edges cause large stress-concentrations even under relatively small forces; hence, the average fracture strength of a body is controlled by the effect of such micro-cracks. With respect to their propagation, large-scale fracture cracks and Griffith cracks can be discussed on the same basis, considering that a fracture crack is in fact a large Griffith crack; at the leading edge of a fracture crack in an elastic medium the conditions are the same as in a Griffith crack since no special properties are attached to Griffith cracks which are assumed to be large enough to justify a continuum mechanical approach in the treatment of a single crack. On the other hand, the analysis of the discrepancy between atomic and technical strength in brittle materials is based on the assumption of a multitude of Griffith cracks of random size and orientation,

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and the technical strength is based on their assumed statistical distribution and the resulting probability of occurrence of cracks of extreme size, the propagation of which determines the technical strength.

Thus the problem is, in fact, transformed into a statistical problem in which the mechanical considerations only form the underlying concept of the "weakest link" leading to the statistical concept of the extreme value distribution of technical strength.

In phenomenological fracture mechanics, however, the mechanism of propagation of an individual crack in a continuum is considered, utilizing methods of mechanics of solids. This phenomenological theory has been fairly successful in establishing certain laws of crack propagation for brittle (elastic) solids, in introducing effects of plasticity, and in dealing with the dynamics of crack propagation. There is one obvious limitation of phenomenological fracture mechanics: it can only be applied to cracks that are sufficiently large in comparison to the inter-atomic distances in the solid. Because of this requirement, which can be satisfied in the theory of propagation of existing cracks, problems of crack nucleation are not accessible to phenomenological methods.

This paper deals with the limited group of problems of fracture mechanics that may be described as problems of equilibrium of cracks. The conditions which cause a crack to propagate or to stop will be investigated under the assumption that all changes of the state of a body are quasi-static. Special attention will be paid to the effect of inelastic properties. The following aspects are dealt with:

1. General criteria of crack propagation (review).
2. The effect of plasticity (time-independent energy dissipation).
3. The effect of time and time-dependent dissipation.

2. ENERGY CRITERIA OF EQUILIBRIUM

The energy equation for a body with a propagating crack can be written in the form

$$\dot{W} = \dot{U}_r + \dot{U}_d + \dot{K} + \dot{S} \quad (2.1)$$

where W is the work of the external forces, U_r the reversible part of internal (mechanical) energy, U_d the irreversible part of internal energy, K the kinetic energy, and S the surface energy of the crack. The existence of the surface energy has been postulated by Griffith.¹ For quasi-static problems discussed in this paper, it is assumed that $K \equiv 0$. Equation (2.1) may be considered as the law of conservation of energy in a form containing the surface energy S . To obtain a criterion for the equilibrium of a crack of specified dimension, a certain property of the energy of a body with a crack has to be assumed. Let c denote the characteristic dimension of the crack, such as its length or radius, so that W , U_r , U_d , K , and S can be considered as functions of c . Denoting by δ the variations corresponding to an infinitesimal change δc of the characteristic dimension c , the condition for an equilibrium crack can be written in the form

$$\delta W = \delta U_r + \delta U_d + \delta S . \quad (2.2)$$

The variations are taken for constant values of all the parameters characterizing the state of the body. The physical meaning of condition (2.2) is the following: the increments of the work of the external forces and of the released strain

energy, $\delta W - \delta U_r$, corresponding to an increment of the crack dimension δc , are equal to the increment of the dissipated energy and of the surface energy, $\delta U_d + \delta S$, corresponding to the same increment of the crack dimension.

If the equality sign in (2.3) is replaced by an inequality sign, the crack is not in equilibrium. In particular, if

$$\delta W - \delta U_r < \delta U_d + \delta S \quad (2.3)$$

the characteristic dimension of the crack is larger than that corresponding to the state of equilibrium. If

$$\delta W - \delta U_r > \delta U_d + \delta S \quad (2.4)$$

the characteristic dimension of the crack is smaller than that corresponding to the state of equilibrium.

For an elastic (brittle) solid, $U_d \equiv 0$ and condition (2.2) becomes

$$\delta W - \delta U_r = \delta S \quad (2.5)$$

The quantity $-\delta W + \delta U_r$ represents the variation of the potential energy of the body. Equation (2.5) expresses thus the Griffith criterion which states that the decrement of the potential energy is equal to the increment of the surface energy.

The criterion of Orowan for elastic-plastic solids can be derived from Eq. (2.2) by assuming that the total dissipation of energy occurs in a thin layer at the surface of the crack; the variation of energy dissipated in this surface

will be denoted by $\delta U_d^{(s)}$. $U_d^{(s)}$ may be considered as a material characteristic in a similar manner as the surface energy S . Thus, Eq. (2.2) becomes

$$\delta W - \delta U_r = \delta U_d^{(s)} + \delta S \quad (2.6)$$

which expresses Orowan's condition that the decrement of the potential energy is equal to the increments of the surface plastic work and the surface energy. Orowan notes that for highly ductile solids $\delta U_d^{(s)} \gg \delta S$ and, consequently, δS may be neglected in Eq. (2.6).

Consider now a solid with time-dependent dissipation mechanism, for which $\delta U_d^{(s)}$ is zero or does not depend on time, such as, for instance, in certain viscoelastic solids in which the extension of a crack does not produce an instantaneous dissipation of energy. Then, three cases are possible.

(a) δU_r decreases with time in such a way that $\delta W - \delta U_r$ increases in time, i.e. the amount of potential energy to be released during crack propagation increases. An equilibrium crack becomes at a certain instant of time a non-equilibrium crack and tends to propagate with increasing time. This process is reflected in the decrease of strength under sustained loading.

(b) δU_r increases in time and $\delta W - \delta U_r$ decreases. In this case the opposite effect can be expected: the strength under slowly applied load is larger than if the load is applied at a high rate.

(c) If neither δU_r nor $\delta W - \delta U_r$ depend on time, the viscoelastic properties of the solid do not influence the process of crack propagation.

The argument presented above can be applied to the consideration of the effect of viscoelastic creep and relaxation in a body as a whole. However, since the viscoelastic phenomena in the vicinity of a leading edge of a crack depend on the character and distribution of the bond stresses, the energy criteria are not quite independent of certain stress hypotheses discussed in the following section.

Another problem of time dependent crack propagation might arise in the case of variation of the surface energy S , or of the specific surface energy s per unit area, as a function of time. The same result may, however, be obtained by either assuming variation of $\delta U_d^{(s)}$ or an opposite variation of δS with time. Certain conclusions may be derived on the basis of the simplifying assumption that S is constant in time.

3. STRESS CRITERIA OF EQUILIBRIUM

The stress criteria of equilibrium are based on the comparison of the actual state of stress at the leading edge of a crack with a limiting state associated with a given solid. An exact approach to this problem would require a knowledge of the bond forces developing at the edge of a crack. However, it is possible to assume certain characteristics of the state of stress which are introduced as measures of the tendencies of a crack.

In a method developed by Barenblatt⁴ it is assumed that at the leading edge of a crack large (but finite) cohesion forces exist, the distribution of which is controlled by the character of bond forces, but is unknown. Now the following assumptions are made concerning the effect of these forces: (a) the leading zone of the crack is very small compared to the characteristic dimension of the crack; (b) the displacements at the leading edge of a crack are unique for an equilibrium state and do not depend on the type of loading (for cleavage type fracture); (c) the crack closes smoothly, i.e. the slope of the surfaces is zero at the end.

Let us apply the above proposition to a circular (axially symmetrical) crack in an infinite body (Fig. 1). In a cylindrical coordinate system r, θ, z , in which, because of symmetry, θ does not enter into the equations, let R denote the radius of the crack, d the width of the leading zone, and $g(r)$ the loading on the surfaces of the crack. The total loading $g(r)$ consists of an arbitrary pressure $p(r)$ and of the cohesion forces $G(r)$

$$\begin{aligned}
g(r) &= p(r) \quad , \quad 0 \leq r \leq R - d \\
g(r) &= -G(r) \quad , \quad R - d < r \leq R
\end{aligned}
\tag{3.1}$$

The z-component of the displacement of the surfaces of the crack⁵

$$w = \frac{4(1 - \nu^2)R}{\pi E} \int_{\rho}^1 \frac{\mu d\mu}{\sqrt{\mu^2 - \rho^2}} \int_0^1 \frac{xq(x|R)dx}{\sqrt{1 - x^2}} \tag{3.2}$$

where $\rho = r/R$, E and ν are the Young modulus and Poisson ratio, respectively.

The condition

$$\left(\frac{\partial W}{\partial r}\right)_{r=R} = 0$$

and Eq. (3.1) result in the equation

$$\int_0^{1-d/R} \frac{x p(xR)}{\sqrt{1 - x^2}} dx = \int_{1-d/R}^1 \frac{x G(xR)}{\sqrt{1 - x^2}} dx \tag{3.3}$$

The integral on the right-hand side of Eq. (3.3) can be transformed into the form

$$J = \frac{1}{\sqrt{2R}} \int_0^d \frac{F(s)ds}{\sqrt{s}} \tag{3.4}$$

where $F(s) = G(r)$, $s = R - r$.

Equation (3.3) becomes

$$\int_0^{1-d/R} \frac{x p(xR)}{\sqrt{1-x^2}} dx = \frac{1}{\sqrt{2R}} \int_0^d \frac{F(s)}{\sqrt{s}} ds \quad (3.5)$$

or

$$\int_0^{R-d} \frac{r p(r)}{\sqrt{R^2-r^2}} dr = \sqrt{\frac{R}{2}} K \quad (3.6)$$

The quantity

$$K = \int_0^d \frac{F(s)}{\sqrt{s}} ds \quad (3.7)$$

thus characterizes the distribution of the unknown cohesion forces $F(s)$. It is introduced as a new material constant; its role is similar to that of the specific surface energy (per unit area). In fact, it can be shown that in order to obtain the same crack-resistance from the energy theory and the stress theory the relation should hold

$$K_{\max}^2 = \frac{\pi E s}{1-\nu^2} \quad (3.8)$$

For a given loading $p(r)$ and with a known cohesion modulus K , Eq. (3.6) determines the radius R corresponding to the equilibrium state of the crack. A similar relation has been derived by Barenblatt for the case of plane stress

$$\int_a^b p(x) \frac{b-x}{x-a} dx = \sqrt{2l} K \quad (3.9)$$

with notations indicated in Fig. 2.

Another approach to the same problem is based on the assumption that no cohesive forces act even near the leading edges of the surface of the crack while the material remains perfectly elastic beyond these edges. For any given loading, for instance for uniform pressure on the surfaces of the crack, the state of stress can be determined. All stress components have a singularity at the edge of the crack. In particular, the normal stress σ_z is

$$\sigma_z = \frac{C}{\sqrt{2\xi}} \quad , \quad \xi > 0 \quad (3.10)$$

where $\xi = r - R$ for a circular crack, and $\xi = x - b$ or $\xi = a - x$ for a crack in plane strain (Figs. 3 and 4). For an equilibrium crack, the constant C reaches its maximum value C_{\max} which depends on the type of the material. The criterion represented by (3.10) gives the same results as the energy criterion for brittle solids if the constant C_{\max} is related to the surface energy s in the following way

$$C_{\max}^2 = \frac{2Es}{\pi(1 - \nu^2)} \quad (3.11)$$

The shapes of cracks analyzed in this way are different from those based on the previous theory. The surfaces of a crack do not close smoothly. In the case of uniform pressure, the profile of the crack is elliptical (Figs. 3 and 4).

Consider now certain generalizations of the above considerations. L. M. Kachanov gave recently⁷ an estimate of the time effect for linear viscoelastic solids in relation to Barenblatt's theory. He introduced a new material constant characterizing the rate of change of the cohesion modulus K with increasing deformation.

In this note a different approach is proposed. The time dependence of crack resistance will be related to the surface energy s and the viscoelastic properties of the solid. The argument used here is valid for either one of the above stress criteria.

Considering an arbitrary linear viscoelastic solid, the simplifying assumption is made that v is approximately constant, although it is possible to extend all considerations to the case of a variable v . Consider a crack whose characteristic dimension at an initial time $t = t_0$ is larger than that corresponding to the state of equilibrium, i.e. the parameters K or C are smaller than their limit values K_{\max} and C_{\max} , respectively. In order to explain the fact that at a certain time t the crack reaches its equilibrium state and starts to propagate, it has to be assumed that the limit values K_{\max} and C_{\max} decrease in time. The same conclusion is suggested by the relations (3.8) and (3.11) where a reduced modulus E should be taken into account. At the same time, Eqs. (3.8) and (3.11) give a quantitative measure of K_{\max} and C_{\max} as functions of time. Denoting by $E(t)$ an equivalent modulus controlling deformation under constant load, i.e.

$$E(t) = E_0 \frac{u_0}{u(t)} \quad (3.12)$$

where E_0 is the modulus of initial deformation, u_0 is an arbitrary component of strain or displacement at initial time, $u(t)$ is the same quantity at the time t , the existence of $E(t)$ defined by (3.12) follows immediately from the assumptions and type of the problem discussed. For other problems an approximate value of $E(t)$ should be established. Hence

$$[K_{\max}(t)]^2 = \frac{E(t)\pi s}{1 - \nu^2} \quad (3.13)$$

and

$$[C_{\max}(t)]^2 = \frac{E(t) 2s}{\pi(1 - \nu^2)} \quad (3.14)$$

The above relations can also be written as

$$K_{\max}(t) = \sqrt{\frac{E(t)}{E_0}} K_{0 \max} \quad (3.15)$$

and

$$C_{\max}(t) = \sqrt{\frac{E(t)}{E_0}} C_{0 \max} \quad (3.16)$$

where $K_{0 \max}$ and $C_{0 \max}$ characterize crack resistance under short time loading.

In the processes of fracture of metals at high temperatures, two different effects exist. One of them is the above mentioned effect of creep, reducing crack resistance according to the relations (3.13), ... (3.16). The other effect, not existing in linear viscoelastic solids, is a redistribution of stresses caused by the non-linearity of creep and relaxation.

This results in reduced intensity of stresses near the edge of a crack, and in this way it counteracts the former effect of creep.

These conditions of fracture are implied by the type of inelastic behavior shown by metals in high temperatures. The existence of two opposite effects is confirmed by experiments in so-called static fatigue of metals, which indicate that the reduction of strength in time is much smaller than would be expected from large creep deformations.

4. CONCLUDING REMARKS

The topics discussed in this note represent a formulation of some laws of fracture mechanics in terms of concepts and equations of mechanics of continua. Realizing fully the inadequacy of this phenomenological approach, it is also necessary to keep in mind its merits. Use is made of relatively few assumptions and hypotheses, usually following from experimental evidence. Certain simplifications (as, for instance, the models of linear elastic or linear viscoelastic solids) have a rather definite scope of validity. The predictions made by phenomenological theories proved to be correct, at least qualitatively, and of practical use.

The conclusions derived in this note concern two problems: energy relations for a body with a crack, and stress conditions at the leading edge of a crack. In particular, the effect of inelastic properties has been taken into account. The qualitative discussion of this effect and, for a special case, certain quantitative relations are given.

Further questions arise in connection with the approach presented in this report which require attention:

(a) Experimental investigation of the values of the surface energy the maximum cohesion modulus K_{\max} , and the maximum stress intensity factor C_{\max} ,

(b) Experimental investigation of the time dependence of crack resistance of several types of inelastic materials,

(c) Establishment of a reasonable distribution of bond forces compatible with the considerations of continuum mechanics.

(d) Development of the methods of analysis of stresses, strain, and energies in the presence of cracks under different loading conditions.

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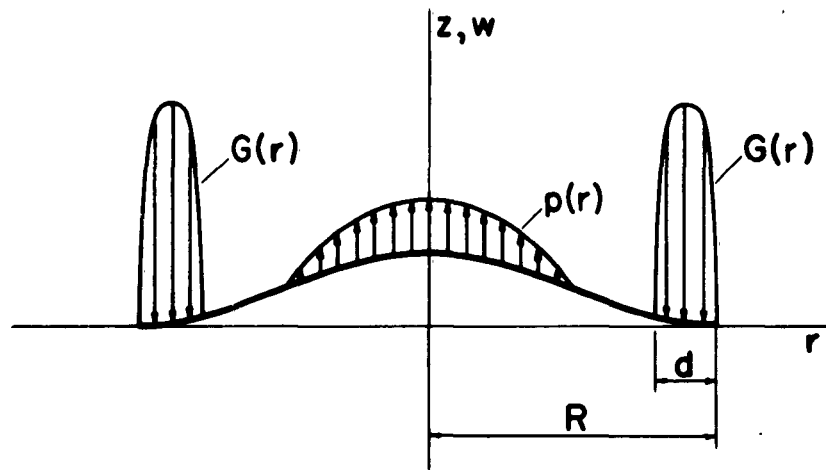


Fig. 1. Stress distribution on the surfaces of a circular crack with smoothly closing edges.

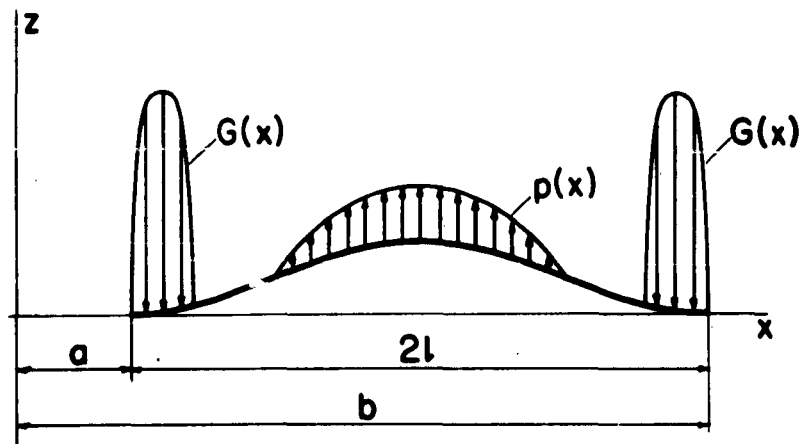


Fig. 2. Stress distribution on the surfaces of an infinitely long crack with smoothly closing edges.

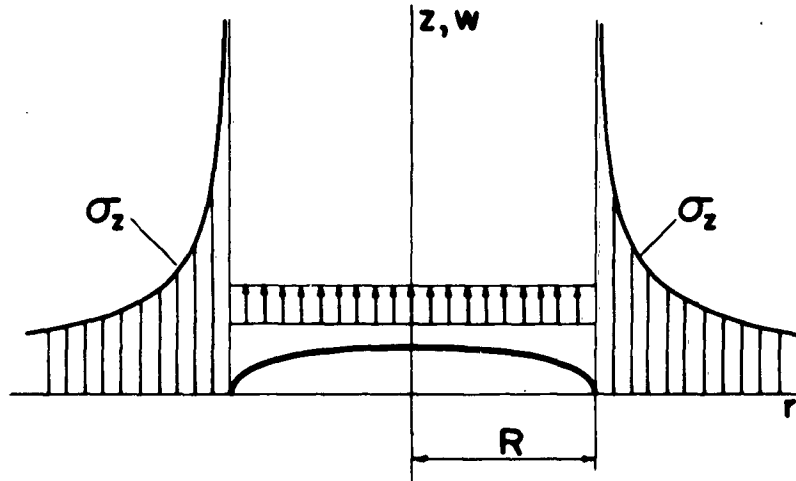


Fig. 3. Stress distribution in the vicinity of a circular crack with elliptical profile.

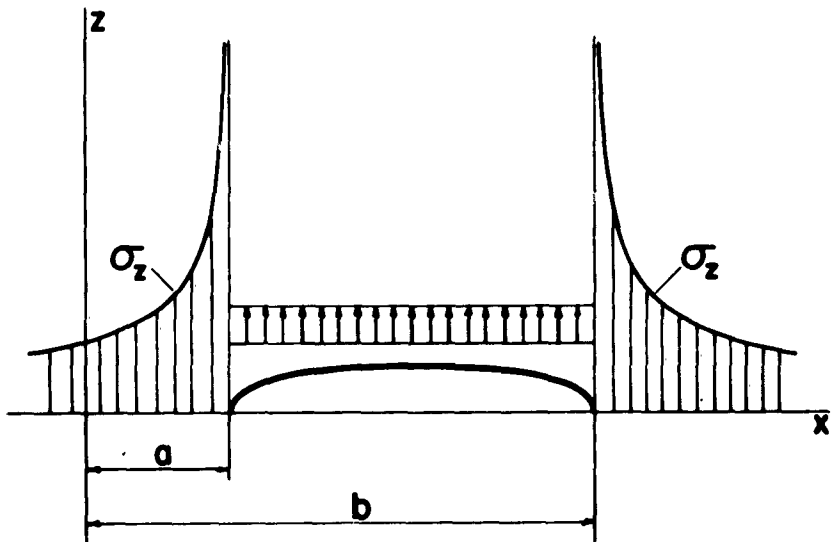


Fig. 4. Stress distribution in the vicinity of an infinitely long crack with elliptical profile.

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