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Technical Report

ELECTROMAGNETIC CONTROL OF THE ATTITUDE OF SPACE VEHICLES

Authors: P. Bruscaglioni, A. Consortini
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CENTRO DI STUDIO PER LA FISICA DELLE MICROONDE
CONSIGLIO NAZIONALE DELLE RICERCHE
FIRENZE - ITALIA

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SUMMARY

A theoretical investigation has been made of the possibility to apply the properties of elliptically polarized waves for the attitude control of space vehicles.

Three different ways of applying electromagnetic radiation has been discussed .

The first one employes an antenna placed on a space vehicle , and radiating with circular polarization . A reaction torque turns out to be applied to the vehicle.

The second one makes use of the the torque experienced by a receiving antenna , placed on the vehicle , when a ground station radiates towards the vehicle . The third one is based on the simultaneous and tuned emission of radiation from an antenna located on the vehicle and a ground antenna.

A comparison between the torque which can be obtained and the torques necessary to the control of the attitude of a space vehicle in actual situations shows that the first procedure can have pratical importance, while the torques obtained by means of the second and third procedures seem to be too weak.

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LIST OF THE SYMBOLS

$\underline{Q}$	resulting momentum of a distribution of charges
$\underline{E}$, $\underline{H}$, $\underline{D}$, $\underline{B}$	electromagnetic field vectors
[illegible]	density of electromagnetic energy
Z	characteristic impedance of the free space
$\underline{S}$	Poynting vector
ω	angular frequency of the waves
λ	wavelength
M	torque acting on a radiating antenna
I	electric current
W	power radiated by an antenna
W_a	power absorbed by a receiving antenna
W_R	power scattered by a receiving antenna
W_S	power radiated by a satellite's antenna
W_g	power radiated by a ground antenna
G	gain of the ground antenna
A	power absorption (or scattering) cross section of a half-wave antenna
K	moment of inertia of a space vehicle about a given axis
K_r, K_p, K_y	moments of inertia of a space vehicle about the roll, pitch, yaw axes respectively
G_r, G_p	torques applied to a space vehicle about the roll or pitch axes respectively, due to the gravitational gradient
e	eccentricity of the orbit of an earth's satellite

ω_o	mean orbital angular velocity of the satellite
T_o	torque applied to a space vehicle due to inertial reactions
ω_s	spin velocity of a spin stabilized space vehicle
C	torque applied to the vehicle, which tends to slow down the spin velocity
P_s	solar radiation pressure
T_s	torque applied to a vehicle due to solar radiation pressure
T_c	control torque applied to a space vehicle
t_r	time necessary for a space vehicle to change its attitude by an angle ι, under the action of the control torque T_c
P_d	aerodynamic pressure

INTRODUCTION

The control of the attitude of a space vehicle is of great importance in many cases . For instance high directivity antennas can be used on communications satellites only if the satellite, or the parts containing the antennas, can be suitably oriented.

The case where the power to be used by the vehicle comes from solar cells, placed on paddles to be directed towards the sun, or from solar thermal devices, which require large mirrors to concentrate light on an absorber, is another example of the necessity of attitude control. In these cases the requested accuracy can be of the order of one degree . However in the case of some satellites for scientific researches the accuracy required is much higher . For example the O.A.O. project provides that the satellite must keep its attitude fixed in an inertial frame of reference , within an angle of a tenth of second of arc.

Two kinds of problems arise : the problem of using a suitable sensor in order to get information about the attitude of the vehicle , and the problem of applying a torque to the vehicle , in order to correct its attitude.

There are a number of different attitude sensors, based on the use of photocells sensitive to visible or infrared light , of gyroscopes, of magnetometers, of telescopes capable of determining the direction of a star with a great accuracy . On can always find a device sufficiently accurate for his purpose .

The attitude control can be effected by several methods, which are generally divided into two classes,namely by passive and active methods , In the first case the control is obtained without an expense of power . An example of this case are the spin stabilized satellites. A passive stabilization can be also obtained by means of the actions of the gravitational gradient on vehicles of elongated shape, or by means of the earth's magnetic field acting on magnetic bars located in the vehicle .

The active methods consist in the application of a torque to the vehicle with expense of some power.

Many methods for applying a torque to a space vehicle have been used or proposed such as (*) : emission of gas jets , controlled motion of indipendent parts of the vehicle (rotating wheels), plasma emissions, excitation of currents in coils interacting with the earth's magnetic field , changes of the mass distribution within the vehicle (effect of the gravitational gradient), use of movable solar sails and rudders.

The torques achievable with the different methods mentioned turn out of different order of magnitude. Thus when a quick maneuver of a vehicle is necessary , the emission of

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(*) see for instance Ref.1

gas jets is considered to be the most suitable device, because of the large value of the torque obtainable. In other cases however very weak torques are sufficient, for instance when the attitude of a vehicle must be kept stable against the action of small external forces.

The object of this research is a discussion of a method based on the use of elliptically polarized electromagnetic waves. Such waves have the property to carry angular momentum about the direction of propagation.

This property was first derived from theoretical considerations by Sadowsky ([2]), Epstein ([3]), and Poynting ([4]). Later it received experimental proof by Beth ([5]), who used visible light , and by Carrara ([6]), who used microwaves.

Three different ways of employing electromagnetic radiation for the attitude control of a space vehicle will be discussed in this report, and will form the subjects of the first, second and third sections respectively. The first is an active method, the second a passive method, the third a mixed method.

1 - REACTION TORQUE ON A RADIATING ANTENNA

1.1 - General remarks

In the present part of this report we will discuss the possibility of employing the torque, experienced by an antenna placed on a space vehicle, to control the attitude of the vehicle.

First, a formula will be derived for the calculation of the flux of angular momentum through a spherical surface. This formula will then be applied to the evaluation of the torques acting on the antennas. In the sequel such torques will be indicated as ''electromagnetic torques'' .

Consider a radiating antenna, which for simplicity will be assumed to be a point source, located at P. It is possible for the electromagnetic radiation issued by the antenna to have an angular momentum about P. This can happen when the radiation is elliptically polarized. In this case it is evident that the antenna must be subjected to a reaction torque.

1.2 - Flux of angular momentum through a spherical surface containing charges and currents .

Let V be a region of space in which charges and currents are present, and $\underline{Q}dV$ the momentum of the charges contained in a small volume dV. Let $\underline{S}$ be the Poynting vector and $\underline{T}$ the Maxwell's tensor defined as :

$$\underline{T} = \underline{E}\,\underline{D} + \underline{H}\,\underline{B} - \mathcal{U}\,\underline{U}$$

where $\underline{E}$, $\underline{H}$ are the electric and magnetic field vectors, $\underline{D}$, $\underline{B}$ are the electric and magnetic induction vectors. $\mathcal{U}$ is the density of electromagnetic energy :

$$\mathcal{U} = \frac{1}{2} (\underline{E}.\underline{D} + \underline{H}.\underline{B})$$

and $\underline{U}$ is the fundamental tensor of the Euclidean space, whose components are :

$$U_{lm} = \delta_{lm} \quad ; \quad \delta_{lm} = 0 \text{ if } l \neq m \quad , \quad \delta_{lm} = 1 \text{ if } l = m$$

By taking into account the action of the field on the charges and currents, it is possible to obtain the following relation (') :

$$(1.2\text{-}1) \qquad \frac{\partial \underline{Q}}{\partial t} = - \frac{\partial}{\partial t} \left(\frac{\underline{S}}{v^2} \right) + \operatorname{div} \underline{T}$$

where v is the velocity of the electromagnetic waves in the medium. For the angular momentum of the charges with respect to a certain fixed point O, one obtains from (1.2-1):

$$(1.2\text{-}2) \qquad \frac{\partial}{\partial t} (\underline{Q} \wedge \underline{r}) = - \frac{\partial}{\partial t} \left(\frac{\underline{S} \wedge \underline{r}}{v^2} \right) + (\operatorname{div} \underline{T}) \wedge \underline{r}$$

where $\underline{r}$ denotes the radius vector of the point of interest with respect to O.

(') see for instance:Ref.7 p.78

The total angular momentum of the charges will be given by:

$$(1.2\text{-}3) \qquad \frac{\partial}{\partial t}\int_V \underline{Q}\wedge\underline{r}\, dV = -\frac{\partial}{\partial t}\int_V \frac{\underline{S}\wedge\underline{r}}{v^2}\, dV + \int_V (\mathrm{div}\ \underline{T})\wedge\underline{r}\, dV$$

Since the following relation is valid for a dyadic $\underline{u}\ \underline{v}$ (see Appendix 1)

$$(1.2\text{-}4) \qquad \left[\mathrm{div}\ (\underline{u}\ \underline{v})\right]\wedge\underline{r} = \mathrm{div}\left[(\underline{u}\wedge\underline{r})\ \underline{v}\right] - \underline{u}\wedge\underline{v}$$

the last integral in (1.2-3) can be written as :

$$(1.2\text{-}5) \qquad \int_V (\mathrm{div}\ \underline{T})\wedge\underline{r}\, dV = \int_V \mathrm{div}\left[(\underline{E}\wedge\underline{r})\ \underline{D}\right] dV +$$

$$+ \int_V \mathrm{div}\left[(\underline{H}\wedge\underline{r})\ \underline{B}\right] dV + \int_V (\mathrm{div}\,\mathcal{U}\,\underline{U})\wedge\underline{r}\, dV$$

If the region V is spherical with its center at O, it is possible to demonstrate that the last term of **Eq.** (1.2-5) is zero (see **Appendix 2**). **Thus, by applying the divergence** theorem, one has :

(1.2-6)
$$\frac{\partial}{\partial t}\int_V \underline{Q}\wedge\underline{r}\,dV = -\frac{\partial}{\partial t}\int_V \frac{\underline{S}\wedge\underline{r}}{v^2}\,dV + \int_\Sigma (\underline{E}\cdot\underline{n})\,\underline{D}\wedge\underline{r}\,d\Sigma + \int_\Sigma (\underline{H}\cdot\underline{n})\,\underline{B}\wedge\underline{r}\,d\Sigma$$

where $\underline{n}$ is the outward normal to the surface Σ containing V. The expression:

$$\underline{m} = \frac{\partial}{\partial t}\int_V \underline{Q}\wedge\underline{r}\,dV$$

is the resulting moment, with respect to 0, of the electromagnetic forces applied to the charges in V.

If $\underline{E}$, $\underline{H}$, $\underline{D}$, $\underline{B}$ are oscillating functions of constant amplitude, the time average $\underline{M}$ of $\underline{m}$ is:

(1.2-7)
$$\underline{M} = \frac{1}{2}\,\mathrm{Re}\int_\Sigma (\underline{E}^*\cdot\underline{n})\,\underline{D}\wedge\underline{r}\,d\Sigma + \frac{1}{2}\,\mathrm{Re}\int_\Sigma (\underline{H}^*\cdot\underline{n})\,\underline{B}\wedge\underline{r}\,d\Sigma$$

where the asterisk * denotes the complex conjugate quantity. The time average of

$$\frac{\partial}{\partial t}\int_V \frac{\underline{S}\wedge\underline{r}}{v^2}\,dV$$

is in fact zero . Some situations can occur , in which **M** is different from zero .

1.3-Torque on two crossed dipoles in quadrature

As a first simple example let us consider the case in which the antenna is formed by two elementary dipoles , perpendicular to each other (Fig.1-1).The currents, assumed to be uniform across both dipoles, have equal amplitudes and their phases differ by $\frac{\pi}{2}$.

The antenna is assumed to be placed in the vacuum, far from other bodies . Thus the field of the antenna, in the region considered for the following calculations, can be assumed as being undisturbed by the presence of other bodies .

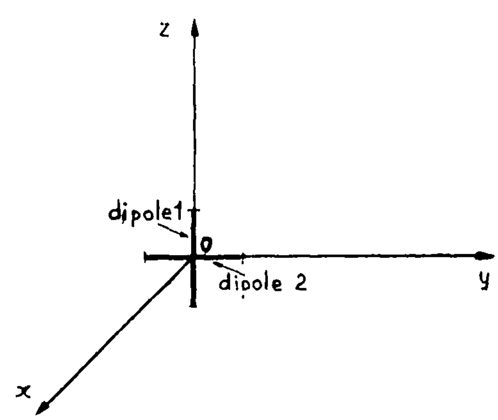

Fig.1-1

All elements of dipole 1 will be labelled by the index 1, all elements of dipole 2, by the index 2. Dipole 1 is assumed to be oriented along the z axis (unit vector **k**) of a rectangular coordinates system, whose center is placed at the point 0 where the dipoles cross each other . Dipole 2 is oriented along the y axis (unit vector **j**) .

The currents of the two dipoles will be represented by

$$I_1 = Ie^{i\omega t}$$

(1.3-1)

$$I_2 = i\, Ie^{i\omega t}$$

where i is the imaginary unit .

Let us consider two systems of polar coordinates: r, θ_1, φ_1, and r, θ_2, φ_2, both with their origin at 0 and with the polar axes along the z and y directions respectively (see Fig.1-2).Let Σ represent a spherical surface with center at 0 and radius r . The antenna radiates with circular polarization in the direction x . The reaction torque applied to the antenna can be evaluated, by means of (1.2-7) , by calculating the flux of angular momentum through the surface Σ .

It is seen at once that in this case the expression :

$$\frac{1}{2}\,\mathrm{Re}\;\mu_o \int_\Sigma (\underline{H}^{*}\cdot\underline{n})\underline{H}\wedge\underline{r}\; d\Sigma$$

which appears in (1.2-7) vanishes, becauses the magnetic field vector is tangent to the surface Σ . Thus (1.2-7) can be

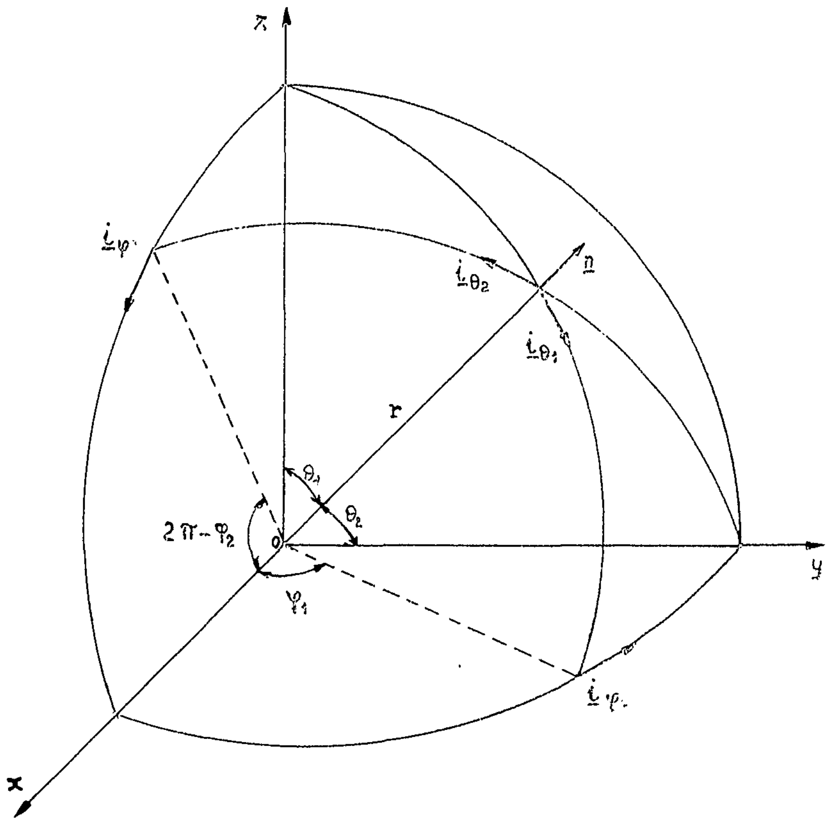

Fig.1-2

Polar coordinates r, Θ_1, φ_1 and r, Θ_2, φ_2.

$\underline{n}$, $\underline{i}_{\Theta 1}$, $\underline{i}_{\varphi 1}$ are the unit vectors of the reference system 1, and
$\underline{n}$, $\underline{i}_{\Theta 2}$, $\underline{i}_{\varphi 2}$ are the unit vectors of the reference system 2.

written as :

$$(1.3\text{-}2) \qquad \underline{M} = \frac{1}{2} \operatorname{Re} \varepsilon_o \int_{\Sigma} (\underline{E}^{*} \cdot \underline{n}) \underline{E} \wedge \underline{r} \, d\Sigma$$

and the component of $\underline{M}$ along the x axis is:

$$(1.3\text{-}3) \qquad M_x = \frac{1}{2} \operatorname{Re} \varepsilon_o \int_{\Sigma} (\underline{E}^{*} \cdot \underline{n}) \ \underline{E} \wedge \underline{r} \cdot \underline{i} \ d\Sigma$$

If E_Θ and E_r represent the transversal and radial components of $\underline{E}$, and s denotes the common length of the dipoles, one has, apart from terms of the order $1/r^3$,

$$\underline{E}_1 = \underline{E}_{1r} \, \underline{n} + E_{1\Theta} \, \underline{i}_{\Theta 1}$$

$$(1.3\text{-}4) \qquad E_{1r} = \frac{Z}{2\pi} \frac{Is}{r^2} \cos\Theta_1 \ e^{i(\omega t - kr)}$$

$$E_{1\Theta} = i \frac{Z}{2} \frac{Is}{\lambda r} \operatorname{sen}\Theta_1 \ e^{i(\omega t - kr)}$$

$$\underline{E}_2 = E_{2r}\,\underline{n} + E_{2\theta}\,\underline{i}_{\theta 2}$$

$$(1.3\text{-}5) \qquad E_{2r} = \frac{iZ}{2\pi}\frac{Is}{r^2}\cos\theta_2\, e^{i(\omega t-kr)}$$

$$E_{2\theta} = -\frac{Z}{2}\frac{Is}{\lambda r}\operatorname{sen}\theta_2\, e^{i(\omega t-kr)}$$

Putting $\underline{E} = \underline{E}_1 + \underline{E}_2$, one has:

$$(1.3\text{-}6) \qquad \underline{E}^{*}\cdot\underline{n} = (\underline{E}_1^{*} + \underline{E}_2^{*})\cdot\underline{n} = E_{1r}^{*} + E_{2r}^{*}$$

besides :

$$(1.3\text{-}7) \qquad \underline{E}\wedge\underline{n} = (\underline{E}_1 + \underline{E}_2)\wedge\underline{n} = -E_{1\theta}\,\underline{i}_{\varphi 1} - E_{2\theta}\,\underline{i}_{\varphi 2} \quad ,$$

$$\underline{r} = r\,\underline{n}$$

Thus (1.3-3) becomes:

$$(1.3\text{-}8) \qquad M_x = -\frac{\varepsilon_o}{2}\operatorname{Re}\int_{\Sigma}(E_{1r}^{*} + E_{2r}^{*})(E_{1\theta}\underline{i}_{\varphi 1} + E_{2\theta}\underline{i}_{\varphi 2})\cdot\underline{i}\; r\; d\Sigma$$

From Fig.1-2 one has:

$$\underline{i}_{\varphi 1} \cdot \underline{i} = -\sin\varphi_1$$

(1.3-9)

$$\underline{i}_{\varphi 2} \cdot \underline{i} = -\sin\varphi_2$$

Hence, with the substitution $d\Sigma = r^2 d\Omega$, (1.3-8) becomes:

$$(1.3\text{-}10)\quad M_x = \frac{\varepsilon_o r^3}{2} \operatorname{Re} \int_\Omega (E^*_{1r} + E^*_{2r})(E_{1\theta}\sin\varphi_1 + E_{2\theta}\sin\varphi_2)\, d\Omega$$

where the integration is over the whole solid angle 4π.
By means of Eq.(1.3-4) and (1.3-5) one has:

$$(1.3\text{-}11)\quad M_x = \frac{\varepsilon_o Z^2}{8\pi\lambda} I^2 s^2 \int_\Omega (-\cos\theta_1 \sin\theta_2 \sin\varphi_2 + \sin\theta_1 \cos\theta_2 \sin\varphi_1)\, d\Omega$$

From Fig.1-2 one can find

$$\cos\theta_2 = \sin\varphi_1 \sin\theta_1$$

(1.3-12)

$$\cos\theta_1 = -\sin\varphi_2 \sin\theta_2$$

Hence (1.3-11) becomes :

$$(1.3\text{-}13) \qquad M_x = \frac{\varepsilon_o Z^2}{4\pi\lambda} I^2 s^2 \int_0^{2\pi} \sin^2\varphi \, d\varphi \int_0^{\pi} \sin^3\Theta \, d\Theta =$$

$$= \frac{4}{3} \frac{\varepsilon_o Z^2}{4\lambda} I^2 s^2 = \frac{1}{3} \mu_o \frac{I^2 s^2}{\lambda}$$

It is seen that the components M_y and M_z of $\underline{M}$ are equal to zero, since their expressions, corresponding to the (1.3-13), are of the form:

$$\int_0^{2\pi} \sin\varphi \cos\varphi \, d\varphi \int_0^{\pi} \sin^3\Theta \, d\Theta$$

The power W radiated by the two dipoles is given by:

$$(1.3\text{-}14) \qquad W = W_1 + W_2 = \frac{1}{3} \mu_o \frac{I^2 s^2}{\lambda} \omega$$

By comparison with (1.3-13) one obtains the relation between the radiated power and the torque applied by reaction to the antenna:

$$(1.3\text{-}15) \qquad M = \frac{W}{\omega}$$

The torque $\underline{M}$ is directed along the normal to the plane of the dipoles.

The antenna tends to rotate in a direction opposite to that of the positive (or negative) charges at the end of the dipoles.

It is interesting to note that result (1.3-15) is exactly the same as one would get in the case of a plane wave with circular polarization , although in our case the polarization is perfectly circular only in the x direction . The apparent paradox can be explained by considering that in our case a non vanishing contribution to angular momentum comes from the radial component of $\underline{E}$, which combined with the tangential component of $\underline{H}$, gives rise to a tangential momentum .

1.4 - Torque acting on a turnstile antenna.

The calculation of the preceding section will now be extended to the case of a turnstile antenna placed in the vacuum and far from other material bodies . The antenna is constituted by two half-wave dipoles fed with currents of equal amplitudes and in quadrature . The arms of the antenna are assumed to be infinitely thin, and the currents to have a sinusoidal distribution . With reference to Fig.1-3 we will write:

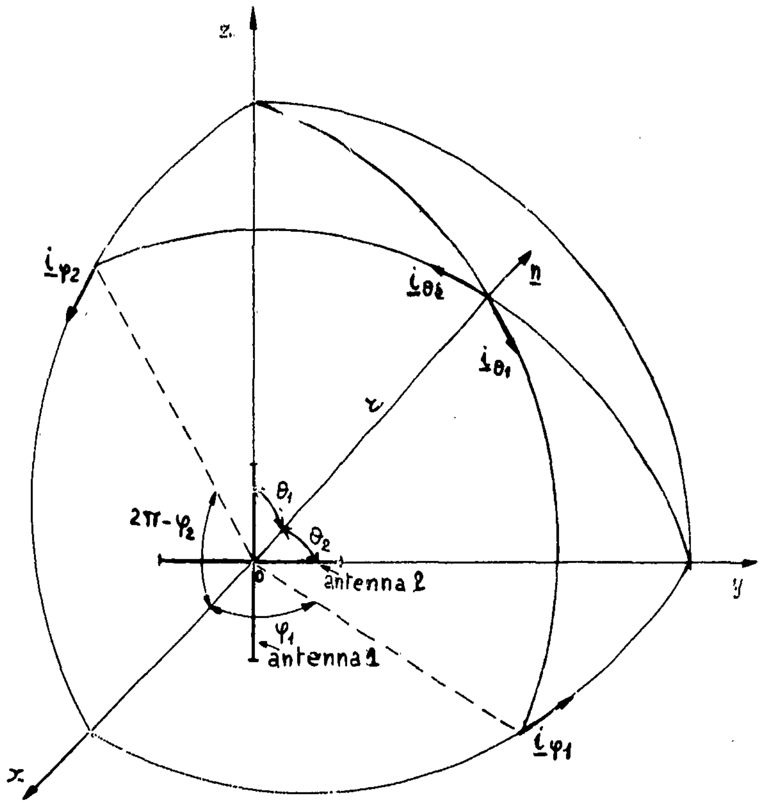

Fig.1-3

Reference systems x, y, z; r, θ_1, φ_1; r, θ_2, φ_2 for the turnstile antenna, formed by the two half-wave antennas 1 and 2.

$$I_1 = I\cos(kz)e^{i\omega t}$$

(1.4-1)

$$I_2 = i\,I\cos(ky)e^{i\omega t} \quad , \qquad k = \frac{2\pi}{\lambda}$$

with a notation analoguos to that of the preceding sections. Eq. (1.2-7) will be used for the evaluation of the torque acting on the antenna.

The expressions of the electric fields are (')

$$\underline{E}_1 = E_{1r}\underline{n} + E_{1\theta}\underline{i}_{\theta 1}$$

(1.4-2)

$$E_{1r} = \frac{ZI\lambda}{8\pi r^2}\sin\left(\frac{\pi}{2}\cos\theta_1\right)e^{i(\omega t-kr)}$$

$$E_{1\theta} = \frac{iZI}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1}\,e^{i(\omega t-kr)}$$

$$E_2 = E_{2r}\underline{n} + E_{2\theta}\underline{i}_{\theta 2}$$

(1.4-3)

$$E_{2r} = \frac{iZI\lambda}{8\pi r^2}\sin\left(\frac{\pi}{2}\cos\theta_2\right)e^{i(\omega t-kr)}$$

$$E_{2\theta} = \frac{-ZI}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_2\right)}{\sin\theta_2}\,e^{i(\omega t-kr)}$$

(') See Ref.8, p.528

With:

$$\underline{E} = \underline{E}_1 + \underline{E}_2$$

$$\underline{E}^{*}\cdot\underline{n} = E^{*}_{1r} + E^{*}_{2r}$$

$$\underline{E}\wedge\underline{n} = -\underline{i}_{\varphi 1}E_{1\Theta} - \underline{i}_{\varphi 2}E_{2\Theta}$$

Eq. (1.2-7) becomes:

$$(1.4\text{-}4) \qquad \underline{M} = -\frac{\varepsilon_o}{2}\,\mathrm{Re}\int_{\Sigma}(E^{*}_{1r} + E^{*}_{2r})(E_{1\Theta}\underline{i}_{\varphi 1} + E_{2\Theta}\underline{i}_{\varphi 2})r\,d\Sigma$$

because the magnetic field vectors are tangential.
Substitution of (1.4-2) and (1.4-3) into (1.4-4) gives:

$$(1.4\text{-}5) \qquad \underline{M} = \frac{\varepsilon_o Z^2 I^2 \lambda}{32\pi^2}\,\mathrm{Re}\int_{\Sigma}\frac{1}{r^2}\left[\sin\left(\frac{\pi}{2}\cos\Theta_1\right) - i\,\sin\left(\frac{\pi}{2}\cos\Theta_2\right)\right]\times$$

$$\times\left[\frac{i\,\cos\left(\frac{\pi}{2}\cos\Theta_1\right)}{\sin\Theta_1}\,\underline{i}_{\varphi 1} - \frac{\cos\left(\frac{\pi}{2}\cos\Theta_2\right)}{\sin\Theta_2}\,\underline{i}_{\varphi 2}\right]d\Sigma$$

or also:

$$(1.4\text{-}6) \qquad \underline{M} = -\frac{\varepsilon_o}{32}\frac{Z^2 I^2}{\pi^2}\lambda \times$$

$$\times \int_{\Sigma} \frac{1}{r^2}\left[\underline{i}_{\varphi 1}\frac{\sin\left(\frac{\pi}{2}\cos\theta_2\right)\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1} - \underline{i}_{\varphi 2}\frac{\sin\left(\frac{\pi}{2}\cos\theta_1\right)\cos\left(\frac{\pi}{2}\cos\theta_2\right)}{\sin\theta_2}\right] d\Sigma$$

By taking into account (1.3-9), with the substitution $d\Sigma = r^2 d\Omega$ one obtains:

$$(1.4\text{-}7) \qquad M_x = \underline{M}\cdot\underline{i} = \frac{\varepsilon_o Z^2}{32\pi^2} I^2 \lambda \times$$

$$\times \int_{\Omega}\left[\frac{\sin\varphi_1 \sin\left(\frac{\pi}{2}\cos\theta_2\right)\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1} - \frac{\sin\varphi_2 \sin\left(\frac{\pi}{2}\cos\theta_1\right)\cos\left(\frac{\pi}{2}\cos\theta_2\right)}{\sin\theta_2}\right] d\Omega$$

where the integration is over the whole solid angle $\Omega = 4\pi$. By introducing (1.3-12), Eq.(1.4-7) becomes:

$$(1.4\text{-}8) \qquad M_x = \frac{\varepsilon_o Z^2}{16\pi^2} I^2 \lambda \int_{\Omega} \frac{\sin\varphi \sin\left(\frac{\pi}{2}\sin\varphi \sin\theta\right)\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\Omega$$

which gives:

$$(1.4\text{-}9) \qquad M_x = \frac{\varepsilon_o Z^2}{16\pi^2} I^2 \lambda \; 2\pi \int_0^{\pi} J_1\left(\frac{\pi}{2}\sin\Theta\right) \cos\left(\frac{\pi}{2}\cos\Theta\right) d\Theta$$

By using the following formula, whose demonstration will be given in Appendix 3 ,

$$(1.4\text{-}10) \qquad \int_0^{\pi} J_1(\alpha \sin\Theta) \cos(\beta \cos\Theta) \, d\Theta = - \frac{2}{\alpha} \cos\left(\sqrt{\alpha^2 + \beta^2}\right)$$

from (1.4-9) one has $(\alpha = \beta = \pi/2)$:

$$\int_0^{\pi} J_1\left(\frac{\pi}{2} \sin\Theta\right)\cos\left(\frac{\pi}{2} \cos\Theta\right) d\Theta = - \frac{4}{\pi} \cos\left(\frac{\pi}{2}\sqrt{2}\right) = - \frac{4}{\pi} \, 0.605699$$

Hence eq.(1.4-9) becomes:

$$(1.4\text{-}11) \qquad M_x = \frac{\varepsilon_o Z^2}{2\pi^2} I^2 \lambda \; 0.605699 = \frac{1}{kc} \frac{Z}{\pi} I^2 \; 0.605699$$

By introducing the radiation resistance of a half-wave component, given by [1]:

$$R = \frac{Z}{4\pi} 2.43765 \qquad (1.4\text{-}12)$$

and the power radiated by the turnstile antenna, one obtains, recalling that $W = I^2R$,

$$M_x = 0.9939 \frac{W}{\omega} \qquad (1.4\text{-}13)$$

The calculation of the torque applied to the antenna has been performed also in another way, namely, by taking into account the mutual actions between the currents and the charges on the four arms of the antenna. This second way is longer and more complicated than the one described above, and the calculations are shown in Appendix 4. Here only the results are given, which were obtained by the use of some approximations.

The value for M_x is

$$M_x = 0.98 \frac{W}{\omega}$$

---

(1) see for instance Ref. 8 p.148 Eq.(16)

Though the result is not exact , this procedure may be more convincing since it is a direct physical approach .

It is seen at once, by considering the mutual forces between charges and currents, that the x and y components of **M** are equal to zero .

1.5 - Application to the attitude control of space vehicles.

Eq. (1.4-13) of the preceding section shows that the reaction torque applied to a turnstile antenna, assumed to be isolated in the vacuum, is proportional to the radiated power and inversely proportional to the frequency . If a turnstile antenna is placed on a vehicle, as shown in Fig.1-4, the whole vehicle is submitted to a torque , when the antenna radiates.

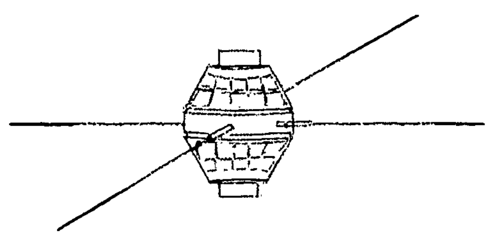

Fig.1-4

Let us see how this source of torque lends itself to be applied to the control of the attitude of the vehicle. First the value of the torque which can be obtained , will be considered .

Eq.(1.4-13) was derived with the assumption that the far field of the turnstile antenna was undirsturbed by the presence of other bodies. If the size of the vehicle is so small, in comparison with the antenna, that the far field of the antenna is not greatly modified, the value of the torque can be assumed to be given by (1.4-13).

In order to obtain numerical values, one must know the level of the electric power available on a space vehicle .

Up to now, solar cells or chemical batteries have been used as power sources on space vehicles. In one case a nuclear thermo-electric generator has been employed.

From the data reported in the literature , it seems that the case in which the highest level of power was used, is that of the Courier (1960 ν) 1 B : 225 watts were available for the transmission to ground stations. The power was supplied by storage batteries , charged by a system of 19152 solar cells, capable of a continuous power of 60 watts.

For the Nimbus, the meteorological satellite which was scheduled for the end of 1962, the power available from batteries, charged by solar cells, was planned to be 250 watts.

In the other cases reported in the literature, at most some tens of watts where used.

The forecast is that , in the future , the electric power necessary for space missions will be much higher. A communication satellite for broadcast service would need approximately 1500 watts of power . 50 kW of power have been indicated for a television transmitter ([10]) .

Snyder ([11]) indicates that tens of kW will be necessary for Space Stations ,hundreds of kW for manned satellites of long life, and up to 20 MW in some cases of big manned and maneuverable vehicles.

Therefore several methods of producing electric power are under study, in addition to those which have been already applied.

Solar cells could give power up to some kW, though they are not considered convenient for power higher than some hundreds of watts.

Solar thermoelectric and thermoionic system are being studied, where a mirror concentrates solar energy on an absorber, in order to heat a junction of a thermocouple , or the cathode of a thermoionic valve.

With these systems, it seem possible to achieve power levels up to 25 kW.

Solar energy can be also employed to feed a turbo-engine by means of mirrors concentrating the solar heat. This system is indicated for power of some tens of kW. As an example of the weight of these systems, one may mention a project,under study at the Sunstrand Denver Corporation ([12]) , for a solar turboelectric engine of 15 kW , having a weight of 820 lb . It is forseen that such a weight/power ratio of 55 lb/kW, can be lowered to a value of 20 lb/kW, by improvement of the technique .

Nuclear energy is considered suitable for producing electrical power on a space vehicle. The satellite Transit IV A used a source of power, consisting of a thermocouple, whose hot junction was placed in a small reactor containing Pt 238. This device delivered 2.7 watts and its weight was 2.1 Kg.

Several systems based on nuclear energy are under study. The availability of the three units: SNAP 10, SNAP 2, SNAP 8 is expected for the years 1963-65. The three systems, will provide 300 watts, 3000 watts,35000 watts, respectively, with weights of the units of 300, 600 and 1500 lb ([13]) .

The availability of the system SNAP 50, delivering 300 kW, is forseen for the year 1970 ([14]) .

From Eq.(1.4-13) it is seen that the torque, which can be obtained by placing a turnstile antenna on a vehicle, is proportional to the wavelength λ . If a good electrical efficiency is desired , an upper limit for λ is set by the dimensions of the antennas which can be placed on the vehicle. The longest antenna placed on a space vehicle, up to now, is the 150 ft. radio antenna of the Canadian satellite Alouette ([15]) . Generally the dimensions of the satellite antennas were much smaller , of the order of one meter.

Table I , calculated by means of (1.4-13) , gives the torques which can be obtained, with power up to some kW, and antennas capable of radiating with wavelenghts up to 100 m.

TABLE I - Values of the electromagnetic torques M

$M = 5.3\ W \lambda$, where M is given in ergs, W in kW, λ in meters.

W (kW)	M (ergs)			
	λ = 5 m	λ = 10 m	λ = 20m	λ = 100 m
0.1	2.65	5.3	10.6	53
1	26.5	53	106	530
5	132.5	265	530	2650

The torques requested for the attitude control of the vehicle vary largely according to the type of vehicle, the mission to be performed and the particular period of its activity .

First the principal causes , which can disturb the attitude of a stabilized vehicle , will be considered .

The principal disturbing actions to be taken into account are the following :

Gradient of the gravity

Inertial reaction forces

Earth's magnetic field

Solar radiation pressure

Meteorites impact

Atmospheric drag

Motion of internal parts

These actions will be discussed one by one, in the following sections.

1.6 - Gravitational gradient and inertial reaction torques

a) Gravitational gradient.

A torque is exerted on a vehicle of non spherical mass-symmetry in the proximity of bodies of great mass. This action occurs when the axis of minimum moment of inertia of the vehicle is not directed towards the center of gravity of the attracting body. In the case of an earth's satellite, this gives rise to torques G_p and G_r about the pitch and roll axes (') and their values are given by (16)

$$G_p = \frac{3}{2}\,\omega_o^2\,\Delta K_p\,\sin 2\Theta_p$$

(1.6-1)

$$G_r = \frac{3}{2}\,\omega_o^2\,\Delta K_r\,\sin 2\Theta_r$$

where ΔK_p, ΔK_r are the differences between the moments of

(') For earth centered references, generally pitch axis indicates the axis passing through the center of mass of the satellite and normal to the orbital plane . The yaw axis in the axis directed along the vertical direction.The roll axis is the axis normal to the preceding two.

inertia about the pitch or roll axes and the moment of inertia about the vertical direction. θ_p and θ_r are the attitude deviation angles from the stable direction, in pitch or roll. ω_o is the mean angular orbital velocity.

In the case $\theta_p = \theta_r = 45^o$ and $\Delta K_p = \Delta K_r = \Delta K$, the expression $G/\Delta K = G_p/\Delta K_p = G_r/\Delta K_r$ has been evaluated, as a function of the altitude. The results are reported in Table II.

TABLE II - Gravitational torque $G/\Delta K$

Altitude (km)	$G/\Delta K$ ($J/kg\ m^2$)
200	$2.1\ 10^{-6}$
400	$1.9\ 10^{-6}$
600	$1.8\ 10^{-6}$
1000	$1.5\ 10^{-6}$
2000	$1\ 10^{-6}$
3000	$0.64\ 10^{-6}$
4000	$0.53\ 10^{-6}$
5000	$0.36\ 10^{-6}$
10000	$1.3\ 10^{-7}$
15000	$0.6\ 10^{-7}$

The effect of the gradient of the gravity on an elongated vehicle can be employed as a means for stabilizing the attitude (example: the Traac satellite) . In general it represents a cause of disturbance for a stabilized satellite, and must be neutralized . From Table II, it is seen that the torque to be balanced can be of the same order of magnitude as the torques obtained by means of emission of circularly polarized radiation.

b) Inertial reaction torques.

In the studies on the attitude control of space vehicles, the consideration of the particular reference system is of primary importance ([1]) . For example an inertial system is suitable for describing the attitude of space astrophysical observatories, while a geocentric system is generally used for communications or meteorological satellites.

Let us suppose that a space vehicle must keep its orientation fixed with respect to a non-inertial reference frame . Then the equations, describing the motion of the vehicle in this reference system , contain some terms, which represent torques applied to the vehicle , due to inertial reactions.

As an example of these inertial reaction torques, let us consider a communications satellite , which must keep an axis (e.g. the maximum directivity axis of an antenna) directed towards the earth . In this case it is useful to consider a rotating reference frame , with angular velocity equal to the angular orbital velocity of the satellite. If the orbit is elliptic , with eccentricity e, the inertial torque T, acting on the satellite is approximately given by,[1]

$$T = 2\,K_p e\,\omega_o^2 \sin\omega_o t = T_o \sin\omega_o t \tag{1.6-2}$$

where K_p is the moment of inertia of the satellite , with respect to the pitch axis , ω_o is the mean orbital angular velocity.

The direction of the torque is normal to the plane of the orbit.

We will evaluate T in some practical cases.

1.7 - Some examples: the synchronous satellite, satellites orbiting at other altitudes

Let us consider a synchronous communications satellite , orbiting with a period of 24 hours at a height of ~ 36000 km. From this height the earth is seen under an angle of $\sim 17^o$. It is therefore advisable to use a good directivity antenna , directed towards the earth.

The torque T is equal to zero , when the orbit is circular . In this case the desired attitude is maintained, if the satellite has been placed in orbit with a spin velocity $= \omega_o$ about the pitch axis .

However , when the orbit is elliptic, a torque must be applied to the satellite to counterbalance the inertial reaction torque .

Let us consider a parallelepiped satellite , with the geometrical situation described in Fig.1-5. It is convenient that the spin axis is the axis of maximum moment of inertia and that the axis of minimum moment of inertia is directed towards the earth, so that the attitude is inherently stable .

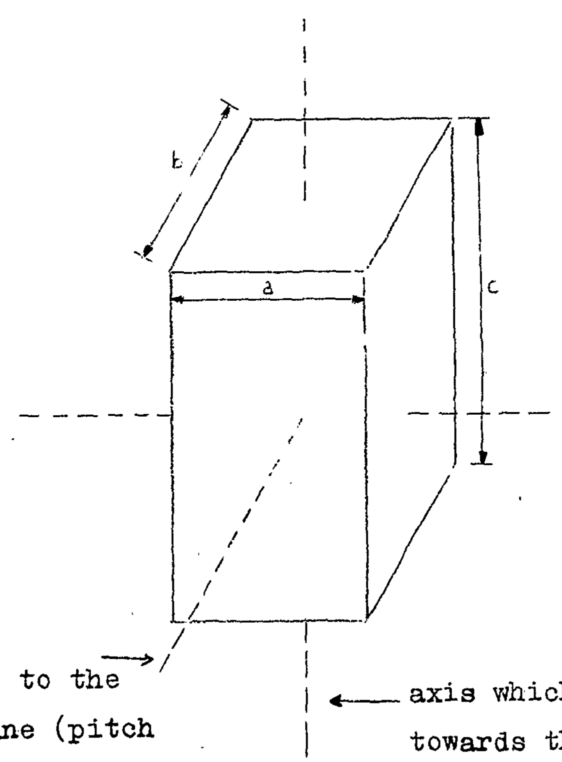

Fig.1-5

Table III gives the maximum value T of the inertial torque, calculated by means of (1.6-2). Several values of the eccentricity, of the size and weight of the satellite are considered.

TABLE III

Maximum torques on the satellite of Fig.5, due to inertial reaction force. The sides a, b, c are given in m, the weight in kg, the torque in J. The eccentricity is denoted by e.

a (m)	b (m)	c (m)	weight (kg)	K_p ($kg\ m^2$)	Torque T_o (J) $e=10^{-3}$	$e=10^{-2}$	$e=10^{-1}$	$e=2.10^{-1}$
1	1	2	500	208	$2.21\ 10^{-9}$	$2.21\ 10^{-8}$	$2.21\ 10^{-7}$	$4.42\ 10^{-7}$
1	1	2	1000	417	$4.42\ 10^{-9}$	$4.42\ 10^{-8}$	$4.42\ 10^{-7}$	$8.84\ 10^{-7}$
1	1	2	2000	834	$8.84\ 10^{-9}$	$8.84\ 10^{-8}$	$8.84\ 10^{-7}$	$1.76\ 10^{-6}$
1	1	3	1000	834	$8.84\ 10^{-9}$	$8.84\ 10^{-8}$	$8.84\ 10^{-7}$	$1.76\ 10^{-6}$
1	1	3	2000	1665	$1.76\ 10^{-8}$	$1.76\ 10^{-7}$	$1.76\ 10^{-6}$	$3.52\ 10^{-6}$
2	2	4	5000	8330	$8.85\ 10^{-8}$	$8.85\ 10^{-7}$	$8.85\ 10^{-6}$	$1.77\ 10^{-5}$
2	2	4	6500	10800	$1.15\ 10^{-7}$	$1.15\ 10^{-6}$	$1.15\ 10^{-5}$	$2.3\ 10^{-5}$
2	2	4	8000	13320	$1.41\ 10^{-7}$	$1.41\ 10^{-6}$	$1.41\ 10^{-5}$	$2.8\ 10^{-5}$
2	2	4	10000	16700	$1.77\ 10^{-7}$	$1.77\ 10^{-6}$	$1.77\ 10^{-5}$	$3.54\ 10^{-5}$

If the axis of minimum moment of inertia makes an angle Θ with the direction of the gravity , the action of the gradient of the gravity results in a torque , applied to the satellite, which tends to lower Θ . Thus if the inertial torque tends to make the satellite deviate from its attitude, a restoring torque G arises from the action of the gradient of the gravity . Precisely G is given by :

$$G = - \frac{3}{2} \omega_o^2 (K_p - K_y) \sin 2\Theta \tag{1.7-1}$$

where K_p and K_y are the moments of inertia with respect to the roll and yaw axes respectively (Fig.1-5).

If other disturbances can be neglected , and there is no attitude control, the equation for Θ can be written as :

$$K_p \ddot{\Theta} = 2 K_p e \omega_o^2 \sin \omega_o t - \frac{3}{2} \omega_o^2 (K_p - K_y) \sin 2\Theta \tag{1.7-2}$$

where dots indicate derivations with respect to time.

The amplitudes of the forced osclillations, calculated with the approximation $\sin 2\Theta = 2\Theta$ can be read in Table IV, for the same satellites considered in Table III.

TABLE IV - Amplitude of the oscillations of the satellite of Fig.1-5 (see Eq.(1.7-2) and Table III .

a (m)	b (m)	c (m)	weight (kg)	amplitude $e = 10^{-2}$	$e = 10^{-1}$
1	1	2	500	1° 25'	14° 20'
1	1	2	1000	1° 25'	14° 20'
1	1	2	2000	1° 25'	14° 20'
1	1	3	1000	45'	8° 35'
1	1	3	2000	45'	8° 35'
2	2	4	5000	1° 25'	14° 20'
2	2	4	6500	1° 25'	14° 20'
2	2	4	8000	1° 25'	14° 20'
2	2	4	10000	1° 25'	14° 20'

Table IV shows that, when the eccentricity of the orbit is not very small, it is necessary to provide the satellite with means for counteracting the natural oscillations.

R.P. Haviland ([10]) states that a synchronous broadcast satellite , weighing 500 lb , with a power demand of 1500 watts, ''is within the reach of present technique'' and that ''a single 50 kW transmitter (weight = 10000 lb) can be installed in a satellite in about 7 years'' .

By comparison of Table I and Table II it is seen that the electromagnetic torques are sufficient to counterbalance the natural torques on a satellite of this kind.

The damping action would be accomplished practically, by placing a turnstile antenna on the synchronous satellite, with the axis of symmetry of the antenna normal to the plane of the orbit (i.e. horizontal), and by making it radiate power at a ratio depending sinusoidally on time. In order to obtain, with a reasonable expense of power, torques sufficient to overcome the disturbing torques indicated by (1.6-2) and table III, a long-arms antenna should be used. This justifies, in this case at least, the assumption, made in the compilation of table I, that the satellite body does not much disturb the field radiated by the antenna.

The device described above would allow control about a single axis. The equatorial position of the satellite would suggest the possibility of obtaining control about the other two axes, by simply placing a big permanent magnet ([18]) on the satellite. The magnet, oriented North-South (magnetic), might overcome the small disturbing torques about the other axes.

So far the inertial reaction torques that a satellite must overcome, when it has to keep its attitude fixed with respect to a geocentric reference system, have been evaluated for the particular case of the synchronous satellite.

Table V gives the torques about the pitch axis, acting on satellites orbiting at other altitudes. The moment of inertia K_p about the pitch axis is assumed to be 100 kg m^2. The table can be easily extended to other values of K_p since the torque is proportional to K_p.

TABLE V - Value of $T_o = 2Ke\omega_o^2$, for different altitudes h and eccentricities e . T_o is given in Joules , h in km.

h (km)	T_o (J) $e = 10^{-2}$	$e = 10^{-1}$	$e = 2 \; 10^{-1}$
500	$2.4 \; 10^{-6}$	$2.4 \; 10^{-5}$	$4.8 \; 10^{-5}$
1000	$2 \; 10^{-6}$	$2 \; 10^{-5}$	$4 \; 10^{-5}$
2000	$1.4 \; 10^{-6}$	$1.4 \; 10^{-5}$	$2.8 \; 10^{-5}$
5000	$5.4 \; 10^{-7}$	$5.4 \; 10^{-6}$	$10.8 \; 10^{-6}$
10000	$1.8 \; 10^{-7}$	$1.8 \; 10^{-6}$	$3.6 \; 10^{-6}$

When the orbit has a certain inclination with respect to the equator , torques about the roll and the yaw axes can also arise in a geocentric reference system , due to the regression of the orbit . A calculation ([1]) shows that, even for satellites of large moments of inertia, these torques have values of a few ergs, at an altitude h = 1000 km, and fall off as $(R + h)^3$, (R is the earth's radius).

1.8 - Other [redacted] forces acting on the attitude of a space vehicle

a) Earth's magnetic field.

The earth's magnetic field can influence the attitude of a space vehicle by different actions. It can interact with currents induced in the moving body , with currents in not well shielded circuits , with induced magnetic moments and with permanent magnetic parts of the vehicle.

As it generally occurs for all of the external actions, the values of the torques depend largely on the particular situation considered .

One can have an idea of the order of magnitude of the magnetic actions by considering some practical cases.

The attitude of the satellite Tiros I was stabilized by means of a rotation about the axis of maximum moment of inertia . It was noticed that the direction of this axis changed with time. This motion was explained ([19]) by taking into account the interaction of the magnetic parts of the satellite with the earth's magnetic field . The calculated magnetic moment, directed along the spin axis , was 896 ergs/G . At the altitudes of Tiros I the magnetic field is about 0.4 G , thus the resulting values of the torque turn out to be of the order of hundreds of ergs .

For the Explorer IV , the calculations of G. Colombo , based on the average diurnal variation of the angular momentum of the satellite , assume a maximum magnetic torque of 300 ergs ([20]) . He also reports that the amount of magnetic torque predicted for the Orbital Astronomical Observatory , is many hundred ergs.

These values of the magnetic torques seem high , when compared with the electromagnetic torques of Table I . It must be observed however , that the magnitude of the earth's magnetic field falls off with the third power of the distance from the center of the earth . Thus, satellites at altitudes higher than those of the Explorer IV (minimum altitude less than 300 km.), of the Tiros I (altitude~700 km) and of the O.A.O. (altitude~800 km) , can be subjected to magnetic torques weaker than those reported above . Besides, the interplanetary magnetic field is very weak (') , and the magnetic torques acting on vehicles travelling far from the earth and other planets would also be very small.

An indirect action of the magnetic field on the attitude of a satellite is also to be considered : a conducting body rotating in a magnetic field is slowed down by the interaction of the magnetic field with the induced currents.

For this reason, spin stabilized satellites lose

(') Due to emission of plasma clouds or of coronal gas from the sun, the solar magnetic field is transported into the interplanetary space,giving rise to magnetic fields of 10^{-3}-10^{-5} G in regions near the sun. Beyond the reach of the solar corpuscolar stream a galactic field of the order of 10^{-5}G is expected to prevail through outer space ([21])

rotation speed and consequently stability of the spin axis.

A study of this braking action on vehicles of simple geometrical shape has been made by Wilson [22].

The slowing torque depends on the electrical properties of the material constituting the satellite and on its shape. For spherical or cylindrical satellites it is proportional to the spin velocity. Such an effect was noted for the Vanguards I and II and for the Tiros I.

The spin velocity ω_s of the Vanguard I (satellite of spherical shape, diameter = 16 cm), was found to vary with time, according to the law (') :

$$\omega_s = 2.72 \exp\left[-5.03\ 10^{-8}(t-t_o)\right] \ r/s$$

where t is expressed in seconds, and t_o denotes the time at which the satellite was placed in orbit.

The moment of inertia I of the satellite was $6.9\ 10^{-3}$ kg m^2.

Hence the slowing torque C results to be

$$C = 3.48\ 10^{-10} \omega_s \ J/s$$

(The proportionality to ω_s indicates the action of the earth's magnetic field).

(') The data for the Vanguard satellites is taken from Wilson's paper (see Ref.22)

Thus , at the beginning of its life, the satellite was submitted to a slowing torque of $9.5\ 10^{-10}$ J.

A more important case is the Vanguard II , a spherical satellite of 9.75 kg with a diameter of 0.51 m . A well determined angular speed was important not only for the stabilization but also for the better performance of the photographic apparatus.

Its spin velocity was found to vary with the law

$$\omega_s = 0.25 \exp\left[1.62\ 10^{-7}(t-t_o)\right] \text{ r/s}$$

The moment of inertia about the spin axis was 1.977 kg m^2 . Hence :

$$C = 0.3205\ 10^{-7}\ \omega_s \text{ J/s}$$

Then the initial slowing torque acting on the satellite was:

$$C_o = 8\ 10^{-9} \text{ J}$$

When an external torque is applied to a spinning satellite , it gives rise to a precession of the spin axis . The magnitude of the precession is proportional to the torque, to the duration of the action , and inversely proportional to the angular momentum of the satellite about the spin axis ([23]). This latter quantity may be taken as an index of the stability of the satellite and in the case of the two Vanguards was reduced to one half of its initial value within 160 and 50 days respectively .

Wilson's analysis shows that the slowing torque and the moment of inertia , for spherical bodies, are proportional to the same power (the fifth) of the vehicle's radius . Thus for satellites of larger dimensions but with similar distribution of mass and materials , as the Vanguards, the time in which the spin velocity, or the inherent stability , is reduced to one half , is of the same order of magnitude (at the same altitudes) .

The slowing down of the spin velocity was taken into account in the post-launch performance of the Tiros satellites.

The Tiros I (of cylindrical shape , diameter = 1.05 m, weight 5 kg) began its working life with a spin velocity of 12 r/min . After 56 days this value was reduced to 9.4 r/min. A pair of rockets , placed around the satellite, brought up the rotation velocity to 12.85 r/min. Two other pairs of rockets were necessary during the life of the satellite.

If one assumes that also in this case ω_s varies exponentially with time

$$\omega_s = \omega_{so} \, e^{-at}$$

and takes into account the value 12.4 kg m^2 of the moment of inertia about the spin axis , one finds

$$C_o = 8 \, 10^{-7} \, J$$

for the value of the initial torque.

b) Solar radiation pressure.

The solar radiation pressure depends on the angle of incidence of the radiation and on the reflectivity of the surface of the vehicle. For complete reflection the pressure can reach a value of $9\ 10^{4}$ dyne/cm^2 at the earth's distance from the sun . For complete absorption , which is generally considered a good approximation , at this distance , the sun pressure P_s reaches a value $P_s = 4.5\ 10^{-4}$ dyne/cm^2 .

In the case of complete absorption the torque T_s which can result is the product of :

the area A presented to the sun,
the radiation pressure P_s
and the distance D between the center of pressure of the area A and the projection of the center of mass of the vehicle on A.
With the above value of P_s,

$$T_s = 4.5.10^{-5}\ AD \qquad \text{Joules}$$

where A is in m^2 , and D in m.

It is seen that for a nearly symmetrical vehicle the perturbing action of the solar pressure on the attitude can have very small values.

c) Meteorites impact.

The impact of a meteorite on a space vehicle can cause an impulsive change of the angular momentum of the vehicle . Whipple ([24]) gives the estimated distribution of the masses and the velocities of small meteorites near the earth . He also gives the calculated mean number per day of meteorites striking a 3 m diameter sphere , placed in the space near the earth . This number is derived with the assumption of 50 °/o shielding by the earth.

Let us consider a spherical space vehicle of this size. The second column of table VI gives the number of particles,of masses in the range indicated in the first column, which strike the vehicle per day . The values are derived from Whipple's data , by neglecting the shielding effect of the earth.

The third column gives the angular momentum I, absorbed by the vehicle when a complete unelastic impact and a distance d = 1 m between the trajectory of the meteorite and the center of mass of the vehicle are assumed . The relative velocity of the meteorite with respect to the vehicle has been assumed to have the value of 80 km/s (near to the estimated maximum) .

For the mass of the meteorite we used the average between the figures given in each row.

TABLE VI - Data on meteorites impact (see text)

range of masses (gr)	number per day	I=change of angular momentum (J-sec/unit of d)
$0.25 - 9.95\ 10^{-2}$	$4.3\ 10^{-5}$	14
$9.95\ 10^{-2} - 3.96\ 10^{-2}$	$9.8\ 10^{-5}$	4.8
$3.96\ 10^{-2} - 1.58\ 10^{-2}$	$2.5\ 10^{-4}$	2.2
$1.58\ 10^{-2} - 6.28\ 10^{-3}$	$6.2\ 10^{-4}$	$9.1\ 10^{-1}$
$6.28\ 10^{-3} - 2.5\ 10^{-3}$	$1.6\ 10^{-3}$	$3.5\ 10^{-1}$
$2.5\ 10^{-3} - 9.95\ 10^{-4}$	$3.9\ 10^{-3}$	$1.4\ 10^{-1}$
$9.95\ 10^{-4} - 3.96\ 10^{-4}$	$9.8\ 10^{-3}$	$4.8\ 10^{-2}$
$3.96\ 10^{-4} - 1.58\ 10^{-4}$	$2.5\ 10^{-2}$	$2.2\ 10^{-2}$
$1.58\ 10^{-4} - 6.28\ 10^{-5}$	$6.2\ 10^{-2}$	$9.1\ 10^{-3}$
$6.28\ 10^{-5} - 2.5\ 10^{-5}$	$1.6\ 10^{-2}$	$3.5\ 10^{-3}$

The table can be extended to other sizes of the vehicle, by considering that the number of particles striking a sphere is proportional to the square of the diameter.

It is seen from table VI that, especially for satellites of long life (') ,the probability of impact with meteorites of considerable mass cannot be excluded . The magnitude of the change of angular momentum can be large . Thus an attitude control device should not ignore the effect of meteorites impact , and should be able to exert large torques, especially if this perturbing action has to be balanced in a short time.

For example, let us consider a spherical vehicle, with diameter = 1 m, which , initially, has its attitude fixed with respect to an inertial reference frame.

Let us assume that a meteorite , having mass = 7.10^{-4} grams and travelling with a velocity = 80 km/s relative to the vehicle , impinges onto the vehicle . From table VI it is seen that the average number of impacts with a similar meteorite is one every 2-3 years, for a vehicle of diameter = 1 m. Let us also assume that the distance d between the center of mass of the vehicle and the trajectory of the meteorite is 0.25 m , and that the impact is unelastic . Then the vehicle , receiving an impulsive acceleration, begins to rotate about an axis normal to the direction of the meteorite , with an angular velocity $\omega = \frac{I}{K}$, where K is the moment of inertia about this axis . In this case , $I = 1.2\ 10^{-2} J\ s$; (see table VI column III) .

If a device must stop this motion before the attitude deviation reaches a certain angle α, the control torque T, assumed to be constant during the control action , must be :

(') For instance, it is assumed that an active communications satellite would be economically convenient, only if its life could be some yers long.

$$T_c = \frac{1}{2}\frac{I^2}{K\alpha}$$

For the example discussed , table VII gives the values of T_c, for several values of α and K . The table also shows the time t_s, necessary to stop the rotation of the vehicle, with the indicated control torques .

TABLE VII – Values of T_c

α	K = 100 kg m^2 T_c (J)	K = 100 kg m^2 t_s (sec)	K = 1000 kg m^2 T_c (J)	K = 1000 kg m^2 t_s (sec)
5^o	$0.8\ 10^{-5}$	$1.5\ 10^{3}$	$0.8\ 10^{-6}$	$1.5\ 10^{4}$
1^o	$4.13\ 10^{-5}$	$2.9\ 10^{2}$	$4.13\ 10^{-6}$	$2.9\ 10^{3}$
10'	$2.48\ 10^{-4}$	$1.93\ 10^{3}$	$2.48\ 10^{-5}$	$1.93\ 10^{4}$
1'	$2.48\ 10^{-3}$	$1.93\ 10^{2}$	$2.48\ 10^{-4}$	$1.93\ 10^{3}$

The table is easily extended to other values of K and T .

After the motion of the vehicle has been stopped, the control device has to readjust the attitude of the vehicle , by making it rotate of an angle α in the opposite direction. To have an idea of the time t_r necessary to obtaining the rotation with a given controlling torque T_c, let us consider the following procedure. The device exerts the torque T_c for a time t_o. Then, for an equal time, the device keeps acting with the same but opposite torque. At the end of these two phases the vehicle has accomplished a rotation α and has been stopped. The angle α is given by:

$$\alpha = \frac{1}{4} \frac{T_c}{K} t_r^2$$

where $t_r = 2t_o$. In Fig.1.6 the angle α (measured in radiants) is plotted

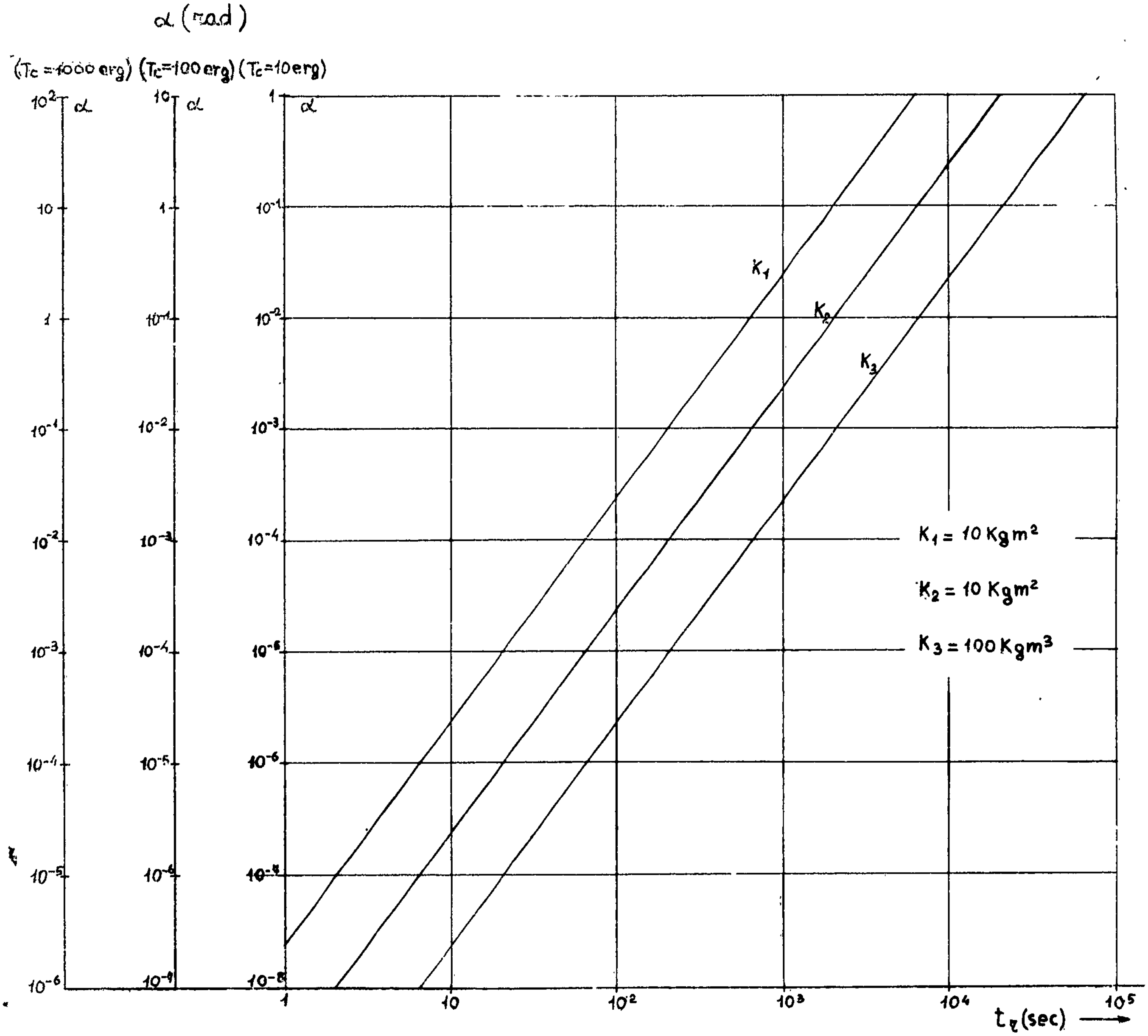

Fig.1.6

versus t_r (in seconds) for different values of K .
The three scales of the ordinate axis correspond to
$T_c = 10^6$, $T_c = 10^{-5}$, $T_c = 10^{-4}$ J.

Of course , during the time t_r, the device must exert a supplementary torque, in order to balance the actions of other sources of disturbance.

d) Atmospheric drag.

The motion of a satellite in the earth atmosphere gives rise to an aerodynamic pressure acting on the vehicle. If the centers of pressure and of mass of the vehicle are not in the same line with the direction of the orbital motion , a torque results applied to the vehicle . The aerodynamic pressure P_d on an orbiting vehicle varies very largely with the altitude . Frye ([25]) indicates the following values of P_d :

at an altitude of 130 km $P_d = 4.5 \; 10^4$ dyne/m^2
at an altitude of 560 km $P_d = 1.4 \; 10^{-3}$ dyne/m^2

It is seen that the aerodynamic pressure can play an important role with regard to the attitude of a vehicle, at low altitudes. At high altitudes however the aerodynamic torques can be neglected in comparison with other causes affecting the attitude . For example it is to be recalled that the solar radiation pressure is of the order of 4 - 8 dyne/m^2.

e) Motion of internal parts.

The motion of internal parts can influence the attitude of a space vehicle.

For example the change of the position of a part of a vehicle with respect to the main body , or the acceleration of rotating parts, can induce a torque on the rest of the vehicle through inertial reaction.

Gyroscopic torques can arise when the attitude of a vehicle, containing rotating parts , has to be changed (or must stay fixed in a non inertial reference frame) .

To have some numerical examples , let us consider the cases : 1) a vehicle, whose only moving parts are the electrical circuits switches , and 2) a vehicle containing an electric turbo-engine .

In the first case one can see that the disturbing actions induced by the the moving parts are very small. In fact, let us consider a small mass m moving along an arc s , of radius r, about the center of mass of the mainbody .

The vehicle , by reaction , will make a rotation α in the opposite direction . If K is the moment of inertia of the vehicle about the axis of rotation , α is given by:

$$\alpha = \frac{smr}{K}$$

For instance , if : $s = 1$ cm , $m = 1$ g , $r = 1$ m , $K = 10$ kg m^2 , α turns out to be about 0,2 seconds of arc.

For the second case, let us consider a vehicle carrying a turbo-electric generator , with a rotor having moment of inertia $K_R = 1$ kg m^2 , rotating at 20.000 r/min .

Let us suppose that the attitude of the vehicle must be changed with respect to an inertial reference frame: the vehicle has to rotate about an axis making an angle α with the axis of the rotor. The angular velocity of this rotation is ω_v. In this case a gyroscopic torque T_g results applied to the vehicle.

$$T_g = \omega_v H \sin\alpha$$

where H is the magnitude of the angular momentum of the rotor with respect to its axis.

If $\omega_v = 10^{-3}$ sec^{-1} (case of a body orbiting around the earth at an altitude h = 1000 km, with an axis constantly directed towards the earth), and $\alpha = 1^\circ$, T_g turns out to be equal to $\sim 6.10^{-3}$ J.

The two preceding examples shows that the motion of internal parts can give rise to torques applied to a vehicle, which vary in a very wide range, according to the particular situation.

1.9 - Conclusions on the applicability of the electromagnetic torques.

In sections 1.6, 1.7 and 1.8 the order of magnitude of the torques applied to a space vehicle, due to external forces or to the motion of internal parts, were discussed.

In general the magnitudes of the disturbing actions depend largely on the particular situation, namely on the dimensions of the vehicle, on its distribution of masses, on its shape, and on the characteristic of the missions to be accomplished. In particular, for an earth's satellite the magnitudes of the forces effecting its attitude vary much with the altitude of the orbit.

Especially when the altitude is high enough, so that the influence of the gravitational field, of the magnetic field and of the atmosphere is not of great importance (see sec.1.6 a) and 1.8 a),b)), the torques which can be obtained by means of emission of circularly polarized radiation are sufficient for balancing external forces. It seems therefore, that the electromagnetic torques can be employed for the control of the attitude of a stabilized vehicle. One should take into particular account the possibility of impact with the meteorites. As shown in section 1.8 b), the impact with a meteorite of substantial mass can be considered a very rare event. However, in some cases of long life vehicles, it may be necessary to furnish the vehicle with a control device to counteract this occurrence. Sec.1.7 b) shows that relatively high changes of angular momentum of the vehicle can ensue. The torques to be exerted by the controlling device depend on the tolerated attitude deviation α, and on the permitted length of the time t_r within which the control device must readjust the attitude. The possibility of employing electromagnetic torques in this case depends on the values of α and t_r, as well as on the power and wavelength available.

In section 1.7 the particular case of a synchronous satellite was considered, together with a possible scheme of a single axis control. For a three axis control three turnstile antennas should be placed on the satellite, each one controlling the attitude about a certain axis .

More simply, a three axis control could be achieved by means of a set of three half wave antennas, disposed as in Figure 1-7. Each of the three possible pairs of antennas, can

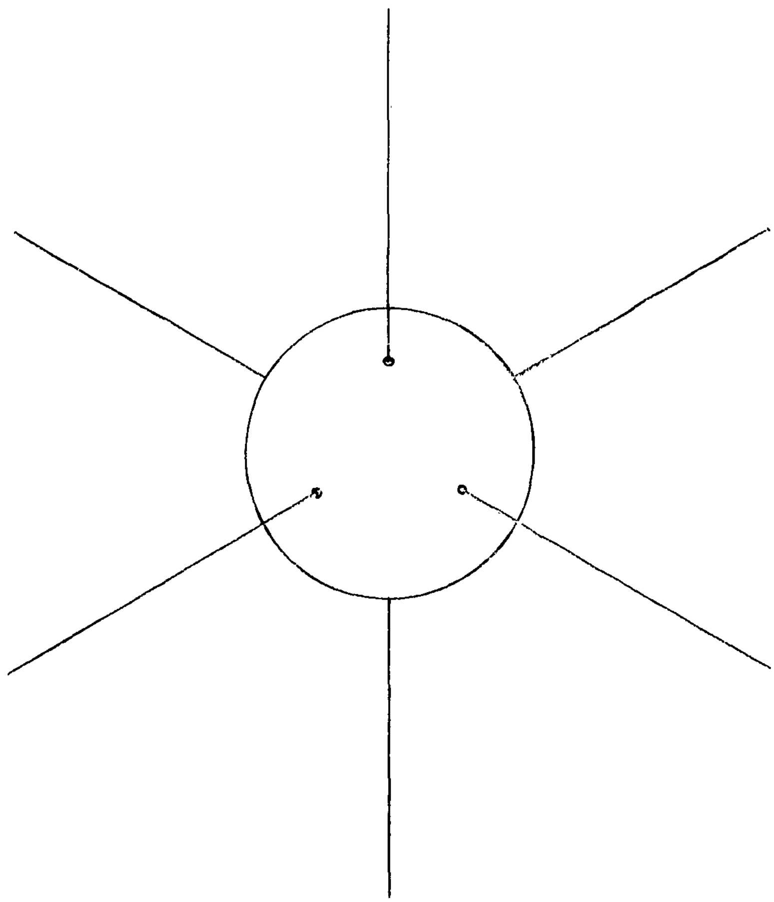

Fig.1-7 - Antennas for a three axis control.

form a turnstile antenna. Different frequencies could be used for the three turnstile antennas, in order to permit an easy separate feeding.

Other types of antennas could be considered. However it is to be expected that in general the reaction torques acting on antennas radiating with circular polarization would be proportional to the radiated power and to the wavelength. Thus it seems that helical antennas or spiral antennas, whose dimensions should be of the order of the wavelength, in order to have a good electrical efficiency, would be more massive and cumbersome than the simple turnstile antenna.

Fig.1-6 shows in what limits a device, capable of supplying weak torques, can be employed when the attitude of a space vehicle must be changed. In fact it shows, for different values of the torques available and of the moment of inertia of the vehicle about the rotation axis, the time necessary to obtain a given rotation α. Of course, during the period of the attitude correction, in addition to the torque necessary to change the attitude of the vehicle, considered free from external actions, the device has to supply an amount of torque sufficient to counteract the external actions, arising from the various causes discussed in sections 1.6 and 1.8.

1.10 - The electromagnetic torque as a disturbing action.

The different sources of disturbing torques, listed in sections 1.6 and 1.8, as well as other of minor importance, have been widely studied in the Literature. So far, the torques which can arise due to the emission of circularly polarized radiation seem not to have been considered.

The power transmitted by the antennas placed on the satellites, launched up to now, was sufficiently low, so that the torques arising in the case of circular polarization were very weak.

As an example, let us consider the case of the satellite Explorer VII, which carried a turnstile antenna radiating 600 mW at 20 Mc.

By applying Eq.(1.4-13) a torque $M = \sim 5\ 10^{-9}$ J turns out to be applied to the satellite.

Explorer VII was stabilized by means of a rotation about the axis normal to the plane of the turnstile antenna. For a comparison with the electromagnetic torque one could consider the torque T which was actually applied to the satellite about this axis. This can be derived from the slowing down of the spin velocity.

By assuming an exponential decrease of the spin velocity ω_s:

$$\omega_s = \omega_o e^{-\alpha t}$$

one has:

$$T = \frac{-\alpha\ \omega_o}{K} e^{-\alpha t}$$

Where K is the moment of inertia. An approximate value of K is obtained by assuming the satellite to be of spherical shape, with a homogeneous distribution of mass. Then, using the data derived from [26], one has:

$$K = \sim 2.4 \text{ kg m}^2$$

the initial spin velocity was 360 r/m. After 50 days it was reduced to 350 r/m. One obtains for M_o (initial torque):

$$M_o = \sim 6 \quad 10^{-7} \text{ J.}$$

This is two orders of magnitude greater that the electromagnetic torque.

In the future however high levels of power will be necessary, especially for communications satellites (as indicated in section 5).

It has been pointed out [27] that for earth-space communications: ''if we have to avoid serious Faraday fading for a wide range of signal frequencies circular polarization or special reception technique are both desiderable and necessary.''.

In sections 1.6 and 1.8 it was shown that the torques applied by rèaction to a space vehicle, when circularly polarized waves of high power are radiated, can be of the same order of magnitude as the other disturbing torques applied to the vehicle. Therefore in the prediction of the post-launch performance of a satellite with an antenna radiating circular polarization, the action of the electromagnetic torques on the attitude of the vehicle will have to be taken into account.

2 - TORQUE ON A RECEIVING ANTENNA

2.1 - Introduction

In the following sections a possible second way will be discussed of applying the properties of elliptically polarized electromagnetic waves, in order to exert a torque on a space vehicle , to control its attitude.

Let us consider the simple case of elliptically polarized radiation impinging on an antenna, placed in the free space and far from other material bodies . The angular momentum transferred from the waves to the antenna per unit time (that is the torque exerted by the radiation) will be evaluated. The torque is equal and opposite to the outgoing flux of the angular momentum of the radiation through the surface Σ of a sphere (') , centered at the antenna . The flux will be evaluated by taking into account both the incident and re-radiated waves.

To get an idea of the order of magnitude of the torque which could be obtained in this way, let as consider a turnstile antenna under the action of a plane circularly polarized wave, propagating in a direction perpendicular to the plane of the antenna . The antenna is assumed to be resonant

(') see sec.1.2

at the **wavelength** λ of the incident wave . Each component antenna has a length = $\frac{\lambda}{2}$, and its terminals are closed through a resistance R equal to the radiation resistance.

With reference to Fig.2-1, let the plane (x, y) be the plane of the turnstile antenna, O the origin of the axes, coincident with the center of the antenna. The two half-wave component antennas lie on the x and y axes respectively.

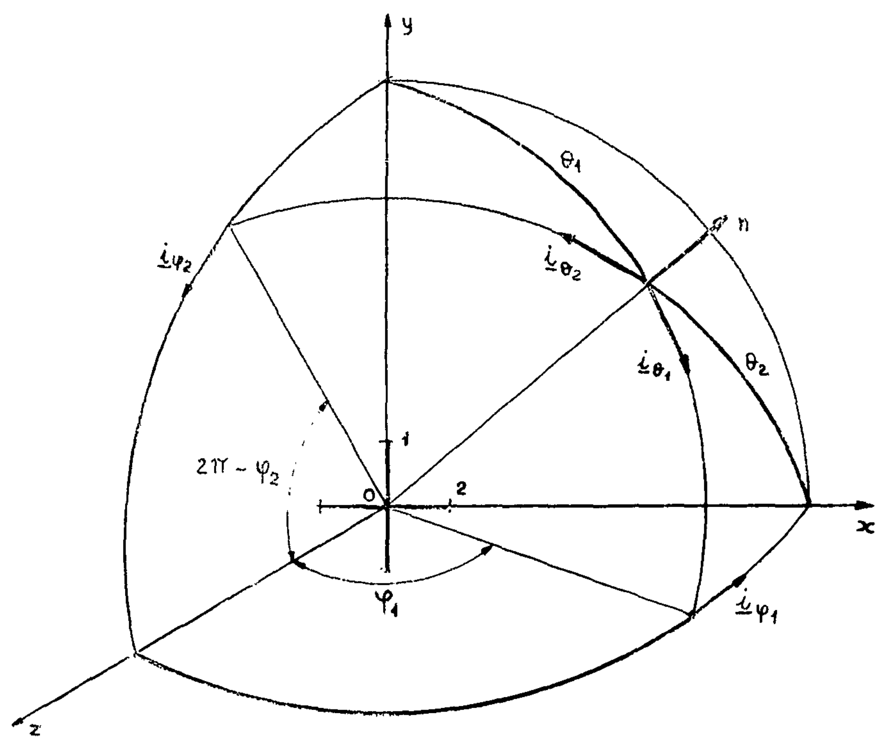

Fig.2.1

Here and in the sequel all elements of the y antenna will be labelled by the subscript 1, and all elements of the x antenna by the subscript 2.

Let the electric and magnetic fields of the incident circularly polarized plane wave be given by:

$$(2.1\text{-}1) \qquad \underline{E}_o = E_o\ (\underline{i} + i\underline{j})\ e^{i(\omega t - kz)}$$

$$(2.1\text{-}2) \qquad \underline{H}_o = \frac{1}{Z}\ \underline{k}\wedge\underline{E}_o = \frac{E_o}{Z}\ (\underline{j} - i\ \underline{i})\ e^{i(\omega t - kz)}$$

Here i denotes the imaginary unit, Z the characteristic impedance of the free space, and $k = 2\pi/\lambda$ the wave number.

The mean value M_z of the component on the z-axis of the torque acting on the antenna is given by the expression (')

$$(2.1\text{-}3) \qquad M_z = \frac{1}{2}\ \mathrm{Re} \int_{\Sigma} \left[(\underline{E}\wedge\underline{n}.\underline{k})(\underline{D}^{*}.\underline{n}) + (\underline{H}\wedge\underline{n}.\underline{k})(\underline{B}^{*}.\underline{n}) \right] r\ d\Sigma$$

where Σ represents a spherical surface with radius $\bar{r}$ centered at O. $\underline{E}$, $\underline{H}$, $\underline{D}$ and $\underline{B}$ are the field vectors; $\underline{n}$ is the outward normal to Σ .

(') see sec.1.2 Eq.(1.2-7)

2.2 - Torque on a turnstile antenna under the action of a circularly polarized plane wave.

The incident wave gives rise to currents on the two half-wave component antennas. If the branches of the antenna are assumed to be infinitely thin, the expressions of the currents are:

$$I_2 = I_o\ e^{i\omega t}\cos(kx) \tag{2.2-1}$$

$$I_1 = iI_o\ e^{i\omega t}\cos(ky) \tag{2.2-2}$$

The total electromagnetic field $\underline{E}$, $\underline{H}$ is the sum of the incident field plus the field re-radiated by the two dipoles. Thus

$$\underline{E} = \underline{E}_o + E_{1\theta}\underline{i}_{\theta 1} + E_{2\theta}\underline{i}_{\theta 2} + E_{1r}\underline{n} + E_{2r}\underline{n} \tag{2.2-3}$$

$$\underline{H} = \underline{H}_o + H_{1\varphi}\underline{i}_{\varphi 1} + H_{2\varphi}\underline{i}_{\varphi 2} \tag{2.2-4}$$

where $\underline{n}$, $\underline{i}_{\theta 1}$, $\underline{i}_{\varphi 1}$ and $\underline{n}$, $\underline{i}_{\theta 2}$, $\underline{i}_{\varphi 2}$ denote the unit vectors of two spherical coordinate systems, having the polar axes on the x and y axis respectively (see Fig.1-2),

$$(2.2\text{-}5) \qquad E_{1\theta} = -\frac{ZI_o}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1}\,e^{i(\omega t - kr)}$$

$$(2.2\text{-}6) \qquad E_{1r} = i\,\frac{ZI_o\lambda}{8\pi r^2}\,\sin\left(\frac{\pi}{2}\cos\theta_1\right)\,e^{i(\omega t - kr)}$$

$$(2.2\text{-}7) \qquad H_{1\varphi} = -\frac{I_o}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1}\,e^{i(\omega t - kr)}$$

$$(2.2\text{-}8) \qquad E_{2\theta} = i\,\frac{ZI_o}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_2\right)}{\sin\theta_2}\,e^{i(\omega t - kr)}$$

$$(2.2\text{-}9) \qquad E_{2r} = \frac{ZI_o\lambda}{8\pi r^2}\,\sin\left(\frac{\pi}{2}\cos\theta_2\right)\,e^{i(\omega t - kr)}$$

$$(2.2\text{-}10) \qquad H_{2\varphi} = i\,\frac{I_o}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta_2\right)}{\sin\theta_2}\,e^{i(\omega t - kr)}$$

By substituting Eqs. (2.2-3) and (2.2-4) into (2.1-3), one derives

$$(2.2\text{-}11)\quad M_z = \frac{\varepsilon_o r}{2}\,\mathrm{Re}\int_\Sigma \left[(\underline{E}_o + E_{1\theta}\underline{i}_{\theta 1} + E_{2\theta}\underline{i}_{\theta 2})\wedge \underline{n}\cdot\underline{k}\right]\left[\underline{E}_o^{*}\cdot\underline{n} + \underline{E}_{1r}^{*} + E_{2r}^{*}\right] d\Sigma +$$

$$+\ \frac{\mu_o r}{2}\,\mathrm{Re}\int_\Sigma \left[(\underline{H}_o + H_{1\varphi}\underline{i}_{\varphi 1} + H_{2\varphi}\underline{i}_{\varphi 2})\wedge \underline{n}\cdot\underline{k}\right](\underline{H}_o^{*}\cdot\underline{n})\, d\Sigma$$

Let us now evaluate the expression (2.2-11). From Fig.2.1 the following relations can be derived:

$$(2.2\text{-}12)\quad \underline{n} = \sin\theta_1 \cos\varphi_1\, \underline{k} + \sin\theta_1 \sin\varphi_1\, \underline{i} + \cos\theta_1\, \underline{j}$$

$$(2.2\text{-}13)\quad \underline{n} = \sin\theta_2 \cos\varphi_2\, \underline{k} - \sin\theta_2 \sin\varphi_2\, \underline{j} + \cos\theta_2\, \underline{i}$$

Hence one has, for the products appearing in Eq.(2.2-11),

$$(2.2\text{-}14)\quad \underline{i}\wedge\underline{n}\cdot\underline{k} = \cos\theta_1 = -\sin\theta_2 \sin\varphi_2$$

$$(2.2\text{-}15)\quad \underline{j}\wedge\underline{n}\cdot\underline{k} = -\sin\theta_1 \sin\varphi_1 = -\cos\theta_2$$

$$(2.2\text{-}16)\quad \underline{i}\cdot\underline{n} = \sin\theta_1 \sin\varphi_1 = \cos\theta_2$$

$$(2.2\text{-}17)\quad \underline{j}\cdot\underline{n} = \cos\theta_1 = -\sin\theta_2 \sin\varphi_2$$

$$(2.2\text{-}18) \qquad (\underline{i}_{\varphi 1} \wedge \underline{n}) \cdot \underline{k} = \underline{i}_{\Theta 1} \cdot \underline{k} = \cos\Theta_1 \cos\varphi_1$$

$$(2.2\text{-}19) \qquad (\underline{i}_{\varphi 2} \wedge \underline{n}) \cdot \underline{k} = \underline{i}_{\Theta 2} \cdot \underline{k} = \cos\Theta_2 \cos\varphi_2$$

$$(2.2\text{-}20) \qquad \underline{i}_{\Theta 1} \wedge \underline{n} \cdot \underline{k} = \sin\varphi_1$$

$$(2.2\text{-}21) \qquad \underline{i}_{\Theta 2} \wedge \underline{n} \cdot \underline{k} = \sin\varphi_2$$

By using Eq.(2.1-1) and (2.1-2) and taking into account Eqs.(2.2-14) to (2.2-17), one has:

$$(2.2\text{-}22) \qquad \mathrm{Re}\left[(\underline{E}_o \wedge \underline{n} \cdot \underline{k})\ \underline{E}_o^{*} \cdot \underline{n}\right] = E_o^2\ \mathrm{Re}\left\{\left[(\underline{i} + i\underline{j}) \wedge \underline{n} \cdot \underline{k}\right](\underline{i} - i\underline{j}) \cdot \underline{n}\right\} =$$

$$= E_o^2\ (\cos\Theta_1\ \sin\Theta_1\ \sin\varphi_1 - \cos\Theta_1\ \sin\Theta_1\ \sin\varphi_1) = 0$$

Analogously: $$\mathrm{Re}\left[(\underline{H}_o \wedge \underline{n} \cdot \underline{k})\ \underline{B}_o^{*} \cdot \underline{n}\right] = 0$$

Eq.(2.2-11), by the help of Eqs.(2.2-18) through (2.2-21), becomes:

$$(2.2\text{-}23) \quad M_z = \frac{\varepsilon_o r}{2} \operatorname{Re}\left\{ \int_\Sigma (E_{1\Theta}\sin\varphi_1 + E_{2\Theta}\sin\varphi_2)\underline{E}_o^{*}\cdot\underline{n}\, d\Sigma + \right.$$

$$\left. + \int_\Sigma (\underline{E}_o \wedge \underline{n}\cdot\underline{k})(E_{1r}^{*} + E_{2r}^{*})\, d\Sigma + \int_\Sigma (E_{1\Theta}\sin\varphi_1 + E_{2\Theta}\sin\varphi_2)(E_{1r}^{*} + E_{2r}^{*})\, d\Sigma \right\} +$$

$$+ \frac{\mu_o r}{2} \operatorname{Re}\left\{ \int_\Sigma (H_{1\varphi}\cos\Theta_1\cos\varphi_1 + H_{2\varphi}\cos\Theta_2\cos\varphi_2)\; H_o^{*}\cdot\underline{n}\, d\Sigma \right\}$$

Let us consider the integral, appearing in Eq.(2.2-23):

$$(2.2\text{-}24) \quad A_1 = \int_\Sigma \left[\underline{E}_o \wedge \underline{n}\cdot\underline{k}\right](E_{1r}^{*} + E_{2r}^{*})\, d\Sigma = r^2 \int_0^{2\pi} d\varphi_1 \int_0^{\pi} \left[\underline{E}_o \wedge \underline{n}\cdot\underline{k}\right] E_{1r}^{*} \sin\Theta_1\, d\Theta_1 +$$

$$+ r^2 \int_0^{2\pi} d\varphi_2 \int_0^{\pi} \left[\underline{E}_o \wedge \underline{n}\cdot\underline{k}\right] E_{2r}^{*} \sin\Theta_2\, d\Theta_2$$

From (2.1-1), (2.2-14) and (2.2-15) one derives:

$$(2.2\text{-}25) \quad \underline{E}_o \wedge \underline{n}\cdot\underline{k} = E_o\, e^{i(\omega t - kr\sin\Theta_1\cos\varphi_1)} \left[\cos\Theta_1 - i\sin\Theta_1\sin\varphi_1\right]$$

or:

$$(2.2\text{-}26)\quad \underline{E}_o \wedge \underline{n}\cdot\underline{k} = E_o\, e^{i(\omega t - kr\sin\Theta_2\cos\varphi_2)}\left[-\sin\Theta_2\sin\varphi_2 - i\cos\Theta_2\right]$$

The second term of the sum on the right side of (2.2-25) is a symmetric function with respect to $\Theta_1 = \pi/2$. This function multiplied by $E_{1r}\sin\Theta_1$ gives rise to an antisymmetric function, whose integral over Θ_1 between 0 and π, in Eq.(2.2-24), vanishes. The same result is obtained by considering the first term of the sum on the right side of (2.2-26).

By substituting (2.2-25) and (2.2-26) into (2.2-24), by taking into account (2.2-6) and (2.2-9), one obtains:

$$(2.2\text{-}27)\quad A_1 = -i\,\frac{E_o Z I_o \lambda e^{ikr}}{8\pi}\int_0^{\pi}\sin\left(\frac{\pi}{2}\cos\Theta\right)\sin\Theta\,\cos\Theta\,d\Theta\int_0^{2\pi}e^{-ikr\sin\Theta\cos\varphi}\,d\varphi$$

The integral over φ is equal to $2\pi\, J_o(kr\sin\Theta)$. Then

$$(2.2\text{-}28)\quad \frac{\varepsilon_o r}{2}\mathrm{Re}\,A_1 = \frac{\varepsilon_o r E_o Z I_o \lambda}{8}\sin(kr)\int_0^{\pi}\sin\left(\frac{\pi}{2}\cos\Theta\right)J_o(kr\sin\Theta)\sin\Theta\,\cos\Theta\,d\Theta$$

With the substitution:

$$(2.2\text{-}29) \qquad \sin\left(\frac{\pi}{2}\cos\theta\right) = \sum_{0}^{\infty}{}_{n} \frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{2}\cos\theta\right)^{2n+1}$$

the integral in (2.2-28) becomes:

$$(2.2\text{-}30) \qquad \int_0^{\pi} \sin\left(\frac{\pi}{2}\cos\theta\right) J_o(kr\sin\theta)\sin\theta\cos\theta\, d\theta =$$

$$= 2\sum_{0}^{\infty}{}_{n} \frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{2}\right)^{2n+1}\int_0^{\pi/2} J_o(kr\sin\theta)\sin\theta\cos^{2n+2}\theta\, d\theta$$

This integral can be evaluated by using Sonine's first formula (') and one derives:

(2.2-31)

$$\frac{\varepsilon_o r}{2}\,\mathrm{Re}\,A_1 = \frac{\varepsilon_o r E_o Z I_o \lambda}{4}\sin(kr)\sum_{0}^{\infty}{}_{n} \frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{2}\right)^{2n+1}\frac{2^{n+1/2}}{(kr)^{n+3/2}}\Gamma\left(n+\frac{3}{2}\right)J_{n+\frac{3}{2}}(kr)$$

(') Ref.28 p.373

When $kr \to \infty$ (or $r \gg \lambda$), one has:

$$\frac{r \sin kr}{(kr)^{n+3/2}} J_{n+3/2}(kr) \to \sqrt{\frac{2}{\pi}} \frac{\sin(kr)\cos\{kr - [(1 + n)/2]\pi\}}{k(kr)^{n+1}}$$

Thus, when $r \gg \lambda$,

$$(2.2\text{-}32) \qquad \frac{\varepsilon_o r}{2} \operatorname{Re} A_1 \to 0$$

Hence this term can be eliminated from Eq.(2.2-23).

Let us now consider the third integral of Eq.(2.2-23), that is

$$(2.2\text{-}33) \quad M'_z = \frac{\varepsilon_o r}{2} \operatorname{Re} \int_\Sigma (E_{1\Theta}\sin\varphi_1 + E_{2\Theta}\sin\varphi_2)(E^*_{1r} + E^*_{2r})\, d\Sigma$$

This expression represents the torque acting on the antenna due to the re-radiated waves. By introducing the expressions of the field, by considering the real part of the integrand

and with the help of Eq.(2.2-16) and (2.2-17) one obtains:

$$(2.2\text{-}34)\quad M'_z = -\frac{Z^2 I^2 \varepsilon_o \lambda}{16\pi^2}\int_0^{2\pi}\sin\varphi\, d\varphi \int_0^{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)\sin\left(\frac{\pi}{2}\sin\theta\,\sin\varphi\right)}{\sin\theta}\sin\theta\, d\theta$$

Apart from the sign, Eq.(2.2-34) is equal to Eq.(1.4-8), (the different sign depends on the different relative phase of the currents on the two half wave-dipoles in the two cases).

By using, for the first term on the right side of (2.2-23):

$$(2.2\text{-}35)\quad \frac{\varepsilon_o r}{2}\,\mathrm{Re}\, A_2 = \frac{\varepsilon_o r}{2}\,\mathrm{Re}\int_{\Sigma}(E_{1\theta}\sin\varphi_1 + E_{2\theta}\sin\varphi_2)\underline{E}_o^{*}\cdot\underline{n}\, d\Sigma,$$

the same procedure as followed in deriving Eq.(2.2-27) from (2.2-24) one has:

$$(2.2\text{-}36)\quad \frac{\varepsilon_o r}{2}\mathrm{Re}\, A_2 = \mathrm{Re}\left\{-\frac{\sqrt{\varepsilon_o\mu_o}}{2\pi}\, I_o E_o\, r^2 e^{-ikr}\int_0^{\pi}\cos\left(\frac{\pi}{2}\cos\theta\right)\sin\theta\, d\theta\, \times \right.$$
$$\left.\times\int_0^{2\pi} e^{ikr\sin\theta\cos\varphi}\sin^2\varphi\, d\varphi\right\}$$

which gives:

$$(2.2\text{-}37)\quad \frac{\varepsilon_o r}{2}\mathrm{Re}\ A_2 = -\sqrt{\varepsilon_o\mu_o}\, I_o E_o\, \frac{r}{k}\cos(kr)\int_0^{\pi} J_1(kr\ \sin\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)d\theta$$

Analogously the last term of the sum on the right side of (2.2-23):

$$(2.2\text{-}38)\quad \frac{\mu_o r}{2}\mathrm{Re}\ A_3 = \frac{\mu_o r}{2}\mathrm{Re}\int_{\Sigma}(H_{1\varphi}\cos\theta_1\cos\varphi_1 + H_{2\varphi}\cos\theta_2\cos\varphi_2)\underline{H}_o^{*}\cdot\underline{n}\ d\Sigma$$

becomes:

$$(2.2\text{-}39)\quad \frac{\mu_o r}{2}\mathrm{Re}\,A_3 = \mathrm{Re}\left\{-\frac{\sqrt{\varepsilon_o\mu_o}}{2\pi}\, I_o E_o r^2 e^{-ikr}\int_0^{\pi}\cos\left(\frac{\pi}{2}\cos\theta\right)\cos^2\theta\ d\theta\ \times\right.$$

$$\left.\times\int_0^{2\pi} e^{ikr\ \sin\theta\ \cos\varphi}\cos\varphi\ d\varphi\right\}$$

which gives:

$$(2.2\text{-}40)\quad \frac{\mu_0 r}{2}\mathrm{Re}A_3 = -\sqrt{\varepsilon_0\mu_0}\; I_0 E_0 r^2\; \sin(kr)\int_0^{\pi}\cos\left(\frac{\pi}{2}\cos\theta\right) J_1(kr\;\sin\theta)\cos^2\theta\; d\theta$$

Finally, substitution of Eqs.(2.2-32),(2.2-34),(2.2-37) and (2.2-40) into (2.2-38), gives, for $r \gg \lambda$,

$$(2.2\text{-}41)\quad M_z = M'_z + \sqrt{\varepsilon_0\mu_0} I_0 E_0 r\left\{-\frac{1}{k}\cos(kr)\int_0^{\pi} J_1(kr\;\sin\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)d\theta -\right.$$

$$\left. - r\;\sin(kr)\int_0^{\pi} J_1(kr\;\sin\theta)\cos\left(\frac{\pi}{2}\;\cos\theta\right)\cos^2\theta\; d\theta\right\}$$

The first integral on the right side of Eq.(2.2-41) can be performed by using Eq.(1.4-10). One has :

$$(2.2\text{-}42)\qquad \int_0^{\pi} J_1(kr\;\sin\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)d\theta = -\frac{2}{kr}\cos\left(\sqrt{\frac{\pi^2}{4} + k^2 r^2}\right)$$

When $kr \gg \pi^2/4$:

(2.2-43) $$\int_0^{\pi} J_1(kr\ \sin\Theta)\cos\left(\frac{\pi}{2}\ \cos\Theta\right) d\Theta \to -\frac{2}{kr}\cos(kr)$$

For the second integral one can use the formula (see Appendix 5):

(2.2-44) $$\int_0^{\pi} J_1(\alpha\ \sin\Theta)\cos(\beta\ \cos\Theta)\cos^2\Theta\ d\Theta =$$

$$-\frac{2}{\alpha}\cos\left(\sqrt{\beta^2+\alpha^2}\right) + 2\alpha\ \frac{\cos\left(\sqrt{\beta^2+\alpha^2}\right)}{\alpha^2+\beta^2} - 2\alpha\ \frac{\sin\left(\sqrt{\beta^2+\alpha^2}\right)}{\left(\alpha^2+\beta^2\right)^{3/2}}$$

from which one has:

(2.2-45) $$\int_0^{\pi} J_1(kr\ \sin\Theta)\cos\left(\frac{\pi}{2}\cos\Theta\right)\cos^2\Theta\ d\Theta =$$

$$= -\frac{2}{kr}\cos\left(\sqrt{\frac{\pi^2}{4}+k^2r^2}\right) + 2kr\ \frac{\cos\left(\sqrt{\frac{\pi^2}{4}+k^2r^2}\right)}{\frac{\pi^2}{4}+k^2r^2} - 2kr\ \frac{\sin\left(\sqrt{\frac{\pi^2}{4}+k^2r^2}\right)}{\left(\frac{\pi^2}{4}+k^2r^2\right)^{3/2}}$$

When $kr \gg \pi^2/4$, Eq.(2.2-45) becomes:

$$(2.2\text{-}46) \qquad \int_0^{\pi} J_1(kr \sin\Theta)\cos\left(\frac{\pi}{2}\cos\Theta\right)\cos^2\Theta \rightarrow -\frac{2}{k^2r^2}\sin(kr)$$

Substitution of (2.2-43) and (2.2-46) into (2.2-41), gives, (for $r \gg \lambda$):

$$(2.2\text{-}47) \qquad M_z = M'_z + 2\,\frac{\sqrt{\mu_o \varepsilon_o}}{k^2}\, I_o E_o$$

It is to be noted that the integrals of Eq.(2.2-41) can be performed in an approximate way, by substituting

$$\sin\Theta \qquad \text{for} \qquad \frac{\cos\left(\frac{\pi}{2}\cos\Theta\right)}{\sin\Theta}$$

This amounts to the same as substituting the radiation pattern of the elementary dipole in place of the radiation pattern of the half wave antenna.

The resulting integrals

$$E = \int_0^{\pi} J_1(kr\ \sin\theta)\sin^2\theta\ d\theta \tag{2.2-48}$$

$$F = \int_0^{\pi} J_1(kr\ \sin\theta)\cos^2\theta\ \sin^2\theta\ d\theta \tag{2.2-49}$$

can be performed by using Sonine's first formula ('), obtaining:

$$E = \sqrt{\frac{2\pi}{kr}}\ J_{3/2}(kr) \tag{2.2-50}$$

$$F = \frac{\sqrt{2\pi}}{(kr)^{3/2}}\ J_{5/2}(kr) \tag{2.2-51}$$

When $r \gg \lambda$, one has:

$$\begin{aligned} J_{3/2}(kr) &\rightarrow -\sqrt{\frac{2}{\pi kr}}\ \cos(kr) \\ J_{5/2}(kr) &\rightarrow -\sqrt{\frac{2}{\pi kr}}\ \sin(kr) \end{aligned} \tag{2.2-52}$$

(') Ref.28 p.373

Substitution of (2.2-52) into (2.2-50) and (2.2-51) shows that the approximate expressions (2.2-50) and (2.2-51) of the integrals appearing in (2.2-41), have values which, for large r, tend to the values of the exact expressions (2.2-43) and (2.2-46).

2.3 - Evaluation of M_z.

Let us evaluate M_z by means of Eq.(2.2-47).

The expression I_o can be obtained by considering the mean Poynting vector $\underline{S}$ of the incident wave, whose magnitude is:

$$S = \frac{E_o^2}{Z} \tag{2.3.1}$$

The energy W_a absorbed by a single component antenna is given by

$$W_a = A\,\frac{S}{2} = \frac{AE_o^2}{2Z} \tag{2.3-2}$$

where A is the absorption cross section of a single component antenna.

By considering the resistance in series between the terminals of the antenna, which was assumed to be equal to the radiation resistance R, one has

(2.3-3) $$W_a = \frac{1}{2} I_o^2 R$$

From (2.3-2) and (2.3-3) one obtains:

(2.3-4) $$\frac{2\sqrt{\varepsilon_o \mu_o}}{k^2} I_o E_o = 4 \frac{W_a}{ck^2} \sqrt{\frac{Z}{AR}}$$

Since ('):

(2.3-5) $$AR = Z/k^2$$

Eq.(2.3-4) becomes:

(2.3-6) $$\frac{2\sqrt{\varepsilon_o \mu_o}}{k^2} I_o E_o = 4 \frac{W_a}{\omega}$$

For the term M'_z appearing in (2.2-47), one has, from Eq.(2.2-34)

(') see Ref.8 p.500

and Eq.(1.4-13):

$$(2.3\text{-}7) \qquad M'_z = -\frac{W_r}{\omega}\, 0.9939$$

where W_r is the power re-radiated by the turnstile antenna. Since the antenna is resonant, the scattered power is equal to the absorbed power. Then $W_r = 2W_a$, hence

$$(2.3\text{-}8) \qquad M_z = 4\,\frac{W_a}{\omega} - 1.9878\,\frac{W_a}{\omega} = 2.0122\,\frac{W_a}{\omega}$$

From (2.3-2) one has:

$$(2.3\text{-}9) \qquad 2\,W_a = A\,S$$

and (2.3-8) becomes:

$$(2.3\text{-}10) \qquad M_z = \frac{AS}{\omega}\, 1.0061$$

S/ω represents the flux per unit time of the angular momentum of the circularly polarized plane wave across the unit area normal to the direction of propagation (').

Eq.(2.3-10) gives a simple relation between the torque acting on the antenna and the flux of the angular momentum of the incident waves. Here A is the absorption cross section of a single component antenna.

Let us now consider the case of circularly polarized plane waves impinging on a half-wave dipole antenna, that is on one only of the two components of the turnstile antenna. In this case one obviously has:

$$M_z' = 0 \tag{2.3-11}$$

because in this case the antenna re-radiates linearly polarized waves.

Thus M_z is:

$$M_z = \frac{\sqrt{\varepsilon_o \mu_o}}{k^2} I_o E_o = \frac{2W_a}{\omega} = \frac{AS}{\omega} \tag{2.3-12}$$

as can be seen by putting $\underline{E}_2 = 0$ in the formulas of the preceding sections.

Eq.(2.3-12) shows that the torque acting on a single half-wave antenna is almost exactly the same as the torque acting on a turnstile antenna. This is due to the fact that the turnstile antenna, re-radiates circularly polarized waves, hence

(') see for instance Ref.29

angular momentum.

2.4 - The case of the elementary dipoles.

Let us now suppose that the antenna is constituted by two elementary dipoles at right angles and in quadrature. The incident field is given by (2.1-1) and (2.1-2) .

The currents of the two dipoles are of the type:

$$I_1 = I_o\, e^{i\omega t}\, \underline{i} \tag{2.4-1}$$

$$I_2 = i\, I_o\, e^{i\omega t}\, \underline{i} \tag{2.4-2}$$

Where I_o depends on the incident field, and on the absorption coefficient. The dipoles are assumed to be tuned at the wavelength of the incident radiation.

The electromagnetic field radiated by the two dipoles is given by

$$E_{1\theta} = - \frac{I_o sZ}{2\lambda r}\, e^{i(\omega t - kr)}\, \sin\theta_1 \tag{2.4-3}$$

$$E_{1r} = i\, \frac{I_o sZ}{2\pi r^2}\, e^{i(\omega t - kr)}\, \cos\theta_1 \tag{2.4-4}$$

$$H_{1\varphi} = - \frac{I_o s}{2\lambda r} e^{i(\omega t - kr)} \sin\theta_1 \tag{2.4-5}$$

$$E_{2\theta} = i \frac{I_o sZ}{2\lambda r} e^{i(\omega t - kr)} \sin\theta_2 \tag{2.4-6}$$

$$E_{2r} = \frac{I_o sZ}{2\pi r^2} e^{i(\omega t - kr)} \cos\theta_2 \tag{2.4-7}$$

$$H_{2\varphi} = i \frac{I_o s}{2\lambda r} e^{i(\omega t - kr)} \sin\theta_2 \tag{2.4-8}$$

where s denotes the common length of the dipoles.
By introducing these expressions into (2.2-11) and following step by step the procedure described in section 2, one obtains exactly:

$$M'_z = - \frac{\mu_o}{3} \frac{(I_o s)^2}{\lambda} \tag{2.4-9}$$

and

$$M_z = M'_z + \frac{\sqrt{\varepsilon_o \mu_o}}{k} IsE_o \tag{2.4-10}$$

Besides, recalling that for the dipole one has:

$$A = \frac{3}{8\pi} \lambda^2 \tag{2.4-11}$$

$$R = \frac{2\pi}{2} Z \frac{s^2}{\lambda^2} \tag{2.4.12}$$

one derives from (2.3-11)

$$Is = \sqrt{\frac{3W_a \lambda^2}{\pi Z s^2}} \tag{2.4-13}$$

which, introduced into (2.4-9), gives:

$$M'_z = - \frac{2W_a}{\omega} \tag{2.4-14}$$

Finally, by expressing E_o in terms of W_a from Eq.(2.3-11), and using (2.4-13), one derives

$$M_z = \frac{2W_a}{\omega} \tag{2.4-15}$$

By taking into account (2.3-15), Eq.(2.4-15) becomes

$$M_z = A \frac{S}{\omega} \tag{2.4-16}$$

In this case the coefficient appearing in the relation between the torque acting on the crossed dipoles antenna, the absorption cross section A and the flux of the angular momentum is exactly 1. With the aid of considerations similar to those made at the end of the preceding section, it is possible to show that the same torque acts on a single elementary dipole.

2.5 - Numerical values and conclusions.

Let a transmitting station be located on the ground, and radiate a power P, towards a space vehicle. The space vehicle is assumed to carry a half-wave antenna tuned at the wavelength λ. A torque results to be applied to the vehicle. The order of magnitude of the torque which can be obtained in this way, will now be discussed.

Let G be the gain of the radiating antenna, h the distance from the vehicle to the station.

The Poynting vector at the vehicle is given by

$$S = \frac{PG}{4\pi h^2} \tag{2.5-1}$$

Let us assume that the receiving antenna is placed in a plane normal to the direction of the wave. By substituting (2.5-1) into (2.3-12) and recalling that $A = 0.13\ \lambda^2$ for the half-wave antenna, one obtains:

$$M = 0.13 \frac{PG\lambda^3}{8\pi^2 h^2 c} \qquad (2.5\text{-}2)$$

For h = 1000 km, λ = 30 m and G = 100 (which corresponds to a parabolic antenna of diameter D $\sim$ 300 m), Eq.(2.5-2) gives:

$$M = 1.39 \; 10^{-17} J/W$$

In this case a power of 10^{10} W should be radiated, to obtain a torque of the order of magnitude of 1 erg.

If h = 100 km, λ = 30 m and G = 100, Eq.(2.5-2) gives:

$$M = 1.5 \; 10^{-15} J/W$$

and a power of 10^8 W should be radiated, to obtain a torque of 1 erg. These simple examples show that this way of employing circular polarized waves to control the attitude of space vehicles, is not of pratical interest. It seems therefore unnecessary to examine different situations (for instance the general case in which the vehicle's antenna is not in the plane of the incident wave) or to consider the effects of the attenuation by the atmosphere, of Faraday rotation, or other particular questions.

3 - RADIATING ANTENNA UNDER THE ACTION OF AN EXTERNAL FIELD.

3.1 - Introduction

A third way of applying a torque to a space vehicle, by means of electromagnetic radiation will be discussed.

Let a ground antenna radiate linearly polarized waves towards a satellite. Let the satellite antenna, on its turn radiate linearly polarized waves. Denote by γ the angle between the direction of polarization of the incident wave and the direction of the satellite antenna.

The combination of the fields radiated by the ground antenna and by the satellite gives rise to elliptically polarized radiation. The degree of polarization at any given point will depend on the phase difference between the two components, on the relative amplitude, and on γ.

Let us examine the case of a thin half-wave dipole antenna in the free space, on which the distribution of current is given as:

$$I = I_0 \cos k_1 \xi \, e^{i\omega_1 t} \tag{3.1-1}$$

where $k_1 = \frac{2\pi}{\lambda_1}$, and ξ is the distance from the midpoint.

A plane electromagnetic wave is assumed to impinge on the antenna. A reference system centered at the center O of the antenna (**Fig.3.1**), is chosen, in such a way that the field vectors of the plane wave are given by:

$$\underline{E} = E_o \, e^{i(\omega t - kz + \varphi)} \underline{i} \tag{3.1-2}$$

$$\underline{B} = \frac{E_o}{c} \, e^{i(\omega t - kz + \varphi)} \underline{j} \tag{3.1-3}$$

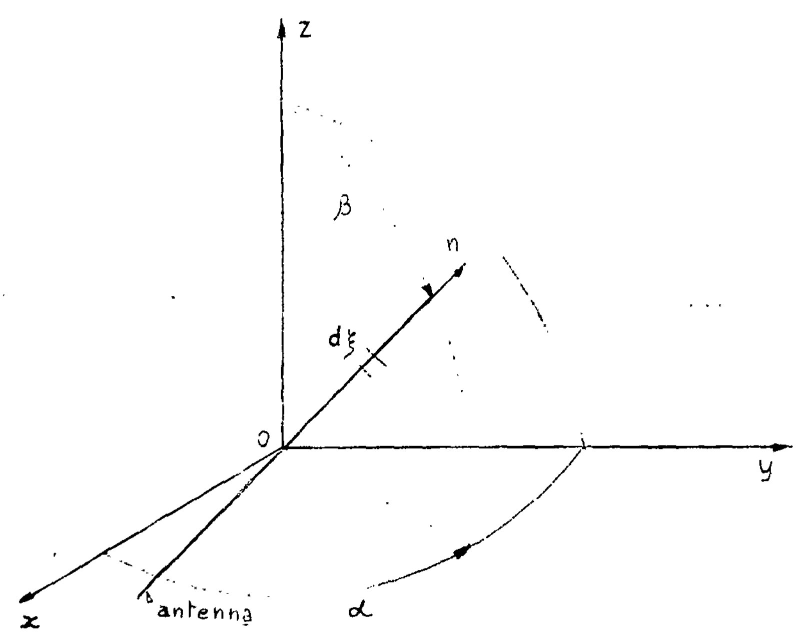

Fig.3.1

The force $\underline{F}$ acting on the element $d\xi$ of the antenna is obtained by summing the forces acting on the elementary charge $\rho\, d\xi$ where ρ denotes the linear charge density, and on the current element $\underline{I}\, d\xi$.

$$(3.1\text{-}4) \qquad \underline{F} d\xi = \frac{1}{2} \mathrm{Re}\left\{\rho\, \underline{E}^{*} + \underline{I} \wedge \underline{B}^{*}\right\} d\xi$$

The torque acting on the antenna is

$$(3.1\text{-}5) \qquad \underline{M} = \frac{1}{2} \mathrm{Re} \int_{-L}^{L} \underline{F} \wedge \underline{n}\, \xi\, d\xi$$

where $\underline{n}$ is a unit vector, directed along the antenna and $L = \lambda/4$. By introducing Eq.(3.1-4) into (3.1-5) one derives:

$$(3.1\text{-}6) \qquad \underline{M} = \frac{1}{2} \mathrm{Re} \int_{-L}^{L} \left[\rho\, \underline{E}^{*} \wedge \underline{n} + (\underline{I} \wedge \underline{B}^{*}) \wedge \underline{n}\right] \xi\, d\xi$$

3.2 - Evaluation of the torque acting on the antenna.

The torque $\underline{M}$, given by Eq.(3.1-6), is evaluated, recalling that:

$$(3.2\text{-}1) \qquad \underline{I} = I\, \underline{n}$$

and that by the equation of continuity $dI/d\xi = -\partial\rho/\partial t$, One derives, by using Eq.(3.1-1):

$$\rho = -\frac{ik_1}{\omega_1} I_o \sin\left(k_1\xi\right)e^{i\omega_1 t} \tag{3.2-2}$$

Further one has:

$$\underline{n} = \cos\beta\ \underline{k} + \sin\beta\cos\alpha\ \underline{i} + \sin\beta\sin\alpha\ \underline{j} \tag{3.2-3}$$

where (see Fig.3.1) β is the angle between the antenna and the z axis and α the angle between the projection of the antenna on the x y plane and the x axis.

Substitution of (3.1-2), (3.2-2), (3.2-3), (3.1-1), (3.2-1) and (3.1-3) into Eq.(3.1-6) gives:

$$\begin{aligned}(3.2\text{-}4)\quad \underline{M} = \frac{1}{2}\,\mathrm{Re}\Bigg\{&(-\cos\beta\ \underline{j} + \sin\beta\sin\alpha\ \underline{k})\left(-i\frac{I_o}{c}E_o\right)\times\\
&\times e^{i[(\omega_1-\omega)t+\varphi]}\int_{-L}^{L}\sin\left(k_1\xi\right)e^{ik_1\xi\cos\beta}\xi\,d\xi -\\
&-\left[\sin\beta\sin\alpha\cos\beta\ \underline{k} + \sin^2\beta\cos\alpha\sin\alpha\ \underline{i} + (\sin^2\beta\sin^2\alpha-1)\underline{j}\right]\times\\
&\times I_o\frac{E_o}{c}e^{i[(\omega_1-\omega)t+\varphi]}\int_{-L}^{L}\cos\left(k_1\xi\right)e^{ik_1\xi\cos\beta}\xi\,d\xi\Bigg\}\end{aligned}$$

By inspection of (3.2-4) it is apparent that, if $\omega_1 \neq \omega$, $\underline{M}$ is an oscillating function whose time average vanishes. Therefore we will suppose $\omega_1 = \omega$.

For the integral

$$(3.2\text{-}5) \qquad \int_{-L}^{L} \sin(k_1\xi) e^{ik_1\xi\cos\beta} d\xi = \frac{S}{k^2}$$

appearing in Eq.(3.2-4), one obtains after some elaboration

$$(3.2\text{-}6) \quad S = \frac{1}{\cos^2\beta - 1}\left[\pi \sin\left(\frac{\pi}{2}\cos\beta\right)\cos\beta + 2\cos\left(\frac{\pi}{2}\cos\beta\right)\frac{\cos^2\beta + 1}{\cos^2\beta - 1}\right]$$

Similarly for:

$$(3.2\text{-}7) \qquad \int_{-L}^{L} \cos(k_1\xi) e^{ik_1\xi\cos\beta} d\xi = \frac{iT}{k^2}$$

one obtains

$$(3.2\text{-}8) \quad T = \frac{1}{\cos^2\beta - 1}\left[\pi \sin\left(\frac{\pi}{2}\cos\beta\right) + 4\cos\left(\frac{\pi}{2}\cos\beta\right)\frac{\cos\beta}{\cos^2\beta - 1}\right]$$

By substituting (3.2-5) and (3.2-7) into Eq.(3.2-4), by taking the real part, one derives

$$(3.2\text{-}9) \qquad \underline{M} = \frac{cI_o E_o}{2\omega^2} \sin\varphi \left\{ \sin^2\beta \cos\alpha \sin\alpha \, T \, \underline{i} \; + \right.$$

$$+ \left[(\sin^2\beta \sin^2\alpha - 1) \, T - \cos\beta \, S \right] \underline{j} \; +$$

$$\left. + \left[\sin\alpha \sin\beta \cos\beta \, T + \sin\beta \sin\alpha \, S \right] \underline{k} \right.$$

where T and S depend only on α and β and are given by Eqs.(3.2-6) and (3.2-8).

When $\beta = \frac{\pi}{2}$, the antenna lies in the plane of $\underline{E}$ and $\underline{B}$. In this case one has:

$$(3.2\text{-}10) \qquad S = 2 \qquad T = 0$$

and the torque becomes

$$(3.2\text{-}11) \qquad \underline{M} = \frac{cI_o E_o}{\omega^2} \sin\varphi \sin\alpha \, \underline{k}$$

When $\varphi = \alpha = \pi/2$ one has:

(3.2-12) $$\underline{M} = \frac{cI_o E_o}{\omega^2} \underline{k}$$

In the case $\beta = 0$ one has

(3.2-13) $$S = \frac{\pi}{4} \quad T = \frac{\pi}{4}$$

and $\underline{M}$ is given by

(3.2-14) $$\underline{M} = - \frac{\pi c I_o E_o}{4\omega^2} \sin\varphi \; \underline{j}$$

3.3 - Numerical examples and conclusions

If the ground antenna emits power W_g, with a gain G, one has:

(3.3-1) $$E_o = \frac{1}{h} \sqrt{\frac{Z W_g G}{2\pi}}$$

where h denotes the distance between the ground antenna and the satellite.

If W_s indicates the power radiated by the satellite's antenna (which is assumed to be tuned at the wavelength λ) and R the radiation resistance, one has:

$$(3.3\text{-}2) \qquad I_o = \sqrt{\frac{2W_s}{R}}$$

Let us suppose now that the phase of the satellite antenna is locked to that of the impinging wave with $\varphi = \pi/2$ and that $\alpha=\beta=\frac{\pi}{2}$. Thus Eq.(3.2-12) becomes

$$(3.3\text{-}3) \qquad \underline{M} = \frac{c}{\omega^2 h} \sqrt{\frac{ZW_g GW_s}{\pi R}}$$

If the ground antenna is assumed to be parabolic with diameter D and with uniform illumination (thus $G = \pi^2 D^2/\lambda^2$), Eq.(3.3-3) becomes:

$$(3.3\text{-}4) \qquad M = \frac{D}{2\omega h} \sqrt{\frac{ZW_g W_s}{\pi R}}$$

From (3.3-4) one can obtain the order of magnitude of the value which the torque can reach.

Let us assume:

$D = 100$ m, $\lambda = 30$ m, $W_g = 100$ kW, $W_s = 10$ W. If: $h = 300$ km:

$$M = 3.4 \quad 10^{-9} \quad J.$$

It seems therefore that this third way of applying electromagnetic radiation, to control the attitude of a space vehicle, cannot have practical importance, since the torque is too small. On the other hand, the appearence of the powers W_i and W_s under the square root sign leaves little hope of significantly improving the situation, even with higher powers.

CONCLUSION

In this report the possibility has been discussed of applying elliptically polarized radiation to the attitude control of a space vehicle.

In order to get a criterion to assess the effectiveness of the different methods and procedures, an analysis and a discussion have been made of the different factors which may disturb the orientation of the vehicle during actual flight. Typical orders of magnitude have been given for the torques due to all these physical factors.

In the first section an active procedure has been discussed. The space vehicle sends out elliptically polarized radiation and is acted upon by a reaction torque. In particular, the case of a transmitting turnstile antenna has been analysed in detail. A simple relation was found between the reaction torque, the radiated power and the frequency.

The power levels available on space vehicles are rapidly increasing year after year. Reasonable forecasts can be made about the power which will become available in the near or even far future. A comparison between the torques obtainable with such powers and the torques due to external or internal physical factors which may disturb the orientation is not at all discouraging. It seems therefore advisable to keep in mind the possibility of this device of attitude control in future designs, especially where a continuous and small action is to be anticipated for very long periods of time.

As a by-product of this investigation, we point out the necessity for the future designer of taking into account the torque produced by the emission of elliptically polarized radiation, which may be undesired and represent a cause of disturbance for the maintainance of a given orientation.

In the second section the passive method has been investigated, where a ground station transmits circularly polarized radiation, which is received by the vehicle. A relation has been given between the resulting torque and the relevant parameters, namely the power of the ground station, the gain of the transmitting antenna and the absorption cross section of the receiving antenna. The torques obtainable turn out to be extremely weak.

Finally, in the third section a mixed procedure has been investigated. A ground station transmits linearly polarized radiation, which at the receiving end is utilised to control the frequency and the phase of the linearly polarized radiation emitted by the space vehicle. Both fields, that emitted by the ground station and that emitted by the space vehicle, combine to give an elliptically polarized field. The torque generated in this way, by assuming reasonable values for the powers, turns out again to be extremely small, though somewhat better than in the previous case.

In conclusion, it seems that the active device described in the first section is both the simplest and the most effective, and the only one to present a practical interest.

<u>APPENDIX 1</u>

In section 1.2 the following relation has been used:

$$(A1-1) \qquad \left[\mathrm{div}\,(\underline{u}\ \underline{v})\right]\ \underline{r} = \mathrm{div}\left[(\underline{u} \wedge \underline{r})\underline{v}\right] - \underline{u} \wedge \underline{v}$$

where $(\underline{u}\ \underline{v})$ is a dyadic. In order to demostrate Eq.(A1-1), let us apply the following formula (\'):

$$(A1-2) \qquad \mathrm{div}\,(\underline{u}\ \underline{v}) = \underline{u}\ \mathrm{div}\ \underline{v} + (\mathrm{grad}\ \underline{u})\cdot\underline{v}$$

By applying (A1-2) to (the two parts of) (A1-1), one has:

$$(\underline{u}\ \mathrm{div}\ \underline{v}) \wedge \underline{r} + \left[(\mathrm{grad}\ \underline{u})\cdot\underline{v}\right] \wedge \underline{r} = (\underline{u} \wedge \underline{r})\mathrm{div}\ \underline{v} + \left[\mathrm{grad}\ (\underline{u} \wedge \underline{r})\right]\cdot\underline{v} - \underline{u} \wedge \underline{v}$$

By introducing rectangular coordinates x, y, z (unit vectors = $= i_1, i_2, i_3$), one has:

$$(\mathrm{grad}\ \underline{u})\cdot\underline{v} = \frac{\partial \underline{u}}{\partial x}\ \underline{i}_1\cdot\underline{v} + \frac{\partial \underline{u}}{\partial y}\ \underline{i}_2\cdot\underline{v} + \frac{\partial \underline{u}}{\partial z}\ \underline{i}_3\cdot\underline{v}$$

thus:

$$\left[(\mathrm{grad}\ \underline{u})\cdot\underline{v}\right] \wedge \underline{r} = \underline{i}_1\cdot\underline{v}\ \frac{\partial \underline{u}}{\partial x} \wedge \underline{r} + \underline{i}_2\cdot\underline{v}\ \frac{\partial \underline{u}}{\partial y} \wedge \underline{r} + \underline{i}_3\cdot\underline{v}\ \frac{\partial \underline{u}}{\partial z} \wedge \underline{r}$$

(') Ref.7 p.26

Besides:

$$\operatorname{grad}(\underline{u}\wedge\underline{r}) = \frac{\partial(\underline{u}\wedge\underline{r})}{\partial x}\underline{i}_1 + \frac{\partial(\underline{u}\wedge\underline{r})}{\partial y}\underline{i}_2 + \frac{\partial(\underline{u}\wedge\underline{r})}{\partial z}\underline{i}_3$$

hence:

$$\text{(A1-4)} \qquad \left[\operatorname{grad}(\underline{u}\wedge\underline{r})\right]\cdot\underline{v} = \frac{\partial(\underline{u}\wedge\underline{r})}{\partial x}\underline{i}_1\cdot\underline{v} + \frac{\partial(\underline{u}\wedge\underline{r})}{\partial y}\underline{i}_2\cdot\underline{v} + \frac{\partial(\underline{u}\wedge\underline{r})}{\partial z}\underline{i}_3\cdot\underline{v} =$$

$$= \underline{i}_1\cdot\underline{v}\left[\frac{\partial\underline{u}}{\partial x}\wedge\underline{r} + \underline{u}\wedge\frac{\partial\underline{r}}{\partial x}\right] + \ldots\ldots$$

By making use of (A1-3) and (A1-4), Eq.(A1-1) becomes:

$$\text{(A1-5)} \qquad 0 = \left[\left(\underline{i}_1\cdot\underline{v}\right)\underline{u}\wedge\frac{\partial\underline{r}}{\partial x} + \left(\underline{i}_2\cdot\underline{v}\right)\underline{u}\wedge\frac{\partial\underline{r}}{\partial y} + \left(\underline{i}_3\cdot\underline{v}\right)\underline{u}\wedge\frac{\partial\underline{r}}{\partial z}\right] - \underline{u}\wedge\underline{v}$$

but r is the distance from the origin of the reference system, then:

$$\frac{\partial\underline{r}}{\partial x} = \underline{i}_1 \ , \ \frac{\partial\underline{r}}{\partial y} = \underline{i}_2 \ , \ \frac{\partial\underline{r}}{\partial z} = \underline{i}_3$$

and the term in the parenthesis in (A1-5) becomes:

$$v_x\underline{u}\wedge\underline{i}_1 + v_y\underline{u}\wedge\underline{i}_2 + v_z\underline{u}\wedge\underline{i}_3 = \underline{u}\wedge\underline{v}$$

and (A1-5), hence (A1-1) is verified.

APPENDIX 2

It is to be demonstrated that:

$$\int_V (\mathrm{div}\, \underline{\underline{U}}) \wedge \underline{r}\; dV = 0$$

when V is a spherical volume centered at O (see p.5 for the notations). The integral can be written:

$$\int_V \left\{ \mathrm{div} \left[\frac{\underline{\underline{E}}.\underline{\underline{D}} + \underline{\underline{H}}.\underline{\underline{B}}}{2} \sum_{l,k} \delta_{lk}\, \underline{i}_l\, \underline{i}_k \right] \right\} \wedge \underline{r}\; dV \quad =$$

$$= \sum_k \int_V \left[\mathrm{div} \left(\frac{\underline{\underline{E}}.\underline{\underline{D}} + \underline{\underline{H}}.\underline{\underline{B}}}{2} \right) \underline{i}_k\, \underline{i}_k \right] \wedge \underline{r}\; dV$$

and because of (A1-1):

$$= \sum_k \int_V \mathrm{div} \left[\left(\frac{\underline{\underline{E}}.\underline{\underline{D}} + \underline{\underline{H}}.\underline{\underline{B}}}{2} \right) \underline{i}_k \wedge \underline{r}\; \underline{i}_k \right] dV - \sum_k \int_V \frac{\underline{\underline{E}}.\underline{\underline{D}} + \underline{\underline{H}}.\underline{\underline{B}}}{2}\; \underline{i}_k \wedge \underline{i}_k\; dV =$$

$$= \sum_k \int_\Sigma \frac{\underline{\underline{E}}.\underline{\underline{D}} + \underline{\underline{H}}.\underline{\underline{B}}}{2} (\underline{i}_k \wedge \underline{r})\, \underline{i}_k \cdot \underline{n}\; d\Sigma$$

where Σ is the boundary surface of the volume V, and $\underline{n}$ the outer normal. The last expression vanishes because:

$$\sum_k (\underline{i}_k \wedge \underline{r})(\underline{i}_k \cdot \underline{n}) = \underline{r} \wedge \sum_k (\underline{i}_k \wedge \underline{n})(\underline{i}_k \cdot \underline{n}) = \underline{r}(\underline{n}\cdot\underline{i}_1 \underline{i}_1 \wedge \underline{n} + \underline{n}\cdot\underline{i}_2 \underline{i}_2 \wedge \underline{n} + \underline{n}\cdot\underline{i}_3 \underline{i}_3 \wedge \underline{n}) = 0$$

APPENDIX 3

In order to perform the integration appearing in Eq.(1.4-10) let us consider the power series development of $J_1(\alpha \sin\theta)$

$$\text{(A3-1)} \qquad J_1(\alpha \sin\theta) = \sum_{0}^{\infty}{}_m \frac{(-1)^m \left(\frac{\alpha \sin\theta}{2}\right)^{2m+1}}{m!(m+1)!}$$

One has

$$\text{(A3-2)} \qquad \int_0^{\pi} J_1(\alpha \sin\theta) \cos(\beta \cos\theta) d\theta =$$

$$= \sum_{0}^{\infty}{}_m \frac{(-1)^m \left(\frac{\alpha}{2}\right)^{2m+1}}{m!(m+1)!} \int_0^{\pi} \sin^{2m+1}\theta \cos(\beta \cos\theta) d\theta =$$

$$= \sqrt{\pi} \sum_{0}^{\infty}{}_m \frac{(-1)^m \left(\frac{\alpha}{2}\right)^{2m+1}}{(m+1)} J_{m+1/2}(\beta) \left(\frac{\beta}{2}\right)^{-(m+1/2)}$$

By putting m + 1 = s, one obtains

$$(A3\text{-}3) \qquad \int_0^{\pi} J_1(\alpha \sin\Theta)\cos(\beta\cos\Theta)\,d\Theta = -\frac{\sqrt{2\pi\beta}}{\alpha}\sum_{1}^{\infty}{}_{s}\frac{\left[-\frac{(\alpha)^2}{2\beta}\right]^s}{s!}\,J_{s-1/2}(\beta)$$

By using Glaisher's formula (') and observing that $J_{-1/2}\left(\frac{\pi}{2}\right) = 0$, one derives

$$(A3\text{-}4) \qquad \int_0^{\pi} J_1(\alpha \sin\Theta)\cos(\beta\cos\Theta)\,d\Theta = -\frac{2}{\alpha}\cos\left(\sqrt{\beta^2 + \alpha^2}\right)$$

APPENDIX 4

Electromagnetic torque on a turnstile antenna, evaluated by means of the mutual actions between charges and currents.

The current distributions on the two half-wave antennas constituting the turnstile of Fig.A4-1 are denoted by:

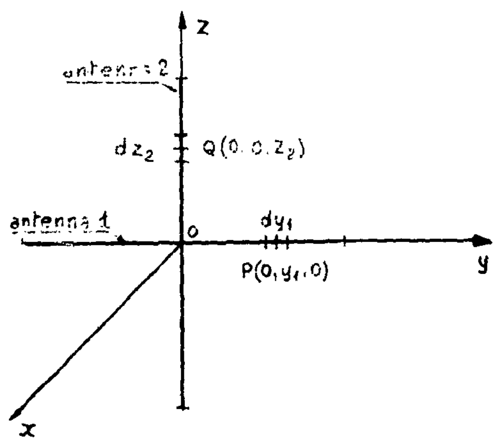

Fig.A4-1

(') see Ref.28 p.140 Eq.(3)

$$i_1 = I_o \cos(ky_1)\, e^{i\omega t}$$

(A4-1)

$$i_2 = i\, I_o \cos(kz_2)\, e^{i\omega t} \qquad k = \frac{2\pi}{\lambda}$$

The distributions of charges are:

$$\rho_1 = \frac{-i}{c} I_o \sin(ky_1)\, e^{i\omega t}$$

(A4-2)

$$\rho_2 = \frac{1}{c} I_o \sin(kz_2)\, e^{i\omega t}$$

Here and in the sequel the convention for the indexes 1 and 2 is the same as established on section 1.3. In (A4-1) and (A4-2) y_1 and z_2 represent the distances of two points, of the antenna 1 and of the antenna 2 respectively, from the origin O of a system of rectangular coordinates. The two antennas cross each other at O. $\underline{i}$, $\underline{j}$, $\underline{k}$ represent the unit vectors of the reference system. The time average $d\underline{F}$ of the force acting on the element dz_2 of antenna 2 at $Q = (0,0,z_2)$, due to the distributions of charges and current on antenna 1 is:

$$\text{(A4-3)} \qquad d\underline{F} = \frac{1}{2} \operatorname{Re}\left[\rho_2(Q)\, \underline{E}_1^*(Q) + i_2(Q)\underline{k} \wedge \underline{B}_1^*(Q)\right] dz_2$$

where $\underline{E}_1(Q)$ and $\underline{B}_1(Q)$ denote the electric vector and the magnetic induction vector, produced at Q by the charges and currents on antenna 1.

Let $\underline{A}_1$ and φ_1 represent the vector and scalar potentials, due to antenna 1. One has, at a point $M \equiv (x, y, z)$:

$$\text{(A4-4)} \qquad A_1 = A_1 \underline{j} = \underline{j}\, \frac{\mu_o}{4\pi}\, e^{i\omega t} \int_{-\lambda/4}^{\lambda/4} \frac{i_1(y_1)}{r}\, e^{-ikr}\, dy_1$$

$$\text{(A4-5)} \qquad \varphi_1 = \frac{1}{4\pi\varepsilon_o}\, e^{i\omega t} \int_{-\lambda/4}^{\lambda/4} \frac{\rho_1(y_1)}{r}\, e^{-ikr}\, dy_1$$

Where:

$$r = \sqrt{x^2 + (y - y_1)^2 + z^2}$$

The field vectors at Q are:

$$\underline{E}_1(Q) = -\,\mathrm{grad}_Q\, \varphi_1 - i\,\omega\, \underline{A}_1(Q)$$

$$\underline{B}_1(Q) = \mathrm{curl}_Q\, \underline{A}_1 \quad .$$

Since Q is located on the z axis:

$$(A4\text{-}6)\qquad \left(\frac{\partial A_1}{\partial x}\right)_Q = \left(\frac{\partial A_1}{\partial r}\right)_Q \left(\frac{\partial r}{\partial x}\right)_Q = \left(\frac{\partial A_1}{\partial r}\right)_Q \frac{x}{r} \quad = 0$$

$$(A4\text{-}7)\qquad \left(\frac{\partial \varphi_1}{\partial x}\right)_Q = \left(\frac{\partial \varphi_1}{\partial r}\right)_Q \left(\frac{\partial r}{\partial x}\right)_Q \quad = 0$$

and:

$$\frac{\partial}{\partial z}\varphi_1 = \frac{\partial}{\partial z}\frac{1}{4\pi\varepsilon_0}e^{i\omega t}\int_{-\lambda/4}^{\lambda/4} \frac{-i}{c} I_0 \sin(ky_1)\frac{e^{-ik\sqrt{x^2+(y-y_1)^2+z^2}}}{\sqrt{x^2+(y-y_1)^2+z^2}}\,dy_1 =$$

$$= \frac{1}{4\pi\varepsilon_0}\, e^{i\omega t}\,\frac{-iI_0}{c}\int_{-\lambda/4}^{\lambda/4} \sin(ky_1)\,e^{-ik\sqrt{x^2+(y-y_1)^2+z^2}}\left(1-\frac{1}{\sqrt[3]{x^2+(y-y_1)^2+z^2}}\right)dy_1$$

At Q, since x and y are equal to zero:

$$\left(\frac{\partial}{\partial z}\varphi_1\right)_Q = \frac{-iI_0}{4\pi\varepsilon_0 c}\int_{-\lambda/4}^{\lambda/4} \sin(ky_1)\,e^{-ik\sqrt{y_1^2+z_2^2}}\left(1-\frac{1}{\sqrt{\left(y_1^2+z_2^2\right)^2}}\right)dy_1$$

The function to be integrated is odd with respect to y = 0, therefore

$$(A4\text{-}8)\qquad \left(\frac{\partial \varphi_1}{\partial z}\right)_Q = 0$$

Thus:

$$\text{(A4-9)} \qquad \underline{B}_1(Q) = -\, i\left(\frac{\partial \underline{A}_1}{\partial z}\right)_Q$$

and:

(A4-10)

$$\underline{E}_1(Q) = -\left(\frac{\partial \varphi_1}{\partial y}\right)_Q - i\omega\, \underline{A}_1(Q)$$

Hence Eq.(A4-3) becomes:

$$\text{(A4-11)} \quad \underline{dF} = \frac{1}{2}\mathrm{Re}\left[-\rho_2(Q)\left(\frac{\partial \varphi_1}{\partial y}\right)_Q^{*} \underline{j} + i\omega\rho_2(Q)\underline{A}_1^{*}(Q) - i_2(Q)\underline{k}\wedge\underline{i}\left(\frac{\partial \underline{A}_1}{\partial z}\right)_Q^{*} dz_2\right] =$$

$$= -\frac{1}{2}\mathrm{Re}\left[\rho_2(Q)\left(\frac{\partial \varphi_1}{\partial y}\right)_Q^{*} - i\omega\rho_2(Q)A_1^{*}(Q) + i_2\left(\frac{\partial A_1}{\partial z}\right)_Q^{*}\right]\underline{j}\, dz_2$$

where ρ_2, i_2, $\frac{\partial\varphi}{\partial y}$, A_1 are evaluated at Q.

One has:

$$\text{(A4-12)} \quad -\frac{1}{2}\mathrm{Re}\, \rho_2\left(\frac{\partial \varphi_1}{\partial y}\right)^{*} = \frac{I_o^2}{8\pi\varepsilon_o c^2} \sin(kz_2)\, \frac{\partial}{\partial y}\int_{-\lambda/4}^{\lambda/4} \sin(ky_1)\, \frac{\sin(kr)}{r}\, dy_1$$

$$(A4\text{-}13) \qquad \frac{1}{2}\,\mathrm{Re}(i\omega\rho_2 A_1^{*}) = -\frac{\mu_o \omega I_o^2}{8\pi c}\,\sin(kz_2)\int_{-\lambda/4}^{\lambda/4} \cos(ky_1)\,\frac{\sin(kr)}{r}\,dy_1$$

and:

$$(A4\text{-}14)$$

$$-\frac{1}{2}\,\mathrm{Re}\; i_2\left(\frac{\partial A_1}{\partial z}\right)^{*} = \frac{\mu_o I_o^2}{8\pi}\,\cos(kz_2)\,\frac{\partial}{\partial z}\int_{-\lambda/4}^{\lambda/4} \cos(ky_1)\,\frac{\sin(kr)}{r}\,dy_1$$

Thus Eq.(A4-11) becomes:

$$(A4\text{-}15) \qquad d\underline{F} = \frac{\mu_o}{8\pi}\,I_o^2\Bigg[\sin(kz_2)\,\frac{\partial}{\partial y}\int_{-\lambda/4}^{\lambda/4} \sin(ky_1)\,\frac{\sin(kr)}{r}\,dy_1 \; -$$

$$-\; k\,\sin(kz_2)\int_{-\lambda/4}^{\lambda/4} \cos(ky_1)\,\frac{\sin(kr)}{r}\,dy_1 \; +$$

$$+\; \cos(kz_2)\,\frac{\partial}{\partial z}\int_{-\lambda/4}^{\lambda/4} \cos(ky_1)\,\frac{\sin(kr)}{r}\,dy_1\Bigg]\,\underline{i}\;dz_2$$

Let us now consider an element dy_1 of antenna 1, placed at the point $P\equiv(0,\, y_1,\, 0)$. The force $d\underline{F}'$ acting on dy_1, due to the charges and currents on the antenna 2 is:

$$(A4\text{-}16) \qquad d\underline{F}' = \frac{1}{2}\,\mathrm{Re}\left[\rho_1(P)\underline{E}_2^*(P) + i_1(P)\underline{j}\wedge\underline{B}_2^*(P)\right] dy_1$$

where notations analogous to those of Eq.(A4-3) have been used: One has:

$$\underline{E}_2(P) = -\,\mathrm{grad}_P\,\varphi_2 - i\omega\underline{A}_2(P)$$

$$\underline{B}_2(P) = \mathrm{curl}\ \underline{A}_2$$

where:

$$(A4\text{-}17) \qquad \underline{A}_2 = A_2\underline{k} = \frac{\mu_o}{4\pi}\,\underline{k}\,e^{i\omega t}\int_{-\lambda/4}^{\lambda/4} \frac{i_2(z_2)}{r}\,e^{-ikr}\,dz_2$$

$$(A4\text{-}18) \qquad \varphi_2 = \frac{1}{4\pi\varepsilon_o}e^{i\omega t}\int_{-\lambda/4}^{\lambda/4} \frac{\rho_2(z_2)}{r}\,e^{-ikr}\,dz_2$$

The following relations can be derived, analogous to Eqs.(A4-6) through (A4-10):

$$(A4\text{-}19) \qquad \left(\frac{\partial A_2}{\partial x}\right)_P = 0$$

(A4-20) $$\left(\frac{\partial \varphi_2}{\partial x}\right)_P = 0$$

(A4-21) $$\left(\frac{\partial \varphi_2}{\partial y}\right)_P = 0$$

Hence :

(A4-22) $$\underline{B}_2(P) = \left(\frac{\partial A_2}{\partial y}\right)_P \underline{i}$$

(A4-23) $$\underline{E}_2(P) = -\left(\frac{\partial \varphi_2}{\partial z}\right)_P \underline{k} - i\omega A_2(P)\underline{k}$$

Eq.(A4-13) becomes:

(A4-24)
$$d\underline{F}' = \frac{1}{2}\,\mathrm{Re}\left[-\rho_1(P)\left(\frac{\partial \varphi_2}{\partial z}\right)^{*}_P \underline{k} + i\omega\rho_1(P)A_2^{*}(P)\underline{k} + i_1(P)\left(\frac{\partial A_2}{\partial y}\right)^{*}_P \underline{j}\wedge\underline{i}\right] dy_2 =$$

$$= -\frac{1}{2}\,\mathrm{Re}\left[\rho_1(P)\left(\frac{\partial \varphi_2}{\partial y}\right)^{*}_P - i\omega\rho_1(P)A_2^{*}(P) + i_1(P)\left(\frac{\partial A_2}{\partial y}\right)^{*}_P\right]\underline{k}\; dy_2$$

Further one has:

(A4-25)
$$-\frac{1}{2}\,\mathrm{Re}\;\rho_1(P)\left(\frac{\partial \varphi_2}{\partial z}\right)^{*}_P = \frac{-I_o^2}{8\pi\varepsilon_o c^2}\,\sin(ky_1)\frac{\partial}{\partial z}\int_{-\lambda/4}^{\lambda/4} \sin(kz_2)\frac{\sin(kr)}{r}\,dz_2$$

$$(A4-26)\quad \frac{1}{2}\mathrm{Re}\ i\omega\rho_1(P)A_2^{*}(P) = -\frac{I_o^2\omega\mu_o}{8\pi c}\sin(ky_1)\int_{-\lambda/4}^{\lambda/4}\sin(kz_2)\frac{\sin(kr)}{r}\,dz_2$$

$$(A4-27)\quad -\frac{1}{2}\mathrm{Re}\ i_1(P)\left(\frac{\partial A_2}{\partial y}\right)_P^{*} = -\frac{-\mu_o I_o^2}{8\pi}\cos(ky_1)\frac{\partial}{\partial y}\int_{-\lambda/4}^{\lambda/4}\cos(kz_2)\frac{\sin(kr)}{r}dz_2$$

Let the points P an Q be chosen at equal distances apart from the center of the antenna.

By comparing Eq.(A4-25) with Eq.(A4-12), Eq.(A4-26) with Eq.(A4-13), Eq.(A4-27) with Eq.(A4-14), and taking into account Eqs.(A4-11) and (A4-24), one has:

$$\underline{dF}\cdot\underline{j} = -\ \underline{dF'}\cdot\underline{k}$$

Fig.A4-2 shows the directions of the forces acting on the

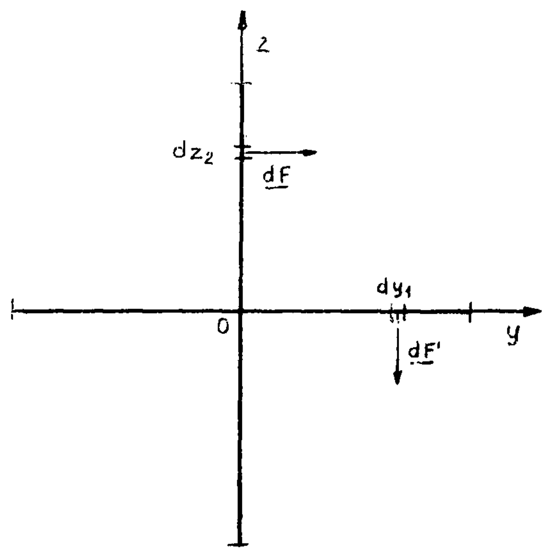

Fig.A4-2

elements dy_1 and dz_2, and indicates that the moments of the two forces with respect to O, are of the same sign.

The integral in (A4-15) will be performed by means of a series approximation of the function $\frac{\sin kr}{r}$:

$$\frac{\sin kr}{r} = k - \frac{1}{6} k^3 r^2 + \frac{1}{5!} k^5 r^4$$

The function and its derivatives with respect to y and z, calculated at point Q, are then approximated as:

$$\left(\frac{\sin kr}{r}\right)_Q = k - \frac{1}{6} k^3 \left(y_1^2 + z_2^2\right) + \frac{1}{5!} \left(y_1^2 + z_2^2\right)^2 k^5 \tag{A4-28}$$

$$\left(\frac{\partial}{\partial y} \frac{\sin kr}{r}\right)_Q = \frac{1}{3} k^3 y_1 - \frac{4}{5!} k^5 \left(y_2^3 + y_1 z_2^2\right) \tag{A4-29}$$

$$\left(\frac{\partial}{\partial z} \frac{\sin kr}{r}\right)_Q = \frac{1}{3} k^3 z_2 + \frac{4}{5!} k^5 \left(y_1^2 + z_2^2\right) z_2 \tag{A4-30}$$

By substituting (A4-28), (A4-29) and (A4-30) into (A4-15) and performing the integrations, one obtains

$$d\underline{F}\cdot\underline{j} = \frac{k^2 I_o^2}{4\pi\varepsilon_o c^2} \left\{ \frac{k^2}{30} z^3 \cos(kz) + \frac{2}{15} kz^2 \sin(kz) - \right. \tag{A4-31}$$

$$\left. - \frac{19}{60} z \cos(kz) - \frac{\sin(kz)}{k} 0.635 \right\}$$

Eq.(A4-31) gives the distribution of the force acting on antenna 2, due to antenna 1. Eq.(A4-31) will now be used to evaluate the resulting moment $\underline{T}$ of the distribution of force, with respect to the center of the antenna.

This moment is:

$$\underline{T} = \int_{-\lambda/4}^{\lambda/4} d\underline{F} \wedge z_2 \underline{k}$$

And one obtains the result:

$$\underline{T} = 1.9 \quad 10^{8} \lambda I^{2} \underline{i} \quad (\mathbf{M} \text{ Joules, } \lambda \text{ meters, I Amperes}) \tag{A4-32}$$

The torque $\underline{M}$ acting on the turnstile antenna is twice the value indicated by Eq.(A4-32); therefore:

$$\underline{M} = 3.8 \quad 10^{-8} \lambda I^{2} \underline{i} = 71.6 \frac{I^{2}}{\omega} \underline{i}$$

By introducing the radiation resistance **R = 73,12 ohm**, of the half-wave antenna, one obtains:

$$\underline{\mathbf{M}} = 0.98 \frac{W}{\omega} \underline{i}$$

where **W** is the power radiated by the turnstile antenna.

This approximate result is to be compared with the exact result given on p. 21.

$$\underline{M} = 0.9939 \frac{w}{\omega} \underline{i}$$

The difference is **practically negligible.**

APPENDIX 5

By following the procedure of Appendix 3, one has, for the second integral on right side of (2.2-41),

$$\text{(A5-1)} \qquad \int_0^\pi J_1(\alpha \sin\Theta)\cos(\beta \cos\Theta)\cos^2\Theta \, d\Theta =$$

$$= \sum_{0}^{\infty} {}_m \frac{(-1)^m \left(\frac{\alpha}{2}\right)^{2m+1}}{m!(m+1)!} \int_0^\pi \sin^{2m+1}\Theta \cos(\beta \cos\Theta) \left[1 - \sin^2\Theta\right] d\Theta =$$

$$= -\frac{2}{\alpha} \cos\left(\sqrt{\alpha^2+\beta^2}\right) - \sqrt{\pi} \sum_{0}^{\infty} {}_m \frac{(-1)^m \left(\frac{\alpha}{2}\right)^{2m+1}}{m!} J_{m+3/2}(\beta) \left(\frac{\beta}{2}\right)^{-(m+3/2)} =$$

$$= -\frac{2}{\alpha}\cos\left(\sqrt{\alpha^2+\beta^2}\right) - \sqrt{2\pi}\,\alpha \sum_{0}^{\infty} {}_m \frac{\left[-\left(\frac{\alpha}{2}\right)^2\right]^m}{m!} (\beta)^{-(m+3/2)} J_{m+3/2}(\beta)$$

Recalling Lommel's formula ('), (A5-1) becomes

$$\text{(A5-2)} \qquad \int_0^{\pi} J_1(\alpha \sin\theta)\cos(\beta \cos\theta)\cos^2\theta \, d\theta =$$

$$= -\frac{2}{\alpha}\cos\left(\sqrt{\alpha^2+\beta^2}\right) - \sqrt{2\pi}\;\alpha \frac{J_{3/2}\left(\sqrt{\alpha^2+\beta^2}\right)}{\left(\alpha^2+\beta^2\right)^{3/4}} =$$

$$= -\frac{2}{\alpha}\cos\left(\sqrt{\alpha^2+\beta^2}\right) + 2\alpha \frac{\cos\left(\sqrt{\alpha^2+\beta^2}\right)}{\alpha^2+\beta^2} - \frac{2\alpha \sin\left(\sqrt{\alpha^2+\beta^2}\right)}{\left(\alpha^2+\beta^2\right)^{3/2}}$$

(') see Ref.28 p.140 Eq.(1)

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Monitoring Agency — Equipment

ELECTROMAGNETIC CONTROL OF THE ATTITUDE OF SPACE VEHICLES P.Bruscaglioni, A.Consortini Project director: G.Toraldo di Francia, Dec.'62

A theoretical investigation has been made of the possibility to apply the properties of elliptically polarized waves for the attitude control of space vehicles. Three different ways of applying electromagnetic radiation has been discussed. The first one employes an antenna placed on a space vehicle, and radiating with

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