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CYBERNETICS AND CONTROL THEORY

-USSR-

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ON THE INVERSION OF FINITE AUTOMATA

[Following is the translation of an article by V.I. Levenshteyn in the Russian-language periodical Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences USSR), Vol 147, No 6, 1962, pages 1300-1303.]

(Presented by Academician M.V. Keldysh 4 July 1962)

Recently, certain studies (refs 6-8) have taken note of the common ground between certain problems in coding theory and the theory of automata. Thus, problems of one-to-one coding correspondence and the possibility of constructing a deconding device with a finite memory have been essentially reduced to the problems of the mutual equivalence of finite automata and the possibility of their inversion. It is true, however, that this has necessitated some extension of the concept of the automatic device, with the elimination of the requirement of synchronism, i.e., the equality of the length of the input and corresponding output words. In his study, Yu.V. Glebskiy (ref 6) found an effective criterion for the recognition of the mutual equivalence of a fully defined finite automaton over a finite-countable set. The present paper deals with effectively checkable necessary and sufficient conditions for the inversion of (asynchronous) partial finite automata, as well as methods of constructing inverse automata. In addition, we construct recognition algorithms for certain other properties of partial finite automata.

1. Let $\mathfrak{U} = \{A, B, S, s_{i_0}, f, \varphi\}$ be a partial finite automaton, where $A = \{a_1, \ldots, a_m\}$ is the input alphabet; $B = \{b_1, \ldots, b_k\}$ is the output alphabet; $S = \{s_0, \ldots, s_N\}$ is the set of states; s_{i_0} is the initial state; f and φ are respectively the transition function and output function of the automaton, defined over a certain subset $\mathfrak{N} \subseteq S \times A$, moreover, $f(s_i, a_j) = s_j^i$, $\varphi(s_i, a_j) = v_j^i$ where $s_j^i \in S$, and v_j^i is a certain (perhaps empty) word in alphabet B. The automaton $\mathfrak{U}$ is called <u>synchronous</u>, if each word v_j^i is a letter in alphabet B. The automaton $\mathfrak{U}$ induces a function $F_{\mathfrak{U}}$

defined on each word of $\alpha = a_{j_1} a_{j_2} \ldots a_{j_\ell}$ for which $(s_{i_{p-1}}, a_{j_p}) \in \mathfrak{R}$, $p = 1, 2, \ldots \ell$, where $s_{i_p} = f(s_{i_{p-1}}, a_{j_p})$, there assuming the value $F_{\mathfrak{U}}(\alpha) = v_{j_1}^{i_0} v_{j_2}^{i_1} \ldots v_{j_\ell}^{i_{\ell-1}}$. In addition, the automaton $\mathfrak{U}$ induces a a function $F_{\mathfrak{U}}^{\infty}$, defined over each infinite sequence $\bar{\alpha} = a_{j_1} a_{j_2} \ldots$, for which $(s_{i_{p-1}}, a_{j_p}) \in \mathfrak{R}$, $p = 1, 2$, where $s_{i_p} = f(s_{i_{p-1}}, a_{j_p})$ and there assumes the value $F_{\mathfrak{U}}^{\infty}(\bar{\alpha}) = v_{j_1}^{i_0} v_{j_2}^{i_1} \ldots$. The defined regions of functions $F_{\mathfrak{U}}$ and $F_{\mathfrak{U}}^{\infty}$ we shall denote by $I_{\mathfrak{U}}$ and $I_{\mathfrak{U}}^{\infty}$ spectively. The state into which the automaton passes from state s_i under the action of the word α from the alphabet will be denoted by $f(s_i, \alpha)$. The two automata $\mathfrak{U}$ and $\mathfrak{V}$ will be considered equivalent if for any word α in alphabet A we have [see note] $F_{\mathfrak{U}}(\alpha) = F_{\mathfrak{V}}(\alpha)$. Inasmuch as we are concerned with automata with an accuracy up to the limit of equivalence, we can assert without any loss of generality that all states of the automaton $\mathfrak{U}$ are distinguishable and for any state $s_i \in S$ there exists a word $\alpha \in I_{\mathfrak{U}}$ such that $s_{\ell} = f(s_{i_0}, \alpha)$. ([Note:] Here, as usual, $F_{\mathfrak{U}}(\alpha) = F_{\mathfrak{V}}(\alpha)$ means that either both functions are undefined for the word α or are defined for this word and take on the smae values).

The automaton $\mathfrak{U}$ will be called <u>mutually-single-valued</u> if for anny distinct words α_1 and α_2 from $I_{\mathfrak{U}}$ we have $F_{\mathfrak{U}}(\alpha_1) \neq F_{\mathfrak{U}}(\alpha_2)$. The automaton $\mathfrak{U}$ will be called <u>mutually-single-valued in the weak sense</u> or an automaton <u>without information loss</u> [see note] if for any distinct words α_1 and α_2 from $I_{\mathfrak{U}}$, such that $F_{\mathfrak{U}}(\alpha_1) = F_{\mathfrak{U}}(\alpha_2)$, the states $f(s_{i_0}, \alpha_1)$ and $f(s_{i_0}, \alpha_2)$ are distinct. The automaton $\mathfrak{U}$ will be called <u>mutually-single-valued over infinity</u> if for any distinct sequences $\bar{\alpha}_1$ and $\bar{\alpha}_2$ from $I_{\mathfrak{U}}^{\infty}$ we have $F_{\mathfrak{U}}^{\infty}(\bar{\alpha}_1) \neq F_{\mathfrak{U}}^{\infty}(\bar{\alpha}_2)$. ([Note:] The latter name was suggested by Huffman (ref 3) who studied such automata in detail for the synchronous case. This name reflects the fact that a fully-defined automaton is mutually-single-valued in the weak sense if and only if there exists an experiment (ref 2) of finite length with automaton $\mathfrak{U}$ processing a random word α (unknown to the experimenter) which makes possible the determination of the word α from the output $F_{\mathfrak{U}}(\alpha)$ and the results of the experiment).

To simplify further formulations, let us impose the following limitation on the region of definition for automaton $\mathfrak{U}$: each word from $I_{\mathfrak{U}}$ is the start of at least two distinct sequences from $I_{\mathfrak{U}}^{\infty}$. With this limitation, the following lemma will hold:

Lemma 1. If the automaton $\mathfrak{A}$ is mutually-single-valued over infinity, it is mutually-single-valued in the weak sense.

It is obvious that a mutually-single-valued automaton is mutually-single-valued in the weak sense. Figure 1 shows a diagram of an automaton mutually-single-valued in the weak sense which is neither mutually-single-valued not mutually-single-valued over infinity.

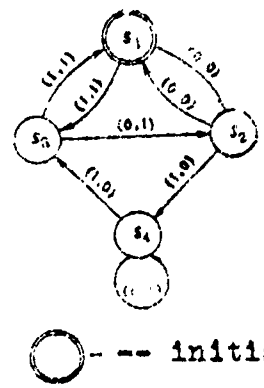

Figure 1

If for the automaton $\mathfrak{A}$ there exists an automaton such that $F_{\mathfrak{B}}^{\infty} = (F_{\mathfrak{A}}^{\infty})^{-1}$, the automaton $\mathfrak{A}$ will be called <u>invertible</u>, and automaton $\mathfrak{B}$ will be called the <u>inverse</u> of $\mathfrak{A}$.

It is easy to see that for a synchronous automaton there exists a synchronous automaton inverse to $\mathfrak{A}$ if and only if automaton $\mathfrak{A}$ is mutually-single-valued (refs 3, 4). The diagram of the inverse automaton is obtained form the diagram of automaton $\mathfrak{A}$ through the juxtaposition of the input and output letters assigned to the connective ribs. Let us note that the mutual single-valuedness of the automaton is not necessary even for the inversion of a synchronous automaton (Figs 2 and 3).

The difficulties arising in the general case are due mostly to the following factors : 1) the class of functions $F_{\mathfrak{A}}^{\infty}$ induced by the automata is considerably wider than the class of functions $F_{\mathfrak{A}}^{\infty}$ induced by the synchronous automata; 2) for any automaton $\mathfrak{A}$ there exists a countable number of paired nonequivalent automata inducing $F_{\mathfrak{A}}^{\infty}$ (as distinct from the unitary automaton with an accuracy up to the equivalence of the automaton in the synchronous case).

2. To construct effective criteria permitting us to recognize the indicated properties of automaton $\mathfrak{A}$, let us

introduce the sets $R_n(\mathfrak{U})$, $(n=1,2,\ldots)$ analogous to the Sardinas and Patterson classes (ref 1). The sets $R_n(\mathfrak{U})$ are determined by induc their elements are ordered triplets of the form (β, i, h), where β is the ending of some word v_j^k, and i and h are state numbers. The set $R_{n+1}(\mathfrak{U})$ is defined as the combination of all elements (β, i, h) for which there exist

$(\beta, i, h) \in R_n(\mathfrak{U})$ and a word v_j^i such that $s_{i'} = s_i, s_{h'} = f(s_i, a_j)$ and $\beta\beta' = v_j^i$ or $s_{i'} = f(s_i, a_j), s_{h'} = s_h$ and $\beta = v_j^i \beta'$.

Lemma 2. The element (β, i, h) belongs to $R_n(\mathfrak{U})$ if and only if there exist numbers $k \geq 1$, $\ell \geq 1$, $k+\ell = n+1$; $i_1, \ldots, i_k, i_{k+1} = i$, $j_1, \ldots, j_k$; $h_1, \ldots, h_\ell, h_{\ell+1} = h$, $d_1, \ldots, d_\ell$ such that $s_{i_{t+1}} = f(s_{i_t}, a_{j_t})$, $t = 1, \ldots, k$; $s_{h_{p+1}} = f(s_{h_p}, a_{d_p})$, $p = 1, \ldots, \ell$; and

$v_{j_1}^{i_1} v_{j_2}^{i_2} \cdots v_{j_k}^{i_k} \beta = v_{d_1}^{h_1} v_{d_2}^{h_2} \cdots v_{d_\ell}^{h_\ell}$ where $i_1 = h_1$, $j_1 \neq d_1$, and the word β is the ending of word $v_{d_\ell}^{h_\ell}$.

Let us establish the following notation: an empty word is Λ, the length of word β in alphabet B is $\lambda(\beta)$, and the maximum length of words v_j^i is λ_{max}. From lemma 2 it follows that there exist not more than $N^2(m\lambda_{max}+1)$ elements which can make up the sets $R_n(\mathfrak{U})$. Consequently, these sets begin to recur periodically at some point.

Theorem 1. Automaton $\mathfrak{U}$ will be mutually-single-valued if and only if all words v_j^i are nonempty and no set $R_n(\mathfrak{U})$ contains words of the form (Λ, i, h) for $n \leq Nm(N\lambda_{max}-1)+N(N-1)/2+2$.

Theorem 2. In order for the automaton $\mathfrak{U}$ to be mutually-single-valued in the weak sense, it is necessary and sufficient that no set $R_n(\mathfrak{U})$ contain elements of the form (Λ, i, i) for $n \leq Nm(N\lambda_{max}-1)+N(N-1)/2+2$.

Theorem 3. In order for the automaton $\mathfrak{U}$ to be mutually-single-valued over infinity, it is necessary and sufficient that all sets $R_n(\mathfrak{U})$ be empty starting with some $n \leq Nm(N\lambda_{max}-1)+N(N-1)2+1$

Theorems 1-3 lead to algorithms for the determination of the the investigated properties of automaton $\mathfrak{U}$ consisting in the ordered construction and study of a finite number of sets $R_n(\mathfrak{U})$. These algorithms are generalizations of the corresponding algorithms in refs 1 and 5.

Theorem 4. Automaton $\mathfrak{U}$ will be invertible if and only if it is mutually-single-valued over infinity.

The necessity of the above condition is evident. Now let automaton $\mathfrak{U}$ be mutually-single-valued over infinity.

Then, by Theorem 3 there exists a minimum number $n(\mathfrak{A})$ such that all sets $R_n(\mathfrak{A})$ are empty for $n \geq n(\mathfrak{A})$. We now proceed to describe methods of constructing automata $\mathfrak{A}'$ and $\mathfrak{A}''$ inverse to $\mathfrak{A}$.

<u>Method for constructing automaton $\mathfrak{A}'$ (Fig 3a)</u>. Let us denote by $\mathfrak{N}'$ the set of all pairs (β, i) such that $\beta = v_{j_1}^{i_1} v_{j_2}^{i_2} \ldots v_{j_k}^{i_k} \gamma_{j_{k+1}}^{i_{k+1}}$, where $k \geq 0$, $\gamma_{j_{k+1}}^{i_{k+1}}$ is the beginning of word and the numbers $i_1 = i, i_2, \ldots, i_k, i_{k+1}, j_1, \ldots, j_k$ are such that $s_{i_{p+1}} = f(s_{i_p}, a_{j_p})$, $p = 1, \ldots, k$. The combination of numbers $\binom{i_1, i_2, \ldots, i_{k+1}}{j_1, j_2, \ldots, j_{k+1}}$ will be called the i-decoding of the word β. Let $\binom{i_1(t), i_2(t), \ldots, i_{k(t)+1}(t)}{j_1(t), j_2(t), \ldots, j_{k(t)+1}(t)}$, $i_1(t) = i$, $t = 1, \ldots, M$ be all i-decodings of the word β and $k'(\beta, i)$ the greatest number such that $k(t) \geq k'$, $i_p(t) = i_p$, $j_p(t) = j_p$, $t = 1, \ldots, M$; $p = 1, \ldots, k'$. By Theorem 3 and Lemma 2 there exists a minimum number $T \leq [n(\mathfrak{A})/2]$ (the same for all pairs $(\beta, i) \in \mathfrak{N}'$) such that if for some $t (t = 1, \ldots, M)$ we have $k(t) \geq 1$ and $\lambda(v_{j_2(t)}^{i_2(t)} \ldots v_{j_{k(t)}(t)}^{i_{k(t)}(t)} \gamma_{j_{k(t)+1}}^{i_{k(t)+1}}) \geq T$, then $k(t) \geq 1$ for each $t (t = 1, \ldots, M)$ and the number $k' > 0$. For the pairs $(\beta, i) \in \mathfrak{N}'$ let us define the following functions: $\psi_1(\beta, i) = i_{k'+1}$, $\psi_2(\beta, i) = v_{j_{k'+1}}^{i_{k'+1}} \ldots v_{j_k}^{i_k} \gamma_{j_{k+1}}^{i_{k+1}}$, $\psi_3 = a_{j_1} \ldots a_{j_{k'}}$. In particular, if $k' = 0$, then $\psi_1(\beta, i) = i$, $\psi_2(\beta, i) = \beta$, $\psi_3(\beta, i) = \Lambda$. We denote by $\mathfrak{M}_i$ the set of words β such that $(\beta, i) \in \mathfrak{N}'$ and $\psi_3(\beta, i) = \Lambda$. As the states of the automaton $\mathfrak{A}'$ we take the symbols $q_{\gamma\beta}^{i}$ where $\beta \in \mathfrak{M}_i$, $i = 1, \ldots, N$ including the symbol $q_{\gamma\Lambda}^{i_0}$ (even if $\Lambda \notin \mathfrak{M}_{i_0}$) which we shall consider the initial state. It is obvious that the number of states does not exceed $N \cdot \frac{\tau^{T + \lambda_{max}} - 1}{\tau - 1}$. The functions f' and φ' of the transitions and outputs of automaton $\mathfrak{A}'$ will be defined on the pairs $(q_{\gamma\beta}^{i}, b_j)$, for which $(\beta b_j, i) \in \mathfrak{N}'$, in the following way: $f'(q_{\gamma\beta}^{i}, b_j) = q_{\gamma\psi_2(\beta b_j, i)}^{\psi_1(\beta b_j, i)}$, $\varphi'(q_{\gamma\beta}^{i}, b_j) = \psi_3(\beta b_j, i)$.

The construction ends with the unification of the undistinguishable states of the automaton and the rejection of those states which cannot be reached from the initial state $q_{\gamma\Lambda}^{i_0}$. ([Note:] From Lemma 1 and the fact that an automaton mutually-single-valued in the weak sense cannot produce more than N-1 empty words in sequence, it follows that the number of i - decondings of the word β is finite).

Method of constructing automaton $\mathfrak{A}''$ (see Fig 3b).

This method differs from the one preceiding merely in a different definition of the functions ψ_1, ψ_2 and ψ_3: if $k'=0$ or $k' \geq 1$ and $\lambda(v_{j_2}^{i_2} \dots v_{j_k}^{i_k} \gamma_{j_{k+1}}^{i_{k+1}}) < T$, then $\psi_1(\beta, i) = i$, $\psi_2(\beta, i) = \beta$, $\psi_3(\beta, i) = \Lambda$; if $k' \geq 1$ and $\lambda(v_{j_2}^{i_2} \dots v_{j_k}^{i_k} \gamma_{j_{k+1}}^{i_{k+1}}) = T$, then $\psi_1(\beta, i) = i_{k''+1}$, $\psi_2(\beta, i) = v_{j_{k''+1}}^{i_{k''+1}} \dots v_{j_k}^{i_k} \gamma_{j_{k+1}}^{i_{k+1}}$, $\psi_3(\beta, i) = a_{j_1} \dots a_{i_{k''}}$ where is the greatest number such that $1 \leq k'' \leq k'$ and $\lambda(\gamma_{j_{k''+1}}^{i_{k''+1}} \dots v_{j_k}^{i_k} \gamma_{j_{k+1}}^{i_{k+1}})$.

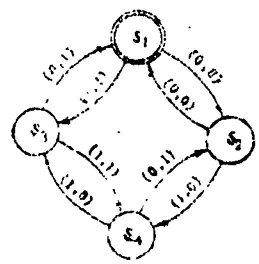

Figure 2. Automaton $\mathfrak{A}$

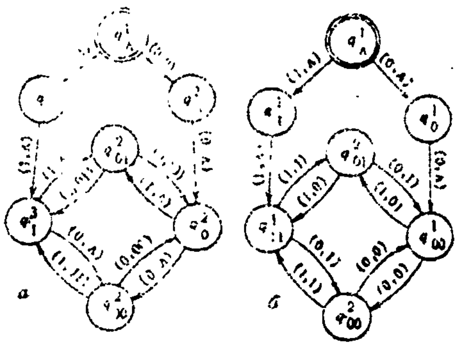

Figure 3. Automatons inverse to automaton $\mathfrak{A}$ (Fig 2). a -- automaton $\mathfrak{A}'$; b -- automaton $\mathfrak{A}''$.

The selection of automata $\mathfrak{A}'$ and $\mathfrak{A}''$ from the countable set of other automata inverse to $\mathfrak{A}$ is explained by their special significance in coding theory. If we assume that automaton $\mathfrak{A}$ is used for message encoding, automata $\mathfrak{A}'$ and $\mathfrak{A}''$ can be considered as deconding automata, the first of which decodes each message with a minimal lag not greater than T, while the second (in the case of non-empty words v_j^{i}) decodes each message with a minimal constant lag T. The automaton $\mathfrak{A}'$, as well as automaton $\mathfrak{A}''$ in the cited instance are determined by these conditions unambiguously with an accuracy up to the limit of equivalence.

Note 1. There exist invertible partial automata (see Fig 3, for example) for which any automata obtained by their further definition without an expansion of the output alphabet, are invertible.

Note 2. From Theorem 4 it follows that if $\mathfrak{A}$ is a finite automaton and the function $(F_{\mathfrak{A}})^{-1}$ is induced by the automaton having an infinite set of states, it is induced by some finite automaton

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SYNTHESIS OF A CLASS OF COMPUTERS

[Following is the translation of an article by M.V. Rybashov in the Russian-language periodical Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences USSR), Vol 147, No 6, 1962, pages 1304-1305.]

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In automatic control systems there often arises the need for devices to carry on automatic tracking of one of the roots $x^*(t) = (x_1^*, \ldots, x_n^*)$ of a system of finite equations

$$f_i(x_1, \ldots, x_n, u_1, \ldots, u_r) = 0 \qquad (i = 1, \ldots, n) \qquad (1)$$

with time-variable parameters $u_1, \ldots, u_r$.

In most cases the variables $x_1, \ldots, x_n$ are not expressed analytically in terms of indiependent variables, so that the problem is usually handled by analog computer methods (ref 1) which make possible functional transformations with functions $x_i = x_i(u_1, \ldots, u_r)$ given implicitly by system (1).

Usually (ref 1) a computer for the construction of implicit functions is designed according to the principle of continuous formation of the shifts $\xi_i = f_i$ in the course of changes in variables $u_1, \ldots, u_r$. Root tracking with the aid of such a device is unavoidably accompanied by an error in the established regime. With considerable speeds of variation of variables $u_1, \ldots, u_r$ the error may turn out to be considerable, which under certain conditions leads to root bifurcation.

In the present article we describe a method of synthesizing systems for tracking roots in the stable regime with an error equal to zero.

1°. Let us suppose that the functions f_i are continuously differentiable with respect to all their arguments in region D covered in the solution of system (1), and the functions $u_k(t)$ are finite and differentiable with respect to t. In addition, the system of functions $f_i (i = 1,\ldots,n)$ has a non-degenerate Jacobian $A = \{\partial f_i / \partial x_k\}$ everywhere in D.

Let us consider the system of equations

$$df_i/dt = \Phi_i(f_1,\ldots,f_n), \qquad i = 1,\ldots,n, \qquad (2)$$

where the functions Φ_i as functions of the arguments $f_1,\ldots,f_n$ everywhere satisfy the Lipschitz conditions and are chosen such that system (2) has a singular asymptotically stable rest point $f = 0$ $(f_1 = f_2 = \ldots = f_n = 0)$ in the phase space F with coordinate axes $f_1,\ldots,f_n$. As such a system we can have, for example, the following:

$$df_i/dt = -\lambda_i f_i, \qquad \lambda_i > 0, \; i = 1,\ldots,n. \qquad (3)$$

System (2) implicitly gives the system of equations with respect to variables $y_1,\ldots,y_n$

$$dy/dt = A^{-1}(\Phi - B du/dt) \qquad (4)$$

where dy/dt, du/dt, Φ are column matrices of the derivatives and functions Φ_i respectively; A^{-1} is the inverse of the Jacobian; B is the matrix (n X r) of derivatives $\partial f_i / \partial u_k$. System (2) has the following properties. Any solution $x_k^*(t)$ (where k is the number of the solution) of system (1) is a special solution of system (4) because the point $f = 0$ is the rest point of system (2). With initial conditions $y(t_0) = x_k(t_0)$ satisfying equations (1), the solution $y(t, y(t_0))$ of the system of equations (4) for all $t > t_0$ will satisfy the system (1), since for $t > t_0$ as a result of (2) we have

$$f_i(y(t, y(t_0)), u(t)) \equiv 0, \qquad i = 1,\ldots,n.$$

If for $t = 0$ not all $\xi_i(t_0) = 0$,

$$\xi_i(t_0) = f_i(y(t_0), u(t_0)),$$

then because of the asymptotic stability at t t_0, all the ξ_i will tend to zero, and as a result of the differentiability of the functions f_i and the finiteness of the function $u_j(t)$, the following evaluation will hold:

$$\left(\sum_{i=1}^{n} (x_i^*(t) - y_i(t))^2 \right)^{1/2} \leq c_{max} \{ \xi_i, \ldots, \xi_n \} ,$$

$$c = const.$$

From the evaluation it follows that the phase trajectories $y(t) \equiv x_k(t)$ (where k is the number of the root of system (1)) are asymptotically stable.

The dynamic error will fall off with time. In practice this means that upon the completion of the transient process the error of the stable regime will be zero. At the same time, there is a fixation of the root.

Equation (4) is realized by means of analog techniques such as computing elements in electronic models.

With the aid of system (4) it is likewise possible to find the roots of finite equations (ref 2). In this case $u_j \equiv const$, $\dot{u}_j \equiv 0$, $j = 1, \ldots, n$. The rest points of the equations

$$dy/dt = A^{1}\Phi \quad , \; dy/dt = (\det A)^{n} C \Phi \quad ,$$

where C is a matrix adjoint to matrix A, are asymptotically stable and coincide with the roots of system (1). With the specification of initial conditions in the region of attraction of a given root, the representing point will henceforth tend toward that root.

2°. Let us consider the matrix equation:

$$df/dt = 0.$$

The rest point $f = 0$ in this equation is Lyapunov-stable, as are the trajectories $y(t) \equiv x_k^*(t)$ of the equation

$$dy/dt = -\bar{A}^{1} B(du/dt). \qquad (5)$$

The computer with equation of motion (5) makes it possible to perform tracking of the root with an error not exceeding ε . Indeed, with the trajectory Lyapunov-stable for a given $\varepsilon > 0$, there exists a $\delta(\varepsilon) > 0$ such that if the error in the initial conditions

$\varepsilon_0 = \left(\sum_{i=1}^{n} (x_i^*(t_0) - y_i(t_0))\right)^{1/2}$ does not exceed δ, then with $t > t_0$ the error will not exceed the specified figure ϵ.

Automation and Telemechanics Institute of the State Committee on Automation and Machine Building of the Council of Ministers USSR and the Academy of Sciences USSR

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