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# WATER WAVES PRODUCED BY IMPULSIVE ENERGY SOURCES (U)
## PART 3: DATA ANALYSIS & SCALING RELATIONS (U)

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Anaheim, California

# (6) WATER WAVES PRODUCED BY IMPULSIVE ENERGY SOURCES ~~(U)~~
# PART 3: DATA ANALYSIS & SCALING RELATIONS (U), -(8)

Prepared for:
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## FOREWORD

This report is the third submittal of an overall study concerning the generation and propagation of surface water waves produced by explosions, conducted for the Office of Naval Research under Contract No. Nonr 3678(00).

The overall study basically is composed of two parts; one part involves analysis of available measurement data, and the second part with development of theory. This progress report is concerned with the former and, for the most part, utilizes wave data from small explosions conducted at Waterways Experiment Station, Vicksburg, Mississippi.

## ABSTRACT

This report is concerned with ~~the analysis~~ of experimental wave data obtained from small scale underwater explosions. ~~In establishing the basis for analysis,~~ consideration was made of all known and available data including those of Project SEAL and others. It was concluded that the only available small-scale field data of sufficient reliability for use in a detailed analysis are those obtained by Waterways Experiment Station scientists in their past and current test series. Accordingly, certain sets of W.E.S. data ~~made available to us~~ have been analyzed in detail with the major objectives being the investigation of scaling laws and empirical wave height prediction formulae.

Much of this report is concerned with an analysis of fourteen wave records obtained from a series of one-half, two, and ten pound explosions in the W.E.S. basin. An empirical formula is derived for predicting the maximum wave height, $H_{max}$, at a distance, r, from the source, resulting from an explosion of weight, W, at an explosion depth, z, and assuming water of constant depth. The relationship between W and wavelength, $\lambda$, of the maximum wave is derived along with $\lambda$ as a function of explosion depth, z. The wave number, $\sigma$, of the maximum wave is obtained as a function of z and of W, while the energy

ABSTRACT (cont'd)

parameter $\left(\frac{RH_{max}}{\sigma_{max}}\right)^2$ is shown as a function of z, where R is the dimensionless distance from the source. In addition, graphs of $H_{max}$ versus r are drawn and the rate of decay of wave height with r found for each value of W.

It is hoped that this work can be extended to take in data from current W.E.S. field tests involving 125 and 385 pound charges, along with data from the 10,000 pound tests of Project HYDRA. The equations, when completed, will be tested with available applicable nuclear test data.

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WATER WAVES PRODUCED BY IMPULSIVE ENERGY SOURCES

PART 3: DATA ANALYSIS AND SCALING RELATIONS

## I. INTRODUCTION

The matter of scaling surface wave characteristics caused by small explosions to those resulting from large explosions has been investigated both theoretically and empirically by many investigators with varied degrees of success and accord.

Clearly, a highly usable scaling law could be formulated easily if there existed relevant phenomenological data covering a wide range of environments and yields. Unfortunately this is not the case. Either the well-documented model or field tests are of small yield or the large yield tests lack control and are not generally well documented. One procedure has been to derive a semi-theoretical scaling relationship for $\eta r$ versus W based on a limited knowledge of the physics of wave generation, and to attempt fitting available experimental or prototype nuclear data thereto. As might be expected, this approach has not been a resounding success.

Initially, when the data analysis was commenced under Contract Nonr 3678(00), the objective was to integrate all available data into a consistent picture, if possible. To this end, data from numerous investigations made in the United States, Britain, New Zealand and Japan were carefully examined in context with the constraints of physics and linear wave theory. After careful evaluation, it was concluded that a majority of the data could not be fruitfully used along the intended lines because of lack of knowledge on generating and environmental control.

Accordingly, it was decided to select those field experiments which were conducted with reasonable control, to analyze such data in detail for prediction relationships, and to test such formulae against certain pertinent tests of other series. It was decided to select certain of the test data obtained by Waterways Experiment Station scientists in their 1961-62 field program for this detailed analysis; the results of which are presented in the following pages.

It is recognized that the W.E.S. data constitute an extremely small range of W; however, it is planned that results of further larger tests, which are expected to be available at a later date, will be incorporated in the analysis for both improvement and testing purposes.

The analysis presented in this report consists of three parts:

A. Review of Scaling Concepts,

B. Analysis of Wave Records, and

C. Development of Empirical Wave Height Prediction Formulae.

A. Review of Scaling Concepts.

Numerous investigators have looked at the problem of scaling underwater explosion characteristics over a wide range of charge weights extending from ounces to megatons. There is, of course, no question that such knowledge would be valuable; however, because of the lack of precise physical theory and reliable data in the high yield range, it is difficult to attach any degree of confidence to derived laws.

Bascially, a scaling analysis consists of determining whether or not similitude can be obtained in a model test, and if so, then finding the scale factors. The basic similitude requirements to be satisfied in such an analysis are given in Table I.

TABLE I: Basic Similitude Requirements

| | | | |
|---|---|---|---|
| 1) | Geometric Similitude | Length Scale Factor | $\lambda$ |
| 2) | Kinematic Similitude* | Velocity Scale Factor | $\phi=\lambda/\tau$ |
| | | Acceleration Scale Factor | $\alpha=\lambda/\tau^2$ |
| 3) | Dynamic Similitude | Pressure Scale Factor | $\pi=\rho\phi^2$ |
| | | Energy Scale Factor | $E=\pi\lambda^3$ |

* $\tau$ is the Time Scale Factor

The list of similitude requirements given in Table I is not complete. Further requirements depend on material (in this case water) constants, of which the density, $\rho$, is the only one listed so far. Other material constants that may affect the explosion process are viscosity, compressibility, surface tension, vapor pressure, and gravity. The influence of these material constants on the scaling picture results in certain characteristic dimensionless numbers that must have the same value in both model and prototype. These characteristic numbers are listed in Table II. (The subscript, m, in the table refers to the model test.)

A. Review of Scaling Concepts (cont'd)

TABLE II: Additional Similitude Requirements

| | | |
|---|---|---|
| 1) Compressibility Effect | Mach No. = $\frac{v}{c}$ | $\phi = \tilde{c} = \frac{c_m}{c}$ |
| 2) Gravitational Effect | Froude No. = $\frac{L}{gT^2}$ or $\frac{v^2}{gL}$ | $\alpha = \tilde{g} = \frac{g_m}{g}$ |
| 3) Condensation Effect | Thomas No. = $\frac{P}{P_{vapor}}$ | $\pi = \tilde{P}_{vapor} = \frac{(P_{vapor})_m}{P_{vapor}}$ |
| 4) Surface Tension Effect | Weber No. = $\frac{PL}{\theta}$ | $\pi\lambda = \tilde{\theta} = \frac{\theta_m}{\theta}$ |
| 5) Viscosity Effect | Reynolds No. = $\frac{vL}{\nu}$ | $\frac{\lambda^2}{\tau} = \tilde{\nu} = \frac{\nu_m}{\nu}$ |

In the above table the notations are:

$\tilde{c}$ = scale factor for velocity of sound,

$\tilde{g}$ = scale factor for gravity,

$\tilde{P}_{vapor}$ = scale factor for vapor pressure,

$\tilde{\theta}$ = scale factor for surface tension,

$\tilde{\nu}$ = scale factor for viscosity,

L = length,

T = time,

A. Review of Scaling Concepts (cont'd)

P = pressure,

v = velocity.

In field tests, the quantities $\tilde{c}$, $\tilde{\rho}$, $\tilde{P}_{vapor}$, $\tilde{\delta}$, and $\tilde{v}$ are all unity since both model and prototype have the same environment. With the five scale factors being unity, the similitude requirements given in Table II clearly are not compatible and, hence, exact scaling is impossible. Consequently, in scaling field tests, one must depend on either a theoretical solution, or an empirical solution derived from extensive small scale tests, unless it is known that some method of approximate scaling will give reasonable results. (Herein it is proposed to revert to an empirical solution using data from small scale tests conducted at Waterways Experiment Station, Vicksburg, Mississippi.)

If there is to be a method of exact scaling, one must turn to small model tests in a tank where the environment can be completely controlled. If one utilizes a high gravity tank and special liquids, it might be possible to obtain an exact scaling situation. It must be realized, also, that there is a basic difference between H.E. and nuclear explosions, since the bubble constituents are radically different. How much, and in what manner, this will effect wave generation, is not known at this time.

A. Review of Scaling Concepts (cont'd)

P = pressure,

v = velocity.

In field tests, the quantities $\tilde{c}$, $\tilde{\rho}$, $\tilde{P}_{vapor}$, $\tilde{\theta}$, and $\tilde{v}$ are all unity since both model and prototype have the same environment. With the five scale factors being unity, the similitude requirements given in Table II clearly are not compatible and, hence, exact scaling is impossible. Consequently, in scaling field tests, one must depend on either a theoretical solution, or an empirical solution derived from extensive small scale tests, unless it is known that some method of approximate scaling will give reasonable results. (Herein it is proposed to revert to an empirical solution using data from small scale tests conducted at Waterways Experiment Station, Vicksburg, Mississippi.)

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A. Review of Scaling Concepts (cont'd)

With respect to scaling of wave characteristics, W. G. Penney derived laws using the fundamental solution of the wave equation for cylindrically expanding infinitesimal gravity waves in water of uniform depth d.

For the condition of an initial surface elevation, Penney obtains the following relationship for surface elevation, $\zeta$,

$$\zeta = \int_0^\infty \cos \sigma t \; J_0(kr)kdk \int_0^\infty f(\alpha) \; J_0(k\alpha)\alpha d\alpha \quad ,$$

where k is the wave number, t is time, and $\sigma^2 = gk \tanh kd$.

Similarly, corresponding to an initial surface impulse, Penney obtained,

$$\zeta = -\frac{1}{g\rho} \int_0^\infty \sigma \sin \sigma t \; J_0(kr)kdk \int_0^\infty F(\alpha) \; J_0(k\alpha)\alpha d\alpha \quad .$$

A review of Penney's scaling analysis follows.

Case I. Initial Surface Impulse

Compare wave systems from charge weights $W_1$ and $W_2$ detonated at heights $L_2$ and $L_1$ above the water. Let the scaling factor be

$$n = (W_2/W_1)^{1/3} \quad .$$

A. Review of Scaling Concepts (cont'd)

The scaling law for $\zeta$ then turns out to be:

(1) $$\zeta_2(nx, t\sqrt{n}) = \sqrt{n}\,\zeta_1(x,t) \quad .$$

That is, impulsively generated wave heights at corresponding distances vary only in the sixth root of the charge ratio.

Case II. Initial Crater and Dome

As before let $n = (W_2/W_1)^{1/3}$ .

One must first solve the equation

(2) $$\frac{h_2{}^3\ (h_2 + Z)}{h_1{}^3\ (h_1 + Z)} = n^3 \quad ,$$

for $\frac{h_2}{h_1}$ , where Z is the atmospheric head.

Assume the solution is $\frac{h_2}{h_1} = m$; then the scaling law for $\zeta$ is

(3) $$\zeta_2(mx,\ t\sqrt{m}) = m\,\zeta_1(x,t) \quad .$$

If a small model experiment is compared with a very large full scale trial, then one may assume $h_1$ is negligible compared with Z, and that Z is negligible compared to $h_2$. Then,

A. Review of Scaling Concepts (cont'd)

(4) $$m = n^{3/4} (Z/h_1)^{1/4} .$$

The heights to be expected from large explosions, therefore scale up from those of a given small explosion as the fourth root of the charge ratio, the corresponding distances and depths being in the same ratio.

The results to be expected from two small scale experiments may be approximated by assuming that $h_1$ and $h_2$ are both negligible compared with Z. Then

(5) $$m = n ,$$

and the wave heights, distances, and depth scale according to the linear dimensions of the charges.

Wave scaling concepts of value also have been presented by H. Kranzer. Kranzer uses the same basic Cauchy-Poisson theory like Penney did, only modified for finite water depths. In this analysis, Kranzer gives solutions for the wave amplitude, η, for high air, surface, and underwater bursts in water of constant depth. These solutions will provide scaling relationships for the maximum wave in each case. The impulse functions used consider vertical forces only. Under some conditions, the horizontal

A. Review of Scaling Concepts (cont'd)

impulse should be considered, but is probably negligible if the air burst is high enough. For surface bursts, likely the horizontal impulse can be compensated for by altering the vertical impulse to give a satisfactory solution.

For an impulsively generated wave from, say, a surface burst or an air burst, the wave amplitude given by Kranzer is

(6) $$\eta(r,t) = \frac{1}{\rho h \sqrt{gh}\, r} \sqrt{\frac{\phi(\sigma) \tanh \sigma}{-\phi'(\sigma)}}\, \sigma\, \bar{I}\left(\frac{\sigma}{h}\right) ,$$

where $\sigma$ is the positive root of the equation

(7) $$\phi(\sigma) = \frac{1}{2}\sqrt{\frac{\tanh \sigma}{\sigma}} + \frac{1}{2\,(\cosh \sigma)^{3/2}}\sqrt{\frac{\sigma}{\sinh \sigma}} = \frac{r}{\sqrt{gh}\, t} \quad \text{for } r \leq \sqrt{gh}\, t.$$

In equations (6) and (7),

$\bar{I}$ = zero-order Hankel transform of the initial impulse distribution,

r = radial distance from ground zero, and

R = effective radius of the impulse.

A. Review of Scaling Concepts (cont'd)

If $I(r) = I_o$ for $r \leq R$ and $I(r) = o$ for $r > R$, Kranzer shows the maximum amplitude for this impulsively generated wave to be

(8) $$\eta_{max}(r) = \frac{0.80\ I_o\sqrt{R}}{\rho\ \sqrt{g}\ r} \quad \text{for } R \ll h \quad ,$$

and

(9) $$\eta_{max}(r) = 0.58\ \frac{I_o R}{\rho\sqrt{gh}\ r} \quad \text{for } R \gg h \quad .$$

The simplest case is that of a surface explosion in water of infinite depth ($R \ll h$).

If Y is the yield of the air burst, the impulse I(r), which is essentially a product of overpressure and duration, scales as $Y^{1/3}$ at a radial distance also scaled as $Y^{1/3}$. Therefore, at a fixed distance r from ground zero we have

(10) $$\eta_{max}(r) \propto Y^{1/3}\ Y^{1/6} = Y^{1/2} \quad .$$

For water of finite depth, the same $Y^{1/2}$ scaling is valid provided $R \ll h$. On the other hand if $R \gg h$, then

(11) $$\eta_{max}(r) \propto Y^{1/3}\ Y^{1/3} = Y^{2/3} \quad .$$

A. Review of Scaling Concepts (cont'd)

For an underwater burst, Kranzer gives a scaling relationship assuming that each explosion takes place at its optimum or lower critical depth. The solution for underwater bursts assumes that the waves are displacement generated, and is

$$\eta(r,t) = \frac{1}{rh}\,\overline{E}\left(\frac{\sigma}{h}\right)\sqrt{\frac{\sigma\,\phi(\sigma)}{-\phi'(\sigma)}} \quad , \tag{12}$$

where $\overline{E}$ is the zero-order Hankel transform of $E(r)$, the displacement of the water surface at time $t=o$. Again assuming that $E(r) = E_o$ for $r \leq R$ and $E(r) = o$ for $r > R$, we obtain

$$\eta_{max}(r) = 0.82\,\frac{E_o R}{r} \qquad \text{for } R \ll h \quad , \tag{13}$$

and

$$\eta_{max}(r) = 0.32\,\frac{E_o R^2}{rh} \qquad \text{for } R \gg h \quad . \tag{14}$$

Essentially, the latter equation need never be considered, since if $R \gg h$, any underwater explosion essentially becomes a surface explosion and can be treated as such.

A. Review of Scaling Concepts (cont'd)

Therefore, for R<<h underwater explosions of different yields, each detonated at their respective optimum depths, scale as,

(15) $$\eta_{max}(r) \propto Y^{2/3} ,$$

since $E_o$ and R both scale as $Y^{1/3}$.

Several other authors have written papers on scaling, but for the most part they are along the same lines as the work of Penney and Kranzer. Other articles have shown that no exact scaling laws existed for field tests, and have stated that one will have to be content with some kind of approximate scaling.

In view of the above, it is seen that $W^{1/6}$, $W^{1/2}$, and $W^{2/3}$ scaling has been proposed for impulsively-generated wave systems, while $W^{1/4}$ and $W^{2/3}$ scaling has been proposed for underwater-explosions or displacement-generated wave systems. Also one fact is important, there are no wave scaling relationships for detonations at depths other than the optimum or lower critical depth, unless R>>h.

A. Review of Scaling Concepts (cont'd)

The immediate conclusion to be reached in view of these various scaling relationships is that without extensive testing, one theory is as good (or bad) as the other. It appears therefore, that an empirical solution derived from analysis of well controlled small tests might be very valuable in determining which, if any, scaling relationship is most closely allied to the actual phenomena.

B. Analysis of Wave Records

The objective of this analysis is to study the nature of the wave characteristics in wave trains resulting from the underwater explosion tests conducted at Waterways Experiment Station. In this preliminary analysis of W.E.S. data, fourteen wave records were used covering charge weights of 1/2 lb., 2 lbs., and 10 lbs. Shot geometry for these records is shown in Table III (page 38). In these tests, the first few wave gauges seemed to be too near the source and were measuring breaking waves; therefore, only records from the four gauges most distant from the source were used on the 1/2 lb. and 10 lb. shots, and records from the last five gauges on the 2 lb. shots.

These small explosions were conducted in a basin of constant depth. The pertinent variables can be expressed in terms of the depth, h, as:

$R = r/h$,

$T = t\sqrt{g/h}$,

$\sigma = kh = 2\pi h/\lambda$,

$\lambda = 2\pi/T = 2\pi/\tau\sqrt{g/h}$ ; where

r = distance from the disturbance,

R = dimensionless distance,

t = time after explosion,

T = dimensionless time,

B. Analysis of Wave Records. (cont'd)

$g$ = acceleration of gravity,

$\lambda$ = wavelength,

$k$ = wave number,

and $\sigma$ = dimensionless wave number.

It is assumed, here, that the ensuing wave behavior is described by the linear theory of gravity waves, since the parameter $A\lambda^2/h^3$ is much less than unity. Therefore, the following fundamental relationships are obtained:

(16) $$C = \frac{dR}{dT} = c/\sqrt{gh} = \left(\frac{\tanh \sigma}{\sigma}\right)^{1/2} ,$$

(17) $$V = \frac{R}{T} = v/\sqrt{gh} = \frac{1}{2} C(1 + 2\sigma/\sinh 2\sigma) ,$$

where $C$ = dimensionless wave velocity,

$V$ = dimensionless group velocity,

$c$ = wave velocity,

and $v$ = group velocity.

From (16) and (17) one obtains:

(18) $$V = \frac{R}{T} = \frac{1}{2} \left(\frac{\tanh \sigma}{\sigma}\right)^{1/2} \left(1 + \frac{2\sigma}{\sinh 2\sigma}\right) ,$$

(19) $$V = \frac{R}{T} = \frac{1}{2} \left(\frac{\tanh \sigma}{\sigma}\right)^{1/2} \left[1 + \frac{\sigma(1 - \tanh^2 \sigma)}{\tanh \sigma}\right] ,$$

B. Analysis of Wave Records (cont'd)

and

(20) $$V = \frac{R}{T} = \frac{1}{2}\left[\left(\frac{\tanh \sigma}{\sigma}\right)^{1/2} + \left(\frac{\sigma}{\tanh \sigma}\right)^{1/2} - (\sigma \tanh^3 \sigma)^{1/2}\right].$$

One now can plot the relationships, $\frac{R}{T}$ vs. $\sigma$, $\frac{R}{T}$ vs. C, $\frac{R}{T}$ vs. $\lambda$, and $\frac{R}{T}$ vs. $\gamma$, since $\gamma = \sigma C$ is an identity (see Figure 1). Figure 2 is a graph of a typical wave train on a R vs. T plot.

The variables of primary interest in an explosion are the wave height H, the wave length $\lambda$, the group velocity V, and the energy parameter $\left(\frac{RH_{max}}{\sigma_{max}}\right)^2$. One must realize, however, as is easily seen in Figure 1, that for any one wave train, $\lambda$ is a function of $\frac{R}{T}$, and that a particular value of $\lambda$ is really an instantaneous value. Therefore, for convenience, all analysis of the data will be made in reference to the group envelope maximum. That is, $H_{max}$ will be the wave height at the envelope maximum, $\lambda_{max}$ will be the average wavelength associated with the envelope maximum (not the max. $\lambda$ in any envelope), $\sigma_{max}$ is $\sigma$ at the envelope maximum, $V_{max}$ is V at the envelope maximum, and $\left(\frac{RH_{max}}{\sigma_{max}}\right)^2$ is the energy parameter at the envelope maximum.

B. Analysis of Wave Records (cont'd)

Plots of log $\lambda_{max}$, log $V_{max}$, and log $\sigma_{max}$, all vs. log $W$ are shown in Figure 3 for a surface explosion. As seen, the loci appear to be straight lines. The corresponding relationships obtained from least squares fits are:

(21) $$\lambda_{max} = 15.0\ W^{0.275} \quad ,$$

(22) $$\sigma_{max} = 8.38\ W^{-0.275} \quad , \text{ and}$$

(23) $$V_{max} = 0.17\ W^{0.141} \quad .$$

Figures 4, 5, 6 show plots of $\lambda_{max}$, $\sigma_{max}$, and $\left(\frac{RH_{max}}{\sigma}\right)^2$ versus z, the depth of explosion measured positive downward, with W as a parameter. Here, one easily can see that there are not enough data points to obtain a true picture of the function's behaviour; however, the plots seem to be similar to plots of $H_{max}$ versus z, which will be shown in the next section. The $\lambda_{max}$ and $\left(\frac{RH_{max}}{\sigma_{max}}\right)^2$ plots seem to have two maxima; one maximum near the lower critical depth, and another near the upper critical depth. The maximum near the lower critical depth seems to increase more rapidly than the maximum near the upper critical depth. On the other hand, due to the relationship between $\sigma_{max}$ and $\lambda_{max}$, the relationship between $\sigma_{max}$ and z is approximately the inverse of that shown for $\lambda_{max}$ versus z.

B. Analysis of Wave Records (cont'd)

Tables IV, V, and VI that follow, contain a summary of the analysis of these wave records. As can be seen, it would be most beneficial to have records from other explosion depths to analyze.

TABLE IV: Results of Analysis of Wave Records from W = 0.5 Lb. Shots.

| z in feet | 0 | 0.04' | 1.78' | 2.17' | 4.74' |
|---|---|---|---|---|---|
| $V_{max}$ | 0.155 | 0.159 | 0.164 | 0.173 | 0.158 |
| $\sigma_{max}$ | 10.2 | 9.70 | 9.20 | 8.30 | 10.0 |
| $\lambda_{max}$ | 12.31 | 12.95 | 13.66 | 15.13 | 12.60 |
| $KH_{max}$ | 0.520 | 0.624 | 0.607 | 0.603 | 0.254 |
| $(\frac{KH_{max}}{\sigma_{max}})^2$ | $2.71 \times 10^{-3}$ | $4.13 \times 10^{-3}$ | $4.35 \times 10^{-3}$ | $5.28 \times 10^{-3}$ | $0.646 \times 10^{-3}$ |

TABLE V: Results of Analysis of Wave Records from W = 2.0 Lb. Shots.

| z in feet | 0 | 0.06' | 2.21' | 4.72' |
|---|---|---|---|---|
| $V_{max}$ | 0.192 | 0.198 | 0.209 | 0.209 |
| $\sigma_{max}$ | 6.75 | 6.30 | 5.61 | 5.61 |
| $\lambda_{max}$ | 18.61 | 19.94 | 22.39 | 22.39 |
| $KH_{max}$ | 1.84 | 1.84 | 1.02 | 1.21 |
| $(\frac{KH_{max}}{\sigma_{max}})^2$ | $7.40 \times 10^{-2}$ | $8.53 \times 10^{-2}$ | $3.35 \times 10^{-2}$ | $4.67 \times 10^{-2}$ |

B. Analysis of Wave Records (cont'd)

TABLE VI: Results of Analysis of Wave Records from W = 10.0 Lb. Shots.

| z in feet | 0 | 0.11' | 2.15' | 4.84' | 10.87' |
|---|---|---|---|---|---|
| $V_{max}$ | 0.237 | 0.246 | 0.243 | 0.249 | 0.235 |
| $\sigma_{max}$ | 4.47 | 4.13 | 4.23 | 4.08 | 4.52 |
| $\lambda_{max}$ | 28.10 | 30.41 | 29.69 | 30.78 | 27.79 |
| $RH_{max}$ | 2.29 | 2.97 | 2.43 | 3.56 | 1.32 |
| $\left(\frac{RH_{max}}{\sigma_{max}}\right)^2$ | 0.263 | 0.518 | 0.329 | 0.759 | 0.085 |

One observation of considerable interest noted from the test records, is that as the explosion depth increased, a phase shift of 90° in the wave train occurred at identical range stations. This can be seen in Figure 7 (for W = 2 lbs.). This phase shift also occurred for the 1/2 lb. and the 10 lb. series. The phase shift seemed to occur rather abruptly since the first two records available had identical phases, whereas every record of an explosion at greater depths showed the same 90° phase shift.

C. Development of Empirical Wave Height Prediction Formulae

In addition to the 14 wave records used in the preceding section, tables of averaged wave height recorded for each shot position and each charge weight were used in the following analysis.

Table III (page 38) shows the charge positions (z) used for each charge weight W, while the charge positions marked $\square$ indicate the records analyzed in the previous section. Table VII (page 39) shows the ranges at which wave gauges were placed, and Table VIII (page 40) shows the number of shots (at each charge position and for each charge weight) that were averaged to obtain the wave heights in the tables referred to above. These explosions all occurred in water of constant depth (20 feet).

The goal of this analysis is to find some prediction formula of the form $H_{max} = f(r,W,z)$. Assume that

(24) $$f(r,W,z) = \frac{1}{r^{a}} g(W,z) \quad ,$$

where a is some constant to be determined by making plots of log $H_{max}$ vs. log r for each charge weight and for each charge position. Figures 8, 9, 10, 11 and 12 show these plots for W = 10 lbs., the lines being determined by making a least squares fit. The probable error was found for each charge weight, and also for a combination of the three charge weights. These results are shown in Table IX below.

TABLE IX: Wave Height Decay Rate

| Charge Weight (lbs.) | Wave Height Decay Rate |
|---|---|
| 0.5 | $H_{max} = r^{-0.830 \pm 0.128}$ |
| 2.0 | $H_{max} = r^{-0.840 \pm 0.102}$ |
| 10 | $H_{max} = r^{-0.718 \pm 0.081}$ |
| 0.5, 2, 10 | $H_{max} = r^{-0.802 \pm 0.119}$ |

The next step is to find the function $g(W,z)$. Plots of $H_{max}$ vs. $z$, with $W$ as a parameter appear in Figures 13, 14, 15, 16. In the figures, the first maxima is defined as the upper critical depth and the second as the lower critical depth. Note that $H_{max}$ seems to decay, more or less, exponentially with depth from some point b[illegible] the lower critical depth. Utilizing this observation, let $g(W,z)$ be defined as:

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

$$g(W,z) = \begin{cases} g_o(W,z) & \text{for } o \leq z < z' \\ g_1(W,z) & \text{for } z' \leq z \end{cases} , \tag{25}$$

where z', (to be determined later) is that depth below the lower critical depth at which $H_{max}$ appears to fall off exponentially.

To determine the function $g_o(W,z)$ one would like to express $H_{max}$ as a polynomial in z with constants that are functions of W. Also, one would like to force this polynomial to have maxima of the correct height at the lower critical depth and at the upper critical depth, and to pinpoint the minimum between these two maxima. It would be desirable, furthermore, for $H_{max}$ at z=o to be the correct value. Figures 17 and 18 show plots of z at the upper critical depth ($z_{u.c.d.}$), z at the lower critical depth ($z_{l.c.d.}$), z at the minimum between l.c.d. and u.c.d. ($z_{min.}$), $H_{max}$ at z=o ($H_{max,o}$), $H_{max}$ at the upper critical depth ($H_{max,\ u.c.d.}$), $H_{max}$ at the lower critical depth ($H_{max,\ l.c.d.}$), and $H_{max}$ at the minimum ($H_{max,\ min}$) each as a function of W. These results are shown below in Table X, the values being normalized to r = 50 feet.

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

TABLE X: Conditions to be Satisfied by $g_o(z,W)$

| | |
|---|---|
| At u.c.d. | $z_{u.c.d.} = \alpha_1(W) = 4.45 \times 10^{-2} W^{0.135}$ <br> $H_{max,u.c.d.}(r) = \beta_1(W) = 2.70W^{0.443}$ |
| At min. | $z_{min} = \alpha_2(W) = 0.591\ W^{-0.146}$ <br> $H_{max,min}(r) = \beta_2(W) = 0.201W^{0.562}$ |
| At l.c.d. | $z_{l.c.d.} = \alpha_3(W) = 2.51W^{0.376}$ <br> $H_{max,l.c.d.}(r) = \beta_3(W) = 0.377W^{0.492}$ |
| At surface | $z = o$ <br> $H_{max,\ o}(r) = \beta_o(W) = 0.702W^{0.276}$ |

As can be seen from the above table, seven conditions must be satisfied, requiring a sixth degree polynomial.

Let $g_o(z,W)$ be of the form

(26) $$AH_{max}(r) = z^6 + Bz^5 + Cz^4 + Dz^3 + Ez^2 + Fz + G \quad ,$$

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

where A, B, C, D, E, and F are all functions of W, to be determined from the above seven conditions.

Then:

(27) at $z=o$, $H_{max}(r) = \frac{G}{A} = \beta_o(W)$ ,

and

(28) $$A \frac{dH_{max}(r)}{dz} = 6z^5 + 5Bz^4 + 4Cz^3 + 3Dz^2 + 2Ez + F \quad .$$

Setting $\frac{dH_{max}(r)}{dz} = o$, it follows that at least three real positive roots must exist; namely $\alpha_1(W)$, $\alpha_2(W)$, and $\alpha_3(W)$, while the other two roots may be real negative, complex or also real positive.

A somewhat justifiable criticism at this point would be that one is not assured that the final function $g_o(z,W)$ will not have another maximum and minimum somewhere in the interval $o \leq z < z'$; however, this is not held to be very likely, and probably will never occur. To find the exact behavior of the final function $g_o(z,W)$ requires a computer program, as it is somewhat complicated. This task is planned for the future.

C. Development of Empirical Wave Height Prediction Formulae (Cont'd)

Proceeding on the search for $g_o(z,W)$ we have,

$$A\frac{dH_{max}(r)}{dz} = o = 6z^5 + 5Bz^4 + 4Cz^3 + 3Dz^2 + 2Ez + F =$$

(29)

$$= [z-\alpha_1(W)][z-\alpha_2(W)][z-\alpha_3(W)]\ (6z^2 + \gamma z + \delta)\quad ,$$

where $\gamma$ and $\delta$ are functions of W, yet to be determined.

It follows that:

(30) $$AH_{max}(r) = \int\{[z-\alpha_1(W)][z-\alpha_2(W)][z-\alpha_3(W)]\ (6z^2 + \gamma z + \delta)\}dz\quad ,$$

and

(31)

$$\begin{cases} AH_{max}(r) = z^6 + \dfrac{\{-6[\alpha_1(W)+\alpha_2(W)+\alpha_3(W)] + \gamma\}}{5} z^5 \\ + \dfrac{\{6[\alpha_3(W)\{\alpha_1(W)+\alpha_2(W)\} + \alpha_1(W)\alpha_2(W)-\gamma[\alpha_1(W)+\alpha_2(W)+\alpha_3(W)]+\delta\}}{4} z^4 \\ + \dfrac{\{-6\alpha_1(W)\alpha_2(W)\alpha_3(W)+\gamma[\alpha_3(W)\{\alpha_1(W) +\alpha_2(W)+\alpha_1(W)\alpha_2(W)]-\delta[\alpha_1(W)+\alpha_2(W)+\alpha_3(W)]\}}{3} z^3 \\ + \dfrac{\{-\alpha_1(W)\alpha_2(W)\alpha_3(W)\gamma+\delta[\alpha_3(W)\{\alpha_1(W)+\alpha_2(W)\}+\alpha_1(W)\alpha_2(W)]\}}{2} z^2 \\ + \{-\delta\alpha_1(W)\alpha_2(W)\alpha_3(W)\}\ z + G\quad , \end{cases}$$

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

where the coefficients of $z^5$, $z^4$, $z^3$, $z^2$ and $z$ are B, C, D, E, and F respectively, and G = $A\beta_o(W)$.

Now, let

(32) $$A_1 = \alpha_1(W) + \alpha_2(W) + \alpha_3(W) \quad ,$$

(33) $$A_2 = \alpha_3(W)\,[\alpha_1(W) + \alpha_2(W)] + \alpha_1(W)\,\alpha_2(W) \quad ,$$

(34) $$A_3 = \alpha_1(W)\,\alpha_2(W)\,\alpha_3(W) \quad ,$$

then

(35) $$\left\{\begin{aligned} AH_{max}(r) &= z^6 + \frac{-6A_1 + \gamma}{5} z^5 + \frac{(6A_2 - A_1\gamma + \delta)}{4} z^4 \\ &+ \frac{(-6A_3 + A_2\gamma - A_1\delta)}{3} z^3 + \frac{(-A_3\gamma + A_2\delta)}{2} z^2 - A_3\delta z + A\beta_o(W) \quad , \end{aligned}\right.$$

or

(36) $$\left\{\begin{aligned} & A[-H_{max}(r) + \beta_o(W)] + \gamma\left[\frac{z^5}{5} - \frac{A_1 z^4}{4} + \frac{A_2 z^3}{3} - \frac{A_3 z^2}{2}\right] \\ & + \delta\left[\frac{z^4}{4} - \frac{A_1 z^3}{3} + \frac{A_2 z^2}{2} - A_3 z\right] = \left[-z^6 + \frac{6A_1 z^5}{5} - \frac{6A_2 z^4}{4} + \frac{6A_3 z^3}{3}\right] . \end{aligned}\right.$$

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

The above equation, (36), has three unknowns, A, $\gamma$, and $\delta$. There are three conditions yet to be used; these are

(37) $$z = \alpha_1(W) \quad , \quad H_{max}(r) = \beta_1(W) \quad ,$$

(38) $$z = \alpha_2(W) \quad , \quad H_{max}(r) = \beta_2(W) \quad ,$$

(39) $$z = \alpha_3(W) \quad , \quad H_{max}(r) = \beta_3(W) \quad .$$

Therefore, the system of three equations and three unknowns becomes,

(40) $$\phi_{11} A + \phi_{12} \gamma + \phi_{13} \delta = \phi_1 \quad ,$$

(41) $$\phi_{21} A + \phi_{22} \gamma + \phi_{23} \delta = \phi_2 \quad ,$$

(42) $$\phi_{31} A + \phi_{32} \gamma + \phi_{33} \delta = \phi_3 \quad ,$$

where,

(43) $$\begin{cases} \phi_{11} = [-H_{max}(r) + \beta_o(W)]_{H_{max}(r)=\beta_1(W)} \\ \phi_{21} = [-H_{max}(r) + \beta_o(W)]_{H_{max}(r)=\beta_2(W)} \\ \phi_{31} = [-H_{max}(r) + \beta_o(W)]_{H_{max}(r)=\beta_3(W)} \quad . \end{cases}$$

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

$$\phi_{12} = [\frac{z^5}{5} - \frac{A_1 z^4}{4} + \frac{A_2 z^3}{3} - \frac{A_3 z^2}{2}]_{z=\alpha_1(W)}$$

$$\phi_{22} = [\frac{z^5}{5} - \frac{A_1 z^4}{4} + \frac{A_2 z^3}{3} - \frac{A_3 z^2}{2}]_{z=\alpha_2(W)}$$

$$\phi_{32} = [\frac{z^5}{5} - \frac{A_1 z^4}{4} + \frac{A_2 z^3}{3} - \frac{A_3 z^2}{2}]_{z=\alpha_3(W)}$$

$$\phi_{13} = [\frac{z^4}{4} - \frac{A_1 z^3}{3} + \frac{A_2 z^2}{2} - A_3 z]_{z=\alpha_1(W)}$$

(43)
$$\phi_{23} = [\frac{z^4}{4} - \frac{A_1 z^3}{3} + \frac{A_2 z^2}{2} - A_3 z]_{z=\alpha_2(W)}$$

$$\phi_{33} = [\frac{z^4}{4} - \frac{A_1 z^3}{3} + \frac{A_2 z^2}{2} - A_3 z]_{z=\alpha_3(W)}$$

$$\phi_1 = [-z^6 + \frac{6A_1 z^5}{5} - \frac{6A_2 z^4}{4} + \frac{6A_3 z^3}{3}]_{z=\alpha_1(W)}$$

$$\phi_2 = [-z^6 + \frac{6A_1 z^5}{5} - \frac{6A_2 z^4}{4} + \frac{6A_3 z^3}{3}]_{z=\alpha_2(W)}$$

$$\phi_3 = [-z^6 + \frac{6A_1 z^5}{5} - \frac{6A_2 z^4}{4} + \frac{6A_3 z^3}{3}]_{z=\alpha_3(W)} .$$

and

(44)
$$A = \left| \frac{\begin{matrix} \phi_1 & \phi_{12} & \phi_{13} \\ \phi_2 & \phi_{22} & \phi_{23} \\ \phi_3 & \phi_{32} & \phi_{33} \end{matrix}}{\begin{matrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{matrix}} \right| .$$

(45)
$$\gamma = \frac{\begin{vmatrix} \phi_{11} & \phi_{1} & \phi_{13} \\ \phi_{21} & \phi_{2} & \phi_{23} \\ \phi_{31} & \phi_{3} & \phi_{33} \end{vmatrix}}{\begin{vmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{vmatrix}} ,$$

and

(46)
$$\delta = \frac{\begin{vmatrix} \phi_{11} & \phi_{12} & \phi_{1} \\ \phi_{21} & \phi_{22} & \phi_{2} \\ \phi_{31} & \phi_{32} & \phi_{3} \end{vmatrix}}{\begin{vmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{vmatrix}} .$$

The above equations are sufficient to determine the original constants, A, B, C, D, E, F, and G.

To summarize:

1. A is given by equation (44),

2. $B = \dfrac{-6A_1+\gamma}{5}$ ,

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

3. $C = \frac{6A_2 - A_1\gamma + \delta}{4}$ ,

4. $D = \frac{-6A_3 + A_2\gamma - A_1\delta}{3}$ ,

5. $E = \frac{-A_3\gamma + A_2\delta}{2}$ ,

6. $F = -A_3\delta$ ,

and

7. $G = A\beta_o(W)$ .

The formula for prediction of $H_{max}$ from an explosion of charge weight W at explosion depth z and range r, then, is,

(47) $$H_{max} = \frac{(50)^\alpha \{z^6 + Bz^5 + Cz^4 + Dz^3 + Ez^2 + Fz + G\}}{A\, r^\alpha} \quad \text{for } o \leq z < z' ,$$

where $\alpha = 0.802$ (Table IX).

It is to be noted that coefficients A, B, C, D, E, and F must be evaluated for each different charge weight W. Also, depth z is measured positive downward, and water depth is assumed constant.

C. Development of Empirical Wave Height Prediction Formulae (cont'd)

All that remains now is to find the function $g_1(z,W)$ for $H_{max}(r)$ when $z \geq z'$. Figures 19 and 20 show plots of $z'$ vs. $W$, and $H_{max,z'}(r)$ vs. $W$ at $R = 21.8$ ft., the least squares fit giving:

(48) $$z' = 4.27\ W^{0.386} \quad ,$$

(49) $$H_{max,\ z'}(r) = 0.525\ W^{0.439} \quad ,$$

(50) $$\therefore\ H_{max} = \frac{(21.8)^{\alpha}(0.525)\ W^{0.439}e^{z'-z}}{r^{\alpha}} \quad \text{for } z \geq z' \quad ,$$

where again $\alpha = 0.802$ (Table IX).

D. Conclusions

It is not possible at this time to derive any conclusions from the above analysis because of the limited number of wave records available for this treatment. However, definite trends are indicated and a further analysis of the remaining W.E.S. data, and of data from the present test series should provide worthwhile prediction formulae.

From the available data, it appears that the upper critical depth seems to become less important as the charge weight increases, and apparently vanishes when the charge weight is somewhere between 500 lbs. and 800 lbs. As the charge weight increases further, the lower critical depth continues to be the optimum depth for generation of large amplitude waves. The wavelength at the envelope maximum, plotted as a function of explosion depth z, also has a maximum in the region of the lower critical depth, as does the energy parameter of the envelope.

Figure 3 shows how $\lambda_{max}$, $\sigma_{max}$, and $V_{max}$ vary as functions of W for a surface explosion, and Figures 4, 5, and 6 show how $\lambda_{max}$, $\sigma_{max}$, and $\left(\frac{RH_{max}}{\sigma_{max}}\right)^2$ vary as a function of depth with W as a parameter. To obtain an exact picture

D. Conclusions (cont'd)

of how these parameters are effected by W and z, one needs more data encompassing larger charge weights. The present test series being conducted by W.E.S. should help in filling in the exact picture.

As shown in Figure 7, a 90° phase shift was observed as the explosion depth increased. This is probably due to a difference in basic generation of the waves from a surface burst and an underwater burst. In one case the primary forcing function is important, in the second case, the secondary forcing function is important.

The prediction formula (47) and (50), will be improved when more data are made available. The constants are complicated functions of W; therefore, examination of the exact nature of these functions necessitates use of a computer. This step is planned when the formula have been improved with additional data.

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TABLE III: CHARGE POSITIONS

| CHARGE POSITION | CHARGE DEPTH Z (FT) FOR CHARGE WEIGHT W (LB) | | | | | |
|---|---|---|---|---|---|---|
| | $\frac{1}{2}$ = B | 2 = A | 10 = C | 125 = D | 385 = E | 1000 = F |
| 1. | +0.08 | +0.13 | - | - | - | - |
| 2. | +0.04 | +0.06 | - | - | - | - |
| 3. | 0 | 0 | 0 | 0 | - | - |
| 4. | -0.04 | -0.06 | -0.11 | -0.25 | -0.37 | - |
| 5. | -0.08 | -0.13 | -0.22 | -0.50 | - | - |
| 6. | -0.16 | -0.25 | -0.43 | - | - | - |
| 7. | -0.32 | -0.50 | -0.86 | -2.00 | - | - |
| 8. | -0.47 | -0.76 | -1.29 | - | - | - |
| 9. | -0.79 | -1.26 | -2.15 | -5.00 | - | - |
| 9A. | -0.99 | -1.58 | -2.69 | - | - | - |
| 9B. | -1.18 | -1.89 | -3.23 | -7.50 | - | - |
| 9C. | -1.38 | -2.21 | -3.76 | - | - | - |
| 10. | -1.58 | -2.52 | -4.31 | -10.00 | -14.6 | - |
| 10A. | -1.78 | -2.84 | -4.84 | -11.25 | -16.4 | -20.9 |
| 10B. | -1.98 | -3.15 | -5.38 | -12.50 | -18.3 | - |
| 10B'. | - | - | - | -13.00 | - | - |
| 10C. | -2.17 | -3.46 | -5.91 | -13.75 | -20.1 | - |
| 11. | -2.37 | -3.78 | -6.46 | -15.00 | -21.9 | - |
| 12. | -2.57 | -4.09 | -7.00 | - | - | - |
| 13. | -2.77 | -4.41 | -7.54 | - | - | - |
| 14. | -2.96 | -4.72 | -8.08 | - | - | - |
| 15. | -3.16 | -5.04 | -8.62 | -20.00 | - | - |
| 16. | -3.56 | -5.67 | -9.68 | - | - | - |
| 17. | -3.95 | -6.30 | -10.87 | -25.00 | - | - |
| 18. | -4.35 | -6.93 | - | - | - | - |
| 19. | -4.74 | -7.56 | - | - | - | - |
| 20. | -5.53 | -8.82 | - | - | - | - |

TABLE VII: WAVE GAUGE POSITIONS

RANGES, ft.

| Charge Weight, lb. | | | | | |
|---|---|---|---|---|---|
| 0.5 | 2 | 10 | 125 | 385 | 1000 |
| 13.90 | 15.75 | 21.50 | 75 | 73 | 73 |
| 15.88 | 18.90 | 26.88 | 98 | 110 | 110 |
| 17.47 | 22.05 | 32.25 | 205 | 146 | 146 |
| 19.85 | 34.65 | 37.63 | 310 | 219 | 219 |
| 21.84 | 47.25 | 53.75 | 410 | 365 | 365 |
| 23.82 | 50.40 | 59.13 | 540 | 511 | 511 |
| 25.41 | 53.55 | 64.50 | 800 | 800 | 800 |
| 27.79 | 56.70 | 80.60 | | | |
| 29.78 | 59.85 | 86.00 | | | |
| 31.76 | 63.00 | | | | |

TABLE VIII: SUMMARY OF EXPLOSIONS

| W | No. Shots Fired | z |
|---|---|---|
| 0.5 | 5 | +0.08 |
| 0.5 | 6 | +0.04 |
| 0.5 | 5 | 0 |
| 0.5 | 7 | -0.04 |
| 0.5 | 5 | -0.08 |
| 0.5 | 2 | -0.16 |
| 0.5 | 3 | -0.32 |
| 0.5 | 2 | -0.47 |
| 0.5 | 2 | -0.79 |
| 0.5 | 6 | -0.99 |
| 0.5 | 4 | -1.18 |
| 0.5 | 3 | -1.38 |
| 0.5 | 5 | -1.58 |
| 0.5 | 5 | -1.78 |
| 0.5 | 5 | -1.98 |
| 0.5 | 5 | -2.17 |
| 0.5 | 7 | -2.37 |
| 0.5 | 5 | -2.57 |
| 0.5 | 5 | -2.77 |
| 0.5 | 5 | -2.96 |
| 0.5 | 5 | -3.16 |
| 0.5 | 4 | -3.56 |
| 0.5 | 3 | -3.95 |
| 0.5 | 3 | -4.35 |
| 0.5 | 3 | -4.74 |
| 0.5 | 3 | -5.53 |
| 2 | 5 | +0.13 |
| 2 | 5 | +0.06 |
| 2 | 5 | 0 |
| 2 | 7 | -0.06 |
| 2 | 5 | -0.13 |
| 2 | 3 | -0.25 |
| 2 | 3 | -0.50 |
| 2 | 3 | -0.76 |
| 2 | 3 | -1.26 |
| 2 | 3 | -1.58 |
| 2 | 3 | -1.89 |
| 2 | 3 | -2.21 |
| 2 | 4 | -2.52 |
| 2 | 5 | -2.84 |
| 2 | 5 | -3.15 |
| 2 | 5 | -3.46 |
| 2 | 7 | -3.78 |
| 2 | 5 | -4.09 |

TABLE VIII: Summary of Explosions (cont'd)

| W | No. Shots Fired | z |
|---|---|---|
| 2 | 5 | -4.41 |
| 2 | 4 | -4.72 |
| 2 | 5 | -5.04 |
| 2 | 3 | -5.67 |
| 2 | 5 | -6.30 |
| 2 | 3 | -6.93 |
| 2 | 3 | -7.56 |
| 2 | 3 | -8.82 |
| 10 | 4 | 0 |
| 10 | 4 | -0.11 |
| 10 | 5 | -0.22 |
| 10 | 3 | -0.43 |
| 10 | 3 | -0.86 |
| 10 | 3 | -1.29 |
| 10 | 3 | -2.15 |
| 10 | 3 | -2.69 |
| 10 | 3 | -3.23 |
| 10 | 3 | -3.76 |
| 10 | 5 | -4.31 |
| 10 | 5 | -4.84 |
| 10 | 5 | -5.38 |
| 10 | 5 | -5.91 |
| 10 | 7 | -6.46 |
| 10 | 5 | -7.00 |
| 10 | 5 | -7.54 |
| 10 | 5 | -8.08 |
| 10 | 5 | -8.62 |
| 10 | 3 | -9.68 |
| 10 | 6 | -10.87 |
| 125 | 1 | 0 |
| 125 | 2 | -0.25 |
| 125 | 1 | -0.50 |
| 125 | 2 | -2.00 |
| 125 | 2 | -5.00 |
| 125 | 3 | -7.50 |
| 125 | 2 | -10.00 |
| 125 | 3 | -11.25 |
| 125 | 3 | -12.50 |
| 125 | 4 | -13.75 |
| 125 | 3 | -15.00 |
| 125 | 1 | -20.00 |
| 125 | 2 | -25.00 |

TABLE VIII: Summary of Explosions (cont'd)

| W | No. Shots Fired | z |
|---|---|---|
| 385 | 1 | -0.37 |
| 385 | 2 | -14.6 |
| 385 | 2 | -16.4 |
| 385 | 2 | -18.3 |
| 385 | 2 | -20.1 |
| 385 | 1 | -21.9 |
| 1000 | 1 | -20.9 |

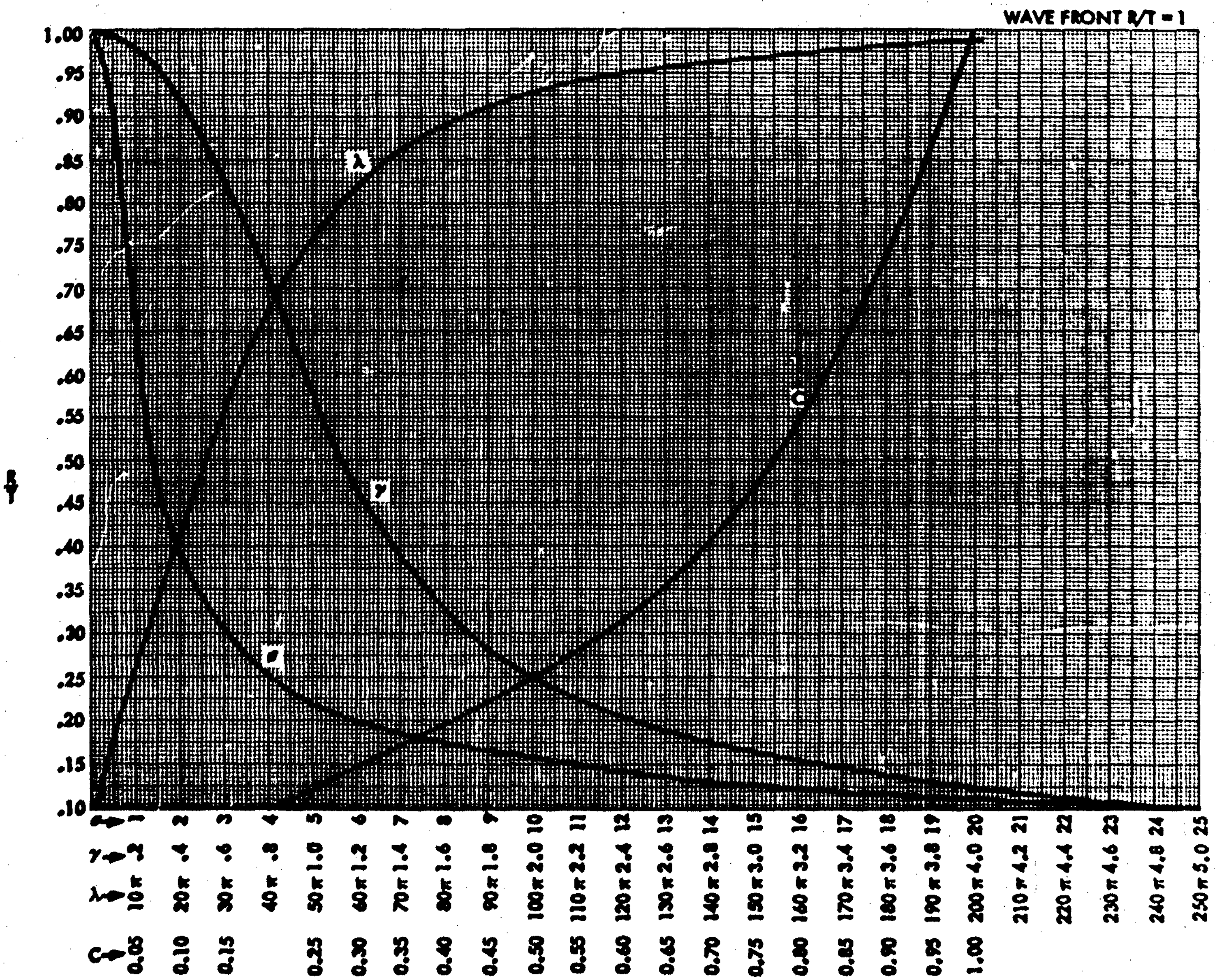
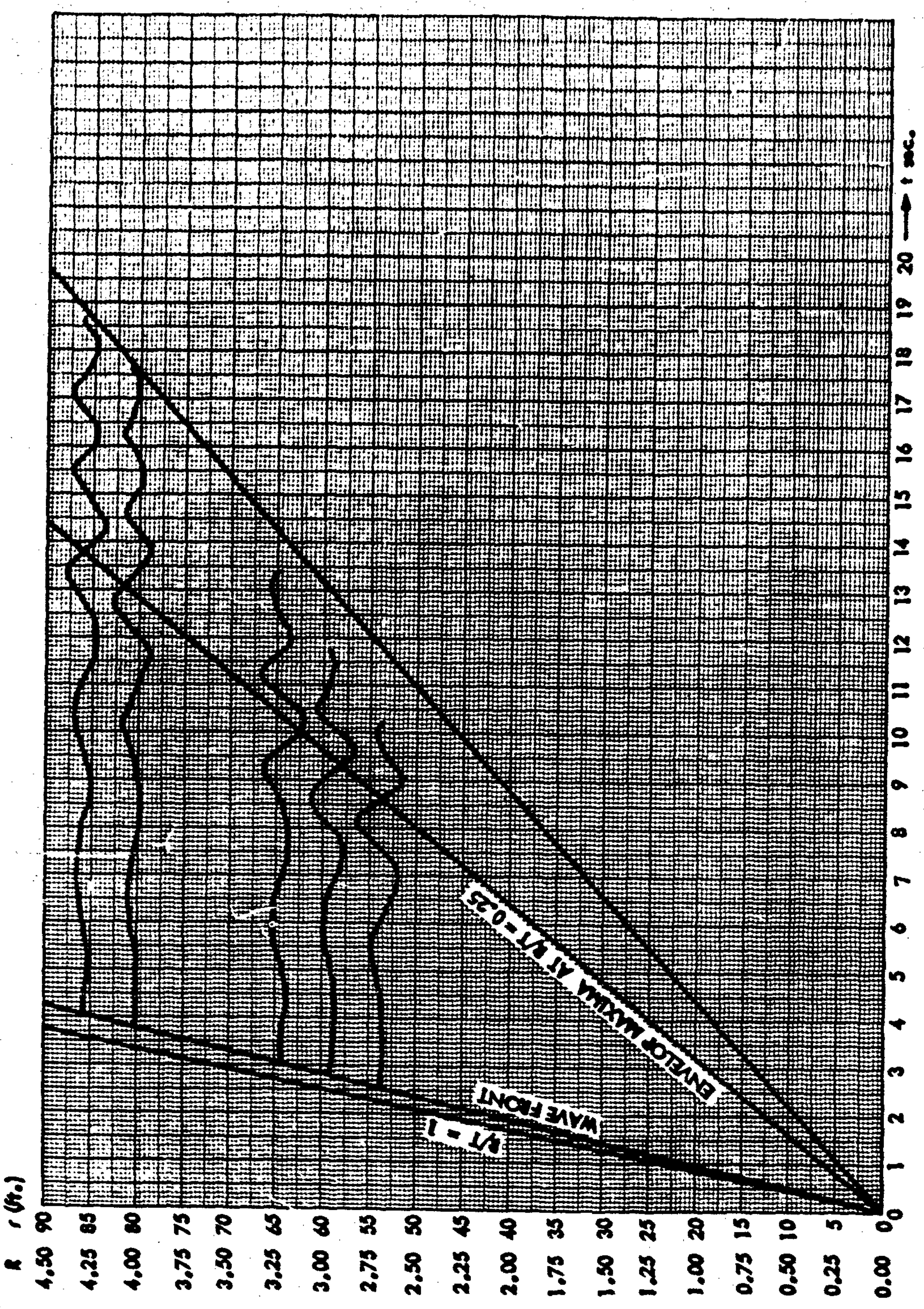



FIGURE 1 PERTINENT VARIABLES AS FUNCTION OF R/T




$T = t\sqrt{g/h} = 1.26t$

FIGURE 2 TYPICAL WAVE TRAIN, SHOT C-4, W = 10 lb, Z = 0.11

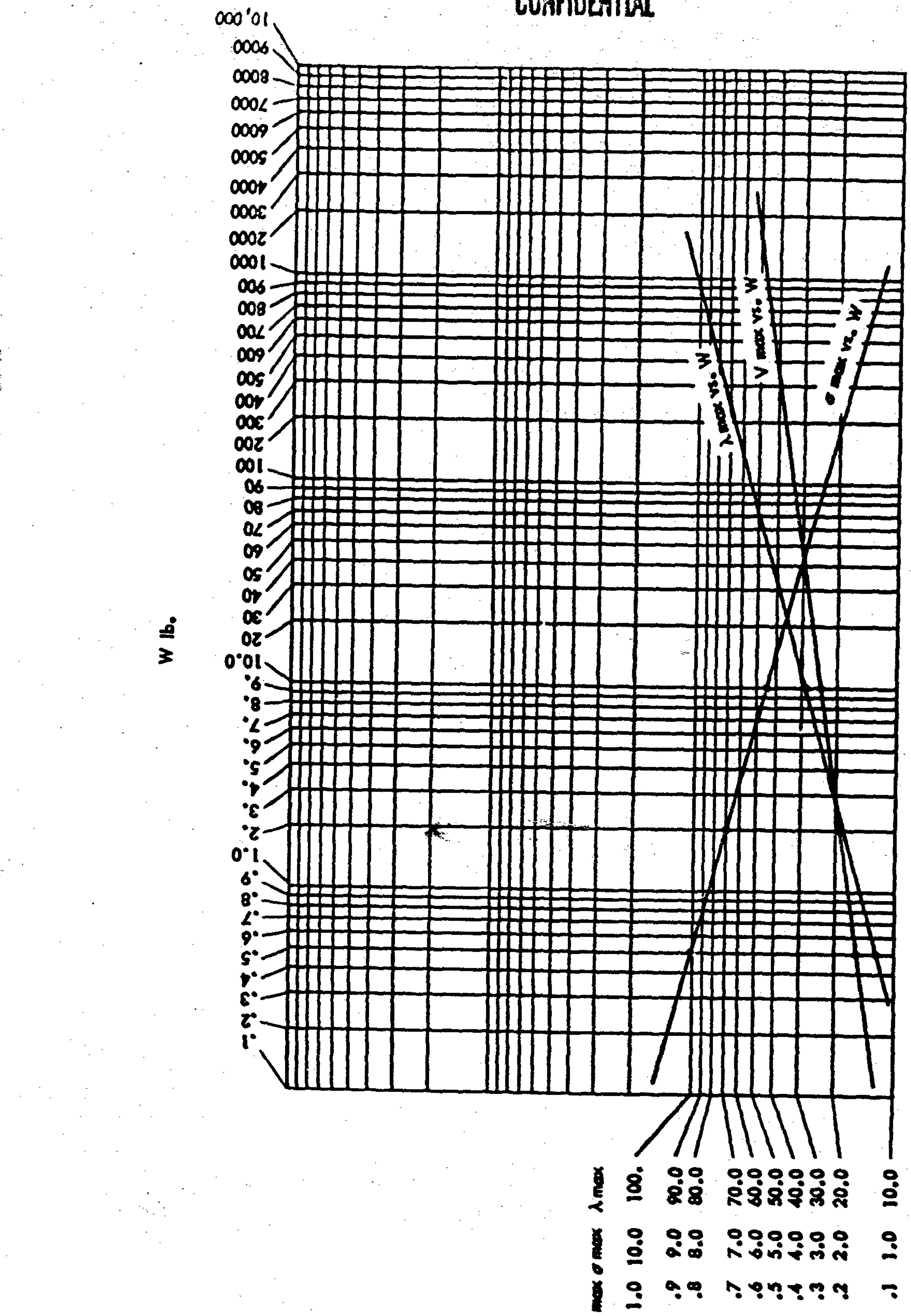



FIGURE 3 V max, σ max, AND λ max AS A FUNCTION OF W, Z = 0

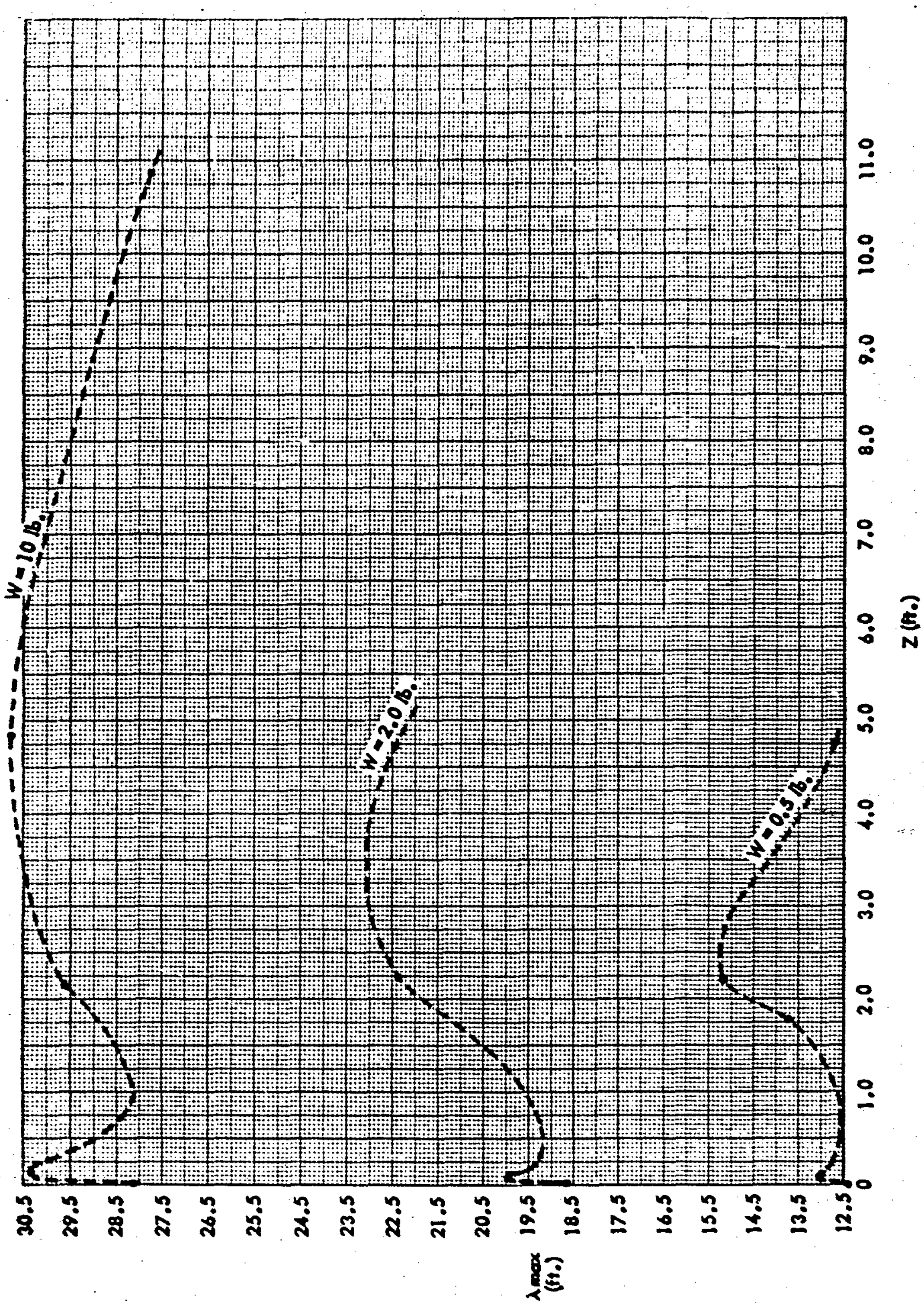



FIGURE 4 WAVE LENGTH AS A FUNCTION OF EXPLOSION DEPTH

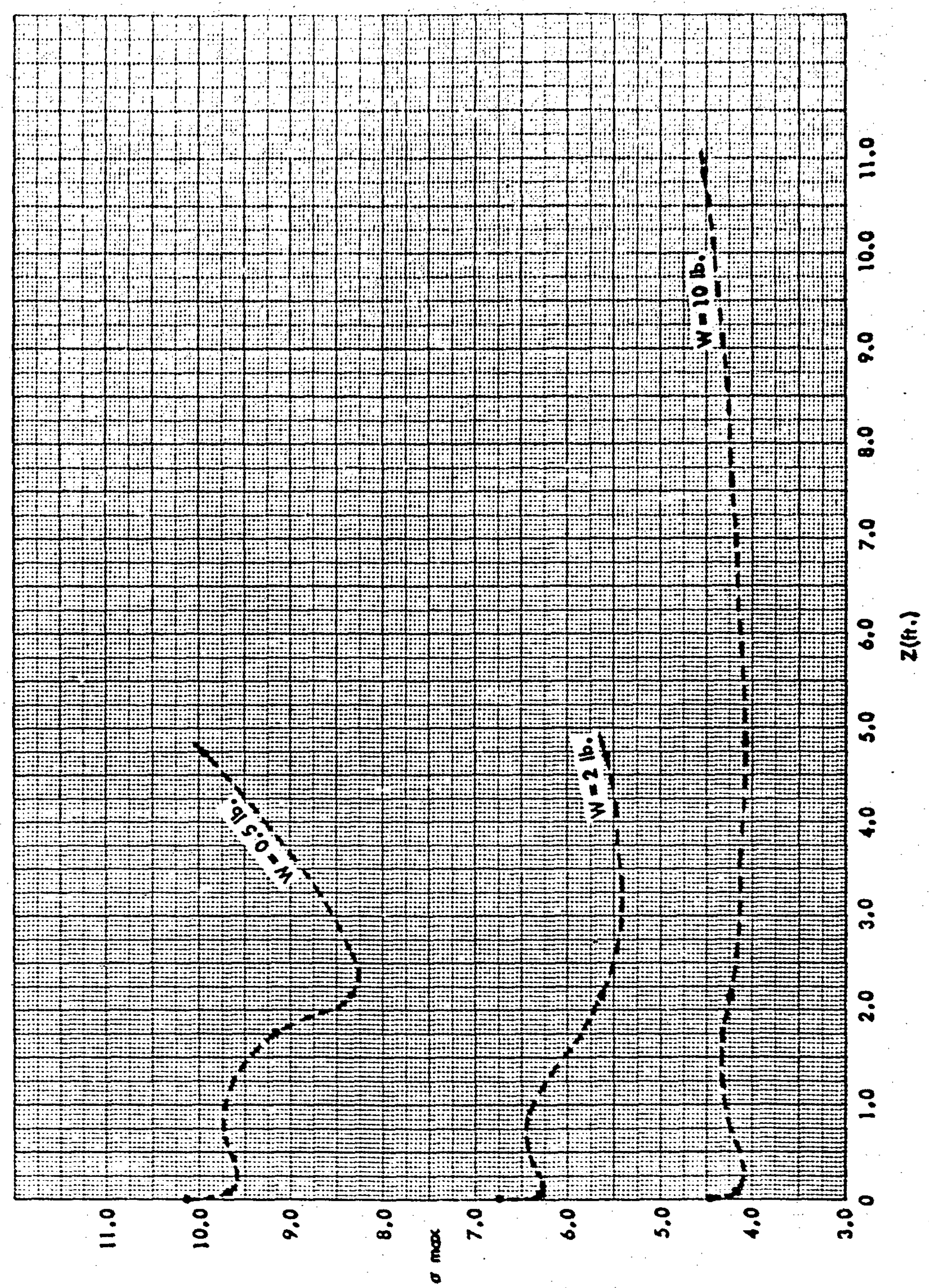



FIGURE 5 DIMENSIONLESS WAVE NUMBER AS A FUNCTION OF EXPLOSION DEPTH

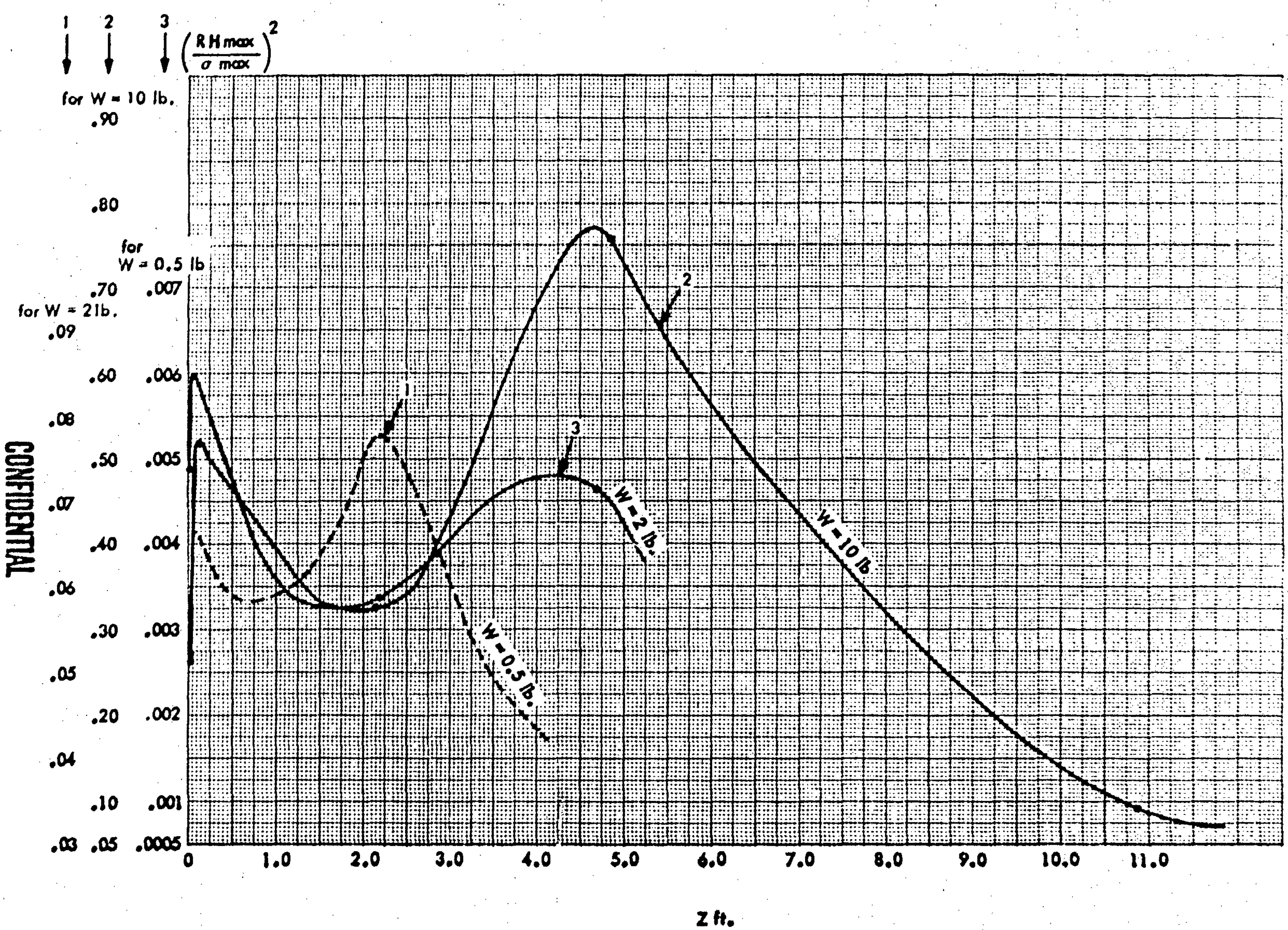



FIGURE 6 ENERGY PARAMETER AS A FUNCTION OF EXPLOSION DEPTH

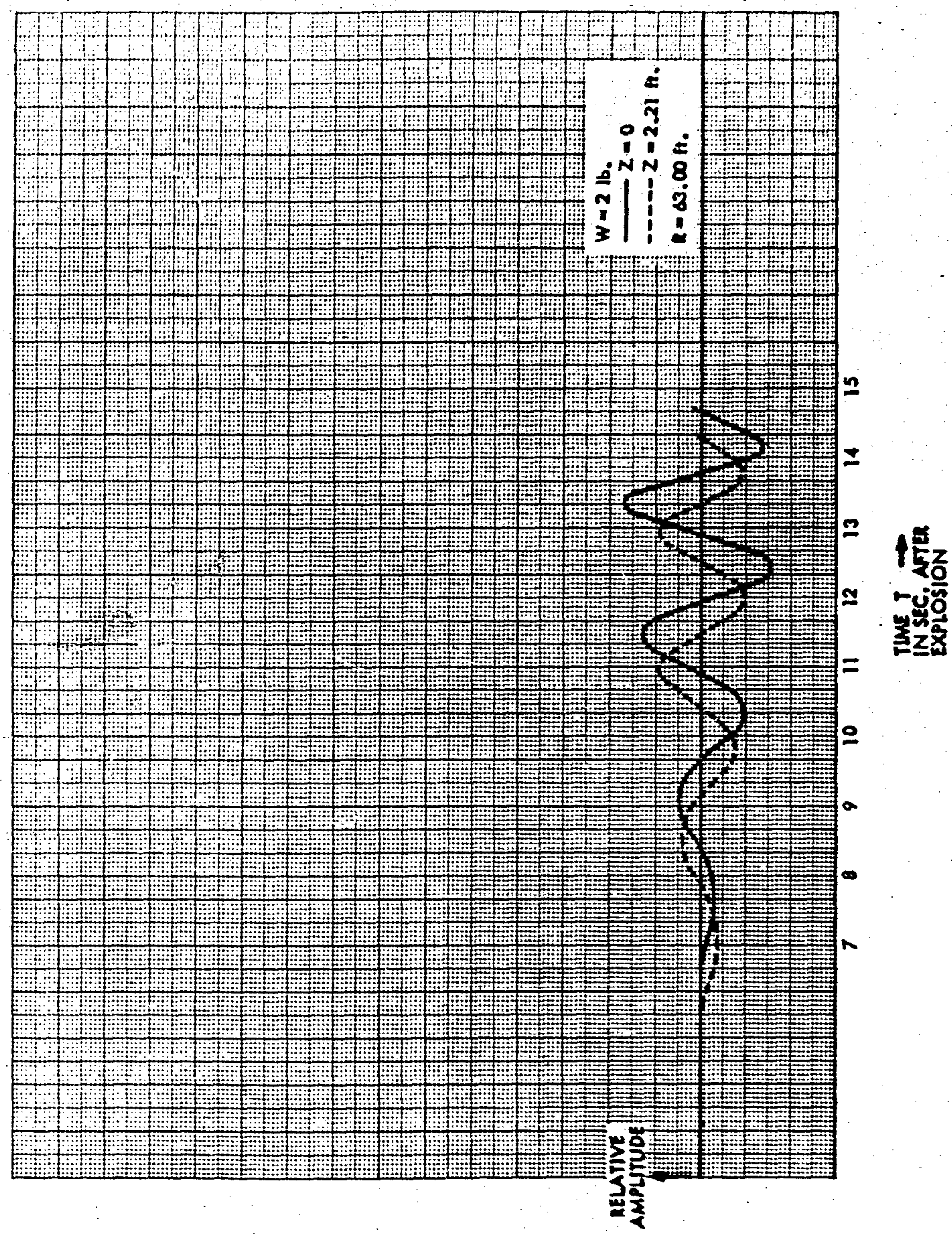



FIGURE 7 PHASE SHIFT OF THE WAVE TRAIN WITH DEPTH OF EXPLOSION

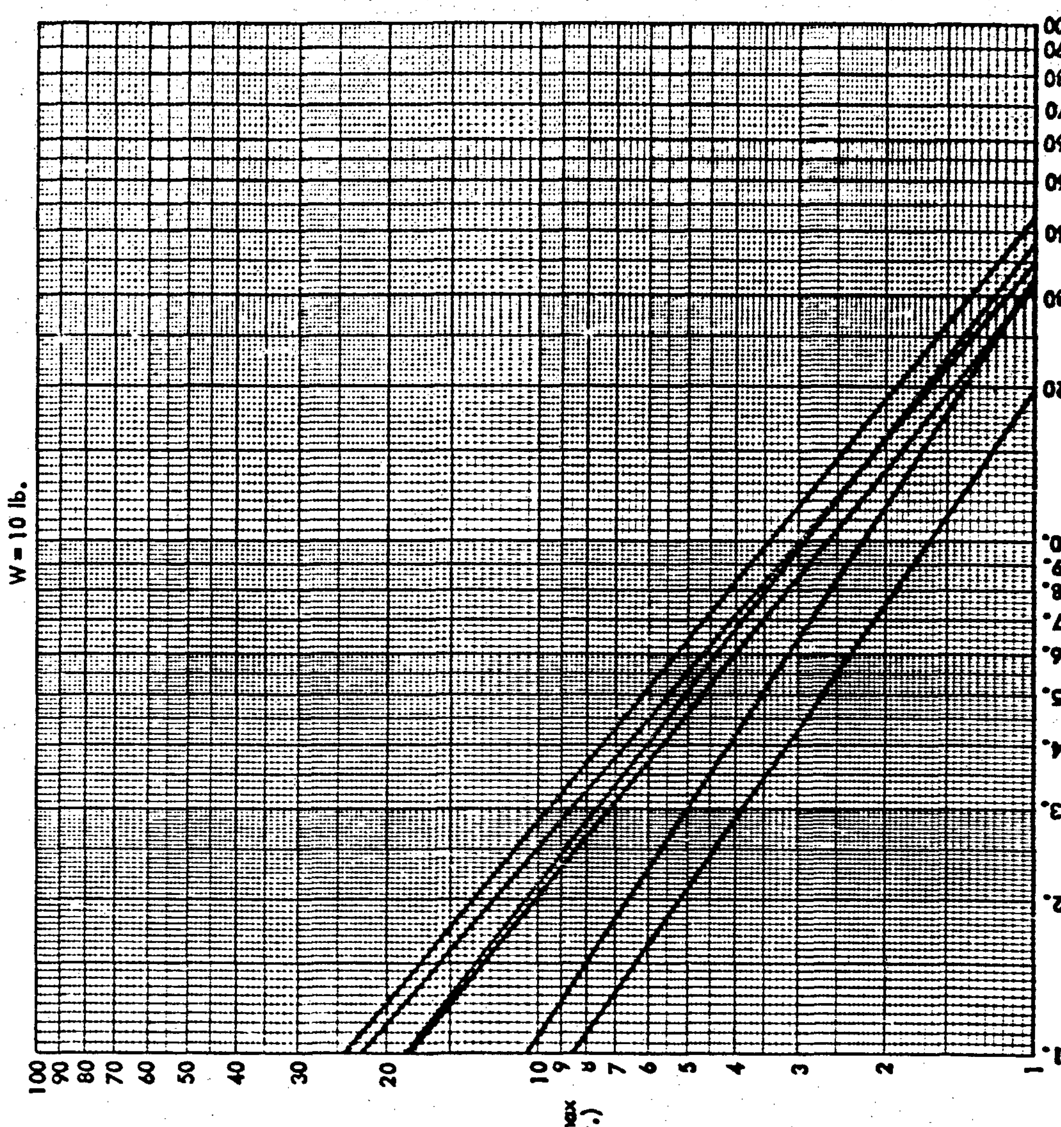



FIGURE 8 WAVE HEIGHT DECAY WITH DISTANCE

W = 10 lb.

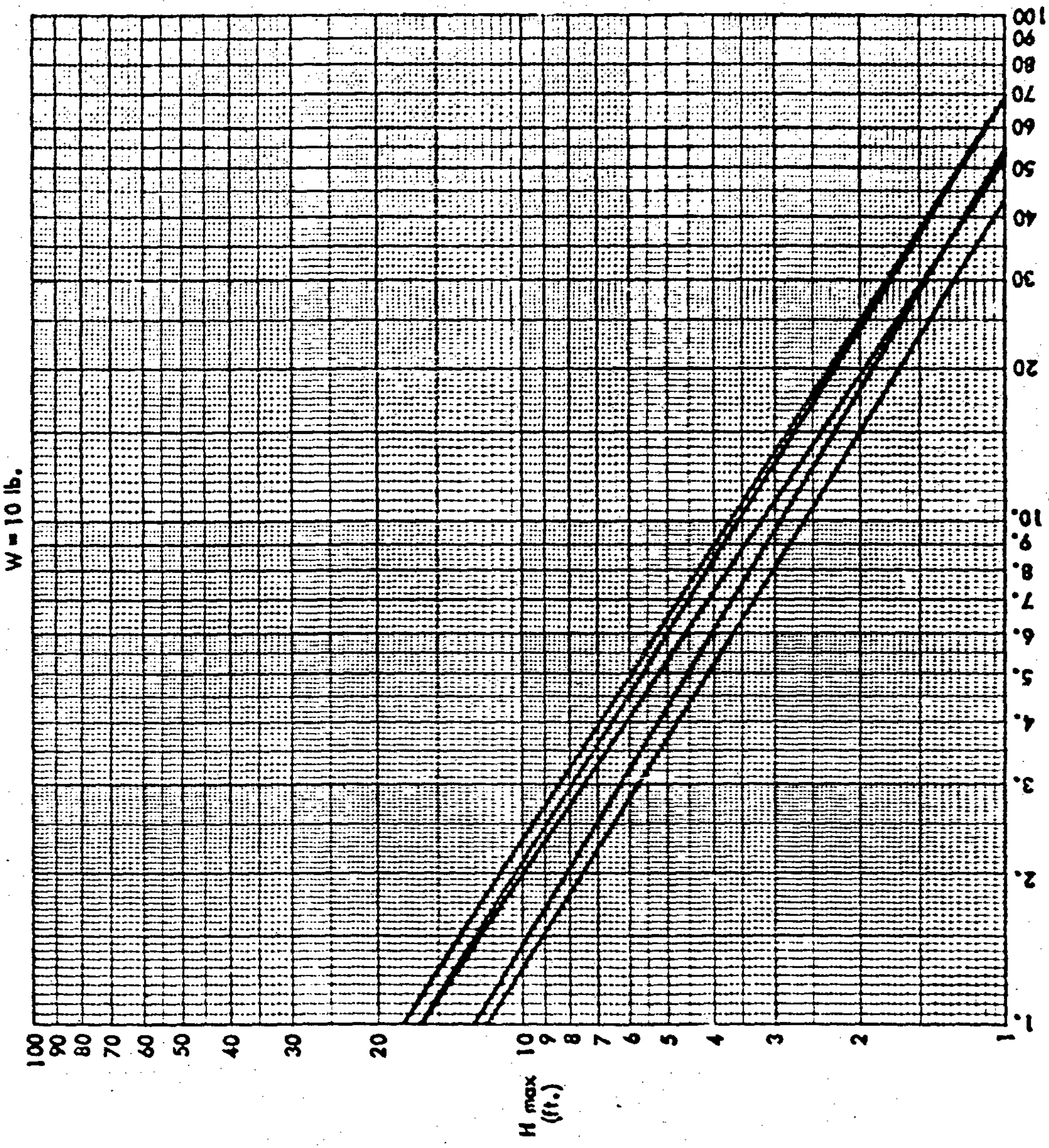



FIGURE 9 WAVE HEIGHT DECAY WITH DISTANCE

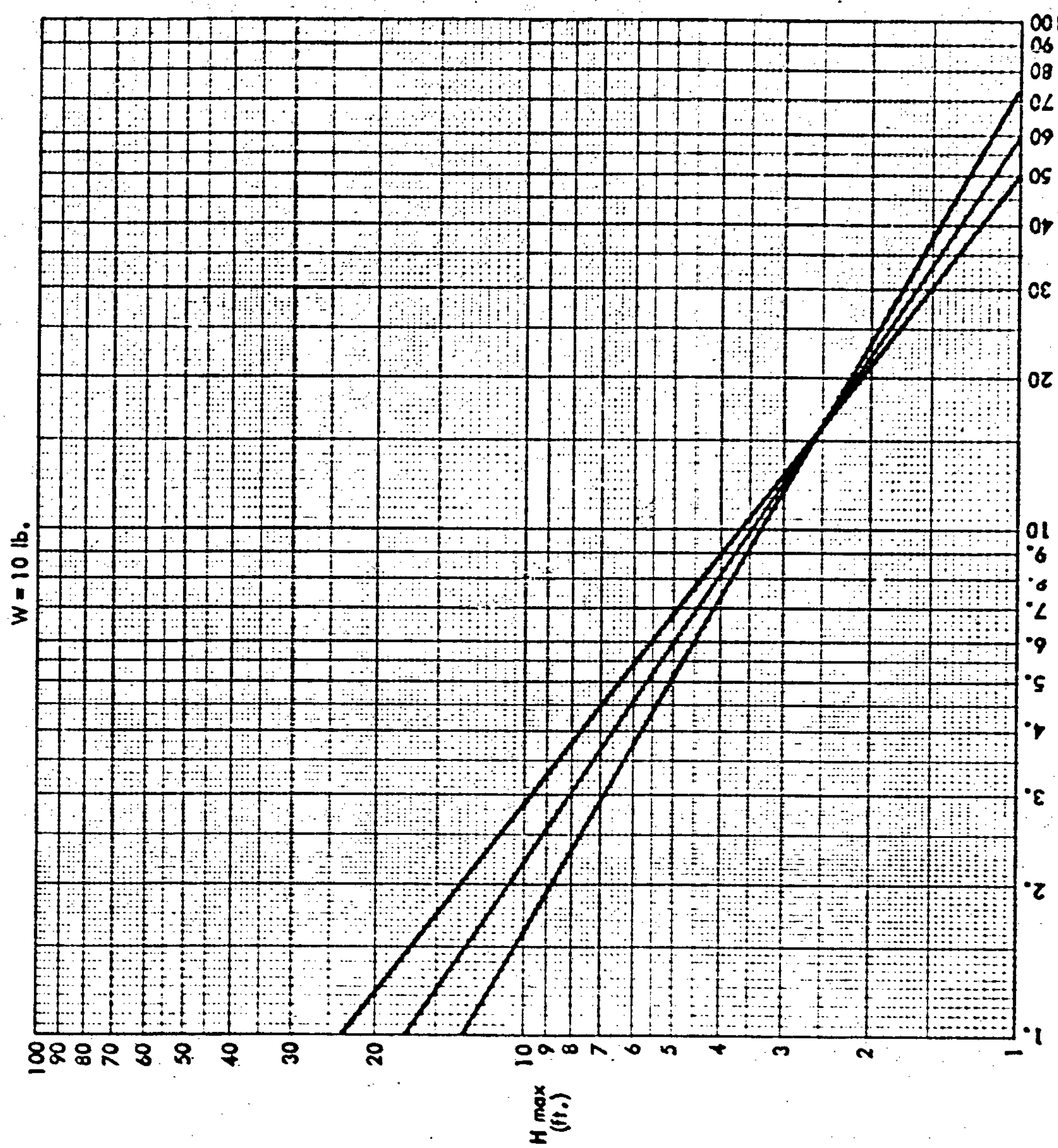



FIGURE 10 WAVE HEIGHT DECAY WITH DISTANCE

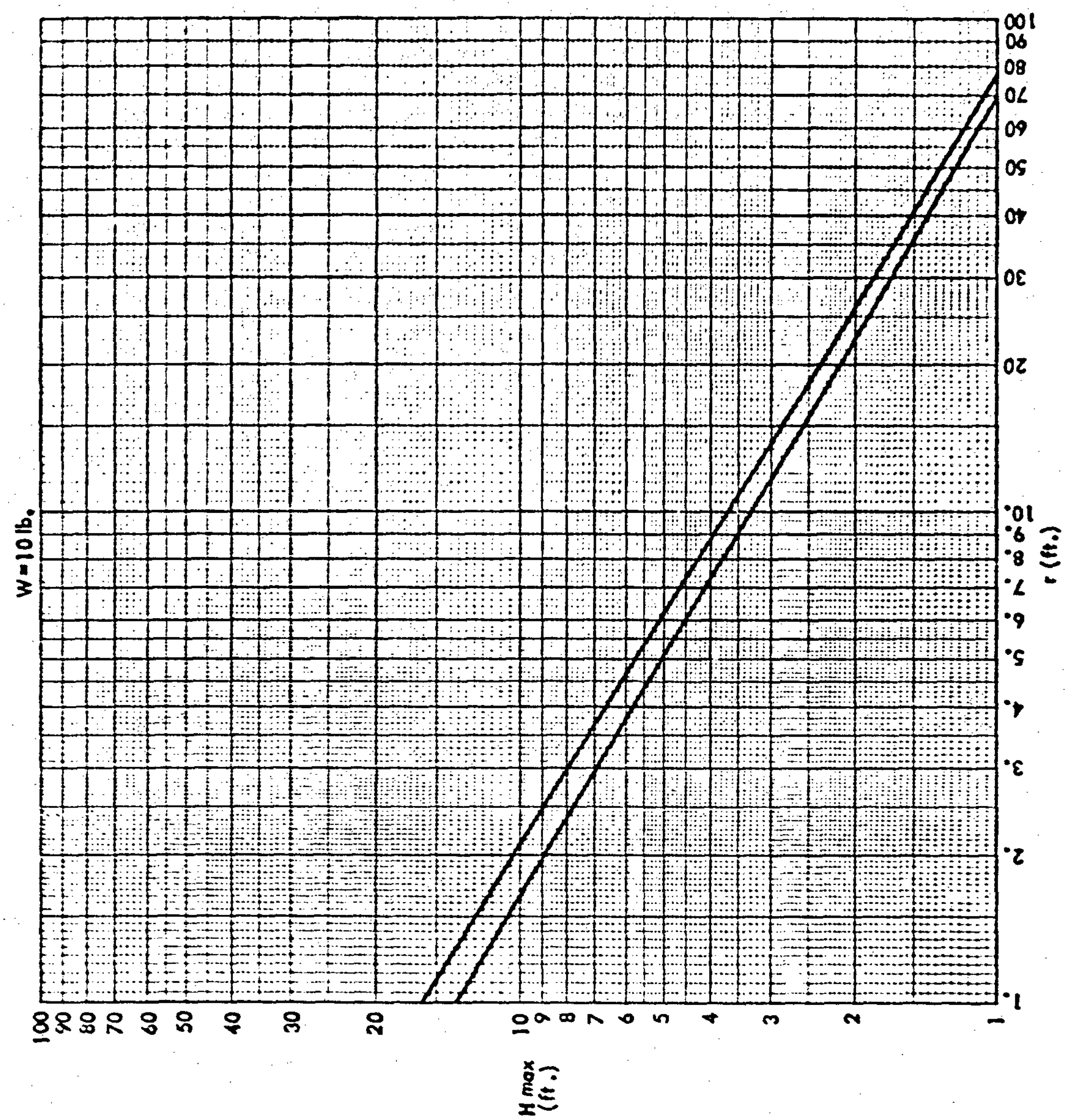



FIGURE 11 WAVE HEIGHT DECAY WITH DISTANCE

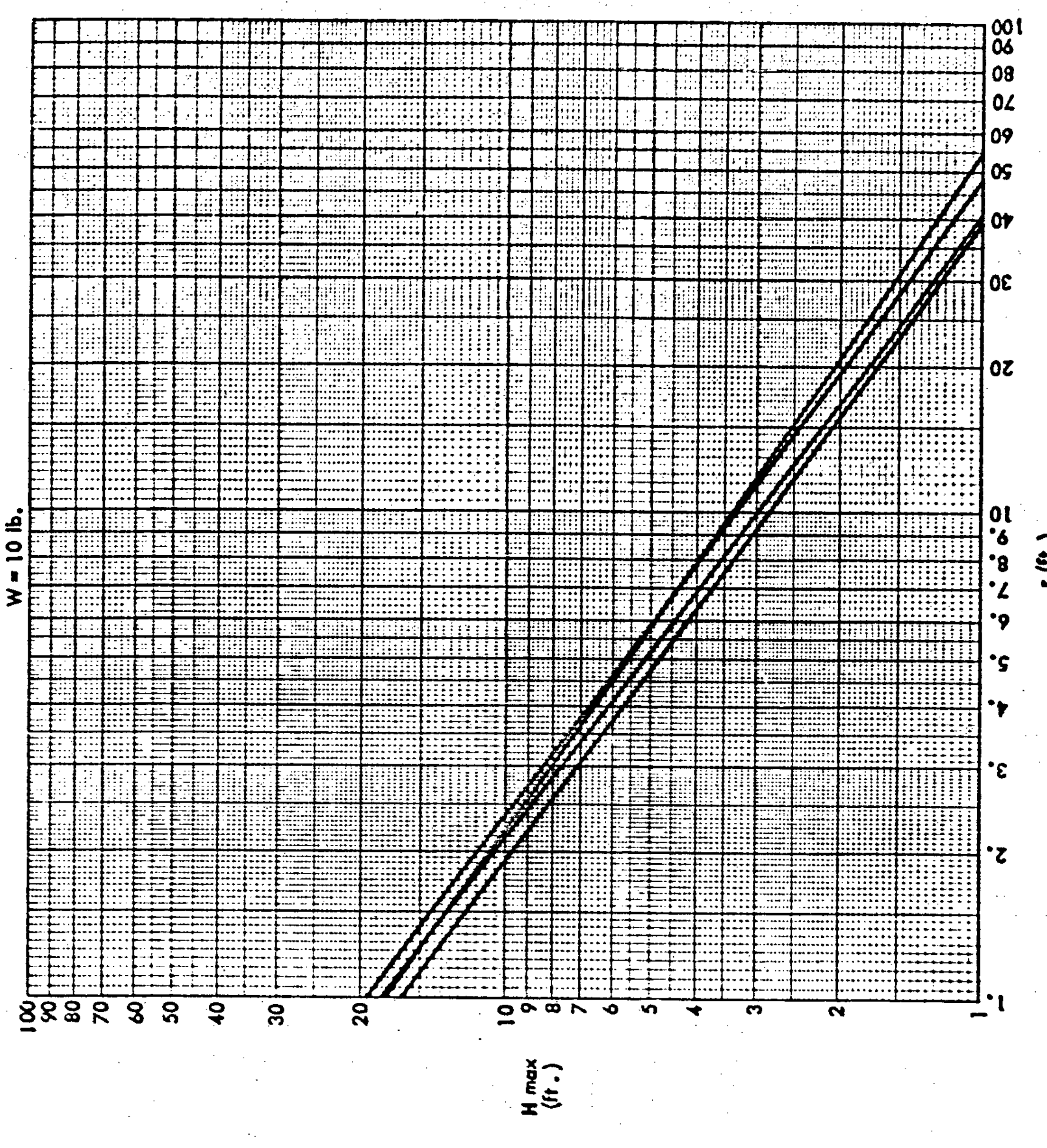



FIGURE 12 WAVE HEIGHT DECAY WITH DISTANCE

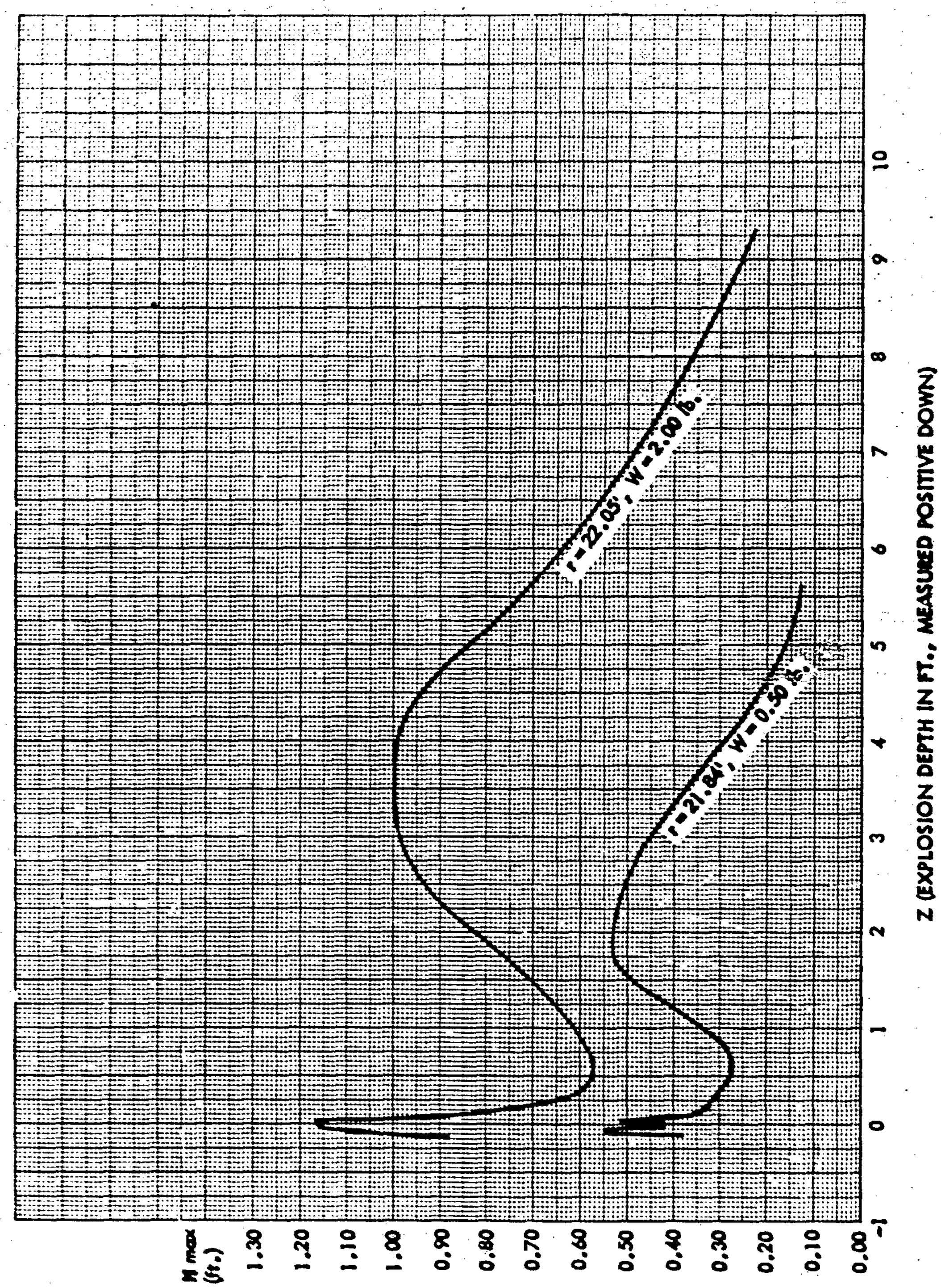



FIGURE 13 WAVE HEIGHT AS A FUNCTION OF EXPLOSION DEPTH

CONFIDENTIAL



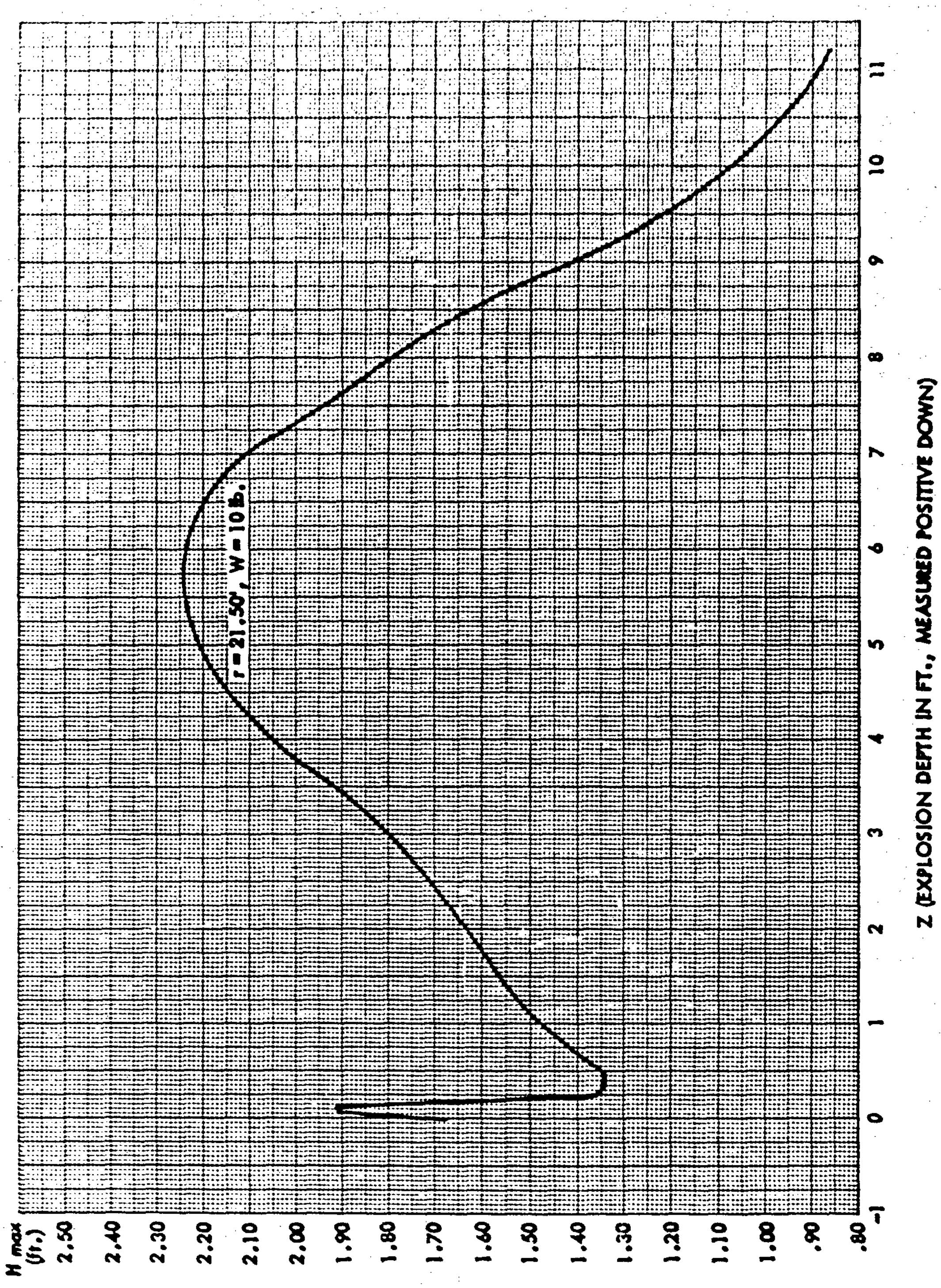



FIGURE 14 WAVE HEIGHT AS A FUNCTION OF EXPLOSION DEPTH

CONFIDENTIAL

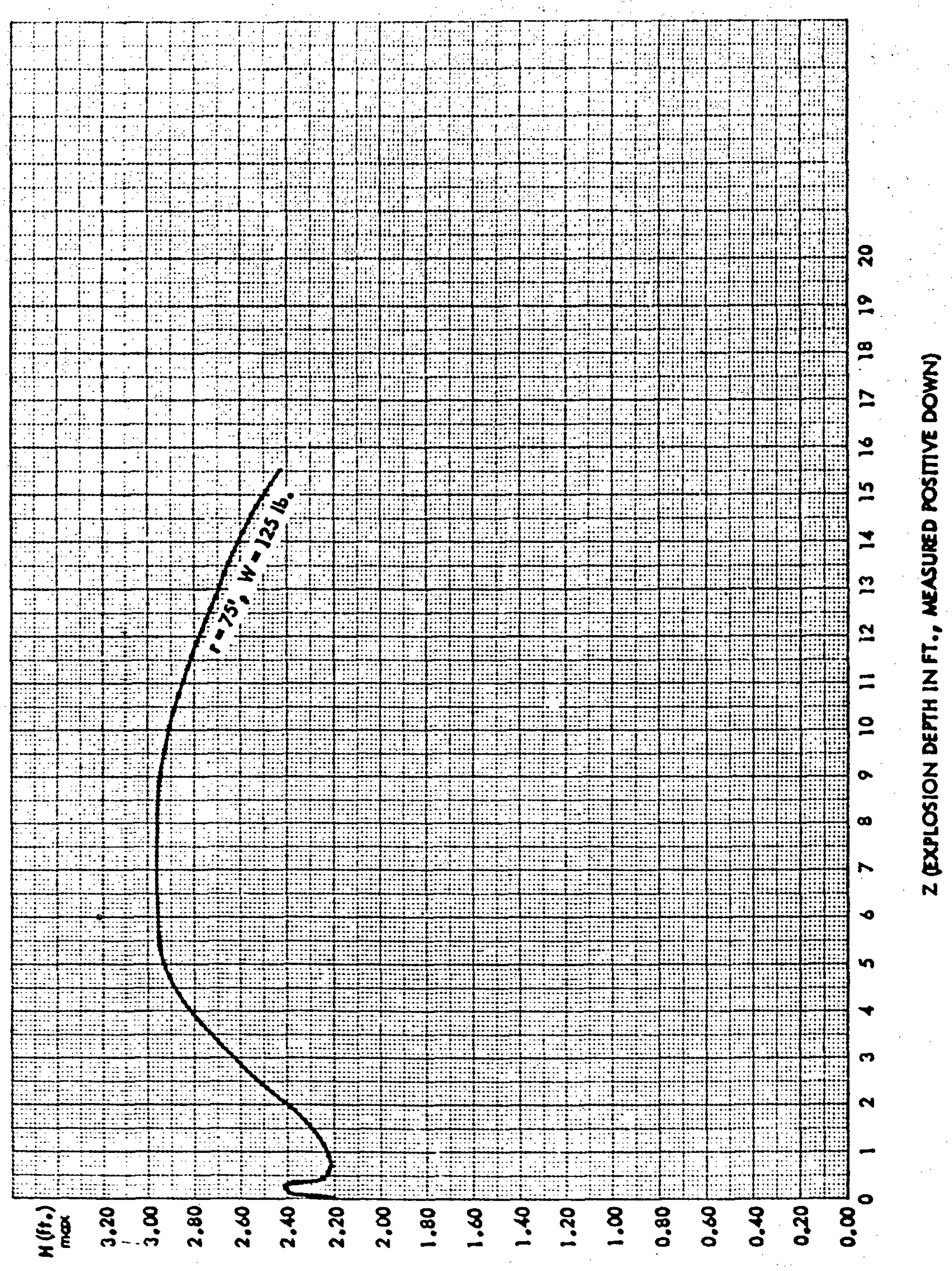



FIGURE 15 WAVE HEIGHT AS A FUNCTION OF EXPLOSION DEPTH

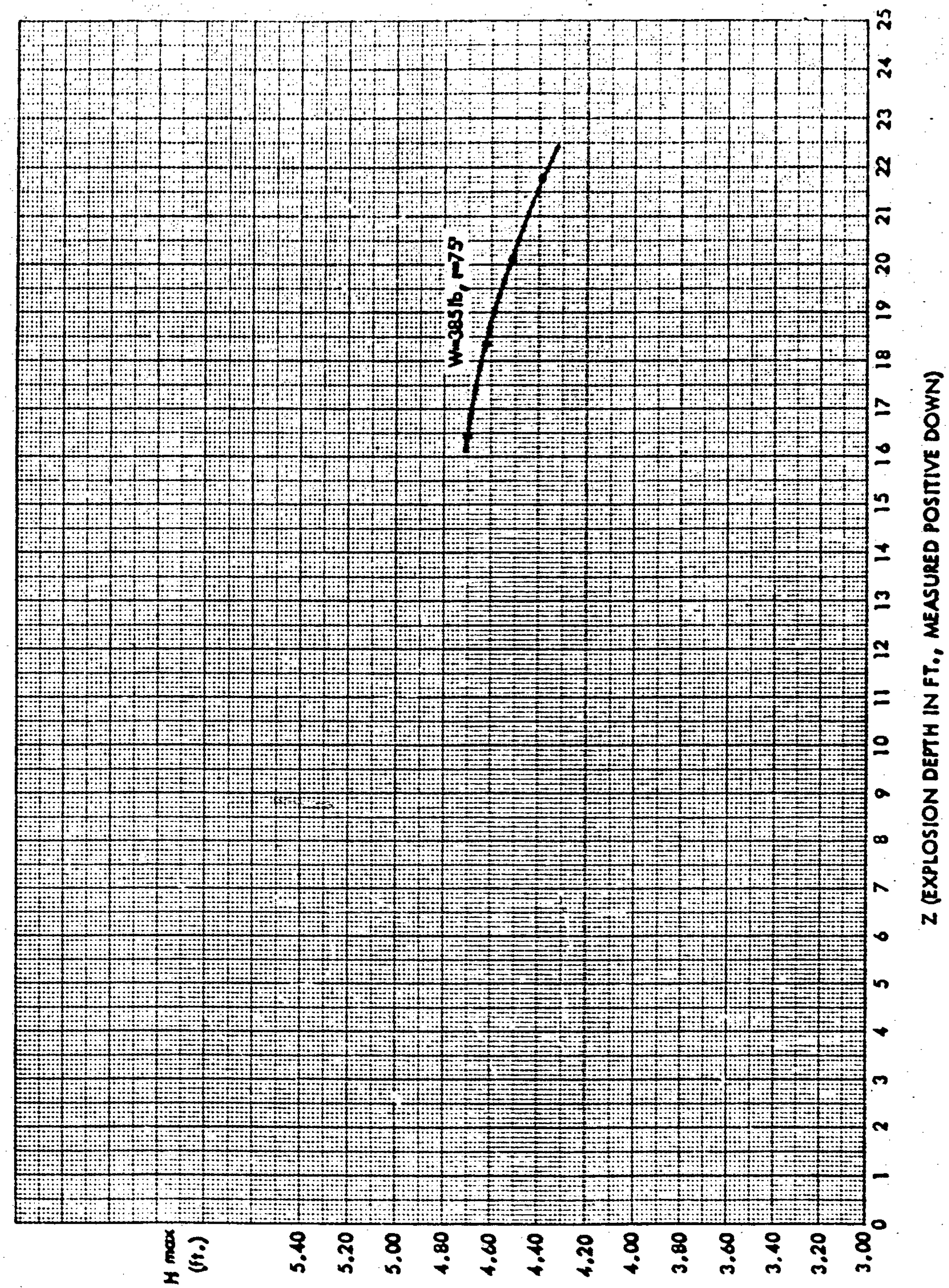



FIGURE 16 WAVE HEIGHT AS A FUNCTION OF EXPLOSION DEPTH

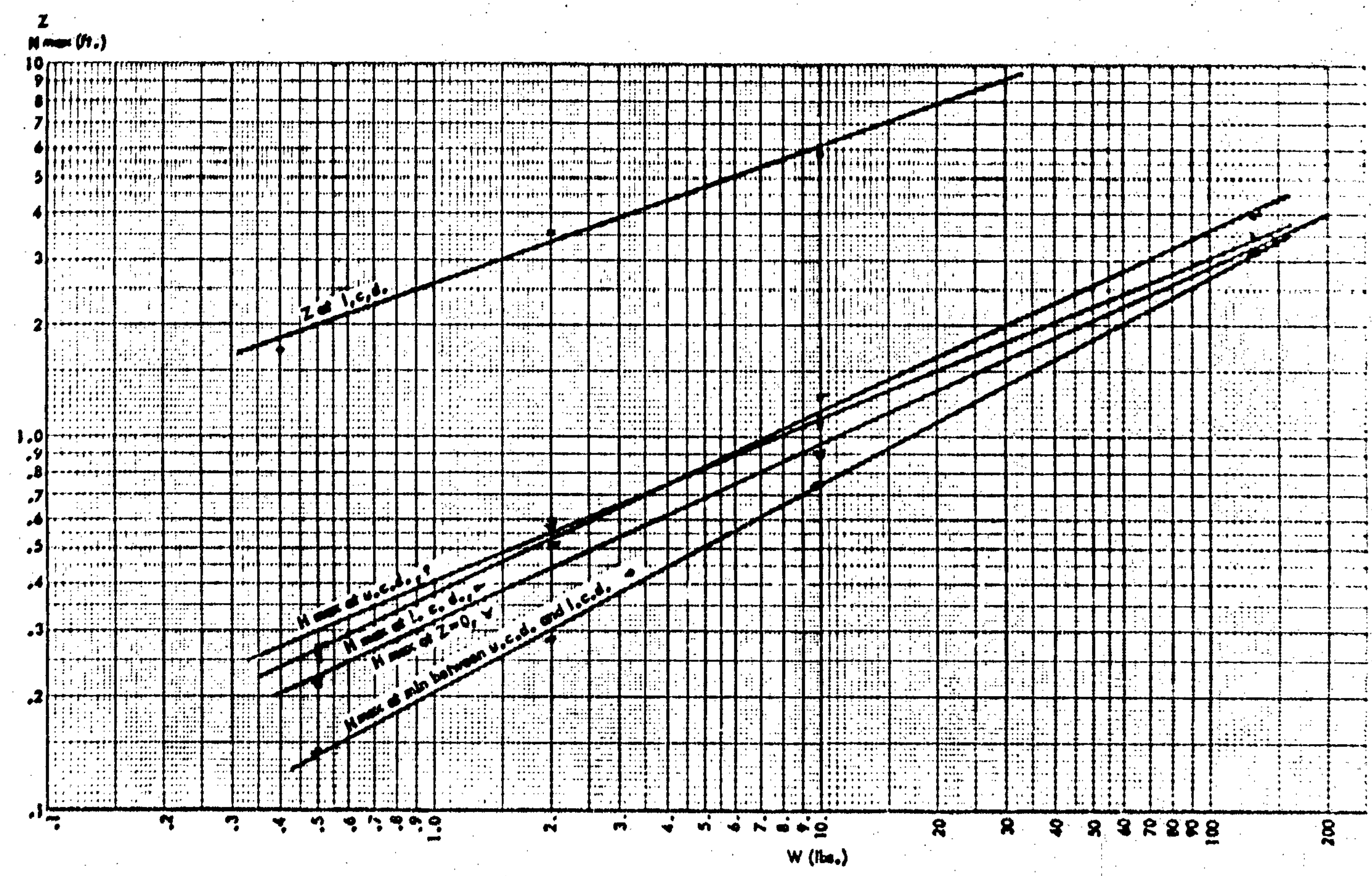



FIGURE 17 Hmax, u.c.d., Hmax, min., Hmax, l.c.d., H max, Z = 0, and Zu.c.d. AS A FUNCTION OF CHARGE WEIGHT

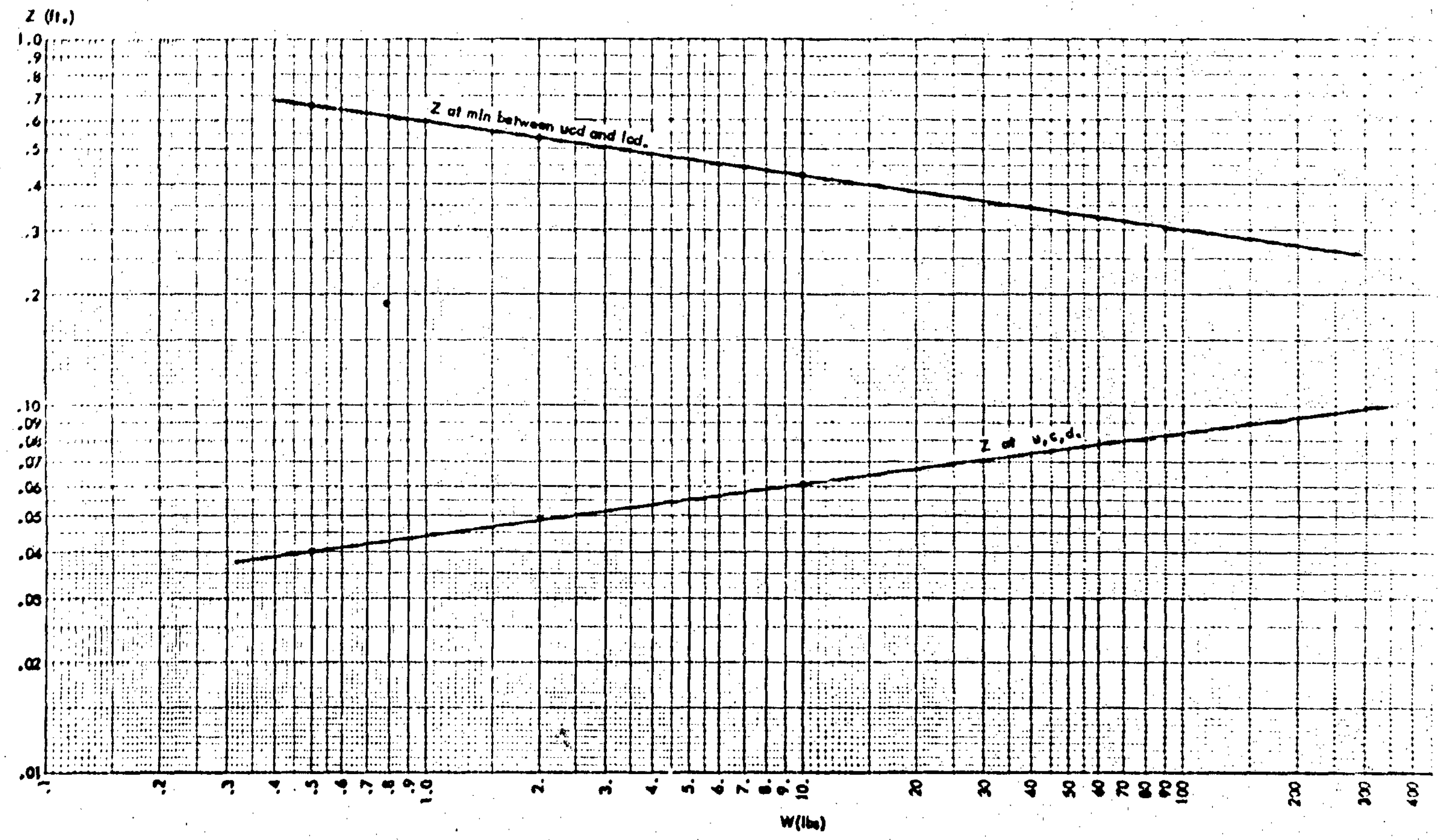



FIGURE 18 Zl.c.d. and $Z_{min}$ AS A FUNCTION OF CHARGE WEIGHT

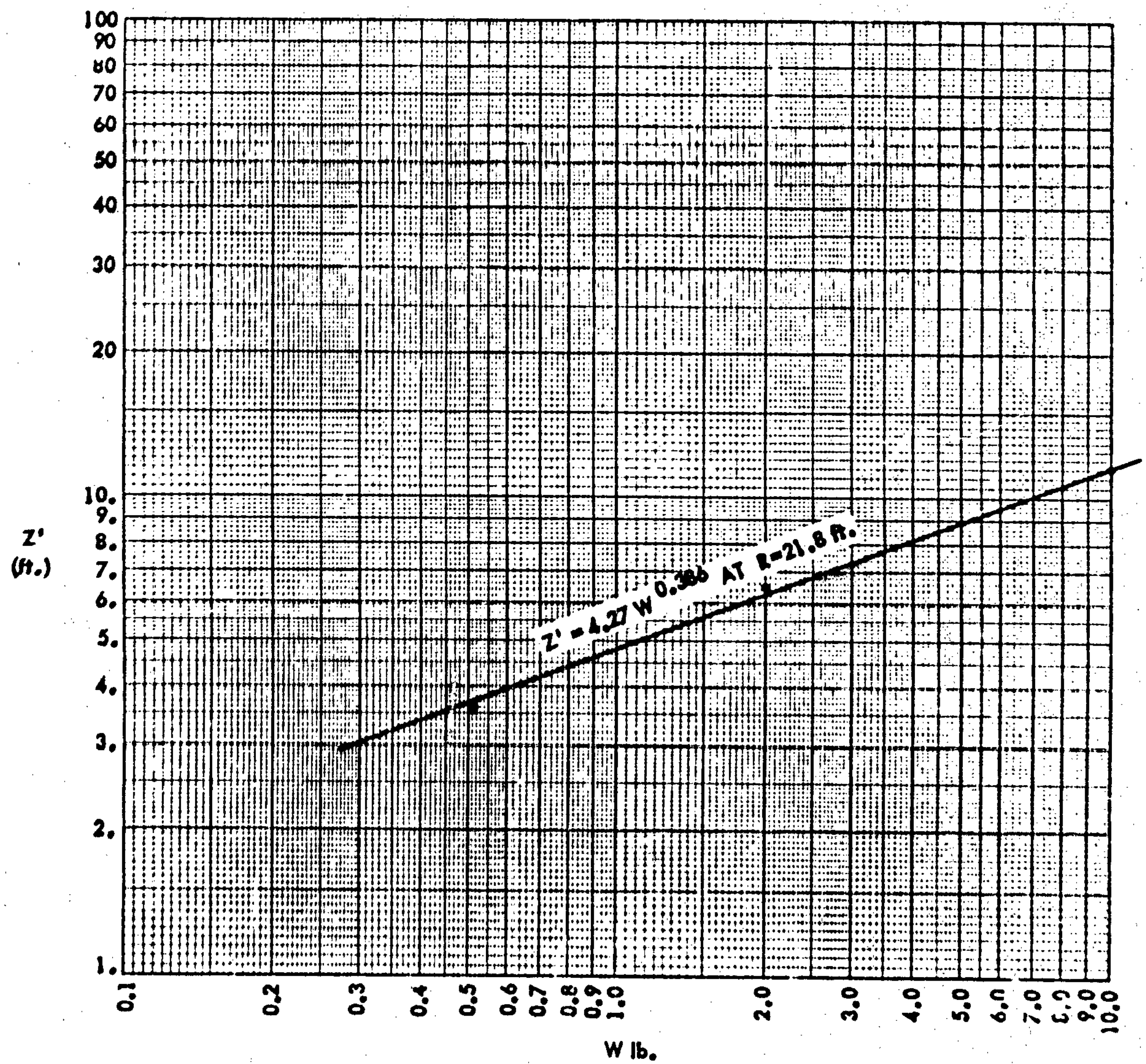
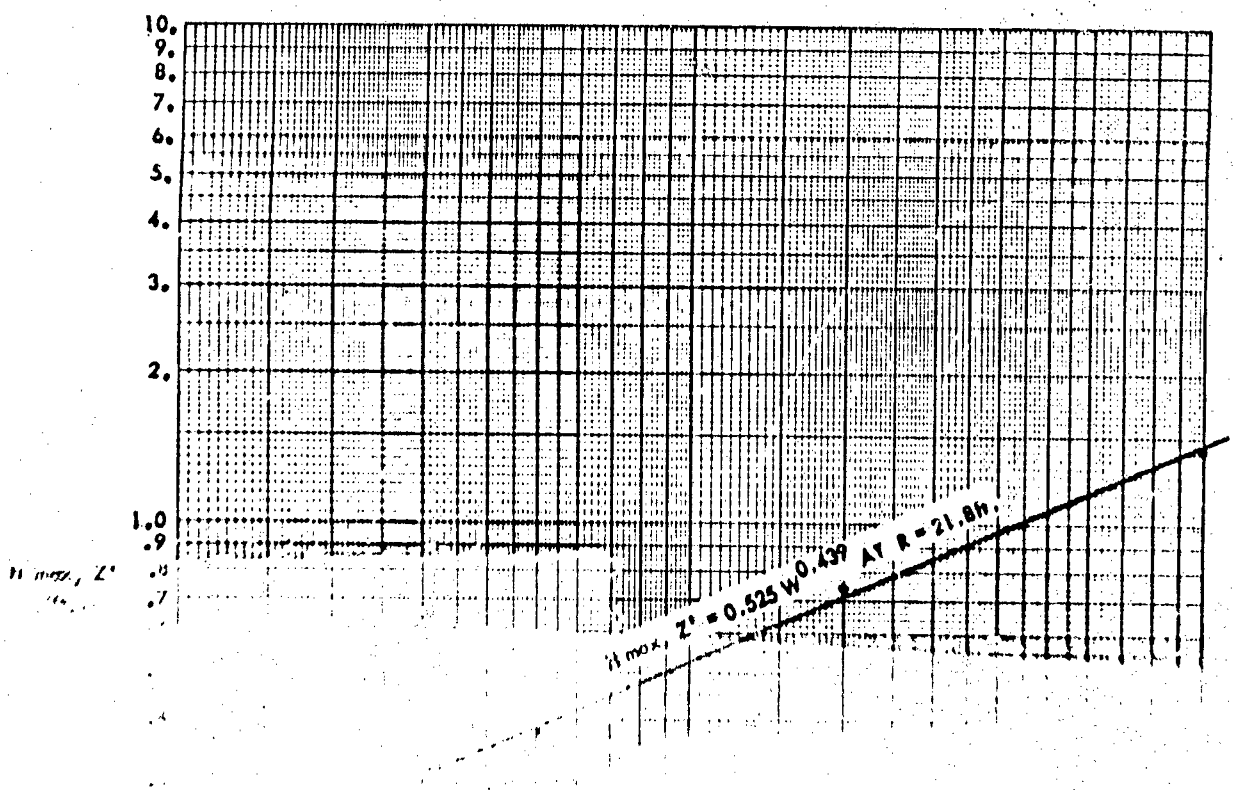



FIGURE 19 Z' AS A FUNCTION OF W



û max, Z' = 0.525 W^0.439 AT R = 21.8h.
10.
9.
8.
7.
6.
5.
4.
3.
2.
1.0
.9
.8
.7
û max, Z'