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# ROYAL AIRCRAFT ESTABLISHMENT

(FARNBOROUGH)

TECHNICAL NOTE No. G.W. 594

## SOME IMPLICATIONS OF ASYMMETRY IN BALLISTIC MISSILE RE-ENTRY HEADS

by

G. S. GREEN, M.A.

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SUMMARY

Asymmetry in a missile is classified under two headings (a) aerodynamic (b) inertial. Two issues are considered:- (1) the problem of spin stabilisation in free space for which (b) is relevant, (2) the interplay with increasing aerodynamic forces during the descent for which (a) and (b) are both relevant.

A full treatment of (2) is elusive mathematically and only a simplified version is attempted here.

The paper does not purport to be fundamentally original, but is rather a re-presentation, in the context of ballistic missile design.

Some general conclusions on asymmetry are drawn.

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Angular dispersion in missiles which have the moments of inertia A, B, C, substantially equal	1



## 1 INTRODUCTION

A ballistic missile re-entry head is normally thought of as an axially symmetrical body of revolution. It may be, however, that it will possess some sort of asymmetry - either by accident or design. The purpose of this paper is to explore some of the implications of this in relatively simple terms - it is not a complete study of the re-entry motion of such a body, this being, apparently, a difficult task.

It is important to clarify, in the first place, what is meant by 'asymmetry' in a ballistic missile head (subsequently referred to simply as a missile). It can refer to the inertial properties of the missile, or to the aerodynamic properties, or involve both simultaneously. Now, it is a fundamental property of a rigid body that, at any point there is always a set of principal axes, i.e. a set for which the products of inertia are zero. The inertial properties of the body are then specified simply by moments of inertia  $A$ ,  $B$ ,  $C$ , about these axes. Also the missile will have an aerodynamic axis through the centre of gravity - the axis which lies in the stream direction when the body is freely pivoted about its centre of gravity in an air-stream.

Asymmetry can mean:-

- (a) The missile is perfectly symmetrical aerodynamically and the roll principal axis ( $A$ ) coincides with the aerodynamic axis. The asymmetry consists of unequal pitch ( $B$ ) and yaw ( $C$ ) moments of inertia, i.e. we have  $A$ ,  $B$ ,  $C$ , instead of the normal  $A$ ,  $B$ ,  $B$ .
- (b) The missile is symmetrical inertially, i.e. we have moments of inertia  $A$ ,  $B$ ,  $B$ , but the aerodynamic axis does not coincide with the roll inertia axis.
- (c) Both these effects can be present together - which is the general case.

We consider first, (section 2), case (a) in free space. This is the spin stabilisation problem in a ballistic missile flight outside the atmosphere. Secondly (section 3), some of the implications of the presence of aerodynamic forces arising during the descent are considered, for all the cases.

The work is not fundamentally original. It is essentially a representation, from a ballistic missile design point of view, of ideas gathered from various sources, some of them quite old.

## 2 FREE SPACE SPIN STABILISATION OF AN ASYMMETRIC BODY

In some ballistic missile systems a control system (or 'turn-over set') is operated on the re-entry body shortly after it separates from the boost. The function of the turn-over set is to orientate the missile roll axis in the re-entry direction, to give the missile a spin about the roll axis, and thereafter, to leave the missile to travel freely until the final re-entry stage takes over. It is virtually certain that, apart from the spin in roll, there will be some angular velocity about the pitch and yaw axes, albeit much less than the roll. The problem is to assess how effective such a process of re-orientation and spin stabilisation will be.

We consider a missile which is asymmetric in the sense of case (a) section 1. The aerodynamic aspects do not matter in this context because we have essentially a free space motion. We have therefore a body with moments of inertia  $A$ ,  $B$ ,  $C$ , about principal axes, free to pivot about its centre of gravity. The angular problem with which we are concerned, can be completely dissociated from the translation of the body.



We have, relative to principal axes, fixed in the body,

moments of inertia     $A, B, C,$   
angular velocities     $p, q, r.$

Since there are no moments,

energy (T) is constant

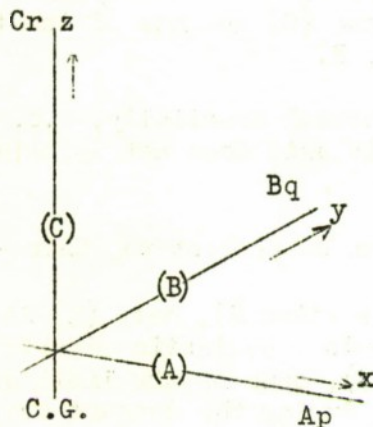
$$Ap^2 + Bq^2 + Cr^2 = 2T \quad (1)$$

angular momentum (H) is constant in magnitude (and direction),

$$A^2 p^2 + B^2 q^2 + C^2 r^2 = H^2 \quad (2)$$

The fixed direction in space of the angular momentum is usually called the 'invariable line'. The resolutes of H along the three axes are  $Ap, Bq, Cr,$  which are not individually constant. Accordingly, relative to the body, the invariable line moves, and describes the 'invariable cone'. If we set up a Cartesian co-ordinate system  $x, y, z,$  in relation to the axes, then on the invariable cone

$$x : y : z = Ap : Bq : Cr$$



Hence, in virtue of equation (1) and (2), the equation defining the invariable cone is

$$\left(1 - \frac{H^2}{2AT}\right) x^2 + \left(1 - \frac{H^2}{2BT}\right) y^2 + \left(1 - \frac{H^2}{2CT}\right) z^2 = 0 \quad (3)$$

Thus, from (3), we can investigate how the invariable line moves relative to the body, and hence, reciprocally, we shall know how the body moves (in space) relative to space-fixed invariable line.

Let us take the case

$$A < B < C \quad (4)$$

since ballistic missiles almost always satisfy the condition of smallest moment of inertia about the roll axis.

In that case,

$$\left. \begin{array}{l} H^2 > 2AT \\ H^2 < 2CT \end{array} \right\} \quad (5)$$

and nothing can be said as to the relation between  $H^2$  and  $2BT$ . We consider next the intersection of the invariable cone [(3), above] with a unit sphere round the origin, whose equation is

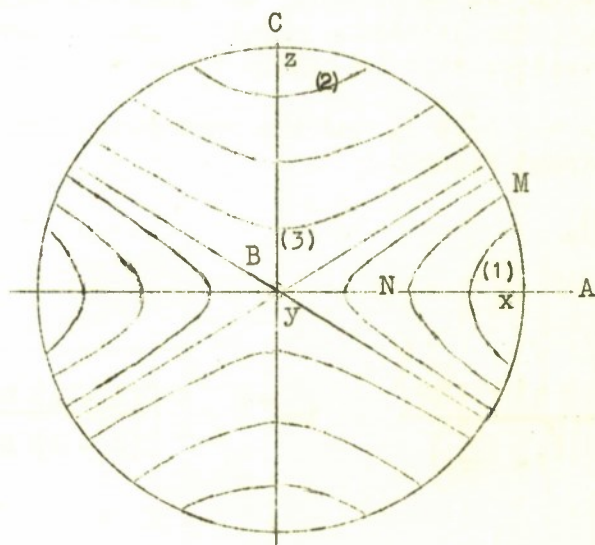
$$x^2 + y^2 + z^2 = 1 \quad (6)$$

The projections of the intersection onto the  $yz$ ,  $zx$ ,  $xy$  planes are

$$\left. \begin{array}{l} \frac{H^2}{2T} \left( \frac{1}{A} - \frac{1}{B} \right) y^2 + \frac{H^2}{2T} \left( \frac{1}{A} - \frac{1}{C} \right) z^2 = \frac{H^2}{2AT} - 1 \quad (\text{ellipse}) \\ \frac{H^2}{2T} \left( \frac{1}{C} - \frac{1}{B} \right) z^2 + \frac{H^2}{2T} \left( \frac{1}{A} - \frac{1}{B} \right) x^2 = 1 - \frac{H^2}{2BT} \quad (\text{hyperbola}) \\ \frac{H^2}{2T} \left( \frac{1}{A} - \frac{1}{C} \right) x^2 + \frac{H^2}{2T} \left( \frac{1}{B} - \frac{1}{C} \right) y^2 = 1 - \frac{H^2}{2CT} \quad (\text{ellipse}) \end{array} \right\} \quad (7)$$

In view of the inequalities (4), (5), these can be classified, as indicated in brackets, into ellipse or hyperbola.

Thus we have the following picture of the movement of the invariable line relative to the body (actually its intersection with the unit sphere)



Which particular track is followed by the invariable line depends on the initial conditions. If we start with  $p$  much larger than  $q$  or  $r$ , then  $H^2$  will be only a very little greater than  $2AT$ , and we shall get a 'tight' track round the  $x$  axis, (moment of inertia  $A$ ), such as (1) in the figure. This would be spin stabilisation about  $A$ . On the other hand, if  $r$  is much larger than  $p$  or  $q$ , there would be a 'tight' track (2) round the  $z$  axis ( $C$ ),  $H^2$  being only a little less than  $2CT$ . This would be spin stabilisation about  $C$ .



But the axis of B, the middle moment of inertia, is differently placed. There are no 'tight' tracks round it, and if  $q$  is large in relation to  $p$ ,  $r$  a curve such as (3) is followed. Thus the invariable line moves right away from the B axis, which, from a space point of view, means that the B axis moves over a wide range (up to  $180^\circ$ ). Spin stabilisation above B, the middle moment of inertia is therefore impossible.

The curves are separable into those which go round A, and those which go round C. The division occurs when  $H^2 = 2BT$ , the hyperbola of equation (7) then degenerating into two lines.

If  $H^2 = 2BT$ , then

$$A^2 p^2 + B^2 q^2 + C^2 r^2 = ABp^2 + B^2 q^2 + BCr^2$$

i.e.

$$p^2 A(B - A) = r^2 C(C - B) \quad (8)$$

If

$p^2 A(B - A) > r^2 C(C - B)$  we have a curve round A,

$p^2 A(B - A) < r^2 C(C - B)$  we have a curve round C.

These relations apply throughout the motion, not just 'initially'.

We shall concentrate, as an example, on cases for which  $p$  is large in relation to  $q$  or  $r$ , since for ballistic missile applications, with a pitch over process after separation this is likely to be the case.

The angle between the axis A, and the invariable line, varies between A - M and A - N (c.f. inset figure).

From equation (3),

$$A - M = \tan^{-1} \frac{-\left(1 - \frac{H^2}{2AT}\right)}{\left(1 - \frac{H^2}{2CT}\right)} = \tan^{-1} \frac{\frac{C}{A} \left\{ \frac{(B - A) Bq^2 + (C - A) Cr^2}{(C - A) Ap^2 + (C - B) Bq^2} \right\}}{\dots} \quad (9)$$

$$A - N = \tan^{-1} \frac{-\left(1 - \frac{H^2}{2AT}\right)}{\left(1 - \frac{H^2}{2BT}\right)} = \tan^{-1} \frac{\frac{B}{A} \left\{ \frac{(B - A) Bq^2 + (C - A) Cr^2}{(B - A) Ap^2 - (C - B) Cr^2} \right\}}{\dots} \quad (10)$$



In principle, either of these could be the greater, but in the example taken, with  $p$  much larger than  $q$  or  $r$  it is  $A - N$  which is critical since the denominator in the expression might be small.  $A - M$  can never be large. Moreover, the unwanted disturbance, producing  $q$  or  $r$ , will produce the largest effect when it is wholly 'r', since the coefficient of  $r^2$  in (10) is greater than that of  $q^2$ .

Thus the significant angular dispersion of the axis is given by

$$\phi = 2 \tan^{-1} \sqrt{\frac{BC (C - A) r^2}{A \{(B - A) A p^2 - (C - B) C r^2\}}} \quad (11)$$

[The factor 2 enters because the complete excursion involves twice the angle  $A - N$ .] This is the essential result in this paragraph, giving the angle to which the axis  $A$  will wander away when a spin rate  $p$  is applied about it, and there is an initial disturbing angular velocity  $r$ .

In practice, the interest in this question usually arises for a type of ballistic missile in which it is very likely that  $A$ ,  $B$  and  $C$  will be close together. As we have already seen, it would be fatal to attempt spin stabilisation about  $A$  if it happened to fall between  $B$  and  $C$ . The extent to which it should lie outside them can be found from (11).

If  $A$ ,  $B$  and  $C$  are very nearly equal, then

$$\phi = 2 \tan^{-1} \sqrt{\frac{(C - A) r^2}{(B - A) p^2 - (C - B) r^2}}, \text{ very closely.}$$

$\phi$ , obtained from this formula, is plotted in Fig.1, against  $\frac{C - B}{B - A}$ , for different values of  $\frac{p}{r}$ . A practical value of  $\frac{p}{r}$  might be 10 (or more). For  $\frac{p}{r} = 10$ ,  $\phi = 38^\circ$  for  $\frac{C - B}{B - A} = 10$ ,  $\phi = 54^\circ$  for  $\frac{C - B}{B - A} = 20$ , and  $\phi = 180^\circ$  for  $\frac{C - B}{B - A} = 100$ .

Thus unless  $A$  is very much closer to  $B$  than  $B$  is to  $C$ , spin stabilisation about  $A$  will be quite satisfactory for a missile in which  $A$ ,  $B$  and  $C$  are close together.

For the more usual type of missile,  $A$  is considerably less than  $B$  or  $C$  which are nominally equal. In these circumstances, equation (11) shows that the fact that  $B$  and  $C$  may turn out to be not quite equal in practice is of no importance so far as spin stabilisation is concerned.

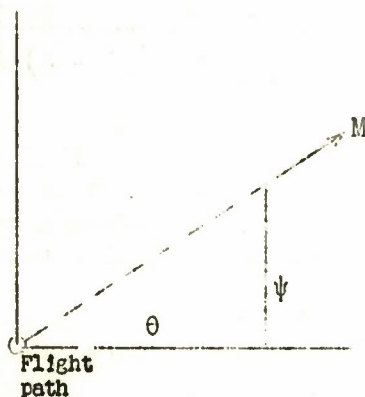
### 3 SIGNIFICANCE OF ASYMMETRY IN THE PRESENCE OF AERODYNAMIC FORCES

We confine ourselves to cases in which the aerodynamic effects are represented solely by a moment about the centre of gravity. Moreover we shall vary the moment linearly with incidence, but not in any other way. In the descent of a ballistic missile, of course, there is much more to the aerodynamics than this - effect of lift, damping moment, etc., and variation with time also. However, in the case of a symmetrical missile it has been found that whilst these affect the size of the motion, they do not affect the frequency. As it is essentially with the frequency that we shall be concerned, it is believed that the conclusions are significant.

### 3.1 Symmetrical missile

It is simplest if we look first at the symmetrical case, that is the body has moments of inertia of A in roll and B in pitch or yaw, and also has aerodynamic symmetry.

We are concerned only with small angular disturbances, so that we can take constituent disturbances on a rectangular basis of  $\theta$ ,  $\psi$ . These are relative to fixed space axes.



The missile has spin P, in roll, and P will be constant.

Equations of motion are

$$B \ddot{\theta} + AP \dot{\psi} - M_a \theta = 0, \quad (12)$$

$$B \ddot{\psi} - AP \dot{\theta} - M_a \psi = 0, \quad (13)$$

where  $M_a$  is the dispersing moment per unit of incidence.  $M_a$  is negative for positive restoring moment. (12) + i(13) gives

$$B \frac{d^2}{dt^2} (\theta + i\psi) - iAP \frac{d}{dt} (\theta + i\psi) - M_a (\theta + i\psi) = 0. \quad (14)$$

If  $\theta + i\psi$  has a solution of the form  $\theta + i\psi = K e^{i\lambda t}$ , then

$$-B\lambda^2 + AP\lambda - M_a = 0.$$

$$\lambda = \frac{AP}{2B} \pm \sqrt{\frac{A^2 P^2}{4B^2} - \frac{M_a}{B}}.$$

For a stable motion of the missile, the values of  $\lambda$  must be real, i.e.,

$$\frac{A^2 P^2}{4B} > M_a \quad (15)$$

If, as is normal,  $M_a$  is negative (aerodynamic stability) this is automatically satisfied, and the values for  $\lambda$  are of opposite sign. In that case, the missile

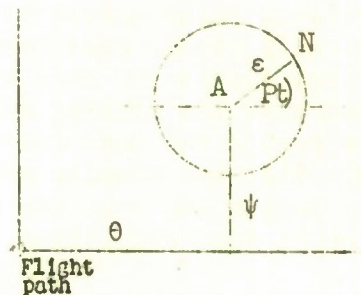
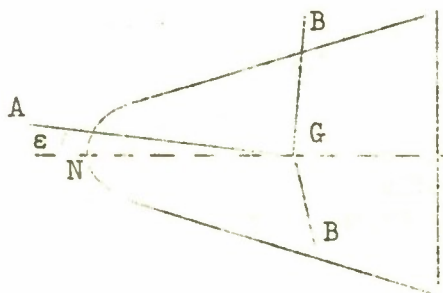


angular motion is the composite effect of two rotating arms, whose directions of rotation are opposite. If on the other hand  $M_a$  is positive (destabilising aerodynamics), stability can be achieved by applying sufficient spin to satisfy equation (15). In this case, the two arm constituents of the motion rotate in the same direction.

This is, of course, well known.

### 3.2 Aerodynamic asymmetry

Now we introduce aerodynamic asymmetry, as in case (b), section 1.



The principal roll axis (A) is not aligned with the aerodynamic axis, there being a small angle  $\epsilon$  between them. N represents the nose of the missile, from an aerodynamic point of view.  $\theta$ ,  $\psi$  again represent the angular disturbance, from the flight direction, of the axis A. In the  $\theta$ ,  $\psi$  plane we have N rotating around A at a radius  $\epsilon$  and rate P.

The equations of motion now are:-

$$B \ddot{\theta} + AP \dot{\psi} - M_a (\theta + \epsilon \cos Pt) = 0 \quad (16)$$

$$B \ddot{\psi} - AP \dot{\theta} - M_a (\psi + \epsilon \sin Pt) = 0 \quad (17)$$

The solution differs from 3.1 only in that we have an additional term (particular integral)

$$\theta + i\psi = - \frac{M_a \epsilon}{M_a + (B - A) P^2} e^{iPt} + \text{complementary function} \quad (\text{as in section 3.1}).$$

The motion is now, therefore, tri-cyclic.

In the normal missile design  $M_a$  is negative and  $B > A$ . Hence the particular integral will be infinite at that stage of the descent when

$$(B - A) P^2 = - M_a \quad (18)$$

On current designs this occurs at some considerable height, 200,000 ft perhaps. In practice the neglected aerodynamic forces would serve to soften the effect, and also, as  $M_a$  is progressively varying, the condition will be

passed through fairly quickly. Nevertheless it might be important. It is frequently referred to as a 'resonance condition' because the condition (18) is equivalent to a coincidence between the frequency in the basic case ( $\lambda$  in section 3.1) and the spin  $P$ .

The only publication seen by the writer which properly investigates the numerical significance of this in a ballistic missile descent is Ref.2. The authors feel that a mathematical solution for the angular disturbance is unlikely, and present the problem to a digital calculating machine. They describe the results obtained in four ad hoc cases - two of which are relevant to this paragraph, and two to the next.

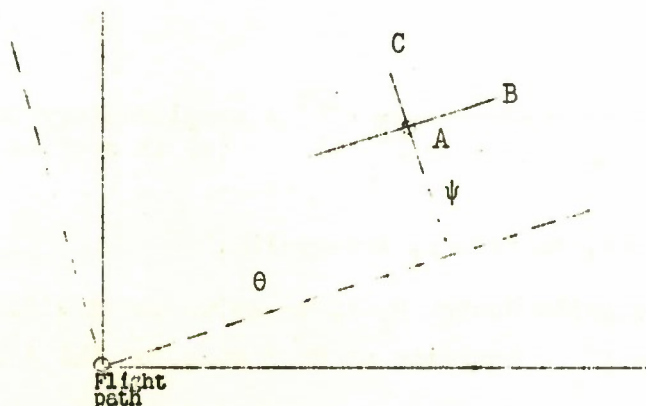
In the cases which apply here, the authors took a value of  $\epsilon$  of  $10^\circ$  - rather a large value unless it is intentional in the design. Their missile was statically stable nose forward or nose backward (unstable somewhere between) but distinctly more strongly stable forward than backward. They found that with the missile descending essentially nose forward, in the resonance region the nose was driven outwards, but not too far and then returned. With the missile descending substantially nose backwards, at the resonance region the nose was driven right away into the nose forward position onto which it then stabilised.

Thus, how important this phenomenon is depends on the individual case. Note that it could not arise if  $A$  were greater than  $B$ , for a missile with aerodynamic stability.

The authors of Ref.2 make the speculation that it might be possible to exploit such an asymmetry if a balance can be struck such that the missile is always driven away from the nose rearward position but never from the nose forward one. A turn over set might not then be necessary from considerations of the dynamics of the body.

### 3.3 Inertial asymmetry

Here we have the case given as (a) in section 1. The modification to section 3.1 is that the moments of inertia are all different,  $A, B, C$  instead of  $A, B, B$ .  $\theta, \psi$ , are now referred to body axes.  $P$  is again constant (to the first order) for small perturbations.



Equations of motion now are:-

$$\begin{aligned} C(\ddot{\theta} - P\dot{\psi}) - (A - B)P(-\dot{\psi} - P\theta) &= M_a \theta \\ B(-\ddot{\psi} - P\dot{\theta}) - (C - A)P(\dot{\theta} - P\psi) &= -M_a \psi \end{aligned} \quad (19)$$



We shall have a solution of the form

$$\theta = K_1 e^{i\lambda t}, \quad \psi = K_2 e^{i\lambda t},$$

if,

$$[B\lambda^2 + (C - A) P^2 + M_a] K_2 - iP\lambda (B + C - A) K_1 = 0,$$

$$[C\lambda^2 + (B - A) P^2 + M_a] K_1 + iP\lambda (B + C - A) K_2 = 0.$$

Hence,

$$BC \lambda^4 + [\{A(B + C - A) - 2BC\} P^2 + M_a (B + C)] \lambda^2 + \{(C - A) P^2 + M_a\}$$

$$\{(B - A) P^2 + M_a\} = 0. \quad (20)$$

This is a quadratic in  $\lambda^2$ , and stability of the angular motion of the body depends on the sign of the two roots in  $\lambda^2$ . For stability, both roots must be positive.

Let us represent the quadratic (20) by

$$ax^2 + bx + c = 0 \quad (21)$$

in which, therefore,

$$a = BC,$$

$$b = \{A(B + C - A) - 2BC\} P^2 + M_a (B + C),$$

$$c = \{(C - A) P^2 + M_a\} \{(B - A) P^2 + M_a\},$$

$$x = \lambda^2.$$

Since 'a' is necessarily positive, the conditions for positive roots are

$$b < 0, \quad c > 0, \quad b^2 - 4ac > 0.$$

All of these must be satisfied.

In general, there are various cases depending on the relationship between  $A$ ,  $B$ ,  $C$ , and the sign of  $M_a$ , but we will concentrate for the moment on the case which usually arises in practice:-

### 3.3.1 A < B < C and $M_\alpha$ negative

In this case

$$\begin{aligned} 'b' &= \{A(B + C - A) - 2BC\} P^2 + M_\alpha (B + C) \\ &= \{-(B - A)(C - A) - BC\} P^2 + M_\alpha (B + C) . \end{aligned}$$

Hence,  $b < 0$  is satisfied.

For ' $c$ ' > 0, the two factors  $(C - A) P^2 + M_\alpha$  and  $(B - A) P^2 + M_\alpha$  must be of the same sign. Hence, to satisfy this condition,  $-M_\alpha$  must not lie between  $(C - A) P^2$  and  $(B - A) P^2$ .

The quantity ' $b^2 - 4ac$ ' can be evaluated by straightforward algebra with the result

$$\begin{aligned} 'b^2 - 4ac' &= A^2 (B + C - A) P^4 + 2 (B + C - A) \\ &\quad (4BC - AB - AC) P^2 (-M_\alpha) + (B - C)^2 (-M_\alpha)^2 . \end{aligned}$$

This expression is necessarily positive for negative values of  $M_\alpha$ , so that the third stability condition is satisfied. Thus, for the normal case in which

$$A < B < C, \quad M_\alpha < 0,$$

the condition for instability is

$$(C - A) P^2 > -M_\alpha > (B - A) P^2 . \quad (22)$$

Thus for this case, namely inertial asymmetry, there is some similarity with the aerodynamic asymmetry of the previous paragraph. Whereas in that case however, there was a single critical relationship for instability (18), here the differing values of the inertias B, C, open this out into a region of instability.

### 3.3.2 A < B < C and $M_\alpha$ positive

This is the case in which the inertia relationship conforms, as above, to the typical missile, but the aerodynamic moment is destabilising.

The condition ' $b$ ' < 0 requires

$$M_\alpha < \frac{\{(B - A)(C - A) + BC\} P^2}{B + C} . \quad (23)$$



The condition ' $C$ ' > 0 is always satisfied in this case.

The condition ' $b^2 - 4ac$ ' > 0 is always infringed for a region of  $M_\alpha$  when positive values of the latter are being considered. To achieve ' $b^2 - 4ac$ ' > 0 we require  $M_\alpha$  must lie outside the two roots of

$$(B - C)^2 M_\alpha^2 - 2(B + C - A)(4BC - AB - AC) P^2 M_\alpha + A^2 (B + C - A)^2 P^4 = 0 \quad \text{.....(24)}$$

The conditions (23) and (24) are necessary and sufficient for stability in this case.

The stability conditions can be simplified if the further restriction is added that B and C are nearly equal.

In these circumstances the one dominant condition will be that  $M_\alpha$  must be less than the smaller root in equation (24).

This means (very closely)

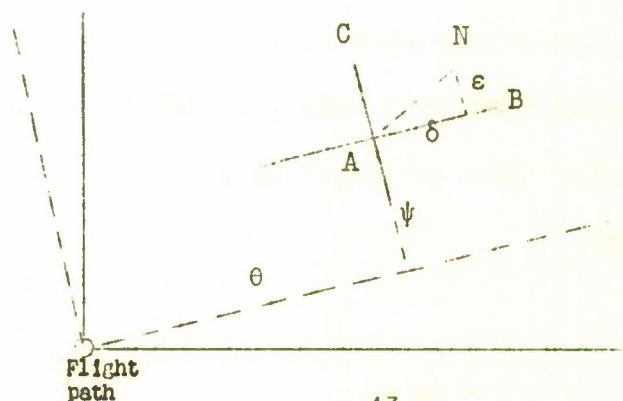
$$M_\alpha < \frac{A^2 (B + C - A) P^2}{2(4BC - AB - AC)} \text{ for stability.}$$

3.3.3 As stated above, the normal case with ballistic missiles, in respect of inertial asymmetry is as in 3.3.1 including of course B - C. In this case, there will be a region during the descent when the motion is unstable. As in 3.2 this will occur at some considerable height and the closer B and C are together the less severe will it be. Two cases in Ref.2 bear on this. They had a small difference between B and C and different strengths of  $M_\alpha$ . In each case, the missile starting at about  $45^\circ$  incidence, spirals in, passing this region without any apparent trouble. Since in ballistic missiles B and C will normally be very nearly equal, it would seem as if this sort of asymmetry may not be of much importance.

3.3.4 Note that if the roll moment of inertia (A) is the largest of the three (not very likely in practice, of course) then if the aerodynamic moment is stabilising ( $M_\alpha$  negative), the conditions for stability are always satisfied.

#### 3.4 Aerodynamic and inertial asymmetry together

In this case we have a combination of the effects in 3.2 and 3.3. Compared with 3.3, the nose of the missile (aerodynamic axis) is displaced  $\delta, \epsilon$ , in terms of missile axes, from the axis A.



The equations of angular motion in this case differ from (19) only in respect of their right-hand sides which now become

$$M_a (\theta + \delta), \text{ and } -M_a (\psi + \epsilon) \text{ respectively.}$$

Over and above the situation in section 3.3 there is therefore simply an additional angular displacement (particular integral)

$$\theta = \frac{-M_a \delta}{M_a + (B - A) P^2}, \quad \psi = \frac{-M_a \epsilon}{M_a + (C - A) P^2}.$$

Thus the aerodynamic asymmetry produces exactly the same effects on an inertially asymmetric missile as on a symmetric one. No further comments arise, therefore, in this case.

#### 4 CONCLUSIONS

The results presented in this note suggest that the following considerations apply when an attempt is being made to stabilise a re-entry body nose first by spin stabilisation applied for the space flight part of the trajectory:-

- (i) The value of the roll moment of inertia, A, should never be between the moments of inertia in pitch or yaw, B, C.
- (ii) A need not be much different from B, C.
- (iii) If A can be made the largest of the three moments of inertia, no difficulty will arise on the score of asymmetry for a missile with static stability.
- (iv) If, as will normally be the case, A is the smallest of the three, there is some danger of a sharp increase in the angle of incidence due to asymmetry, in the 'resonance' region. If the asymmetry is merely due to manufacturing imperfections in a nominally symmetrical missile (and therefore small), and particularly if the missile has considerable static stability, it seems unlikely that this will be serious. At the present time a numerical check on this could only be made for any particular case in question on a digital calculating machine.

#### NOTATION:

A, B, C	moments of inertia about principal axes
H	angular momentum
$M_a$	static stability moment coefficient
P	angular velocity about roll axis (A) (constant)
p, q, r	angular velocity about principal axes
T	kinetic energy



NOTATION (Cont'd)

$\delta, \epsilon$	angles defining degree of asymmetry
$\theta, \psi$	angles defining orientation of roll axis
$\phi$	angle of dispersion of spin-stabilised missile
$\lambda$	frequency in standard form $e^{i\lambda t}$ .

LIST OF REFERENCES

<u>No.</u>	<u>Author(s)</u>	<u>Title, etc.</u>
1	Green, G.S., Weaver, A.K.	The three-dimensional oscillations of a spinning re-entry body. G.W. Tech Note to be published.
2	Welch, J.D., Shik, S.L.	The dynamics and certain aspects of control of a body re-entering the atmosphere at high speed. Preprint 818 (Jl. Aero/Space Sci), presented at 26th Annual Meeting, January, 1958.

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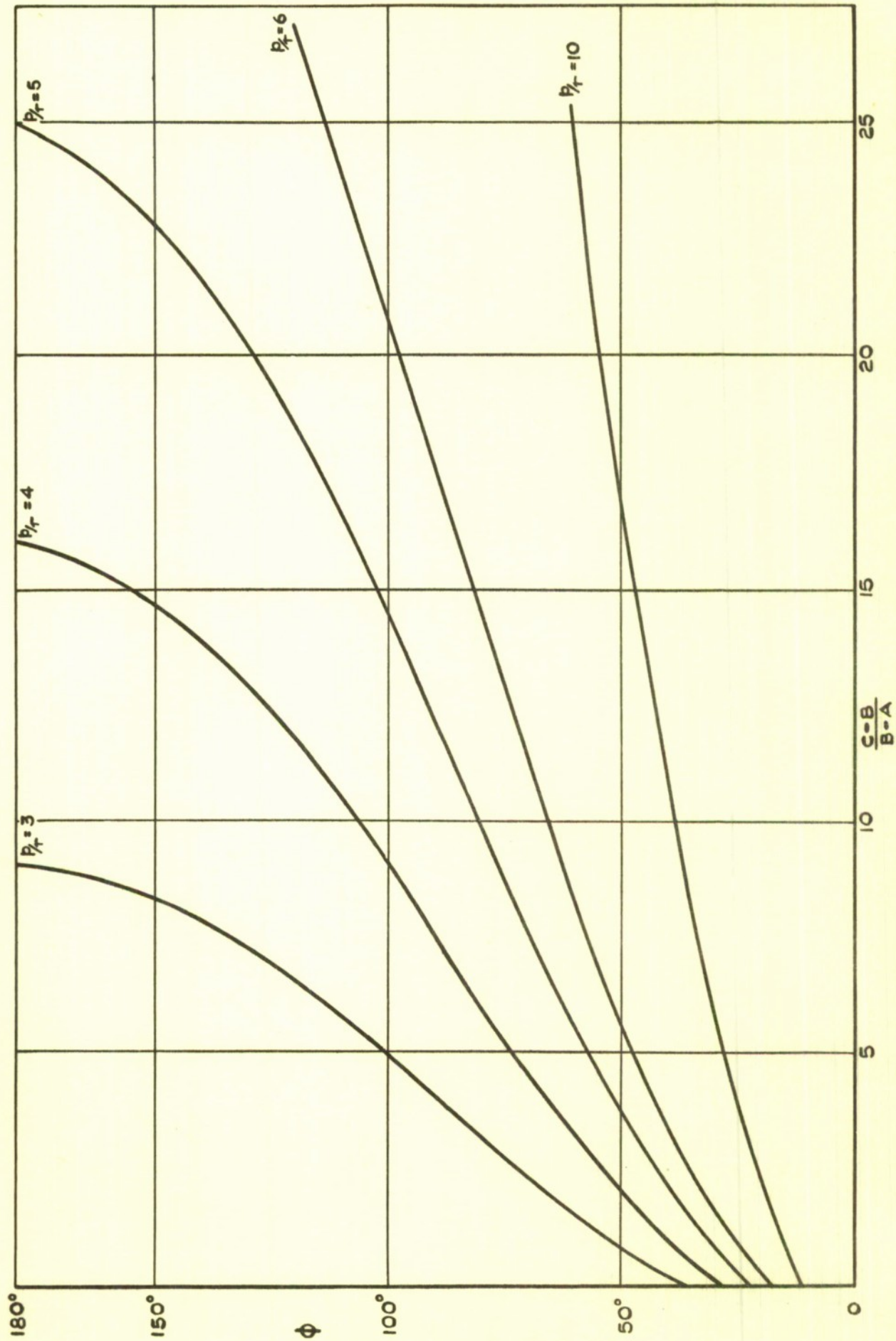


FIG. I. ANGULAR DISPERSION IN MISSILES WHICH HAVE THE MOMENTS OF INERTIA, A,B,C, SUBSTANTIALLY EQUAL.





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