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TECHNICAL REPORT No. 87

Mass Transport in the Wake Of a Submerged Submarine

H. A. Miranda, Jr.,
Howard Schimmel, and P. S. Rooney

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Columbia University
Hudson Laboratories
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Technical Report No. 87

MASS TRANSPORT IN THE WAKE
OF A SUBMERGED SUBMARINE (y)

by

H. A. Miranda, Jr. Howard Schimmel, and P. S. Rooney

April 6, 1961

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ABSTRACT

An experimental technique has been developed to study mass transport in the wake of a submerged submarine. A radioactive tracer is dispensed from the keel of a moving submarine, and the wake is probed with a towed vertical array of scintillation counters that provides a cross-sectional view of the marked waters.

Preliminary experiments done with the USS ALBACORE indicate little tendency for the marked waters to rise or fall. In addition, it appears that local turbulence induced by the ship's screws is a relatively minor factor in the spreading of the submarine wake, except for the initial few minutes. The major portion of the work was done in essentially isothermal waters where the density increase with depth was approximately 0.05 percent in the first 200 m. Further work in situ without a submarine is of course necessary in order to substantiate the above conclusions.

A horizontal coefficient of eddy diffusion has been computed from the data and is seen to increase with time throughout the experiment, approaching an asymptotic value. On the basis of certain assumptions, the data yields asymptotic values for both the scale of turbulence and the intensity of turbulence. The latter quantity, in turn, has been used to compute a radius of curvature due to Coriolis forces at the latitude of 30 deg. This value agrees with the asymptotic scale of turbulence to within 50 percent. It is therefore speculated that Coriolis forces may be a mechanism which tends to limit the scale of turbulence.
INTRODUCTION

It has been realized for some time that a wake detection scheme capable of noting the passage of a submerged submarine might find considerable usefulness in a barrier type of operation. A number of fundamental questions, however, must be answered before any practical system can be given serious consideration. Among these is, for example, the persistence of a wake as affected by turbulent diffusion. In short, what is required is a working definition of the wake of a submerged submarine, including its space and time distribution. This paper describes the technique developed for studying mass transport in the wake and presents the results of work carried out to date.

TECHNIQUE

Tank studies were not thought to be suitable for a number of reasons:

(1) Scaling factors always present serious problems; in this case they are virtually unknown. (2) Unless one uses self-propelled models (in themselves costly and cumbersome), one must drag a model through the water, thus introducing a net momentum into the medium precisely equal to and opposite that of the backward-moving screw-wash—a rather important factor in the wake. (3) Tank experiments had been performed elsewhere.(1) Moreover, it was felt that a technique for operating in situ would be more generally useful for oceanographic work. Therefore, a radioactive tracer technique seemed to be ideally suited for such an application, save for the fact that commercially available gamma detecting apparatus is so limited in sensitivity that a very large quantity of radiotracer is required. Fortunately a new and easily handled scintillation liquid was available, (2) allowing the construction of a far more sensitive detector. Thus the amount of radiotracer required was reduced to less than 1 C—an amount presenting no serious technical problems.

The technique evolved is straightforward in concept: A small quantity of a suitable radiotracer is dispensed from the keel of a submerged
submarine moving in a straight line. The line is then crisscrossed by a surface ship towing an array of eight scintillation counters spaced 10 ft apart. In addition, the thermal structure of the wake water is viewed, using eight thermistors, one affixed to each counter.

**SIGNAL CABLE AND TOWED RIG**

Each scintillator can was provided with a pair of tabs for attachment to the strain cable, as shown in Fig. 1. The axis of each can was parallel to the strain cable when attached to it. Both the drag and the dynamic instability of each can were reduced by means of a demountable fairing, which rendered the towing much smoother.

The signal cable, consisting of some 28 wires, was sheathed with a zipper jacketing made of Fiberglas-reinforced vinyl and tied to the strain cable every 10 ft or so with 3M electrical tape. Care was taken that the lead-in to each can was not strained in any way when under tow. The strain cable was weighted at the bottom with a faired "fish" weighing 1000 lb.

In order to learn more about the effectiveness of the fairing, the drag of a scintillator can was measured as a function of speed with and without fairing. The drag of a long pipe 1 in. in diameter was also measured. It can be seen from the results, shown in Fig. 2, that the fairing is effective only at speeds below 5 knots. A towing speed of 3 knots was chosen since under these conditions the drag of each can was not more than twice that of an equal length of signal cable.

The shape of the entire rig under tow was studied in a series of calibration runs made at sea with a dummy rig. The depth of each can was measured at various towing speeds with a Bourns pressure transducer. Wiper-arm hysteresis was eliminated by attaching to each transducer a buzzer which was actuated whenever a depth measurement was desired. Under this arrangement an accuracy of ±1 ft was achieved for all depths. The results of these runs confirmed that for our purposes the shape under tow could be considered a catenary. On this basis, the depth of each can...
ASSEMBLED SCINTILLATION CAN

ELECTRICAL CABLE

THERMISTOR

STRAIN CABLE

SCINTILLATION CAN

FAIRING

FIG. 1
FIG. 2
DRAG OF CAN AS A FUNCTION OF SPEED

PIPE PLUS CYLINDER AND FAIRING

DRAG (POUNDS)

PIPE PLUS CYLINDER
PIPE ALONE

SPEED (KNOTS)
under tow, as well as its distance behind the point of tow, was computed. These curves are shown in Fig. 3, with ship's speed as a parameter.

Accelerometer measurements, also made during this series of runs, indicated that the rig did not sway from side to side under tow.

RADIATION COUNTING APPARATUS

The radiation detectors were of the scintillation counter type. Approximately 1.6 liters of scintillation liquid were placed in a specially designed Lucite cylinder 5 in. i.d. and 5 in. in height. A 2-in. thick slab of Lucite formed the top of this cylinder and also acted as a light-pipe to minimize spread in pulse height. The light-sensitive surface of a Dumont 6292 multiplier phototube was pressed against the window of the light-pipe by means of a Lucite cover screwed into the light-pipe.

Two holes were drilled in the light-pipe and fitted with seal screws. These holes served a dual purpose: they were used as filling-ports, as well as expansion chambers. Since the liquid scintillant and Lucite have different coefficients of thermal expansion and since temperature changes of as much as 55°F were anticipated in the field, some expansion chamber scheme was necessary. After a number of abortive attempts to construct a suitable expansion chamber, it was found that the various vapors (including dissolved nitrogen) present in the liquid scintillant filled the volume of the two holes or receded into the liquid as required by the temperature conditions, preventing pressures inside the cylinder from becoming too great. In placing this scintillant package inside the pressure can for field work, care had to be taken to put these holes aft of the vertical axis of the cylinder. In this position these holes were oriented so that they were uppermost when the can was affixed to the strain cable and the entire rig was under tow. In this manner no bubbles formed in the optical path directly in front of the multiplier phototube.

The outside surface of the Lucite cylinder was coated with a diffusely reflecting coating prepared in the following manner: A small amount of 325-mesh Brazilian quartz was thoroughly cleaned in a boiling solution
CATENARY CURVES

FIG. 3

DEPT (FT)

0 40 80 120

DISTANCE AFT OF SHIP (FT)

0 20 40 60 80 100

5.0 KNOTS

4.5

3.0

2.5

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of Sparkle® detergent, rinsed at least five times in boiling water, thoroughly dried, and then mixed with H-94, an acrylic cement, to form a thin slurry. This slurry was painted on the outer surface of the cylinder with a clean camel’s hair brush. After two such applications, the surface was quite rough and diffusely transmitting, and the cylinder then was carefully wrapped in heavy-duty Reynolds Wrap aluminum foil, whose reflecting surface had been well cleaned with detergent and rinsed with distilled water. This coating of the Lucite cylinder has been found slightly superior to the usual MgO - TiO₂ paints. In addition, no sign of aging has been noted over a one-year period.

This scintillator package was carefully wrapped in 1/4-in. sponge rubber sheet and placed inside a 5-in. in diameter aluminum can 16 in. high, to which a cover was fitted in the usual manner. The can was designed to be used down to a depth of about 300 ft. An exploded view of the entire assembly is shown in Fig. 4.

For ease of maintenance, it was decided to house no electronic equipment (save for the multiplier phototube) in the can (see functional block diagram, Fig. 5). Because of this, the high voltage required for the dynode system of the multiplier phototube was delivered via a polyethylene-jacketed wire. A single high-voltage wire was used for all eight tubes, which were wired in parallel. As a result, the wire passed through the cover of each can twice. A third lead passing through the cover was the signal lead (RG 62B/U), a 93-ohm coaxial cable, terminated at the laboratory end by its characteristic impedance.

The scintillation pulses were fast, having rise times of the order of 30 μsec. These pulses, received in the ship’s laboratory, were amplified by a variable-gain amplifier. They triggered a monostable multivibrator,

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† Schwartz Chemical Company, Inc., 326 West 70 Street, New York, N. Y.
‡ Eighteen gauge, standard wire, No. B-4401, purchased from the Vector Cable Company.

 Because the individual multiplier phototubes differed widely in amplification, a voltage dropping resistor was placed in series with the dynode voltage divider. This tended to equalize their response.
FIG. 4
EXPLODED VIEW OF SCINTILLATION CAN

- MAGNETIC SHIELD
- TUBE SOCKET WITH ENCAPSULATED PROTECTIVE TUBE CAP
- PHOTOMULTIPLIER TUBE ELECTROSTATIC SHIELD
- LUCITE LIQUID-SCINTILLATION CHAMBER WRAPPED WITH FOAM RUBBER
- SCINTILLATION CAN
which in turn developed a single square pulse of a fixed amplitude for every incoming pulse of more than a certain predetermined value. The pulse length at the output of the multivibrator could be varied in four steps from 100 μsec to 1000 μsec. This pulse was then fed through any one of three integrating time constants to a Honeywell Visicorder oscillographic galvanometer. Thus an average current was derived which was proportional to the average counting rate. In the case of very low counting rates when the dc level was difficult to ascertain, the time constants could be suitably adjusted so as to record the individual pulses as they arrived. This special feature of the counting system provided a very wide dynamic range. A schematic diagram of the circuitry is shown in Fig. 6. It must be noted that the entire electronic system functioned admirably throughout the field work. This we ascribe mainly to the absence of electronic components in the water.

The counting apparatus was calibrated by immersing the cans into a large tank (4 ft × 4 ft × 5 ft) of sea water containing a known quantity of radioactive tracer. (It can be shown that each detector derives 95 percent of its response from a spherical volume approximately 3 ft in diameter.)

Figure 7 is a typical plot of the recorded dc level vs the amplifier gain, with tracer concentration as a parameter. From this family of curves were plotted the curves of dc level vs concentration, with gain as a parameter; a typical curve is shown in Fig. 8. As can be seen from this figure, the response of the system was linear throughout the range of concentrations found. In order to equalize the response of the detectors in the field, the particular gain for each was determined by placing a 1-μC source of Cs$^{137}$ at a selected spot on each can and adjusting the respective gains so as to yield the same dc level. With this system it was possible to detect a radiotracer concentration in the ocean of $2 \times 10^{-13}$ C/ml in a 5-sec sampling period.
Fig. 7
D.C. Level vs GAIN for Can # 3

D.C. Level (inches)

Gain (nominal)

0.8 x 10^-11 cu/cc
1.6 x 10^-11 cu/cc
FIG. 8

D.C. LEVEL VS CONCENTRATION
FOR CAN # 3

- Concentration vs D.C. Level
- GAIN = 17
- GAIN = 15
- GAIN = 14
- GAIN = 12
- GAIN = 11
- GAIN = 9

D.C. LEVEL (INCHES)

CONCENTRATION (X 10^{11} cu/cm^3)

- Actual
- Interpolated
TEMPERATURE APPARATUS

Thermistors were chosen for this work because of their ruggedness, short thermal time constant, good sensitivity, and relative ease of operation. Each glass thermistor was cast by means of an epoxy resin within a cylindrical protective case of anodized aluminum 3/4 in. in diameter. A mounted thermistor is shown in Fig. 9.

The protective case extended about 3/8 in. beyond the end of the bead and was open to the sea water. A number of holes were drilled along the side of the case to allow easy flow of water past the bead. The leads were spliced to a pair of Signal Corps wires (WD 1/TT), which formed part of the signal cable. Each thermistor package was then taped to the side of its corresponding scintillator can in such a way that, under tow, the thermistor protruded into the flow stream ahead of the can. Thus the scintillator can could not affect the measured water temperature.

Pressure dependent effects were not encountered with the thermistors and mountings, nor were any expected, because the depths were relatively shallow (less than 300 ft).

The thermistor circuitry was straightforward and relatively simple. Figure 10 is a schematic diagram of this circuitry. Each thermistor formed one arm of a balanced bridge; another arm consisted of a 10-turn helipot. Once it had been calibrated, an absolute temperature measurement was easily achieved by simply noting the helipot reading required to null the bridge. Small deviations from this temperature caused a current to flow in the Visicorder galvanometer.

Calibration was carried out in a temperature controlled bath. A typical calibration curve is shown in Fig. 11. Sensitivity of better than ±0.02°C was achieved in the field with this arrangement, and all units functioned very well. Absolute values were reproducible to ±0.01°C in the laboratory. In order to check this point in the field, a number of BT's were taken simultaneously with thermistor measurements. In all cases, both techniques agreed within the experimental error or the BT's.
FIG. 9
MOUNTED THERMISTOR
DISPENSING SYSTEM AND OPERATION

A dispensing system was designed and constructed whose function was to eject a radiotracer from the keel of a submarine at a known flow rate and in a continuous manner. The operating principle was simple in concept: A pump caused sea water to flow through a long hose at a rate of about 10 gal/min. This hose was wrapped circumferentially about the submarine, ejecting its contents under the keel. The radiotracer was injected into this flow by means of gas pressure.

Figure 12 shows the dispensing mechanism. On the right is the pig which contained the radiotracer. When in transit the two manual valves on the pig were protected by a steel cover, shown in Fig. 13. The cylinder on the left in Fig. 12 housed the water pump which operated continuously during the dispensing period. The central cylinder contained a single pressure-regulating valve and two solenoid valves connected in parallel as a precaution against failure. The pressure valve allowed a predetermined gas pressure to be applied to the radiotracer in the pig. The solenoid valves permitted the radiotracer, under gas pressure, to enter the exit line of the pump.

For obvious reasons it was essential that the tracer be a liquid. However, at Brookhaven National Laboratory, where the elements were activated, liquids could not be put into the reactor. This necessitated having the radioactive material initially in powder form and converting it to a liquid prior to the dispensation.

For the first experiment the isotope chosen was lanthanum, which has a half-life of about 40 hr. Its decay scheme is shown in Fig. 14. The 1.6-Mev gamma ray was the only one of interest for this experiment. A 20-g sample of lanthanum oxide (La₂O₃), when irradiated for a 2 1/2-hr period in a flux of 2×10¹² neutrons/cm²/sec, yielded approximately 2.7 C of gamma activity. Upon its removal from the reactor the sample was placed in the pig, which served both as a shipping container and as an integral part of the dispensing system.
FIG. 13

SHIPPING CONTAINER
FIG. 14
DECAY SCHEME OF La$^{140}$

La$^{140}$

Ce$^{140}$

0.83 MEV 12%
1.10 26%
1.34 45%
1.67 10%
2.15 7%

2.92
2.65
2.53
2.42
2.09
1.60

3.75 MEV

0
The interior of the pig consisted of a countersunk sample holder of monel about 3 in. in diameter and about 4 1/2 in. in height surrounded on all sides by at least 5 1/2 in. of lead. Two 1/8-in. stainless steel tubes entered the sample holder through a lead-filled plug. The first (entrance) tube terminated at the base of the plug. It was through this tube that nitrogen pressure was exerted on the liquid. The second (exit) tube terminated at the lowest point in the sample holder.

When the pig and its contents had been transported to the site of the experiment, the process of getting the solid sample into solution was begun. 110 cc of concentrated hydrochloric acid added to the sample holder through the exit tube generated the following reaction:

$$\text{La}_2\text{O}_3 + 6 \text{HCl} \rightarrow 2 \text{LaCl}_3 + 3 \text{H}_2\text{O}.$$  

Thus the insoluble lanthanum oxide was converted to highly soluble lanthanum chloride. The remaining volume was filled by adding distilled water.

Rubidium $^{86}$ with an 18.7-day half-life was chosen for the second experiment. Its decay scheme is shown in Fig. 15. As may be noted from the decay scheme, only 10 percent of the indicated activity is due to the 1.08-Mev gamma ray.

For the irradiation 180 g of rubidium carbonate ($\text{Rb}_2\text{CO}_3$), wrapped in aluminum foil, were inserted in a standard aluminum reactor capsule and then placed in the reactor. A 2-week irradiation yielded approximately 40 C of total activity, 10 percent of which consisted of the desired gamma radiation. Upon removal from the reactor the aluminum foil and its contents were taken out of the reactor capsule and placed in the pig for shipment.

Once again hydrochloric acid was used to obtain a water-soluble compound, i.e.,

$$\text{Rb}_2\text{CO}_3 + 2 \text{HCl} \rightarrow 2 \text{RbCl} + \text{CO}_2 \uparrow + \text{H}_2\text{O}.$$  

In this case the reaction was a violent one, and a gas ($\text{CO}_2$) was released in the process. A considerable amount of hydrogen gas was also liberated in the reaction with the aluminum foil:

$$2 \text{Al} + 6 \text{HCl} \rightarrow 2 \text{AlCl}_3 + 3 \text{H}_2 \uparrow.$$  

-22-  
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FIG. 15

DECAY SCHEME OF Rb\textsuperscript{86}

\begin{align*}
\beta^-_{1} & : 0.698 \text{ MEV} \sim 10\% \\
\beta^-_{2} & : 1.776 \text{ MEV} \sim 90\% \\
\gamma & : 1.078 \text{ MEV} \sim 10\%
\end{align*}

STABLE Sr\textsuperscript{86}
If these reactions had been allowed to proceed rapidly, the gases formed could not have been vented adequately through the small diameter tubing. The resultant pressure build-up would then have expelled some of the radioactive material. To prevent this the acid was added in small amounts over a 3-hr period.

On the submarine the dispensing system and a high-pressure nitrogen bottle were secured inside the superstructure. In this position they were external to the pressure hull yet protected from the seas (see Fig. 16). The intake and exit hoses for the pump were banded to a 5/8-in. wire rope which was wrapped around the submarine in bellyband fashion. The intake was 6 ft below the water line on the port side, and the exit was located beneath the keel and approximately 80 ft forward of the screw. A diverging nozzle assured rapid diffusion of the radioactive material.

The dispensing procedure was as follows: The two manual valves on the pig were not opened until the submarine was on station. The manual valve on the nitrogen bottle was then opened, thus pressurizing the pig. At this point the submarine submerged, and electrical power was applied to the switches controlling the salt water circulating pump and the solenoid discharge valves. When the submarine had leveled off at the preassigned depth, the pump was started. Dispensing was initiated by opening the solenoid valves and terminated at the conclusion of the run by closing the solenoid valves and then stopping the pump.

After the final run when the pig had been entirely emptied of its contents, the valve of the nitrogen bottle was closed. The pressure on the system was relieved by venting the gas through the solenoid valves. The entire system was then flushed with distilled water until the level of radioactivity was low enough to permit personnel to work on it for an indefinite period.

Due to the low flow rates required (1 to 1.5 ml/sec) a conventional needle valve could not be used in the exit line. Instead a small stainless steel orifice (0.010 in. in diameter) was placed in series with the solenoid valves. The flow rate through this orifice was determined for fixed fore pressure and varying back pressure. This was done because the fore
FIG. 16
DISPENSER MOUNTED ON SUB
pressure could not be changed during the experiment due to the high level of radioactivity existing in the vicinity of the dispenser once the lines had been charged. It was desirable to vary the back pressure because it was proportional to depth and not all runs were to be made at the same depth. The liquid used for this calibration was chemically identical to that used in the actual experiment, although it was not radioactive. The calibration curve, a graph of flow rate vs differential pressure (depth), is shown in Fig. 17.

**EXPERIMENTAL OPERATIONS**

A preliminary experiment was carried out in the Gulf of Maine during October, 1959, using the USS ALBACORE (SSN 569) as dispensing vessel and Hudson Laboratories' T-boat as research vessel. An operating area 500 ft deep was chosen just off Jeffrey's Ledge. A bottom-anchored surface buoy provided by the U. S. Coast Guard served as a crude navigational reference.

The operating plan, Fig. 18, called for the T-boat to lie to at a distance 500 ft from the buoy. The ALBACORE then maneuvered in such a manner as to pass parallel to the line of the T-boat and the buoy at a distance of about 500 ft. The direction was to coincide with that of the subsurface current at keel depth, determined in a crude manner just prior to the experiment. The ALBACORE began to dispense the radiotracer when its range to the buoy was approximately 2000 ft, at which time a smoke flare was released. A second flare was released at the end of the run.

On sighting the second flare, the T-boat approached the dispensing line until the radiotracer signal was perceived. At this time it dropped a sea-anchored buoy and commenced executing a series of crisscrossing maneuvers. This was continued until the radiotracer was no longer detectable.

In this preliminary experiment La$^{140}$ was used as a tracer element. Its relatively high-energy gamma ray (1.6 Mev) was very desirable, because it greatly simplified setting the bias of the detecting apparatus. Since this was a first try and the experimental parameters were not well known (the
OPERATING PLAN FOR PRELIMINARY EXPERIMENT

FIG. 18
width of the wake, the concentration expected at the initial crossing, etc.), the choice of gamma-ray energy tended to be conservative.

The use of La^{146} exacted a rather serious price, however, owing to its relatively short half-life (40 hr). The operating area dictated by this choice of tracer was highly unsuitable, due to the complex character of the water, and the entire operation had to be executed under great pressure of time, which placed a heavy strain on personnel. Also, the operation could not wait for good weather; as it happened, it was far from ideal for all of the runs, with winds of approximately 15-20 knots and sea state of about 3. For a small boat such as the T-boat, these were rather strenuous conditions, since it was difficult, for example, to see the various buoys from the bridge. In addition, the operation was severely handicapped by the lack of a dead-reckoning tracer, which would have provided some knowledge of our relative whereabouts.

For the second experiment, a much longer-lived radiotracer (Rb^{86}) was employed. Its 19-day half-life allowed the choice of a better operating area as well as better weather. Its main disadvantage was the fact that the principal gamma ray was of lower energy (about 1.0 Mev), rendering the elimination of unwanted noise and background radiation much more difficult. After dealing with the disadvantages of short-lived material in the preliminary experiment and learning more about some of the operating parameters, it was felt that this choice of radiotracer was a far more prudent one.

In this operation, the USNS GIBBS was the research vessel. It rendezvoused with the USS ALBACORE at sea in a region where the temperature gradient to a depth of 400 ft was approximately 1°C. The GIBBS lay to, lowered the experimental rig, and otherwise got itself in readiness. A BT was obtained; at this point the ALBACORE was notified to proceed with the dispensation.

The ALBACORE got into a position determined by the direction that the GIBBS had assumed about a mile off its starboard bow. The ALBACORE then proceeded towards the GIBBS in such a way that it would pass the GIBBS about 500 ft to its starboard. Having set its course on the surface, the ALBACORE released a smoke bomb, submerged to the prescribed
depth, and continued on this course at a speed of 6 knots. As soon as the ALBACORE neared the bow of the GIBBS, the radioactive tracer was dispensed at a rate calculated to produce a trace approximately 1500 ft long. At the beginning of the dispensation as well as at the end a 1/2-min smoke flare, together with a specially constructed dye-release mechanism,\* was ejected. As soon as possible after the end of dispensation, the ALBACORE surfaced, stopped all engines, and released a second smoke bomb. Needless to say, at all times the operation was plagued by the malfunctioning of these flares and bombs.

The purpose of the smoke flares was to direct attention to the point of the tracer's initial release, marked by the appearance of the green dye on the surface. The end of the radiotracer was marked in the same manner. The purpose of the smoke bombs at the beginning and end of the run was to guide the approach of the GIBBS to the line joining these two bombs at a right angle.

At the appearance of the first dye marker, the GIBBS proceeded to make a right-angle turn in order to cross the wake just inside the marker, and the recording apparatus was turned on. This apparatus simultaneously recorded the signals from the eight scintillation cans. When, upon crossing the radioactive wake, one or more of the detectors showed a signal, the operator notified the bridge, and immediately a floating buoy was released. When the signals returned to background, the recorder was secured, and the GIBBS proceeded on its course for some 500 ft, made a 180-deg turn, and repeated the operation. The attempt was made to lay five such buoys along the course of the radioactive wake. During this crucial period, the smoke bombs proved invaluable in orienting the GIBBS. Once the five buoys were laid, the GIBBS could proceed on its crisscrossing maneuvers in a more relaxed fashion.

It should be noted at this point that the marker buoys could easily be seen for at least 1/2 mile from the bridge of the GIBBS. Each of these

\* An aluminum can painted brilliant red, which was designed to pop open upon reaching the surface, releasing a batch of fluorescein dye.
buoys was attached by means of a thin nylon line to a sea anchor placed about 10 ft above keel depth of the ALBACORE. In this manner the marker buoys followed the course of the wake water. It is interesting to note that even with winds of 15 knots the buoys did not drift more than 200 ft away from the wake during a 4-hr period.

After the first few crossings, the thermistor information was switched onto the recording. These eight tracers had been blanked out during the initial crossings in order that the operator be presented with the least confusing picture possible. (During this initial period the main problem was simply to locate the wake and mark it with the buoys; temperature information could be sacrificed to this end.)

Throughout the operation, a dead-reckoning tracer kept a record of the GIBBS' movements, and, in addition, the ship was guided during the initial phase to some extent by the ALBACORE, which computed the position relative to the marked wake and advised the GIBBS. The DRT record proved useful in determining the angle made between the GIBBS' line of motion and that of the marked wake during each of the crossings.

Two such runs were carried out in the open sea under practically isothermal conditions during the latter part of March and the early part of April, 1960. The first run was made at a keel depth of 50 ft, and the second, 100 ft. During both these runs, wind averaged about 15 knots, and sea state was about 3. It was possible to track the wake for a period of about 4 hr for both runs.

The preliminary experiment had been carried out under somewhat different conditions. Although sea state and wind were about the same in the two experiments, the tidal currents in the Gulf of Maine are rather confused, and large salinity variations are to be expected. In addition, due to the bobbing of the small T-boat, the temperature information was much more confused.*

* It should be noted that during the preliminary experiment, a strong temperature gradient existed. The cyclical plunging of the array through this gradient showed up quite clearly in the records.
RESULTS AND DISCUSSION

1) Preliminary Experiment: The results obtained in the Gulf of Maine verified our choice of design parameters. The wake was clearly visible up to 1/2 hr after passage.

A strong temperature gradient existed, and there was evidence that this was disturbed by the passing vessel. However, due to the continual bobbing of the rig, it was impossible to establish this quantitatively. As mentioned before, these are confused waters; we saw ample experimental evidence of differential currents in the top 150 ft of water—a fact which is not surprising in view of the bottom contour and tidal currents in this locality.

In addition, the wake waters seem to become stratified in two out of three runs. We wish to stress, however, that this inference is made on the basis of one assumption which is open to question, namely, that the marked wake which was 1000 ft long at the initiation of the run does not alter its length appreciably during the first half-hour. The reasoning is as follows: Realizing that the half-life of La$^{140}$ is long compared to the 1/2-hr period, we have

\[ C_n V_n = \text{constant}, \]

where

- \( C_n \) = average concentration of radiotracer at the \( n \)th crossing, and
- \( V_n \) = volume of marked fluid at the \( n \)th crossing.

Consider, for simplicity, a rectangular cross section, such that

\[ V_n = H_n W_n L_n, \]

where

- \( H_n \) = height of the marked fluid at the \( n \)th crossing,
- \( W_n \) = " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " &n
It follows that

\[ \frac{C_n}{C_1} = \frac{(H_1 W_1)}{(H_n W_n)} \cdot \frac{(L_1)}{(L_n)} . \]

From the above-mentioned assumption, \((L_1/L_n)\) is unity, and we have

\[ \frac{W_n}{W_1} = \left( \frac{C_1}{C_n} \right) \cdot \left( \frac{H_1}{H_n} \right) . \]

The factor \((C_1/C_n)\) is obtained from the Visicorder traces directly. The factor \((H_1/H_n)\) is obtained from the number of cans which show a signal above background.

From this data the curves of Fig. 19 were plotted for each of three runs. As can be seen from this figure, stratification occurred in two of the runs (Bravo 1 and Charlie II). As was mentioned, these curves are predicated upon the constancy of \(L\). Although this may not be strictly valid, it is probably true that the \(L\)'s do not vary by more than a factor of 2. Certainly the value of \(L\) does not decrease with time, so that the curves of Fig. 19 are upper extremes.

2) Second Experiment: The second experiment was carried out in the open waters of the Atlantic Ocean in a region where the vertical temperature gradient was much smaller. Our thermistors indicated a difference of \(1/3^\circ C\) between the surface and 140 ft.

A single successful run was made--this at a keel depth of 100 ft. (Due to inadequate calibration the first run, made at 50-ft keel depth, did not present fruitful data.) Eleven crossings in all were made over a period of about 3 hr. At the end of this period, contact was lost rather abruptly.

In Fig. 20 are shown the isoconcentration contours of a vertical cross section of the wake for each of the crossings. Each of these has been corrected for our nonperpendicularity of approach, as obtained from the DRT records.

Unfortunately our first contact with the marked wake waters did not take place until 36 min after dispensation. Thus no information on the initial wake spreading was obtained. However, the isoconcentration curves of Fig. 20 for Crossings 1, 3, and 4 show that the peak concentration remained at a depth of 100 ft for at least the first 80 min. In addition, it may be co-
FIG. 19
ILLUSTRATION OF OCCURRENCE
STRATIFICATION IN
PRELIMINARY EXPERIMENT

CONFIDENTIAL
FIG. 20
WAKE CROSSINGS

CONFIDENTIAL
cluded by a qualitative inspection of these curves that vertical diffusion due to turbulence is negligible compared with horizontal diffusion during this interval of time. The measured vertical temperature gradient is 1/4°C per 100 ft. This agrees very well with measurements made during April of 1954 in the same locality, \(^6\) whereby it had been computed that the density increased by about .05 percent in the first 200 m. According to Sverdrup et al., \(^7\) this condition would be expected to reduce greatly the vertical coefficient of eddy diffusivity. Thus, our results are in qualitative agreement with what might be expected.

It is also noted that an abrupt change in the structure occurs between Crossings 4 and 5. The peak concentration at 100 ft is no longer evident, and the cross sectional patch size seems to have become quite a bit larger. The wind increased steadily throughout the experiment, being about 5 knots at the initiation and 15 knots at the time of Crossing 8. Whitecaps were appearing by this time. It is possible that the abrupt changes noted at the 100-ft depth are the result of turbulence due to horizontal shear near the surface.

It had been hoped at the outset that possibly coefficients of diffusivity could be obtained as an added dividend from these experiments. As was mentioned above, however, our data is extremely poor in this respect, necessitating a number of assumptions which render the results tenuous. Nevertheless, in what follows the analysis will be presented for what it may be worth.

Figure 21 is a plot of the volume occupied by the marked fluid vs time. The ordinate of each point was computed from a knowledge of the concentration distribution, the area bounded by the least isoconcentration curve, and the total quantity of radioactive material released. To a first approximation, these points indicate that the volume occupied by the marked fluid increases linearly with time. The line drawn through the experimental points is a least-squares fit.

One can then write the expression

\[ V(t) = H(t) L(t) W(t) , \]  

(1)
\[ V_0 = 0.148 \times 10^{12} \text{ cm}^3 \]
\[ \frac{dv}{dt} = 0.75 \times 10^{10} \text{ cm}^3 / \text{min} \]
where
\[ H(t) \equiv \text{height of marked fluid at any time}, \]
\[ L(t) \equiv \text{length}, \]
\[ W(t) \equiv \text{width}, \]
and for \( t = 0 \), the expression
\[ V_o = H_o L_o W_o. \]
Since \( L_o \) is known, assuming \( H_o \cong W_o \), \( W_o \) may be computed:
\[ W_o = \sqrt{\frac{V_o}{L_o}}. \]
Differentiating Eq. (1) with respect to time, one obtains
\[ \frac{dV}{dt} = H_o \left[ W \frac{dL}{dt} + L \frac{dW}{dt} \right], \tag{2} \]
where it has been assumed that
\[ \frac{dH}{dt} = 0. \]

If one assumes isotropic turbulence in two dimensions, then
\[ \frac{dL}{dt} \cong \frac{dW}{dt} * \]
and
\[ \frac{1}{H_o} \frac{dV}{dt} \cong (L + W) \frac{dW}{dt}, \tag{3} \]
since \( L \gg W \).

Using a numerical technique, \( W(t) \) was determined from Eq. (3) and is shown in Fig. 22.

* The point may be raised as to the validity of this assumption, on the basis of the different lengths compared to the scale of turbulence. It is asserted, however, that the final result is not appreciably affected by the choice of the scale of turbulence compared with the value of \( L \).
WIDTH OF MARKED FLUID VS TIME

FIG. 22

TIME (MIN)

W (CM X 10^-3)

20 16 12 8 4 0

0 40 80 120 160 200 240
Now, Batchelor\(^{(8)}\) has shown that the coefficient of eddy diffusivity \(E(t)\) is expressible in terms of the displacement \(X(t)\) of a fluid particle over an interval of \(t\) sec as follows:

\[
E(t) = \frac{1}{2} \frac{dX^2}{dt},
\]

where the bar indicates an average for a large number of particles. (In Eq. (4), we have restricted ourselves to one dimension.) He has further shown that the dispersion of the mean concentration is independent of the shape of the cloud of marked fluid and is identical with that for a single fluid particle.

Applying this result to the experiment at hand one can thus be confident in asserting that, for all times,

\[
dW = \left(\bar{X^2}\right)^{1/2},
\]

from which:

\[
W \frac{dW}{dt} = \frac{1}{2} \frac{dX^2}{dt} = E(t)
\]

may be derived.

Figure 23 is a semilogarithmic plot of \(E(t)\) vs \(\frac{E(t)}{t}\). It is observed from the figure that for times \(t > 85\) min the function \(E(t)\) can be expressed as follows:

\[
E(t) = E_0 e^{-\frac{t}{\kappa}} E(t),
\]

where \(E_0\) is the asymptotic value of \(E(t)\) for \(t \rightarrow \infty\) and \(\kappa\) has the dimensions \(L^{-2} T^{-2}\). It is tempting, therefore, to define the intensity of turbulence as

\[
\frac{1}{\sqrt{\kappa}};
\]

computing from the value \(\kappa = 6.08\) sec\(^2\) cm\(^{-2}\) given in Fig. 23, the intensity of turbulence is found to be 1.28 cm/sec.
PLOT OF THE LOGARITHM OF THE COEFFICIENT OF EDDY DIFFUSIVITY (E) VS \( \frac{t}{E} \)

FIG. 23

\( log (E \times 10^{-4}) \)

\( t = 85 \, \text{MIN} \)
The asymptotic value \( E_0 \) obtained from Fig. 23 is \( 29.2 \times 10^3 \text{ cm}^2 \text{ sec}^{-1} \). If one writes\(^{(9)}\)

\[
E_0 = VS,
\]

where

\( V \) is the intensity of turbulence, and

\( S \) is scale of turbulence (i.e., the length scale in which a particle moves substantially in one direction),

then \( S \) is found to be \( 2.27 \times 10^4 \text{ cm} \) or 227 m.

It is interesting to note that at 30-deg latitude the radius of curvature of a particle having a linear speed of 1.28 cm/sec would be deflected by Coriolis forces into a circular course having a radius of 350 m. This leads one to speculate on the possibility that Coriolis forces may be a mechanism which tends to limit the scale of turbulence.

**FUTURE WORK**

The overall system that has emerged from the work to date is reliable and can be used to good advantage. Some improvements can possibly be achieved through a fast readout (perhaps automation of the isoconcentration contour plotting, which would greatly facilitate decisions which must be made in the field). We are looking into this possibility.

The experimental results to date seem to indicate that wake-induced turbulence decays in a relatively short time, after which the existing oceanic turbulence is the dominant factor in the spread of submarine wake constituents. It would seem reasonable, therefore, to study in greater detail the normal oceanic turbulence and its effects on diffusion before continuing with experimental work on submarine wake studies. In particular one would like to know the effect of vertical density gradients and of wind-generated horizontal shear upon the diffusion coefficients.

The experimental technique which has been developed can be employed to this end very easily by simply dispensing the radiotracer from a towed line. It might be added that this would at the same time greatly reduce the experimental effort and simplify the logistics of the experiment.
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