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NEWTONIAN DRAG OF COATED CYLINDERS

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ABSTRACT

Newtonian impact theory is used to examine the variation in ballistic coefficient of a coated cylinder as the thickness of the coating decreases.

It is shown that when the density of the coating is less than one-half that of the core-material, an initial value of coating-thickness can be chosen which results in a steady increase of the ballistic coefficient as the thickness of the coating tends to zero.

The largest such value of coating-thickness, the corresponding minimum value for the ballistic coefficient, and the rate at which the ballistic coefficient rises to that of the bare core are shown to be simple functions of the particular density-ratio.

INTRODUCTION

In order to predict the atmospheric deceleration of a cylinder whose axis lies in a plane normal to the direction of motion, it is necessary to consider the ballistic coefficient $\beta = W/C_D A$; W here is the weight of the cylinder and $C_D A$ the effective "drag area" upon which the dynamic pressure acts to oppose the motion of the body.†

Since relatively low ballistic coefficients are associated with lightweight cylinders at large angles of attack, maximum deceleration and heating may occur rather early following re-entry.[1] Survival of such atmospheric heating may be enhanced not only by an appropriate choice of materials and configuration, but also by the use of ablative coatings.

We are concerned in the discussion which follows only with the variation in ballistic coefficient which occurs with mass-loss when such a protective coating is used.

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† Other orientations may also be of interest, and for such cases β should be replaced in the sequel by $\beta \sin^3 \alpha$, where α is the "angle of attack" measured between the cylinder axis and the direction of flight. (See Appendix.)

THE BALLISTIC COEFFICIENT

If a cylindrical core of length L, radius r, and density ρ is covered by an ablative coating of thickness $R-r$ and density $\rho' < \rho$, then the Newtonian ballistic coefficient of the coated cylinder when oriented normally to the flight path (with $\alpha = \pi/2$) is given approximately by the expression†

$$\beta = \frac{W}{C_D A} = 1.2\,[\rho' R + r^2 (\rho - \rho')/R]$$

where (1)

$$W = \pi r^2 L \rho + \pi (R^2 - r^2) L \rho'$$

and

$$C_D A = (8/3) R L \quad .$$

Two limiting cases are of immediate interest for purposes of comparison: (1) a bare cylinder of density ρ and radius R, since it occupies the same volume as the coated cylinder; and (2) a bare cylinder of density ρ and radius r, since this is the core which the coating is intended to protect.

The first of these has the ballistic coefficient

$$\bar{\beta} = 1.2 R \rho \qquad (2)$$

and the ballistic coefficient for the second is

$$\beta^* = 1.2 r \rho \quad , \qquad (3)$$

† This formula for the ballistic coefficient and the ones following are derived in the Appendix.

where $r < R$ (and $\alpha = \pi/2$ has been assumed in each instance).

A. Case 1

It is apparent from Formula (1) for the coated cylinder that

$$\beta \rightarrow \overline{\beta} \quad \text{as} \quad \rho' \rightarrow \rho \quad \text{and as} \quad r \rightarrow R \quad .$$

Indeed, the defect from the limiting value is

$$\begin{aligned} \overline{\beta} - \beta &= 1.2\,(\rho - \rho')(R^2 - r^2)/R \\ &= \overline{\beta}\,(1 - \rho'/\rho)[1 - (r/R)^2] \quad , \end{aligned} \tag{4}$$

and under the assumption that the coating is so thin that $(R + r) \sim 2R$, we obtain the simple approximation that the defect varies directly with the thickness of the coating and the difference in density of the materials:

$$\overline{\beta} - \beta \sim 2.4\,(\rho - \rho')(R - r)$$

or (5)

$$\overline{\beta} - \beta \sim 2\overline{\beta}\,(1 - \rho'/\rho)(1 - r/R) \quad .$$

A direct comparison of the coated cylinder with outside diameter 2R and the bare one of the same dimensions may conveniently be expressed in terms of a "weighting function"

$$\overline{g}\,(\rho', R) = \beta/\overline{\beta} \tag{6}$$

where r and ρ are assumed fixed .

Formula (1) may be rewritten to make the form of $\bar{g}$ apparent:

$$\beta = [(\rho'/\rho) + (r/R)^2(1-\rho'/\rho)]\bar{\beta} \quad . \tag{7}$$

The weighting-function $\bar{g}$ is plotted in Fig. 1 for several different values of (ρ'/ρ) and appears as an increasing function of the parameter (r/R) .

The penalty in ballistic coefficient which a given density and thickness of coating may impose in relation to a given choice of core may thus be assessed directly. (The densities of some representative materials are given in Table 1.)

However, in viewing Fig. 1 with the dynamic situation in mind, it must be remembered that while $\bar{g} \to 1$ as $(r/R) \to 1$ for each value of (ρ'/ρ) , $\bar{\beta}$ is decreasing as $R \to r$ and consequently, so is β .

B. Case 2

Survival problems which arise from atmospheric heating may make the use of a protective coating attractive in spite of the lowered ballistic coefficient (compared to $\bar{\beta}$ and β^*) which results. In this connection, a noteworthy fact may easily be established:

> For each choice of core and for each coating with density ratio $\rho'/\rho < 0.5$, an initial value of coating-thickness may be chosen so that the ballistic coefficient rises steadily toward β^* as ablation proceeds.

Once again, it is convenient to rewrite Formula (1) and introduce a new weighting-function g^* :

$$\beta = g^*(\rho', R)\beta^* \tag{8}$$

where

$$g^*(\rho', R) = [(\rho'/\rho)(R/r) + (1-\rho'/\rho)(r/R)] \tag{9}$$

for fixed r and ρ. The assertion then follows from the fact that for each value $\rho'/\rho < 0.5$, g^* as a function of r/R has an absolute minimum less than one.

Proof:– g^* has the form $cx + d/x$ where $x = r/R$, $d = \rho'/\rho$ is the density ratio, $c = 1-d>0$, and cd is constant for a given choice of coating. The identity

$$[cx + d/x]^2 = 4cd + [cx - d/x]^2$$

shows that an absolute minimum is attained when the r.h. term vanishes:

$$\min_{0<x<1} [cx + d/x] = \sqrt{4cd} = 2cx^*$$

where the critical value of x which gives the minimum is:

$$x^* = \sqrt{d/c} \quad . \qquad \text{(End of proof.)}$$

It follows that g^* has the minimum value

$$\min g^* = \sqrt{4d(1-d)} \tag{10}$$

where $d = \rho'/\rho$ and that this minimum is attained for

$$r/R = \sqrt{d/(1-d)} \quad . \tag{11}$$

Moreover, the simple identity

$$4d(1-d) = 1 - (1-2d)^2 \tag{12}$$

shows that the conditions

$$\min g^* < 1 \quad \text{with} \quad r/R < 1 \tag{13}$$

are compatible only for $\rho'/\rho = d < 1/2$.†

An initial choice of r/R thus dictated by the density-ratio $d = \rho'/\rho$ minimizes g^* , so different values of r/R encountered as $R \to r$ cause an increase in ballistic coefficient along with $g^* \to 1$.

Of course, this argument shows that larger values of $r/R < 1$ --which is to say, smaller values of R relative to a fixed r --than that suggested by Equation (11) may be chosen with the same result, an increasing ballistic coefficient. Equation (11) merely gives the critical value below which a period of initially decreasing β must be tolerated.

The corresponding critical value of coating-thickness is easily found from Equation (11):

$$R - r = r\sqrt{\rho/\rho' - 1} - r \quad . \tag{14}$$

† As a function of two variables, g^* has a saddle-point at $(\rho'/\rho) = 1/2$, $(r/R) = 1$.

Several plots of g^* are shown in Fig. 2 for values of ρ'/ρ in the range 0.01 to 0.5. These all lie above a straight line from the origin to the point (1, 1) corresponding to the degenerate case $g^*(0, R) = r/R$ where $\rho' = 0$.

The rise of g^* from its minimum is seen to be almost linear for low values of ρ'/ρ and may be estimated for such cases of interest by evaluating the derivative at $r/R = 1$:

$$\frac{d}{dx} g^*(\rho', x)\Big|_{x = 1} = 1 - 2(\rho'/\rho) \quad . \tag{15}$$

(This expression occurred once before in Equation (11) and thus relates the rate of growth to the depth of the minimum.) For low values of ρ'/ρ, β thus approaches β^* at a rate proportional to β^* itself:

$$\frac{d\beta}{d(r/R)} \sim [1 - 2(\rho'/\rho)]\beta^* \quad .$$

The critical values of r/R and the corresponding minima of g^* for different density-ratios ρ'/ρ are plotted in Fig. 3.

SUMMARY AND CONCLUSIONS

1. For the usual case where the coating has lower density than the material being protected, a coated cylinder of given outside dimensions has a lower ballistic coefficient than an uncoated cylinder having these dimensions. The difference in ballistic coefficient is directly proportional to the difference in densities and to the coating-thickness when the latter is small.

2. The ballistic coefficient of a coated cylinder can vary with coating thickness in different ways relative to the ballistic coefficient of the (smaller) bare cylinder being protected:

(a) A coating whose density exceeds one-half that of the core gives a higher ballistic coefficient than that of the core alone for all (positive) coating-thicknesses.

(b) A coating whose density is less than one-half that of the core gives a ballistic coefficient which may be either higher or lower than that of the core and either increasing or partly decreasing, depending on the range of values of coating-thickness considered.

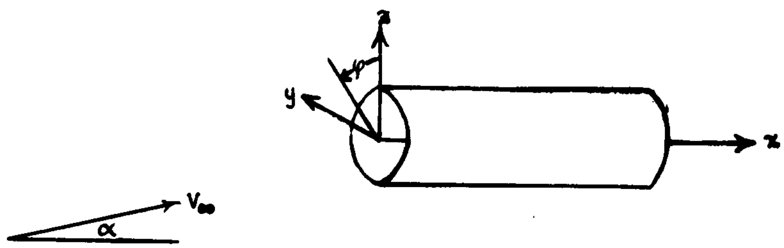
3. When the density of the coating is less than one-half that of the core, initial values of coating-thickness can be chosen so that the ballistic coefficient rises steadily toward that of the bare core as ablation proceeds. The density-ratio determines a maximum allowable coating-thickness for this rise to occur; if a thicker coating is used, the ballistic coefficient first decreases to a minimum which is less than the ballistic coefficient of the core. This minimum is also determined by the density-ratio and corresponds to the critical value of coating-thickness.

APPENDIX

C_DA FOR A CYLINDRICAL SURFACE

The central concept of Newtonian impact-theory is the assumption that fluid particles hitting an exposed surface lose entirely their component of momentum normal to the surface while the tangetial component remains unchanged.

We denote by $\vec{n}$ the inner normal to the surface of the cylinder with the coordinate system shown below:



The free-stream velocity $\vec{v}_\infty$ has the direction of

$$\vec{u} = [\cos\alpha , 0 , \sin\alpha] \quad ;$$

$$\vec{n} = [0 , -\sin\varphi , -\cos\varphi] \quad ,$$

and the component of $\vec{v}_\infty$ normal to the surface is therefore of magnitude

$$v_n = v_\infty . n = (-\sin\alpha \cos\varphi) v_\infty \quad .$$

The volume of fluid striking the element of area

$$dS = R\,d\varphi\,dx$$

in unit time (where the cylinder has radius R) is simply $v_n dS$, so if the fluid has density ρ_∞ , the change in the normal component of momentum in unit time corresponding to the mass-change dm is

$$v_n dm = v_n(\rho_\infty v_n dS) = \rho_\infty (v_\infty . n)^2 dS \quad .$$

The resulting pressure-force on the surface element dS , according to the Newtonian approximation is, therefore,

$$d\vec{F} = (\rho_\infty v_\infty^2 \sin^2\alpha \cos^2\varphi \, R\,d\varphi\,dx)\,\vec{n} \quad .$$

The component in the direction of the z-axis is $(\cos\varphi)dF$; symmetry with respect to φ over the surface cancels out the y-components, and the x-component of v_n is zero; integration over the surface exposed to the flow thus leads to a resultant force acting normally to the cylinder axis:

$$F_N = \rho_\infty v_\infty^2 R \sin^2\alpha \int_0^L \int_{\pi/2}^{3\pi/2} \cos^3\varphi \, d\varphi \, dx$$

where R is the radius, and L is the length.

The integrand is a periodic, even function, and since

$$\cos^3\varphi = \cos\varphi - \sin^2\varphi\cos\varphi \quad ,$$

$$\int_{\pi/2}^{3\pi/2} \cos^3\varphi\, d\varphi = -2\int_0^{\pi/2} \cos^3\varphi\, d\varphi = -4/3 \, .$$

Thus, in terms of the dynamic pressure

$$q_\infty = \frac{1}{2}\rho_\infty v_\infty^2 \quad ,$$

Newtonian impact theory predicts a resultant force (neglecting end-effects) represented by:

$$F_N = q_\infty\left(\frac{8}{3} RL\sin^2\alpha\right) \quad .$$

The drag force is the component acting in the direction of the relative free-stream velocity v_∞ :

$$D = q_\infty C_D A = (\sin\alpha)\, F_N \quad .$$

Hence, the Newtonian expression for the drag-area of a cylindrical surface--neglecting end-effects--is given by the formula

$$C_D A = \frac{8}{3} RL\sin^3\alpha \quad .$$

A solid cylinder of density ρ, radius R, and length L, has weight

$$W = \pi R^2 L \rho \quad .$$

Its ballistic coefficient accordingly is

$$W/C_D A = 1.2 R \rho / \sin^3 \alpha$$

where

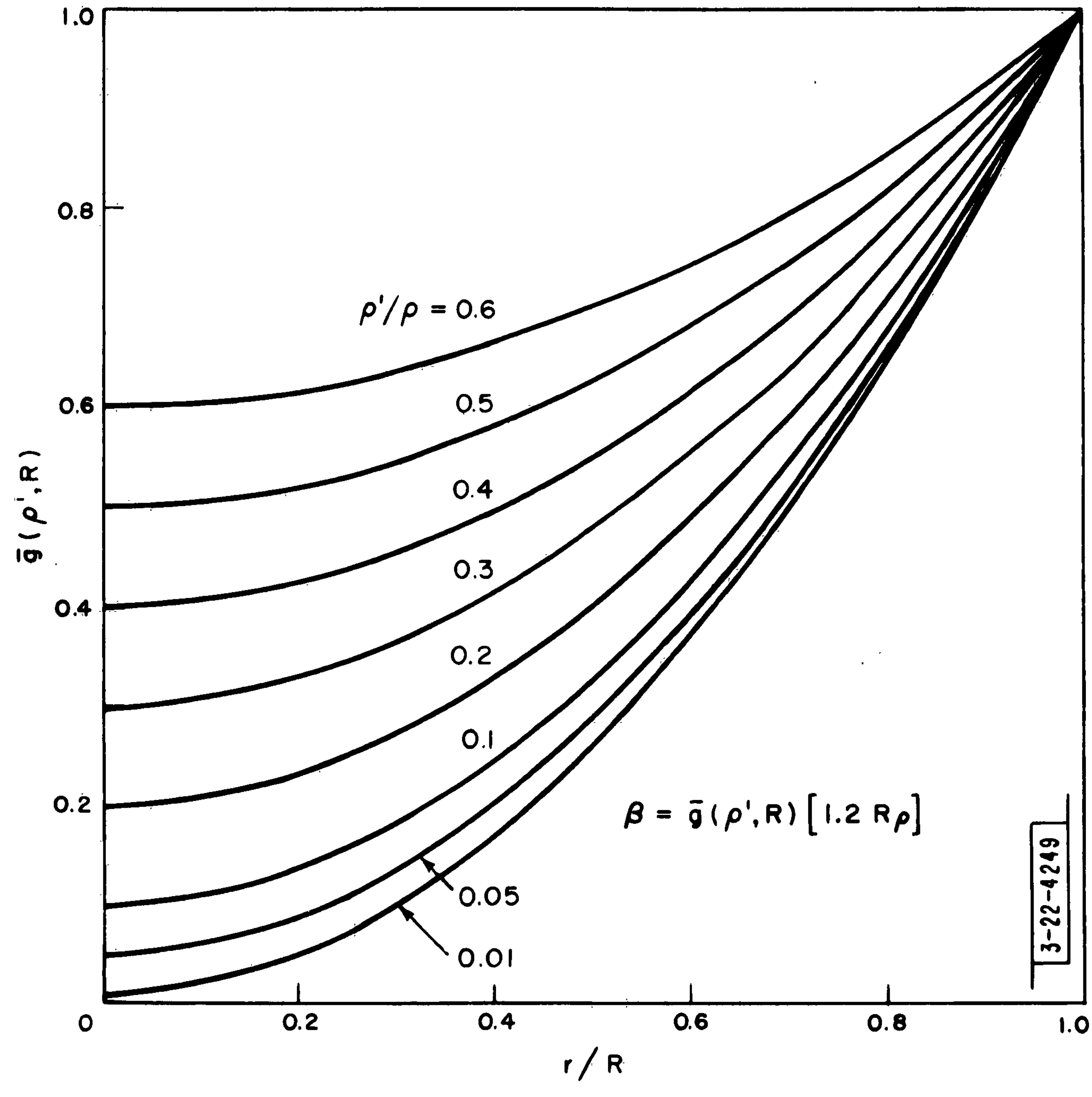
$$3\pi/8 \sim 1.2 \quad .$$

TABLE 1

APPROXIMATE DENSITIES

(lbs/ft^3)

Aluminum	170	Nylon	70
Beryllium	115	Polyethylene	55
Chromium	410	Sealing Wax	110
Columbium	525	Tantalum	1035
Copper	555	Teflon	130
Gold	1200	Tungsten	1200
Graphite	140	Uranium	1170



1.0
0.8
0.6
0.4
0.2
0
$\bar{g}(\rho', R)$
$\rho'/\rho = 0.6$
0.5
0.4
0.3
0.2
0.1
0.05
0.01
$\beta = \bar{g}(\rho', R)\,[1.2\,R\rho]$
3-22-4249
0.2
0.4
0.6
0.8
1.0
r / R

FIGURE 1

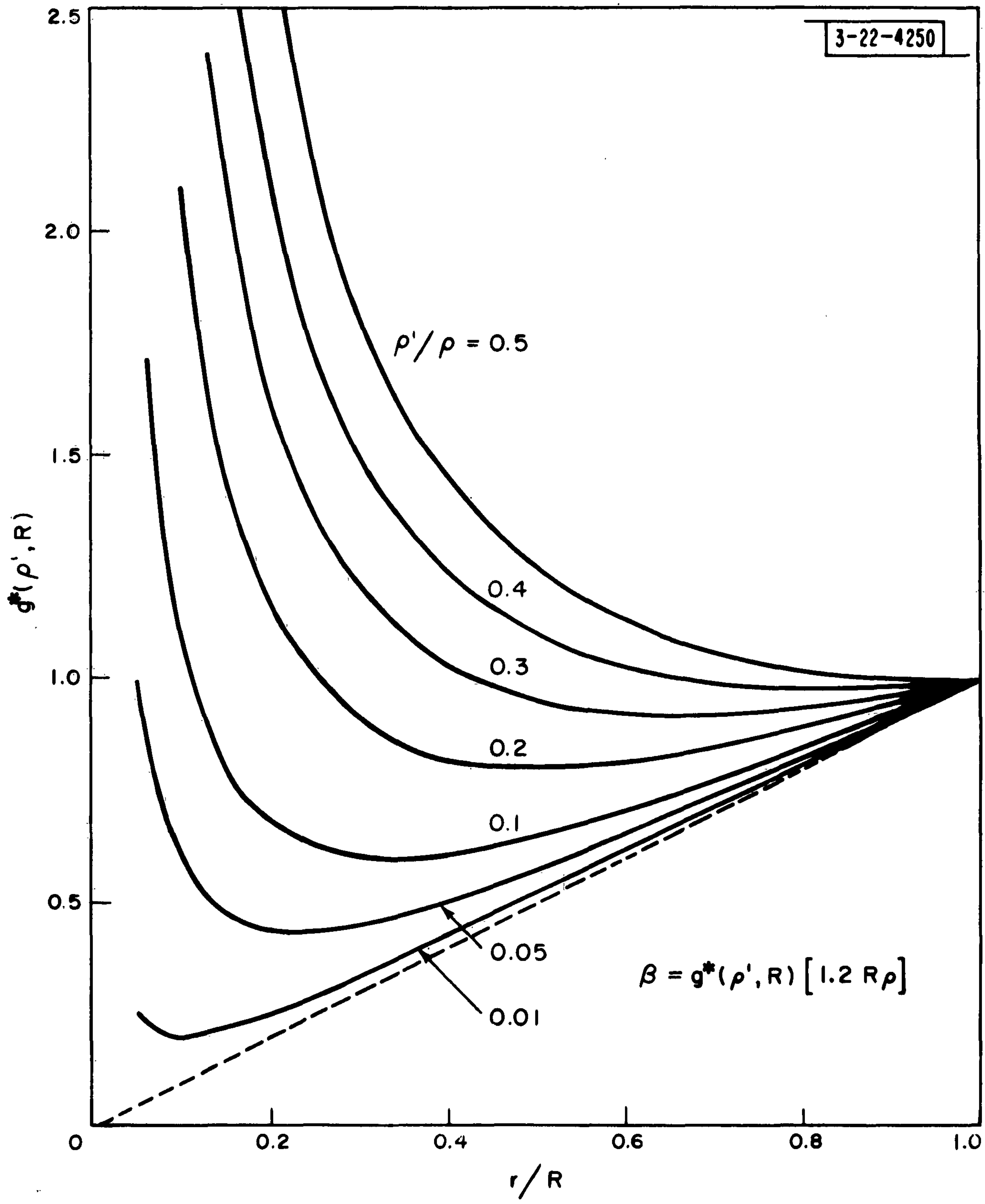

3-22-4250
2.5
2.0
1.5
1.0
0.5
0
0.2
0.4
0.6
0.8
1.0
$g^*(\rho', R)$
r/R
$\rho'/\rho = 0.5$
0.4
0.3
0.2
0.1
0.05
0.01
$\beta = g^*(\rho', R)\,[1.2\,R\rho]$

FIGURE 2

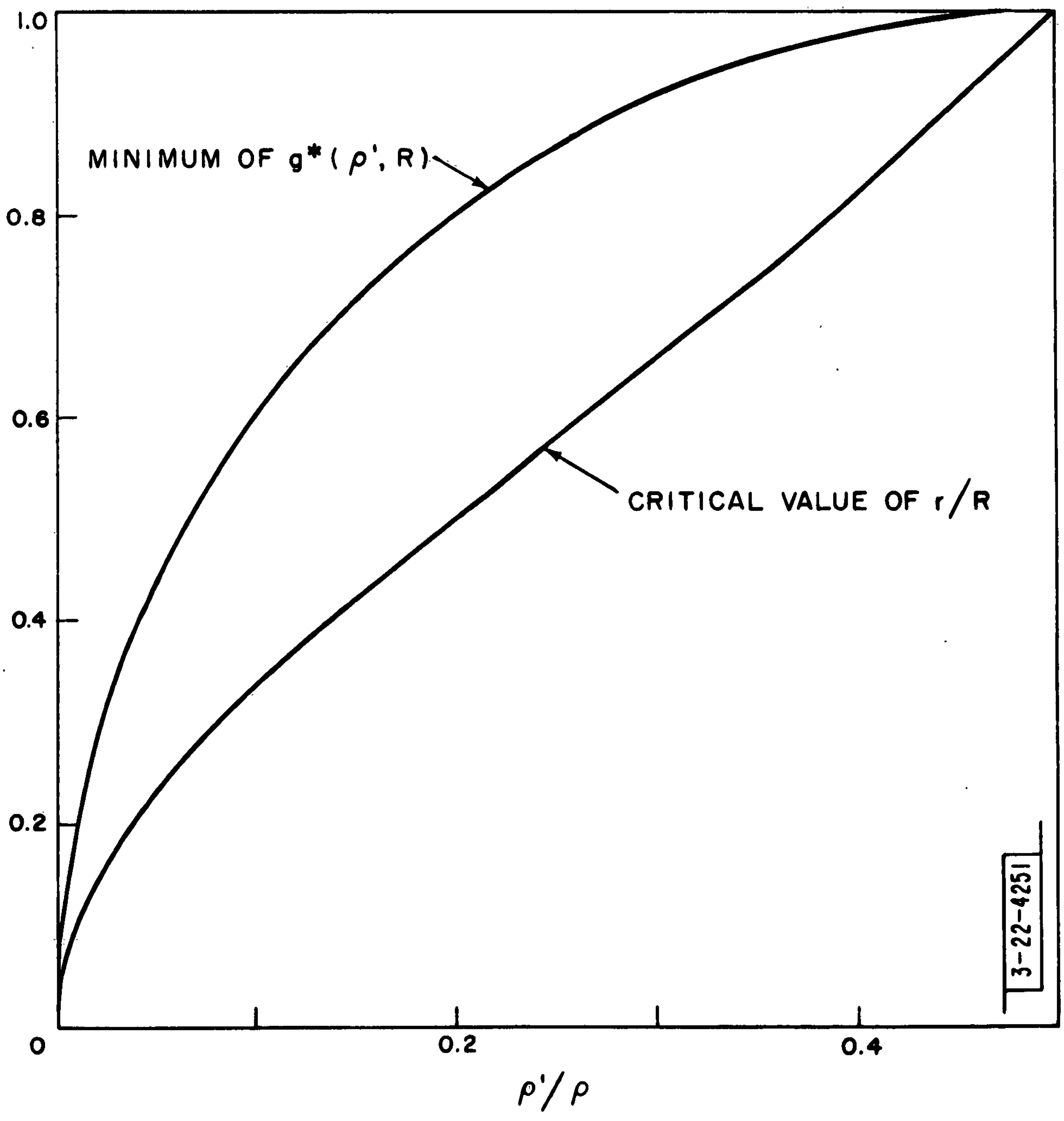

1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
MINIMUM OF g*(ρ', R)
CRITICAL VALUE OF r/R
3-22-4251
ρ'/ρ

FIGURE 3

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