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BLOCKS IN ERROR AND PARETO'S LAW

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Figure 1 shows the distribution of inter-error intervals for two HF radio teletype circuits and one telephone circuit. An inter-error interval is defined as the number of correct blocks plus one, between an error and the next following error. The graphs are plotted on doubly logarithmic paper to check against Pareto's Law behavior (indicated by a straight line). * The curve for the high-speed data telephone circuit is straight except for the kink for an inter-error interval of two. This indicates that inter-error intervals of one occurred considerably more frequently than indicated by Pareto's Law; however, the method of obtaining the statistics may have produced a considerable distortion of the statistics. The HF teletype data for the Bermuda link is reasonably straight. The Johannesburg curve shows the greatest departure from linearity, but the experimental procedures were least satisfactory in the case of the Johannesburg statistics. A brief account of the method of recording bit error statistics and their subsequent re-assembly into block error statistics follows. Possible distortions of the statistics produced by this process are mentioned in the following paragraphs.

Compilation of Data

1. A test character (containing 16 bits in the case of the telephone link data and, effectively, 5 bits in the case of the HF teletype link) was continuously transmitted over a channel. The received pattern was examined for errors which were recorded together with their time of occurrence on paper tape. Since the time required to punch the necessary data for a received character containing errors is longer than the time required for transmission of that character, in the case of the telephone circuit high-speed data, some provision was necessary to handle successive characters with errors. The

* See note, page 4.

error-recording apparatus was provided with storage for three characters. If characters with errors are received when these three storage registers are in use, a counter, called the "excess error" counter, is used to count the number of such characters, even though the error pattern in these characters cannot be saved. When a storage register is again available for error data, the contents of the "excess error" counter are punched onto tape and the counter reset to zero. These small transmitted test characters were then combined to give sequences of blocks of 255 bits (corresponding to the block length in one of the Bose-Chaudhuri codes). When the three buffer registers are full, the machine records only that a block or number of blocks were received in error. Under these conditions there is no record of the number or locations of bits in error in the block in error. It was therefore arbitrarily assumed that a number of erroneous characters corresponding to the "excess error" count occurred consecutively after the last character in the buffer. Their error patterns were assumed to be identical with the error pattern of the last character in the buffer. The effect of this recording procedure is only felt in the case of the high-speed data, since there is no problem of recording data at teletype speeds. It is not possible to predict the distortion that this produces on the statistics.

2. If more than 384 successive characters contained errors, the recording equipment was automatically turned off and could only be turned on again by test personnel.

3. For the HF teletype link in addition to the five bits of the test character that were examined for errors, there was a preceding start signal (the same length as one of the five bits) and a stop signal (1-1/3 -bit lengths). Thus the link is being examined for only 5/7ths of the time.

4. In any system that attempts to detect bit errors, there is the problem that when the link is open some bits appear to be correctly received when, in fact, chance coincidences between a known sent pattern of bits and a randomly generated pattern of bits are occurring.

New Data

Tests have been concluded on another high-speed data link. The test equipment included a considerably larger buffer than in the previous equipment and thus an almost exact record of bits in error has been obtained. This data will be analyzed in the near future to see if Pareto's Law applies for blocks of various lengths and also to the individual bits.

Note

Let the probability that an inter-error interval of length greater than or equal to N blocks occurs be written $Pr(n \geq N)$, then if these inter-error intervals obey the Law of Pareto $Pr(n \geq N) = N^{-\alpha}$. It is assumed that there is no dependence between the length of an interval and that of any of the preceding intervals. The Law has been found applicable to problems in economics. The present investigation has been suggested by some recent work of Benoit Mandlebrot* on the error distributions for telephone lines. This work dealt with the intervals between bits in error and found that their distribution conformed well to Pareto's Law. Values of α, in the vicinity of 1/4, were found. Referring to Fig. 1, curve A gives a value for α of about 2/3, curve C gives a value of α of less than 1/3. It is interesting to note that for $0 < \alpha < 1$ there are no moments of any order. For $1 < \alpha < 2$ only the first moment is finite. Thus in the present case there is no average value or variance of the inter-error intervals.

* International Business Machines, Thomas J. Watson Research Center

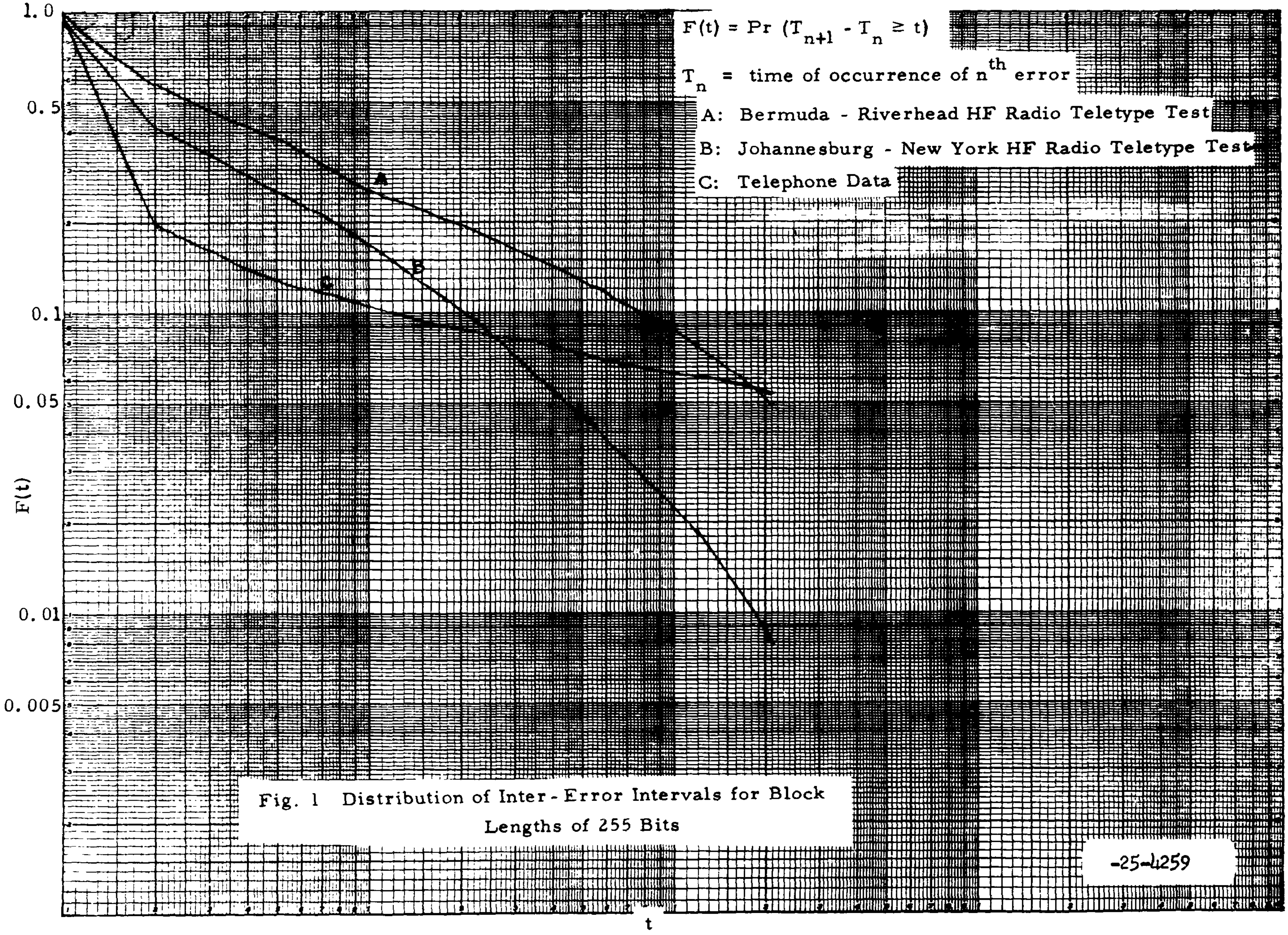

Fig. 1 Distribution of Inter - Error Intervals for Block Lengths of 255 Bits

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ADDENDUM

to 25G-15

The reader will be interested in a more complete reference to the work that motivates this report, "A New Model for the Clustering of Errors in Telephone Circuits," by Dr. J. M. Berger, Advanced Systems Development Division, IBM, and Dr. B. Mandelbrot, T. J. Watson Research Center, IBM, Yorktown Heights, published as an IBM report and shortly to appear in the IBM Journal of Research and Development. An early version of the work of Dr. Berger and Dr. Mandelbrot was presented by the latter at an M. I. T. Seminar in November, 1962.

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