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RADIATION WITH TEMPERATURE DEPENDENT THERMAL PROPERTIES

by

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PROG. N. FOGLIO N.

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CONTENTS

PROG. N. FOGLIO N.

ABSTRACT

The paper is devided into two parts.

In part I the problem of radiation from a generic body with variable thermal coefficients is considered and a general equation of Volterra's type is derived. Explicit expression is also given for the spherical shell, whence, as particular case, (i) the solid sphere, (ii) the flat plate, (iii) the indefinite body, can be studied. Also a pratical solution for numerical application is deviced.

In part II the theory is applied to particular laws of variation of thermal coefficients, and the effects of such coefficients is investigated.

INTRODUCTION

The analysis of radiative heat conduction problems has received considerable attention in recent years, by reason of the many technical problems referring to it. However, none of the solutions available in the literature seems to be complete, since, in general, the simplified case of constant thermal coefficients (conductivity and specific heat) is considered. It is well knew, on the contrary, that radiation is efficient only at very hight temperatures, where the effects of variable coefficients cannot be ignored.

This paper deals with the application to radiation problems of the particular method of analysis described in Ref. 1.

The method is based on the use of Green's function through which it is possible to reduce any heat conduction problem to an integral equation of Volterra's type. In this latter the only independent variable is time, whereas space coordinates are eliminated.

In Ref. 1, Green's function is determined for a body of whatever shape; its analytical expression is:

$$v(P, P_*; t-\lambda) = \sum_{0}^{\infty}{}_n U_n(P)\, U_n(P_*)\, e^{-p_n^2(t-\lambda)} \qquad (1)$$

where U_n and p_n are the orthogonal eigenfunctions and the eigenvalues of the problems:

$$\left.\begin{array}{ll} K_i \nabla U_n + c_i p_n^2 U_n = 0 & \text{in the solid volume } V \\ \dfrac{\partial U_n}{\partial \nu} = 0 & \text{at the boundary } W \end{array}\right\} \qquad (2)$$

Eq. (1), as well known provides the raise in temperature at P_*, at time t due to a unit heat pulse acting at P_*, at time t.

It should be pointed out that the problem described by (2) refers to the real body of the problem, but with constant thermal coefficients; it is therefore, a linear problem and, at a certain extent, an elementary one. The introduction of thermal coefficients variable with temperature can be however be performed exactly starting from the functions (2), as it will be shown in the subsequent numbers.

PART I

GENERAL THEORY

1 - General Equations.

The heat conduction equation for an isotropic body is, as well known:

$$\operatorname{div}\left[K(T)\operatorname{grad}T\right] = c\frac{\partial T}{\partial t} \qquad (3)$$

Setting (see also Ref. 2)

$$\frac{K(T)}{K_i} = \frac{dT'}{dT} \qquad (4)$$

(where K_i is the conductivity at any reference temperature, f.i., the initial one), Eq. (3) reduces to:

$$\nabla T' = \frac{c}{K}\frac{\partial T'}{\partial t} \qquad (3')$$

Setting again:

$$\frac{c/K}{(c/K)_i} = 1-\eta_1(T) = 1-\eta(T') \qquad (5)$$

the foregoing equation yields:

$$K_i\nabla T' + q = c_i\frac{\partial T'}{\partial t} \qquad (6)$$

where:

$$q = c_i \eta(T') \frac{\partial T'}{\partial t} \tag{7}$$

It is seen, therefore, that the problems is identical with that of constant coefficients (c_i and K_i), provided that the sources (7) be introduced. The problem is, of course, still non-linear, but every non-linearity has been introduced into the fictitious sources q.

In radiation problems, there are also the sources arising from heat losses at the surfaces. It is assumed, as customarily, that such losses be adequately expressed by:

$$Q(P_w) = F_1(T_w) = F(T'_w) \tag{8}$$

where T_w denotes the temperature of the generic point of surface W. It is usually admitted that $F_1(T_w) = -\varepsilon\sigma T_w^4$; but no restriction is placed here, also in view of the fact that no simplification would occur in general, once the temperature T' is introduced.

The temperature T' at any point of the body is then: (Ref. 3)

$$T'(P,t) = \int_0^t d\lambda \left\{ \int_V v(P,P_*;t-\lambda)\, q(P_*,\lambda)\, dV_* - \int_W v(P,P_{W_*},t-\lambda)\, F(T'_w)\, dW_* \right\} + T'_i \tag{9}$$

where dV_* is the element surrounding P_* and W_* the surface element surrounding P_{W_*}, T'_i is the initial transformed temperature, assumed to be constant.

Eq. (9) is the general equation of the problem. It is, of course, non-linear (its non-linearities arise from the form of q and F).

It is, however, well suitable for numerical integration, since it is, an integral equation in an open variable, such as time.

It is also seen that, once the functions describing the radiation mechanism (F_1) and the material behaviour (η_1 and $K(T)$) are given, the solution is essentially depending on the body shape.

The following Arts. are essentially devoted to write Eq. (5) for the most common shapes. For other shapes, see Ref. 2.

2 - The Spherical Shell

Consider a spherical shell of outer radius a and inner radius $a\beta$; let $s=a(1-\beta)$ be its thickness.

The eigenfunctions U_n for the case of radial flow are:

$$\left.\begin{aligned} U_n &= \frac{C_n}{\sqrt{2\pi c_i s}} \; \frac{\cos\left\{\omega_n x + \tan^{-1}\frac{1-\beta}{\omega_n}\right\}}{a\left[1-x(1-\beta)\right]} \qquad n=1,2,\ldots\ldots \\ U_o &= \frac{C_o}{\sqrt{4\pi c_i s}} \; \frac{1}{a} \end{aligned}\right\} \tag{10}$$

and the eigenvalues p_n :

$$p_n = \omega_n \sqrt{\frac{K_i}{c_i s^2}} \tag{10'}$$

(for the values of C_n and ω_n see Appendix); x is the nondimensional coordinate represented in Fig. 1.

From Eqs. (1) - (10), one has now:

$$v(P,P_*;t-\lambda) = \frac{1}{4\pi c_i a^2 s} \; \frac{3}{1+\beta+\beta^2} \; +$$

$$+\frac{1}{2\pi c_i a^2 s\left[1-x(1-\beta)\right]\left[1-x_n(1-\beta)\right]}\sum_{1}^{\infty}{}_n C_n^2 \cos\left\{\omega_n x+\tan^{-1}\frac{1-\beta}{\omega_n}\right\}\cos\left\{\omega_n x_n+\tan^{-1}\frac{1-\beta}{\omega_n}\right\}\times$$

$$\times\; e^{-\omega_n^2(t-\lambda)\,K_i/c_i s^2} \qquad (11)$$

and, so, Eq. (9) is written:

$$T'(x,t)=\int_0^t d\lambda\left\{\int_0^1\left[C_0^2\left\{1-x_n(1-\beta)\right\}^2+\right.\right.$$

$$+\,2\,\frac{1-x_n(1-\beta)}{1-x(1-\beta)}\sum_{1}^{\infty}{}_n C_n^2\cos\left(\omega_n x+\tan^{-1}\frac{1-\beta}{\omega_n}\right)\cos\left(\omega_n x_n+\tan^{-1}\frac{1-\beta}{\omega_n}\right)e^{-\omega_n^2(t-\lambda)\frac{K_i}{c_i s^2}}\Big]\times$$

$$\times\left[\eta(T')\frac{\partial T'}{\partial t}\right]_{x_n,\lambda}dx_n-F(T'_{w_n},\lambda)\left[\frac{C_0^2}{c_i s}+\frac{2}{c_i s\left[1-x(1-\beta)\right]}\sum_{1}^{\infty}{}_n C_n^2\cos\left(\omega_n x+\tan^{-1}\frac{1-\beta}{\omega_n}\right)\times\right.$$

$$\left.\left.\times\frac{\omega_n}{\sqrt{(1-\beta)^2+\omega_n^2}}\;e^{-\omega_n^2\frac{K_i}{c_i s^2}(t-\lambda)}\right]\right\}+T_i' \qquad (12)$$

Eq. (12) can be further be made nondimensional; Setting:

$$\left.\begin{aligned}&t=\theta\frac{c_i s^2}{K_i}\;;\quad \lambda=\mu\frac{c_i s^2}{K_i}\;;\quad \tau=\frac{T'}{T_i}\;;\quad \tau_i=\frac{T_i'}{T_i}\\&F(T_{w_n})=\varepsilon\sigma T_i'^4 f(T_i'\,\tau_w)\;;\quad \frac{\varepsilon\sigma T_i'^3 s}{K_i}=\alpha\end{aligned}\right\}\qquad(13)$$

it is obtained:

$$\tau(x,\theta)=\int_0^\theta d\mu\left[\int_0^1\left\{\varphi_0(x_n)+2\sum_{1}^{\infty}{}_n\psi_n(x,x_n)e^{-\omega_n^2(\theta-\mu)}\right\}\left\{\eta(\tau T_i')\frac{\partial\tau}{\partial\theta}\right\}_{x_n,\mu}dx_n\right.$$

$$\left.-\,\alpha\left[\varphi_0(0)+2\sum_{1}^{\infty}{}_n\psi_n(x,0)\,e^{-\omega_n^2(\theta-\mu)}\right]f(\tau_w T_i')\right]+\tau_i \qquad (14)$$

where the following further positions have been made:

$$\left.\begin{aligned} \varphi_0(x) &= C_0^2 \left[1 - x(1-\beta)\right]^2 \\ \psi_n(x, x_*) &= \frac{1 - x_*(1-\beta)}{1 - x(1-\beta)}\; C_n^2 \cos\left(\omega_n x + \tan^{-1}\frac{1-\beta}{\omega_n}\right) \cos\left(\omega_n x_* + \tan^{-1}\frac{1-\beta}{\omega_n}\right) \end{aligned}\right\} \quad (15)$$

Eq. (14) is the required one. The space variable x appearing in it is merely a parameter: more exactly, Eq. (14) yields an infinite set of integral equations in the ~~variable~~, ϑ each corresponding to a value of x .

3 - The Solid Sphere.

The equation for the solid sphere is formally identical to (14), with $\beta=0$ In this case, the equation for the ω_n is

$$\tan \omega_n = \omega_n \,, \quad (16)$$

and the coefficients C_0, C_n are given in Appendix. The functions φ_0, ψ_n have now the expressions:

$$\left.\begin{aligned} \varphi_0(x) &= C_0^2 \left[1 - x\right]^2 \\ \psi_n(x) &= \frac{1 - x_*}{1 - x}\; C_n^2 \sin\left[\omega_n (1-x)\right] \sin\left[\omega_n (1-x_*)\right] \end{aligned}\right\} \quad (17)$$

4 - The Flat Plate.

For the flat plate, $\beta = 1$, and $\omega_n = n\pi$. One has therefore (App.)

$$\left.\begin{aligned} \varphi_o(x) &= C_o^2 = 1 \\ \psi_n(x) &= C_n^2 \cos n\pi x \cos n\pi x_* = \cos n\pi x \cos n\pi x_* \end{aligned}\right\} \quad (18)$$

and Eq. (14) is written:

$$\tau(x,\theta) = \int_0^\theta d\mu \left[\int_0^1 \left[1 + 2\sum_{1}^{\infty} {}_n \cos n\pi x \cos n\pi x_* \, e^{-n^2\pi^2(\theta-\mu)}\right] \left\{\eta(\tau T_i')\frac{\partial \tau}{\partial \theta}\right\}_{x_*,\mu} dx_* \right.$$

$$\left. - \alpha\left(1 + 2\sum_{1}^{\infty} {}_n (\cos n\pi x)\, e^{-(n\pi)^2(\theta-\mu)}\right) f\left(\tau_w \, T_i'\right)\right] + \tau_i \quad (19)$$

5 - The Semi-Indefinite Solid.

Eq. (19) holds also as $s \to \infty$. However, a re-arrangement of the equation is necessary, since the characteristics length s is now missing.

With the well-known technique of Fourier's integral, Eq. (19) yields:

$$\tau(x_1,\theta) = \frac{2}{\pi}\int_0^\theta d\lambda_1 \int_0^\infty d\xi \left[\int_0^\infty dx_{1*} \cos(\xi x_1) \cos(\xi x_{1*})\, e^{-\xi^2(\theta_1-\mu_1)} \left\{\eta(\tau T_i')\frac{\partial \tau}{\partial \theta_1}\right\}_{x_{1*},\mu_1}\right.$$

$$\left. - \cos(\xi x_1)\, e^{-\xi^2(\theta_1-\mu_1)} f(\tau_w \, T_i')\right] + \tau_i \quad (20)$$

when now the nondimensional quantities are:

$x_1 = \frac{\varepsilon \sigma s T_i^3}{K} \times X$, where X is the dimensional abscissa computed from the surface. (21)

$$\left.\begin{matrix}\theta_1 \\ \mu_1\end{matrix}\right\} = \left(\frac{\varepsilon \sigma T_0^3}{K}\right)^2 \chi \times \text{dimensional time.}$$

6 - The Case of Constant Diffusivity.

When $\eta = 0$, a great semplification occurs. Firstly, the term in brackets vanishes; secondly, the integral equation needs only be written at the surface, since the surface temperature is depending on the surface temperature itself. Eq. (14) becomes:

$$\tau_w(\theta) = -\alpha \int_0^\theta \left[C_0^2 + 2 \sum_{1}^{\infty} C_n^2 \frac{\omega_n^2}{(1-\beta)^2 + \omega_n^2} e^{-\omega_n^2(\theta-\mu)} \right] f(\tau_w T_i') d\mu + \tau_i \qquad (22)$$

Which also applies, obviously, for the solid sphere and for the slab. For the semi-indefinite solid, since,:

$$\int_0^\infty e^{-\xi^2(\theta-\mu)} d\mu = \frac{\sqrt{\pi}}{2\sqrt{\theta-\mu}} \qquad (23)$$

Eq. (23) yields:

$$\tau_w(\theta) = \tau_i - \frac{2}{\sqrt{\pi}} \int_0^\theta \frac{f(\tau_w T_i')}{\sqrt{\theta-\mu}} d\mu \qquad (24)$$

When $T' = T$, i. e., when also the effects of variable conductiv ity are ignored, Eqs. (21) - (22) are substantially the same as those of Ref.3

Eqs. (21) - (22) lend themselves to a straight numerical integration. As $\theta \to 0$, an asymptotic solution must be devised (Ref.3)

7 - Approximate Solution.

Coming back to Eq. (3'), it is seen that, letting:

$$\frac{dt'}{dt} = \frac{(c/K)_i}{(c/K)} \tag{25}$$

the same equation yields the constant coefficients one:

$$K_i \nabla T' = c_i \frac{\partial T'}{\partial t'} \tag{26}$$

to which, according to the body shape, Eqs. (22) - (23) can be applied.

The introduction of t' does not take away, of course, the non-linearity of the problem, since the relationship between T' and t' is not known (and is not even unique, since it depends on the space-coordinate). In radiation problem, however, the highest values of $\frac{\partial T'}{\partial t}$ are near the surface; this means that the main importance is that of the sources q near the wall. A good approximation is therefore to assume:

$$\frac{dt'}{dt} = \frac{1}{1-\eta(\tau_w T_i')} \tag{27}$$

In this way, the procedure of the calculation is to determine τ_w from Eqs. (21) or (22), and so to determine $\tau_w = \tau_w(t')$ Then Eq. (25) provides $t' = t'(t)$, and Eq. (2) $T = T'(\tau_w, T_i')$ and so the problem is fully solved.

PART II

ILLUSTRATIVE EXAMPLES

The foregoing theory has been applied to numerical cases. Results are collected in Figs. 2 - 3 - 4 - 5 - 6.

Figs. 2 and 3 refer to the radiation of semi-infinite solid with thermal coefficient varying according to the law:

$$\frac{K}{K_i} = \left(\frac{T}{T_i}\right)^m \; ; \qquad \frac{c}{c_i} = \left(\frac{T}{T_i}\right)^s \qquad (a)$$

λ

Fig. 2 gives the variation of nondimensional wall temperature vs. nondimensional time for several values of m and s. The influence of specific heat is seen to be greater, in general, than that of conductivity.

Fig. 3 provides - for the same case - the variation of surface heat flow versus time.

X

Figs. 4 and 5 refer to the solid sphere, with laws analegous to (a). Fig. 4 (which is of the same type of Fig. 2) provides the variation of wall temperature vs. time and comparison is made with the case of constant coefficients. Fig. 5 gives, for two values of nondimensional times, the space-variation of temperature in the sphere; also here comparison is made with the case of constant coefficients.

Fig. 6 refers to the case of the hollow cilinder (whose theory has not been given in this report; see Ref. 4). The heating conditions are:

$$Q = h\left(T_o - T_W\right) \quad \text{in the inner wall}$$

$$Q = -\varepsilon\sigma T_W^4 \quad \text{in the outer wall}$$

with h and T_o given in Fig. 6. The problem simulates an element of radiator of a space powerplant. The temperature in the hot side are given by the plot of Fig.6

APPENDIX

1 - For a spherical shell, of radius a and thickness $s = a(1-\beta)$, the eigenvalues of the problem are obtained as said in Ref. 4, from the trascendental equation:

$$\tan \omega_n = \frac{(1-\beta)^2 \omega_n}{(1-\beta)^2 + \beta \omega_n^2} \qquad n = 1, 2, \ldots\ldots$$

There is a root $\omega_n = 0$, corresponding to the first term of r.h.s. of Eq. (12). The constants C_n of Eq. (10) are obtained through the normality condition:

$$\int_V C_0 U_n^2 \, dV = 1$$

which in this case yields:

$$\frac{1}{C_n^2} = 1 + \frac{\sin\left\{2\left[\omega_n + \tan^{-1}\frac{1-\beta}{\omega_n}\right]\right\}}{2\omega_n} - \frac{\frac{1-\beta}{\omega_n^2}}{1+\left(\frac{1-\beta}{\omega_n}\right)^2} \quad ; \qquad n = 1, 2, \ldots\ldots$$

$$C_0^2 = \frac{3}{1+\beta+\beta^2}$$

2 - For the solid sphere one has simply to set $\beta = 0$, and so:

$$\tan \omega_n = \omega_n$$

$$\frac{1}{C_n^2} = 1 + \frac{\sin\left[2\left(\omega_n + \tan^{-1}\frac{1}{\omega_n}\right)\right]}{2\omega_n} - \frac{1}{1+\omega_n^2} \quad ; \qquad n = 1, 2, \ldots\ldots$$

3 - For the flat plate, one has simply to set $\beta = 1$, and so:

$$\tan \omega_n = 0 \qquad \omega_n = n\pi$$

$$\frac{1}{C_n^2} = 1 \qquad n = 0, 1, 2, \ldots..$$

4 - For the semi-indefinite solid, let Eq. (19) be written with dimensional quantities X and t and let s approach infinity. Thus

$$\tau(X,t) = \lim_{s\to\infty} \int_0^t d\lambda \left[\int_0^s \frac{dX_*}{s} \Sigma_n \left(\frac{n\pi}{s} X \cos \frac{n\pi}{s} X_* \right) e^{-n^2\pi^2 \frac{K_i}{c_i s}(t-\lambda)} \right] \left[\eta(\tau T_i') \frac{\partial \tau}{\partial t} \right]$$

$$- \frac{\varepsilon \sigma T_i'^3 s}{K_*} \int_0^t \frac{K_i}{c_i s^2} d\lambda \left[1 + 2 \sum_{1}^{\infty}{}_n \cos \frac{n\pi X}{s} e^{-n^2\pi^2 \frac{K_i}{c_i s^2}(t-\lambda)} \right] f(\tau_w T_i') + \tau_i$$

As well known, as s approaches infinity, one has to take only such values of n, for which $\frac{n\pi}{s}$ is finite with the positions (21), and letting furthermore.

$$\xi = \frac{K}{\varepsilon \sigma T_i^3} \frac{n\pi}{s}; \quad \left(\text{whence } d\xi = \frac{K}{\varepsilon \sigma T_i^3} \frac{\pi}{s} \right)$$

Eq. (20) is obtained.

GLOSSARY OF SYMBOLS

INTRODUCTION AND PART I

- a - Radius of outer sphere
- c - Specific heat
- f - Function defining radiation Eq. (13) .
- p_n - Generic eigenvalues
- q - Heat source intensity
- s - Shell thickness
- t - Dimensional time
- t' - Transformed time Eq. (27) .
- v - Green's function
- x - Nondimensional co-ordinate (Fig. 1)
- C_0, C_n - Normalization constants
- F, F_1 - Functions defining radiation
- K - Thermal conductivity
- P, P_n - Points in the body
- P_w, P_{w_n} - Points at the surface
- Q - External heat flow
- T - Temperature
- T' - Transformed temperature Eq. (4) .
- U_n - Eigenfunction
- V - Body volume
- W - Body surface
- α - See Eqs. (13)
- $1-\beta$ = Thickness / radius
- ε - Emissivity
- η - Function defining variation of diffusivity
- ϑ - Nondimensional time

χ - Diffusivity

λ - Time

μ - Nondimensional time

ν - Outer normal to W

σ - Boltzmann's constant

τ Nondimensional temperature

φ_o, ψ_n - See Eq. (15)

ω_n - See Eq. (10')

∇ - Laplace operator

SUBSCRIPTS

o, n - Order of eigenfunctions

$'$ - Transformed times and temperature

i - Initial values

w - Surface values

$*$ - Dumming variation

PART II

m, s - exponents of laws defining variation of K and c .

T_o - Total temperature

h - Coefficient of transmission

REFERENCES

1 - Luigi BROGLIO — Transient Temperatures and Thermal Vibrations in space Structures. Proceedings of DURAND Centennial Congress, Stanford, August 1959.

2 - Luigi BROGLIO — Heat Conduction in Solids at Hypersonic Speed. Proceedings of AGARD Meeting, Copenhagen, 1958.

3 - Saul S. ABARBANEL — On Some Problems in Radiative Heat Transfer. OSR Technical Note No 59-531 MIT Fluid Dynamics Research Group Report No 59-1, April 1959

4 - Luigi BROGLIO — Thermal and Structural Analysis of Space Radiator Systems. SIARgraph No 59 bis, Contract No AF - 61 (052) - 429, TN 3, August 1961, Rome.

LIST OF ILLUSTRATION

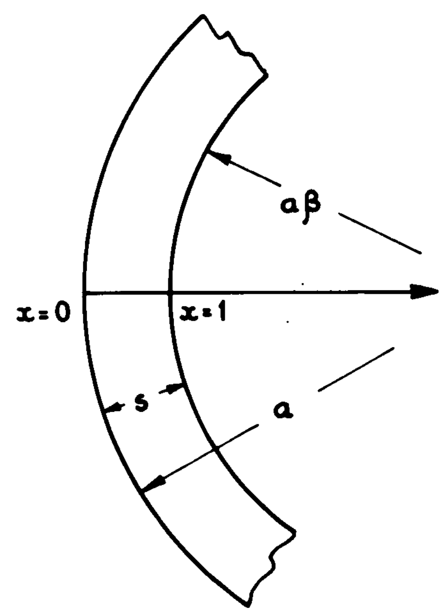
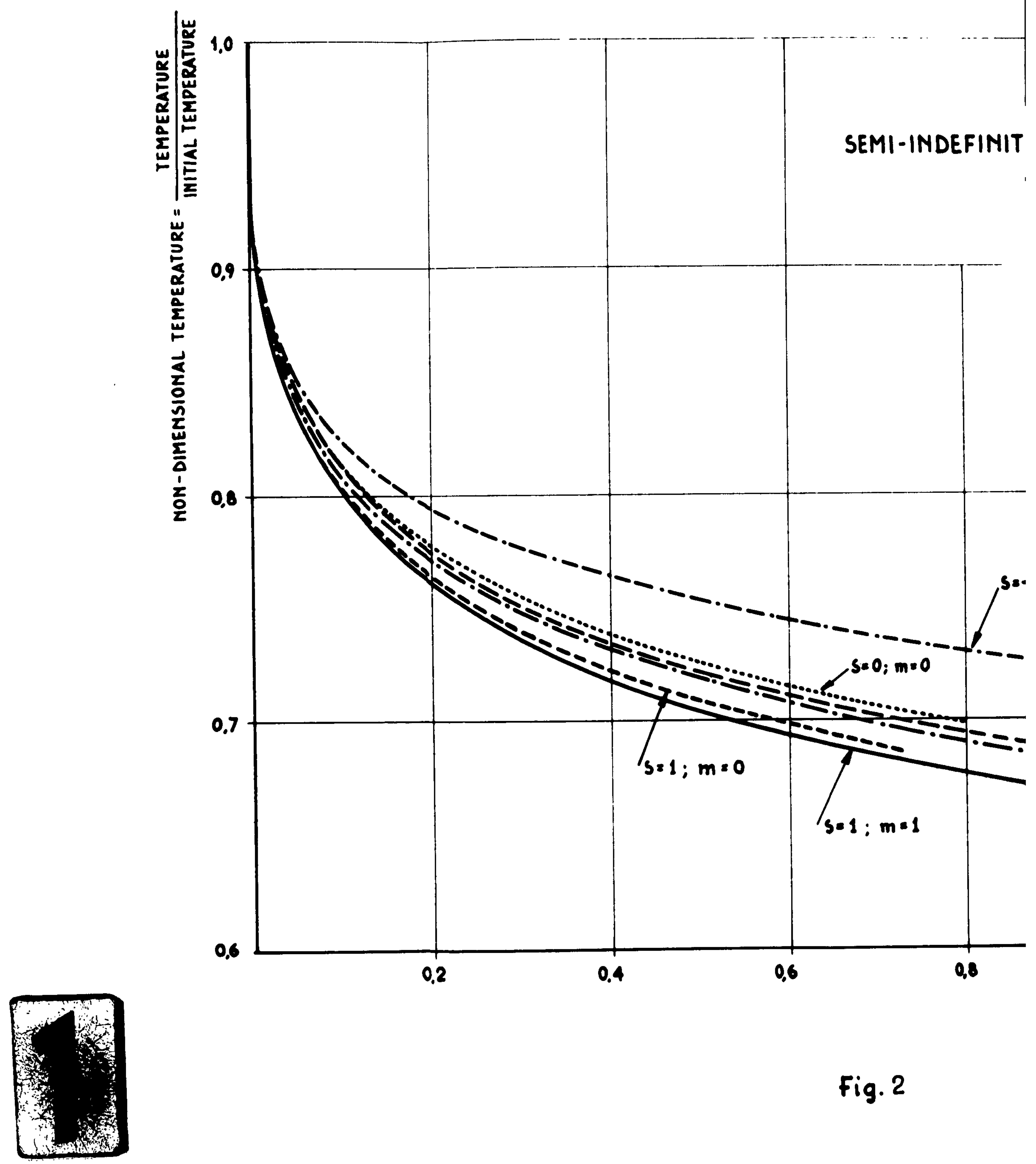


Fig. 1



NON-DIMENSIONAL TEMPERATURE = TEMPERATURE / INITIAL TEMPERATURE
1,0
0,9
0,8
0,7
0,6
0,2
0,4
0,6
0,8
SEMI-INDEFINIT
S=0; m=0
S=1 ; m=0
S=1 ; m=1

Fig. 2

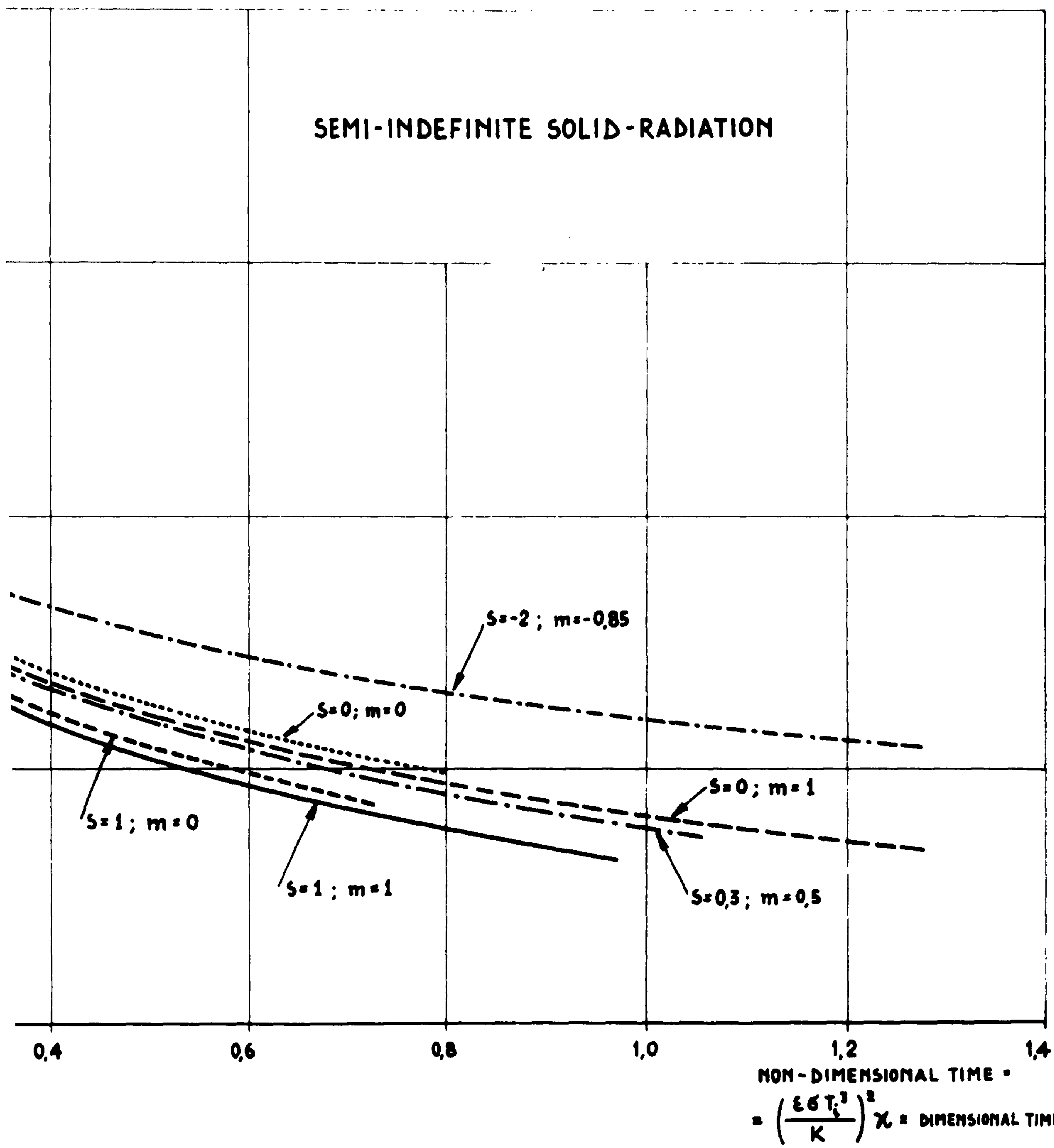
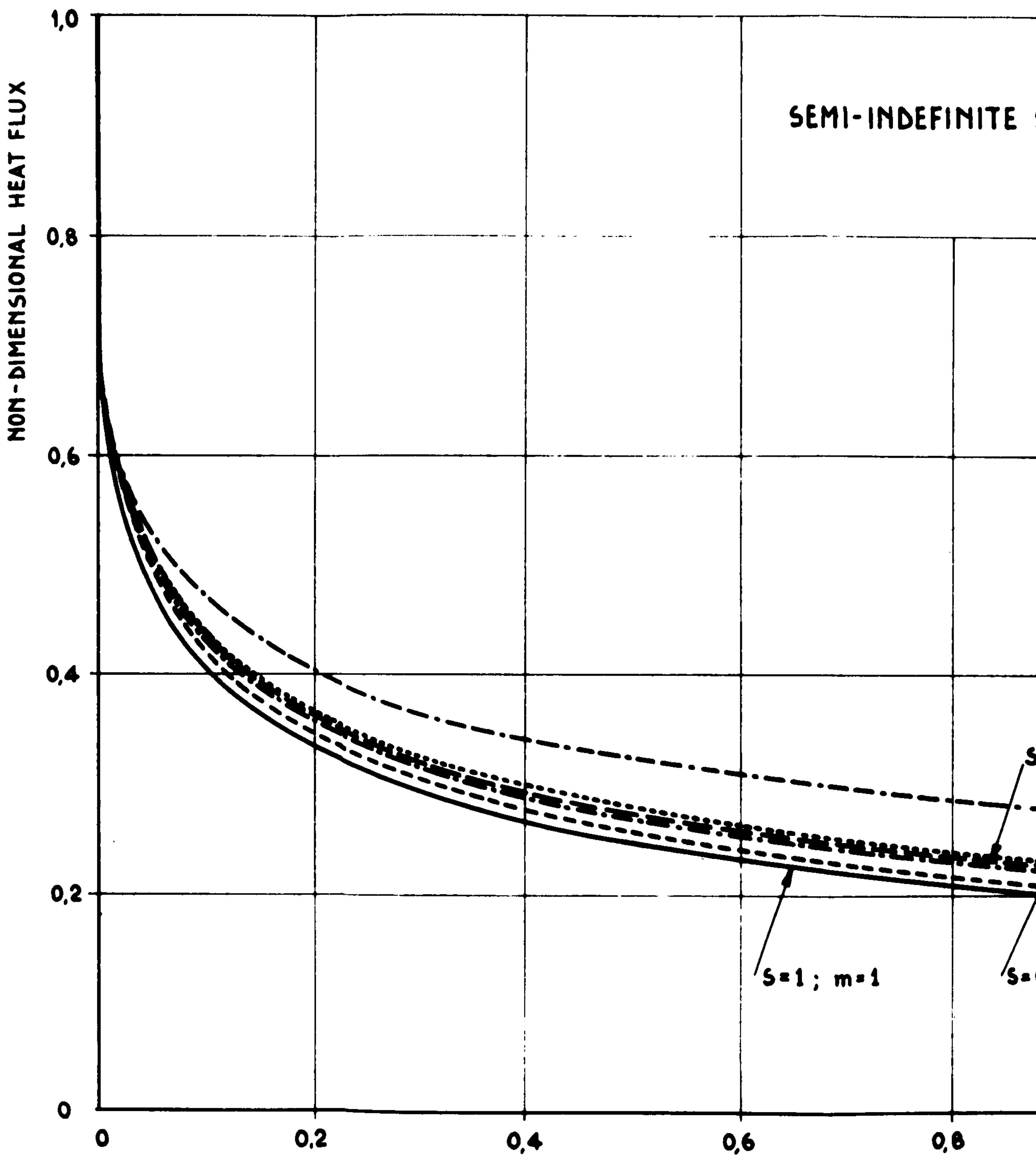

SEMI-INDEFINITE SOLID-RADIATION
S=-2 ; m=-0,85
S=0; m=0
S=0 ; m=1
S=1 ; m=0
S=1 ; m=1
S=0,3 ; m=0,5
0,4
0,6
0,8
1,0
1,2
1,4
NON-DIMENSIONAL TIME =
$= \left(\frac{\varepsilon \sigma T_i^3}{K}\right)^2 \chi$ = DIMENSIONAL TIME

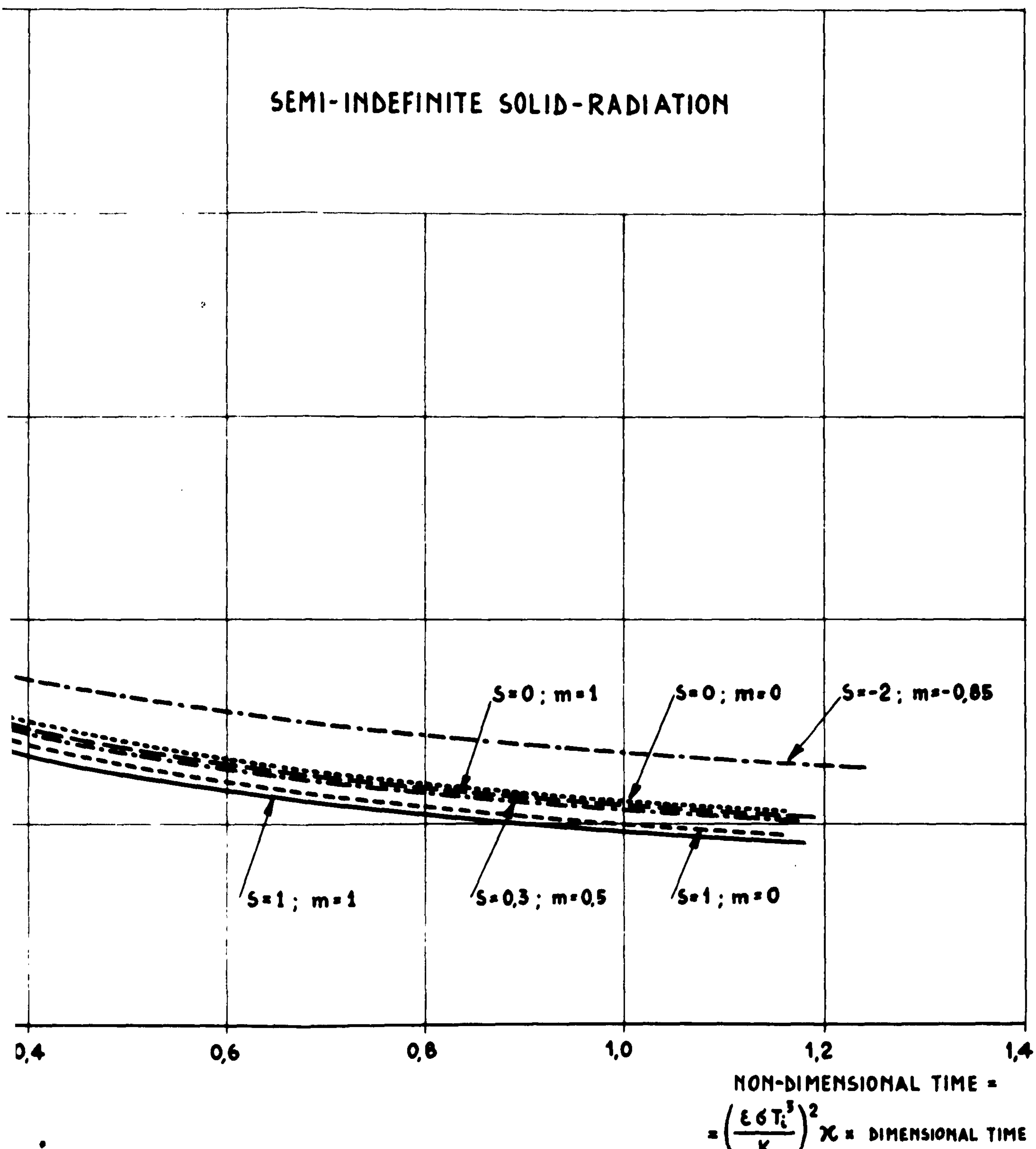
Fig. 2

2



SEMI-INDEFINITE S
NON-DIMENSIONAL HEAT FLUX
1,0
0,8
0,6
0,4
0,2
0
0,2
0,4
0,6
0,8
S=1 ; m=1

Fig. 3



SEMI-INDEFINITE SOLID-RADIATION
S=0 ; m=1
S=0 ; m=0
S=-2 ; m=-0,85
S=1 ; m=1
S=0,3 ; m=0,5
S=1 ; m=0
0,4
0,6
0,8
1,0
1,2
1,4
NON-DIMENSIONAL TIME =
$= \left(\frac{\varepsilon \sigma T_i^3}{K}\right)^2 \chi \times$ DIMENSIONAL TIME

Fig. 3

2

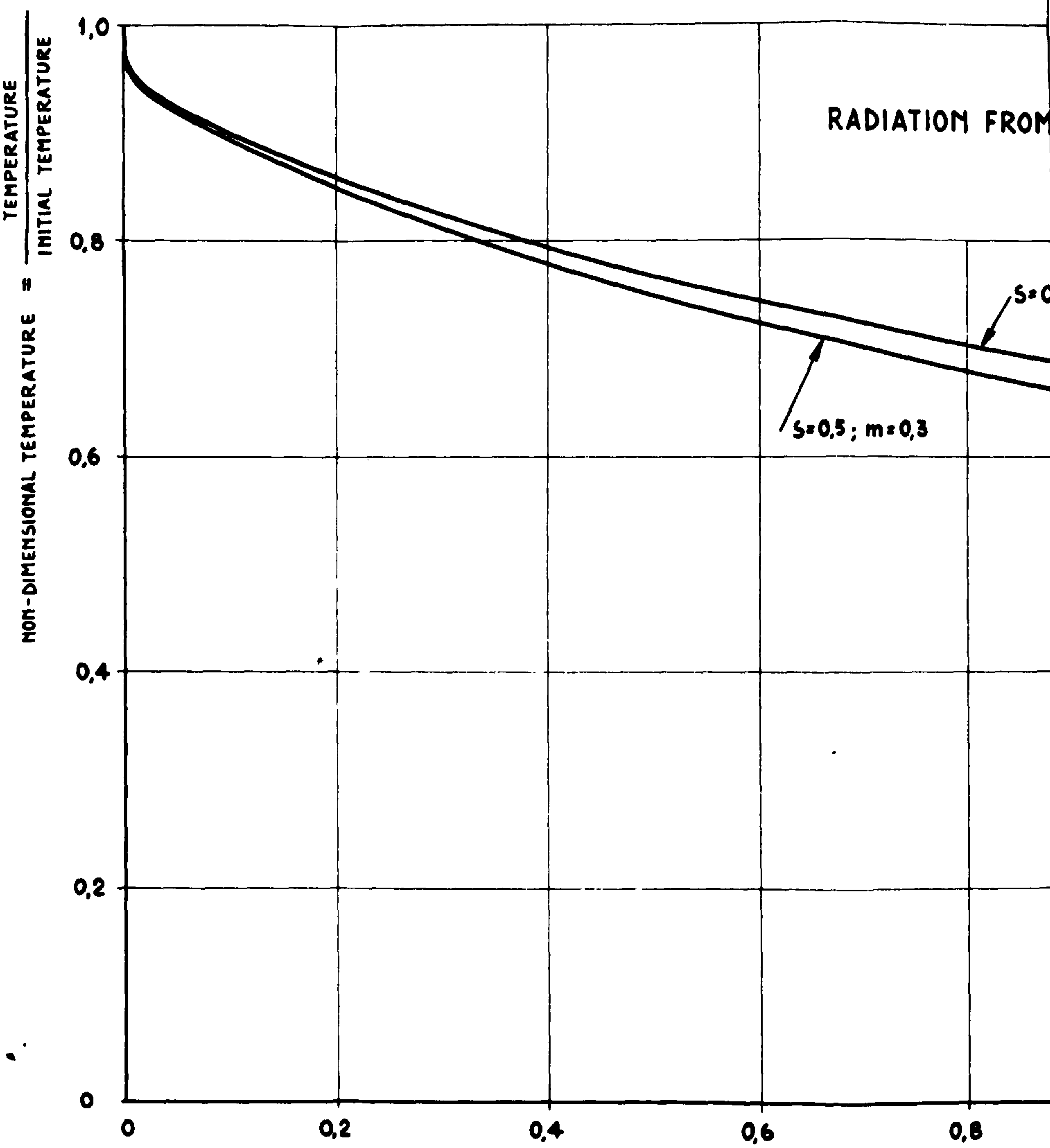
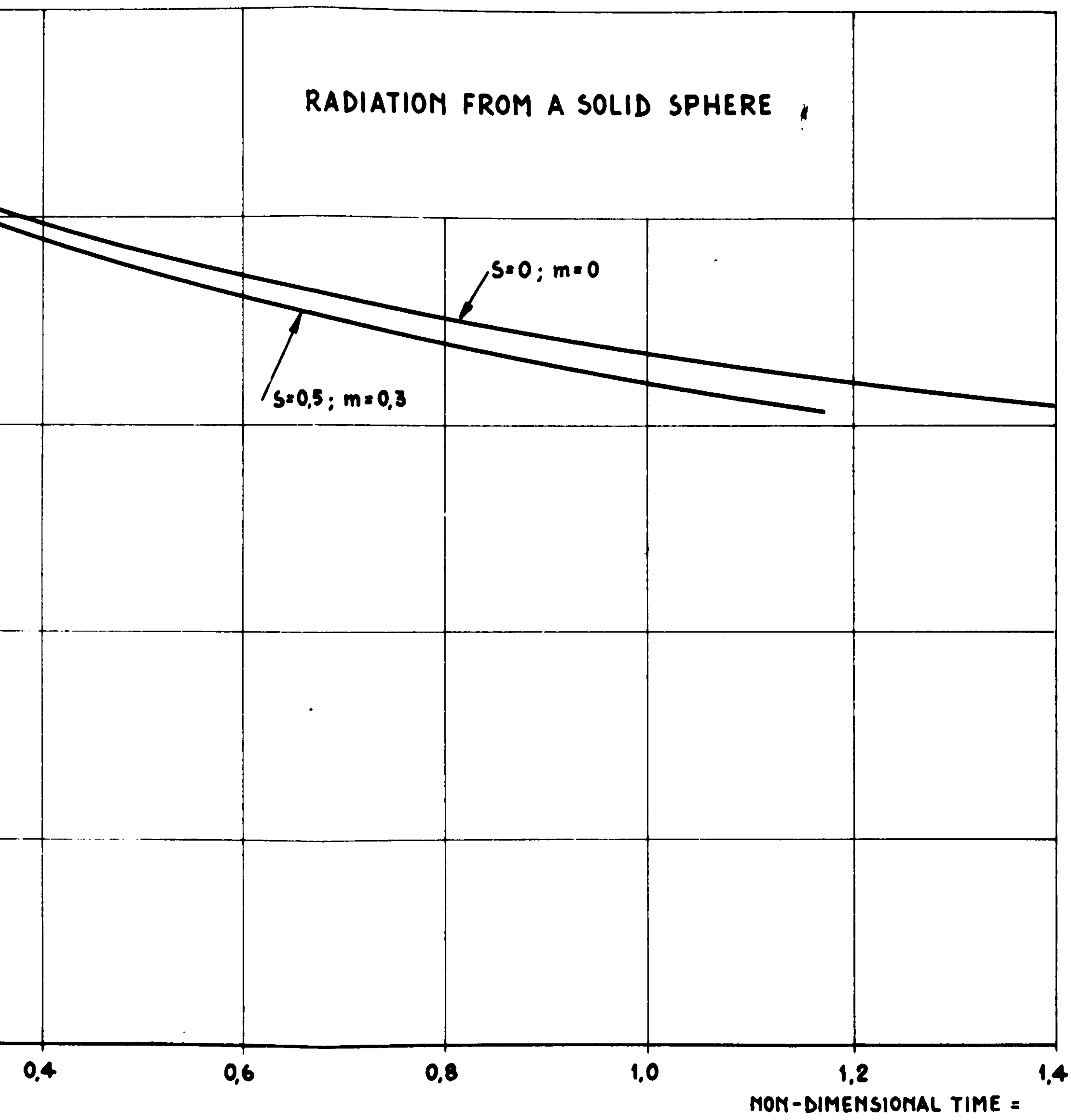

RADIATION FROM
S=0
S=0,5 ; m=0,3
NON-DIMENSIONAL TEMPERATURE = TEMPERATURE / INITIAL TEMPERATURE
1,0
0,8
0,6
0,4
0,2
0
0
0,2
0,4
0,6
0,8

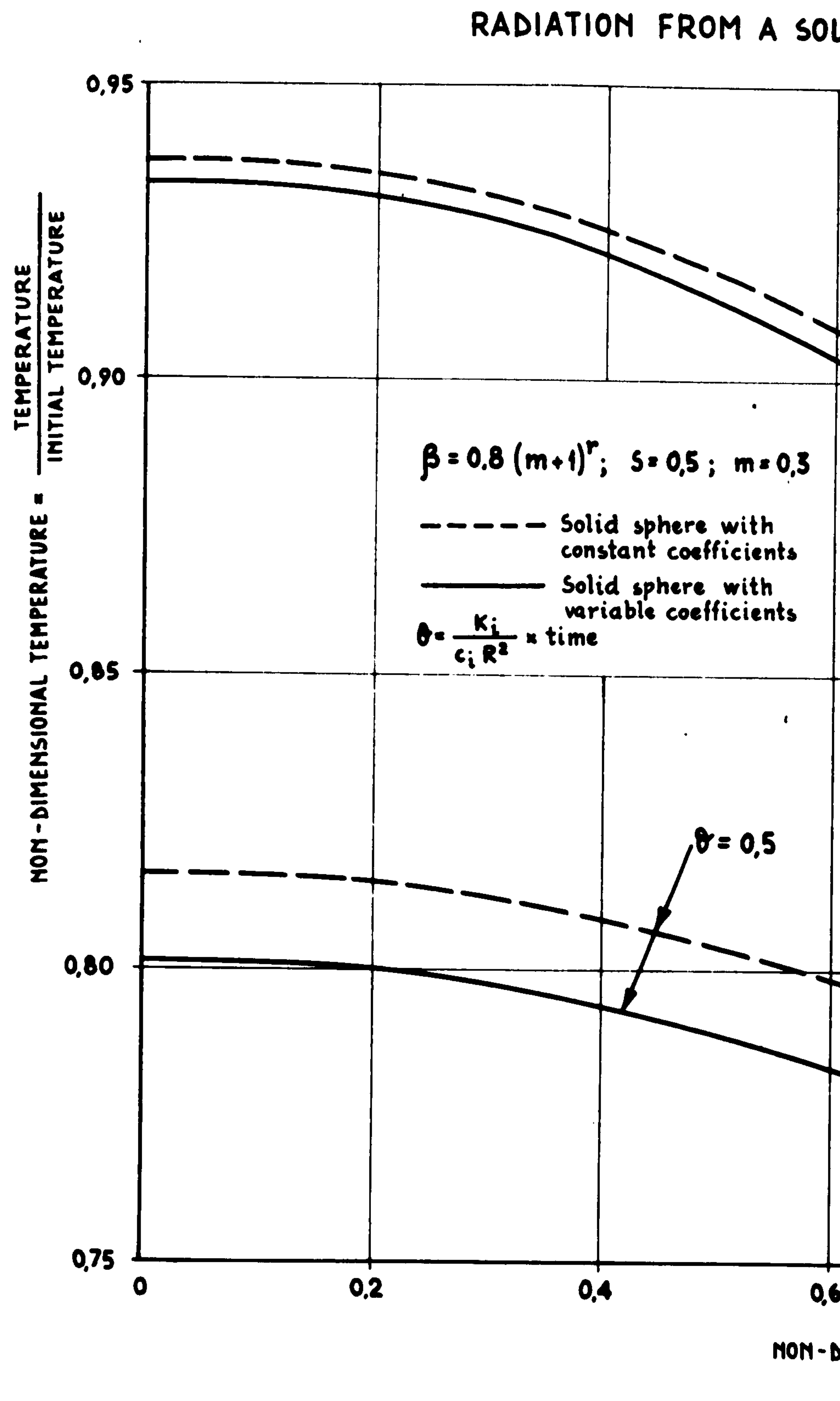
fig. 4



RADIATION FROM A SOLID SPHERE
S=0 ; m=0
S=0,5 ; m=0,3
0,4
0,6
0,8
1,0
1,2
1,4
NON-DIMENSIONAL TIME =
$= \frac{K_i}{c_i R^2} \times$ TIME

fig. 4

2



RADIATION FROM A SOL
0,95
0,90
0,85
0,80
0,75
NON-DIMENSIONAL TEMPERATURE = TEMPERATURE / INITIAL TEMPERATURE
β = 0,8 (m+1)^r; S = 0,5; m = 0,3
Solid sphere with constant coefficients
Solid sphere with variable coefficients
θ = K_i / (c_i R²) × time
θ = 0,5
0
0,2
0,4
0,6
NON-DI

Fig. 5

RADIATION FROM A SOLID SPHERE

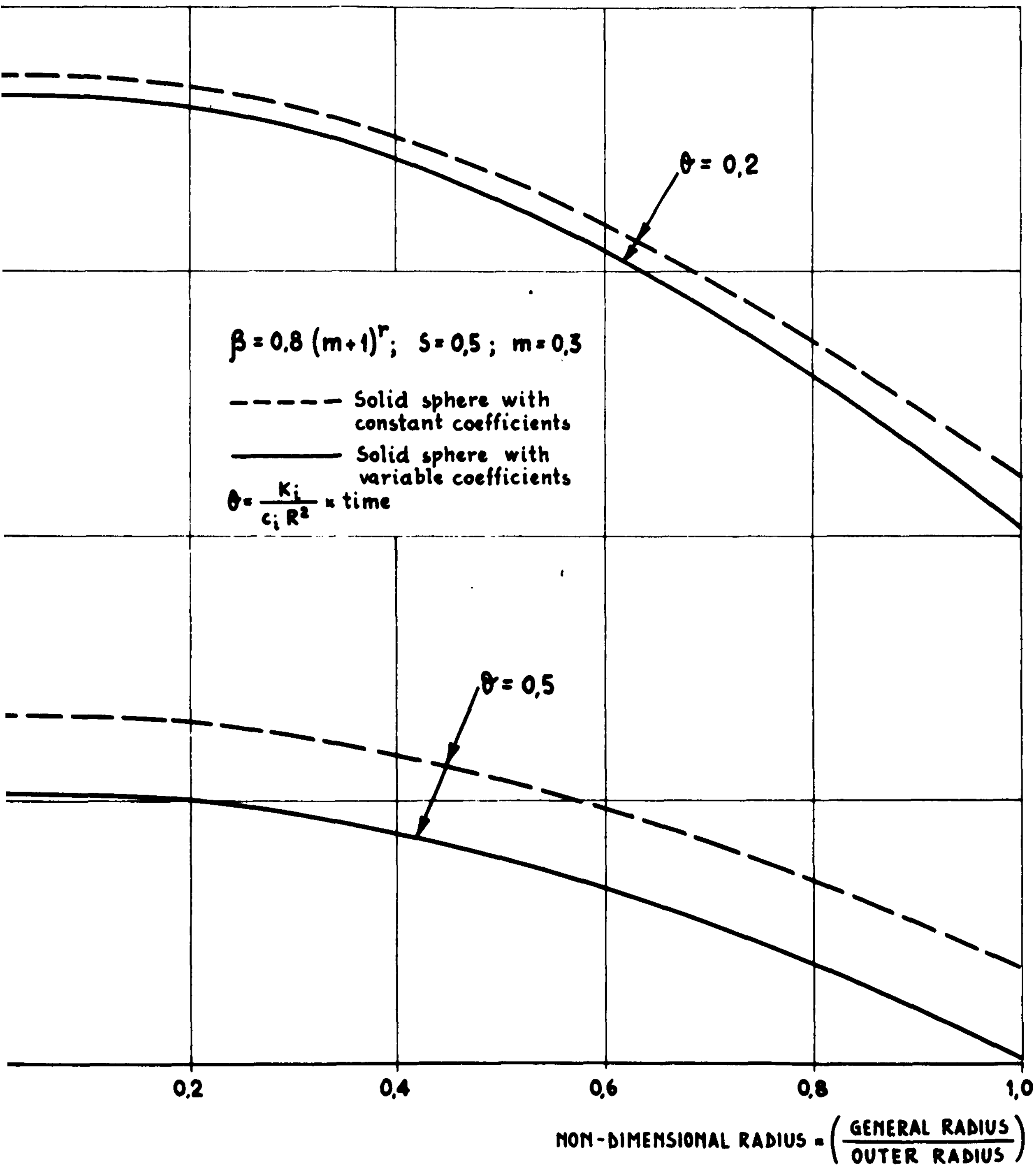


Fig. 5

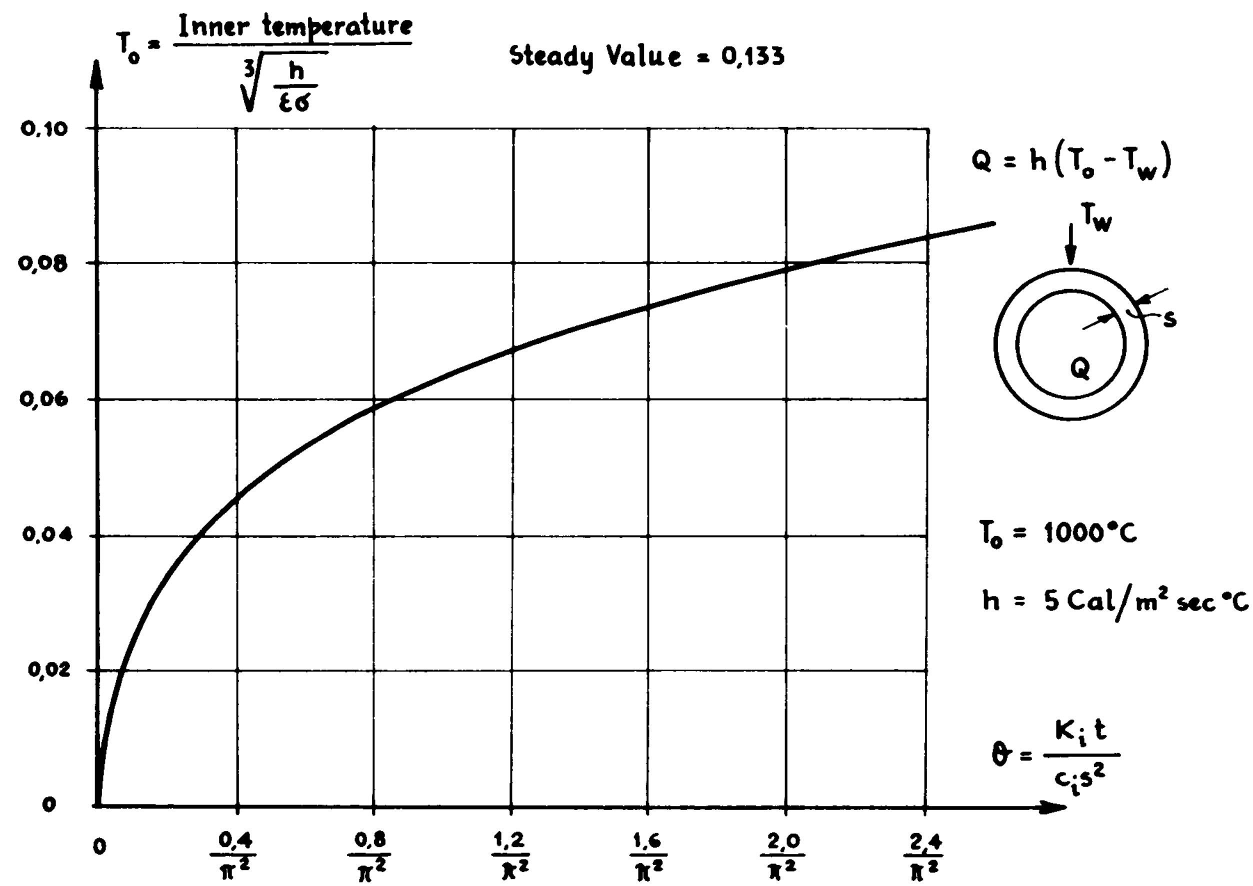


Fig. 6 - Radiating cylindrical shell-temperature on hot side.