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ASD-TDR-62-1005

GCA TECHNICAL REPORT NO. 62-24-A

INVESTIGATION OF PHYSICAL PHENOMENA FOR THE NAVIGATION OF SPACE VEHICLES AND EARTH SATELLITES

TECHNICAL DOCUMENTARY REPORT ASD-TDR-62-1005

DECEMBER 1962

AERONAUTICAL SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

PROJECT NO. 9(610-5219)

TASK NO. 50865

PREPARED UNDER

CONTRACT NO. AF 33(616)-7413

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GEOPHYSICS CORPORATION OF AMERICA
BEDFORD, MASSACHUSETTS

NOTICES

FOREWORD

This study was undertaken as part of an investigation of navigation within the solar system by optical means. The objective of the investigation is to evaluate the suitability of various physical phenomena as sources of navigational information and to estimate the accuracy of navigational information obtained by various techniques. The work was supported under Contract No. AF 33(616)-7413 by the Navigation and Guidance Laboratory, Aeronautical Systems Division, Air Research and Development Command, United States Air Force. Mr. William Harmon was project monitor for the Navigation and Guidance Laboratory. The Contractor's report number is GCA 62-24-A. This is the final report under AF 33(616)-7413.

ABSTRACT

(1) Available information concerning the moon, which is likely to be of interest to designers of navigational aids for lunar vehicles, is given. This includes data on phases, spectral distribution, size and figure and the intensity of Lyman alpha reflected from the moon.

(2) Preliminary design and operation of an automatic sextant for use in interplanetary vehicles is discussed. The sextant is capable of measuring angles, usually not exceeding 5°, with a precision of 0.2".

(3) Various aspects of satellite navigation by terrestrial occultations of stars are discussed. A condition for occultation in terms of stellar coordinates and the orbital elements of the satellite is derived. Also an occultation equation relating the time of occultation to the orbital elements is derived. The expected frequency of occultations is discussed. A method of "virtual occultation" using extinction of starlight in the atmosphere is discussed. An error of approximately ½ km in the satellite position measurement is anticipated due to unpredictable variations of atmospheric density. Finally the interference due to the brightness of earth's atmosphere is discussed.

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SECTION 1

INTRODUCTION

The purpose of this investigation is to study various physical phenomena which may be suitable sources of navigational information for space vehicles within the solar system, including the earth satellites. The scope is limited to phenomena which can be studied at optical wavelength by means of instruments which are totally contained in the space vehicle. During the year 1960-1961, eleven technical reports were prepared and published and the results were summarized in the Final Technical Documentary Report entitled "Investigation of Physical Phenomena for Space Navigation" by Stubbs (for a complete list see Bibliography).

This investigation was continued during 1961-1962, and the topics studied fall into three categories: (1) lunar photometry and Lyman-alpha intensity near the lunar limb, (2) theory and design of an automatic sextant for space navigation and (3) satellite navigation by terrestrial occultations of stars. Detailed results of these studies have already been published in the form of six technical reports. The purpose of this final report is to summarize these results. For details as well as for bibliographical material the reader is referred to the original technical reports.

Manuscript released by the author December, 1962 for publication as an ASD Technical Documentary Report.

SECTION 2

LUNAR PHOTOMETRY

Levy (GCA Tech. Rep. 62-10-A) has summarized the available information on the phases, spectral distribution and size and figure of the moon. Figure 2.1 and 2.2 give the total brightness and the limb brightness of the moon at various phases. The changes in brightness with the phase can be seen to be very rapid. At quarter phase when the moon appears practically as a semi-circle its brightness is roughly 1/9 times the brightness of the full moon. The brightness of any feature on the moon is proportional to

$$j \sim \frac{\cos i \; I \text{ (limb)}}{\cos i + \lambda \cos \epsilon} \qquad (2.1)$$

where i and ϵ are the angles of incidence and reflection respectively. $\lambda = 0.225 \, (1 + \tan^2 \frac{\alpha}{2})$ and I (limb) is the intensity at the limb.

Even the hemisphere of the moon which is in the sun's shadow is faintly illuminated by the sunlight reflected by the earth. This is particularly noticeable near the new moon, and it is possible to measure the lunar limb by this so-called "ashen light". Optimally the surface brightness of the earthlit moon is 9,100 times less than that of the sunlit full moon.

Since the terminator of the moon is usually jagged, all measurements should preferably be made at the limb and not at the terminator.

The reflectivity of the moon has been measured at several optical wavelengths and is found to increase gradually from the blue to the red region of the spectrum. As a result the relative spectral intensity distribution of the moonlight (Figure 2.3) is not exactly the same as for the sunlight, but resembles the latter fairly well.

The figure of the moon in the past was thought to be roughly a triaxial ellipsoid, but recent measurements due to Schrutka-Rechenstamm and Hopkins indicate that within the mean error of the altitude measurements used, ±1.2 km, the best figure for the moon is that of a sphere of radius

$$a_o = 1738.1 \pm 0.2 \text{ km} . \qquad (2.2)$$

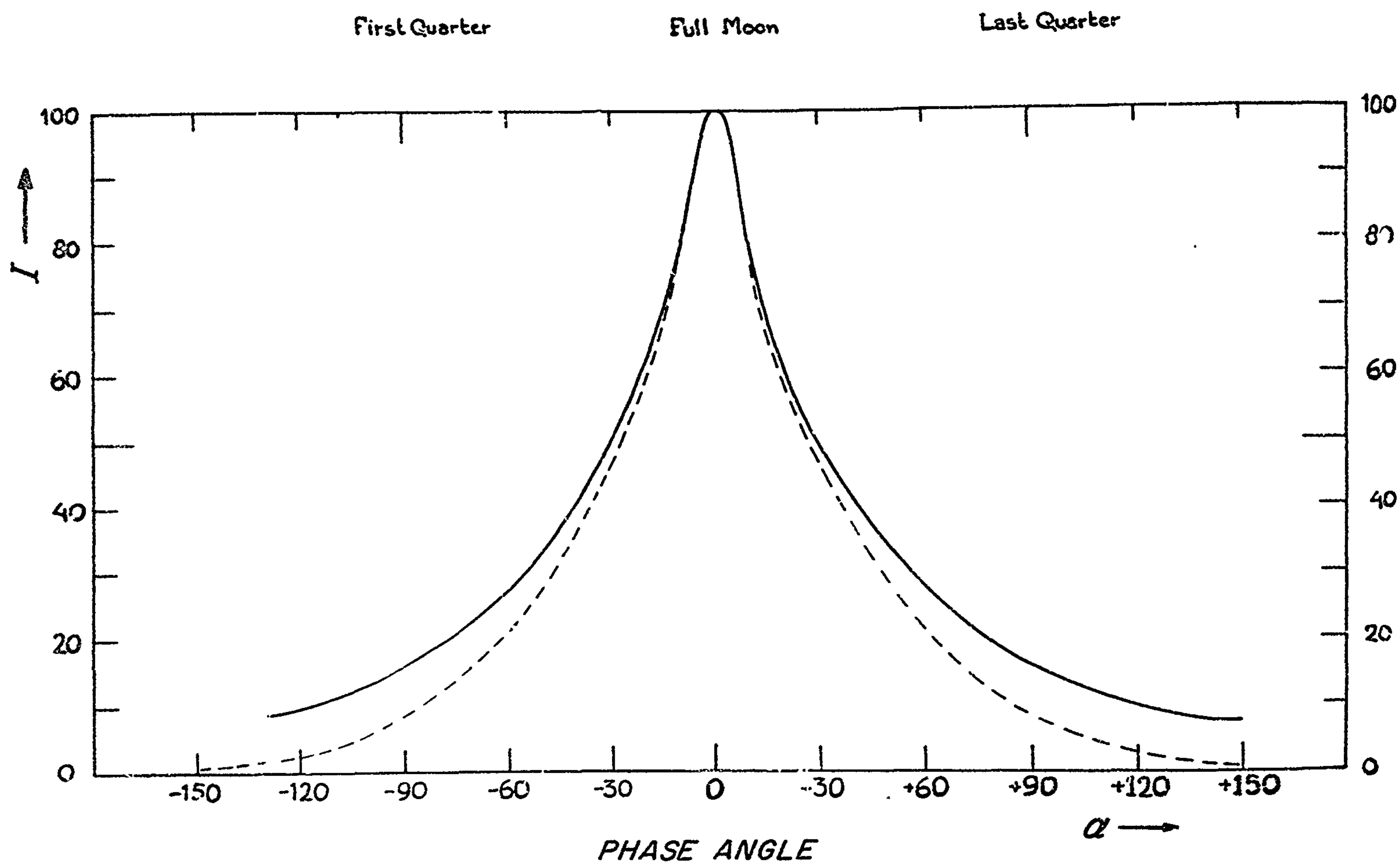


Figure 2.1. Phase variation of (a) total moonlight (dashed curve), (b) average brightness of illuminated fraction (solid curve); both relative to full phase.

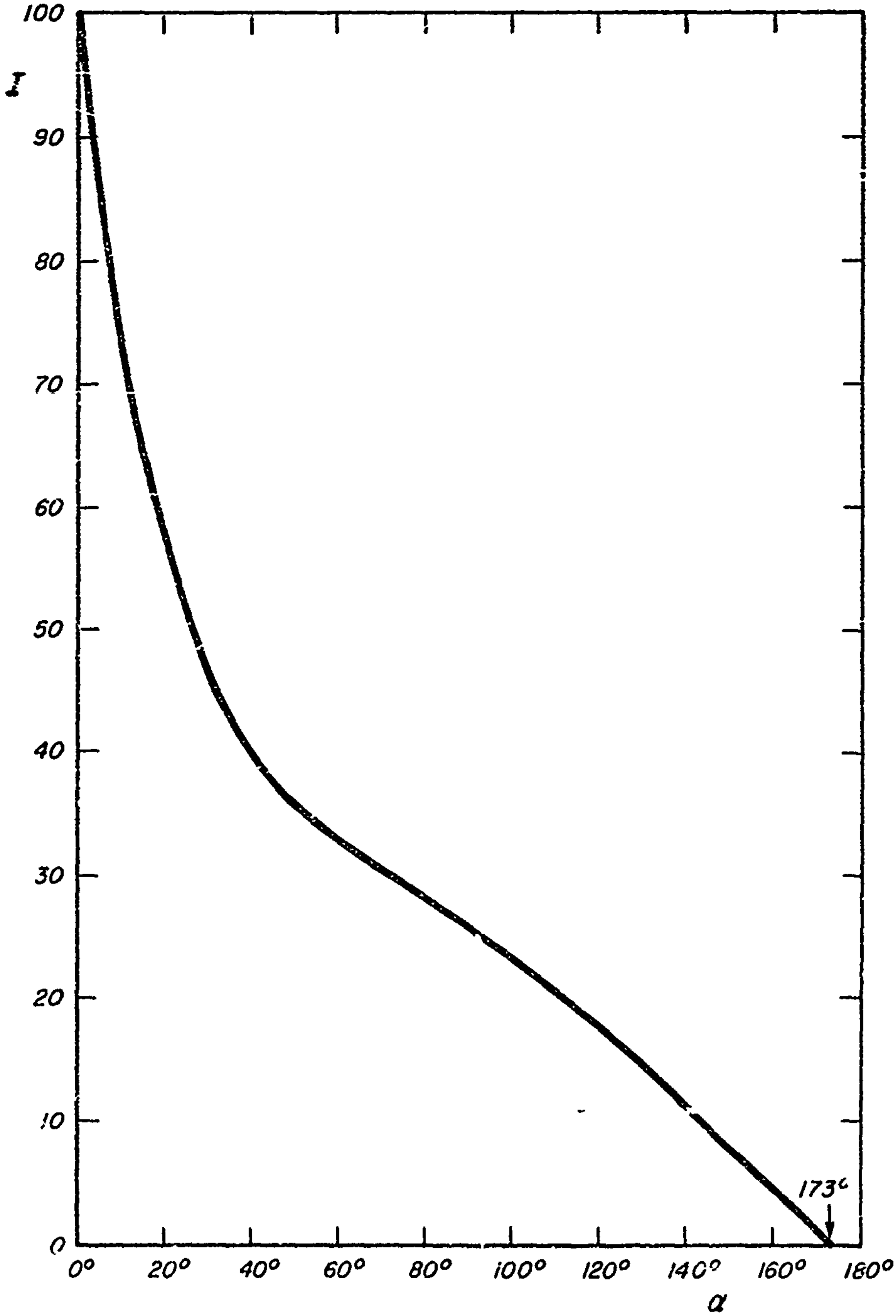


Figure 2.2. Phase variation of lunar limb brightness, according to Danjon.

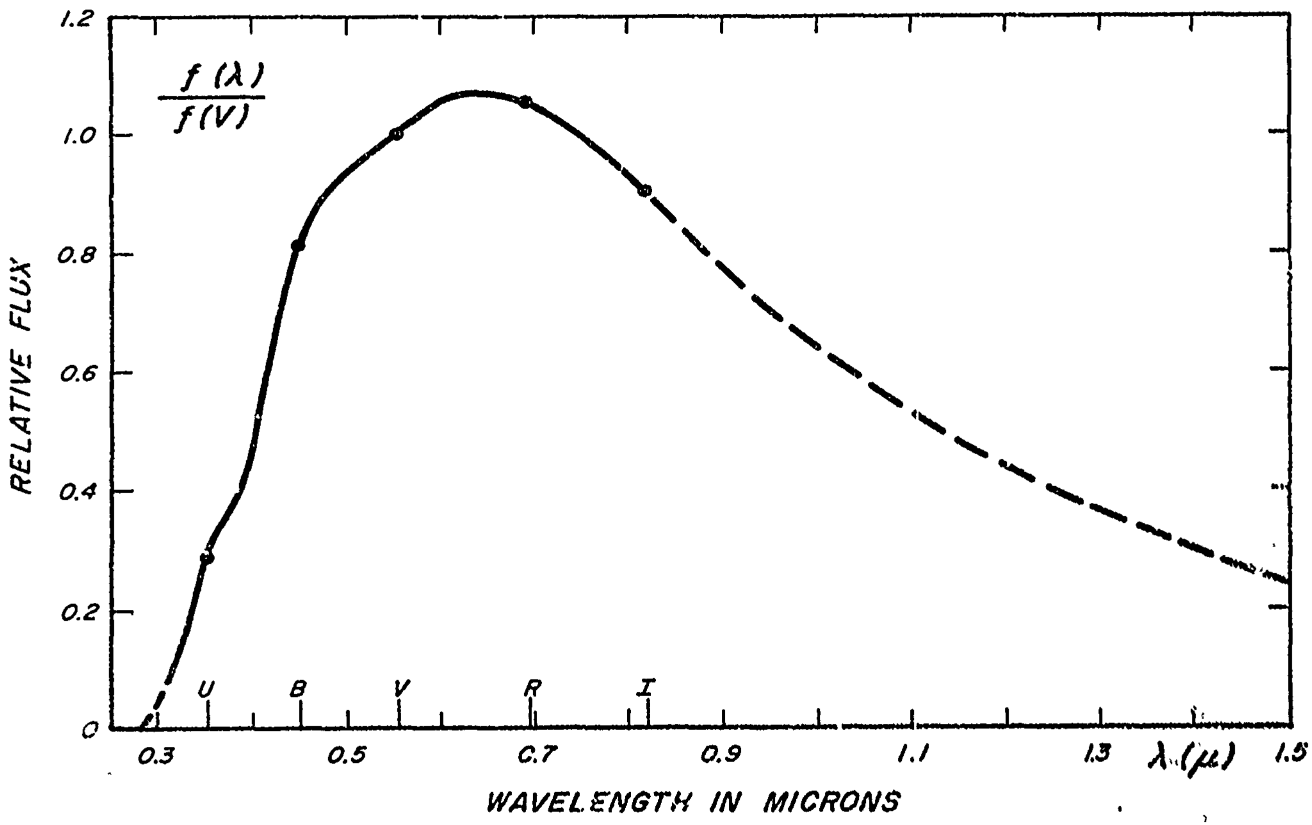


Figure 2.3. Spectral energy distribution of moonlight (relative to energy at $\lambda_V \approx 5540Å$).

Superimposed on this sphere are local elevations and depressions of up to ±5 km, with ±3 km quite common. The lunar maria are an average of 2 km lower than the mean level, continents 2 km higher.

SECTION 3

LYMAN-ALPHA AND THE LUNAR LIMB

Levy (GCA Tech. Rep. 62-13-A) has made rough calculations of the intensity of the solar Lyman-alpha radiation reflected by the moon. He finds an upper limit for the lunar limb brightness in Lyman-alpha to be 4×10^{-4} ergs cm^{-2} sec^{-1} $ster^{-1}$. The Lyman-alpha glow of the night sky, according to measurements of Kupperian et al, is 2.7×10^{-3} ergs cm^{-2} sec^{-1} $ster^{-1}$ in the antisolar direction and slightly higher in other directions. In the Lyman-alpha the moon will, therefore, appear silhouetted against a brighter night sky background.

SECTION 4

AN AUTOMATIC SEXTANT FOR SPACE NAVIGATION

The theory, preliminary design and operation of an automatic sextant has been discussed by Stubbs (GCA Tech. Rep. 62-17-A). The sextant is expected to measure angles, usually not exceeding 5 degrees, with an accuracy of 0.2" (1 microradian). This order of accuracy is achieved by using a prism pair, capable of rotation around a common axis. Large relative rotations of the prism, which can be measured with good precision by the usual mechanical methods, produce very small deflections of the light beam. This is illustrated in Figure 4.1 for a prism of 1.5° deflection angle. In this manner up to 60 times improvement in accuracy can be achieved over that of the usual mechanical methods. Figures 4.2 and 4.3 illustrate the arrangement of the optical components of the sextant, which consists of two telescopes mounted parallel to each other on the same rigid platform. Relative rotations of the two prism pairs bring the images of the star and the planet at the center of each of the trihedral mirrors. The angle between the star and the planet is calculated from the relative rotations of the prisms.

The design requirements for an overall accuracy of 1 microradian in the measurement are summarized below:

Deflection of each element of prism pair	3°
Prism rotation controlled to	8"
Telescope axis controlled to	20"
Maximum angle between light sources	5°
Minimum angle between light sources	30'
Parallelism of the two telescopes	0.2" (1 microradian)

The objective of the program is the accurate measurement of four celestial angles; the angles between a first planet and each of two nearby stars and a second planet and each of two nearby stars. Knowing these four angles and the angle between the two stars it is possible to derive navigational information. The identification of individual stars is not required. All that is necessary is to know the angle between the two stars. This information concerning nearly 5,300 stars will be carried in the memory of the computer. It turns out that the angles between all possible combinations of stars are not needed. It is sufficient to store in the computer memory the angles between three times as many pairs as the number of stars chosen, namely about 16,000 angles This requirement can be met easily.

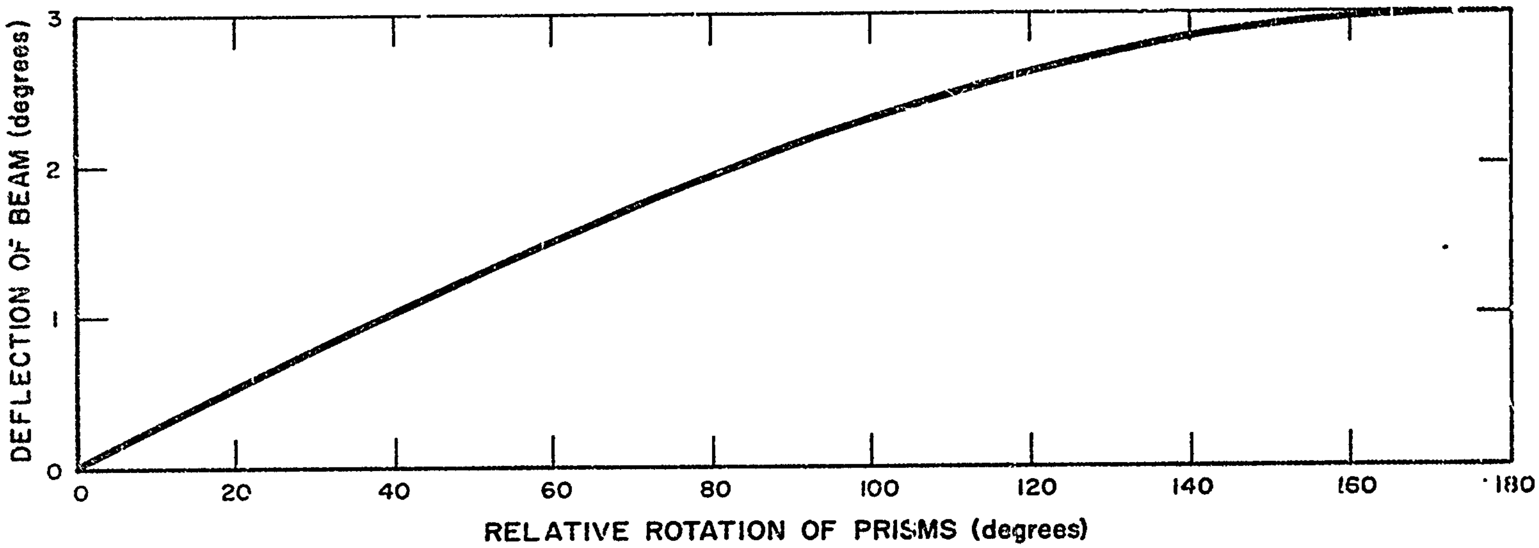
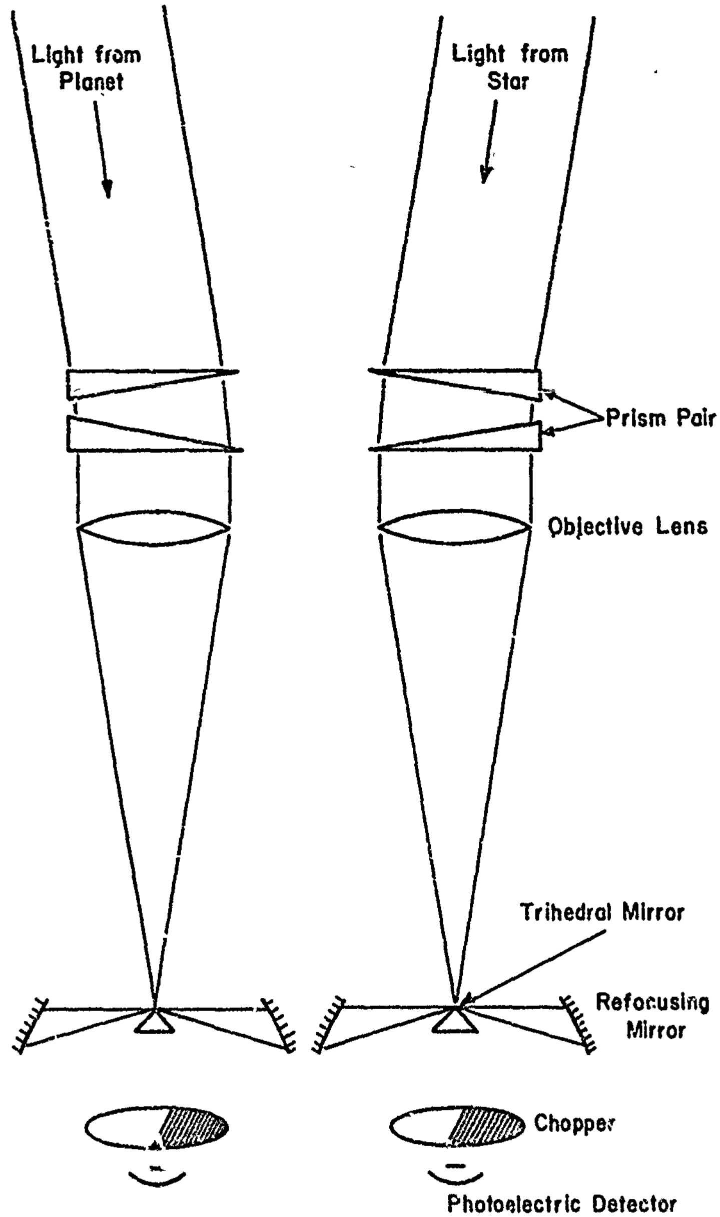


Figure 4.1. Deflection of light beam as a function of relative rotation angle of two identical prisms of 1.5° deflection angle.



NOTE: The trihedral mirrors and refocusing mirrors have three-fold symmetry

Figure 4.2. Arrangement of optical components.

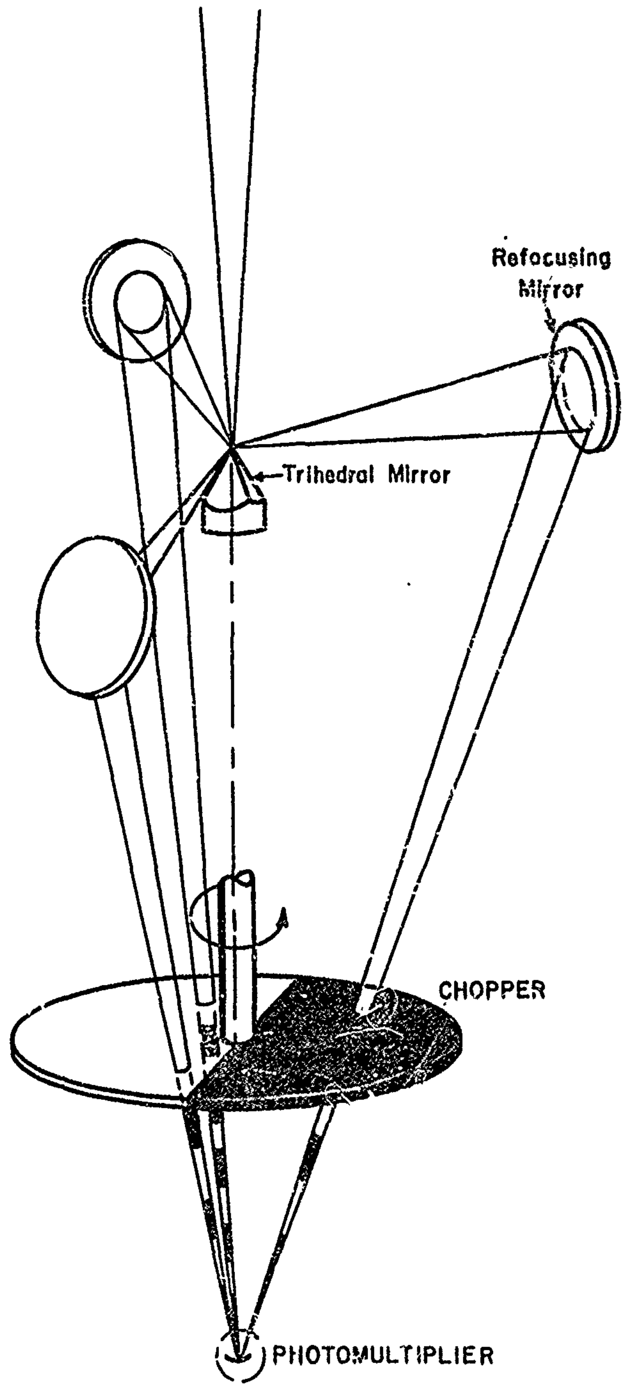


Figure 4.3. Arrangement of optical components.

SECTION 5

SATELLITE NAVIGATION BY TERRESTRIAL OCCULTATION OF STARS

Let r_i denote the impact parameter of a ray in a parallel beam of starlight, which is received by the satellite at an instant of time t. For an undeflected ray (no refraction) r_i equals the distance of closest approach of the ray measured from the earth's center.

The ingress or egress in a terrestrial occultation of a star is characterized by the condition that the impact parameter r_i equals the radius of the earth a_o. The signal for the occurrence of ingress (or egress) is provided by the disappearance (or re-appearance) of the star at the edge of the earth. This method, which works well for lunar occultations, however, fails for terrestrial occultations, primarily on account of the fact that the star can just as easily disappear behind an absorbing cloud, whose unknown height above the earth's surface may be as much as 18 km, as behind the true edge of the earth.

This difficulty can be surmounted if we consider a ray with a known impact parameter, say (a_o + 20) km, thus avoiding the unpredictable clouds. For a spherically symmetric atmosphere and satellites above 100 km, it turns out that deflection and extinction of the light beam, produced in the atmosphere, are, for all practical purposes, functions of the impact parameter only. The impact parameters can therefore be calculated from the change in direction or intensity of light. We will call the latter method, the method of "virtual occultations".

The following equation (GCA Tech. Rep. 62-18-A) holds between the satellite orbital elements, the equatorial coordinates, α and δ, of the star and the time of observation t.

$$
\begin{aligned}
-\left[1 - \left(\frac{r_i(1 + e\cos v)}{a(1 - e^2)}\right)^2\right]^{\frac{1}{2}}
&= \cos i \cos\delta \sin(\alpha - \Omega)\sin(v + \omega) \\
&\quad + \sin i \sin\delta \sin(v + \omega) \\
&\quad + \cos\delta \cos(\alpha - \Omega)\cos(v + \omega) \qquad (5.1)
\end{aligned}
$$

where a, e and i are respectively the semi major-axis, eccentricity and inclination to the equatorial plane of the satellite orbit. v, called the true anomaly, is a known function of a, e, T (time of perigee passage) and the time of observation t. r_i is the impact parameter of the observed ray. For a true occultation r_i equals the earth's radius a_o, and for a "virtual occultation" it is derived from changes in direction, intensity, etc., of the light ray.

Six observations of occultation time for different stars of known coordinates are required to obtain the six orbital elements, but due to numerous uncertainties a much greater number is recommended.

In most cases an approximate orbit of the satellite is known; sometimes it is the pre-determined orbit before the launch. The condition that an occultation of a star, whose coordinates are α and δ, will occur sometime during an orbital revolution of the satellite is given by

$$\frac{a}{r_i}(1 - e^2)(\cos i \sin \delta - \sin(\alpha - \Omega) \sin i \cos \delta)$$

$$+ e \cos \tan^{-1} \frac{\sin i \sin \delta + \sin(\alpha - \Omega) \cos i \cos \delta}{\cos(\alpha - \Omega) \cos \delta}$$

$$\leq 1 \quad . \qquad (5.2)$$

(For derivation see GCA Tech. Rep. 62-18-A). This equation makes it possible to calculate the stars which will be occulted and the number of such stars.

In Figure 5.1 the number of stars required (and the corresponding limiting stellar magnitudes) are plotted against the radius of the satellite orbit for 25 occultations per period (N_{25}) and per hour (N^*_{25}) for a circular orbit and a uniform distribution of stars over the sky. This figure does not take two effects into account. First the occultation observations must be confined to the night side of the earth, and second, the starlight, at the time of observation, is reduced in intensity, by say 50%, due to atmospheric extinction. The net effect is that the scale giving the number of stars in Figure 5.1

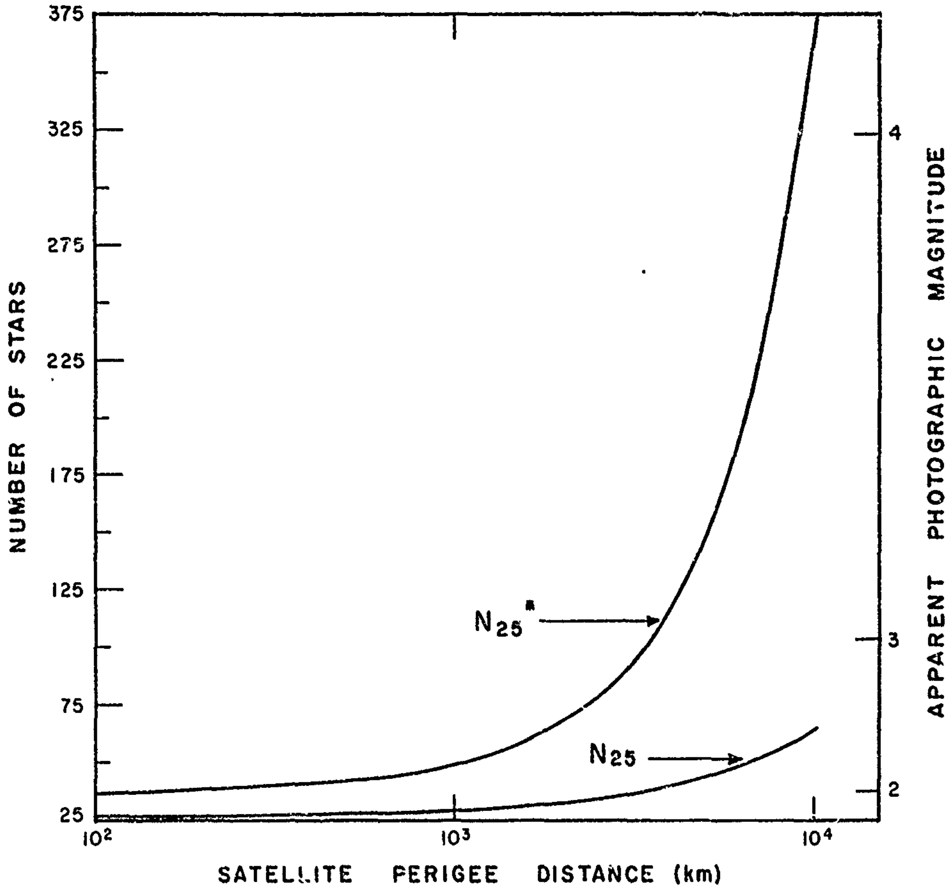


Figure 5.1. Number of stars required and the corresponding apparent photographic magnitudes for 25 occultations per period (N_{25}) and per hour (N^*_{25}) for a circular orbit ($e = 0$).

should be multiplied by two and 1.4 should be added to the magnitude scale. These changes are incorporated in Table 5.1, which gives the number of stars and the corresponding limiting magnitudes for circular as well as elliptical orbits for two values of perigee distances.

The interference due to brightness of the earth's atmosphere is discussed in GCA Tech. Rep. 62-22-A from which Table 5.2 is reporduced. The interference due to day airglow, noctilucent clouds and Rayleigh scattering of sunlight can be ignored if occultation observations are confined to the night side of the earth. The interference due to aurorae can be large at times, but is variable. The interference due to night airglow and scattered moonlight, particularly near full moon, are both quite large and should be further investigated. These sources of atmospheric radiance can be partially eliminated by using techniques in which the atmospheric radiance is subtracted from the total radiance due to the star and the atmosphere. Nevertheless, night airglow and scattered moonlight may be quite troublesome in occultation observations.

The method of measuring extinction of starlight to derive information regarding the impact parameter of the light ray is preferred over the method in which the deflection of the ray, due to refraction, is measured. This method is discussed in GCA Tech. Rep. 62-21-A. The wavelength region 0.3 to 0.55 μ is chosen for extinction measurements, since the atmosphere has negligible true absorption in this region, and extinction can be predicted rather well. Figure 5.2 shows percentage extinction of the atmosphere as a function of impact parameter for two wavelengths (0.375 and 0.520 μ). Seasonal and latitudinal variations of the atmospheric density have not been considered in these calculations. This and several other improvements, listed in the above-mentioned technical report, are required. After various suggested improvements have been incorporated in the calculations, the major source of error is expected to be the deviance of the actual atmospheric densities from the standard model used. The error in position measurement of the satellite due to these deviations is of the order of 1/2 km.

TABLE 5.1

NUMBER OF STARS REQUIRED AND THE CORRESPONDING INSTRUMENTAL LIMITING MAGNITUDE FOR 25 OCCULTATIONS PER PERIOD (N_{25}) AND PER HOUR (N^*_{25}) FOR A FEW SELECTED ORBITS (INCLUDING ATMOSPHERIC EFFECTS)

Perigee Height h_p = 200 km

Eccentricity	semi-major axis a (km)	Period (hours)	Number of Stars N_{25}	Limiting Magnitude (photographic)	N^*_{25}	Limiting Magnitude (photographic)
0	6578	1.475	52	3.0	76	3.3
.01	6644	1.497	52	3.0	78	3.3
.5	13156	4.171	78	3.4	322	4.8

Perigee Height h_p = 2000 km

Eccentricity	semi-major axis a (km)	Period (hours)	Number of Stars N_{25}	Limiting Magnitude (photographic)	N^*_{25}	Limiting Magnitude (photographic)
0	8378	2.106	66	4.2	140	4.0
.01	8463	2.152	66	4.2	144	4.0
.5	16756	5.995	98	4.5	590	5.3

TABLE 5.2

ATMOSPHERIC RADIANCE IN THE 0.3 TO 0.5μ REGION

(COMPARISONS WITH TWO BRIGHT STARS)

Source	Radiance N (watts/cm^2 ster)	Stellar Signal to Atmospheric Noise Ratio S/N (field-of-view = 10^{-5} ster) Sirius*	Betelgeuse*	m_{pg} for S/N = 1 Uncorrected	For Atmospheric Extinction Corrected
Airglow (night)	2.9×10^{-9}	150	3.8	3.9	3.1
(day)	3.1×10^{-7}	1.4	0.036	-1.2	-2.0
Noctilucent Cloud	4.0×10^{-7}	1.1	0.028	-1.4	-2.2
Aurora	3.5×10^{-8}	13	0.31	1.2	0.4
Rayleigh Scattering					
(Sun)	3×10^{-3}	1.5×10^{-4}	3.7×10^{-6}		
(Full Moon)	6×10^{-9}	73	1.8	3.1	2.3

Stellar Irradiance

Sirius = 4.4×10^{-12} watts/cm^2

Betelgeuse = 1.1×10^{-13} watts/cm^2

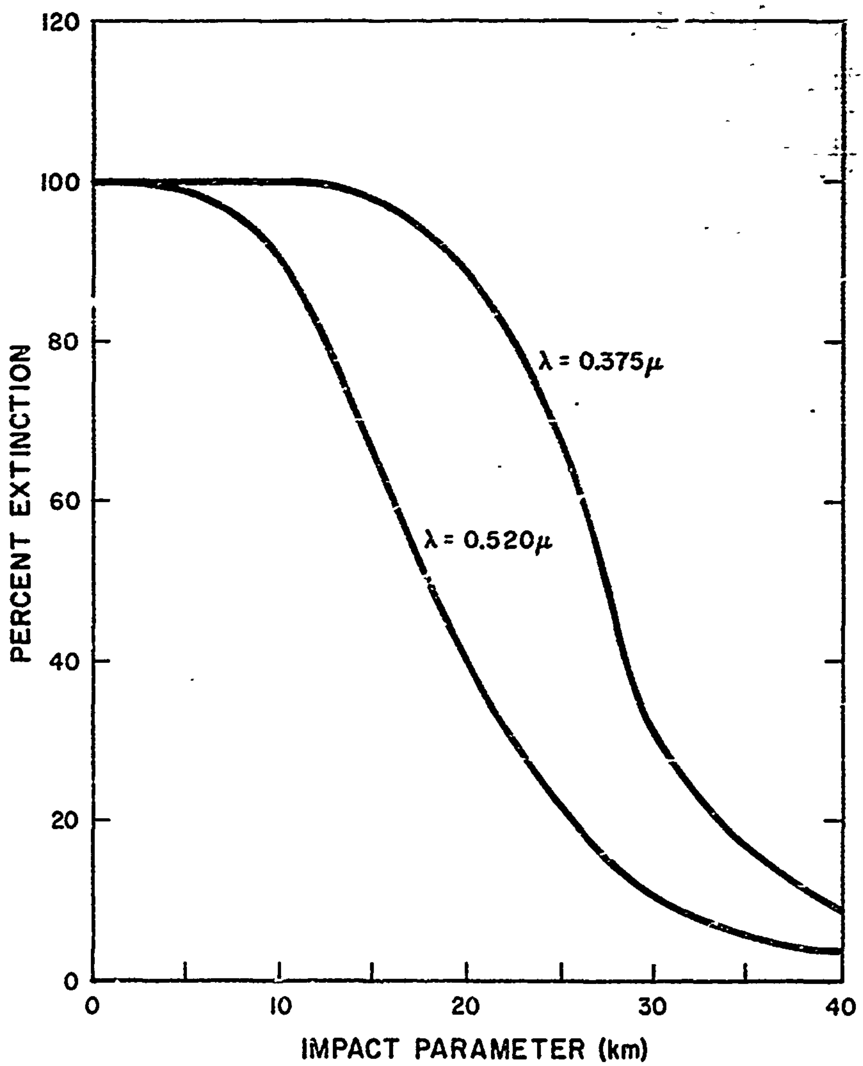


Figure 5.2. Extinction due to Rayleigh scattering for various impact parameters and two wavelengths.

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GCA Technical Report No.	Authors and Title
Quarterly Report Sept. 1960	R. Levy and H.E. Stubbs, "The Diameter of the Sun in Interplanetary Navigation Calculations"
61-14-A	G. de Vaucouleurs, "The Astronomical Unit of Distance, Solar Parallax and Related Constants"
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61-25-A	G. de Vaucouleurs, "Interplanetary Navigation Studies: Light Centers of Terrestrial Planets"
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62-22-A	R.C. Jones and A.M. Naqvi, "Satellite Navigation by Terrestrial Occultations of Stars III: Interference Due to Brightness of Earth's Atmosphere"

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Aeronautical Systems Division, Dir/Avionics, Navigation and Guidance Lab, Wright-Patterson AFB, Ohio.
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(1) Available information concerning the moon, which is likely to be of interest to designers of navigational aids for lunar vehicles, is given. This includes data on phases, spectral distribution, size and figure, and the intensity of Lyman-alpha reflected from the moon.
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