

UNCLASSIFIED

AD NUMBER

AD116554

CLASSIFICATION CHANGES

TO: unclassified

FROM: confidential

LIMITATION CHANGES

TO:

Approved for public release, distribution unlimited

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors; Specific Authority; 8 Jun 50. Other requests shall be referred to U.S. Dept. of the Air Force, Public Affairs Office/STINFO, Washington, DC.

AUTHORITY

U.S. Dept. of the Air Force via ltr., dtd March 27 1969.; U.S. Dept. of the Air Force via ltr., dtd March 27 1969.

THIS PAGE IS UNCLASSIFIED

UNCLASSIFIED

AD 165544

Armed Services Technical Information Agency

Reproduced by

DOCUMENT SERVICE CENTER

KNOTT BUILDING, DAYTON, 2, OHIO

This document is the property of the United States Government. It is furnished for the duration of the contract and shall be returned when no longer required, or upon recall by ASTIA at the following address: Armed Services Technical Information Agency, Document Service Center, Knott Building, Dayton 2, Ohio.

...rished for the du-
...on recall by ASTIA
...nation Agency,
...hio.

NOT TO BE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY APPLICANT OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MAKE ANY USE OF ALL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERE TO

OR OTHER DATA DEFINITELY RELATED HEREBY INCURS THE FACT THAT THE SUPPLIED THE DATA BY THE HOLDER OR ANY OTHER PERSON OR ANY OTHER TO MAKE ANY USE OF ALL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERE TO

UNCLASSIFIED

UNCLASSIFIED NOT FOR OPEN PUBLICATION

~~CONFIDENTIAL~~

U.S. AIR FORCE

Project

RAND

RESEARCH MEMORANDUM

AVERAGE PERCENT OF TARGET AREA COVERED (U)

A. M. Mood

RM-404

ASTIA Document Number AD 116554

8 June 1950

Copy No. _____

This material contains information affecting the national defense of the United States within the meaning of the espionage laws, Title 18 U.S.C., Secs. 793 and 794, the transmission or the revelation of which in any manner to an unauthorized person is prohibited by law.

SEE TITLE PAGE FOR DISTRIBUTION RESTRICTIONS

This is a working paper. Because it may be expanded, modified, or withdrawn at any time, permission to quote or reproduce must be obtained from RAND. The views, conclusions, and recommendations expressed herein do not necessarily reflect the official views or policies of the United States Air Force.

AD NO 116554
ASTIA FILE COPY

Classification changed to UNCLASSIFIED
changed to
by RAND Corp. dtl 20724.57
Date 7/21/52

The RAND Corporation
SANTA MONICA • CALIFORNIA

UNCLASSIFIED

~~CONFIDENTIAL~~

57AA

UNCLASSIFIED NOW FOR OPEN PUBLICATION

~~CONFIDENTIAL~~

U.S. AIR FORCE

Project

RAND

FC

RESEARCH MEMORANDUM

AVERAGE PERCENT OF TARGET AREA COVERED (U)

A. M. Hood

RM-404

ASTIA Document Number AD 116554

8 June 1950

Copy No. _____

This material contains information affecting the national defense of the United States within the meaning of the espionage laws, Title 18 U.S.C., Secs. 793 and 794, the transmission or the revelation of which in any manner to an unauthorized person is prohibited by law.

SEE TITLE PAGE FOR DISTRIBUTION RESTRICTIONS

This is a working paper. Because it may be expanded, modified, or withdrawn at any time, permission to quote or reproduce must be obtained from RAND. The views, conclusions, and recommendations expressed herein do not necessarily reflect the official views or policies of the United States Air Force.

Classification ~~changed to~~ UNCLASSIFIED

Date 11/20/86
By 2010
Reason: 1957
AD No. 116554
ASTIA FILE COPY

The RAND Corporation

SANTA MONICA • CALIFORNIA

UNCLASSIFIED

~~CONFIDENTIAL~~ FEB 10 1950

57AA

6076

~~CONFIDENTIAL~~

PROJECT RAND

RESEARCH MEMORANDUM

AVERAGE PERCENT OF TARGET AREA COVERED (U)

A. M. Mood

RM-404

ASTIA Document Number AD 116554

8 June 1950

Copy No. _____

UNCLASSIFIED

No restrictions on further distribution, other than those imposed by security regulations.

RAND Corporation

485 AVENUE OF THE STARS, WASHINGTON, D. C. 20007

~~CONFIDENTIAL~~

AVERAGE PERCENT OF TARGET AREA COVERED

A. R. Mood

1. Introduction

This memorandum presents two numerical tables. The first gives the average percent of target area covered at least once for a wide range of bomb pattern sizes and target sizes. The second table is an auxiliary table which makes it easy for one to compute the average percent of target area covered exactly or at least once, twice, three times, etc. Both pattern and target areas are squares, and the number of bombs ranges from one to ten.

2. Basic Formulas

If a square pattern of area P^2 is aimed with normally distributed error at the center of a square target (with the same orientation) of area T^2 , the probability that a point (x, y) in the target will be covered is

$$\int_{y-\frac{1}{2}P}^{y+\frac{1}{2}P} \int_{x-\frac{1}{2}P}^{x+\frac{1}{2}P} \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}(u^2+v^2)} du dv = g(x)g(y)$$

where it is assumed that aiming errors in range and deflection are independent and have the same standard deviation σ .

Supposing n bombs are dropped, the probability that the point (x, y) will be covered exactly k times is

$$\binom{n}{k} [g(x)g(y)]^k [1 - g(x)g(y)]^{n-k}$$

which may be put in the form

$$\begin{aligned} b_n^k(x, y) &= (-1)^{n-k} \binom{n}{k} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} [g(x)g(y)]^{n-1} \\ &= (-1)^{n-k} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{k} \binom{n}{i} [g(x)g(y)]^{n-1}. \end{aligned} \quad (1)$$

If this expression is summed on k from r to n , one obtains the probability that (x, y) is covered at least r times

$$c_n^r(x, y) = \sum_{k=r}^n \sum_{i=0}^{n-k} (-1)^{n-k+i} \binom{n-1}{k} \binom{n}{i} [g(x)g(y)]^{n-1}$$

and by interchanging the order of summation the sum on k is easily determined from the identity

$$\sum_{a=r}^m (-1)^a \binom{m}{a} = (-1)^r \binom{m-1}{r-1}.$$

One finds that

$$\begin{aligned} c_n^r(x, y) &= \sum_{i=0}^{n-r} (-1)^{n+1+r} \binom{n-1-i}{r-1} \binom{n}{i} [g(x)g(y)]^{n-1} \\ &= \sum_{j=r}^n (-1)^{j+r} \binom{j-1}{r-1} \binom{n}{j} [g(x)g(y)]^j. \end{aligned}$$

The expected proportion of the target area covered at least r times is now found by integrating this last expression over the

target area. Letting

$$\sqrt{J_1} = \frac{1}{T} \int_{-\frac{1}{2}T}^{\frac{1}{2}T} [g(x)]^2 dx, \quad (2)$$

the expected r -fold coverage may be written

$$h_n^r = \sum_{i=r}^n (-1)^{i+r} \binom{i-1}{r-1} \binom{n}{i} J_1^i. \quad (3)$$

3. Description of Tables

Table I tabulates h_n^1 defined by equation (3); that is, it gives the expected proportion of target area covered at least once. Table II tabulates J_1 defined by equation (2); this table together with equation (3) enables one to calculate directly the expected proportion of target area covered at least r times. Also, by substitution of J_{n-1} for $[g(x)g(y)]^{n-1}$ on the right of equation (1), one can compute directly the expected proportion of target area covered exactly k times.

The unit of length is taken to be one circular probable error (CEP). If the aiming errors in range and deflection are independent and have the same standard deviation σ , then

$$1 \text{ CEP} = \sqrt{2 \log 2} \sigma \approx 1.177 \sigma.$$

(One mean radial error (MRE) = $\sqrt{\pi/2} \sigma \approx 1.253 \sigma \approx 1.064 \text{ CEP}.$)

Thus the lengths of the target side and pattern side are denoted by

$$t = T/\text{CEP}$$

$$p = P/\text{CEP}$$

in the tables.

TABLE I
Average Percent of Target Area Covered at Least Once

p	t	Number of Bombs									
		1	2	3	4	5	6	7	8	9	10
.0625	.0625	.0009	.0017	.0026	.0034	.0043	.0052	.0060	.0069	.0077	.0086
	.125	.0009	.0017	.0026	.0034	.0043	.0051	.0060	.0069	.0077	.0086
	.25	.0009	.0017	.0026	.0034	.0043	.0051	.0060	.0068	.0077	.0085
	.50	.0008	.0017	.0025	.0033	.0042	.0050	.0058	.0067	.0075	.0083
	1.00	.0008	.0015	.0023	.0031	.0038	.0046	.0054	.0061	.0069	.0077
	2.00	.0006	.0011	.0017	.0023	.0028	.0034	.0040	.0045	.0051	.0056
	4.00	.0002	.0005	.0007	.0009	.0012	.0014	.0016	.0019	.0021	.0023
	8.00	.0001	.0001	.0002	.0002	.0003	.0004	.0004	.0005	.0005	.0006
16.00	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
.125	.0625	.0034	.0069	.0103	.0137	.0171	.0205	.0238	.0272	.0305	.0339
	.125	.0034	.0069	.0103	.0137	.0171	.0204	.0238	.0272	.0305	.0338
	.25	.0034	.0068	.0102	.0136	.0170	.0203	.0237	.0270	.0303	.0336
	.50	.0033	.0067	.0100	.0133	.0166	.0199	.0232	.0264	.0297	.0329
	1.00	.0031	.0061	.0092	.0122	.0153	.0183	.0213	.0243	.0273	.0303
	2.00	.0023	.0045	.0068	.0090	.0112	.0135	.0157	.0179	.0201	.0223
	4.00	.0009	.0019	.0028	.0038	.0047	.0056	.0065	.0075	.0084	.0093
	8.00	.0002	.0005	.0007	.0010	.0012	.0014	.0017	.0019	.0022	.0024
16.00	.0001	.0001	.0002	.0002	.0003	.0004	.0004	.0005	.0005	.0006	
.25	.0625	.0137	.0272	.0405	.0536	.0666	.0793	.0919	.1044	.1166	.1287
	.125	.0137	.0271	.0404	.0536	.0665	.0792	.0918	.1042	.1165	.1286
	.25	.0136	.0270	.0402	.0533	.0661	.0788	.0914	.1037	.1159	.1279
	.50	.0133	.0264	.0394	.0522	.0648	.0772	.0895	.1016	.1136	.1254
	1.00	.0122	.0243	.0363	.0481	.0597	.0712	.0826	.0938	.1049	.1158
	2.00	.0090	.0179	.0268	.0355	.0442	.0528	.0613	.0697	.0780	.0863
	4.00	.0038	.0075	.0112	.0149	.0185	.0222	.0258	.0293	.0329	.0364
	8.00	.0010	.0019	.0029	.0038	.0048	.0057	.0066	.0076	.0085	.0094
16.00	.0002	.0005	.0007	.0010	.0012	.0014	.0016	.0019	.0021	.0023	

TABLE I (cont'd)

Average Percent of Target Area Covered at Least Once

P	t	Number of Bombs									
		1	2	3	4	5	6	7	8	9	10
.5	.0625	.0536	.1043	.1523	.1977	.2407	.2813	.3198	.3563	.3908	.4234
	.125	.0535	.1041	.1521	.1974	.2404	.2810	.3195	.3559	.3904	.4230
	.25	.0532	.1036	.1513	.1965	.2393	.2797	.3181	.3544	.3887	.4213
	.50	.0521	.1015	.1484	.1927	.2348	.2747	.3125	.3483	.3823	.4145
	1.00	.0480	.0937	.1372	.1786	.2181	.2556	.2913	.3252	.3576	.3884
	2.00	.0356	.0698	.1027	.1343	.1648	.1941	.2223	.2494	.2755	.3007
	4.00	.0150	.0296	.0438	.0575	.0710	.0840	.0967	.1091	.1211	.1329
	8.00	.0039	.0077	.0114	.0150	.0185	.0219	.0253	.0285	.0317	.0348
	16.00	.0010	.0019	.0028	.0037	.0045	.0053	.0060	.0068	.0075	.0082
1	.0625	.1970	.3552	.4822	.5842	.6661	.7319	.7847	.8271	.8612	.8885
	.125	.1968	.3548	.4818	.5837	.6656	.7314	.7843	.8267	.8608	.8882
	.25	.1958	.3533	.4799	.5818	.6637	.7295	.7825	.8251	.8593	.8869
	.50	.1921	.3473	.4727	.5740	.6558	.7219	.7753	.8184	.8533	.8815
	1.00	.1782	.3245	.4447	.5434	.6244	.6911	.7458	.7908	.8278	.8583
	2.00	.1349	.2505	.3497	.4349	.5083	.5715	.6261	.6732	.7140	.7494
	4.00	.0593	.1125	.1604	.2036	.2417	.2783	.3108	.3405	.3678	.3930
	8.00	.0156	.0297	.0425	.0540	.0646	.0743	.0831	.0913	.0989	.1059
	16.00	.0039	.0074	.0106	.0135	.0161	.0186	.0208	.0228	.0247	.0265
2	.0625	.5789	.8227	.9253	.9686	.9868	.9944	.9977	.9990	.9996	.9998
	.125	.5784	.8223	.9251	.9684	.9867	.9944	.9976	.9990	.9996	.9998
	.25	.5765	.8206	.9240	.9678	.9864	.9942	.9976	.9990	.9996	.9998
	.50	.5689	.8141	.9198	.9654	.9851	.9936	.9972	.9988	.9995	.9998
	1.00	.5396	.7875	.9016	.9543	.9787	.9901	.9953	.9978	.9990	.9995
	2.00	.4407	.6810	.8147	.8905	.9342	.9599	.9752	.9844	.9901	.9937
	4.00	.2257	.3781	.4860	.5659	.6270	.6752	.7142	.7463	.7732	.7960
	8.00	.0625	.1064	.1391	.1645	.1850	.2022	.2168	.2296	.2409	.2511
	16.00	.0156	.0266	.0348	.0411	.0463	.0505	.0542	.0574	.0602	.0628

TABLE I (cont'd)

Average Percent of Target Area Covered at Least Once

p	t	Number of Bombs										
		1	2	3	4	5	6	7	8	9	10	
4	.0625	.9632	.9986	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.125	.9631	.9986	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.25	.9624	.9986	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.50	.9599	.9984	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.00	.9494	.9974	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.00	.9028	.9890	.9986	.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	4.00	.6899	.8744	.9418	.9708	.9845	.9915	.9951	.9971	.9983	.9990	
	8.00	.2493	.3541	.4169	.4612	.4951	.5225	.5454	.5650	.5821	.5972	
	16.00	.0625	.0889	.1047	.1160	.1246	.1316	.1375	.1426	.1470	.1509	
8	.0625	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.125	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.00	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	4.00	.9974	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	8.00	.8378	.9438	.9766	.9892	.9947	.9972	.9985	.9992	.9995	.9997	
	16.00	.2500	.3063	.3372	.3581	.3738	.3864	.3968	.4056	.4133	.4201	
16	.0625	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.125	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	4.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	8.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16.00	.9171	.9735	.9897	.9956	.9980	.9990	.9995	.9997	.9999	.9999	

TABLE II (cont'd)
Values of J_1

P	t	1																								
		1	2	3	4	5	6	7	8	9	10															
.25	.0625	.0136845	.0001873	.0000026	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000						
	.125	.0136662	.0001868	.0000026	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000					
	.25	.0135930	.0001848	.0000025	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000				
	.50	.0133059	.0001771	.0000024	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000			
	1.00	.0122389	.0001506	.0000019	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000		
	2.00	.0090078	.0000872	.0000009	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	
	4.00	.0037596	.0000267	.0000002	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	
	8.00	.0009686	.0000066	.0000001	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	
	16.00	.0002121	.0000024	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
	.5	.0625	.0535711	.0028702	.0001538	.0000082	.0000004	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
.125		.0535037	.0028626	.0001532	.0000082	.0000004	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	
.25		.053234	.0028328	.0001508	.0000080	.0000004	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	
.50		.0521229	.0027176	.0001447	.0000074	.0000004	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
1.00		.0480272	.0023179	.0001124	.0000055	.0000003	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
2.00		.0355541	.0013547	.0000547	.0000023	.0000001	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
4.00		.0149988	.0004177	.0000149	.0000006	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
8.00		.0039062	.0001047	.0000038	.0000002	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
16.00		.0009690	.0000376	.0000015	.0000001	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000

TABLE II (cont'd)
Values of J_1

P	t	i									
		1	2	3	4	5	6	7	8	9	10
1	.0625	.1970043	.0388107	.0076459	.0015063	.0002967	.0000585	.0000115	.0000023	.0000004	.0000001
	.125	.1967673	.0387174	.0076183	.0014990	.0002950	.0000580	.0000114	.0000022	.0000004	.0000001
	.25	.1958239	.0385473	.0075095	.0014706	.0002880	.0000564	.0000110	.0000022	.0000004	.0000001
	.50	.1921086	.0369154	.0070955	.0013642	.0002623	.0000505	.0000097	.0000019	.0000004	.0000001
	1.00	.1782003	.0318858	.0057281	.0010330	.0001870	.0000340	.0000062	.0000011	.0000002	.0000000
	2.00	.1349030	.0193056	.0029039	.0004551	.0000737	.0000123	.0000021	.0000004	.0000001	.0000000
	4.00	.0597405	.0061492	.0008111	.0001200	.0000187	.0000031	.0000005	.0000001	.0000000	.0000000
	8.00	.0156248	.0015422	.0002030	.0000298	.0000047	.0000010	.0000002	.0000000	.0000000	.0000000
	16.00	.0039063	.0003857	.0000507	.0000075	.0000012	.0000002	.0000000	.0000000	.0000000	.0000000
	2	.0625	.5789112	.3351382	.1940153	.1123176	.0650219	.0376419	.0217913	.0126153	.0073031
.125		.5784277	.3345788	.1935298	.1119432	.0647512	.0374740	.0216645	.0125314	.0072485	.0041928
.25		.5764990	.3325537	.1916046	.1104625	.0636836	.0367170	.0211671	.0122035	.0070358	.0040564
.50		.5683664	.3296501	.1841662	.1048022	.0596485	.0337553	.0199294	.0110055	.0062669	.0035690
1.00		.5596120	.2917682	.1580701	.0858018	.0466614	.0254222	.0138753	.0075861	.0041546	.0022790
2.00		.4406502	.2002614	.0935567	.0447806	.0218915	.0108988	.0055116	.0028248	.0014645	.0007667
4.00		.225616	.0733043	.0288878	.0126350	.0058758	.0028427	.0014336	.0007173	.0003696	.0001896
8.00		.0624945	.0185453	.0072333	.0031594	.0014690	.0007107	.0003514	.0001793	.0000924	.0000482
16.00		.0156250	.0046363	.0019083	.0007899	.0003672	.0001777	.0000884	.0000448	.0000231	.0000121

TABLE II (cont'd)
Values of J_1

p	t	1									
		1	2	3	4	5	6	7	8	9	10
4	.0625	.9632280	.9278083	.9936910	.8608282	.8291739	.7986835	.7693144	.7410252	.7137763	.6875297
	.125	.9630719	.9275075	.8932564	.8602702	.8285021	.7979972	.7684421	.7400651	.7127360	.6864161
	.25	.9624458	.9263022	.8915162	.8580368	.8258149	.7948033	.7649564	.7362307	.7085838	.6819754
	.50	.9599179	.9214470	.8845223	.8490814	.8150645	.7821144	.7510758	.7209960	.6921243	.6644120
	1.00	.9494467	.9015277	.8561015	.8130346	.7722005	.7334802	.6967608	.6619357	.6289043	.5975714
	2.00	.9027744	.8165771	.7400001	.6718309	.6110259	.5566828	.5080209	.4643659	.4251258	.3897915
	4.00	.6898762	.5053195	.3881747	.3094629	.2539129	.2130421	.1819048	.1574863	.1378731	.1218039
	8.00	.2493391	.1445796	.1026608	.0793127	.0642014	.0535417	.0455891	.0394181	.0344878	.0304593
	16.00	.0625000	.0361455	.0256652	.0198282	.0160504	.0133854	.0113973	.0098545	.0086219	.0076148
	8	.0625	.9999950	.9999900	.9999850	.9999800	.9999751	.9999701	.9999651	.9999601	.9999551
.125		.9999949	.9999899	.9999848	.9999797	.9999747	.9999696	.9999645	.9999595	.9999544	.9999493
.25		.9999946	.9999892	.9999838	.9999785	.9999731	.9999677	.9999623	.9999569	.9999515	.9999461
.50		.9999932	.9999865	.9999797	.9999730	.9999662	.9999594	.9999527	.9999459	.9999392	.9999324
1.00		.9999859	.9999719	.9999578	.9999437	.9999296	.9999156	.9999015	.9998874	.9998734	.9998593
2.00		.9999127	.9998253	.9997380	.9996507	.9995635	.9994762	.9993889	.9993017	.9992145	.9991273
4.00		.997561	.9947276	.9921182	.9895232	.9869440	.9843306	.9818328	.9793005	.9767836	.9742424
8.00		.8377602	.7317008	.6584013	.6052128	.5649567	.533745	.5078365	.4866583	.4687235	.4532683
16.00		.2500000	.1936904	.1682263	.1526591	.1417776	.1335659	.1270535	.1217055	.1171989	.1133251

TABLE II (cont'd)
Values of J_1

P	t	1																							
		1	2	3	4	5	6	7	8	9	10														
16	.0625	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000					
	.125	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000				
	.25	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000			
	.50	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000		
	1.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000		
	2.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	
	4.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	
	8.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
	16.00	.9170662	.8606227	.8203100	.7902803	.7670572	.7485061	.7332724	.7204691	.7094978	.6999994	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993	.6999993