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ELECTRONICS RESEARCH LABORATORY

MODE SELECTIVE DIRECTIONAL COUPLERS

by

H. A. Judy

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by

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ABSTRACT

This is a theoretical and experimental study of directional couplers which have both the \( TE_{10} \) and \( TE_{20} \) modes propagating in the rectangular primary wave guide. The coupling slots are oriented so they are only excited by one of these modes. Results are shown of a practical -20 dB directional coupler for each mode, with a coupling of -50 dB for the undesired mode.

H. A. Bethe's small hole coupling theory is placed in a form to handle general coupling situations in which there are different wave guides and modes being considered.

It is shown from the results of S. R. Cohn's small aperture polarizability experimental study that a narrow slot will essentially couple to only a parallel magnetic field. Narrow slots were placed so only the desired mode had an adjacent parallel magnetic field. The tolerance of the slot position with respect to the null of the field of the undesired mode is the limiting factor in rejecting the undesired mode.

Three types of mode selective directional couplers were studied. A directional coupler of the two hole type was built and studied for each mode. Both had a mode selection of 30 db and could be used in a practical two mode microwave system. An advantage of this type coupler is that the design can be extended to a multi-hole binomial or Tschebyscheff array to obtain high directivity over a broad frequency range.

A reverse type directional coupler was designed which is similar to the cross guide coupler. The difference is that the secondary wave guide crosses the primary wave guide at an acute angle, and dissimilar wave guides are used. This design is applicable to mode selection although its practical application is limited.
I. INTRODUCTION

Directional couplers have long been an integral part of most microwave systems. This is particularly true where work of an experimental nature is being carried on. While directional couplers have been developed to a high degree for the TE\textsubscript{10} mode in rectangular wave guide, their extension to multi-mode systems has not been required in the past. There is a large amount of weight and space being occupied by a multiplicity of microwave systems on modern airplanes and ships. The minimum size of wave guides is limited by machinability and voltage breakdown at high power. Other space saving techniques have been brought to the foreground. One of these techniques is the multi-mode system. In this system one wave guide could carry energy from two or more different sources, perhaps at different frequencies, with a minimum of interaction, by using a different mode of propagation for each energy source (see Fig. 1).

The purpose of this study is twofold. First, to study mode selective methods of coupling wave guides with apertures, and second, to develop directional couplers which will react only to the desired mode and cause a negligible amount of cross coupling between the propagating modes.

About a dozen basic configurations for mode selective directional couplers have been studied for various combinations of TE\textsubscript{10}, TE\textsubscript{20}, and TE\textsubscript{30} energy propagations. Of those, the three most promising were studied experimentally and theoretically with the TE\textsubscript{10} and TE\textsubscript{20} modes present.
FIELD DISTRIBUTION

TE10 AND TE20 MODES IN RECTANGULAR WAVEGUIDE

TE10 MODE

TE20 MODE

TOP VIEW

END VIEW

ELECTRIC FIELD
MAGNETIC FIELD

VERTICAL ELECTRIC FIELD
TRANSVERSE MAGNETIC FIELD
LONGITUDINAL MAGNETIC FIELD

FIG. 1
II. COUPLING THEORY

The coupling of two rectangular wave guides transmitting $\text{TE}_{MN}$ waves may be calculated from formulas developed by H. A. Bethe. Although the rigorous validity of this work has been questioned by Bouwkamp, the practical value of the formulas has been amply demonstrated experimentally. This "small hole theory" is based on the following assumptions.

1. The aperture or hole is small with respect to the wave length. The results of these formulas are very accurate where the major dimension of the aperture is less than one-sixth of a free space wave length. A dielectric in the aperture would increase the effective size of the aperture in this regard.

2. The aperture is in an infinite plane wall. This requires that the aperture not be near a corner of the wave guide. A situation such as this showed a slight loss of coupling in the $\text{TE}_{20}$ directional coupler discussed in section 6C.

3. The wall containing the aperture is infinitely thin. The calculations for the effect of wall thickness will be shown later.

The amount of coupling is defined as the ratio of the power in the auxiliary wave guide to the power in the main wave guide, expressed in dB. Coupling may be of two types, electric or magnetic. This coupling may be visualized as the fringing of the electric and/or magnetic field adjacent to the aperture, through the aperture and into the secondary.

$8, 11, 1, 2, 3$ Superscripts refer to the numbered bibliography listed at the end of the paper.
wave guide. The aperture is excited by an incident electric or magnetic field. It in turn radiates as if it were an electric or magnetic dipole in the aperture. Essentially, the coupling is proportional to the field strength which excites the aperture. It is easy to see from reciprocity, that the mode which will be excited in the auxiliary wave guide, is one which would excite the aperture in the same manner. Also from reciprocity, the amount of coupling must be proportional to the relative strength of this unperturbed field at the aperture for the given mode.

The aperture will have a certain "polarizability" or ability to couple or transmit which is only a function of the type of exciting field, and in the case of excitation by a magnetic field, the orientation of the field. According to Bethe¹; "Any small iris can be characterized by three polarizabilities p, m₁, and m₂ which determine its transmission wherever the iris may be used. This means that we need not treat the problem of the wave guide or cavity with the iris, but we may treat the iris by itself and the normal modes of wave guide or cavity by themselves and obtain the required results by a combination of the quantities relevant for the individual parts of the system." The values p, m₁, and m₂, respectively, represent the polarizability to the electric field, magnetic field parallel to the major axis of the aperture, and magnetic field perpendicular to the major axis of the aperture. Polarizabilities have been calculated for simple shapes. For a round hole they are:

\[ p = \frac{d^3}{12}, \quad m_1 = \frac{d^3}{6} \quad \text{and} \quad m_2 = \frac{d^3}{6} \]

where d is the diameter of the hole. For odd shapes, such as slots, S. B. Coon⁵,⁶ has found the polarizabilities experimentally, using static tests, in terms of \( pd^3 \), \( m_1 d^3 \), and \( m_2 d^3 \) where p, m₁, and m₂ are a function of the slot shape, and the d is the long dimension of the slot.
There are two important corrections which must be made to this coupling theory. The first is for the attenuation due to the finite thickness of the wall containing the aperture. The finite thickness of the aperture may be considered as a length of wave guide propagating below cutoff (see Appendix). In general, attenuation is greatest for electric coupling. The electric coupling propagates as TM_{11} through a rectangular slot or TM_{10} through a circular slot. The only difference is the coordinate system which defines the aperture shape and hence the mode which propagates through it. Magnetic coupling propagates as TE_{10} through a rectangular aperture. When determining the cutoff wave length for this mode, the long dimension of the slot must be used for a parallel magnetic field, and the short dimension for a perpendicular exciting magnetic field.

The second correction is that for the proximity to resonance of the slot. As stated before, this theory is valid only for small apertures, but the calculations are fairly accurate in the case of a slot whose length is of the order of 3/8 of the free space wave length. Once the increased coupling is determined for a large slot in terms of its shape and proximity to resonance, that factor should apply equally well to all coupling situations with the same slot parameters. An exception to this would be in the case where the slot couples an appreciable fraction of the power being propagated in the main wave guide, so that there is a "loading" effect. Where the coupling is less than -10 db this effect is quite small.

The primary emphasis of the small hole theory has been its application to coupling between two similar wave guides with a common hole at the center of the adjacent broad or narrow face of the wave
guides. Bethe, however, has the formulation in a form that is readily expanded to handle a general coupling situation. The general form, as expanded in this paper, will allow calculation of coupling with an aperture at any position with respect to either wave guide. Also, the aperture may couple, in the case of magnetic coupling, to the transverse field of one wave guide and the longitudinal field of the other wave guide.

Bethe's formulas for the coupling of two parallel wave guides by an aperture are:

\[ A_2 = \frac{\pi i}{\lambda_0 S_2} \left[ -m_1 H_{1t} H_{2t} - im_2 H_{1l} H_{2l} + pE_1 E_2 \right] \]

\[ B_2 = \frac{\pi i}{\lambda_0 S_2} \left[ +m_1 H_{1t} H_{2t} - im_2 H_{1l} H_{2l} + pE_1 E_2 \right] \]

where the (2) subscript refers to the secondary wave guide and the (1) subscript refers to the primary wave guide. \( A_2 \) and \( B_2 \) refer to the amplitudes of the electric field of the waves excited in the forward and reverse directions, respectively, with a unit electric field incident. The signs and "i" terms denote the relative phase of the coupled fields.

The quantities \( m_1, m_2 \) and \( p \) refer to the polarizabilities of the aperture to the transverse magnetic field, longitudinal magnetic field, and electric field, respectively. Note that for flexibility, a different definition of these terms is used everywhere else in this paper, such that \( m_1 \) refers to whichever magnetic field is parallel to the slot.

\( S_2 \) is a normalizing factor, with respect to the secondary wave guide, equal to \( \lambda_0 a_2 b_2 / 2 \lambda_z \). It is the term used to normalize electric field strength to power flow. It will be shown later that his term is necessary so the formula will satisfy reciprocity conditions.

\( E_1 \) and \( E_2 \) represent the ratio of the electric field at the aperture center to the maximum electric field for the corresponding wave guide.
$H_{lt}$ and $H_{2t}$ represent the ratio of the transverse magnetic field at the center of the aperture to the maximum transverse magnetic field multiplied by the term $(\lambda_0/\lambda_g)$ which relates the maximum transverse magnetic field to the maximum electric field for any TE mode.

$H_{lt}$ and $H_{2t}$ represent the ratio of the longitudinal magnetic field at the center of the aperture to the maximum longitudinal magnetic field multiplied by the term $(\lambda_0/\lambda_{co})$ which relates the maximum longitudinal magnetic field to the maximum electric field.

If the $S_2$ normalization term were used to normalize one of the magnetic fields (rather than the electric field) to the power flow, the $H$ and $E$ factors would relate their corresponding field to that magnetic field instead of to the electric field as in the above example.

In order to convert these formulas from field strength ratios to power ratios it is necessary to take the square and multiply by the normalization term for the secondary wave guide and divide by the normalization term for the primary wave guide. These normalization terms are with respect to the same type of field as $S_2$ in the formula.

Handling each exciting field separately and converting to power ratios, the formulas become for an incident field of unit power:

$$P_2 = \frac{n^2}{\lambda_0} \frac{S_2}{S_1} \left[ H_{lt}^2 / H_{2t}^2 \right]$$

$$P_2 = \frac{n^2}{\lambda_0} \frac{S_2}{S_1} \left[ H_{lt}^2 / H_{2t}^2 \right]$$

$$P_2 = \frac{n^2}{\lambda_0} \frac{S_2}{S_1} \left[ E_1^2 / E_2^2 \right]$$

For these formulas the $S$ terms are the normalization terms for the corresponding field exciting or being excited, and the $H$ and $E$ terms then need only by the ratio of the adjacent field to the maximum of that field.
Two important facts should be noted at this time. All terms in the power equation refer to each wave guide in an equivalent manner, thereby satisfying the necessary reciprocity conditions. It is also quite proper for a slot to couple to the transverse magnetic field of one wave guide and the longitudinal magnetic field of the other wave guide. In the case of perpendicular wave guides, the major magnetic polarizability of the slot (m₁) would do just this. The formula would be:

$$ P_2 = \frac{\pi^2}{\lambda_0^2} \frac{S_1}{S_2 S_1} \left[ m_1^2 \cdot H^2_{1t} \cdot H^2_{2s} \right] $$

where the S and H terms would refer to the corresponding field being coupled in each wave guide. The development of the formulas in the complete form to handle each of the four coupling situations follows.

In the case of electric coupling, the aperture is excited by an incident electric field perpendicular to the guide wall containing the aperture. The aperture will then reradiate and excite any modes in either wave guide which would have an electric field perpendicular to the wall at the center of the aperture.

Electric Couplings may be calculated as follows:

$$ C = \frac{P_2}{P_1} \text{ (db)} = 10 \log_{10} (A^2 N N, D_1 D_2) - \text{Att} $$

The factor "A" represents the coupling of the aperture to the normal electric field.

$$ A = \frac{\pi^2 \cdot \lambda_0^3}{\lambda_0^3} = \frac{\pi d^3}{\lambda_0^3} = \frac{\pi p}{\lambda_0} $$

where r is the radius of a round hole,

d is the diameter of a round hole,

l is the long dimension of a slot or round hole,

p is the electric polarizability of the slot or round hole,

\( \lambda_0 \) is the free space wavelength.
1/N normalizes the maximum electric field to the power flow in the corresponding wave guide.

\[ N_1 = \frac{2 \delta_1}{\lambda_0 a_1 \beta_1} \quad \text{and} \quad N_2 = \frac{2 \delta_2}{\lambda_0 a_2 b_2} \]

where \( \delta_1 \) is the guide wave length in the primary (1st) wave guide,
\( a_1 \) is the wide dimension of the primary or main rectangular wave guide,
\( h_1 \) is the narrow dimension of the primary or main rectangular wave guide,
\( a_2 \) is the wide dimension of the secondary or auxiliary wave guide.

\( D \) is a correction factor for the displacement of the slot from the position of the maximum electric field in the corresponding wave guide.

\[ D_1 = \sin^2 \left( \frac{\text{m} x_1}{a_1} \right) \quad D_2 = \sin^2 \left( \frac{\text{m} x_2}{a_2} \right) \]

where \( x_1 \) is the distance of the aperture center from the side of the main wave guide.

\( \text{Att}_e \) is attenuation below cutoff due to the finite thickness of the aperture (in db).

\[ \text{Att}_e = \frac{\delta_1 t}{\lambda_{coa}} \left( \frac{\lambda_{coa}}{\lambda_0} \right)^2 \]

where \( t \) is the aperture thickness,
\( \lambda_{coa} \) is the cutoff wave length of the aperture for the proper mode,
\( \lambda_0 \) is the free space wave length.

For a slot aperture, the electric coupling is:

\[ G = \log_{10} \left( \frac{\pi^2}{16} \frac{2 \delta_1}{\lambda_{coa}} \frac{2 \delta_2}{\lambda_{coa}} \sin^2 \left( \frac{\text{m} x_1}{a_1} \right) \sin^2 \left( \frac{\text{m} x_2}{a_2} \right) - 5 \delta_1 \delta_2 \sqrt{1 - \left( \frac{\lambda_{coa}}{\lambda_0} \right)^2} \right) \]

where \( \lambda_{coa} = \frac{2 \delta_1 \delta_2}{\sqrt{\delta_1^2 + \delta_2^2}} \) for electric coupling.
The second type of coupling is magnetic coupling. The aperture or slot is excited by the magnetic field in the plane of the slot and radiates like a magnetic dipole in the plane of and parallel to, the slot. Coupling can be either by a transverse "series" slot or a longitudinal "shunt" slot. A slot at an angle can act as both a series and a shunt slot. No slots discussed in this paper will combine series and shunt coupling to the same wave guide, although some may be series coupled to one wave guide and shunt coupled to the other.

Magnetic coupling may be calculated as follows:

\[ C = \frac{P_2}{P_1} \left( \text{db} \right) = 10 \log_{10} \left( \frac{A^2}{1^2 1^2 1^2 1^2} \right) - \text{Att} \]

The factor \( A \) is the coupling of the aperture to the magnetic field in the wave guide. This is normally the magnetic field which is parallel to the slot which corresponds to the wall surface current cut at right angles to the slot. By using \( m_2 \), the polarizability of the slot to the orthogonal field, the smaller coupling to the orthogonal field can be computed.

\[ A = \frac{m_1}{m_0} \]

where \( m_1 \) is the magnetic polarizability of the slot to the parallel field and \( m_2 \) the polarizability to the orthogonal field.

The factor \( H_1 \) normalizes the maximum transverse magnetic field (at the center of the wave guide for \( TE_{10} \)) to the power flow in the corresponding wave guide.

\[ H_1 = \frac{2 \lambda_0}{\lambda_{g1} a_1 b_1} \]

The factor \( R_1 \) is the ratio of shunt to series coupling in the corresponding wave guide. \( R \) is only inserted in the formula where the slot is shunt coupled to the longitudinal magnetic field.
The factor $D_1$ is the correction factor for the displacement of the slot from the position of the maximum field of the type to which the slot is coupling, i.e., transverse or longitudinal magnetic.

$$D_1 = \sin^2\left(\frac{n m x_1}{a_1}\right)$$

for a series coupled slot.

$$D_2 = \sin^2\left(\frac{n m x_1}{a_2}\right)$$

for a shunt coupled slot.

The factor $G_1$ is the correction factor for the angle between the slot and the adjacent magnetic field to which the slot is coupling in the corresponding wave guide.

$$G_1 = \cos^2 \varphi_1$$

where $G_1$ is the angle between the slot and the adjacent magnetic field in the corresponding wave guide.

For a slot aperture, series-series magnetic coupled:

$$C = 10 \log_{10} \left[ \frac{\mu_0^2 \mu^2}{\lambda_0^2} \frac{2 \lambda_0}{\lambda_1 a_{1} b_{1}} \frac{2 \lambda_0}{\lambda_2 a_{2} b_{2}} \sin^2 \left(\frac{n m x_1}{a_1}\right) \sin^2 \left(\frac{n m x_2}{a_2}\right) \cos^2 \varphi_1 \cos^2 \varphi_2 \right]$$

$$\frac{5 \mu_0 \epsilon_0}{\lambda_{coa}} \sqrt{1 - \left(\frac{\lambda_{coa}}{\lambda_0}\right)^2}$$

For a slot aperture shunt-series magnetic coupled:

$$C = 10 \log_{10} \left[ \frac{\mu_0^2 \mu^2}{\lambda_0^2} \frac{2 \lambda_0}{\lambda_1 a_{1} b_{1}} \frac{2 \lambda_0}{\lambda_2 a_{2} b_{2}} \cos^2 \left(\frac{n m x_1}{a_1}\right) \sin^2 \left(\frac{n m x_2}{a_2}\right) \cos^2 \varphi_1 \cos^2 \varphi_2 \right]$$

$$\frac{5 \mu_0 \epsilon_0}{\lambda_{coa}} \sqrt{1 - \left(\frac{\lambda_{coa}}{\lambda_0}\right)^2}$$

For a slot aperture shunt-shunt magnetic coupled:

$$C = 10 \log_{10} \left[ \frac{\mu_0^2 \mu^2}{\lambda_0^2} \frac{2 \lambda_0}{\lambda_1 a_{1} b_{1}} \frac{2 \lambda_0}{\lambda_2 a_{2} b_{2}} \cos^2 \left(\frac{n m x_1}{a_1}\right) \cos^2 \left(\frac{n m x_2}{a_2}\right) \cos^2 \varphi_1 \cos^2 \varphi_2 \right]$$

$$\frac{5 \mu_0 \epsilon_0}{\lambda_{coa}} \sqrt{1 - \left(\frac{\lambda_{coa}}{\lambda_0}\right)^2}$$
The cutoff wave length must be calculated using the slot dimension parallel to the magnetic field whose coupling is being computed. For a rounded slot:

\[ \lambda_{\text{coa}} = 2\ell - 0.546 \, w \]

where \( \ell \) and \( w \) are the length and width of the slot.

See Appendix for tables for rapid calculation of coupling.

III. PRINCIPLES OF MODE SELECTION

Mode selection is the ratio of the coupling to the desired mode to that of the undesired mode. It is just the difference in db of the coupling to each mode, using the coupling formulas previously developed.

The primary problem of mode selection is to so orient and shape the coupling aperture that it is only excited by the desired mode. As long as the aperture is not excited by an undesired mode, it cannot excite the undesired mode and hence will not cause any cross coupling of modes, which will be the major problem of any multi-mode system. Obviously, the first step in mode selective directional coupler design is mode selective aperture design.

The coupling of any aperture to energy propagating in a wave guide is proportional to the field strength adjacent to the aperture, with the aperture removed. In the limiting case where the aperture's large dimension is about one-sixth of a wave length or less, this assumption is justified, for the aperture has a negligible perturbing effect upon the field configuration in the wave guide. Experimental data with resonant half-wave length slot radiators has shown that this assumption is accurate with only minor perturbations for these fairly large apertures.
A resonant slot radiating into half space from a wave guide has approximately a sine squared relation to transverse position with respect to power radiated.

A small aperture may be excited by either an electric or a magnetic field adjacent to it. Consequently, to obtain mode selection, the aperture must either be placed where neither type of field exists for the undesired mode, or where one of the fields is zero and the other is very poorly coupled due to the shape and orientation of the slot. There is no place on the boundary of a wave guide where neither the electric nor magnetic field exists for a TE mode. The problem then becomes one of diminishing the coupling of the undesired field by the shape and orientation of the aperture. The logical solution to this requirement is the narrow slot.

The narrow slot couples strongly only to a magnetic field which is parallel to the long dimension of the slot. Consequently a slot will effectively reject electric coupling and coupling by the perpendicular magnetic field in favor of the parallel magnetic field. According to small hole theory, a round hole aperture couples 6 db closer to a magnetic field than to an electric field. According to an experimental study of magnetic and electric aperture polarizability (ability to couple to adjacent fields) by S. B. Cohn, changing a round hole to a slot has the following effects. Coupling to the electric field is decreased rather sharply as is the coupling to magnetic fields which are perpendicular to the long dimension of the slot. The coupling to magnetic fields parallel to the slot is only slightly decreased. A slot with semi-circular ends and a length to width ratio of seven is less sensitive than a round hole to the electric
field by a factor of 27.1 db and to the parallel magnetic field by 7.1 db. For this situation, the slot couples 25.7 db more strongly to the parallel H field than to the E field. The coupling to the electric field is further reduced by the attenuation below cutoff due to the finite thickness of the slot between the two wave guides being coupled. In order to calculate this attenuation, the slot may be considered to be a wave guide of approximately rectangular cross section, propagating the coupled energy below cutoff of the mode excited by the incident field. It can be seen that the parallel magnetic field would excite the TE10 mode in the slot while the normal electric field would excite the TM11 mode. The TM11 mode is much farther below cutoff than the TE10 mode, and this difference increases with decreasing slot width.

A typical case in the experiments to follow is a slot with length 0.220 inches, width 0.060 inches, and thickness .090 inches. For a frequency of 9375 mc, the attenuation of the magnetic coupling is only 2.6 db while that of the electric coupling is 25.1 db causing an additional separation of 20.5 db between the two types of coupling.

From the foregoing it can be seen that for a slot of only moderate length to width ratio, the coupling to the normal electric field and the perpendicular magnetic field is quite small with respect to the coupling to the parallel magnetic field, the mode selection problem is much simplified. Mode selection may be obtained by placing the slot where a parallel magnetic field exists for the desired mode and does not exist for the undesired mode. Since all TE modes have a longitudinal H field adjacent to the narrow side of the wave guide, good mode selection here is out of the question. Higher modes would couple somewhat more closely because of stronger longitudinal magnetic fields due to the proximity to cutoff, but the mode selection would not be sufficient.
A longitudinal slot placed at the center of the wave guide (like the slot for a conventional sliding probe) will not couple to the $TE_{10}$ mode, since there is no longitudinal magnetic field at the center of the wave guide (see Fig. 2). This slot will couple, however, to the $TE_{20}$ mode which has a maximum longitudinal $H$ field at this point. The weakest link in the mode selection of this slot is not the rejection of the strong $TE_{10}$ electric field, as might be expected, but the tolerance with which the slot is machined at the center of the wave guide. A slot slightly off center will couple to the longitudinal $H$ field of the $TE_{10}$ mode which has a cosine variation in the transverse direction. This is the factor limiting the mode selection of this type of slot. The mode selection is often aided by the fact that the $TE_{20}$ mode, which is closer to cutoff, has a proportionately stronger longitudinal $H$ field than the $TE_{10}$ mode.

In order to calculate mode selection, it is necessary to calculate all possible types of coupling which may contribute. In the case of this paper, all coupling apertures are slots of medium length, which couple predominantly to the magnetic field parallel to the slot. Coupling of the undesired mode could occur through electric coupling, coupling of the magnetic field orthogonal to the slot, or coupling of the magnetic field parallel to the slot. Even though the slot is placed at a null of the magnetic field parallel to the slot, calculations have shown that if the slot position is only displaced by 0.002", this type of undesired coupling will predominate.
MODE SELECTIVE SLOTS

LONGITUDINAL SLOTS AND DISTRIBUTION OF LONGITUDINAL MAGNETIC FIELD.

**TE₂₀ SLOT**

**TE₁₀ SLOT**

BY PLACING A SLOT AT THE NULL OF THE PARALLEL MAGNETIC FIELD OF THE UNDESIGNED MODE, MODE SELECTION MAY BE OBTAINED.

TRANSVERSE SLOT AND DISTRIBUTION OF TRANSVERSE MAGNETIC FIELD.

**TE₁₀ SLOT**

**TE₁₀ TRANSVERSE MAGNETIC FIELD**

**TE₂₀ TRANSVERSE MAGNETIC FIELD**

FIG. 2
IV. APPLICATION OF MODE SELECTIVE SLOTS TO DIRECTIONAL COUPLERS

To obtain mode selection, the following requisites are necessary. The coupling aperture must be the general shape of an ellipse or rectangle in order to couple to the preferred magnetic field much more closely than the electric field or magnetic field orthogonal to the slot. Calculations show that to minimize undesired coupling, the length to width ratio of the aperture should be at least 7:1 and 10:1 is preferable. With this type of slot, which will couple strongly to only the parallel magnetic field, it is necessary to find a slot position and orientation where there is an adjacent magnetic field of the desired mode and not the undesired mode (see Fig. 2). An even mode can always be coupled in preference to an odd mode by a centered longitudinal slot. An odd mode can always be coupled in preference to an even mode by a center of a series (transverse) or shunt (longitudinal) slot will be at a null. To obtain high mode selection it is desirable to use a shunt slot for the higher mode and a series slot for the lower mode. A higher mode will be closer to cutoff and have stronger longitudinal field while the reverse is true of a lower mode.

Once slots have been found which will select modes, a directional array may be chosen which will allow proper excitation of the auxiliary or secondary wave guides. The two hole type directional coupler is simplest and will always allow coupling to the same mode in the auxiliary guide. It will also permit excitation of the first mode which seems most practical for a system once the modes have been separated. This type of coupler can also be extended to a binominal or Tschebyscheff array for broadbending purposes. A reverse type directional coupler will reject any given even mode.
7. EXPERIMENTAL PROCEDURES.

The coupling in the forward or preferred direction, coupling in the backward or non-preferred direction, and coupling of the undesired mode in the forward direction were measured directly in db. The results for each coupler were plotted on a single graph as a function of frequency. The directivity is simply the difference in db between the backward coupling and the forward coupling of the desired mode.

The appropriate mode was excited in the primary guide of the directional coupler with the transducer shown in Fig. 3a,b. Energy fed into arm 1 of the transducer is propagated as the TE\textsubscript{10} mode while energy fed into arm 2 of the transducer is propagated as TE\textsubscript{20} at the output arm 3, which was connected to the coupler. The cross coupling of the transducer was less than -50 db and was neglected in the experiment.

The primary guide of the coupler was terminated with a sliding load which had a cross coupling of -65 db and a VSWR of less than 1.02 for both modes.

Untuned Hewlett-Packard L15A probes and crystals were used as power measuring devices. These were used in conjunction with a HP L15A meter. Where crystals were used, they were preceded by a pad attenuator, as shown in Fig. 1a, b to keep the standing wave ratio low. The sensitivity of the power measuring devices, relative to a probe chosen as a standard, was measured in db by varying an attenuator so each probe or crystal gave the same reading of -55 db on the meter. The difference in attenuator reading was the relative sensitivity of the probe or crystal to the standard probe. All coupling measurements were essentially the same. A probe or crystal was placed at the desired output of the coupler and a probe was placed at the input to the transducer. The calibrated
A $TE_{10}$ MODE WAVE INCIDENT AT ARM 1 WILL EXCITE THE $TE_{10}$ MODE IN THE WIDE WAVEGUIDE WHICH WILL TRAVEL TO ARM 3.

A $TE_{20}$ MODE WAVE INCIDENT AT ARM 2 WILL EXCITE THE $TE_{20}$ MODE IN THE WIDE WAVEGUIDE WHICH WILL TRAVEL TO BOTH ARM 3 AND ARM 1.

THE $TE_{20}$ MODE IS BELOW CUTOFF IN ARM 3 AND WILL BE ATTENUATED VERY RAPIDLY.

EITHER MODE MAY BE EXCITED IN ARM 3 BY PROPER CHOICE OF THE INPUT ARM.

FIG 3a
Fig. 3b TE$_{10}$ - TE$_{20}$ Mode Transducer
Fig. 4b Experimental Set-up
The attenuator was varied so each probe or crystal had the same meter reading as was used originally to find its relative sensitivity. The coupling of the coupler being measured was the extra sensitivity of the output probe plus the difference in attenuator reading. The basic standard for all measurements was the calibrated attenuator. The probes, crystals, and meter were always read the same level, so variations of characteristics with power level had no effect on the experimental results.

To measure the forward coupling of a coupler, a probe was placed at the forward output arm and another was placed at the input to the transducer with the transducer input matched. The proper arm of the transducer was chosen so the desired mode was fed to the primary guide of the coupler.

To measure the coupling of the undesired mode, energy was fed to the corresponding matched arm of the transducer with a crystal at the output arm of the coupler and a probe at the input arm of the transducer.

VI. EXPERIMENTAL MODE SELECTIVE DIRECTIONAL COUPLERS AND RESULTS

A. TE20 Mode Two Hole Type Coupler.

The experimental results show this coupler to be a practical mode selective directional coupler (see Figs. 5 and 6). The forward coupling agrees fairly well with the theory. The forward coupling varies from two to four db closer than small hole theory predicts as frequency is increased, but this is expected since the slots used were about 2/3 resonant length. For this slot length of 0.440", the proximity to resonance of the slot does not cause enough frequency sensitivity of coupling to detract from the value of the directional coupler for most uses. In future designs the resonance effect of the slots could be considered approximately constant (in db) so the desired forward coupling should be obtained within an error of ± 1 db.
From the results of this study it is not possible to calculate precisely the slot spacing for a given optimum frequency of directivity. Conventional "X" band directional couplers with a forward coupling of -20 dB have been designed using a spacing of 0.973 \( \lambda_\text{g} / \lambda \). The empirical constant is different for this type coupler using dissimilar wave guides. To complicate matters, the constant apparently varies with the desired optimum frequency. This experimental coupler with a slot spacing of 0.357" and an optimum frequency of 9300 mc has a constant of 0.873. Another coupler of the same design and forward coupling had a slot spacing of 0.381" and an optimum frequency of 9020 mc which would correspond to a constant of 0.913. Since this empirical constant varies considerably, a rather detailed study would be necessary to make it possible to design a coupler for a given optimum frequency accurately on the first try. The directivity characteristics of the coupler also deviate slightly from the calculated values, assuming the theoretical curve is centered on the actual optimum frequency of the coupler. Since this deviation is not serious, the theoretical curve gives an approximate picture of the directivity characteristics of this type coupler. An important feature of this coupler design is that there is no limit to the number of slots that may be used so a binomial type array could be used which would improve the frequency characteristics of the directivity considerably.

The mode selection of this coupler is 30 db. The calculated mode selection for a slot tolerance of \( \pm 0.001" \) is 28 db. Transverse slots have inherently good mode selection when coupling TE\(_{10}\) and rejecting TE\(_{20}\). This is because the TE\(_{10}\) mode has a stronger transverse magnetic field, slightly stronger than the TE\(_{20}\) mode. It would be difficult to obtain a greater mode selection since machining tolerances of less than \( \pm 0.002" \) are difficult and expensive to obtain.
TK10 MODE TWO HOLE TYPE DIRECTIONAL COUPLER

SLOT LENGTH 0.440"
SLOT WIDTH 0.060"
SLOT SPACING 0.357"
CENTER TO CENTER
SLOT THICKNESS 0.050"

TERMINATION IN SMALL WAVEGUIDE

FIG. 5
TE$_{10}$ MODE TWO HOLE TYPE
DIRECTIONAL COUPLER

---

**THEORETICAL**

---

**EXPERIMENTAL**

---

**SLOT LENGTH 0.440"**

---

**FREQUENCY (MC)**

---

**COUPLING (dB)**

---

**UNDESIRED MODE COUPLING**

---

**BACKWARD COUPLING**

---

**FORWARD COUPLING**

---

FIG. 6
B. TE_{10} Mode Reverse Type Coupler.

The experimental results show that this type of coupler is limited in its practical value (see Figs. 7, 8, and 9). Two couplers were studied using different values of forward coupling and optimum frequency of directivity.

A definite disadvantage of this type coupler is that the TE_{10} mode has weak longitudinal magnetic fields in the broad wave guide, since the mode is far from cutoff. This requires a long slot to obtain coupling of -20 db. In the case of the model "A" coupler (Fig. 9) the slots used were 0.530" long which is about 80% of the resonant length at 9375 mc. The slot's proximity to resonance causes a very high frequency sensitivity of forward coupling which causes a variation of 12 db of the coupling in the frequency range between 8800 and 10000 mc. If the coupler is to be used at one frequency only, or over a small frequency range, this frequency sensitivity of coupling will not necessarily be a disadvantage. In the case of the model "B" coupler, where the forward coupling is -29 db, the variation of forward coupling is less than 4 db over the broad frequency range. For this coupling the slot length is 0.450" which is not close enough to resonance to cause trouble. This characteristic of this design must be evaluated in terms of the use for which the coupler is being designed.

The electrical path lengths should be equal toward arm 3 for complete cancellation. The angle "\theta", of the secondary wave guide, is adjusted to meet this condition at the desired optimum frequency of directivity. By changing the angle away from 90°, due to the different guide wave lengths, a frequency sensitivity of directivity is introduced. The utility of this coupler with respect to directivity is quite dependent on the
conditions of its use. The frequency sensitivity of this directional coupler depends on the difference of the change in guide wave length as the frequency is changed from that of optimum directivity. If the cutoff frequency for the modes being coupled in the wave guides are not too far apart, the directivity will be good over a broad band. Also, if the frequency is far above cutoff, the difference between the change in guide wave lengths will be small and directivity will remain high. The "B" coupler, which has a higher frequency of optimum directivity has a broader range of good directivity. This is because at the higher frequency the guide wave length is not affected a great deal by the proximity to cutoff in the two wave guides. If the true cross guide design were being used with similar wave guides, the directivity would have no essential frequency sensitivity and would be high all across the frequency band.

The frequency of optimum directivity is lower than the theoretical value, as in the other directional couplers. This is most likely due to a shift in phase in the primary wave guide due to the presence of the coupling slots.

The theoretical mode selection of this type coupler is not as good as that of the two hole couplers. The reason is that longitudinal slots are used and the undesired TE20 mode has a stronger longitudinal magnetic field than the TE10 mode because it is closer to cutoff. The experimental mode selection is 24 db and 20 db for models "A" and "B", respectively, which is 6 to 10 db less than the mode selection of the TE10 two hole couplers. The theoretical difference is about 10 db. A relative variation of a few db is to be expected; since the mode selection is dependent upon the position tolerance of the slot which is about ± 0.001".
MODIFIED CROSS GUIDE TYPE DIRECTIONAL COUPLER (A) AND (B)

MODEL B

\( \theta = 36.5^\circ \)

SLOT LENGTH 0.450"

SLOT WIDTH 0.060"

MODEL A

\( \theta = 39^\circ \)

SLOT LENGTH 0.330"

SLOT WIDTH 0.060"

SLOT THICKNESS 0.020"

FIG. 7
TE$_{10}$ MODE REVERSE TYPE
DIRECTIONAL COUPLER
MODEL B

FIG 8
TE$_{10}$ MODE REVERSE TYPE
DIRECTIONAL COUPLER

TYPE A

FREQUENCY (MC)

COUPLING (-dB)

BACKWARD COUPLING

UNDESIRED MODE COUPLING

FORWARD COUPLING

SLOT LENGTH 0.530''

FIG. 9
G. TE\textsubscript{10} Mode Two Hole Type Coupler.

The experimental results and calculations show this coupler design to be the best of the designs studied (see Figs. 10 and 11). The forward coupling decreases (greater negative db) by about 1 db from 8800 mc to 10000 mc. The theoretical forward coupling decreases more rapidly than this but is partially compensated for by the frequency dependence of the slot. The experimental forward coupling is almost identical to the calculated forward coupling, but this is probably due to two compensating factors causing errors, each having an effect of about 2 db. In other coupling situations, the proximity to resonance for a slot of this length (0.440") causes an increased coupling of about 2 db. Since the slots couple into the secondary wave guide very close the narrow side a slight loss in coupling would be expected, which appears to be the compensating 2 db.

The directivity of this coupler is very close to the calculated values. The optimum frequency of directivity is the same as that calculated using the conventional empirical constant of 0.973 in the formula for slot spacing. Judging from the results of the TE\textsubscript{10} mode two hole type coupler, this factor may not necessarily hold for different frequencies or different combinations of wave guide sizes. The frequency sensitivity of the directivity is very close to the theoretical as shown on Fig. 11. This coupler could also be designed with a greater number of coupling slots in a binomial type array to increase the frequency band of high directivity.

The mode selection of this coupler is about 30 db. Since the longitudinal magnetic fields are weaker in the TE\textsubscript{10} mode, mode selection is aided by a factor of over 3 db. The slot position is least critical when rejecting the TE\textsubscript{20} mode because there is only one-half cosine variation of field and hence less change of field strength with respect
to position near the null. The $TE_{20}$ mode has a full cosine transverse variation of field and is twice as sensitive to slot position.
TE_20 MODE TWO HOLE TYPE DIRECTIONAL COUPLER

SLOT LENGTH 0.428"
SLOT WIDTH 0.060"
SLOT THICKNESS 0.050"
SLOT SPACING 0.468"
CENTER TO CENTER
SLOT SPACING 0.543"
BETWEEN ADJACENT ENDS

TERMINATION IN SMALL WAVEGUIDE

FIG. 10
FIG. 11

TE$_{20}$ MODE TWO HOLE TYPE
DIRECTIONAL COUPLER

<table>
<thead>
<tr>
<th>FREQUENCY (MC)</th>
<th>COUPLING (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>24</td>
</tr>
<tr>
<td>9000</td>
<td>18</td>
</tr>
<tr>
<td>10000</td>
<td>12</td>
</tr>
<tr>
<td>11000</td>
<td>6</td>
</tr>
<tr>
<td>12000</td>
<td>0</td>
</tr>
</tbody>
</table>

REVERSE COUPLING
UNDESIRED MODE COUPLING
FORWARD COUPLING

THEORETICAL
EXPERIMENTAL

SLOT LENGTH 0.420
VII. CONCLUSIONS

Small hole coupling theory may be applied to general coupling situations where dissimilar wave guides and modes are present. In the case where slot apertures are used which are longer than the limit for strict application of small hole theory, but less than 3/8 of a free space wave length, a small correction factor may be used. This increased coupling due to the proximity to resonance of the slot is of the order of 4 db or less, but increases rapidly as the slot length is made longer than 3/8 of a wave length.

Narrow slots with a length to width ratio of seven or more and a thickness of 0.050" will couple at least 40 db more closely to the parallel magnetic field than to the perpendicular magnetic field or the electric field. By placing such a slot where there is a null of the parallel magnetic field of a given propagating mode, the slot will essentially reject that mode in favor of one with a strong parallel field at that position.

It has been shown that mode selection of the order of 30 db is possible in practical directional couplers where the T_{30} and T_{20} modes are present.

The procedures outlined in this paper may be applied to other modes (T_{30}, T_{40}, etc.). Mode selection of these higher modes is poorer since the fields build up more rapidly on either side of a null position.

The two hole type directional coupler, or multi-hole couplers of the same basic design, has the best all around characteristics with respect to directivity, mode selection, and frequency sensitivity of forward coupling.

For a practical operating system, mode selection of greater than 30 db would not be necessary. Other discontinuities in the system such as joints and bends normally cause cross coupling between the modes of this order of magnitude. A photograph of three couplers is shown in Fig. 12.
Fig. 12

Typical Mode Selective Directional Couplers
APPENDIX I

Sliding Load Measurement Technique

When measuring power at some point, often the measurement is affected by mismatched terminations at one or more places in the circuit. By using sliding terminations, Fig. 13, the power at the point of measurement may be determined for the case of ideal terminations. Two readings are taken, one with all reflections from the terminations adding directly to the "ideal" field strength, and another with all reflections subtracting from the "ideal" field strength. A practical systematic procedure which may be followed for such a measurement is outlined here.

First, successively adjust all loads until the reading is a maximum at the point of measurement. With a large number of loads, this will require a large number of adjustments of each load. Next, adjust each load in turn by moving approximately one-half a guide wave length and adjusting for a minimum at the point of measurement. Each sliding load should be adjusted only once. If the nth load adjusted causes the measured power to pass through zero at positions each side of the new position one-half a guide wave length from the original position, that load, and all further loads, adjusted, should be adjusted for a maximum reading. The second reading is always less than the first, regardless of whether the final adjustment is to obtain a maximum or a minimum. The difference of these two readings should be determined in decibels. By knowing the difference of these two readings in db, and whether the final adjustment of the second reading was for a maximum or minimum, the strength of the "ideal" power reading may be determined from the accompanying graph, Fig. 14.
Fig. 13 Sliding Termination
The sum of all the load reflections added together is the "ideal" reading. It is considered as a separate single signal. By determining the difference in db between the two signals adding and subtracting, the ratio of each signal to the sum or maximum reading may be determined on the graph. If varying the loads causes a null, then the "ideal" reading is the smaller of the two, while if the final adjustment is for a minimum, the "ideal" reading is the larger of the two.

The values for the graph were obtained by assuming a signal of unit field strength and smaller signal of varying values ($V'$). For each $V'$ three things are determined. These are: the range or difference in readings in decibels, the ratio of the maximum reading to the larger signal in decibels, and the ratio of the maximum to the smaller reading in decibels.

Example: For a large signal of unity and a small signal ($V'$) of 0.3:

\[
1 + V' = 1.3 = 2.3 \text{ (db)} \\
1 - V' = 0.7 = -3.1 \text{ (db)}
\]

The range will be 5.4 (db).

It can easily be seen that the large unit signal is the maximum reading multiplied by $1/(1 + V')$. In this case the unit signal is 2.3 db weaker than the maximum reading. The small signal is the maximum field multiplied by:

\[
V'/ (1 + V') = 0.3 / 1.3 = -12.8 \text{ db}.
\]
FIG. 14
APPENDIX II

Calculations for Attenuation Due to Finite Thickness of Coupling Slot

Since the small hole theory of aperture coupling of wave guides requires an infinitely thin wall between the two wave guides, a finite thickness causes a decrease in coupling. The coupling may be considered to be a wave guide mode propagating through the aperture below cutoff. For a slot aperture considered to be similar to a rectangular wave guide, electric coupling propagates as TM_{11} and magnetic coupling propagates as TE_{10}. The attenuation or decrease in coupling is calculated for a length of wave guide equal to the thickness of the slot. The formula for the attenuation is as follows:

$$A_{tt} = \frac{54.6 t}{\lambda_{co}} \sqrt{1 - \frac{\lambda_{co}^2}{\lambda_o^2}} \text{ db}$$

where $t$ is the thickness of the aperture,

$\lambda_{co}$ is the free space cutoff wave length of the aperture for the mode corresponding to the type of coupling,

$\lambda_o$ is the free space wave length.

For the TM_{11} mode and a rectangular slot or a narrow slot with rounded ends:

$$\lambda_{co} = \frac{2w}{\sqrt{w^2 + l^2}}$$

For the TE_{10} mode and a slot with rounded ends:

$$\lambda_{co} = 2 (1 - 0.273w)$$

where $l$ is the length of the slot,

$w$ is the width of the slot.
In the case of coupling by a magnetic field perpendicular to the slot length, the coupling through the slot is as a TE\textsubscript{10} mode with the cutoff wave length equal to twice the slot width. For a slot 0.050 in wide and 0.050 in thick the attenuation is 19.2 db.

See Fig. 15 for curves of attenuation below cutoff.
ATTENUATION BELOW CUTOFF DUE TO FINITE THICKNESS OF SLOT.

ELECTRIC COUPLING
\[ t = 0.050' \]

MAGNETIC COUPLING
\[ t = 0.050' \]

Frequency
- Freq. = 10000 Mc
- Freq. = 5575 Mc
- Freq. = 6800 Mc

FIG. 15
APPENDIX III

Coupling Calculations and Tables

To simplify the calculation of coupling using the formulas developed in part II, the formulas may be broken down into parts. Where there are a number of terms being multiplied under the logarithm, the terms can be converted to decibels separately and added. It may be noted that there is a natural grouping of terms as they refer to slot parameters, waveguide dimensions, type of field coupled to, etc. By tabulating these natural groups, the theoretical coupling may be obtained by merely adding the values (in db) of the proper combination of groups.

The formula for electric coupling may be broken down into six parts where:

\[ C_e = A_1 + A_2 + B_e - \text{Att}_e + D_1 + D_2 \] (for one slot)

1. Term \( A_1 \) includes terms involving frequency and the primary waveguide's dimensions.
   \[ A_1 = 10 \log_{10} \left( \frac{2\pi \lambda_{g_1}}{\lambda_0 \ a_1 \ b_1} \right) \]

2. Term \( A_2 \) includes terms involving frequency and the secondary waveguide's dimensions.
   \[ A_2 = 10 \log_{10} \left( \frac{2\pi \lambda_{g_1}}{\lambda_0 \ a_2 \ b_2} \right) \]

3. Term \( B_e \) includes terms involving the slot size and shape.
   \[ B_e = 20 \log_{10} (p) \]
   where \( \ell \) is the length of the slot and
   \( p \) is the polarizability of the slot's shape.

4. Term \( \text{Att}_e \) is the attenuation beyond cutoff of the slot. This may be obtained from Fig. 15 in Appendix II.
5. Term \( D_1 \) is the term involving the displacement of the slot with respect to the primary wave guide.

\[
D_1 = 20 \log_{10} \sin \left( \frac{\pi x_1}{a_2} \right)
\]

where \( n \) corresponds to the \( \text{TE}_{10} \) mode,

\( x_1 \) is the distance of the slot center from the edge of the wave guide,

\( a_2 \) is the width of the wave guide.

6. Term \( D_2 \) is the term involving the displacement of the slot with respect to the secondary wave guide.

The terms \( D_1 \) and \( D_2 \) are normally unity under the logarithm and equal to zero or. These will not be tabulated. Note that some terms are positive decibels and others are negative decibels and must be added accordingly.

**Electric Coupling Term \((A)\) in decibels.**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Narrow Guide</th>
<th>Wide Guide</th>
<th>Wide Guide</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \text{TE}_{10} ) Mode</td>
<td>( \text{TE}_{10} ) Mode</td>
<td>( \text{TE}_{20} ) Mode</td>
</tr>
<tr>
<td>8500</td>
<td>12.9 dB</td>
<td>9.1 dB</td>
<td>11.3 dB</td>
</tr>
<tr>
<td>9000</td>
<td>12.9</td>
<td>9.2</td>
<td>11.2</td>
</tr>
<tr>
<td>9200</td>
<td>12.9</td>
<td>9.2</td>
<td>11.2</td>
</tr>
<tr>
<td>9375</td>
<td>12.9</td>
<td>9.3</td>
<td>11.0</td>
</tr>
<tr>
<td>9500</td>
<td>12.9</td>
<td>9.4</td>
<td>11.0</td>
</tr>
<tr>
<td>9600</td>
<td>12.9</td>
<td>9.5</td>
<td>11.0</td>
</tr>
<tr>
<td>9800</td>
<td>12.9</td>
<td>9.6</td>
<td>11.0</td>
</tr>
<tr>
<td>10000</td>
<td>12.9</td>
<td>9.6</td>
<td>11.0</td>
</tr>
</tbody>
</table>

**Slot Term \((B_e)\) in decibels. Slot Width 0.060".**

<table>
<thead>
<tr>
<th>Slot Length</th>
<th>( B_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.060&quot;</td>
<td>-96.8 dB</td>
</tr>
<tr>
<td>0.120</td>
<td>87.3</td>
</tr>
<tr>
<td>0.150</td>
<td>79.7</td>
</tr>
<tr>
<td>0.200</td>
<td>76.6</td>
</tr>
<tr>
<td>0.300</td>
<td>71.5</td>
</tr>
<tr>
<td>0.360</td>
<td>72.6</td>
</tr>
<tr>
<td>0.420</td>
<td>71.2</td>
</tr>
<tr>
<td>0.480</td>
<td>69.8</td>
</tr>
<tr>
<td>0.540</td>
<td>68.7</td>
</tr>
<tr>
<td>0.600</td>
<td>67.7</td>
</tr>
<tr>
<td>0.660</td>
<td>66.7</td>
</tr>
</tbody>
</table>
The formulas for magnetic coupling may be handled in the same

where:

\[ G_m = M_1 + M_2 + B_n - \text{Att}_n + A_1 + D_2 + C_1 + C_2 \] (for one slot).

1. Term \( M_1 \) includes terms involving frequency, the primary wave guide's dimensions, and the primary guide wave length. This term may be chosen to represent coupling to either the transverse or the longitudinal magnetic field.

   When coupling to the transverse magnetic field
   \[ M_1 = 10 \log_{10} \frac{2\pi}{\lambda_{g1} a_1 b_1} \]

   When coupling to the longitudinal magnetic field
   \[ M_1 = 10 \log_{10} \frac{2\pi}{\lambda_{c1} a_1 b_1} \]

2. Term \( M_2 \) is the same as \( M_1 \) except that it refers to the secondary wave guide.

3. Term \( B_n \) includes terms involving the slot size and shape. \( B_n \) may be calculated using the polarizability \( (m_2) \) with respect to the magnetic field perpendicular to the slot length as well as \( (m_1) \), the magnetic polarizability with respect to the parallel magnetic field.

   \[ B_n = 20 \log_{10} (a) \]

4. \( \text{Att}_n \) is the attenuation beyond cutoff of the slot. The values for computing the coupling of the magnetic field parallel to the slot are found on Fig. 2.b. For the coupling of the perpendicular magnetic field, the \( \text{Att}_n \) is 0.8 db for each 0.010" thickness for a long slot 0.060" wide.

5. The \( D \) and \( C \) terms refer to the position and angle of the slot.

These are defined in part II, but are normally equal to zero db.
Magnetic Coupling Term (M) in Decibels (positive).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transv. Field</td>
<td>Longit. Field</td>
<td>Transverse Field</td>
</tr>
<tr>
<td></td>
<td>TE(_{10})</td>
<td>(\frac{\pi}{a})</td>
<td>TE(_{10})</td>
</tr>
<tr>
<td>8800</td>
<td>+.94</td>
<td>+10.3</td>
<td>+8.2</td>
</tr>
<tr>
<td>9000</td>
<td>9.6</td>
<td>10.1</td>
<td>8.3</td>
</tr>
<tr>
<td>9200</td>
<td>9.8</td>
<td>9.9</td>
<td>8.5</td>
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<tr>
<td>9375</td>
<td>10.0</td>
<td>9.8</td>
<td>8.6</td>
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<td>9600</td>
<td>10.2</td>
<td>9.6</td>
<td>8.7</td>
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<tr>
<td>9800</td>
<td>10.3</td>
<td>9.4</td>
<td>8.8</td>
</tr>
<tr>
<td>10000</td>
<td>10.5</td>
<td>9.3</td>
<td>8.9</td>
</tr>
</tbody>
</table>

Slot Term (\(B_h\)) in Decibels. Slot width 0.060\(a\).

<table>
<thead>
<tr>
<th>Slot Length</th>
<th>Parallel Field</th>
<th>Perpendicular Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.060(a)</td>
<td>-88.8 db</td>
<td>-88.8 db</td>
</tr>
<tr>
<td>0.120</td>
<td>73.4</td>
<td>73.4</td>
</tr>
<tr>
<td>0.180</td>
<td>61.5</td>
<td>79.9</td>
</tr>
<tr>
<td>0.240</td>
<td>58.3</td>
<td>77.2</td>
</tr>
<tr>
<td>0.300</td>
<td>53.2</td>
<td>75.2</td>
</tr>
<tr>
<td>0.360</td>
<td>49.2</td>
<td>73.6</td>
</tr>
<tr>
<td>0.420</td>
<td>45.8</td>
<td>72.2</td>
</tr>
<tr>
<td>0.480</td>
<td>43.0</td>
<td>71.0</td>
</tr>
<tr>
<td>0.540</td>
<td>40.0</td>
<td>69.3</td>
</tr>
<tr>
<td>0.600</td>
<td>37.6</td>
<td>69.0</td>
</tr>
<tr>
<td>0.660</td>
<td>35.4</td>
<td>68.3</td>
</tr>
</tbody>
</table>

As an example consider the coupling of the parallel field in the following situation. The slot is parallel to the longitudinal field of the TE\(_{20}\) mode of the wide primary wave guide. The slot is parallel to the transverse field of the narrow secondary wave guide. The frequency is 9375 ac and the slot is 0.420\(a\) long and 0.050\(a\) thick.

\(M_1 = +8.9, M_2 = +10.0, B_h = -45.8, -Atth = -2.6.\)

The coupling for one such slot is -29.5 db. Whenever coupling is being calculated for two slots that are adding, the coupling is 6 db closer. Two equal fields adding, double the fields and quadruple the power flow.
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