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THE RESPONSE OF A SUBMERGED CYLINDRICAL SHELL TO AN AXIALLY PROPAGATING ACOUSTIC WAVE

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The Response of a Submerged Cylindrical Shell to an Axially Propagating Acoustic Wave

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1. Introduction. In this report, we shall discuss the dynamic response of a submerged elastic cylindrical shell when a plane acoustic wave propagates relative to the shell as indicated in Fig. 1. We shall be concerned only with the deformation and stress in certain localities where these quantities are large. The results are found in a very concise form and are of more general applicability than is implied by the title.

2. The interaction problem. The motion of the shell indicated in Fig. 1 is governed by the usual conservation laws as applied to the shell and to the surrounding fluid. For axially symmetric deformations of the shell, the conservation of momentum requirements imply that

$$EIu^{IV} + \sigma hu'' + \rho hu_{tt} - \rho Iu''_{tt}$$
$$+ (Eh/R^2)u = - p(R,x,t). \qquad (2.1)$$

Here, E, ρ, h, R, u are, respectively, the elastic modulus, density, thickness, radius, and radial displacement of the shell; σ is the average over the shell thickness of the axial compressive stress [i.e.,

$$\sigma = - \int_{R-h/2}^{R+h/2} (\sigma_x/h)dr], \quad I = h^3/12,$$

p is the externally applied over-pressure, and primes denote differentiation with regard to the axial coordinate x. It is to be noted that we have retained the rotational inertia terms in the bending theory. Our object, in doing so, is to demonstrate that this term has no appreciable effect on the results even in high-speed impact problems. It should also be noted that these equations are pertinent to an unstiffened shell. We shall consider the stiffened shell in a later section.

The pressure p is associated with the acoustic field surrounding the shell. This consists of a contribution from the incident wave plus that associated with the shell motion. Although we could readily formulate the problem in a form which accounts for the detailed acoustic field, it is convenient to approximate p(R,x,t) by

$$p = p_o + \rho_f a u_t(x,t) \tag{2.2}$$

where ρ_f is the fluid density, a the fluid acoustic speed, and p_o is the overpressure in the incident acoustic wave. The justification of this approximate formula is found in the fact that the pressure associated with the motion of a plane obstacle normal to itself is $\rho_f aF(t)$, where F(t) is the normal velocity.

The equation governing u(x,t) then becomes

$$EIu^{IV} + \sigma hu'' + \rho hu_{tt} - \rho Iu''_{tt} + \rho_f au_t + (Eh/R^2)u = - p_o S(t - x/a) \tag{2.3}$$

where S(t - x/a) is the conventional step function.

We shall be interested in two basic solutions of this equation. The first is that for which $u(x,t) \equiv w(t - x/a)$.

This function must satisfy the ordinary differential equation (where $\xi = t - x/a$)

$$\left(\frac{EI}{a^4} - \frac{\rho I}{a^2}\right)w^{IV} + h\left(\rho + \frac{\sigma}{a^2}\right)w'' + \rho_f a w' + \left(\frac{Eh}{R^2}\right)w = -\, p_o S(\xi). \qquad (2.4)$$

The desired solution has the form

$$w = \begin{cases} -\left(\frac{p_o R^2}{Eh}\right)\left[1 + a_1 e^{\lambda_1 \xi} + a_2 e^{\lambda_2 \xi}\right], & \xi > 0 \\ \left(\frac{p_o R^2}{Eh}\right)\left[a_3 e^{\lambda_3 \xi} + a_4 e^{\lambda_4 \xi}\right], & \xi < 0 . \end{cases} \qquad (2.5)$$

The λ_j are the roots of

$$\left(\frac{EI}{a^4} - \frac{\rho I}{a^2}\right)\lambda^4 + h\left(\rho + \frac{\sigma}{a^2}\right)\lambda^2 + \rho_f a \lambda + \frac{Eh}{R^2} = 0, \qquad (2.6)$$

and the a_j are given by[1]

$$a_j = \prod_{k \neq j}' \frac{\lambda k}{\lambda_j - \lambda_k} . \qquad (2.7)$$

Equations (2.5) and (2.7) are valid, of course, only if the λ_j are distinct and are such that λ_1, λ_2 have negative real parts, and λ_3, λ_4 have positive real parts.

An example of some interest is one for which:

$$R = 8', \quad h = 1'', \quad a = 5000'/\text{sec}, \quad E/\rho = (17{,}000'/\text{sec})^2,$$

$$\sigma = 25{,}000\ \#/\text{in}^2, \quad \rho_f/\rho = 1/8, \quad E = 3.10^7 \#/\text{in}^2.$$

For these dimensions (in units, sec^{-1}), $\lambda_1 = -\,650$, $\lambda_2 = -\,7500$, $\lambda_{3,4} = 7300 \pm 66000i$. The coefficient, $-a_1$, is

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[1] These are chosen so that u, u', u'', u''', are continuous at $\xi = 0$.

near unity and $|a_2| = O(|\lambda_1/\lambda_2|)$, $|a_3| = O(|\lambda_1\lambda_2/2\lambda_3^2|)$.

The quantities of interest are w and w''/a^2 since these are directly related to the hoop and bending stresses, respectively. The orders of magnitude of these terms are given by

$$|w|_{max} = - p_oR^2/Eh$$

$$|w''/a^2|_{max} = O[\lambda_1\lambda_2\ p_oR^2/Eha^2].$$

It is now convenient to turn to the second basic solution of Eq. (2.3). We wish, in fact, to find the deformation of the shell which is associated with the conditions $p_o = 0$, $u(x,t) = 0$ for $t < 0$, $u(0,t) = f(t)$, and $u'(0,t) = 0$. It is evident that a combination of this solution with the foregoing will allow a treatment of practical problems wherein the shell is supported in one manner or another.

Our purpose is most readily accomplished by introducing the Laplace transform of u,

$$\bar{u}(x,s) = \int_0^\infty e^{-st}u(x,t)dt.$$

The conventional use of this transform leads to the equation

$$EI\bar{u}^{IV} + (\sigma h - \rho Is^2)\bar{u}'' + (\rho hs^2 + \rho_f as + Eh/R^2)\bar{u} = 0.$$

The pertinent solution of this ordinary differential equation is

$$\bar{u}(x,s) = \frac{\bar{f}(s)}{\eta_2 - \eta_1}\,[\eta_2(s)e^{-\eta_1 x} - \eta_1(s)e^{-\eta_2 x}]. \qquad (2.8)$$

Here, $\bar{f}(s)$ is the transform of the given motion u(0,t), and

$$\eta_{1,2}(s) = [M \pm (M^2 - N)^{1/2}]^{1/2},$$

where

$$-M = (\sigma h - \rho I s^2)/2EI, \quad N(s) = (\rho h s^2 + \rho_f a s + \frac{Eh}{R^2})/EI.$$

This transform would be rather difficult to invert with any degree of precision but the most useful piece of information is readily extracted. The deformation (and stress) of major concern is that associated with the bending near $x = 0$. However, using Eq. (2.8),

$$\bar{u}_{xx}(0,s) = - N^{1/2}\bar{f} \qquad (2.9)$$

and this can be inverted to give

$$u_{xx}(0,t) = - (\rho h/EI)^{1/2} L[f(t)], \qquad (2.10)$$

where $L[f(t)]$ can be written

$$L[f(t)] = f_t + \alpha f - k^2 \int_0^t e^{-\alpha(t-\tau)} \frac{I_1[k(t-\tau)]}{k(t-\tau)} f(\tau) d\tau$$

$$= f_t + \epsilon f + k \int_0^t e^{-\alpha(t-\tau)} (I_0[k(t-\tau)] + I_1[k(t-\tau)]) \{f'(\tau) + \epsilon f(\tau)\} d\tau. \qquad (2.11)$$

The essential feature of this result is that each of the integrals in Eqs. (2.11) is positive when $f(t)$ and $f'(t)$ are positive in the time interval of interest, e.g., for $0 < t < t_0$. Thus,

$$- (EI/\rho h)^{1/2} u_{xx}(0,t)$$

lies between the values $f_t + \epsilon f$ and $f_t + \alpha f$ where ϵ is the smaller root of $N(-s)=0$, $2k$ is the positive difference of the two roots, and 2α is the sum of the roots, i.e., $\rho_f a/\rho h$. In the example used earlier, $\alpha \simeq 3750 \text{ sec}^{-1}$ and $\epsilon \simeq 650 \text{ sec}^{-1}$.

3. Properties of the foregoing results. Some general observations associated with the work of section (2) can now be stated. We note first that the rotational inertia term has a negligible effect on the solution of the first problem. In fact, its inclusion affects the size of λ_3 and λ_4 by about 5% and λ_1 and λ_2 not at all. In the latter result, this term has no effect whatever (rigorously). The deformation at $x \neq 0$ may be affected by this term (must be, in fact) but at $x = 0$, no contribution of this item is present. As a matter of fact, a simple boundary layer type analysis will demonstrate that only near $x = (E/\rho)^{1/2}t$ and for $x > (E/\rho)^{1/2}t$ can this term seriously modify the result.

We should also note that, for a beam vibration problem of this type, the terms in u and u_t would be absent and $u_{xx}(0,t)$ is precisely given by

$$u_{xx}(0,t) = -(\rho h/EI)^{1/2}u_t(0,t). \tag{3.1}$$

We should also note that the term representing the axial membrane contribution (i.e., $\sigma u''$) plays a completely negligible role. This is fortunate for the investigator since, in many problems of interest, σ will vary considerably during the interesting time interval[2]. If σ were to exceed the yield stress, of course, this remark (and the foregoing analysis, as well) would be invalid. Note, however, that during the deformation, the axial membrane deformation contributed by the bending is such as to decrease the value of σ.

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[2] As implied earlier, all of our remarks apply to the time interval where u and u_t are positive.

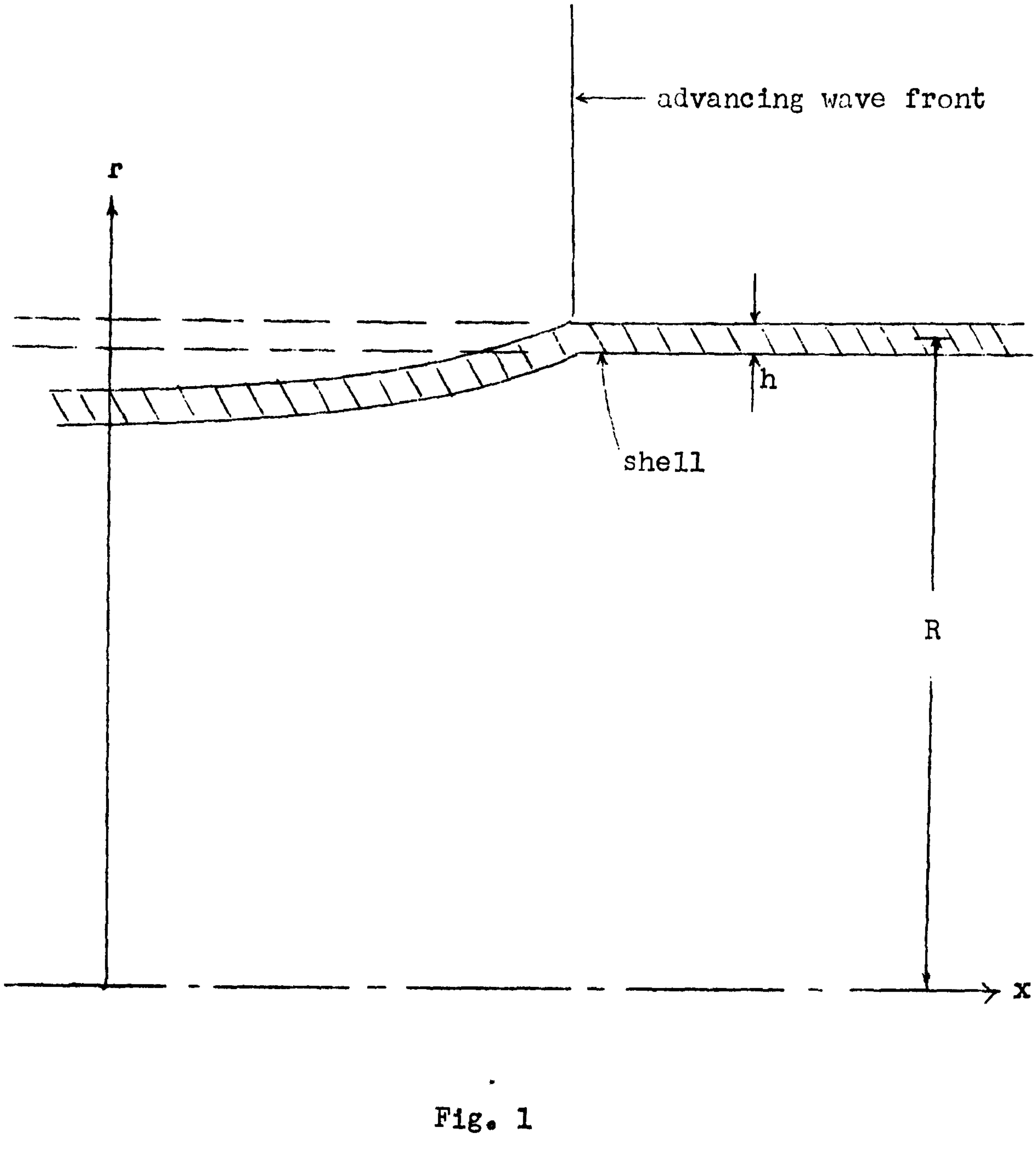

Fig. 1

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