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This paper describes the identification of some of the root causes of one of the components of tank gun accuracy errors, namely occasion-to-occasion variability. Occasion-to-occasion variability is the shift of the mean impact point for a given tank/gun tube combination from one firing occasion to another. Unless the cannon and fire control system of the tank can be calibrated for each firing event through live fire zeroing, its occasion-to-occasion variability will have an adverse effect on the accuracy of tank cannons. Unfortunately, live fire zeroing before each firing event is impractical for several reasons. First and foremost is the cost. Not only does each ammunition type require a separate zero, but each ammunition type requires a minimum of three rounds to achieve calibration. Even given unlimited ammunition, the logistical requirements of providing this much ammunition to each tank unit is beyond the sustaining capability of those units. Additionally, the tactical situation will rarely allow the time, space or security needed to zero each of the unit's tanks. Finally, there is currently no way to determine the end and start of new firing occasions. A calibration zero may be minutes or days long, depending on many different variables.

Many factors influence the magnitude of occasion-to-occasion variability. Identification and analysis of these factors is critical to understanding and solving the occasion-to-occasion variability problems in the M1 series tank. Over the last several years, an effort by the Army Research Laboratory has identified and quantified several components to this error source. These include: errors associated with optical alignment of the cannon to the fire control system, uncorrected gun jump which is dependent on the propellant temperature of the ammunition, variation in the linear recoil motion which couples into the angular motion of the gun and lot-to-lot variations of the ammunition used in the tank.
Identification and Quantification of Sources of Occasion-to-Occasion Elevation Variability in Tank Gun Accuracy

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1. INTRODUCTION

During the ground offensive of the 1991 Gulf War, one of the most spectacular examples of the high tech advantage enjoyed by the Coalition Forces was the accuracy of the M1 series tank. Historically, tank battles have been fought at ranges under 1000 meters. This has been due to a combination of intervisibility problems and the inability of tanks to accurately engage at longer ranges. This all changed during the Gulf War. Intervisibility problems were minimized by the flat desert terrain and the M1A1 tank was able to successfully engage targets out to three kilometers and beyond.

As a result of the example of the Gulf War, developers of tanks around the world can be expected to continue efforts aimed at improving the accuracy of their tanks. The implication for the U.S. Army is that we must also continue to improve the accuracy of our own tanks or lose the advantage that we currently enjoy. Given the current political and fiscal realities this means improving the M1 series tank for a number of years. Research efforts in this area can also extend to almost any other cannon system, direct and indirect fire, that may be employed on future U.S. weapons. This paper describes the identification and possible fixes for some of the root causes of one of the components of tank gun accuracy errors, namely occasion-to-occasion variability.

2. COMPONENTS OF TANK GUN ACCURACY

Generally, tank cannon accuracy is broken into several components for ease of analysis. The statistical means and standard deviations of groups of shot impacts on targets are used to describe these components of accuracy. The accuracy components generally used are round-to-round dispersion, occasion-to-occasion variability, central tendency and tank-to-tank variability. These various terms are described below and illustrated with Figure 1.

Figure 1 represents a target with the aim point located at the origin. In this example, rounds were fired over three occasions. On each occasion, three rounds were fired and the target impacts were recorded.

Each shot fired is represented on the target with open squares, circles or triangles to distinguish the impacts of the three firing occasions. The average of each of these groups is represented by a solid symbol of the same
kind. This average value is known as the occasion zero and the standard deviations associated with these three-round groups are known as the round-to-round dispersion. On Figure 1, the estimate of the round-to-round dispersion for each occasion is indicated by the box surrounding each occasion's average impact point. In this example, the average value of the round-to-round dispersion is 0.14 milliradians (mrad) in azimuth and 0.27 in elevation. These are typical values, although this component of accuracy is very much round-type dependent.

The standard deviation of the means of the three-round shot groups represents occasion-to-occasion variability. Occasion-to-occasion variability in this example is 0.05 mrad in azimuth and 0.37 mrad in elevation. Generally, occasion-to-occasion variability is estimated to average 0.25 mrad in azimuth and elevation across the tank fleet and across ammunition types.

The mean value of all nine shots for this tank is an estimate of this tank's central tendency, represented by an X in Figure 1. In this example, the tank has an estimated central tendency of 1.26 mrad in azimuth and 0.23 mrad in elevation.

Finally, tank-to-tank variability for a particular ammunition type is a measure of the dispersion of the average central tendencies across the fleet of tanks. Tank-to-tank variability is estimated at 0.25 mrad in both azimuth and elevation. The mean central tendency across the fleet is normally referred to as the fleet zero value for an ammunition type.

3. SOURCES OF OCCASION-TO-OCCASION ERROR

Determination of the sources of occasion-to-occasion error is primarily a process of looking for those events that cause a change between firing occasions, either in the armaments system of the tank itself or in the ammunition that will be fired. Some of these events, such as improper maintenance of the cannon system or damage to the ammunition, are obvious.
While the effect of such causes can be significant in the field, they will not be considered in this paper as they are correctable through training and proper procedure.

The other root causes of occasion-to-occasion variability are more subtle. They may happen slowly over time or by abrupt changes in the tank or environment. These are problems over which the soldier in the field can exercise little to no control. These problems must be corrected through hardware changes in the tank itself or by accounting for them in the fire control solution calculated for each round.

3.1 Definition of a Firing Occasion

Different firing occasions are defined in terms of time, i.e. a long period (hours or days) between rounds; or in terms of significant events between rounds, such as maintenance on the weapon, large environmental changes or moving the tank to new firing positions. For the purposes of this paper, a firing occasion is defined to include all of those rounds fired from the same tank during a time period in which no significant events (with the exception of firing the weapon) have occurred that could affect the fire control system on the tank, the cannon system, or ammunition.

3.2 Muzzle Velocity Variation

Muzzle velocity variation affects accuracy by varying the projectile's time of flight to the target. The time of flight variation, in turn, varies the gravity drop of the projectile during its trajectory and hence there is an elevation error. Occasion-to-occasion variations in muzzle velocity are primarily caused by two distinct sources - the daily temperature cycle and a lot-to-lot variation in the average muzzle velocity for an ammunition type.

3.2.1 Muzzle Velocity Variations Due to Temperature Differentials

The ignition and burning rate of ammunition propellant varies with temperature. Warmer propellant ignites and burns at a faster rate than cooler propellant. This means that projectiles are accelerated more quickly when the propellant is warm and more slowly when the propellant is cool. This results in a relationship between muzzle velocity and propellant temperature. This relationship is well understood and is accurately modeled with a second degree polynomial. Therefore, if the propellant temperature of the ammunition is known, the muzzle velocity can be accurately calculated.

In modern tanks, ammunition temperature is estimated by measuring the air temperature in the ammunition storage compartment (hereinafter referred to as the bustle). There are two problems with this approach. First, there is a difference in the round temperature based on the round's location in the storage compartment. Because of this difference, one bustle air temperature measurement cannot be accurate for all the ammunition. Rounds stored near the top of the bustle tend to heat and cool more quickly due to thermal radiation transfer through the top of the turret, while ammunition in the bottom of the bustle is insulated to a greater degree by the air and ammunition above it. The second problem with measuring bustle air temperature to estimate ammunition temperature is that air and ammunition change temperature at different rates, producing a phase shift between the diurnal temperature cycles of the bustle air and the
ammunition in the compartment. Thus, during the day, the temperature cycle for ammunition near the top of the bustle precedes the bustle air temperature cycle by about 1.5 hours while the ammunition temperature cycle at the bottom lags the

![Diagram of ammunition and compartment temperature cycles](image)

**Figure 2. Ammunition and Ammunition Compartment Temperature Cycles**

...
Training Practice (HEAT-TP) and M865 Training Practice, Cone Stabilized, Discarding Sabot (TPCSDS) ammunition respectively. AV is the difference between the average and calculated muzzle velocities.

A statistical analysis of the AV provides an estimated mean AV over the entire time of consideration (72 hours, or three complete cycles, for this analysis) and a standard deviation. When the mean AV is not equal to zero, the muzzle velocity calculation based on the bustle air temperature is biased. A positive mean for AV indicates that the muzzle velocity estimates tend to be underestimated, while a negative mean indicates an overestimation. Mean-squared error (MSE) is a measure of closeness that takes into consideration not only the variance of an estimator, but also the bias of that estimator. It is given by the formula, \( \text{MSE} = (\text{Bias})^2 + \sigma^2 \). MSE is used here as a measure of the muzzle velocity estimation error. Table 1 lists the mean AV, the standard deviation and the velocity MSE resulting from temperature measurement errors.
An estimate of the occasion-to-occasion elevation error caused by calculating muzzle velocity based on bustle air temperature is found by calculating two trajectories. One trajectory is calculated for a round launched at the muzzle velocity estimated with the bustle air temperature. The other trajectory is calculated with the same muzzle velocity plus or minus the error term from Table 1. The trajectories are differenced and an angular measure of the error is calculated as a function of range. The two trajectories were calculated using the Ballistic Research Laboratory General Trajectory Program [2]. Figure 5 plots the temperature related muzzle velocity error as a function of range for both the M831 and M865.

![Figure 5. Error Due to Temperature Related Muzzle Velocity Variation](image)

Notably, this error is range dependent and increases with range. The M831's error is 0.06 mrad at 3000 meters, while the error for the M865 is 0.04 mrad at the same range. The M831's greater sensitivity to muzzle velocity variation is due primarily to its lower initial velocity and its greater retardation (loss of velocity as a function of range). The high retardation is the result of the M831's high drag shape. It should be pointed out that the M865 is cone stabilized and is also a relatively high drag projectile. The occasion-to-occasion temperature related muzzle velocity error associated with a fin stabilized, service KE projectile is, therefore, significantly lower than that of the M865, due to the lower drag associated with fin stabilization.
3.2.2 Lot-to-Lot Muzzle-Velocity Variations

Service ammunition is generally manufactured in lots of several thousand rounds. The quality of each lot is tightly controlled and is verified through lot acceptance testing. One way in which quality is controlled is through the use of single lots of component parts in the manufacture of completed rounds of ammunition. For example, only one lot of propellant will be used in the manufacture of a lot of completed rounds. While this reduces variability within a lot, variation between lots is to be expected.

For ease of accountability and accuracy, tank ammunition is normally issued to a unit from the same lot. This means that accuracy errors occurring as a result of lot-to-lot variations normally show up as occasion-to-occasion errors, rather than as round-to-round errors. One ammunition characteristic that varies from lot-to-lot is the average muzzle velocity for each lot of ammunition. Therefore, lot-to-lot muzzle velocity variations manifest themselves as occasion-to-occasion accuracy errors.

The mean muzzle velocity for a lot of ammunition can be found in the lot acceptance test records for each lot of ammunition. Records for a total of 36 lots of M831 HEAT-TP ammunition and 29 lots of M865 TPCSDS ammunition were examined. The mean muzzle velocity and the standard deviation about the mean was calculated for each ammunition type to get an estimate of the lot-to-lot muzzle velocity variation. When the mean muzzle velocity is not equal to the required muzzle velocity, bias is introduced into the superelevation correction for the gun. As with the temperature related muzzle velocity variation, a mean-squared error term is used to estimate error in order to account for the bias and the variation. Table 2 is a listing of the required muzzle velocity, the mean muzzle velocity across the lots of tested ammunition, the standard deviation and the MSE resulting from the difference between the required and actual muzzle velocities.

Figure 6. Error Due to Lot-to-Lot Muzzle Velocity Variation
Table 2. Lot-to-Lot Muzzle Velocity Statistics

<table>
<thead>
<tr>
<th></th>
<th>Standard MV</th>
<th>Mean MV</th>
<th>Mean ΔMV</th>
<th>(σ_{MV})</th>
<th>(\sqrt{MSE})</th>
</tr>
</thead>
<tbody>
<tr>
<td>M831</td>
<td>1140.0</td>
<td>1139.03</td>
<td>-0.97</td>
<td>3.95</td>
<td>4.07</td>
</tr>
<tr>
<td>M865</td>
<td>1700.0</td>
<td>1700.90</td>
<td>0.90</td>
<td>4.65</td>
<td>4.74</td>
</tr>
</tbody>
</table>

All Units Are Meters/Second

An estimate of the occasion-to-occasion elevation error caused by lot-to-lot muzzle velocity variation may be found in a manner similar to that used to find the temperature related muzzle velocity variation error. Figure 6 plots the lot-to-lot error as a function of range for both the M831 and M865. This error is also range dependent and increases with range. The M831’s lot-to-lot error is 0.19 mrad at 3000 meters, while the error for the M865 is 0.07 mrad at the same range. For the same reasons stated above, the M831’s greater sensitivity to muzzle velocity variations is evident. Again, service KE ammunition can be expected to be even less sensitive to muzzle velocity variations than M865.

3.3 Gun Dynamics

A change in the pointing angle of the muzzle during the shot process will cause a projectile to exit the cannon at a different launch angle than was initially laid. When this effect is predictable, it may be accounted for in modern fire control computers with a computer correction factor (CCF). In fact, one component of the CCF in the M1 series tank is the average value of the muzzle pointing angle at shot exit. Variation in the average muzzle pointing angle at shot ejection between rounds and from firing occasion to firing occasion makes it impossible to provide a precise value for the CCF; therefore, finding the cause of muzzle angle variation between firing occasions is needed to reduce occasion-to-occasion variability.

The dynamics of the cannon are known to affect occasion-to-occasion variability in at least two significant ways. First is a coupling of the linear recoil of the cannon into its angular motion, hence a coupling of linear recoil variation to muzzle angle variation [3]. Second, the temperature of the ammunition determines the amount of time it takes the projectile to travel the length of the cannon. If the cannon has any angular motion during this in-bore time, ammunition fired at different temperatures will exit the cannon with varying muzzle angle conditions [4].

In large tank cannons, there are several forces which create turning moments about the trunnions during firing. The dominant moment is due to a breech mass imbalance, which causes the center of gravity of the recoiling mass to be lower than the centerline of the gun. This offset acts as a lever arm when the pressure of the burning propellant accelerates the gun by pushing against the breech block along the centerline. The resultant couple, known as the powder pressure couple, induces a torque and subsequent rotation about the trunnion of the gun system [5]. In addition to the powder pressure couple, forces exerted by the radially expanding gun tube against the gun tube bearings and the resistance of the elevating mechanism to rotation create additional turning moments (Figure 7).
Simply modeling these forces results in the following equation of rigid body motion for the cannon:

$$\ddot{\theta} = \frac{\ddot{r}_1 r_2 m + \sum_{i=5}^{n} F_i r_i - [b r_3 \dot{\theta} + k r_3 \dot{\theta}]}{I_{cg} + m (r_1^2 + r_2^2)} = \frac{\ddot{r}_1 r_2 m + \sum_{i=5}^{n} F_i r_i - [b r_3 \dot{\theta} + k r_3 \dot{\theta}]}{I_T}$$

where $\theta$ is the angular rotation of the cannon about the trunnion. The powder pressure couple is modeled by $r_1$, the magnitude of the linear recoil of the gun along its centerline; $r_2$, the offset between the center of gravity of the gun and the centerline of the cannon; and $m$, the mass of the cannon. The resistive forces at the elevating mechanism are modeled by $b$ and $k$, the damping and spring constants of the elevating mechanism, and $r_3$, the distance between the elevating mechanism and the trunnion. The forces of gun tube expansion against the gun bearings are modeled by $F_i$, a distance of $r_i$ from the center of gravity. $I_{cg}$ equals the moment of inertia of the cannon about the center of gravity, and $I_T$ equals the moment of inertia about the trunnion [3].

![Figure 7. Simple Tank Cannon Model](image)

3.3.1 Recoil Variation.

It is important to note that the equation of the angular motion has a term, $r_1 r_2 m$, that is dependent on the recoil acceleration. This implies that a change in recoil motion will be seen as a change in the angular motion of the cannon. Recoil motion can vary for a number of reasons. These include maintenance of the recoil system, changes in recoil hydraulic fluid temperature and viscosity, and varying hydraulic pressures between occasions. Also of note is the fact that this same term depends on the magnitude of the vertical offset, $r_2$, between the center of gravity and the centerline of the gun. By moving the cannon's center of gravity with balancing weights, the offset between the center of gravity and the centerline can be eliminated. With zero offset, all recoil loads act along the centerline of the cannon. Thus, the angular motion due to the powder pressure couple is eliminated. By comparing the motion of the cannon with and without the balancing masses, it is possible to gain some insight into the variability of the angular motion that results from changes in recoil motion.
A test was conducted in which the center of gravity of the recoiling mass of an M256 cannon was moved to the centerline of the gun [6]. This was accomplished by adding mass to the top of the breech of an M256 tank cannon (Fig. 8). This configuration will be referred to as the balanced breech. Reference to the standard breech will indicate that the weights were not attached to the breech. During this test, the muzzle angle of the cannon was measured with proximity probes [7]. Two different gun tubes were used for this test and both were configured with and without the balancing weights.

Figure 9 plots the muzzle pointing angle of one of the cannons during four firing occasions with M831 HEAT ammunition - two occasions with the balancing weights and two without. Shot exit time is at 0.0 milliseconds (msec). The occasions were separated by several days and movement of the tank. The muzzle pointing angle plots of the two occasions fired in the standard configuration fall into two populations, while those of the balanced firings cannot really be separated.
Figure 10. Average Muzzle Pointing Angle. 2 x Firing Occasion. Balanced Breech and, 2 x Firing Occasion. Standard Breech

This is seen more clearly in Figure 10, where the plots of individual shots in each firing occasion are averaged together. The averaged plots for the balanced configuration's two occasions nearly lie on top of one another, while those in the standard configuration are quite distinct.

These same trends are clear with the other gun tube and ammunition type [3]. Figures 11 and 12 are plots of the muzzle pointing angle variation for a cannon firing, respectively, M831 HEAT and a kinetic energy (KE) round whose in-bore characteristics are similar to M865.

Figure 11. Variation in Vertical Muzzle Pointing Angle. HEAT Ammunition. 4 Rounds/Occasions
Four rounds of each ammunition type were fired per gun to obtain estimates of muzzle pointing angle variation. Combinations of time, movement of the tank, and temperature change broke up the four round groups into firing occasions, though each round did not necessarily represent a new occasion. The elevation variation of muzzle pointing angle at shot exit for the standard configuration is approximately .19 mrad with M831 HEAT-TP and .14 mrad with the KE ammunition. The balanced configuration has variations at shot exit of .07 and .04 mrad respectively for M831 and KE ammunition. The difference in variation between the two configurations is due to the decoupling of the linear recoil motion and the angular motion of the muzzle.

The residual muzzle angle variability seen in the balanced configuration is a round-to-round effect. Assuming that the causes of the residual variability are independent of the recoil effects, they may be removed in a root-sum-square sense:

\[ \sigma_{\text{rv}}^2 = \sigma_{\text{rv}}^2 + \sigma_{\text{rr}}^2 \]

For HEAT ammunition
\[ \sigma_{\text{rv}} = 0.177^2 = \sqrt{0.19^2 - 0.07^2} \]

For KE ammunition
\[ \sigma_{\text{rv}} = 0.134^2 = \sqrt{0.14^2 - 0.04^2} \]

where \( \sigma_{\text{rv}} \) = total muzzle pointing angle variation and \( \sigma_{\text{rr}} \) = the residual round-to-round muzzle velocity variation.

The muzzle angle variation that is the result of recoil variation has both round-to-round effects and occasion-to-occasion recoil effects. While it is not possible to separate the round-to-round and occasion-to-occasion effects, the information gives some idea about the magnitude of the two. Even assuming that the occasion-to-occasion effects make up only half of the total, this represents an error of .13 and .09 mrad for HEAT-TP and KE ammunition respectively.
3.3.2 Propellant Temperature.

As mentioned earlier, propellant burn rates are a function of propellant temperature. As a result, muzzle velocity and in-bore time varies with temperature. In addition to the muzzle velocity errors, there is a significant impact bias for certain ammunition types that is dependent on ammunition temperature and is independent of the change in gravity drop associated with the muzzle velocity variation.

In order to determine the muzzle angle at shot exit, equation (1) is integrated twice. The muzzle angle at shot exit therefore depends on the limits of this integration, which are defined by the in-bore time of the projectile. Since the propellant temperature directly affects the in-bore time of the projectile, it will also affect the muzzle angle at shot exit [4].

Figure 13 is a plot of the muzzle pointing angle for firings of M831 HEAT ammunition which was conditioned to three different temperatures. Of note in these plots is that the general shapes of the curves are very similar. The major difference between the plots appears to be a time shift. Note that the pointing angle curve around shot exit time (0.0 msec) has a very steep slope. This causes a significantly different muzzle pointing angle at shot exit between the three conditions due to the time shift.

Currently, the muzzle pointing angles of only a limited number of ammunition types conditioned to different temperatures before firing have been measured. There are, however, computational methods for determining the pointing angles for different ammunition types across a range of temperatures [4]. There is also target data (i.e., measured holes in targets) for most current ammunition types that were fired after being conditioned to different temperatures. It is this target data that is used here to estimate the occasion-to-occasion effects of muzzle angle dependence on propellant temperature [8].

![Figure 13. Muzzle Pointing Angle, M831, 3 Propellant Temperatures](image)
Target impact data was collected over a period of several years. This data included ammunition type, temperature of the propellant and the impact point on the target. A regression analysis of propellant temperature versus mean impact point was used to determine the linear relationship between these two variables for each ammunition type. Because of the many differences in the tests themselves, only the simplest linear trends were sought in this analysis [8].

Figure 14 shows the plot of corrected target impact in elevation versus ammunition temperature for M831 HEAT ammunition. Visual inspection indicates a positive relationship between the two variables over the entire temperature range examined; a statistically significant slope confirmed the trend. The linear regression equation is given as:

$$\hat{y} = -0.43 + 0.0046T$$

where \(\hat{y}\) is the expected elevation impact point in mrad, -0.43 is the intercept in mads, 0.0046 is the slope in mads/°F, and \(T\) is the ammunition temperature in °F.

![Regression Line](image)

**Figure 14. Target Impact Elevation vs. Temperature, M831**

Figure 15 shows the plot of elevation impact versus ammunition temperature for M865. The interesting feature of this plot is the lack of a temperature dependency on elevation impact. This is due to the fact that around shot exit time for M865, the muzzle pointing angle is at a maximum point. This means that the time shift of the pointing angle plot does not result in large pointing angle differences for this ammunition type. Typically though, service KE ammunition has a temperature dependency whose slope is similar to M831 instead of M865.

Using the M831 regression equation as an estimate of the muzzle pointing angle dependence on propellant temperature, an approximate value of the occasion-to-occasion variability due to propellant temperature can be calculated. Temperature data in some likely area of operations is used to calculate muzzle pointing angles for a period of one year. From this population
of pointing angles the standard deviation is determined to provide an estimate of the error. Since tanks are used all year and combat occurs 24 hours a day, this approach seems reasonable.

Temperature values were taken every three hours in a climate typical of the middle latitudes in the northern hemisphere. Muzzle pointing angles were calculated for each of these temperature values and the standard deviation of the set of pointing angle data was calculated. The standard deviation thus calculated provides an estimate of the occasion-to-occasion dispersion due to propellant temperature of 0.093 mrad.

Calculating an estimate of the dispersion due to propellant temperature in this way is necessarily dependent on the climate in the part of the world that is used. Equatorial regions do not experience as much temperature variation during the year and the dispersion should be correspondingly lower. High-desert regions, on the other hand, experience greater temperature variation and the dispersion may be greater. Since U.S. tanks have a world-wide contingency mission, using a climate that is between these two extremes is reasonable.

One final note, the variation in ammunition temperature across the bustle implies a round-to-round muzzle angle variability. From Figure 2, the maximum temperature difference between rounds of ammunition is found at 21.45 hours. The hottest round, at the top of the bustle is 118.3 °F and the coldest, at the bottom of the bustle, is 104.9 °F. Applying equation 5 gives a maximum muzzle angle spread of 0.06 mrad. The variability of ammunition temperature is much smaller than the maximum spread. Over the three days of testing, the pooled ammunition temperature variability was only 3.4 °F. Using this temperature variability with equation 5 provides an estimate of the overall muzzle pointing error produced by ammunition temperature variability within the bustle. This error is only 0.02 mrad.
3.3.3 Lot-to-lot Muzzle Pointing Angle Variation

A muzzle angle error, similar to the temperature dependent error just described, occurs as a result of lot-to-lot muzzle velocity variations. Since the variable muzzle velocity implies a variable in-bore time for the projectile, the pointing angle at shot exit will vary with the lot-to-lot muzzle velocity variations.

It is possible to estimate the pointing angle error due to lot-to-lot muzzle velocity variations. This is accomplished by a similarity method. This method starts with the assumption that the general shape of the pointing angle-time plot will be similar for rounds shot at various muzzle velocities, but that the plot will shift in time. This assumption appears to be reasonably accurate for the muzzle velocity variations seen. Figure 13 above is an example of this phenomenon when the muzzle velocity variations are the result of propellant temperature variation, as opposed to lot-to-lot variation.

The idea behind the similarity solution assumes that the in-bore projectile velocity/time curve will be similar in shape across some small variation of muzzle velocities. With this assumption it is possible to write an algebraic equation that determines in-bore time. The dependent variable of the similarity function is a ratio of projectile velocity divided by muzzle velocity. At shot exit, this ratio is always equal to one. The independent variable is a ratio of the time \( t \) from propellant ignition, divided by some reference time \( t_{ref} \). \( t_{ref} \) is defined to be the gun length \( L \) divided by the muzzle velocity for that particular lot of ammunition \( v_m \). Since the in-bore time vs. velocity curve is assumed similar for projectiles with variable muzzle velocities, the ratio of \( t/t_{ref} \) is a constant at shot exit for all lots. Using the values of a standard lot of M831 HEAT-TP \( t = 9.4 \text{ msec}, \ t_{ref} = L/v_m = (4.83 \text{ m})/(1140 \text{ m/sec}) = 4.237 \text{ msec} \), the constant equals 2.219. This constant \( C \) is now used to find \( \Delta t \), the time shift due to lot-to-lot muzzle velocity variation. \( t_1 \) is the in-bore time for a projectile that has a muzzle velocity \( v_{ml} \) equal to the standard muzzle velocity plus the lot-to-lot muzzle velocity variation from Table 2.

\[
\begin{align*}
V_{ml} & = V_m + \sigma_v_{lot-to-lot} \\
t_1 & = C \cdot \frac{L}{V_{ml}} = 9.368 \text{ msec} \\
\Delta t & = t - t_1 = 0.032 \text{ msec}
\end{align*}
\]

From Figure 13, the value of \( d\theta/dt \) near shot exit time is found to be \(-.94 \text{ mrad/msec}\). The lot-to-lot muzzle pointing error is therefore calculated to be:

\[
|\Delta \theta| = \left| \frac{d\theta}{dt} \right| \cdot \Delta t = \left| -.94 \text{ mrad/msec} \cdot 0.032 \text{ msec} \right| = 0.03 \text{ mrad}
\]

3.4 Boresight Calibration.

In addition to the desire that the muzzle angle change from shot start to shot exit be the same from occasion to occasion, calibration between the muzzle of the cannon and the fire control optics used to point the cannon must be consistent from occasion to occasion. Currently, the U.S Army calibrates the cannon to the fire control optics with a muzzle boresight device. This is an optical device that is placed in the muzzle of the cannon. The muzzle boresight is used to aim the cannon at some target point. The fire control optics are then aimed at the same point to align them with the muzzle of the cannon. Any
inability to point the cannon at the same spot or to align the fire control optics at the same spot as the cannon will show up as occasion-to-occasion dispersion.

Recent tests have indicated that tank cannons may be aimed with an accuracy of 0.06 mrad (standard deviation) using the boresights currently in use with the U.S. Army. Included in this figure is the error associated with the boresight itself (i.e. parallax, reticule lines that obscure the target, etc.), the error associated with a slightly different placement of the boresight in the muzzle on each occasion and the error associated with moving the cannon [9].

The error inherent in the boresight itself (i.e. the inability to read the same spot with the boresight, independent of inserting the boresight in the muzzle and moving the cannon) was measured to be 0.02 mrad during the same test. This value can be used to make an estimate of the error associated with laying the fire control optics on the same spot as the muzzle of the cannon. Since the power of the optics in the fire control system and the boresight is the same (10 power), the ability to see the target spot should be about the same or less with the fire control optics (parallax problems inherent in the boresight are not applicable to the fire control optics). Combining the errors associated with aiming the cannon through use of the boresight (0.06 mrad) and those with placing the fire control optics on the target point (<0.02 mrad) leaves the total boresighting error at approximately 0.063 mrad.

IV. CONCLUSION

These six sources; temperature related muzzle velocity variation, lot-to-lot muzzle velocity variation, recoil variation, muzzle pointing angle dependence on propellant temperature, muzzle pointing angle dependence on lot-to-lot muzzle velocity variation and boresighting variation, represent major sources of occasion-to-occasion error that have been recently investigated and are now better understood. With this better understanding comes an increased ability to correct the problems. Reduction or elimination of these problems can significantly improve both the occasion-to-occasion error and the overall error for both current and future tanks.

V. REFERENCES

