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STATEMENT OF GOVERNMENT INTEREST

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BACKGROUND OF THE INVENTION

(1) Field of the Invention.

The present invention relates generally to a system and method for estimating the geodetic position of acoustic sensors, and more particularly to a system and method for estimating position which considers timing and sound velocity biases as parameters to be estimated, thus precluding biasing errors from propagating to the sensor coordinates.

(2) Description of the Prior Art

Underwater acoustic tracking ranges, as typically operated, utilize the well known principle of hyperbolic multilateration similar to radio frequency (RF) navigation systems. The accuracy and limitations of these systems has been well documented. The underwater acoustic environment, however, creates some unique
differences. For example, both the RF and acoustic systems make
timing measurements and convert them to range measurements. The
nature of underwater acoustic wave propagation results in non-
dispersive wave propagation, significantly smaller propagation
velocities, shorter propagation distances, longer transit times
and severe refraction through the stratified ocean. In addition,
both types of tracking systems require precise knowledge of the
relative and geodetic location of the reference sensors
(transmitters and receivers). For a land based RF system, this
is accomplished using well known, conventional terrestrial survey
techniques. For a satellite based RF system, it requires
techniques adapted for that environment. Historically, two
methods have been employed for surveying underwater acoustic
sensors, commonly referred to as the Vanderkulk and Spherical
Least Squares methods, the main difference between the two
methods being the use of geodetic position information for the
acoustic sources. However, both techniques are extensions of the
conventional, well known trilateration survey technique. A
trilateration survey consists of making range measurements from
reference points to points to be surveyed. The ranging
measurements are obtained by making timing measurements and
converting them to range measurements based upon a presumed
knowledge of the propagation velocity of the transmitted acoustic
signal. The major source of error for the traditional survey
methods is systematic error or bias. This type of error can manifest itself both in the timing measurements and in the assumptions of the sound velocity. Timing biases are normally removed by making system timing measurements and removing the systematic component before processing, the systematic component normally being a timing delay. Systematic errors in the acoustic propagation velocity, however, are more difficult to deal with because they cannot be measured and are generally functions of both space and time. For underwater acoustic multilateration tracking systems, the nature of the acoustic propagation is such that ray theory is considered valid. If one further restricts the propagation to direct monotonic acoustic ray paths, the propagation velocity is characterized by an Effective Sound Velocity (ESV). The ESV is that velocity which when multiplied by the transit time between two underwater points, yields the geometric or slant range between them. To avoid systematic (non-random) errors in the ESV calculation, an unbiased measurement for the Sound Speed Profile (SSP) is prerequisite. However, a typical measured SSP may have a 2 meter/second bias error. Consequently, the traditional methods may suffer significant errors on estimating the sensor coordinates.
SUMMARY OF THE INVENTION

Accordingly, it is an object of the present invention to provide a system and method which can be used to more accurately estimate the geodetic position of acoustic sensors.

Another object of the present invention is to provide a system and method for estimating the geodetic position of acoustic sensors which considers timing and sound velocity biases as parameters to be estimated.

Still another object of the present invention is to provide a system and method for estimating the geodetic position of acoustic sensors which precludes biasing errors from propagating to the sensor coordinates.

Other objects and advantages of the present invention will become more obvious hereinafter in the specification and drawings.

In accordance with the present invention, a system and survey method for estimating the geodetic position of acoustic sensors placed at fixed but unknown locations on the seafloor is disclosed. Bottom mounted sensors are surveyed using an extension of the well known trilateration survey technique, i.e., making ranging measurements from reference points to the point to be surveyed. For acoustic sensors, these ranging measurements are obtained by transmitting an acoustic signal from a near surface projector and making corresponding timing and/or position
measurements, requiring calculation of the Effective Sound Velocity. Bias errors are precluded from propagating to the sensor coordinates by considering timing and sound velocity biases, in addition to the sensor coordinates, as parameters to be estimated.

BRIEF DESCRIPTION OF THE DRAWINGS

A more complete understanding of the invention and many of the attendant advantages thereto will be readily appreciated as the same becomes better understood by reference to the following detailed description when considered in conjunction with the accompanying drawings wherein corresponding reference characters indicate corresponding parts throughout the several views of the drawings and wherein:

FIG. 1 is a flow chart of the method of the present invention;
FIG. 2 is a schematic representation of a hydrophone location to be estimated;
FIG. 3 is a Sound Velocity Profile corresponding to FIG. 2; and
FIG. 4 is a Slant Range Residual Plot for data from FIGS. 2 and 3.
DESCRIPTION OF THE PREFERRED EMBODIMENT

To formulate the geodetic position estimation problem for underwater acoustic sensors, prior art methods utilize a spherical positioning model given by:

\[ R_i = c_i \left( t_i^a - t_i^r \right) \quad ; \quad i = 1 \ldots N \]

(1)

where \( R_i \) is the geometric slant range between the source and receiver such that

\[ R_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \]

(2)

The source coordinates are denoted \( \{x_i, y_i, z_i\} \) and the sensor coordinates (to be estimated) are denoted \( \{x, y, z\} \). The ESV is denoted \( c_i \) and the arrival times and emission times of the acoustic signal are denoted \( t_i^a \) and \( t_i^r \), respectively. Under the assumption of a stratified sound velocity medium, \( c_i \) is calculated as a function of \( z \), \( z_i \) and the horizontal distance

\[ H_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} \].

Technically the depth of the source and receiver is used vice the tangent plane \( z \), careful consideration must be given to geodetic and Cartesian coordinate systems and their transformations. When a direct, monotonic ray tracing method is followed, the value of \( c_i \) can be uniquely determined based on the measured (SVP). If an additional parameter representing an unknown system timing bias \( b_i \) is added, the so-called hyperbolic positioning model, used in many well known land based and satellite based methods of position estimation, is created. Any systematic timing errors can now be estimated.
without propagating to the position estimate of the underwater
acoustic sensor. The hyperbolic model is thus:
\[ R_i = c_i (t_i^* - t_i^0 + b_i) ; \quad i = 1 \ldots N. \] (3)

This positioning model has not normally been employed for
acoustic sensor position estimation because one has the ability
to synchronize the transmitter and receiver and perform timing
measurements so that this term is known a priori. However, the
present invention extends the concept further. A fifth parameter
representing an unknown effective sound velocity bias \( b_c \) is
added. Consequently, a new positioning model with effective
sound velocity bias estimation is thus:
\[ R_i = (c_i + b_c) (t_i^* - t_i^0 + b_i) ; \quad i = 1 \ldots N. \] (4)

It is noted that higher order terms could be added to the model
if the mathematical nature of the ESV error dictates.

The models shown in Eq. (1), (3) and (4) are nonlinear
systems of equations. All of the models shown above are nonlinear
systems of equations. Eq. (1) contains three parameters to be
estimated, namely \((x,y,z)\). Eq. (3) and (4) contain four
\((x,y,z,b_c)\) and five \((x,y,z,b_c,b_t)\) parameters, respectively. A
common approach to solve this kind of nonlinear problem is based
on a well known iterated Newton-Raphson method, suitable for
implementation within a computer, as will be explained further
hereinafter.

Defining a new variable \( t^e \) as the transit time where
\[ t^e = t^o - t^e \] and denoting a reference point \((x_0, y_0, z_0, b^0_c, b^0_t)\) for \((x, y, z, b_c, b_t)\), a direct use of the Taylor series expansion for \(t^e_i\), as a function of \((x, y, z, b_c, b_t)\), yields:

\[ t_i' = t'_0 + \frac{\partial t_i}{\partial x} |_{0} (x - x_0) + \frac{\partial t_i}{\partial y} |_{0} (y - y_0) + \frac{\partial t_i}{\partial z} |_{0} (z - z_0) + \frac{\partial t_i}{\partial b_c} |_{0} (b_c - b^0_c) + \frac{\partial t_i}{\partial b_t} |_{0} (b_t - b^0_t), \]  

(5)

where a super- or sub-script "0" indicates that the value is calculated at the reference point \((x_0, y_0, z_0, b^0_c, b^0_t)\). For instance,

\[ t'_0 = \frac{R^0_i}{c^0_i + b^0_c} - b^0_t, \]  

(6)

in which:

\[ R^0_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \]  

(7)

and \(c^0_i\) is the ESV between \((x_i, y_i, z_i)\) and \((x_0, y_0, z_0)\).

From EQ. (5), one can easily show

\[ \frac{\partial t_i}{\partial b^0_c} |_{0} = -\frac{R^0_i}{(c^0_i + b^0_c)^2} \]  

(8)

and

\[ \frac{\partial t_i}{\partial b^0_t} |_{0} = -1. \]  

(9)

Due to the fact that both \(R_i\) and \(c_i\) are dependent on \(x\), one writes:

\[ \frac{\partial t_i}{\partial x} |_{0} = \frac{1}{c^0_i + b^0_c} \left( \frac{\partial R_i}{\partial x} |_{0} - \frac{R^0_i}{c^0_i + b^0_c} \frac{\partial c_i}{\partial x} |_{0} \right). \]  

(10)
Since

$$\frac{\partial R_i}{\partial x_i} \left|_{x_i} = \frac{x_0 - x_i}{R_i^0} \right.$$ (11)

one can rewrite EQ. (10) to be

$$\frac{\partial R_i}{\partial x_i} \left|_{x_0} = \frac{1}{c_i^0 + b_i^0} \left( \frac{x_0 - x_i}{R_i^0} - \frac{R_i^0}{c_i^0 + b_i^0} \frac{\partial c_i}{\partial x_i} \right) \right.$$ (12)

Because there is no explicit expression available for $c_i$ as a

function of $x$, one cannot analytically evaluate

$$\frac{\partial c_i}{\partial x_i} \left|_{x_i}.$$ (13)

Therefore, one has to rely on a numerical method to obtain it.

One possible way to obtain an approximation for $\frac{\partial c_i}{\partial x_i} \left|_{x_0}$ is stated below. First, one estimates the ESVs corresponding to the target position at $(x_0 + \epsilon, y_0, z_0)$ and at $(x_0 - \epsilon, y_0, z_0)$, respectively, denoted as $c_i^0(x_0^+)$ and $c_i^0(x_0^-)$, where $\epsilon$ is a small length quantity (say 1 meter). Then, one calculates $\frac{\partial c_i}{\partial x_i} \left|_{x_0}$ as:

$$\frac{\partial c_i}{\partial x_i} \left|_{x_0} = \frac{c_i^0(x_0^+) - c_i^0(x_0^-)}{2\epsilon} \right.$$ (13)

It is well known in solving Eq. (10) to assume that $\frac{\partial c_i}{\partial x_i} \left|_{x_0}$
is negligibly small. This assumption is particularly true where
the elevation angle associated with \((x_i, y_i, z_i)\) and \((x_0, y_0, z_0)\) is
large. As a result, one can simplify Eq. (12) to be:

\[
\frac{\partial \alpha_i}{\partial x_0} = \frac{1}{c_i^0 + b_c^0} \frac{x_0 - x_i}{R_i^0} .
\]

Exercising the same mathematics used in deriving Eq. (12), one
obtains:

\[
\frac{\partial \alpha_i}{\partial y_0} = \frac{1}{c_i^0 + b_c^0} \left( \frac{y_0 - y_i}{R_i^0} - \frac{R_i^0}{c_i^0 + b_c^0} \frac{\partial \alpha_i}{\partial y_0} \right)
\]

and

\[
\frac{\partial \alpha_i}{\partial z_0} = \frac{1}{c_i^0 + b_c^0} \left( \frac{z_0 - z_i}{R_i^0} - \frac{R_i^0}{c_i^0 + b_c^0} \frac{\partial \alpha_i}{\partial z_0} \right),
\]

and following the same numerical procedure as in Eq. (13):

\[
\frac{\partial \alpha_i}{\partial y_0} = \frac{c_i^0(y_0^+) - c_i^0(y_0^-)}{2\varepsilon}
\]

and

\[
\frac{\partial \alpha_i}{\partial z_0} = \frac{c_i^0(z_0^+) - c_i^0(z_0^-)}{2\varepsilon},
\]

where, following the same definitions as given for \(c_i^0(x_0^+)\) and
\(c_i^0(x_0^-), c_i^0(y_0^+), c_i^0(y_0^-), c_i^0(z_0^+)\) and \(c_i^0(z_0^-)\) are ESVs
corresponding to the target positions at \((x_0, y_0+\varepsilon, z_0),\)
\((x_0, y_0-\varepsilon, z_0), (x_0, y_0, z_0+\varepsilon)\) and \((x_0, y_0, z_0-\varepsilon)\), respectively.
Simplifying Eq. (15) and Eq. (16) when the associated elevation angle is large yields:

\[
\frac{a'_i}{\partial y_i} \bigg|_0 = \frac{1}{c_i^0 + b_c^0} \frac{y_i - y_i}{R_i^0} 
\]

and

\[
\frac{a'_i}{\partial z_i} \bigg|_0 = \frac{1}{c_i^0 + b_c^0} \frac{z_i - z_i}{R_i^0}.
\]

The well known iterated Newton-Raphson method can now be applied. Referring now to FIG. 1, there is shown a flow chart of the method of the present invention. Writing the linearized N equations (Eq. 5) in five unknowns in matrix vector notation, step 100 yields:

\[
J\Delta x = J\Delta R,
\]

where \( J \) is the well known Jacobian matrix, formed using Equations 14, 19, 20, 8 and 9 for \( t \) with respect to \( x, y, z, b_c \) and \( b_t \), respectively, and using data points 1 ... \( N \); \( \Delta x \) is the state error vector; and \( \Delta R \) is the difference between the measurement and the reference state vector. A first linearization point, \( x_0, y_0, z_0, b_c^0 \), \( b_t^0 \), is chosen to obtain the least squares estimate, given by:

\[
\Delta \bar{x} = (J^TJ)^{-1} J\Delta R.
\]

as provided for in the Newton-Raphson method at step 200. The least squares estimate is added to the linearization point to yield an improved estimate at step 300. If the error vector is greater than a predetermined tolerance value, step 400 returns to
step 100, using the updated parameter values. If the error
vector is less than the predetermined tolerance value, step 400
exits with the improved estimate. As noted previously, computer
algorithms for implementing the Newton-Raphson method for solving
linear equations in multiple unknowns are well known in the art.

As also noted earlier, the term $\frac{\partial c_i}{\partial \alpha}$ was taken to be
negligibly small, especially for large elevation angles. In
practice, the evaluation of the spatial dependence of $c_i$ is
incorporated into the iteration of the linearized state variable
and is performed automatically regardless of the elevation angle.

The following numerical simulation verifies the solution
algorithm and clearly demonstrates the capability of this new
method without the introduction of noise and uncertainty of field
data. Referring to FIG. 2, consider hydrophone 10 cabled to
shore (not shown) which is placed at a fixed location "L" on the
seafloor such that its geodetic coordinates are 24.50° N
latitude, 77.50° W longitude and an ellipsoid height of -1628.94
meters. The geoid height is known to be -28.94 meters, thus the
hydrophone 10 depth (orthometric height) is -1600.00 meters.
Assume that a surface vessel, or ship 12, transited on a ship
path relative to the hydrophone as shown by arrows 14 and 16,
where (without loss of generality) the coordinate system has been
transformed to an East, North, Up local tangent plane coordinate

12
system. The ship 12 is transmitting acoustic signals which
propagate to hydrophone 10 as indicates by lines 18 in FIG. 2 and
are detected by electronics (not shown) on shore. The signals
correspond to data points 1 ... N for use in forming the Jacobian
matrix. FIG. 3 shows a SVP plot for the area in question. Using
this SVP as a true SVP, the true transit times from the ship 12
to hydrophone 10 at each position can be calculated. If a
systematic timing bias error of 2 ms and an ESV bias error of
0.63 m/s is incorporated, a new set of (erroneous) acoustic
transit times can be calculated. Starting from a reference
location \( (x_0, y_0, z_0) \) several hundred meters from the true location,
the position of hydrophone 10 is estimated using both data sets
and all three positioning models (spherical least squares, SLS;
hyperbolic least squares, HLS; and the method of the present
invention referred to as hyperbolic least squares with bias
estimation, HLSBE) as summarized in Table 1 as follows:

**TABLE 1: Numerical Simulation Results**

<table>
<thead>
<tr>
<th>Model</th>
<th>( x(\text{m}) )</th>
<th>( y(\text{m}) )</th>
<th>( z(\text{m}) )</th>
<th>( b_t(\text{ms}) )</th>
<th>( b_c(\text{m/s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SLS</td>
<td>0.00</td>
<td>0.00</td>
<td>-1600.00</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>HLS</td>
<td>0.00</td>
<td>0.00</td>
<td>-1600.00</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>HLSBE</td>
<td>0.00</td>
<td>0.00</td>
<td>-1600.00</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>SLS</td>
<td>0.00</td>
<td>0.00</td>
<td>-1606.78</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>HLS</td>
<td>0.00</td>
<td>0.00</td>
<td>-1597.09</td>
<td>4.1</td>
</tr>
<tr>
<td>6</td>
<td>HLSBE</td>
<td>0.00</td>
<td>0.00</td>
<td>-1600.00</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Estimates numbered 1 through 3 correspond with the three different positioning models for acoustic data containing no bias error. As shown in Table 1, if no biases are present, each positioning model and its associated solution algorithm is sufficient for estimating the sensor location. Estimates 4 through 6 utilize data having the timing delay and ESV bias. In this situation, however, only the method of the present invention, HLSBE, correctly estimates the sensor location in the presence of both a system timing bias and ESV bias.

Traditionally, any systematic bias was estimated and eliminated through the analysis of slant range residuals. The residuals for estimates 4 through 6 associated with each model are shown in FIG. 4. Use of a residual plot shows that a bias is present, but gives no immediate insight as to the source or relative magnitude of the bias errors. Historically, errors were removed by undertaking a laborious and time consuming trial and error approach. The situation is further complicated for field data as the residual plot will have noise superimposed on any systematic trend event in the residuals. It is noted that the HLSBE model of the present invention yields not only a correct estimate for the sensor location, but also correct estimates for the timing and ESV biases.

The invention thus described provides an improved survey method for estimating the geodetic position of sensors placed at
fixed but unknown locations on the sea floor developed. This method eliminates the largest sources of error, timing and sound velocity bias error, by considering them as additional unknown parameters to be estimated. The method is shown to be very robust and insensitive to the uncertainties of SVP and timing bias error when estimating acoustic sensor location from experimental field data. In addition to providing accurate estimates of geodetic position, the method also provides correct estimates for the timing and ESV biases. For this new method, as well as all previous methods, care must be taken to simultaneously ensure that the measurements, mathematical model, and solution algorithm are compatible. Specifically, the new method will usually require a wider spatial coverage while collecting time of arrival measurements, because the successful use of this new model needs diverse effective sound velocities associated with those measurements.

Although the present invention has been described relative to a specific embodiment thereof, it is not so limited. For example, as noted previously, higher order terms could be added to the model if the mathematical nature of the ESV error so dictated.

Thus, it will be understood that many additional changes in the details and steps which have been herein described and illustrated in order to explain the nature of the invention, may
be made by those skilled in the art within the principle and scope of the invention.
GEODETIC POSITION ESTIMATION FOR UNDERWATER ACOUSTIC SENSORS

ABSTRACT OF THE DISCLOSURE

A system and survey method for estimating the geodetic position of acoustic sensors placed at fixed but unknown locations on the seafloor is disclosed. Bottom mounted sensors are surveyed using an extension of the well known trilateration survey technique, i.e., making ranging measurements from reference points to the point to be surveyed. For acoustic sensors, these ranging measurements are obtained by transmitting an acoustic signal from a near surface projector and making corresponding timing and/or position measurements, requiring calculation of the Effective Sound Velocity. Bias errors are precluded from propagating to the sensor coordinates by considering timing and sound velocity biases, in addition to the sensor coordinates, as parameters to be estimated.
FIG. 1

100. DETERMINE JACOBIAN MATRIX

200. OBTAIN LEAST SQUARES ESTIMATE

300. ADD TO LINEARIZATION POINT

400. NO

ERROR VECTOR GREATER THAN TOLERANCE VALUE?

YES

EXIT WITH IMPROVED ESTIMATE
FIG. 3

FIG. 4