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The Perturbation of Pendulum and Gyroscope Instruments by Acceleration of the Vehicle

Die Stoerung von Pendel-und Kreiselapparaten durch die Beschleunigung des Fahrzeuges

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Gyros	Machine Elements (14)
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"THE PERTURBATION OF PENDULUM AND GYROSCOPE INSTRUMENTS BY ACCELERATION OF THE VEHICLE"

By

M. Schuler

Translated by W. E. Frye (1952)

T-24

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"THE PERTURBATION OF PENDULUM AND GYROSCOPE INSTRUMENTS BY ACCELERATION OF THE VEHICLE"

(Die Storung von Pendel-und Kreiselapparaten durch die Beschleunigung des Fahrzeuges)

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"THE PERTURBATION OF PENDULUM AND GYROSCOPE INSTRUMENTS BY ACCELERATION OF THE VEHICLE"

M. Schuler, Physik. Zeitschr. 24, 344(1923).

In the following I will briefly discuss the effect of accelerations of a vehicle on pendulum and gyro equipments and show how such disturbances can be avoided.

In the past I was led to these considerations through the gyrocompass on whose development I worked in association with Anschutz-Kaempfe in Kiel. The initiative for this work is due especially to a paper by C. Martienssen,* in which he calculated the errors of a gyrocompass due to North-South accelerations of a ship.

In computing several examples, Martienssen ended up with very large errors in the compass and concluded from this that the gyrocompass was useless as a true direction indicator for navigation. Therefore I posed the question: Can acceleration errors of this kind be eliminated by suitable mechanization?

The answer is: Yes! And the solution is almost trivial. On the other hand, it is so universally applicable to any arbitrary pendulum and any arbitrary gyro configuration that I take the liberty of presenting it here in brief form. Naturally, the separation of gravity and acceleration is impossible; nevertheless, by the construction of a mechanical apparatus which is subject to the influence of gravity, one can so choose the parameters that only such torques can act on the apparatus, when accelerated, as it requires for the

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* Physik Zeitschr. 7, 535(1906).

change of its state of motion so that the condition of equilibrium is maintained permanently.

I will not lead the reader along the cloudy way through the equations of the gyrocompass which I had to follow myself, to find the solution, but I will begin with the simple pendulum.

The equilibrium position of an ordinary simple pendulum is not affected by vertical accelerations. On the other hand, it is set into oscillation by horizontal accelerations of its supporting point, even if it had previously hung at rest. For example, as soon as a railroad train is set in motion, a pendulum hanging in it is thrown backwards. In addition a change in the direction of motion is sufficient to produce a strong deflection of the pendulum, even if the speed remains unchanged. Obviously it would be otherwise only if the pendulum had a length equal to the earth's radius. Then one could move arbitrarily over the surface of the earth without having the pendulum thereby disturbed in the least. For its bob stays always at the center of the earth and hence at rest. Actually, the earth is not exactly a sphere but an ellipsoid. Strictly speaking, the pendulum must have a length equal to the radius of curvature. This is somewhat different at different latitudes, and at the Equator too the radii of curvature in the East-West and North-South directions are not quite equal. However, the differences are only magnitudes of the second order, so that in the following discussion the earth can be assumed to be a sphere.

Naturally such a simple pendulum is unrealizable, although one can achieve the same result with a physical pendulum which has the same period as a simple pendulum of earth's radius length. This yields the condition:

$$T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ minutes} \qquad (1)$$

For proof examine Figure 1. The vehicle moves with an arbitrary velocity v in the plane of the paper. Further let:

R = radius of the earth

ω = angular velocity over the earth

$v = R \cdot \omega$ = velocity of the vehicle

$\gamma = R \cdot \frac{d\omega}{dt}$ = acceleration of the vehicle (2)

Θ = moment of inertia of the physical pendulum

A = suspension point of the physical pendulum

S = center of gravity of the physical pendulum

a = distance: suspension point — center of gravity

ω_1 = rotational velocity of the pendulum

From this there results

$$T = 2\pi\sqrt{\frac{\Theta}{mga}} = \text{period of the pendulum} \qquad (3)$$

and

$$\frac{d\omega_1}{dt} = \frac{\gamma ma}{\Theta} \qquad (4)$$

i.e. because of the acceleration, the pendulum experiences definite changes of its rotational velocity.

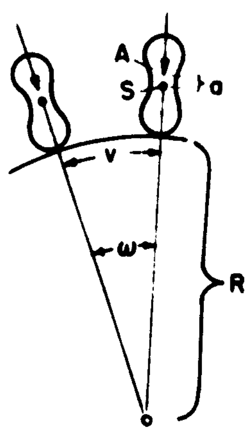

Fig. I

An observer on the earth considers a pendulum at rest only if the rotational velocity of the pendulum always equals the rotational velocity of the earth's gravity vector, i.e. $\omega_1 = \omega$. If now at the beginning of the vehicle's motion the pendulum is at rest and hence this condition is satisfied, then this is still the case if $\frac{dv_1}{dt} = \frac{dv}{dt}$. However by Equations (2) and (4) this requires:

$$\gamma \frac{ma}{\Theta} = \gamma \frac{1}{R}$$

This condition is always satisfied for the case $\gamma=0$; that is: If the vehicle moves with uniform velocity in a definite plane, then every pendulum is on equal footing. Disturbances cannot arise. However, if accelerations of the vehicle occur, then

$$\frac{\Theta}{mga} = \frac{R}{g} \qquad (5)$$

must hold, otherwise the condition is not satisfied. The period of the pendulum is by Equation (3):

$$T = 2\pi\sqrt{\frac{g}{R}} = 84 \text{ minutes}$$

Such a pendulum, if it once comes to rest, remains always in its equilibrium position and indicates the plumb line which prevails at that particular place on the earth's surface regardless of how the vehicle moves. Such a physical pendulum is, however, also not realisable, for even with 20,000 kg and 4 m radius of gyration only a 0.6 micron displacement of the center of gravity would be obtained.

At this point, Nature, through the laws of the gyro, comes to our aid, and I maintain that a gyro, which is kept in its equilibrium position by gravity, is not displaced from equilibrium by arbitrary motions on the surface of the earth if it has a period of 84 minutes. In addition, the configuration of the gyro is completely immaterial. The only requisite is the correct choice of the period.

Also, this is the only possibility for eliminating the effect of accelerations. I will prove this with two examples: A gyro with a vertical axis (a "gyro pendulum" or "artificial horizon") and a gyro with a horizontal axis (a "gyro-compass").

A gyro with a vertical axis is shown in Fig. 2. The gyro rotor turns

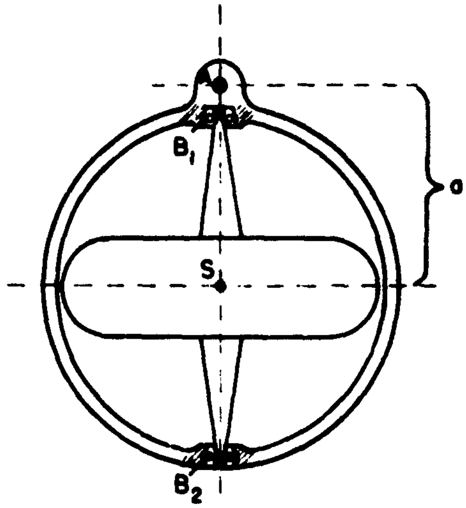

Fig. 2

in two ball bearings B_1 and B_2. The suspension point, which is designated by A, forms a point or a ball so that the gyro pendulum can swing fre-ly in any direction. S is the center of gravity of the pendulum, a the distance of the suspension point from the center of gravity. Hence, for an angle of inclination β with respect to the vertical, the gyro axis is drawn to the vertical direction by the gravitational moment mga sin β. If the gyro is driven by a motor and held at a constant speed of rotation, then it has a definite momentum J, and the pendulum, by the laws of gyro action follows a conical surface with the period:

$$T = 2\pi \frac{J}{mga} \qquad (6)$$

We will now for a moment ignore the rotation of the earth in order to avoid the carrying along of constant terms in the calculations. As before, any velocity v of the vehicle corresponds to an angular velocity ω of the line to the center of the earth. In order for the gyro to assume this rotation, a torque in the perpendicular plane must be applied to it, which can be calculated by Eq. (7):

$$mga \sin \beta_1 = \omega J \cos \beta_1 \qquad (7)$$

One can see this best from Fig. 3. The angles ω and β_1 form a spherical

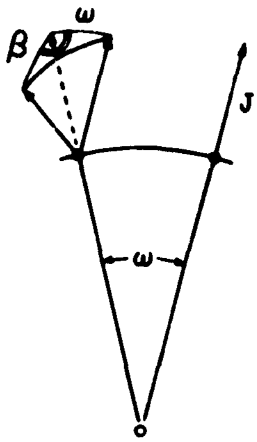

Fig. 3

triangle, in which β_1 is perpendicular to ω. From Eq. (7) one can calculate for each velocity v the corresponding equilibrium position of the gyro:

$$\tan \beta_1 = \frac{J}{mga} \cdot \frac{v}{R}$$

The differentiation of this equation gives, with sufficient accuracy, for small angle β_1:

$$\frac{d\beta_1}{dt} = \frac{J}{mga} \frac{d\omega}{dt} \qquad (8)$$

On the other hand, the acceleration forces act on the suspension point, while the gravity forces act on the center of gravity. There results a torque on the gyro:

$$\sqrt{ma} = maR \frac{d\omega}{dt}$$

in the plane of the angle ω.

This torque causes a precession of the gyro in the plane perpendicular to ω and hence in that of the angle β_1, which is given by:

$$\frac{d\beta_2}{dt} = \frac{Rma}{J} \frac{d\omega}{dt} \qquad (9)$$

If now, with an acceleration of the vehicle, the precession of the gyro is always equal to the change of the equilibrium position, then the gyro cannot go into oscillation, i.e. for

$$\frac{d\beta_1}{dt} = \frac{d\beta_2}{dt}$$

the equilibrium is maintained. Equations (8) and (9) give for this special case the condition

$$\frac{J}{mga} \frac{d\omega}{dt} = \frac{Rma}{J} \frac{d\omega}{dt}$$

For $\frac{d\omega}{dt} = 0$ this equation is always satisfied. Hence for the case of uniform rotation, every gyro pendulum satisfies the condition. However, if accelerated motion occurs, then

$$\left(\frac{J}{mga}\right)^2 = \frac{R}{g} \qquad (10)$$

must be true for equilibrium to exist. But that means, if we recall Equation (6) for the period:

$$T = 2\pi \sqrt{\frac{R}{g}} = 84 \text{ minutes}$$

One sees from the form of the equation that this is the only possible solution. If a different value for the period is chosen, then disturbances to the gyro must always appear. On the other hand it is immaterial in what sense the gyro is rotating and whether the center of gravity lies above or below the point of support, since both J and a occur only as the square. A top which dances on its point is therefore completely equivalent to the gyro pendulum shown in Fig. 2, if only its cone of precession is traversed in 84 minutes. According to this principle one can construct an artificial horizon. To be sure, the gyro axis would be always inclined to the vertical at an angle β_1 which can be calculated from the velocity of motion. For the gyro of 84 minute period one obtains:

$$\beta_1 = \frac{v}{\sqrt{Rg}} = \frac{v(m/sec)}{7900}$$

The greatest part of this angle arises from the rotation of the earth, since the gyro does not differentiate between the velocity due to earth rotation and the motion of the vehicle over the earth. For the Equator one calculates, e.g.:

$$\beta_A = 3^\circ 20'$$

This is already a completely observable angle.

If one wishes to avoid the calculation of this correction, he can use two identical gyros of which the first rotates in the right hand and the other in the left hand sense. Then β_1 would be positive in the first case and negative in the other case, and for the average the angle β_1 would be eliminated. Thus we would obtain the true plumb line. On the other hand the difference of the two angles is proportional to the velocity of the vehicle with respect to inertial space.

Finally, I come to the most complicated case: The gyro with horizontal axis which finds application in the gyrocompass. The configuration is shown in Fig. 4. The gyro (K)

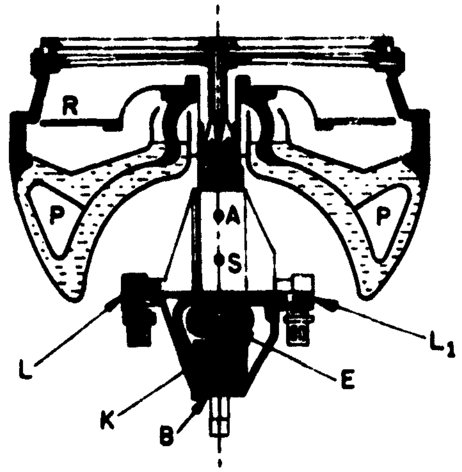

Fig. 4

which turns on the ball bearings (L and L_1), is driven by an electric induction motor (E) and is completely inclosed in a cap (B). The gyro system is suspended from a hollow ring (P) of iron which floats as free of friction as possible in mercury, so that the gyro axis is held always in a horizontal position by the force of gravity, since the center of buoyancy (A) of the float lies a fixed distance a above the center of gravity of the system (S). The rotation of the earth acts on such a gyro so that its position of equilibrium is along the meridian. The gyro swings back and forth about this equilibrium position until it is brought to rest by an added damping device. The course can be read on the rose (R) on which the direction of the gyro axis is designated N-S. The oscillation period of such a gyro compass about the meridian is (1):

$$T = 2\pi \sqrt{\frac{J}{mga\ u\ \cos \varphi}} \qquad (11)$$

u is the angular velocity of the earth's rotation and φ is the geographic

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(1) c.f. F. Klein and A. Sommerfeld, "On the Theory of the Gyroscope," Vol. IV. p. 845.

latitude.

I claim now that a gyrocompass operates free of perturbations only with a period of oscillation of 84 minutes. The proof can be demonstrated in a manner similar to that given previously for the gyro pendulum. However, now the output angle of the gyro, which one must observe, lies in the horizontal plane.

The equilibrium position of the gyro compass on a moving ship is turned through an angle δ with respect to the N-S direction as shown in Fig. 5, which represents a view of the horizontal plane.

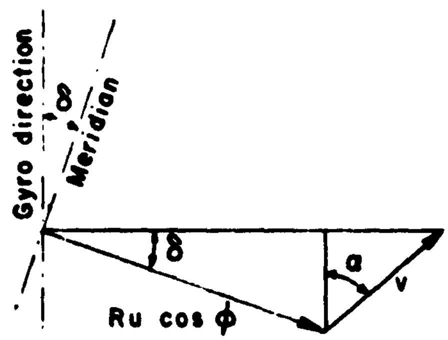

Fig. 5

Ru cos φ is velocity of motion due to the earth's rotation only. By this the compass is aligned in the N-S direction.

If now there is added the ship's velocity v at a course angle α indicated on the compass, then the compass positions itself perpendicular to the resultant of both velocities. There results a course error:

$$\sin \delta = \frac{v \cos \alpha}{Ru \cos \varphi} \qquad (12)$$

This angle depends only on the motion of the earth and the ship but not on the construction of the gyro compass.

On the other hand an acceleration γ_N in the direction of the gyro axis causes a turning moment in the vertical plane and thereby a precession of the gyro in the horizontal plane. The angle of precession μ is given by the following equation:

$$\sin \mu = \frac{ma}{J} \int_0^T \gamma_N \, dt$$

The integral is evaluated for us by Nature. If we measure time from the departure of the ship, then

$$\int_0^T \gamma_N \, dt = v \cos \alpha$$

is equal to the velocity of the ship in the direction of the gyro axis. Then finally we obtain:

$$\sin \mu = \frac{ma}{J} v \cos \alpha \qquad (13)$$

If equilibrium is to continue, then $\mu = \delta$ must hold. From this there results:

$$\frac{J}{mga\ u \cos \varphi} = \frac{R}{g}$$

i.e. an oscillatory period of the gyrocompass of 84 minutes. This necessary condition, to be sure, can only be fulfilled for a single geographic latitude since the period depends on the cosine of the geographic latitude. Hence one tunes the compass to a mean geographic latitude at which it would be normally operated. The gyro compass is to be built according to this rule by the German firm, Anschütz and Co., as well as by foreign firms, as a result of my proposing this condition. In Fig. 4 one recognizes that the float is pulled down over the gyro in a peculiar way. The purpose of this is to obtain just that lowering of the center of gravity which gives an oscillation period of 84 minutes. In comparison, as I well know, an artificial horizon has still not been successfully built which possesses the full 84 minutes oscillation period. The longest periods which I have yet obtained in the laboratory are about 30 minutes. Nevertheless, only purely technical fabrication difficulties have to be overcome. Once the way is found to attain the goal, it would soon be possible by the proper gyro configuration to realize the ideal.

It was shown above that a gyro horizon with two counter-rotating gyros also indicated at the same time the velocity of the vehicle. And an aviator's ideal would be attained if he had an undisturbed pendulum and at the same time could read off the velocity of his motion. The gyro compass gives him the direction of his motion. This is simply the E-W line of the compass rose. A difficulty arises here in that the earth's rotation must be taken into account since the flyer wishes to know only his velocity with respect to the earth. The gyro equipment, however, gives the total velocity and he must subtract the earth's motion from it in order to obtain the velocity of the aircraft with respect to the earth. But to do that he must know the direction and inclination of the earth's axis. To make these determinations with a magnetic needle is completely unfeasible since the accuracy is not sufficient by far. There remains only the employment of star or sun measurements. To do that, however, the flyer requires:

a chronometer

an apparatus for measuring angles

an almanac with the precalculated star positions

the gyro pendulum which serves as an artificial horizon

Thus it is clear that the flyer requires the same aids as the seaman for his fix, only the latter uses the sea as a horizon while the flyer requires an artificial horizon. To wish to do more e.g. to compute the velocity from acceleration forces is a completely impossible undertaking. For the difficulty lies in this - that the direction of the earth's axis must be determined, and this is clearly only possible by observation of the stars.

The great significance of an artificial horizon is this - that with its help a position fix can always be made. Thus it is possible on an aircraft, just as now on a ship, to carry out true navigation with determination of drift and with course correction. It is to be expected that, in 20-30 years, flights from Europe to America will be no rarity. Such great distances can only truly be covered if good navigation is guaranteed by a gyro horizon. It is to be hoped that the construction on the above principles of an artificial horizon of sufficient accuracy can be accomplished in a significantly shorter time. Also such an equipment would be of great value for shipboard use, for then it would be possible to obtain position fixes with a poorly visible water horizon or at night.

SUMMARY

In conclusion, if the above observations are collected, the interesting result appears that, only for a characteristic oscillation period of 84 minutes, do the various mechanical apparatuses considered undergo no perturbations by arbitrary motions of the vehicle. Herein lies clearly a general law which perhaps reads as follows:

A mechanical system capable of oscillating, on whose center of gravity a central force acts, will not be excited into oscillation by arbitrary motions on a spherical shell about the center of force, if its oscillation period is equal to that of a pendulum of the length of the sphere's radius in the effective force field. To be sure, I must pass up the general proof of this law. In this work

I have developed proofs only for the most important typical configurations. The difficulty of a general proof can be recognised from this - that for the individual systems completely different initial equations must be used as a basis. The principal equations are assembled in convenient form in the accompanying table.

TABLE OF PRINCIPAL EQUATIONS

1. Physical Pendulum	2. Gyro Pendulum	3. Gyrocompass
$T = 2\pi\sqrt{\frac{\Theta}{mga}}$	$T = 2\pi \frac{J}{mga}$	$T = 2\pi\sqrt{\frac{J}{mga\ u\ \cos\varphi}}$
$\frac{d\omega}{dt} = \frac{\gamma}{R}$	$\frac{d\beta_1}{dt} = \frac{J}{mga}\frac{d\omega}{dt}$	$\sin\delta = \frac{v\cos\gamma}{Ru\cos\varphi}$
$\frac{d\omega_1}{dt} = \frac{\gamma ma}{\Theta}$	$\frac{d\beta_2}{dt} = \frac{Rma}{J}\frac{d\omega}{dt}$	$\sin\mu = \frac{ma}{J} v\cos\alpha$
for $d\omega = d\omega_1$	for $d\beta_1 = d\beta_2$	for $\delta = \mu$
$\frac{\Theta}{mga} = \frac{R}{g}$	$\left(\frac{J}{mga}\right)^2 = \frac{R}{g}$	$\frac{J}{mga\ u\ \cos\varphi} = \frac{R}{g}$

In the case of the simple pendulum the law is obvious from inspection. For the physical pendulum the rotation of the earth's radius vector in the direction of motion of the vehicle yields the conditional equations, while for the gyro pendulum the deflection angle in the vertical plane perpendicular to the motion is the decisive factor. In the case of the gyrocompass, on the other hand, the precession of the gyro in the horizontal plane must be compared with the course error of the compass.

It is perhaps significant for human activity that I was led through computation from the complicated and involved equations of the gyro compass to the knowledge of these simple and clear relationships.