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THIS PAGE IS UNCLASSIFIED
IMPROVED LIFE PREDICTION OF TURBINE ENGINE COMPONENTS USING A FINITE ELEMENT BASED SOFTWARE CALLED ZENCRACK

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16. SECURITY CLASSIFICATION OF:

a. REPORT
Unclassified
b. ABSTRACT
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17. LIMITATION OF ABSTRACT
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18. NUMBER OF PAGES
266

19. NAME OF RESPONSIBLE PERSON
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20. TELEPHONE NUMBER (include area code)
937-255-9229

21. OTHER COMMENTS:

This document contains color.
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1 Executive Summary

During the SBIR Phase II program, several enhancements were incorporated in the commercially available finite element based 3D non-planar crack growth software Zencrack (Ref. 1). These enhancements enable improved prediction of crack growth life of aerospace components under realistic loading conditions.

Zencrack is capable of generating truly 3-D crack surfaces that may twist and turn under generalized mixed-mode loading. This functionality is not usually available in conventional 3-D crack growth software. Also, by using an energy release rate approach Zencrack can model the changing state of stress along a crack front without making assumptions that have to be made in calculating stress intensity factors using CTOD methods.

One development of extreme importance has been the inclusion of the effect of residual stress fields during crack growth. Residual stress may significantly affect the shape of crack development and the life of the component. Zencrack can predict these possibly dramatic shape changes and the altered life. This capability is not generally available in conventional crack growth methods.

The following are the major analysis capabilities developed during this program to improve the modeling of non-planar 3D crack growth and the prediction of crack shapes similar to those observed in experiment and service:

- incorporation of residual stress distributions
- combination of static load systems (e.g. residual stresses) with external cyclic stress using the LEFM principal of superposition
- user defined crack front shapes which can also address breakthrough of corner-type cracks to through crack fronts
- enhanced crack growth integration scheme with error control
- development of transition elements for large crack-blocks to correctly model stress gradients at the interfaces with surrounding elements
- boundary shifting routines to resize crack-blocks and minimize element distortion
- automatic “flipping” of through crack-blocks during crack growth
- a mesh relaxation algorithm to reduce distortion of elements surrounding crack-blocks
- implementation of spectrum loading
• implementation of time dependent crack growth (and combined time / fatigue growth)
• basic and generalized Willenborg retardation models (with or without Chang acceleration due to under-loads)
• fatigue crack growth data specified as a function of both stress ratio and temperature
• time dependent crack growth data specified as a function of temperature
• user subroutines for crack growth data and threshold
• a pre-processor utility to generate tabular data from TANH crack growth equations
• incorporation of crack-blocks in a tetrahedral finite element mesh
• 8-noded solid element crack-blocks
• crack growth in a preferred direction for orthotropic materials such as single crystals
• rainflow counting software of raw load spectrum data
• CTOD method to calculate stress intensity factors from displacements in LEFM
• modeling effects on crack growth of changing state of stress along crack fronts
• development of a Zencrack interface to Ansys
• development of a GUI pre-processor for Microsoft Windows XP or Windows 2000
• post-processing utilities to plot crack growth versus cycles and crack growth profiles

All the above features have been incorporated in the general 3D fracture mechanics software system, Zencrack (Ref. 3) which is interfaced to Abaqus, MSC.Marc and Ansys finite element codes (Ref. 10, Ref. 11 and Ref. 12).

The methodology used in Zencrack is based on replacement of one or more elements in an uncracked mesh by a detailed region of brick elements (crack-blocks) that include one or more crack fronts. With load history and crack growth data supplied by the user, the crack in a 3D model can be advanced to simulate the crack growth over a number of load cycles.

The main issues which were addressed relate to fracture mechanics, development of numerical integration algorithms and the topology of 3D finite element meshes as the crack fronts advance in order to avoid excessive distortions of FE mesh. Major fracture mechanics issues studied and implemented in Zencrack software include the effects of crack closure, dependency of crack growth formulation on stress ratio and temperature, time dependent crack growth, reverse crack growth, presence of residual stresses and introduction of user defined crack fronts. To evaluate crack growth
and its direction in order to simulate 3D crack propagation Zencrack can now use both the J-integral (3D) at the crack front and CTOD-LEFM methods. The integration schemes have been modified to take into account spectrum loading, the generalized Willenborg retardation model, temperature dependent loading and the presence of residual stress distributions. Issues relating to the topology of the 3D FE mesh involved development of large crack-blocks with transition elements, FE mesh ‘relaxation’, crack-block boundary shifting and boundary flipping.

The validation of Zencrack software was carried out by comparing results against the commercially available software, Afgrow (Ref. 2), theoretical solutions and, where possible, against experimental crack growth data.
2 Introduction

Zentech was awarded the SBIR Phase II contract (F33615-01-C-5211) in August 2001 to conduct research entitled “Improved Life Prediction of Turbine Engine Components” and implement certain capabilities in the commercially available software Zencrack, the feasibility of which were assessed in the SBIR Phase I.

The following is the summary of the major activities of research carried out in Phase II and the features that have been implemented in Zencrack software. For details see the respective individual interim reports identified in the table below:

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<td></td>
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<td>No. 2</td>
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<tr>
<td>1.03</td>
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<td></td>
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<tr>
<td>2.02</td>
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<td>2.22</td>
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<td></td>
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<td>Nos. 1,4,5</td>
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<tr>
<td>4.00</td>
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<tr>
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All these features have been incorporated in the commercially available software Zencrack version 7.1.

All Abaqus demonstration and benchmark finite element analyses described in this report have been carried out using Abaqus/Standard version 6.2-1, 6.3-1 or 6.4-1 (Ref. 10).

All Ansys demonstration and benchmark finite element analyses described in this report have been carried out using Ansys/Ed Version 6.1 (Ref. 12).

This report comprises of 12 sections:

- In section 3 fracture mechanics related issues such as effects of retardation, dependency of crack growth formulation on stress ratio and temperature, reverse crack growth and presence of residual stresses are discussed. Also issues related with the use crack tip opening
displacements (CTOD) and J-integral at crack front to evaluate 3D crack growth and its direction are discussed here.

- In section 4 issues connected with numerical integration schemes to address spectrum loading, generalized Willenborg retardation, time and temperature dependent loading, presence of residual stress are described.
- In section 5 a number of topology related issues that needed to be addressed in order to limit the distortion of a FE mesh as a general 3D crack progresses in a structure are discussed. These involved development of re-meshing facilities, large crack-blocks with transition elements, crack-block boundary shifting and boundary flipping.
- Section 6 describes the development of a graphical user interface (GUI).
- Section 7 describes the development of the Ansys interface.
- In section 8 to 10 discussion of the validation exercise carried out to test various features implemented in Zencrack software have been presented. This validation has been carried out against Afgrow software, theoretical solutions, and where possible, against test results supplied by the client. The development of Zencrack software has been carried out under strict company Quality Assurance (QA) standards complying with ISO 9001 and accordingly both individual algorithms and systems have been checked and tested thoroughly.
- Section 11 includes discussion and conclusions
- Section 12 - References

2.1 What is Zencrack?

Zencrack uses the finite element method via interfaces to Abaqus, MSC.Marc and as a result of this project, Ansys, to provide a versatile and powerful analysis tool for fracture mechanics applications. Users have at their disposal all of the capabilities within these finite element programs and may, if desired, include any number of non-linear features in an analysis in addition to being able to carry out a “standard” linear elastic fracture mechanics analysis.

A fully featured demonstration version of Zencrack and user documentation is available at:-
http://www.zentech.co.uk/zencrack.htm.

Zencrack provides two levels of analysis capability. For industries where static loading is important the program can be used to evaluate stress intensity factors using energy release rate and nodal
displacement methods. For thermal transients, the instantaneous stress intensities may be evaluated through the transient to steady state conditions.

The second level of capability provides a facility for 3D non-planar crack growth prediction for cases of fatigue loading. This includes several options for crack growth data definition and a flexible “load system” approach for defining load spectra.

Section 2.2 describes the way that the program works in more detail. Some of the capabilities described have been incorporated as a result of this project and are covered in more detail elsewhere in this report.

### 2.2 How does Zencrack work?

Zencrack works by inserting one or more cracks into a user-supplied finite element mesh of an intact component. Zentech's proprietary “crack-block” approach is used in this process. The cracked mesh is then submitted for analysis in the user's finite element program. As mentioned above, interfaces are available to Abaqus, Ansys and MSC.Marc for this purpose. The results of the finite element analysis are processed automatically to provide energy release rate and stress intensity factor distributions along the crack front(s).

For analyses where crack growth is modeled, adaptive meshing algorithms are integrated into Zencrack to simulate crack growth by updating nodal positions in and around the crack region.

Figure 2-1 gives a high-level overview of the program flow.
A crack-block approach is used to generate 3D meshes containing one or more crack fronts. The term crack-block refers to a collection of brick elements stored as a unit cube. The arrangement of these crack-blocks is such that in their unit cube form they contain either a quarter circular or through crack front on one face. Part of this face is allowed to open up under loading giving the opening crack face within the crack-block. Examples of unit cube crack-blocks are shown in item 2 of Figure 2-2. The opening crack faces are shaded for clarity.
Figure 2-2 - ZenCrack meshing and crack growth procedure

The meshing procedure is one of replacement of one or more 8 or 20 noded brick elements in a user supplied uncracked mesh by crack-blocks. During the mapping process to introduce the crack-blocks the user can control the size and shape of the generated crack front section for each crack-block. The initial crack front derived from a quarter circular crack-block may be elliptic, for example. Crack-blocks can be connected together to form distinct crack fronts of the required size in the cracked mesh. In Figure 2-2, two crack-blocks have been merged together to form a single crack front. In a more general case there may be two or more distinct crack fronts in a model.

For crack growth analysis the updated crack front position and any non-planar crack history are used to re-insert the cracks to create an updated position in the mesh. The crack shape that develops as the crack grows is a function of the initial shape, specimen geometry, applied loading and materials data.

The elements forming the crack front in each crack-block are modeled using collapsed 8 or 20 noded brick elements. When these are inserted into a 20 noded element mesh, the user has an option to control the midside node positions extending radially from the crack front to allow generation of
quarter point nodes if required. The crack front itself is seen as a line of nodes on the crack-block surface. The internal mesh of the crack-block coarsens away from the crack front such that some of the crack-block faces can be matched with standard brick elements to allow them to be merged with a user-supplied mesh. These crack-blocks are referred to as standard crack-blocks. The other highly populated faces can be left as free surfaces, symmetry surfaces, or connected to other compatible crack-blocks. In some crack-blocks all of the external surfaces are highly populated surfaces. These special crack-blocks, referred to as large crack-blocks, must be tied to the rest of the user-supplied mesh.

In the simplest case of all, a cracked mesh may contain only a single crack-block (and therefore a single crack front). If there is a single crack-block then only one side of the crack (i.e. one crack face) is modeled and symmetry constraints should be applied. These can be applied in the uncracked mesh and automatically updated by Zencrack.

If both sides of the crack (i.e. both crack faces) are to be modeled, then pairs of crack-blocks are used with a face-to-face match of the crack-blocks. An example with crack-blocks used in a face-to-face match is shown in Figure 2-3. This example uses large crack-blocks that are tied to the surrounding mesh. The figure shows a displaced plot with the visible part of one crack face shaded for clarity.

![Figure 2-3 - Example with both sides of the crack modeled](image)

All merging of crack-blocks with one another and with the uncracked mesh is carried out automatically to create a new finite element mesh containing the required crack fronts. If requested,
this new cracked mesh can be submitted for finite element analysis or a crack growth analysis can be carried out with this mesh as the starter crack.

A number of options are available to assist in reducing element distortion in and around the crack-blocks when standard crack-blocks are used. These options include boundary shifting and mesh relaxation. These options allow cracks to be placed in meshes where element distortion would otherwise preclude the analysis of the mesh. A simple example of the effect of boundary shifting and relaxation is given in Figure 2-4. These features are extremely useful in parametric studies of different crack sizes in the same uncracked mesh.

![Figure 2-4 - Example of boundary shifting and mesh relaxation](image)

For crack fronts consisting entirely of standard through crack-blocks, the large growth capability allows crack-block transfer from one set of element positions to another in the mesh. This requires the use of the boundary shifting and relaxation algorithms. The effect of the update of the crack region and surrounding elements allows significant growth through a model in these cases. Examples are included in section 2.3.

Zencrack uses a “load system” approach to define the loading spectrum applied to the component. Several basic load system types are available, including:

- static e.g. residual stress, centrifugal load
- cyclic - constant amplitude
- cyclic - load spectrum with “blocks” of loading at different levels and stress ratios.
A superposition load system is available to combine a set of static and cyclic load systems. A rainflow counting utility is available to generate counted spectrum data from random uncounted data.

2.3 Examples

A number of examples are used throughout this report. In general, and by necessity, these are based on simple models. This section highlights some of the new capabilities on more demanding geometries and/or crack shapes. These examples are intended to give a feel for the new capabilities without detailing the full analysis specifications.

2.3.1 Example 1 - Crack at turbine disk bolt hole

During Phase I of the project (Ref. 5), a spin test analysis was used to demonstrate potential of an automated large crack growth capability based on data from Ref. 6. In the Phase I work, considerable problem-specific hard coding was needed to achieve this. At the end of Phase II, the equivalent analysis is now fully automated. A quarter model of the disk has been generated and significant growth of an initial corner crack at a bolt hole has been calculated.

This example is shown in Figure 2-5 to Figure 2-8.

2.3.2 Example 2 - Elliptic corner crack in DEN(T) specimen

This DEN(T) specimen model was provided during Phase II by W.P.A.F.B. The model was not running well in the interim release of Zencrack that was made available in June 2002. At the end of Phase II, the model runs successfully and can be used for corner and through crack phases of the growth. Additional growth could be modeled in the uncracked mesh was modified slightly (see section 5.9).

This example is shown in Figure 2-9 and Figure 2-10.
2.3.3 Example 3 - Semi-circular surface crack with residual stress

This example demonstrates the burrowing effect that is seen in crack growth profiles when residual stress distributions are included in the analysis. The example show results for constant amplitude external load at R=0 and R=0.75 with and without the inclusion of a residual stress distribution. The effect of residual stress on crack profile development and life are clearly demonstrated. A detailed explanation of the techniques uses to model residual stress effects for the Zencrack / Abaqus interface is given in the Appendix.

This example is shown in Figure 2-11 to Figure 2-15.

2.3.4 Example 4 - Stiffened panel

This example demonstrates the potential for cracks branching and dividing due to geometric discontinuities. The analysis is completed in three stages:

- growth to the stiffener
- growth across the “T”
- continued growth in the panel with a second crack in the stiffener

This example is shown in Figure 2-16 to Figure 2-18.
Figure 2-5 - Example mesh during corner crack phase
Figure 2-6 - Example mesh during through crack phase
Figure 2-7 - Example meshes during through crack phase
Figure 2-8 - Crack profiles
Figure 2-9 - Cracked mesh and growth profiles
Figure 2-10 - Meshes during corner and through crack phases
Figure 2-11 - Initial mesh and crack region
Figure 2-12 - Crack region during growth with residual stress for $R=0.75$
Figure 2-13 - Crack region during growth with residual stress for $R=0.75$
profiles with external load only  

profiles with external load and residual stress

Figure 2-14 - a vs N and profiles plots for R=0.75 with and without residual stress
Figure 2-15 - a vs N and profiles plots for R=0.0 with and without residual stress
Figure 2-16 - Initial crack and part-grown crack
Figure 2-17 - Part-grown crack(s) after 1st and 2nd break-through
Figure 2-18 - Part-grown cracks and crack profiles
3 Fracture Mechanics Issues

3.1 Residual stress and other forms of static loading

The presence of residual stress, (e.g. due to either shot peening, laser shock processing, low plasticity burnishing, welding etc.) affects the fatigue crack growth rate (FCGR) under cyclic loading.

The principles involved in addressing residual stress distributions are described in a number of reference documents published by Zentech (e.g. Ref. 8). These papers are available on our website at http://www.zentech.co.uk/zencrack_papers.htm. A detailed explanation of the techniques uses to model residual stress effects for the Zencrack / Abaqus interface is given in the Appendix (which is Ref. 8). An overview is given below.

In order to evaluate this effect and to develop an algorithm for incorporation in Zencrack software for 3D crack growth the following activities were performed:

- Review of relevant literature
- Decision regarding crack size to be analyzed
- Review of Afgrow residual stress capabilities
- Determination of required density of FE mesh
- Assess requirement for new residual stress crack-blocks
- Incorporation of residual stress and/or static loading capability
- System check
- Benchmarking of Zencrack against Afgrow and other test data.

These activities were carried out over a number of months and details have been presented in the interim reports. At the heart of the residual stress implementation is the necessity to combine a static load with a cyclic load. A summary of this important “load system” methodology is described below.

The load system methodology is summarized as follows:
• The number of unique load conditions is identified and an analysis step is set up for each condition in the finite element model (e.g. tension and bending as two load conditions).
  o For a given crack size this provides:
    ▪ One crack front stress state for each load condition.
    ▪ One set of energy release rate data (magnitudes and growth directions) for each load condition.
  • A number of load systems are defined each of which is based on the basic load conditions. A load system may be static, constant amplitude, spectrum, minimum or maximum.
  • A load system must use the energy release rate data for the appropriate load condition.
  • More than one load system may be based on one load condition.
  • A superposition load system may be constructed by combining a static and cyclic system or a minimum and maximum load system.
  • The total load spectrum is defined by a sequence of load systems.

A schematic example of this approach is shown in Figure 3-1.

In the general case the growth direction will be different from one load condition to the next. Hence the growth at a node will zigzag. This is accounted for during integration. The final position of the crack front that is used for the next finite element analysis assumes the growth is the vectorial sum of the zigzag path. This is shown schematically for one integration step in Figure 3-2.

In the superposition type of system the energy release rates from one load condition can be superimposed onto the energy release rates from another load condition to generate a range for crack growth. This allows a residual stress (or static system) to be combined with a cyclic system.
Load Conditions

Load condition 1
Load condition 2

Load Systems

Load system 1
Associated with load condition 1

Load system 2
Associated with load condition 2

Load system 3
Associated with load condition 2

Load Spectrum

Complete spectrum consists of load system sequence 1, 2, 3, 2

Figure 3-1 - Relationship between load conditions, systems and spectrum
Figure 3-2 - Effect of multiple load systems on non-planar crack growth

Seven types of load systems have been implemented:

**Static load system**
The static load system allows definition of a load system that is either static (i.e. constant) or time varying from one level to another. This type of system can be used, for example:

- to define the static portion of an overall combined static and cyclic load scenario that is later combined as a method 1 superposition system e.g. residual stress with cyclic load in a fatigue analysis
- to define a load time history in a time dependent analysis

**Constant amplitude load system**
The constant amplitude load system is the simplest cyclic load system to define. The finite element analysis of a load condition is carried out at one load level. This generates a set of energy release rate data. The input for the constant amplitude load system consists of two load scale factors to scale the finite element load level to the required minimum and maximum load levels. The energy release rate data is scaled by Zencrack to provide the range “delta root G”.
A constant amplitude system is treated internally as a spectrum with one block of load data (i.e. one load level range). It must therefore have a number of cycles associated with it. The default is $10^9$ cycles. This can be modified by the input of an optional value for the number of cycles in the block.

For a combined fatigue and time dependent analysis there must also be a time scale associated with the system to allow cyclic events to be positioned within the timeline.

**Characteristic K load system**

The characteristic K method can predict the fatigue crack growth for stationary variable amplitude loading that has a short recurrence period. The sequence of stationary loading must be deterministic or for random loading statistically invariant with time. A short recurrence between peak loads is required so that the crack is always growing through residual stresses from peak loads. The crack growth rate can then be considered as approximately regular. It is correlated to a characteristic stress intensity factor that is considered representative of the complete load history.

$$\Delta K_{rm} = m \sqrt{\sum (\Delta K_i)^m n_i}$$  \hspace{1cm} \text{Equation 3-1}

$$\sum n_i = 1.0$$  \hspace{1cm} \text{Equation 3-2}

where:

$i$ ranges from 1 to the number of load levels

$n_i$ is the proportion of the load cycles that corresponds to the stress intensity range $\Delta K_i$

$m$ is the exponent in the Paris equation representing the experimental crack growth data.

Derivation of $\Delta K_i$ must account for crack closure and follow a rational procedure of cycle counting.

As with a constant amplitude load system the finite element analysis of a load condition is carried out at one load level. This generates a set of energy release rate data. The input for the characteristic K load system consists, for each of the load levels, of two load scale factors to scale the finite element load level to the required minimum and maximum load levels and a value of the proportion of the number of cycles. Re-writing Equation 3-1:
\[ \Delta K_{rm} = \sqrt[\text{N}]{\sum_{i} \left( \left( S_{\text{max}i} - S_{\text{mini}} \right) K \right)^m n_i} \]  

Equation 3-3

where:

- \( K \) is the stress intensity factor at the load level in the finite element analysis
- \( S_{\text{max}i} \) is the factor on \( K \) to give \( K_{\text{max}} \) for the \( i \)th load level
- \( S_{\text{mini}} \) is the factor on \( K \) to give \( K_{\text{min}} \) for the \( i \)th load level
- \( n_i \) is the proportion of the total number of load cycles for the \( i \)th load level (\( \Sigma n_i = 1 \))

The processing of the load levels produces an equivalent \( \Delta K \) term and hence can therefore be written with a maximum and minimum scale factor i.e. as a constant amplitude system. A characteristic \( K \) system is treated internally as a constant amplitude spectrum with one load level. It must therefore have a number of cycles associated with it. The default is \( 10^9 \) cycles. This can be modified by the input of an optional value for the number of cycles in the block.

**Spectrum load system**

The spectrum load system can be considered as a series of constant amplitude systems with varying numbers of cycles and load ranges. Each constant amplitude system is referred to as a load block within the spectrum. The finite element analysis of a load condition is carried out at one load level. This generates a set of energy release rate data. The input for the spectrum load system consists of a series of two load scale factors to scale the finite element load level to the required minimum and maximum load levels and the associated number of cycles in the block. The energy release rate data is scaled by Zencrack to provide the range “delta root G”. The format of load spectrum data files consists of spectrum and sub-spectrum files.

If time dependent effects are to be included in the analysis, then the cyclic spectrum must include time data to allow each cyclic event to be positioned within the timeline.

**Minimum load system**

A minimum load system only needs to be defined if it is required to define a superposition load system using combination method 2. In this case the superposition consists of a minimum and maximum load system. The minimum load system represents conditions at the “bottom” end of the load range. For example, the loading may be constant residual stress plus a cyclic load. Then the
minimum load system refers to a load case in the f.e. analysis that includes the residual stress and minimum cyclic load.

**Maximum load system**

A maximum load system only needs to be defined if it is required to define a superposition load system using combination method 2. In this case the superposition consists of a minimum and maximum load system. The maximum load system represents conditions at the “top” end of the load range. For example, the loading may be constant residual stress plus a cyclic load. Then the maximum load system refers to a load case in the f.e. analysis that includes the residual stress and maximum cyclic load.

**Superposition load system**

A superposition load case is intended for use when a cyclic load is superimposed on a constant load e.g. cyclic loading with a residual stress distribution. A superposition load case always consists of one cyclic system and one static system. The two systems are combined to give an effective stress ratio that is a function of crack size.

\[
R_{\text{effective}} = \frac{K_{\text{external min}} + K_{\text{residual}}}{K_{\text{external max}} + K_{\text{residual}}}
\]

Please note that the value of deltaK is unchanged in LEFM Method 1.

Two methods are available for superposition load systems:\(^1\):

**Method 1 combination**

Method 1 is an LEFM approach and requires one linear FEA with two load steps. No crack face contact conditions should be used in this method. The first load step is with static load only (e.g. residual stress applied to the crack faces through a user subroutine such as DLOAD in Abaqus). The second load step is with the maximum value of the cyclic external load only. The external load

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\(^1\) Both of these methods can use LEFM and EPFM type of singularities along the crack fronts.
results can be scaled and combined with the static load result to give a solution at any desired external load level. This method can be used with constant amplitude or spectrum external loading.

**Method 2 combination**

Method 2 has been included to allow crack face contact to be incorporated in an analysis with combined static and cyclic loading. For crack growth prediction the method is limited to constant amplitude external loading in which results are generated at the minimum and maximum external loads in the cycle.

In Method 2 the first load step is with static load and minimum external load applied together. This minimum external load may be non-zero and provides a solution for the point of minimum external load in the load cycle. The second load step is with static load and maximum external load applied together and provides a solution for the point of maximum external load in the load cycle. It is noted that if the second load step is broken into a number of distinct points, the full curve between minimum and maximum external load can be generated.

**Combination equations**

The \( G_{\text{max}} \), \( G_{\text{min}} \) and \( R \) for method 1 combination are calculated as shown below. A similar approach is used for method 2 combination.

\[
G_{\text{new max}} = \left\{ \left( G_{\text{max static}}^{0.5} + G_{\text{max cyclic}}^{0.5} \right)^2 \right\} \\
G_{\text{new min}} = \left\{ \left( G_{\text{max static}}^{0.5} + G_{\text{min cyclic}}^{0.5} \right)^2 \right\} \\
R_{\text{new}} = \left( \frac{G_{\text{new min}}}{G_{\text{new max}}} \right)^{0.5}
\]

Equations 3-4 through 3-6

Note that Zencrack treats a “closed” crack front condition by using a negative sign on \( G_{\text{max}} \). For brevity the effect of sign is not shown in the equations above or below.

The combined growth direction is taken as a weighted ratio of the growth direction for the base and associated systems:
\[
(x_{\text{new}})_i = \frac{C_{\text{max, static}}^{0.5}(x_{\text{static}})}{G_{\text{max, new}}^{0.5}} + G_{\text{max, cyclic}}^{0.5}(x_{\text{cyclic}}), \quad i = 1,3
\]

where \(x_i\) for \(i=1,3\) represents the growth direction

### 3.2 Combining cyclic loading and load time history

A sample flight spectrum given as load vs time is shown in Figure 3-3. **It is important to note that the flight spectrum should start and end with zero loads so that the ground-to-air cycle is properly counted.** This is counted using Zencrack’s Rainflow pre-processing program to generate a cycle-by-cycle counted fatigue spectrum (see section 3.8). The time position at which each cycle occurs is stored in the counted data along with the minimum and maximum load levels for the cycle. The counted cycles will occur at the time of the peak for the cycle in the original time history. A user option is included in the Rainflow counting program to specify if the cycle peak is at the start or end of a hold period. The counted spectrum with positional information derived from Figure 3-3 is shown in Figure 3-4.

In the general case of the inclusion of time and cyclic effects, the analysis is controlled by the time line, with discrete cyclic “events” deemed to instantaneously increase the crack size.

In addition to supplying a cyclic spectrum file with time information, the analysis must be provided with the raw load versus time information.

The time history capability includes an option to also specify a temperature time history. This can be used to:

- select the correct temperature curve for \(da/dt\) versus \(K_{\text{max}}\) time-dependent crack growth data.
- select the correct temperature curve for \(da/dn\) versus \(\Delta K\) crack growth data.
- use appropriate temperature-dependent values of Young’s Modulus, Poisson Ratio (and expansion coefficient if relevant).
"Raw" time dependent load levels

Figure 3-3 - “Flight spectrum” load time history

Counted cycles with time positions

Figure 3-4 - Counted “fatigue spectrum” with cycles at known time points
3.3 Retardation effects

The scope of the SBIR requires inclusion of a retardation model such as the generalized Willenborg model under variable amplitude loading. This has been implemented in Zencrack.

The following are implemented:

- Crack growth with no retardation effects (the default).
- Basic Willenborg model.
- Generalized Willenborg model.
- Optional inclusion of Chang acceleration effects due to periodic under-load when the Willenborg model is requested.

3.4 Materials data

3.4.1 Crack growth data

The use of crack growth data during the crack growth integration scheme is essentially a look-up to evaluate da/dn or da/dt for known values of relevant parameters such as \( \Delta K \), stress ratio, temperature and time.

The following are implemented for da/dn:

- Paris equation.
- Walker equation.
- Tabular input.
- User subroutine passing in \( \Delta K \) as primary input.
- User subroutine passing in \( \Delta \sqrt{G} \) as primary input.

The following are implemented for da/dt:

- Paris equation.
- Tabular input.
- User subroutine passing in K as primary input.
- User subroutine passing in G as primary input.
3.4.2 Threshold

The threshold limit is the value of $\Delta K$ below which there is no growth. The method for defining the threshold limit depends partly upon the select format of the crack growth data.

The following are implemented for $\Delta K_{th}$ during cyclic crack growth:

- Single value of $\Delta K_{th}$.
- Single value of $\Delta K_{th}$ at stress ratio of zero (for Walker crack growth data only).
- Harter-t method (strictly only valid for tabular data that was derived using the Harter-t method).
- Points method with $\Delta K_{th}$ as a function of stress ratio.
- Using a $da/dn_{low}$ cut-off value (for tabular data).
- User subroutine.

The following are implemented for $K_{th}$ during time dependent crack growth:

- Single value of $K_{th}$.
- Using a $da/dt_{low}$ cut-off value (for tabular data).
- User subroutine.

3.4.3 Failure

Failure is determined by the value of $K$ exceeding the plane stress or plane strain fracture toughness of the material.

The following are implemented for $K_{th}$ during time dependent crack growth:

- use of plane stress fracture toughness
- use of plane strain fracture toughness

3.4.4 Temperature dependency in materials data

Temperature dependent crack growth data may be included when using the Paris, Walker or tabular options to define crack growth data. In addition, the value of temperature is passed into all four of the crack growth data user subroutines.
Temperature dependent threshold data may be included when using the single value or \( \frac{da}{dn_{\text{low}}} \) and \( \frac{da}{dt_{\text{low}}} \) cut-off tabular methods of threshold input. In addition, the value of temperature is passed into both threshold user subroutines.

Temperature dependency may also be included in the following materials data:

- Young’s modulus
- Poisson ratio
- Yield stress
- Plane strain fracture toughness
- Plane stress fracture toughness
- \( \Delta K_{\text{th}} \) at stress ratio of zero (for use in Willenborg retardation)

### 3.5 Using nodal displacements instead of the J-Integral

Zencrack software was originally designed to use J-integral values to evaluate the direction and magnitude of crack growth in a 3D structure. The capability to evaluate the J-integrals in a 3D FE model is only available in a limited number of commercially available FE software systems such as Abaqus (Ref. 10) and MSC.Marc (Ref. 11). In order to generalize the usage of the software and to develop an interface to Ansys, it was necessary to implement a method for calculating stress intensity factors from the displacement solution of the finite element analysis. The generic name for such techniques is “crack opening displacement” or COD methods. More specifically, the method implemented uses displacements near the crack tip, sometimes being referred to as the “crack tip opening displacement” or CTOD method. The method is only applicable to linear elastic material behavior.

The following are implemented:

- Calculation of stress intensity factors \( K_I, K_{II} \) and \( K_{III} \) from nodal displacements.
- Conversion of the stress intensity factors to an equivalent energy release rate magnitude.
- Calculation of a direction for crack growth based on the stress intensity factors.
- Crack growth prediction using either:
  - j-integral based results in the Abaqus and MSC.Marc interfaces, or,
  - displacement based results in the Abaqus, MSC.Marc and Ansys interfaces.
### 3.5.1 Using displacements to determine the open or closed status of the crack front

In any type of finite element analysis that does not include contact definitions between the crack faces it is possible for the loading to cause part or all of a crack front to close. This would be seen in a displaced post-processor plot as penetration of one side of the crack into the other side. In 3D models, particularly of a fully embedded crack, it can sometimes be difficult to determine the open and closed regions by inspection of such plots. In addition, the calculation of the energy release rate does not provide any information on whether the crack is open or closed.

Zencrack generates an “open” or “closed” status for:

- Each corner node on the crack front.
- Each node on the crack face.

Zencrack generates an “open” or “closed” status for each corner node on the crack front by extracting and processing nodal displacements. Each crack front node and the associated crack face quarter point nodes in the collapsed crack front elements are examined. A local coordinate system is created to allow calculation of relative opening displacements of pairs of nodes on either side of the crack face in local mode I, II and III orientations. The local mode I direction is into side 1 of the crack. An example is shown in Figure 3-5. The local mode I opening displacement, if positive, indicates an open crack at that point. If the displacement is negative the crack is closed.

When a crack front node is found to be closed a negative sign is applied to the energy release rate and stress intensity factors for that node to indicate a closed status. Any “closed” crack front node is forced to have zero growth. A report will be provided that the node is below threshold for a complete spectrum pass.
3.5.2 General relationships between displacements and stress intensity factors

Displacements at the quarter point node positions are used to calculate $K_I$, $K_{II}$ and $K_{III}$ from the relative opening displacements described in section 3.5.1.

The general displacement expressions for a point on the crack face for an elastic material are:

$$V_i = \frac{(1 + \nu)(1 + \kappa)}{2E} \sqrt{\frac{2r}{\pi}} K_I + O(r)$$  \hspace{1cm} \text{opening} \hspace{1cm} \text{Equation 3-8}

$$V_{ii} = \frac{(1 + \nu)(1 + \kappa)}{2E} \sqrt{\frac{2r}{\pi}} K_{II} + O(r)$$  \hspace{1cm} \text{sliding} \hspace{1cm} \text{Equation 3-9}

$$V_{iii} = \frac{2(1 + \nu)}{E} \sqrt{\frac{2r}{\pi}} K_{III} + O(r)$$  \hspace{1cm} \text{tearing} \hspace{1cm} \text{Equation 3-10}

where:
\[ \kappa = 3 - 4\nu \quad \text{for plane strain} \]
\[ \kappa = \frac{3 - \nu}{1 + \nu} \quad \text{for plane stress} \]

and \( r \) is distance from the crack front.

Re-writing these equations and re-arranging, the usual expressions, neglecting the \( O(r) \) terms, for \( K_I \), \( K_{II} \) and \( K_{III} \) are taken to be:

\[
K_I = \frac{E V_i}{4B} \sqrt{\frac{2\pi}{r}} \\
\text{Equation 3-11}
\]
\[
K_{II} = \frac{E V_{ii}}{4B} \sqrt{\frac{2\pi}{r}} \\
\text{Equation 3-12}
\]
\[
K_{III} = G V_{iii} \sqrt{\frac{\pi}{2r}} \\
\text{Equation 3-13}
\]

where:

\[
B = 1 - \nu^2 \quad \text{for plane strain} \]
\[
B = 1 \quad \text{for plane stress} \]
\[
G = \frac{E}{2(1+\nu)}
\]

An equivalent energy release rate term can be calculated from the stress intensity factors. This is defined as:

\[
G_{equiv} = \frac{B}{E} \left( K_I^2 + K_{II}^2 \right) + \frac{1}{2G} K_{III}^2 \\
\text{Equation 3-14}
\]

The local out of plane growth angle is defined using the following relationship, which allows the global growth direction to be calculated:

\[
\theta = \tan^{-1} \left( \frac{K_{II}}{K_I} \right) \\
\text{Equation 3-15}
\]
3.5.3 Distance between a quarter point node and the crack front

In order to calculate $K$ from nodal displacement of a single quarter point node (see section 3.5.2) it is required that the distance from the crack front to the node is known. In many cases element edges are normal to the crack front and the distance required will simply be the distance to the nearest crack front node. However, it is not guaranteed that an edge will be normal to the crack front e.g. Figure 3-6. In such cases the distance, $r$, as shown in Figure 3-7, is calculated by considering the shape function along the crack front. The algorithm samples 100 points along each element edge on the crack front to find the closest sampling point to the node. This is deemed to be the required distance.

Having calculated the stress intensity values at these “closest” crack front locations, the values are then extrapolated to the nodal positions. These nodal positions are reported and used in any subsequent crack growth calculations when displacement based results are used to drive the crack growth integration. An example of a typical report of interpolated values is given below. It is noted that the interpolated values are always based on the plane strain values.
Figure 3-6 - Element edges normal (top) and not normal (bottom) to the crack front
3.6 Stress state constraint factor

The preference in Zencrack is to use energy release rate rather than stress intensity factors wherever possible. There are, however, a number of situations in which the conversion between \( G \) and \( K \) is necessary. This requires knowledge of the state of stress. To assist in these conversions, a stress state constraint factor is defined at each node along a crack front and describes a state of pure plane stress, pure plane strain or something in between.

The stress state constraint factor is used for three calculations in the program:

- To calculate the apparent fracture toughness as a function of plane stress and plane strain fracture toughness:

\[
K_{\text{app}} = K_{IC} + \left( \frac{6 - SS_{\text{factor}}}{4} \right) (K_{IC} - K_C)
\]

Equation 3-16

where \( K_{IC} \) is the plane strain fracture toughness and \( K_C \) is the plane stress fracture toughness. Under this definition the stress state constraint factor, \( SS_{\text{factor}} \), ranges from 2 for plane stress to 6 for plane strain.
• During Willenborg retardation as described in section 4.4. The selected method of surface constraint definition affects this calculation due to the way that the empirical retardation model is formulated.
• During the report for “automatic” $K_i$ when $K$ is converted from energy values. This is purely a report and does not affect the analysis results.

It is noted that when the solution for crack growth uses displacement based stress intensity factors, the values are based on a plane strain condition regardless of the method chosen for definition of the constraint factor.

Several methods are available to define values for the stress state constraint factor. These are described in the following sections. In general the constraint factor is a function of surface proximity, specimen thickness and the applied load.

- Pure plane strain at all crack front nodes
- Pure plane stress at all crack front nodes
- User subroutine
- Thickness method

The thickness method is based on work by Harter as implemented in Afgrow (Ref. 2). The stress state constraint factor is defined by:

$$ SS_{\text{index}} = 6.7037 - \left( \frac{1.4972}{t} \right) \left( \frac{K_{\text{max}}}{\sigma_y} \right)^2 $$

subject to the plane strain and plane stress upper and lower bounds:

$$ 6 \geq SS_{\text{index}} \geq 2 $$

where $t$ is thickness and $\sigma_y$ is yield stress. Harter derived this relationship from test results for aluminum, titanium and steel aircraft alloys.

- Distance method

The surface method allows definition of a constraint factor based on the distance along the crack front from a free surface. This method requires input of two distances, $S_1$ and $S_2$,
along the crack front as shown in Figure 3-8. Distance S1 is the distance from the free surface over which plane stress conditions are assumed. Distance S2 is the distance over which transition occurs to plane strain. Plane strain conditions are assumed outside these regions. The following restrictions should be noted with regard to this method:

1. The distances S1 and S2 are applied at the “ends” of all crack fronts in the model regardless of whether or not there is a free surface at the end of the crack front. This presents difficulties if one end of a crack front is at a free surface and the other lies on a symmetry plane.
2. If there is more than one crack front the values of S1 and S2 are applied to all crack fronts.

Figure 3-8 - Surface constraint using the distance method
3.7 Reverse crack growth (failure analysis)

Reverse crack growth is implemented in Zencrack using *GROWTH CONTROLS, type=reverse.

Test analyses of reverse crack growth have shown instabilities in the crack growth profiles although it has been demonstrated that a stable solution can be obtained by imposing constant energy release rate along the crack front. These matters were discussed fully in the document SBIR Phase 1 Progress report to 13th September 2000.

3.8 Rainflow load spectrum cycle counting program

The rainflow counting software is implemented as a stand-alone program utility. It has been further developed to accept time signatures in the uncounted fatigue spectrum for use in time dependent fatigue growth. It is possible for the user to define whether the maximum stress during a hold period occurs at the beginning or end of the hold period. Intermediate time points for the maximum stress during hold can be obtained by introducing a stress level slightly above the holding stress. The software determines whether time dependency data is present contextually.

3.9 Material orthotropy

Preferred crack growth direction can be specified using the Zencrack *GROWTH CONTROLS keyword with parameter DIRECTION=initial or DIRECTION = preferred.

It is assumed by definition that the starter crack forms in the preferred growth plane direction. Therefore the FEM should model the starter crack in the preferred growth plane.

If the “DIRECTION = preferred” option is used, crack growth is calculated in the same way as “DIRECTION=initial” but performs checks that the initial crack plane is in the same plane or parallel to that specified by the user on the appropriate data lines.

3.10 Batch processing

This activity has been reviewed to look at a batch process to define a distribution of initial crack front starter sizes according to some statistical distribution. The conclusion was that it would be much simpler to modify Zencrack input files and create multiple Zencrack analyses.
4 Integration Scheme Issues

An integration method has been developed to provide maximum crack growth prediction accuracy with a minimum number of finite element analyses (FEA). The objective is to remove control of the integration from the user and to allow a fully automated solution in which the integration steps adapt depending upon the accuracy of the solution. This is done by tracking the potential errors in energy release rates from one finite element analysis to the next and modifying the allowed steps size, \( da \), accordingly.

The integration algorithm processes each crack front corner node in sequence and ensures that all nodes are integrated to the same \( dn \) and/or \( dt \), subject to a maximum allowed change in \( da \).

The following factors that further affect the integration scheme are described in the subsequent sections:

- Use and definition of materials data
- Spectrum loading with varying stress ratios
- Residual stress
- Retardation
- Time dependent crack growth prediction

The majority of these effects may be combined in one analysis e.g. spectrum loading, retardation, residual stress and temperature dependent material properties. The exception is that retardation is not implemented if time dependent crack growth is requested.

Zencrack uses results from finite element analysis of discrete crack positions together with a crack growth integration scheme to provide a general crack growth capability. The integration scheme is based on the use of energy release rate, \( G \). This section describes issues related to the integration scheme and accuracy of the analysis.

4.1 Overview of integration scheme

For any crack front that is analysed by Zencrack the result in terms of data for crack growth prediction is an energy release rate magnitude and growth direction at each crack front node. The
integration is required to calculate the amount of growth, da, that occurs over the next n cycles (or time t) within certain limits on da and/or dn and/or dt.

For a fixed load magnitude the energy release rate is constantly changing as the crack grows due to changes in the crack size and shape i.e. dG/da≠0. Hence the growth scheme should make some allowance for this during the integration.

The preferred assumption used by Zencrack is that for given load the rate of change of energy release rate with crack size, dG/da, is constant as the crack grows. A known rate of change calculated from analysis of previous crack positions is assumed to occur over the next integration step to a new crack position. This assumption is based on simple fracture mechanics theory for a crack in a plate:

\[ K = f(\text{geometry}) \sigma \sqrt{\pi a} \]  

Equation 4-1

Neglecting effects of the geometry function gives an approximation that:

\[ K^2 a^2 \text{ and therefore } G \propto a \]  

Equation 4-2

In the integration scheme this is generally written as:

\[ G_{\text{max final}} = G_{\text{max initial}} + \left( \frac{dG_{\text{max}}}{da} \right) da \]  

Equation 4-3

Two growth schemes are available based on assumptions about dG/da. The schemes are:

- The “constant G” scheme.
  This is the simplest but least accurate scheme. It assumes that G is constant i.e. dG/da is zero. During the integration of each cyclic load block the energy release rate is assumed to remain constant. At the end of each cyclic load block there is no update of G, i.e. \( G_{\text{final}} = G_{\text{initial}} \). During integration of time segments the energy release rate is assumed to remain constant.

- The “forward predictor” or “constant dG/da” scheme.
This scheme assumes that $G$ varies linearly i.e. $\frac{dG}{da}$ is constant. Past history is used to forward predict the variation of $G$ through the next integration step. During the integration of each cyclic load block the energy release rate is assumed to vary linearly with crack size. At the end of each load block the estimate of $\frac{dG}{da}$ and the accumulated growth for the block are used to update the value of $G$ for the new crack position, i.e. $G_{\text{final}} = G_{\text{initial}} + \frac{dG}{da} \times da$. During integration of time segments the energy release rate is updated based upon the accrued growth.

The forward predictor scheme is the preferred scheme. After the first finite element analysis no value of $\frac{dG}{da}$ is available and the constant $G$ scheme is always used.

### 4.1.1 Constant $G$ integration

The assumption in the constant $G$ integration scheme is that the energy release rate, $G$, is constant as the crack grows. The known energy release rate at the start of the integration is assumed to occur over the entire integration step. This is clearly not correct but provides a numerically simple calculation.

Given a value of delta root $G$ for one crack size the growth over an increment of $dN$ cycles is given by using the general form of the growth equation:

$$
\frac{da}{dN} = f \left( G_{\text{max}}^{\frac{1}{2}} - G_{\text{min}}^{\frac{1}{2}} \right)
$$

so

$$
da = \left[ \frac{da}{dN} \right] dN
$$

Equation 4-4

Generally $G$ increases with $a$. Hence:

- for a given $dN$ this scheme will under estimate the crack growth
- for a given $da$ this scheme will over estimate the number of cycles.

The constant $G$ integration scheme is used in the following circumstances:

- When the constant $G$ growth scheme is requested
- After the first finite element analysis when the forward predictor growth scheme is selected
- For a node that has $dG/da=0$ if the forward predictor scheme with closed form solution is requested.
4.1.2 Constant dG/da integration

Two variants of the forward predictor scheme are available:

- The “closed form” solution is valid for crack growth data that can be described as one or more Paris segments for each value of R. It also requires that the stress ratio does not change as the crack advances. This is not the case, for example, if static and cyclic loads are combined.

- The “numerical” solution is valid for any type of crack growth data. This integrates numerically from one crack position to the next.

These variants are described in sections 4.1.2.1 and 4.1.2.2. The present input options for crack growth data allow the use of closed form or numerical solution schemes for all types of fatigue crack growth data input except the user subroutine option. If crack growth data is entered using a user subroutine the closed form solution forward predictor method is not available.

For spectra with a large number of load blocks with low numbers of cycles per block the numerical forward predictor may require long run times.

4.1.2.1 Constant dG/da integration - closed form solution

The closed form solution assumes crack growth data in terms of Paris segments. The objective of the closed form scheme is to determine the number of cycles required to move from a known initial position to a final position along one or more Paris segments of the type shown in Figure 4-1. The stress ratio is constant as the crack grows.
For known position $i$ and required crack growth magnitude $da_i$ determine the final position $f$ (which may be on a subsequent segment) and the number of cycles required to reach that position

**Figure 4-1 - Objective of the forward predictor scheme**

The line segment is described by:

\[
\frac{da}{dN} = C(\Delta K)^n = C'(\Delta \sqrt{G})^n
\]

Equation 4-5

At the initial point $i$ all values are known. Hence the line segment can be described by:

\[
\frac{da}{dN} = \left( \frac{da}{dN} \right) \left( \frac{\Delta K}{\Delta K_i} \right)^n = \left( \frac{da}{dN} \right) \left( \frac{\Delta \sqrt{G}}{\Delta \sqrt{G_i}} \right)^n
\]

Equation 4-6

Integrating this equation in the delta root $G$ form gives:

\[
\int_i^f dN = \int_i^f \frac{1}{\left( \frac{da}{dN} \right) \left( \frac{\Delta \sqrt{G}}{\Delta \sqrt{G_i}} \right)^n} da
\]

Equation 4-7
Changing integration variables on the right hand side and assuming that R is constant throughout the integration:

\[
\int_{i}^{f} dN = \int_{i}^{f} \frac{1}{a'_i} \left( \frac{\Delta \sqrt{G}}{\Delta \sqrt{G_i}} \right)^{n} \left( \frac{1 - R}{2 \Delta \sqrt{G}} \right) d\left( \Delta \sqrt{G} \right)
\]

Equation 4-8

Assuming that \( \frac{dG_{\text{max}}}{da} \) is constant gives:

\[
G_{\text{max}, f} = G_{\text{max}, i} + \left( \frac{dG_{\text{max}}}{da} \right) da_{i/j}
\]

Equation 4-9

Re-arranging, integrating and simplifying:

\[
\Delta N = \frac{2 \left( \Delta \sqrt{G_i} \right)^2}{a'_i (1 - R)^2 (n - 2)} \left[ 1 - \left( \frac{\Delta \sqrt{G_i}}{\Delta \sqrt{G_f}} \right)^{n-2} \right]
\]

Equation 4-10

Due to the \((n-2)\) term in the denominator of the final expression the closed form solution is not recommended for Paris segments with a power close to 2. The program issues a warning if this condition arises.

4.1.2.2 Constant dG/da integration - numerical solution

The objective of the numerical scheme is to determine the number of cycles required to move from a known initial position to a final position along some arbitrary crack growth curve(s).

Generalising Equation 4-5 as:

\[
\frac{da}{dN} = f \left( \Delta \sqrt{G} \right)
\]

Equation 4-11

allows a general integral to be written:

\[
\int_{i}^{f} dN = \int_{i}^{f} \frac{1}{f \left( \Delta \sqrt{G} \right)} da
\]

Equation 4-12

This expression is integrated numerically if the numerical scheme is selected:
The numerical scheme is the default forward predictor setting if crack growth data is entered using user subroutine user_dadn.

The numerical scheme takes due account of changing stress ratio between the integration limits and therefore should be used for problems where residual stresses are modeled with superimposed cyclic loading. If an attempt is made to use the closed form solution for a load spectrum containing a superposition load system, the program issues a warning and switches to the numerical solution.

4.2 Spectrum loading

The integration scheme allows definition of spectrum load consisting of blocks of constant amplitude loading. Each block may have 1 or more cycles thus allowing random load spectra to be analyzed. Each block is integrated in turn until the limit on the allowed crack growth is reached.

There is no upper limit on the number of levels in the load spectrum. For extremely large spectra (i.e. a large number of load levels), memory limitations may require that part of the spectrum is stored on disk in scratch files. This is under the control of the user.

The integration scheme is formulated such that finite element analyses are only required at a number of discrete points through the analysis. The accuracy of integration between the finite element analyses is controlled as described in section 4.7.

4.3 Residual stress and other forms of static loading

Two methods are available for including the effects of residual stress in an analysis. These are referred to as "method 1" and "method 2". These methods both involve the use of definition of load systems and the subsequent combination of the load systems within the integration scheme. The load systems and combination equations are described in section 3.1.

Taking method 1 as an example, the result of each finite element analysis is a value of $G_{\text{residual}}$ and $G_{\text{external}}$ for each crack front node. Hence there are two forms of the general variation of $G$ through the integration as described by Equation 4-3: one for $G_{\text{residual}}$ and one for $G_{\text{external}}$. The combination equations (Equation 3-4 to Equation 3-6) are used to combine these values during integration to determine a “current” effective stress ratio. Since this combination means that the stress ratio is a
function of crack growth, the numerical integration scheme must be used rather than the closed form integration scheme if the forward predictor integration scheme is used.

A reference on the Abaqus interface for residual stress analyses is given in section 3.1.

4.4 Retardation

4.4.1 General discussion

The inclusion of the generalized Willenborg retardation model within the integration scheme means that an inner loop of cycle-by-cycle integration is required during a retardation phase. This is necessary in order to track the plastic zone size and determine the end of the retardation condition. One consequence of this is that analysis run time may be significantly increased when retardation is included.

The optional use of Chang acceleration due to under-load effects uses the method implemented in Afgrow. This method was selected in order to allow comparative analyses to be conducted.

4.4.2 Implementation issues

Two main areas of difficulty have been identified during the implementation of the Willenborg retardation model. These are described in the following sections.

4.4.2.1 Conversion of G to K and vice versa

The integration scheme in Zencrack works with energy release rate rather than stress intensity factor. The Willenborg retardation model uses equations that involve K terms. For example:

The plastic zone size: $R_y = \frac{K_{\text{max}}}{\sigma_y SS_{\text{index}}} \frac{1}{\pi}$

where $SS_{\text{index}}$ is the stress state index from 2 for plane stress to 6 for plane strain

$\sigma_y$ is the yield stress

The effective stress ratio: $R_{\text{eff}} = \frac{K_{\text{MINeff}}}{K_{\text{MAXeff}}}$

where $K_{\text{MINeff}} = K_{\text{MIN}} - K_r$
\[ K_{\text{MAXeff}} = K_{\text{MAX}} - K_r \]
\[ K_r = \phi \left( K_{\text{MAXol}} \left( 1 - \frac{da}{dR} \right)^{0.5} - K_{\text{MAX}} \right) \]
\[ \phi = \frac{1 - \frac{\Delta K_{\text{th(R=0)}}}{K_{\text{MAX}}}}{\text{shutoff ratio} - 1} \]

The definition of a stress state index for each crack front node in the Zencrack analysis is implemented by several user options, including an iterative procedure to provide an “automatic stress state” according to the method available in Afgrow. The Willenborg implementation converts the “current” energy release rate terms to stress intensity terms using the standard mode I conversion in order that the above equations can be used. The conversion of \( G \) to \( K \) itself requires the calculated stress state constraint factor.

Test analysis against Afgrow have been conducted using a plane stress condition as this is the only possible approach to mimic the 2D nature of the Afgrow analyses. To fully verify the retardation capability in Zencrack it would be necessary to compare results against experimental data. It is recommended that the plane stress constraint factor be selected for any analysis that includes retardation.

4.4.2.2 Cut-offs for \( R_{\text{low}} \) and \( R_{\text{high}} \)

The following modifications are made by Afgrow to cater for the \( R_{\text{low}} \) and \( R_{\text{high}} \) cut-off conditions in crack growth data:

If \( R < R_{\text{low}} \):
- the value of \( \Delta K \) is defined using \( \Delta K = (1 - R_{\text{low}})K_{\text{max}} \)
- the crack growth curve for \( R_{\text{low}} \) is used to determine \( \frac{da}{dN} \)

If \( R > R_{\text{high}} \):
- the value of \( \Delta K \) is defined using \( \Delta K = (1 - R)K_{\text{max}} \)
- the crack growth curve for \( R_{\text{high}} \) is used to determine \( \frac{da}{dN} \)
These conditions are further complicated when retardation is taking place. First the effective stress ratio is calculated as described above. Then the modifications for $R_{\text{low}}$ and $R_{\text{high}}$ are made.

It is noted that these checks can readily be skipped by setting very low and high values for $R_{\text{low}}$ and $R_{\text{high}}$ respectively.

### 4.5 Time dependent loading

#### 4.5.1 General discussion

Time dependent loading may be analyzed with or without cyclic effects. The general case of the inclusion of time and cyclic effects is described below. In this instance, the integration is controlled by the time line, with discrete cyclic “events” deemed to instantaneously increase the crack size.

The combined effect of cyclic and time effects is such that the overall growth is:

$$\Delta a_{\text{mission}} = \Delta a_{\text{cyclic}} + \Delta a_{\text{time}}$$  \hspace{1cm} \text{Equation 4-13}

Equation 4-13 can be expanded to:

$$\Delta a_{\text{mission}} = \left(\frac{da}{dn}\right)_{\text{cyclic}} \cdot \Delta n + \left(\frac{da}{dt}\right)_{\text{ad}} \cdot \Delta t$$  \hspace{1cm} \text{Equation 4-14}

where $\left(\frac{da}{dn}\right)_{\text{cyclic}} = f(K,R,T)$ and $\left(\frac{da}{dt}\right)_{\text{ad}} = g(K_{\text{max}},T)$

In order to implement the new features, a common timeline had to be established between the uncounted load history specified on a timeline basis (designated as “flight spectrum”) and the cyclic mechanical loading (designated as “fatigue spectrum”) extracted by counting the flight spectrum in the Zencrack rainflow counting pre-processor utility program. By definition these two spectra are of the same load system type (e.g. both producing stress levels due to varying engine throttle settings) and can be processed in a single FEA load step and the values scaled to maximum and minimum stresses in the fatigue spectrum and the stress levels in the flight spectrum. The flight spectrum is
used to integrate with respect to time to calculate $\frac{da}{dt}$ between fatigue spectrum cyclic events using the material $\frac{da}{dt}$ versus $K_{max}$ growth data. The instantaneous damage $da_{cyclic}$ associated with each cyclic event in the fatigue spectrum is calculated by integrating with respect to $dn$ (cycle-by-cycle) using the material $\frac{da}{dn}$ versus $\Delta K$ fatigue growth data. The integration scheme can handle a cyclic system and one superimposed static load system (such as residual stress).

4.5.2 Modifications to integration scheme

The task was to integrate through the flight spectrum time history using $\frac{da}{dt}$ and $K_{max}$ data. Fatigue cycles will occur at specified times in the time history as defined by the cycle-by-cycle data in the counted fatigue spectrum file. Whenever the time integration reaches one of these cycles, the effect of $\frac{da}{dn}$ is instantaneously added to the accumulated growth.

At the start of an integration phase the energy release rate associated with the flight spectrum time dependent load level $S_{max}$ is known and is $G_{max}$. The value of $dG_{max}/da$ is also known, based on previous history.

The general relationship for $G_{max}$ over an integration step is taken to be:

$$G_{max} = G_{max,i} + \frac{dG_{max}}{da} \cdot da$$

When time dependency is included this becomes:

$$G_{max} = G_{max,i} + \frac{dG_{max}}{da} \cdot (da_{cyclic} + da_{id})$$

where $da_{cyclic}$ occurs over one cycle and $da_{id}$ occurs between the current fatigue cycle and the previous cycle.

The driver for the integration has been taken to be $da/dt$ with $da/dn$ effects being instantaneously added as each cycle occurs. The integration scheme is as follows:
1. Initialize \( da=0 \) at the start of the integration phase.
2. Find the time to the next counted fatigue spectrum cycle.
3. Integrate \( da/dt \) for that time period and update \( da \).
4. Calculate the instantaneous \( da/dn \) and add the \( da \) for one cycle to the cumulative \( da \).
5. Go to 2 and repeat until the \( da \) tolerance (or some other control) is met.

Any static stress case such as residual stress is treated as an add-on- it just modifies the instantaneous \( R_{eff} \) and \( K_{max} \) and so influences both the cyclic and time dependent parts of the integral.

A stand-alone program was developed to test some time dependent and combined time and cyclic dependent problems against Afgrow. The stand-alone program was developed to use the same spectrum input structure as Afgrow. Using this approach time dependency is incorporated as follows:

- Each fatigue cycle has a time period allocated to it. If the input is of block spectrum type, a single time is allocated to the entire block and distributed evenly to each cycle within the block. Time integration of each cycle is carried out by splitting the cycle into a sine wave defined by 100 strips.
- Ramp up, ramp down and holds periods are defined by the start and end load levels and the duration. Time integration of these periods is based on an average of conditions at the start and end up of the period. Each period of this type is treated as a “cycle” in the reported Afgrow cycle count.
- A single spectrum file referred to as a time dependent spectrum is defined and may contain all types of input i.e. ramp up and down, hold cycles and fatigue cycles.

This approach works by assuming that the analysis is controlled by cyclic events with time integration added as a secondary effect. Ramp up, ramp down and hold periods are defined as “cycles” and the final cycle count includes these cyclic events in addition to the fatigue cycles in the load history. To mimic this procedure in SEnad it the integration of each cycle is done by calculating the cyclic effect (if any) followed by the time effect.
The implementation in Zencrack differs from that used by Afgrow. The Zencrack philosophy is based on the idea of an uncounted spectrum that defines the time history. A time-based counted cyclic spectrum is derived from the uncounted spectrum (by rainflow counting). In this time-based counted spectrum each cycle has a time associated with it at which the cycle peak is deemed to occur. This removes the requirement that is present in Afgrow to associate a time period with a counted fatigue cycle.

Unlike in Afgrow the time integration does not assume a sinusoidal waveform to integrate a fatigue cycle. Instead a linear change in load level is used from one time history point to another. If a sinusoidal waveform is required, the time history must be split into an appropriate number of points to define the variation. For the purposes of comparison in the examples given below, the time history and cyclic data have been constructed in such a way as to mimic the Afgrow analyses i.e. the time history has been defined with 9 points per cycle assuming a sinusoidal variation of load.

In the Zencrack implementation the integration is considered to be controlled by time history effects. Cyclic effects are added as instantaneous increase of \( da \) whenever a cycle peak is encountered in the time history. Each segment of time integration falls into one of the following four categories:

- time integration to the next time point
- time integration to the next cyclic event
- time integration to the next time point starting with a cyclic event
- time integration to the next cyclic event starting with a cyclic event

It is noted that in the Afgrow approach the accuracy of time integration, particularly for ramp up, ramp down and hold periods, depends upon the duration of the period. The only means of improving the accuracy is to split the period manually into multiple smaller periods by modifying the spectrum file. In the Zencrack implementation the accuracy can be controlled by defining parameters that determine the number of segments between any two time history points without having to change the spectrum data itself.
4.6 Closed form and numerical integration

For a Paris-type segment of cyclic crack growth data a closed form integration is possible to determine the number of cycles for a given amount of growth. A numerical scheme has been implemented as an option and MUST be used under the following circumstances:

- When a static load system is included, such as residual stress (because the assumption of \( \frac{dR}{da} = 0 \) in the closed form method is not valid).
- When a user subroutine is used to define \( da/dn \).

It is noted that during a Willenborg retardation phase the condition \( \frac{dR}{da} = 0 \) is not true. However, this phase is integrated and updated cycle-by-cycle so the closed form method can be used (provided there is no residual stress superposition as well).

Further details of the closed form and numerical integration are given in sections 4.1.2.1 and 4.1.2.2.

4.7 Analysis accuracy and control

During development of the integration scheme the method of controlling the step size between finite element analyses has been modified a number of times. In the final version, three schemes are available. The recommended scheme is the “error control” scheme.

After a finite element analysis is completed Zencrack has an “accurate” value for energy release rate, \( G \), at each crack front node. This is denoted by \( G_{\text{maxFEi}} \) for the \( i^{\text{th}} \) f.e. analysis. During integration to obtain the updated crack front position there is an assumption about the way that \( G \) changes over the step resulting in growth magnitude \( da_{FE} \) and the expected value of \( G \) from the next f.e. analysis:

\[
G_{\text{maxESTIMATE}} = G_{\text{maxFEi}} + \left( \frac{dG_{\text{max}}}{da} \right) da_{FE}
\]

Equation 4-15

When the analysis of the updated crack front is completed, an “accurate” value for \( G \) is available for the \( i+1^{\text{th}} \) f.e. analysis, \( G_{\text{maxFEi+1}} \). As Equation 4-15 is just an approximation the differences between \( G_{\text{maxFEi+1}} \) and \( G_{\text{maxESTIMATE}} \) is an indicator of how accurate the integration was for the last step.
If this difference exceeds a user-specified tolerance (which is 2% by default), the allowed da increment for the following step is cut-back. If several steps fall below the tolerance, the step allowed for da can be increased.

It is important to note that this scheme does not repeat any finite element analyses if the tolerance is not met. It has been seen that provided the correct start-up procedure is used to prevent large errors at the beginning of an analysis, the method is robust and provides a stable and accurate solution. A number of control options are available to ensure a smooth start-up.

If the error control scheme is not used and the step sizes are too big the errors in $G$ will tend to increase during the analysis and may oscillate. Oscillations can build up in the crack profile if the step size is too large.

Note: Although described here in terms of $G$, the error control in fact uses $\sqrt{G}$. This is necessary to allow meaningful use of the method when static load systems are superimposed onto cyclic systems.
5 Topology And Finite Element Issues

Zencrack uses a crack-block approach to generate the cracked mesh. This involves replacing the 3D finite element with initial crack with a crack-block consisting of an FE mesh well refined around the crack front in order to obtain an accurate value of energy release rate. As the crack grows the surrounding topology changes and the FE mesh distorts, especially in the case of large crack growth. In order to maintain accuracy it is important that the distortion of the FE mesh is kept to a minimum. A number of techniques developed and incorporated in the software are discussed in this section.

5.1 Alternative schemes for treatment of large crack growth

Currently three methods have been developed to solve large crack growth:

- Large crack-blocks
- Crack-block boundary shifting
- Crack-block boundary flipping

5.2 Large crack-block methodology

The standard method of using Zencrack to generate a cracked mesh is to replace one element of the uncracked mesh with one crack-block. Growth is then limited to remain within the crack-block region unless boundary shifting and crack-block transfer are used. Without these options, the amount of crack growth is limited.

An alternative approach is to replace a group of elements with a single large crack-block. The boundaries of this large crack-block cannot be made to match with the rest of the mesh. Hence tying constraints are required to link the two dissimilar mesh regions. This type of approach is one that has been seen to become increasingly popular as automated methods have become available for tying dissimilar meshes. The approach for fracture mechanics applications relies on a fine mesh in the crack region (slave) being constrained at its boundary to a coarser mesh in the rest of the model (master).
There are a number of benefits in using this method over a one-to-one crack-block approach:

- It is necessary to have a higher element density in the crack region for this method than for the one-to-one approach. This should have benefits for better evaluation of displacements, incorporation of residual stresses and inclusion of extra element rings at the crack front.
- Assistance with project activity 2.17 - incorporating crack-blocks in a tetrahedral mesh.
- It is also noted that a small initial defect can be grown through a relatively large region of the mesh using many of the existing capabilities in Zencrack.

### 5.2.1 New crack-blocks

In order to use a large crack-block capability it was necessary to develop new crack-blocks. A number of variations were tried before settling upon the crack-blocks shown in Figure 5-1 to Figure 5-6. The main requirement in generating these crack-blocks is that of a sufficient nodal density on the outer surface to allow tying as a slave surface to a coarser master surface in the surrounding mesh region. The salient features of these crack-blocks are:

- Semi-elliptic and through crack-blocks that can merge side-to-side.
- A much higher element density than existing crack-blocks due to a requirement for tying to adjacent mesh regions.
- The ideal crack size within the crack-blocks is 0.375 of the edge length of semi-elliptic crack-blocks and 0.5 for through crack-blocks. It is essential that at much smaller and much larger crack sizes the distribution of nodes on the faces remains suitable for tying to the adjacent mesh.
Figure 5-1 - Crack-block sq496x8 at ratio 0.15
Figure 5-2 - Crack-block sq496x8 at ideal ratio 0.375
Figure 5-3 - Crack-block sq496x8 at ratio 0.85
Figure 5-4 - Crack-block st496x8 at ratio 0.15
Figure 5-5 - Crack-block st496x8 at ideal ratio 0.375
Figure 5-6 - Crack-block st496x8 at ratio 0.85
5.2.2 Test cases

Three test cases have been developed to demonstrate the use of the large crack-block methodology. An additional test case was later added as part of the residual stress development work.

The test cases are described very briefly in the following sections. The benefit of the method is that in each case significantly more growth has been achieved than would have been possible using any older method previously available in Zencrack.

5.2.2.1 Semi-elliptic surface crack

In this quarter symmetry model of a semi-elliptic surface crack a 1x2x1 set of elements (i.e. two elements) of the original mesh have been replaced by one large crack-block. The loading is cyclic end tension. The initial defect is shown in Figure 5-7. Two meshes obtained during the growth phase are shown in Figure 5-8.

5.2.2.2 Quarter circular corner crack

In this full model of a circular corner crack in a bar two crack-blocks have been introduced. Each crack-block replaces a set of 2x2x2 elements (i.e. 8 elements) of the original mesh. The loading is cyclic end torsion and tension producing non-planar growth. The initial defect is shown in Figure 5-9. Two meshes obtained during the growth phase are shown in Figure 5-10 and Figure 5-11.

5.2.2.3 Coupon test specimen

In this full model of a circular corner crack a test specimen two crack-blocks have been introduced. Each crack-block replaces a set of 2x3x3 elements (i.e. 18 elements) of the original mesh. The loading is cyclic tension. The model is shown in Figure 5-12. Two meshes obtained during the corner crack and subsequent through crack growth phases are shown in Figure 5-13 and Figure 5-14. (Note: there is no automation between the corner and through phases - see further work described in section 5.9).
Figure 5-7 - Semi-elliptic surface crack - initial defect
Figure 5-8 - Semi-elliptic surface crack - during growth
Figure 5-9 - Corner crack - initial defect
Figure 5-10 - Corner crack - during growth
Figure 5-11 - Corner crack - during growth
Figure 5-12 - Coupon test specimen
Figure 5-13 - Corner crack phase part way through growth
Figure 5-14 - Through crack phase part way through growth
5.2.3 Limitations

The following limitations apply to large crack-blocks:

- The process of setting up an uncracked mesh to accept a large crack-block is time consuming in setting up tying constraints with surrounding elements and replacing a group of elements by a single target “large” crack-block element.
- The crack-blocks can only be mapped into one element. This may impose limitations to the situations where large crack-blocks can be used.
- The crack-blocks must use a form of tying to join with the surrounding mesh which requires strict QA procedures to verify the FEM.

5.3 Crack-block boundary shifting

The elements inside the crack-block must not become too distorted otherwise the J-Integral values become unreliable. Typically this results in an initial and final crack front ratio of about 0.25 to 0.75 of the length of the initial element length for a through crack-block with an ideal aspect ratio of 0.5.\(^2\) A crack-block with a crack front aspect ratio of 0.75 can be seen in Figure 5-15.

By changing the size of the crack-block one can improve the crack front ratio. This is known as crack-block boundary shifting and can be seen in Figure 5-16. Crack-block boundary shifting has the effect of:

- Obtaining better J-Integral values since the elements inside the crack-blocks will be less distorted.
- Allowing crack growth to be undertaken for a larger range of crack sizes.

Boundary shifting is a general procedure that can be used for all FE codes and is suitable for elliptical and straight crack-blocks. If the crack-block size is increased too much the surrounding elements will turn inside out. Subsequent results have shown that accurate results can be obtained by changing the element boundaries from about 50-150% of the initial element size.

\(^2\) Each crack-block has an ideal crack front to element edge ratio. Large discrepancies from the ideal and actual crack front ratio can result in J-Integral errors.
Altering the crack-block size has the effect of distorting the surrounding mesh. A relax algorithm has been developed that improves the quality of the mesh surrounding the crack (see section 5.5). A crack-block with a crack front aspect ratio of 0.75 using boundary shifting and relaxation can be seen in Figure 5-17.
Figure 5-15 - Through crack with an aspect ratio of 0.75

Figure 5-16 - Through crack with an initial aspect ratio of 0.75 with boundary shifting
Figure 5-17 - Through crack with an initial aspect ratio of 0.75 with boundary shifting and relaxation
5.4 Crack-block boundary flipping

Crack-block boundary flipping is an extension of crack-block boundary shifting. All the crack-blocks along the crack front shift into the next row of elements when the boundary-shifted crack-block becomes too large.

Figure 5-18 to Figure 5-21 illustrate the growth of a through crack using flipping and relaxation. From Figure 5-18 to Figure 5-19 the crack has grown using boundary shifting. However the crack-block is becoming very large and is distorting the rest of the mesh. Between Figure 5-19 and Figure 5-20 the crack-block flips into the next element in front of the crack-block. This has the result of allowing the boundary shifting routines to reduce the crack-block size to that of the neighboring element it is moving into in the original uncracked mesh. Further crack growth can now be undertaken.

5.4.1 Limitations of crack boundary flipping

Only through crack-blocks can be transferred to a new element location. All the crack-blocks on the crack front have to flip at the same time. Therefore long crack fronts composed of through cracks and terminated with semi-elliptical crack fronts cannot flip. Also “large crack-blocks” either as semi-elliptic or through cracks cannot flip.
Figure 5-18 - Initial crack size

Figure 5-19 - Through crack before flipping
Figure 5-20 - Through crack after flipping

Figure 5-21 - Further crack growth after flipping
5.5 Relax algorithm

5.5.1 General formulation
A general purpose relax algorithm has been developed to improve the quality of the elements surrounding the crack-blocks. The relax algorithm uses Laplacean smoothing. Laplacean smoothing does not guarantee a better mesh but in most cases it does result in a good mesh.

5.5.2 Handling of free surfaces and edges
Zencrack has an inbuilt topology detection algorithms (see section 5.6). The relaxed nodes are automatically mapped onto the surfaces and edges detected by the topology algorithms. This process guarantees that the corner nodes are mapped onto the surface profile given by the shape functions of the original uncracked mesh. It should result in a mesh that has the same surface profile as the original mesh.

5.5.3 Results of the mesh relaxation
Mesh relaxation usually enables:

- Better quality mesh for awkward crack sizes.
- Better quality mesh during boundary shifting.
- Larger quarter circular cracks to be modeled.

The relaxed mesh in Figure 5-16 can be compared to the un-relaxed mesh in Figure 5-17. Mesh relaxation can also be seen in the non-planar crack growth example shown in Figure 5-23 to Figure 5-28.

In some situations mesh relaxation can result in inside out elements due to the nature of Laplacean smoothing. In this case do one of the following:

- Chose a smaller element set to be relaxed.
- Create a node set of nodes not to be relaxed.
- Turn of the relaxation algorithm.
5.6 Large non-planar 3D crack growth

Note “Large crack growth” refers to the ability to move crack-blocks through the mesh. It is only available for crack fronts composed entirely of through crack-blocks.

5.6.1 Previous 3D capability

The results of analyses of a tension torsion bar model is shown in 5.2.2. This demonstrates the previous Zen crack 6.0 capability (Ref. 1) which was limited to growth within one crack-block.

5.6.2 Implementing 3D non-planar crack growth

The following items were implemented to enhance non-planar crack growth:

- The crack face nodes are placed on the previously calculated crack history.
- In order to cater for the non-planar growth the crack-blocks can now rotate about the crack front. The nodes opposite the crack plane are moved to try and make the crack-block rectangular. This has the effect of improving the quality of the mesh in the crack-block.
- As part of this implementation the movement of the crack through the mesh now works on the current mesh rather than the original mesh. That is, each crack advancement is an incremental change on the previous crack position without reverting to the original mesh. This is necessary to accommodate general non-planar growth. The original mesh is used to define model boundaries (see section 5.7).

5.6.3 Non-planar 3D crack growth test specimen

The specimen in Figure 5-22 was used to test the amount of non-planar growth. The displacement of nodes along the top and bottom of the model are subjected to constraint equations to prevent rotations and provide moment reactions. The model is loaded axially by a pressure load of magnitude (F_{axial}) and a shear load of which is applied to every node on the bottom and top surface (each node has a shear load of F_{shear} applied to it.) By controlling the ratios of F_{shear}/F_{axial} one can control the angle of the non-planar growth.

The initial crack length was taken to be 2.5mm (model width is 30mm and the length is 90mm).
Figure 5-22 - Boundary conditions

The results of the model are shown in Figure 5-23 to Figure 5-28 ($F_{\text{axial}}=110$ MPa and $F_{\text{shear}}=400$ N per node). One can observe that the nodes along the crack wake have been relaxed.
Figure 5-25 - Non-planar model step 20

Figure 5-26 - Non-planar model step 30
Figure 5-27 - Non-planar model step 40

Figure 5-28 - Non-planar model step 47
5.7 Mapping nodes onto surfaces and edges

Due to crack-block boundary shifting or the relax algorithm Zencrack may move nodes that need to be mapped onto the models edges and surfaces. Zencrack uses the shape functions of the elements in the uncracked mesh to map the nodes back onto the surfaces and edges.

A simple algorithm is currently used to detect free surfaces, edges and corners:

1. A corner node is currently defined as a node that is on three neighboring free surfaces of the same element. A node assigned on a corner cannot be moved under any conditions.
2. An edge node is currently defined as a node that is on two neighboring free surfaces of the same element. An edge node must remain along the edge of the detected elements.
3. A surface node is a node on a free surface that is not defined as an edge or corner.

The algorithm is able to successfully detect all the free surfaces, edges and corners on a simple rectangular mesh of hexahedral elements. However the algorithm is not suitable for concave surfaces. Since the algorithm is topology based, it cannot detect some geometric discontinuities, edges or corners (see example 1 - vaulted roof). In order to correct this limitation it is possible to create corner and edge node sets:

- Corner nodes can be assigned manually by selecting the nodes.
- Edge sets can be defined by selecting all the nodes along an edge in sequence.
The two edges, shown by the circled corner nodes, cannot be detected automatically. This is because a topology based edge detection system is used.

The two edges can be added manually.

*Figure 5-29 - Application of required user edge node sets*
5.8 Incorporation of transition elements in crack-blocks

The large crack-blocks do not mate with the surrounding elements nodes. Either surface tying or tied contact must be used to join the crack-block with the surrounding mesh.

Both surface tying and tied contact use a strict master slave algorithm. Care must be taken in determining which surfaces are the master and slave surfaces. If both surfaces in the constraint pair are deformable surfaces, the master surface should be chosen as the surface with the coarser mesh. However this limitation can often be severe:

- The slave surface must be denser than the master surface in all directions.
- The element density in a large crack-block changes depending on the size of the crack (see Figure 5-30 for an example with radical changes in crack-block mesh density due to extensive crack growth).

![Crack block mesh is finer than the surrounding mesh](image1)

![Crack block mesh is coarser than the surrounding mesh](image2)

*Figure 5-30 - Comparison of different element densities due to the crack position*

The generation of transition elements has been completed in order to get around the master slave problems (see Figure 5-32, Figure 5-34, Figure 5-36 and Figure 5-38). The user specifies the number of elements to generate along each edge of the crack-block. The transition elements should
be denser than the crack-blocks and the surrounding mesh. The transition elements will then be the slave surfaces and the crack-block surfaces and the surrounding mesh surfaces will be the master surfaces.

The following items have been programmed into the transition element option:

- Automatic detection of the surfaces to apply the transition elements. The transition elements are generated on any face on the crack-block which has a surface definition in the uncracked mesh.\(^3\)
- Automatic generation of the transition elements and resizing of the crack-blocks.
- Automatic implementation of the tied contact between the transition elements and the crack-blocks.
- The boundary conditions on the crack-blocks are applied to the transition zone.
- Pressure loads on the crack-blocks are applied to the transition zone.
- The option can be used with both 8 or 20 noded meshes (for Abaqus only).
- This option is programmed for multiple crack fronts. Each crack front can have multiple crack-blocks of various types i.e. semi-elliptic or through crack-blocks or a combination.
- Testing has included models that have curved outer surfaces to the crack-blocks.

Stress plots of comparisons with and without transition elements can be seen in Figure 5-31 to Figure 5-38. The even figures have transition elements while the odd figures do not have transition elements. The numbers of transition elements on each crack-block surface are shown on the figure captions in brackets. These analyses include thermal and centrifugal loading. Figure 5-31 and Figure 5-32 show the improvement transition elements can have on a coarse crack-block with varying mesh densities along each side of an edge. Figure 5-33 to Figure 5-38 show the improvement in stresses for varying crack sizes when a more detailed crack-block is used.

The values of the energy release rate \((G_{\text{max}})\) are given in Figure 5-39 to Figure 5-42. The models used are for a crack aspect ratio of 0.333 using the sq496x8 and sq112x4c crack-block with and

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\(^3\) The transition elements will never be generated on faces 1,3 and 4 of a through crack-block or faces 1 and 4 of a quarter circular crack-block. These face will either be on a free surfaces, a plane of symmetry or merged with another crack-block.
without transition elements (see Figure 5-31, Figure 5-32, Figure 5-35 and Figure 5-36). In Figure 5-39 the energy release rate is calculated from the J-Integral in Abaqus. In Figure 5-40 the energy release rate is calculated via stress intensity factors from the displacements. The plain strain values are used everywhere except at the surface where the plain stress values are used. This was performed manually and is not an automatic option in Zencrack. Figure 5-41 shows the energy release rates calculated from the J-Integral and displacements for the runs without transition elements. Figure 5-42 shows the energy release rates calculated from the J-Integral and displacements for the runs with transition elements.

The difference in the energy release rates with and without transition elements is small. There is, however, less variation in the results for the two different crack-blocks when transition elements are included than there is when they are omitted i.e. the transition elements produce a beneficial effect on the energy release rate distributions.
Figure 5-31 - Sq112x4c crack-block with an aspect ratio of 0.333

Figure 5-32 - Sq112x4c crack-block with an aspect ratio of 0.333 using transition elements (24x24)
Figure 5-33 - Sq496x8 crack-block with an aspect ratio of 0.1

Figure 5-34 - Sq496x8 crack-block with an aspect ratio of 0.1 using transition elements (48x48)
Figure 5-35 - Sq496x8 crack-block with an aspect ratio of 0.333

Figure 5-36 - Sq496x8 crack-block with an aspect ratio of 0.333 using transition elements (48x48)
Figure 5-37 - Sq496x8 crack-block with an aspect ratio of 0.8

Figure 5-38 - Sq496x8 crack-block an aspect ratio of 0.8 using transition elements (48x48)
Figure 5-39 - Gmax from J-Integrals

Figure 5-40 - Gmax from displacements
Gmax for the runs without transition elements

Figure 5-41 - Gmax without the transition elements

Gmax for the runs with transition elements

Figure 5-42 - Gmax with the transition elements
5.9 User-defined initial crack shape and its application

Before this task was undertaken the only method of controlling the starting crack shape within a crack-block element was by two edge lengths. For a rectangular through crack-block a straight line is drawn between the two points and for a rectangular quarter circular crack-block an ellipse is drawn between the two points. For cases where the element face containing the initial crack plane has curved edges the user has no direct control of the intermediate crack front nodes for each crack-block as they are positioned using the edge definitions and the element shape functions. This is a severe limitation in defining complex crack fronts or if the user wants to undertake a manual restart. A manual restart is essential if a user wants to model the breakthrough of a quarter circular crack to a through crack (see the example in this section).

For each crack front the user must list a series of points. A cubic spline is fitted to the points and all the nodes on the crack front are mapped onto the spline. The user can use as many points as he wants to define the spline (a minimum of two points must be used). For fully enclosed cracks, the user has to ensure the first and last points on the crack front are the same.

There is no special non-planar functionality built into the user defined crack front shape. It is up to the user to modify the uncracked mesh to ensure the crack will be inserted into the desired plane.

The example used to demonstrate the user defined crack shape is the transition of a corner crack to a through crack in a SEN specimen\(^4\). This example requires three separate Zencrack runs to be undertaken. The steps are shown below:

1. Modeling a quarter circular crack using a large quarter circular crack-block. The uncracked mesh is shown in Figure 5-43. The displaced mesh can be seen in Figure 5-45.
2. Break through from a quarter circular to a through crack. The last crack front from step 1 is modified to cater for the break through. The crack front is modeled using a large through crack with the user defined crack front. The uncracked mesh is shown in Figure 5-43. The displaced meshes can be seen in Figure 5-46 and Figure 5-47.

\(^4\) A Young’s modulus 110.316 GPa and a poisson’s ratio of 0.0 was used.
3. Change of crack-blocks from one large through crack-block to three through crack-blocks to enable crack growth through the entire specimen. The user defined crack shape is used to map the crack front with minimal discrepancy. The uncracked mesh is shown in Figure 5-44. The displaced mesh can be seen in Figure 5-48.

The crack front shapes for all three steps can be seen in Figure 5-50. The user defined crack shape is used twice:

- Between steps 1-2. The last crack profile from step 1 is modified by keeping the top half of the crack front and creating points for the bottom half of the crack front. The new crack front is used for the first run in step 2. The top half of the two crack front profiles lie on top of each other confirming the accuracy of the user defined crack shape.

- Between steps 2-3. The last crack profile in step 2 is used for the initial crack profile in step 3. It is difficult to distinguish between the two crack profiles confirming the accuracy of the user defined crack shape.

Figure 5-43 - Uncracked model used for steps 1 and 2
Figure 5-44 - Uncracked model used for step 3

Figure 5-45 - Final step 1 run that ran to completion
Figure 5-46 - First step 2 run (first run after break through)

Figure 5-47 - Final step 2 run (same crack front shape as in Figure 5-48)
Figure 5-48 - First step 3 run (same crack front shape as in Figure 5-47)

Figure 5-49 - Step 3 run after two crack-block transfers
5.10 Different materials within crack-block

It is possible to specify material properties as a function of spatial position using the Abaqus user subroutine USDFLD.

5.118-noded brick elements in crack-blocks

This activity is complete for the Zencrack interfaces to Abaqus, MSC.Marc and Ansys.

Zencrack can handle meshes of either 20 noded or 8 noded brick elements automatically. However it cannot handle mixed meshes containing both 8 and 20 noded elements.
Zencrack can also handle temperature interpolation of user specified nodal temperature in an 8-nodes or 20-noded mesh (Abaqus interface only).

5.12 Crack-block brick elements in tetrahedral finite element mesh

It is possible to incorporate crack-block elements into a tetrahedral mesh. Some planning is required to provide a surface interface between the crack-blocks and other tetrahedral elements to the outlying tetrahedral structure. Surface tying can be used with “large” crack-blocks with transition elements.
6 Pre & Post Processor Development (GUI)

6.1 Objective

The objective of this task is to create a graphical user interface (GUI) for Microsoft Windows XP or Windows 2000 that will allow the user to input the data and review the results in a windows environment. This should enable accurate data entry and may cut down the time taken to review the results, thus increasing the overall quality of the product. The GUI is written using the Python scripting language (Ref. 9).

6.2 Outline of the Zencrack program

Zencrack requires two input files to run:

- An uncracked FE input file.
- A Zencrack input file that contains information required to create a cracked mesh and may also contain information to undertake crack growth.

The FE input file is either created by hand or using a FE preprocessor (e.g. Femap, Patran, Ideas).

The Zencrack input file is an ASCII file that contains a set of keywords. The keywords have parameters that are separated by either commas or spaces. The keywords file used to be created manually using a text editor. However the user now has the option of creating the file using the GUI.

The GUI should simplify the learning curve into Zencrack by making it easier to create the Zencrack input file. The user can simply select the required options from windows based dialog options and the GUI will automatically create the Zencrack input file.

6.3 General functionality of the GUI

The overall functionality of the GUI can be broadly divided into three steps and each step is briefly explained here.

1. Preparation and editing of the Zencrack input file

   At the time of preparing the input file for Zencrack, the uncracked FE mesh would have been already prepared and the input files (.inp file for example) are available for creating Zencrack
input file. Zencrack is currently interfaced with three FE Programs namely Abaqus, MSC.Marc and Ansys. The GUI helps the user choose various crack initiation and crack growth related options to prepare the final Zencrack input file. Since the basic uncracked mesh is already available, this can be viewed through the GUI. It also provides facilities such as creation of node sets and element sets. These sets are very useful in identifying and introducing the crack-block into the finite element mesh. These sets can also be created dynamically so that the user can easily identify the elements that are going to be replaced with crack-blocks. Zencrack has a built-in crack-block library to replace the uncracked elements with set of crack-block elements. The user can access this library from the GUI. The Zencrack input file can be saved, viewed and edited before submitting the file to Zencrack. These input options, the friendly screens and the dialog are all explained later in this section through screen capture pictures. Detailed documentation of these options is available under the help menu of the GUI.

2. Running of Zencrack Program

Having prepared the final Zentech input file from the above step, Zencrack can be executed from within the GUI itself. The user can select all the required analysis options from the screen and submits the input for analysis. Zencrack creates new mesh by replacing the original selected element(s) in the uncracked mesh with crack-blocks and creates new input file for Abaqus, MSC.Marc or Ansys. This new input file containing the crack-block elements is being sent to the FE Program for execution. Once the execution is complete, Zencrack extracts the relevant information from the outputs of these programs for performing crack growth calculations. This process will continue depending on the crack growth control information provided by the user.
3. Post Processing of Results

Once the analysis is complete, in other words completion of second step, the user can view the results either on the screen by selecting the appropriate output files created by Zencrack or can plot the results by choosing one of the plotting utilities available within Zencrack program. These Utilities can be launched from the interactive Pre/Post Processor.

6.4 Exporting user input data to ASCII format data file

In order to run an uncracked FE mesh, a Zencrack input file (with extension .zcr) is required.

The Zentech Input file is a collection of keywords, parameters through which crack growth analysis can be undertaken.

The GUI provides a graphical interface in the form of dialog to write this ASCII text file.

Once the program is installed it will create a shortcut on the user’s desktop, upon double clicking, the program will be launched.

1. Main Window of the program

To begin working with the GUI the user will need to open either an existing Zencrack input file or create a new one. This can be done using the toolbar icons or using the “File” menu in the main window of the program.

The screenshot below shows the Input menu option which contains the menu items pertaining to all the dialogs used for writing the ASCII file.
For example, consider the keyword “FILES” with a parameter “UNCRAKED” whose value is the uncracked mesh input file (.inp or .dat file) for which the analysis is to be carried out.

Now the user can click on Input > Job Name & Type which opens a dialog as shown below.
Figure 6-2 - The user can specify input uncracked mesh file using Input > Job Options

Likewise the user can set all Zencrack Parameters and values pertaining to all the keywords in the ASCII file. The GUI documentation in the help menu gives the details of all the available keywords and their corresponding dialogs in the GUI.

2. Toolbar of the program
The toolbar of the program looks similar to what is shown below. Icons are described in the following table.
<table>
<thead>
<tr>
<th>No.</th>
<th>Image</th>
<th>Description</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image" alt="File - New" /></td>
<td>File - New</td>
<td>Opens new ZenCrack .zcr file</td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="File - Open" /></td>
<td>File - Open</td>
<td>Opens existing ZenCrack .zcr file</td>
</tr>
<tr>
<td>3</td>
<td><img src="image" alt="File - Save" /></td>
<td>File - Save</td>
<td>Saves current ZenCrack .zcr file</td>
</tr>
<tr>
<td>4</td>
<td><img src="image" alt="File - Save As" /></td>
<td>File - Save As</td>
<td>Saves current ZenCrack .zcr file as new .zcr file</td>
</tr>
<tr>
<td>5</td>
<td><img src="image" alt="File - View" /></td>
<td>File - View</td>
<td>Shows the ZenCrack .zcr file in text editor</td>
</tr>
<tr>
<td>6</td>
<td><img src="image" alt="File - Exit" /></td>
<td>File - Exit</td>
<td>Exits the program</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td><img src="image" alt="Show Geometry" /></td>
<td>Show Geometry</td>
<td>Plots Abaqus geometry view on screen if .INP file is specified on FILES keyword of .zcr file.</td>
</tr>
<tr>
<td>8</td>
<td><img src="image" alt="Dynamic Element Sets" /></td>
<td>Dynamic Element Sets.</td>
<td>Pops up the interface for creation, viewing and Modification of Dynamic element sets.</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td>9</td>
<td><img src="image" alt="3D View" /></td>
<td>3D View</td>
<td>Plots 3D Geometric view</td>
</tr>
<tr>
<td>10</td>
<td><img src="image" alt="XY View" /></td>
<td>XY View</td>
<td>Plots XY Geometric View</td>
</tr>
<tr>
<td>11</td>
<td><img src="image" alt="YZ View" /></td>
<td>YZ View</td>
<td>Plots YZ Geometric View</td>
</tr>
<tr>
<td>12</td>
<td><img src="image" alt="XZ View" /></td>
<td>XZ View</td>
<td>Plots XZ Geometric View</td>
</tr>
<tr>
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<tr>
<td>13</td>
<td><img src="image" alt="Mesh Model" /></td>
<td>Mesh Model</td>
<td>Mesh Mode where the model can be viewed as wire frame.</td>
</tr>
<tr>
<td>14</td>
<td><img src="image" alt="Show Crack Elements" /></td>
<td>Show Crack Elements</td>
<td>Mesh Mode with Hidden Surface Removal Algorithm.</td>
</tr>
<tr>
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<td><img src="image" alt="Show Crack Elements" /></td>
<td>Show Crack Elements</td>
<td>Solid View with Hidden Surface Removal Algorithm.</td>
</tr>
<tr>
<td>16</td>
<td><img src="image" alt="Show Crack Elements" /></td>
<td>Show Crack Elements</td>
<td>Toggles Showing of Crack-block elements in the rendered geometry.</td>
</tr>
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<td></td>
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</tr>
<tr>
<td>17</td>
<td><img src="image" alt="Show Crack Elements" /></td>
<td>Show Crack Elements</td>
<td>Toggles Showing of Crack-block elements in the rendered geometry.</td>
</tr>
<tr>
<td>18</td>
<td><img src="image" alt="Show Selection" /></td>
<td>Show Selection</td>
<td>Toggles Showing of Dynamic Element Sets in the rendered geometry.</td>
</tr>
<tr>
<td>19</td>
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<td>Graphical Blending</td>
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<td>Interface to View Element sets defined in Abaqus Input .inp File.</td>
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<td>Rotate Y Positive</td>
<td>Rotates view in Positive Y direction</td>
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</table>
6.5 OpenGL FEM display facilities for generating cracked mesh

6.5.1 Display of the Geometry

The Abaqus input deck can be viewed in the GUI. The user can look at the model on the screen, pick the crack-block interactively, view where the crack-block is to be placed in the uncracked mesh and all other OpenGL features incorporated in the GUI. Similar effort has been made for Ansys and Marc geometry files but the implementation has not been done in this present version. However the Zencrack input file (.zcr) can be written using the Zencrack GUI for all the three supported FE programs namely Abaqus, Ansys and Marc. Execution of Zencrack can be undertaken within the GUI.

The user can select Abaqus input file by one of the following two ways.

- Open an existing Zencrack input file, which has valid Abaqus input (.inp) file on FILES keyword.
- Start with a new Zencrack input file and specify an Abaqus input file in the dialog “Job Options”
Once the program has valid Abaqus input file name, the user can click on “Draw Geometry” toolbar. The program will read the Abaqus input file. The file has all the information about nodes, nodal coordinates, connectivity details of 8 and 20 noded solid elements. Once this information is retrieved, the program will display 3D view on the screen with OpenGL calls.

![Figure 6-3 - Typical mesh view in GUI](image)

### 6.5.2 Viewing the Model

Once the Abaqus geometry is shown on the screen, it can be viewed in several ways with the toolbars provided on the main screen. The description of the toolbar was already given earlier. The model can be viewed in various fashions:

1. mesh mode where the model is viewed as a wire frame model.
2. mesh mode with the hidden surface removal algorithm implemented.
3. solid view with the hidden surface removal algorithm implemented.

Navigation of the model is done:
1. Panning: Default pan by Left Mouse Move

2. Zooming: This can be done in three ways:
   - Default Zoom In and Out by Right Mouse Move (New Addition)
   - Standard functions for Zoom In and Out.
   - Zoom Window

3. Rotations

The X, Y and Z axes are displayed at the bottom left Corner of the Screen. The axes have the following color convention:

- Red X
- Green Y
- Blue Z

6.5.3 Element sets

The element sets in the Abaqus model are imported when the Abaqus input file is read. The user can also create dynamic element sets by select a particular portion of the model. The user can save the element set. All element sets can be viewed as a separately rendered entity or as a part of the entire model. Viewing the selected set as a part of whole model can be useful in locating the position of the selection set in the entire model.

The dynamic element sets are created by dragging a window or by picking individual elements.

Saving Dynamic Element sets: This facility saves the element selection in *.zcrdyn file that is also saved along with *.zcr file. The user can refer to the created dynamic element sets at anytime by opening the *.zcr file assuming that the *.zcrdyn file is present.

The data in the *.zcrdyn files is stored in a specialized format so it is not readable in a text editor and can only be read inside the program.
6.6 Crack front definition and selection

Once the GUI had displayed the uncracked mesh, it can also be used to define crack fronts. Initially an empty crack front is created. The user can individually add crack blocks to the crack front. Currently each crack front is limited to 10 crack-blocks.

Once the entire Crack Front is defined, it is possible to preview the entire crack front. In Figure 6-4 two penny shaped crack fronts can be seen. Each crack front consists of four elements.

![Figure 6-4 - Penny-shaped crack profile](image)
6.7 Interface to Zencrack analysis module

The Zencrack GUI is interfaced with Zencrack. The user can run/execute Zencrack analysis using the Zencrack Analysis Module.

The Zencrack Analysis Module:

- This interface provides all the command line options that are available for the user to perform the analysis.
- The command formed is shown in the edit Box Below and can be manually edited.
- If a particular option uses an input file like a FORTRAN file or an executable file, the user will be prompted accordingly.

A screen shot of the Submit Analysis Job dialog is shown Figure 6-5.
6.8 Crack growth plotting routines

The user can run the Zencrack utility programs: 3DMesh, Rainflow, Growth and Tanh.

There are several utilities like 3DMesh, Rainflow, Growth etc which the user can select and execute using the following interface. The screen shot of this interface is shown here. By selecting any of the Utility listed there, the user can generate the required information or files that will be used to continue with the Utilities. These Utilities are part of the Zencrack Program. The GUI will help the user in generating the files required for plotting. Otherwise, the user would have run these Utilities at the command prompt.

Run > Submit Analysis Job

![Submit Analysis Job Interface](image)

*Figure 6-6 - Interface to invoke Zencrack utilities*
6.9 Output data report screens

The GUI has an interface though which the user can view and edit the input and output files.

This interface senses the files present in the directory. If a particular file is available then its name will appear in the dialog. Alternatively the user can also browse the files. Once the file is selected, the user can choose either to edit the file or to view the file. The screen shot of the dialog is shown in Figure 6-7.

![Figure 6-7 - Interface to view/edit output and input files through GUI](image_url)
6.10 Help system

The complete documentation of the Dialogs and Keywords and how these keywords are mapped to the Dialog is available though the Help > GUI Documentation Menu.

The documentation is compiled in a help file that talks about the mapping of Zencrack keywords to the dialogs.

Adobe Acrobat Reader is required to view the documentation.

Figure 6-8 - Screenshot of GUI documentation through Help > Dialog documentation
7 Ansys Interface

The Ansys interface to Zencrack has been developed using Ansys/Ed Version 6.1 software and relies on batch file input for analysis. Ansys does not have a 3D J-Integral capability so stress intensity factors (and associated energy release rates for an assumed state of stress) are derived from displacements of 20-noded brick elements quarter-point nodes adjacent to the crack front. This solution is only applicable to LEFM. For 8-noded brick elements the solution uses the adjacent corner node displacements to the crack front.

“Large” crack-blocks have been implemented for the Ansys interface with automatic tying constraints between the target crack-blocks elements in the uncracked mesh to the surrounding elements. Transition elements have not yet been incorporated for the Ansys interface. Large crack growth is supported with multiple load steps in the FEA.

Currently there is no capability in the Ansys interface to interpolate nodal temperatures from the uncracked mesh to the mesh with introduced crack fronts.
8 Validation - Integration Scheme

Many analyses have been undertaken to test new features. Some of these have been reported in the interim reports. Some key results are included in this section.

In order to eliminate uncertainty in the solution arising from the finite element solution, an “equation method” has been implemented in Zencrack to allow comparison with Afgrow results for a single edge through crack geometry. The equation for K used for this geometry is given in section 3.2.3.1.1.19 of the Afgrow manual and is reproduced in Figure 8-1. This method was enhanced by the additional of a residual stress distribution to test that capability. The distribution that has been used is defined by K vs a and shown in Figure 8-2. The chosen specimen width is 30mm with initial crack size 0.5mm. Additional information on the SEN geometry is contained in section 2.1.4 of .

All Afgrow runs in this section of the report are based on cycle-by-cycle evaluation beta and spectrum evaluation.

![Figure 8-1 - K equation for external load in SEN equation method](image-url)

3.2.3.1.1.19 Single Edge Through Crack (Application Defined)

Tension Loading:

The standard solution for the edge cracked case accounts for in-plane bending caused by the specimen geometry as the crack grows. The specimen is assumed to be remotely pin loaded so there is no constraint to the in-plane bending as the crack grows.

\[
\text{Beta} = \frac{0.752 + 2.02(C/W) + 0.37 \left(1 - \sin\left(\frac{\pi C}{2W}\right)\right)^3}{\cos\left(\frac{\pi C}{2W}\right)} \sqrt{\frac{2W}{\pi C} \tan\left(\frac{\pi C}{2W}\right)}
\]

Reference [37]

This solution is valid for the following dimensions:

\(0 < C/W < 1.0\)

This solution is within 0.5% for all crack lengths
8.1 Residual stress

Results from Zencrack qa example 075 are presented in Figure 8-3. This is a constant amplitude analysis with $\sigma_{\text{max}}=150\text{MPa}$ and $R=0$. The crack growth data is in tabular form. The figure shows results for the case with external load only and external load plus residual stress. Zencrack runs use the error control scheme described in section 4.7. Results are presented for an error tolerance of 2% (the default) and 0.05%. All results use the forward predictor integration scheme in which the term $dG/da$ is used during integration. The following points are noted:

- The Afgrow results for the case with external load is accurately matched by both Zencrack runs.
- When residual stress is included, the Zencrack analyses require additional f.e. evaluations to match the Afgrow results. The 0.05% runs give excellent agreement but the runs with 2% error control overestimate the life by about 5%. This need for smaller steps and the greater sensitivity in the solution is typical of analyses in which residual stress is included.
- The method 1 and method 2 analyses give slightly different results. This is due to the effect of the different methods of combination of the loading on the $dG/da$ terms in the forward predictor integration, and the error value being calculated for effectively different terms. If
the constant G integration method is used (i.e. dG/da=0) with fixed step control, then methods 1 and 2 give identical results. However, the use of the constant G scheme is inherently less accurate than the forward predictor scheme.

8.2 Retardation

The stand-alone SEN program that has been used for testing changes in the Zencrack integration scheme against Afgrow was modified to allow testing and implementation of the Willenborg retardation model for introduction into Zencrack. This follows on from the work reported for the Willenborg model in the stand-alone program for Phase I of the SBIR.

The stand alone program has been verified in cycle-by-cycle and block cycle modes against Afgrow using Harter T crack growth data definition. It then acts as a baseline for testing and comparisons of the Zencrack implementation using tabular crack growth data definition.
The tests are based on the SEN geometry described in Ref. 6. The initial crack size is 0.5mm. For the initial implementation and testing in Zencrack the issues of plane stress and plane strain conversions have been avoided by assuming plane stress conditions throughout the analysis. All analyses are set to complete when the plane stress fracture toughness is reached.

A number of load spectra have been used for testing. These are block spectra with various overload blocks. The following are included:

- spectrum with a single non-repeated overload (qa095)
- spectrum with a repeated overload that repeats after retardation from the previous overload has ended (qa096)
- spectrum with a repeated overload that repeats before retardation from the previous overload has ended (qa097)
- spectrum with compressive minimum load levels to test acceleration (qa094)

In order to fully understand the requirements for the implementation and to have confidence in the results it was necessary to first create a cycle-by-cycle version of the SEN program to compare against Afgrow. In this program the crack length and stress intensity factor are updated for every cycle. If a block of cycles is below threshold, da=0 for each cycle.

The analyses were then repeated in the standard SEN program using a tolerance of dN=1 cycle. This provides cycle-by-cycle integration during growth phases but skips straight to the end of a block when a block is below threshold. This simulates the method used in Zencrack for blocks that are below threshold and verifies that block updates are handled correctly. These analyses were all carried out using the Harter T method of defining crack growth data.

Having verified the basic algorithms using cycle-by-cycle integration the next stage was to switch to an integration scheme based on discrete evaluations of stress intensity factor (i.e. using the results of each finite element analysis in Zencrack). It is noted that the “inner” integration loop in Zencrack switches to cycle-by-cycle integration when retardation is taking place, but the finite element evaluations are unaffected by this change in the inner loop. The SEN program is coded with the adaptive stepping algorithm that is available in Zencrack. Hence one set of analyses were conducted using the SEN adaptive method and an equivalent set of analyses using the same stepping controls.
in Zencrack. These two sets of analyses should give the same results if the implementation in both programs is consistent. A tolerance on da of 1mm was used for these analyses.

A further set of Zencrack analyses were set up using an error control stepping scheme with a small tolerance (0.05% error). The aim of these analyses was to demonstrate improved agreement with Afgrow when this preferred scheme is used.

As the Harter T method of defining crack growth data is not available in Zencrack these analyses were conducted using tabular materials data derived from the Harter T data used for the Afgrow and cycle-by-cycle analyses. Some small differences may be expected in the results due to this change. These could be investigated by re-running the cycle-by-cycle programs using tabular data but due to time constraints this has not been undertaken. Further, in view of the results presented, it is not considered necessary.

Final results for the Zencrack runs are shown in Table 8-1 to Table 8-3. It is clear from the agreement of the results that the retardation implementation in Zencrack is consistent with the SEN stand-alone program and Afgrow.

Analyses using the adaptive scheme typically have 15-25 finite element steps and analyses using the error control scheme typically have 60-80 finite element steps.
<table>
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<th></th>
<th>Afgrow</th>
<th>factor on cycles compared to &quot;no retardation&quot;</th>
<th>Zenckrack adaptive scheme</th>
<th>% difference cf Afgrow</th>
<th>Zenckrack error control scheme, 0.05%</th>
<th>% difference cf Afgrow</th>
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*Table 8-1 - Spectrum comparison with Afgrow - no retardation*
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*Table 8-2 - Spectrum comparison with Afgrow - Willenborg SOR=2.5*
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**Table 8-3 - Spectrum comparison with Afgrow - Willenborg SOR=2.7**

### 8.3 Time dependent and combined time / cyclic growth

A number of examples were generated and analyzed in a stand-alone test program and Zencrack for comparison with Afgrow. Final results for the Zencrack analyses are shown in Table 8-4 and Table 8-5 for adaptive growth scheme and error control scheme with 0.05% error. Results are in good agreement.

- **qa69_01**: Constant amplitude, 1s per cycle.
- **qa69_02**: Random cyclic spectrum, 2 load levels with 1 cycle each 1s. One pass is 2s / 2 cycles.
- **qa69_06**: Ramp up (1s), cyclic load (10 levels, 2s), hold period of 20s, cyclic load (4 levels, 8s), ramp down (1s). One pass is 32s / 17 cycles.
Note:

- All analyses use Paris-type cyclic and time dependent crack growth data.
- All analyses are ended when the crack length exceeds 20mm (in a total SEN specimen width of 30mm). The initial crack size is 1mm.
### Table 8-4 - Comparison of time dependent analyses with Afgrow (adaptive)

<table>
<thead>
<tr>
<th></th>
<th>time-only, fatigue-only or both</th>
<th>Cycles Afgrow</th>
<th>Cycles Zencrack</th>
<th>% difference cf Afgrow</th>
<th>Time (s) Afgrow</th>
<th>Time (s) Zencrack</th>
<th>% difference cf Afgrow</th>
<th>Number of f.e. steps for Zencrack</th>
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<td>qa69_01</td>
<td>fatigue</td>
<td>365199</td>
<td>366785</td>
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<td>-</td>
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<td>-</td>
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<td>-</td>
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<td>236296.12</td>
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<td>-</td>
<td>-</td>
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</tr>
<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>335404</td>
<td>336870.12</td>
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</tr>
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<td>qa69_02</td>
<td>both</td>
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<td>194020</td>
<td>0.419</td>
<td>193210</td>
<td>194019.12</td>
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<td>-</td>
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<td>476733.25</td>
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<td>43</td>
</tr>
</tbody>
</table>

- = not applicable

### Table 8-5 - Comparison of time dependent analyses with Afgrow (error control 0.05%)

<table>
<thead>
<tr>
<th></th>
<th>time-only, fatigue-only or both</th>
<th>Cycles Afgrow</th>
<th>Cycles Zencrack</th>
<th>% difference cf Afgrow</th>
<th>Time (s) Afgrow</th>
<th>Time (s) Zencrack</th>
<th>% difference cf Afgrow</th>
<th>Number of f.e. steps for Zencrack</th>
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</thead>
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<td>-</td>
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<td>time</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>335404</td>
<td>335520.12</td>
<td>0.035</td>
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<td>1798967</td>
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<td>-</td>
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<td>474775</td>
<td>-0.616</td>
<td>183</td>
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</tbody>
</table>

- = not applicable
9 Validation - Abaqus Interface

The scope of this section is the validation of the final version of Zencrack for an Abaqus interface.

The validation involves four main models, all under remote tension:

1. A square cross section measuring 80mm x 80mm and 80mm long with a small embedded elliptical crack at the center representing an elliptic crack in an infinite body.
2. A rectangular bar, 10mm x 5mm x 20mm long, with a semi-elliptical surface crack at center, (PST1).
3. A square bar, 5mm x 5mm x 20mm long, with a corner crack (CRT1).
4. A plate, 200mm x 70mm x 6mm thick, with a through thickness crack at one of the 200mm x 6mm edges as used by Kang et al (SENT1).

The first three models used a titanium alloy. The material properties are listed in Table 9-1. The crack growth rate curves are shown in Figure 9-1 for all the stress ratios. The material data for the fourth model, SENT1, is described in section 9.5.

During the crack growth analysis all runs stopped once the crack growth rate exceeded 0.0001mm/cycle (the highest value on the user defined crack growth tabular data). At no stage were fracture toughness limits exceeded.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>112 000 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.33</td>
</tr>
<tr>
<td>Yield stress</td>
<td>900.0 MPa</td>
</tr>
<tr>
<td>( K_{\infty} )</td>
<td>2085 MPa-sqrt(mm)</td>
</tr>
<tr>
<td>( K_{c} )</td>
<td>3613 MPa-sqrt(mm)</td>
</tr>
<tr>
<td>Threshold deltaK at R=0</td>
<td>89.80869 MPa</td>
</tr>
<tr>
<td>R high shift cut off</td>
<td>0.8</td>
</tr>
<tr>
<td>R low shift cut off</td>
<td>-1</td>
</tr>
</tbody>
</table>

*Table 9-1 - Material properties used in section 9.1 to 9.3*
Figure 9-1 - Crack growth curves used in section 9.1 to 9.3

Afgrow reported warnings when the tabular data was input as shown in Figure 9-2.

Figure 9-2 - Afgrow tabular data input warnings

Raju-Newman solutions in the following sections are based on Ref. 13 and Ref. 19.
The definition of the angle PHI used in plots of results for the first three elliptic crack front test cases is shown in Figure 9-3.

\[ \phi = \tan^{-1} \left( \frac{c}{a} \tan \theta \right) \]

\[ \theta = \tan^{-1} \left( \frac{a}{c} \tan \phi \right) \]

**Figure 9-3 - Definition of angles for elliptic crack fronts**
9.1 Embedded elliptical crack

Existing theoretical solutions exist for embedded elliptical or circular cracks (Ref. 17 pg 43 and Ref. 7 pg 185). The theoretical solutions were determined by the use of potential functions and are based on the assumption of a crack in an infinite medium.

Since the theoretical solutions are for an infinite plate, it is essential that the crack size must remain small compared to the dimensions of the bar. A biased mesh has been used to model a square cross section measuring 80mm x 80mm and 80mm long with an elliptical crack in the center. The model is subjected to an external pressure of 600MPa. The model utilizes 3 symmetry planes resulting in a 1/8 model measuring 40mm x 40mm x 40mm. The uncracked 1/8 model is shown in Figure 9-4. The center of the model is a regular square region of 5x5x5 elements each with an element edge length of 1.0mm. This region is setup to be a relaxed region and can be seen in Figure 9-4 and Figure 9-5. All analysed cracks have a and c dimensions up to 1.5mm.

Previous experience has shown that the best results are obtained by limiting element distortion in the crack-blocks. This is done by maintaining good edge aspect ratios\(^5\) within the crack-block. By turning on boundary shifting the crack-blocks are automatically resized to try and maintain good edge ratios. However the boundary shifting algorithm can only change the crack-block size by a factor of 0.5 to 1.5 for an initial regular mesh.

The aim of the runs were to compare the Zencrack results for:

- Different crack sizes which would result in the boundary shift algorithm being unable to maintain the ideal edge ratios.
- Different ellipse sizes and elliptic ratios a/c that will result in element distortion within the crack-block.
- Modeling a large crack with either a large crack-block or one quarter circular crack-block and one through crack-block.

\(^5\) Ratio of the crack length to the crack-block edge length.
Figure 9-4 - Uncracked mesh for the embedded crack

Figure 9-5 - Cracked mesh with a single standard quarter circular crack-block (a=0.75mm, c=0.75mm)

The relaxed region is shown in orange.
Table 9-2 contains a listing of the FEA runs that were undertaken using one ordinary crack-block. The table lists the a/c ratio and the percentage deviation from the ideal a and b ratios. A crack size of 0.3mm x 1.5mm was also modeled using two methods: a large crack-block and a combination of a through and quarter circular crack-block.

<table>
<thead>
<tr>
<th>a₀ (mm)</th>
<th>c₀ (mm)</th>
<th>a₀/c₀</th>
<th>% away from ideal aspect ratio</th>
<th>a ratio</th>
<th>b ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.10</td>
<td>1.0</td>
<td>-13.33</td>
<td>-13.33</td>
<td>-13.33</td>
</tr>
<tr>
<td>0.30</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.75</td>
<td>0.75</td>
<td>1.0</td>
<td>16.67</td>
<td>16.67</td>
<td>16.67</td>
</tr>
<tr>
<td>0.30</td>
<td>0.60</td>
<td>0.5</td>
<td>0.00</td>
<td>6.67</td>
<td>0.00</td>
</tr>
<tr>
<td>0.10</td>
<td>0.50</td>
<td>0.2</td>
<td>-13.33</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 9-2 - Embedded elliptical crack runs using one normal crack-block

Three circular cracks have been modeled with one quarter circular crack-block (sq158x6). The crack sizes are considered to encompass the entire range of sensible crack sizes that can be undertaken without re-meshing or using a large crack-block. The stress intensities are shown in Figure 9-8 to Figure 9-10. All the runs use the sq158x6 crack-block. For a crack size of 0.1mm and 0.3mm the J-Integral results are well within 2% of the theoretical values. For the crack size of 0.75mm some of the points are just outside of the 2% error. The larger discrepancy between the theoretical values may be due to element distortion. The COD results are worse than the J-Integral results for crack sizes of 0.1 and 0.3 mm. However for a crack size of 0.75 mm the COD is better than the J-Integral results.

Results for two different elliptical cracks are shown in Figure 9-11 and Figure 9-12 with the sq158x6 crack-block. For an a/c=0.5 the J-Integral results are well within 2% of the theoretical solution. Even for an a/c=0.2 (which is considered to be very small) the results are just within the 2% error bars of the theoretical solution.

Other methods of modeling low a/c ratios are by using more than one crack-block or using large crack-blocks. Examples of such meshes are shown in Figure 9-6 and Figure 9-7. Figure 9-13 and Figure 9-14 show results for a crack with a=0.3mm and c=1.5mm. The large crack-block run uses the sq496x8 crack-block while the two crack-block run uses one quarter circular crack-block.
(sq103x4) and one through crack (st88x5). The large crack-block model in particular gives good results and most of the J-Integral results are well within the 2% error bars of the theoretical solution. The COD results are slightly worse than the J-Integrals and hover around the 2% error bars of the theoretical solution.

### 9.1.1 Review of results for embedded elliptical cracks

When embedded elliptical cracks are modeled within Zencrack the user can expect the majority of J-Integral results to be within 2% of the theoretical values providing none of the crack-blocks are too distorted and the surrounding mesh is of good quality. The J-Integrals appear to be more consistent and generally give better results than CTOD.
Figure 9-6 - Embedded elliptical crack modeled by two crack-blocks (a=0.3mm, c=1.5mm)

Figure 9-7 - Embedded elliptical crack modeled by a large crack-block (a=0.3mm, c=1.5mm)
Figure 9-8 - Variation of $K$ with $\phi$ for $a=0.1\text{mm}$ and $c=0.1\text{mm}$ (±2% error bars for theoretical values)

Figure 9-9 - Variation of $K$ with $\phi$ for $a=0.3\text{mm}$ and $c=0.3\text{mm}$ (±2% error bars for theoretical values)
Figure 9-10 - Variation of K with phi for a=0.75mm and c=0.75mm (±2% error bars for theoretical values)

Figure 9-11 - Variation of K with phi for a=0.3mm and c=0.6mm (±2% error bars for theoretical values)
Figure 9-12 - Variation of $K$ with $\phi$ for $a=0.1\text{mm}$ and $c=0.5\text{mm}$ ($\pm 2\%$ error bars for theoretical values)

Figure 9-13 - Variation of $K$ from J-Integrals with $\phi$ for $a=0.3\text{mm}$ and $c=1.5\text{mm}$ ($\pm 2\%$ error bars for theoretical values)
Figure 9-14 - Variation of K from COD with phi for a=0.3mm and c=1.5mm (±2% error bars for theoretical values)
9.2 Surface crack model (PST1)

Utilizing double symmetry, the semi-elliptic surface crack PST was modeled as bar 5mm x 5mm in the section of the crack plane and 10mm in the tension direction. The crack has the shape of a quarter of an ellipse located at a corner of the 5mm square face. The external tensile stress was taken as 600MPa for all Zencrack and Afgrow analyses.

Three stress ratios were considered, namely, -0.25, 0 and +0.25. To cover a range of initial shapes (i.e. a/c ratios, where a is the depth and 2c the width of the crack) and sizes for the crack we selected the following six combinations of a₀ and c₀ (in mm): (0.05, 0.1); (0.1, 0.1); (0.2, 0.1); (0.1, 0.2); (0.2, 0.2) and (0.4, 0.2). Residual stress distributions for shot-peen type 8A were chosen at 0% and 30% of the post shot-peening process to allow for stress relaxation in early mission histories (see Figure 9-15). 30% residuals were chosen since previous experience has shown that at 50% residuals some analyses produced zero crack growth.

![Residual stress distributions](image)

**Figure 9-15 - Residual stress distributions**

The Zencrack load superposition option was used where the effects of the external pressure and residual stress are computed separately and then superimposed ignoring any contact at the crack face.
In all cases the standard crack-block sq158x6 was used, along with BOUNDARY SHIFT, TYPE=BOUNDARY option with default Zencrack values for tolerances.

The ‘NORMAL=NO’ option was chosen for MAPPING so that the element edges of the controlled rings were not forced to be normal to the crack front, thereby avoiding excessive element distortion.

The RELAX option was used in all model PST1 analyses.

The (default) error correction integration algorithm was used with a target 2% error between K values at the end of each integration step and the values found from FEA at the beginning of the next integration step.

The numerical integration scheme was adopted for all analyses with residual stress to take correct account of the changing stress ratio over an integration interval. The default forward predictor integration scheme was used for the remaining PST1 model analyses.

All conversions from displacements or energy release rates to stress intensity factors were calculated assuming plane strain conditions at every position along the crack fronts.

In subsequent sections results are presented of crack growth. However, for convenience of discussion we first present in one section the results for \( K_{\text{ext}} \) and \( K_{\text{res}} \) for all the cases analyzed here. These were found for the initial crack size, i.e. on the basis of the results after the first FEA.
9.2.1 Review of $K_{ext,max}$ and $K_{res}$ Results for PST1 Model

We compare here the computed J-integral- and displacement-based SIFs along the initial crack front resulting from the maximum external loading with those given by the Raju-Newman equations and those resulting from residual stress induced by type 8A shot peening with the weight function (WF) based values at the two end nodes. We seek to identify any trends in the nature of disagreement (where they occur) between the results found by the alternative procedures as a function of the model type and the shape and size of the crack. As discussed before, the six initial crack sizes ($a_0,c_0$ in mm) were: (0.05, 0.1); (0.1, 0.1); (0.2, 0.1); (0.1, 0.2); (0.2, 0.2) and (0.4, 0.2).

9.2.1.1 $K_{ext}$

Figure 9-16 to Figure 9-21 show the variation of the stress intensity factors along the crack front for all the six combinations of $a_0$ and $c_0$. The reference Raju-Newman solutions are depicted with ±2% error bars.

The following items were noticed about the SIF results:

- The Zencrack J-Integral SIFs are closer to the Raju-Newman SIFs than the Zencrack COD SIFs.
- The vast majority of the Zencrack J-Integral SIFs are within 2% of the Raju-Newman SIFs.

9.2.1.2 $K_{res}$

Figure 9-22 to Figure 9-27 show the variation of the stress intensity factors along the crack front due to 30% of the residual stress associated with type 8A shot-peening. This is applied on the surface at phi=0 degrees where the crack position is at $c_0$. The two sets of Zencrack results are shown against the Afgrow weight function values which are derived from work by Prof. G. Glinka, University of Waterloo, Ontario, Canada. In the surface crack, $K_a$ is at the deepest point.

The following items were noticed about the SIF results:

- The Zencrack J-Integral SIFs are closer to the Afgrow SIFs than the Zencrack COD SIFs.
- Good agreement is seen between Zencrack J-Integral SIF and Afgrow SIF particularly for circular cracks.
Surface crack, $a=0.05\,\text{mm}, c=0.10\,\text{mm}$

![Graph showing variation of $K_{\text{ext}}$ with angle along crack front at 600 MPa external pressure (Model PST1 - psz0510p00) with ±2% error bars for Raju-Newman values.](image)

**Figure 9-16** - Variation of $K_{\text{ext}}$ with angle along crack front: 600 MPa external pressure (Model PST1 - psz0510p00) (±2% error bars for Raju-Newman values)

Surface crack, $a=0.10\,\text{mm}, c=0.10\,\text{mm}$

![Graph showing variation of $K_{\text{ext}}$ with angle along crack front at 600 MPa external pressure (Model PST1 - psz1010p00) with ±2% error bars for Raju-Newman values.](image)

**Figure 9-17** - Variation of $K_{\text{ext}}$ with angle along crack front: 600 MPa external pressure (Model PST1 - psz1010p00) (±2% error bars for Raju-Newman values)
Surface crack, $a=0.10\text{mm}$, $c=0.20\text{mm}$

Figure 9-18 - Variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure
(Model PST1 - psz1020p00) (±2% error bars for Raju-Newman values)

Surface crack, $a=0.20\text{mm}$, $c=0.10\text{mm}$

Figure 9-19 - Variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure
(Model PST1 - psz2010p00) (±2% error bars for Raju-Newman values)
Surface crack, $a=0.20\text{mm}$, $c=0.20\text{mm}$

![Graph showing variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure.](Model PST1 - psz4020p00) ($\pm2\%$ error bars for Raju-Newman values)

Figure 9-20

Surface crack, $a=0.40\text{mm}$, $c=0.20\text{mm}$

![Graph showing variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure.](Model PST1 - psz4020p00) ($\pm2\%$ error bars for Raju-Newman values)

Figure 9-21
Surface crack, $a=0.05\text{mm}$, $c=0.10\text{mm}$

Figure 9-22 - Variation of $K_{res}$ with angle along crack front: 30% of 8A shot peen residual stress
(Model PST1 - psz0510p30)

Surface crack, $a=0.10\text{mm}$, $c=0.10\text{mm}$

Figure 9-23 - Variation of $K_{res}$ with angle along crack front: 30% of 8A shot peen residual stress
(Model PST1 - psz1010p30)
Surface crack, $a=0.10\text{mm}, c=0.20\text{mm}$

![Graph](image1)

*Figure 9-24 - Variation of $K_{res}$ with angle along crack front: 30% of 8A shot peen residual stress (Model PST1 - psz1020p30)*

Surface crack, $a=0.20\text{mm}, c=0.10\text{mm}$

![Graph](image2)

*Figure 9-25 - Variation of $K_{res}$ with angle along crack front: 30% of 8A shot peen residual stress (Model PST1 - psz2010p30)*
**Figure 9-26** - Variation of \( K_{\text{res}} \) with angle along crack front: 30% of 8A shot peen residual stress

*(Model PST1 - psz2020p30)*

**Figure 9-27** - Variation of \( K_{\text{res}} \) with angle along crack front: 30% of 8A shot peen residual stress

*(Model PST1 - psz4020p30)*
9.2.2 Crack growth prediction

In all Zencrack crack growth analyses, growth was terminated by $da/dn$ exceeding the maximum value of the tabular data. Afgrow continued after this condition occurred but erroneously attributed the maximum value of $da/dn$ in the tabular data to subsequent crack growth. The Afgrow results were duly cropped to achieve the correct basis for comparison with Zencrack.

Results of all analyses are tabulated in the form of cycles to failure and crack length at failure in Table 9-3 to Table 9-8.

Crack growth curves are presented graphically in Figure 9-28 and Figure 9-29 and showing the crack growth behavior for the deepest and the surface node. The figures are for a circular crack with and without residual stress ($a=c=0.2mm$ and $R=0$). The crack growth profiles for the same runs are shown Figure 9-30 and Figure 9-31. The change in shape of the crack profiles due to residual stress can be observed (less of an obvious elliptic shape, with faster initial growth in the depth direction).

Afgrow makes the unrealistic assumption that the crack remains elliptical throughout crack growth. Zencrack does not make this assumption (see Figure 9-30 and Figure 9-31). This distinction is particularly important when residual stress is present. It is to be expected that there should be some variation in results between the Zencrack and Afgrow crack growth analysis.

The crack growth predictions in Figure 9-28 show significant differences in life between AFGROW and Zencrack results. The reason for this is the lower SIFs calculated by Zencrack as shown in Figure 9-32. These lower values are partly due to the non-elliptic shape of the crack front in the Zencrack analyses (as shown in Figure 9-33 after approximately 2630 cycles). Similar observations are made for the corner crack model in section 9.3.3.
### Table 9-3 - Results for $R=-0.25$ at 0% residual stress

<table>
<thead>
<tr>
<th>$a_0$ (mm)</th>
<th>$c_0$ (mm)</th>
<th># FEA for Zencrack</th>
<th>Zencrack 7.1 $a_f$ (mm)</th>
<th>Zencrack 7.1 $c_f$ (mm)</th>
<th>Zencrack 7.1 N (cycles)</th>
<th>Afgrow $a_f$ (mm)</th>
<th>Afgrow $c_f$ (mm)</th>
<th>Afgrow N (cycles)</th>
</tr>
</thead>
<tbody>
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### Table 9-4 - Results for $R=0.0$ at 0% residual stress

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<th>Zencrack 7.1 $a_f$ (mm)</th>
<th>Zencrack 7.1 $c_f$ (mm)</th>
<th>Zencrack 7.1 N (cycles)</th>
<th>Afgrow $a_f$ (mm)</th>
<th>Afgrow $c_f$ (mm)</th>
<th>Afgrow N (cycles)</th>
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### Table 9-5 - Results for $R=0.25$ at 0% residual stress

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<th># FEA for Zencrack</th>
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<th>Zencrack 7.1 $c_f$ (mm)</th>
<th>Zencrack 7.1 N (cycles)</th>
<th>Afgrow $a_f$ (mm)</th>
<th>Afgrow $c_f$ (mm)</th>
<th>Afgrow N (cycles)</th>
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<td># FEA for Zencrack</td>
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<td>Zencrack 7.1 N (cycles)</td>
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<td>Afgrow cf (mm)</td>
<td>Afgrow N (cycles)</td>
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**Table 9-6 - Results for R=0.25 at 30% residual stress**

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<th>Zencrack 7.1 af (mm)</th>
<th>Zencrack 7.1 cf (mm)</th>
<th>Zencrack 7.1 N (cycles)</th>
<th>Afgrow af (mm)</th>
<th>Afgrow cf (mm)</th>
<th>Afgrow N (cycles)</th>
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**Table 9-7 - Results for R=0.0 at 30% residual stress**

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<th>Zencrack 7.1 cf (mm)</th>
<th>Zencrack 7.1 N (cycles)</th>
<th>Afgrow af (mm)</th>
<th>Afgrow cf (mm)</th>
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<td>10325</td>
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</table>

**Table 9-8 - Results for R=0.25 at 30% residual stress**
Surface Crack, $a=0.20$ mm, $c=0.20$mm

![Graph showing crack growth for $a=0.20$ mm, $c=0.20$mm, zero residual stress at $R=0$.

Figure 9-28 - Crack growth for $a=0.20$mm, $c=0.20$mm, zero residual stress at $R=0$]

Surface Crack, $a=0.20$mm, $c=0.20$mm

![Graph showing crack growth for $a=0.20$ mm, $c=0.20$mm, 30% residual stress at $R=0$.

Figure 9-29 - Crack growth for $a=0.20$mm, $c=0.20$mm, 30% residual stress at $R=0$]
Figure 9-30 - Crack growth for \( a=0.20\text{mm}, c=0.20\text{mm}, 0\% \) residual stress at \( R=0 \) (surface is parallel to the ‘1’ direction, depth, \( a \), is in the –ve ‘3’ direction)

Figure 9-31 - Crack growth profiles for \( a=0.20\text{mm}, c=0.20\text{mm}, 30\% \) residual stress at \( R=0 \) (shot-peened surface is parallel to the ‘1’ direction, depth, \( a \), is in the –ve ‘3’ direction)
Figure 9-32 Stress intensity factors during surface crack growth

Figure 9-33 - Surface crack growth profiles
9.2.3 Additional PST1 Zencrack analysis using a large crack-block

The susceptibility of the crack growth predictions to element distortion and mesh density was investigated using the Zencrack “large crack-block” type sq496x8.sup to model a semi-elliptic crack with \( a = c = 0 \) 10 mm at zero residual stress and zero stress ratio.

The re-analysis (of psz1010p00) achieved 11478 cycles in only 38 finite element analyses compared to 12244 in the second case of Table 9-4. This compares more favorably with the Afgrow solution of 10728 cycles than was achieved previously using standard crack-blocks (see Figure 9-34).

![Figure 9-34 - Crack growth using a Zencrack large crack-block](image)

Zencrack has also been benchmarked against experimental data of fatigue crack growth in a plate with a central semi-elliptic surface crack (see Ref. 4, example 4). The cracked model with a large crack-block give excellent agreement with experimental data.
9.3 Corner crack model (CRT1)

For the study of the corner crack a 5mm x 5mm x 10mm long half symmetry model (CRT1) was used representing a square bar 20mm long. This model has the same dimensions as the PST1 model except that the two 5mm x 10mm faces forming the corner were stress free. In place of the standard crack-block sq158x6.sup used for the PST model we have now used the large crack-block, sq496x8.sup. For this purpose eight 1mm cube elements from the corner have been replaced by a single 2mm cube element tied to the surrounding mesh. The orientation of the model is such that the crack length “a” is measured in the X (or 1) direction and the length “c” is measured in the -ve Z (or -ve 3) direction. The mesh for crack size a=0.1mm, c=0.2mm is shown in Figure 9-35. The residual stress distribution is a function of X i.e. as if the Y-Z (or 2-3) surface visible in Figure 9-35 was shot peened.

We present the results for this model covering the same set of initial crack sizes and stress ratios as for the PST1 model. The maximum external load has been taken as 600MPa for the Zencrack and Afgrow runs. Analyses are presented for cyclic external load only and for cyclic external load plus 30% of the 8A shot-peening residual stress. All materials data and the residual stress distribution are the same as for the PST model presented in section 9.2 (see sections 9 and 9.2 respectively).
All Zencrack analyses utilized Method 1 (see section 3.1) for combining the residual stress and external load. The forward predictor algorithm was used during integration with default settings. The error control scheme was used with default values to control step sizes between finite element analyses. For mapping the option ‘NORMAL=NO’ was chosen so that the element edges in the controlled rings were not forced to be normal to the crack front. This was done to help meshing of the elliptic cracks.

All Zencrack analyses terminated when the upper bound of the tabular crack growth data was reached. To allow equivalent tables and curves to be plotted, the Afgrow solutions are reported in the following sections up to the equivalent point of the Afgrow analyses (when da/dn reaches the maximum value specified in the tabular data). Beyond this the Afgrow analyses make the unrealistic assumption that the growth curve becomes horizontal.

As with the PST model we first discuss in one section all the results for the initial $K_{ext,max}$ and $K_{res}$, which is followed by the presentation of the crack growth results.

### 9.3.1 Review of $K_{ext}$ and $K_{res}$ results for CRT1 model

#### 9.3.1.1 $K_{ext}$

Figure 9-36 to Figure 9-41 show the variation of the stress intensity factors along the crack front for all the six combinations of $a_0$ and $c_0$. The crack position at $c_0$ is 0 degrees and at $a_0$ is 90 degrees. The Zencrack solutions and the Raju-Newman results all correspond to an external pressure of 600 MPa. The Raju-Newman curves include 2% error bars. Unlike the PST results, both ends of the curves for these plots (at phi = 0 and 90 degrees) represent surface nodes. Zencrack results at these points are plotted using a plane strain assumption rather than a plane stress assumption. The following observations can be made:

- At all the interior nodes of both the 0.1mm and the 0.2mm radius quarter-circular cracks the J-based values are larger than the displacement-based ones, but the situation in both cases reverses at the surface nodes.
- For the four non-circular crack shapes the J-based values are again larger than the displacement-based values at all the interior nodes, and they are smaller than or equal to the latter at the surface nodes except at the phi = 0 node for the smallest crack ($a_0 = 0.05mm$ and $c_0 = 0.1mm$).
Except for the smallest crack the Raju-Newman results are almost everywhere smaller than the Zencrack values, the difference in most cases being largest at the surface nodes. Further comment on this discrepancy is made in section 9.3.3.

9.3.1.2 $K_{\text{res}}$

Figure 9-42 to Figure 9-47 show the variation of the stress intensity factors along the crack front due to 30% of the residual stress associated with type 8A shot-peening. This is applied on the surface at $\phi=0$ degrees where the crack position is at $c_0$. The two sets of Zencrack results are shown against the Afgrow surface values which are based on the weight function surface values which are derived from work by Prof. G. Glinka, University of Waterloo, Ontario, Canada.

Further validation of $K_{\text{res}}$ values against the weight function results by Shiratori (Ref. 14) for certain one- and two-dimensional polynomial stress distributions are presented in section 9.3.4.

The following observations can be made:

- The $J$-based values are numerically larger than the displacement-based values at both the surface nodes in all cases except for the smallest ($0.05 \times 0.1$mm) crack where they are slightly smaller.

- At the interior nodes the difference between the two sets of results does not appear to follow any pattern; however, it is usually small particularly for the four larger cracks with a dimension of at least 0.2mm on one surface.

- The agreement between Zencrack and Afgrow is generally good except for the 0 degree values for the $a=0.20$mm, $c=0.10$mm (Figure 9-45) and $a=0.4$mm, $c=0.2$mm curves (Figure 9-47). In the former case the Afgrow value at 0 degrees is similar to the 90 degree value. In the latter case the value of $K_{\text{res}}$ from Afgrow is 636MPa-sqrt(mm) and is not plotted on Figure 9-47 as it results in distortion of the y-axis scale. These two Afgrow values are invalid since the weight function approach used in Afgrow is only applicable for $0.2 < a/c \leq 1.0$ and these cases have $a/c = 2.0$. It is noted, however, that no warning is given by Afgrow that the ratio is out of bounds.
Figure 9-36 - Variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure

(Model CRT1_s600_rxx_a05_c10_8A00) (±2% error bars for Raju-Newman values)

Figure 9-37 - Variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure

(Model CRT1_s600_rxx_a10_c10_8A00) (±2% error bars for Raju-Newman values)
**Figure 9-38** - Variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure  
(Model CRT1_s600_rxx_a10_c20_8A00) (±2% error bars for Raju-Newman values)

**Figure 9-39** - Variation of $K_{ext}$ with angle along crack front: 600 MPa external pressure  
(Model CRT1_s600_rxx_a20_c10_8A00) (±2% error bars for Raju-Newman values)
Corner Crack, $a=0.20\text{mm}$, $c=0.20\text{mm}$

Figure 9-40 - Variation of $K_{\text{ext}}$ with angle along crack front: 600 MPa external pressure
(Model CRT1_s600_rxx_a20_c20_8A00) (±2% error bars for Raju-Newman values)

Corner Crack, $a=0.40\text{mm}$, $c=0.20\text{mm}$

Figure 9-41 - Variation of $K_{\text{ext}}$ with angle along crack front: 600 MPa external pressure
(Model CRT1_s600_rxx_a40_c20_8A00) (±2% error bars for Raju-Newman values)
Corner Crack, a=0.05mm, c=0.10mm

Figure 9-42 - Variation of $K_{res}$ with angle along crack front: 30% of 8A shot peen residual stress
(Model CRT1_s600_rxx_a05_c10_8A30)

Corner Crack, a=0.10mm, c=0.10mm

Figure 9-43 - Variation of $K_{res}$ with angle along crack front: 30% of 8A shot peen residual stress
(Model CRT1_s600_rxx_a10_c10_8A30)
Corner Crack, $a=0.10\text{mm}$, $c=0.20\text{mm}$

\[ K (\text{MPa}-\sqrt{\text{mm}}) \]

\[ \Phi (\text{degrees}) \]

Figure 9-44 - Variation of $K_{\text{res}}$ with angle along crack front: 30% of 8A shot peen residual stress
(Model CRT1_s600_rxx_a10_c20_8A30)

Corner Crack, $a=0.20\text{mm}$, $c=0.10\text{mm}$

\[ K (\text{MPa}-\sqrt{\text{mm}}) \]

\[ \Phi (\text{degrees}) \]

Figure 9-45 - Variation of $K_{\text{res}}$ with angle along crack front: 30% of 8A shot peen residual stress
(Model CRT1_s600_rxx_a20_c10_8A30)

See final bullet point in section 9.3.1.2 with regard to Afgrow value at 0 degrees.
Corner Crack, \(a=0.20\text{mm}, c=0.20\text{mm}\)

Figure 9-46 - Variation of \(K_{\text{res}}\) with angle along crack front : 30% of 8A shot peen residual stress
(Model CRT1_s600_rxx_a20_c20_8A30)

Corner Crack, \(a=0.40\text{mm}, c=0.20\text{mm}\)

Figure 9-47 - Variation of \(K_{\text{res}}\) with angle along crack front : 30% of 8A shot peen residual stress
(Model CRT1_s600_rxx_a40_c20_8A30)

See final bullet point in section 9.3.1.2 with regard to Afgrow value at 0 degrees.
9.3.2 Crack growth prediction

Results of all analyses are tabulated in the form of cycles to failure and crack length at failure in Table 9-9 to Table 9-14. For all analyses failure is defined as occurring when the top of the tabular crack growth data is reached for reasons mentioned previously.

Plots of crack length against cycles are presented in Figure 9-48 to Figure 9-59. These results are only plotted for the R=0 analyses. The plots are arranged in pairs of external load only and external load plus residual stress for each of the six starter crack sizes. This allows the effect of the residual stress on the life to be readily seen.

Profile plots from Zencrack are presented for the 0.10mm circular corner crack at R=0 along with plots of a against c. These are shown in Figure 9-60 and Figure 9-61 for 0% and 30% residual stress respectively.

An additional analysis for 40% residual stress has been undertaken to demonstrate the effect of crack shape development as a function of applied residual stress. The profiles for the 40% analysis are shown in Figure 9-62. Comparisons of crack profiles for different amounts of residual stress for “a” approximately equal to 0.35mm are shown in Figure 9-63.

The following points are noted:

- There is good agreement between Afgrow and Zencrack for all 18 analyses without residual stress. For most analyses the difference in the final value of N is less than 2%. The exception is the 0.4mm / 0.2mm elliptic crack in which the difference is over 4% for all three R values.
- For the analyses with residual stress the differences in the number of cycles between Zencrack and Afgrow varies between -11% and +27%. The Zencrack analysis for the 0.2mm / 0.2mm circular crack is the only crack size to give a lower value for N than the corresponding Afgrow analysis.
- Differences between the two programs for the analyses with residual stress are to be expected due to the basic solution methods in Zencrack and Afgrow. The Zencrack analyses are able to predict crack shape development and hence take account of any effects
of the crack shape on the Kres and Kext distributions. The Afgrow analyses on the other hand assume elliptic crack shapes and only the a/c ratio is of consequence. The effect of the crack shape can be seen in the profile and a vs c plots of Figure 9-60 and Figure 9-61. For the external load only, Figure 9-60, the initial circular corner crack maintains its profile. The step size increases and then cuts back when instability develops. From the a vs c plot it is clear that the Zencrack run maintains a=c as would be expected. For the Afgrow analysis there is a small deviation from a=c. For the residual stress case, Figure 9-61, the crack develops more quickly in the a direction than the c direction. This effect is more marked in the Zencrack analysis. The Zencrack profiles are not elliptic and a burrowing or horseshoe shape can be seen at the edge which has been shot peened (the “c” edge on the top of the profile plot). This burrowing effect is more pronounced as the amount of residual stress is increased, as demonstrated in Figure 9-63. This type of shape development is typical of that reported in the literature e.g Ref. 18.

- It is noted that the Afgrow analysis for the 0.40mm / 0.20mm crack with residual stress gives a result that is invalid. The initial Kres value in the c direction is high and positive, resulting in an initial value of da/dn of 1e-4m/cycle. For the remainder of the analysis the Kres value behaves in a similar fashion to the other analyses. This behaviour is due to the invalid a/c ratio for the weight function used in calculating the Kres value. It is noted that the upper limit of a/c = 1.0 for the weight function Kres method is exceeded in many of the Afgrow growth analyses. For example, in the 0.10mm circular crack analysis with residual stress and external stress ratio of zero, the ratio a/c starts at 1.0, falls to 0.96 and increases to 1.35.

### 9.3.3 Additional comments

There is generally good agreement in the life prediction between Zencrack and Afgrow for the analyses without residual stress. This appears to be at odds with the 4% to 5% differences in the stress intensity values reported for the initial defect in section 9.3.1.1 between Zencrack and Raju-Newman equations (noting that Afgrow uses a Raju-Newman solution).

Figure 9-64 gives the variation of stress intensity factor with crack size for the early stages of the analysis. The Afgrow value of crack size and stress intensity is the mean of the a and c values. The Zencrack results are based on j-integral values along the a edge. It is clear that the agreement is good
other than at very small crack sizes, up to about 0.112mm. This explains the good agreement in the predicted life.

The crack front profile from finite element analysis number 5, at which $a=0.112\text{mm}$, is shown in Figure 9-65. It is clear that in the Zencrack analysis the crack shape is not circular any longer. In fact the radius at the 45 degree line is 2.25\% lower than the radius at the surfaces. Although this does not explain the differences in $K$ for the initial circular crack shape, it does demonstrate that a 3D analysis can provide more information than would be possible from a 2D analysis.

Further observations regarding the Raju-Newman solutions are made in section 9.4.
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<tr>
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Table 9-9 - Results for \(R=-0.25\), external load only

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Table 9-10 - Results for \(R=0.0\), external load only

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Table 9-11 - Results for \(R=0.25\), external load only
Table 9-12 - Results for R=-0.25, 30% residual stress 8A

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Table 9-13 - Results for R=0.0, 30% residual stress 8A

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Table 9-14 - Results for R=0.25, 30% residual stress 8A

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Corner Crack, $a=0.05\text{mm}$, $c=0.10\text{mm}$ (external load only)

Figure 9-48 - $a$ vs $N$: $a=0.05\text{mm}$, $c=0.10\text{mm}$, external load only at $R=0$

Corner Crack, $a=0.05\text{mm}$, $c=0.10\text{mm}$ (30% of 8A shot peen stress)

Figure 9-49 - $a$ vs $N$: $a=0.05\text{mm}$, $c=0.10\text{mm}$, external load at $R=0$, 30% of 8A shot peen residual stress
Corner Crack, $a=0.10\text{mm}$, $c=0.10\text{mm}$ (external load only)

Figure 9-50 - $a$ vs $N$: $a=0.10\text{mm}$, $c=0.10\text{mm}$, external load only at $R=0$

Corner Crack, $a=0.10\text{mm}$, $c=0.10\text{mm}$ (30% of 8A shot peen stress)

Figure 9-51 - $a$ vs $N$: $a=0.10\text{mm}$, $c=0.10\text{mm}$, external load at $R=0$, 30% of 8A shot peen residual stress
Corner Crack, \(a=0.10\text{mm}, c=0.20\text{mm}\) (external load only)

Figure 9-52 - \(a vs N: a=0.10\text{mm}, c=0.20\text{mm}\), external load only at \(R=0\)

Corner Crack, \(a=0.10\text{mm}, c=0.20\text{mm}\) (30% of 8A shot peen stress)

Figure 9-53 - \(a vs N: a=0.10\text{mm}, c=0.20\text{mm}\), external load at \(R=0\), 30% of 8A shot peen residual stress
Corner Crack, a=0.20mm, c=0.10mm (external load only)

Figure 9-54 - a vs N : a=0.20mm, c=0.10mm, external load only at R=0

Corner Crack, a=0.20mm, c=0.10mm (30% of 8A shot peen stress)

Figure 9-55 - a vs N : a=0.20mm, c=0.10mm, external load at R=0, 30% of 8A shot peen residual stress
Corner Crack, $a=0.20\text{mm}$, $c=0.20\text{mm}$ (external load only)

Figure 9-56 - $a$ vs $N$: $a=0.20\text{mm}$, $c=0.20\text{mm}$, external load only at $R=0$

Corner Crack, $a=0.20\text{mm}$, $c=0.20\text{mm}$ (30\% of 8A shot peen stress)

Figure 9-57 - $a$ vs $N$: $a=0.20\text{mm}$, $c=0.20\text{mm}$, external load at $R=0$, 30\% of 8A shot peen residual stress
Corner Crack, a=0.40mm, c=0.20mm (external load only)

Figure 9-58 - a vs N : a=0.40mm, c=0.20mm, external load only at R=0

Corner Crack, a=0.40mm, c=0.20mm (30% of 8A shot peen stress)

Figure 9-59 - a vs N : a=0.40mm, c=0.20mm, external load at R=0, 30% of 8A shot peen residual stress
a measured in +ve “1” direction, c measured in -ve “3” direction – c edge is shot peened

Figure 9-60 - Profiles and a vs c plot for a=0.10mm, c=0.10mm, external load only
a measured in +ve “1” direction, c measured in -ve “3” direction – c edge is shot peened

Figure 9-61 - Profiles and a vs c plot for a=0.10mm, c=0.10mm with 30% 8A residual stress
a measured in +ve “1” direction, c measured in -ve “3” direction – c edge is shot peened

*Figure 9-62 - Profiles a=0.10mm, c=0.10mm, with 40% 8A residual stress*
Profiles for a of around 0.35mm

Figure 9-63 – Crack profiles for length a of approximately 0.35mm under 0%, 30%, 40% residual stress
Figure 9-64 - Kext against crack size for growth analysis under external loading only

Figure 9-65 - Crack profile for Zencrack analysis step 5, initial $a=0.1\text{mm}$, $c=0.1\text{mm}$
9.3.4 Further validation of $K_{res}$ results for corner crack models (CRT1 and CRT2)

As mentioned earlier, no comparison with other independent solutions has been possible for the SIFs due to the residual stresses as the results for a corner crack model under the shot-peen induced stress distributions were not available. Therefore, to validate the capability of Zencrack to predict $K_{res}$ for corner cracks, further analyses were performed for uni-directional and bi-directional polynomial distributions of pressure on the crack surface using the weight function (WF) results for these distributions given by Shiratori (Ref. 14).

For this validation we use two different bar sizes and uncracked finite element models to analyze two crack sizes, which are chosen from the range of cases treated by Shiratori. Both the cracks are quarter-circular in shape ($a/c = 1.0$). For the first, the crack radius is chosen as 1.0mm so that our existing CRT1 model, 5mm x 5mm x 10mm half length, gives $a/t = c/w = 0.2$. For this combination the results are available only at the surface nodes. For one case ($a/c = 1$ and $a/t = 0.4$) Shiratori also provides distributions of $K_{res}$ along the crack front. To replicate these results we chose a 5mm wide by 3mm thick bar with a crack radius of 1.2mm. This gives for the second model (CRT2) $c/w = 0.24$, which is slightly different from 0.2 used by the author. For both models a 2mm cube corner element is replaced by the sq496x8 large crack-block.

Following Shiratori, the pressure distribution on the crack face simulating the residual stress is expressed in the following form:

$$\sigma(x,z) = \sigma(0)*[1.0 - x/a]^m *[1.0 - (w-z)/c]^n$$

Equation 9-1

with $\sigma(0)$ taken as 100 MPa. The cross sectional dimensions of the model and the axes of coordinates used in Zencrack are shown in Figure 9-66. Shiratori has given tables for the non-dimensional stress intensity factors at the two surface nodes for 16 pressure distributions corresponding to $m, n = 0, 1, 2$ and 3 and for a range of combinations of $a/t$ and $a/c$. The results for $a/t = 0.2$ and $a/c = 1.0$, used in the first model, are taken from these tables. For the crack with $a/t = 0.4$ and $a/c = 1.0$, treated in our second model, we used the digitized data from the plots given in Figure 12 of the paper for ten of the above 16 pressure distributions.
9.3.4.1 Corner Crack Model (CRT1): $a/c = 1.0$, $a/t = 0.2$, $c/w = 0.2$

Table 9-15 shows the stress intensity factors for all the 16 pressure distributions on the crack face. $K_a$ is for the node on the vertical face ($x = a$, $z = w$) and $K_c$ is that on the horizontal face ($x = 0$, $z = w-c$). The WF results require the term $Q$, which is the square of the complete elliptic integral of the second kind and works out to be 2.464 for the chosen crack. The table shows the differences (‘errors’) between the Zencrack and the WF values as a percentage of the latter. As is evident, the ‘errors’ tend to be quite large when either or both of the indices ($m$ and $n$) equals 2 or 3. However, we should note that in these cases the actual value of $K$ is relatively small so that the errors will work out to be much smaller if the values for the uniform pressure ($m = n = 0$) were used as the base. For this pressure the WF values for $K_a$ and $K_c$ equal $129.74\text{MPa}\sqrt{\text{mm}}$, whereas for $m = n = 2$ and $m = n = 3$ they are respectively $11.52\text{MPa}\sqrt{\text{mm}}$ and $5.31\text{MPa}\sqrt{\text{mm}}$. 

![Figure 9-66 - Sketch of crack section](image-url)
### Table 9-15 Comparison of Zencrack computed $K_{res}$ at surface nodes 'c' and 'a' with the Shiratori WF solution

The distributions of $K$ for the case of uniform pressure on the crack face is shown in Figure 9-67. The effect of this loading on the stress intensity factors is identical to that of a remotely applied uniform pressure for which the Raju-Newman solution is available; these results are also plotted. It appears that the WF results, available only at the surface nodes, differ from the N-R solution as well as the Zencrack solutions.
9.3.4.2 Corner Crack Model (CRT2): a/c = 1.0, a/t = 0.4, c/w = 0.24

Figure 9-68 to Figure 9-77 show the variation of the K values along the crack front for a range of pressure distributions on the crack face. The J-based and displacement-based results obtained from Zencrack are shown along with those presented by Shiratori. The nature of the pressure distribution is shown by designations of the form ‘minj’, indicating that the indices are taken as m = i and n = j. There is good agreement between the Zencrack and the WF results for pressures varying in one direction, but there appears to be considerable difference for the bi-directional pressure distributions, m1n1, m2n2 and m3n3. However, for these cases, and particularly for m3n3, the actual values of the SIF are quite small, and the pattern of the discrepancy and its variation with phi is quite similar for all these distributions.
Figure 9-68 - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1

$(m = 0$ and $n = 0$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2\text{mm}$)

Figure 9-69 - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1

$(m = 1$ and $n = 0$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2\text{mm}$)
Figure 9-70 - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1
($m = 2$ and $n = 0$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2$mm)

Figure 9-71 - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1
($m = 3$ and $n = 0$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2$mm)
Figure 9-72 - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1
($m = 0$ and $n = 1$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2mm$)

Figure 9-73 - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1
($m = 0$ and $n = 2$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2mm$)
**Figure 9-74** - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1

$(m = 0$ and $n = 3$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2\text{mm}$)

**Figure 9-75** - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1

$(m = 1$ and $n = 1$. $a/c = 1.0$, $a/t = 0.4$, $c/w = 0.24$; $a = c = 1.2\text{mm}$)
**Figure 9-76** - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1

$(m = 2$ and $n = 2. \ a/c = 1.0, \ a/t = 0.4, \ c/w = 0.24; \ a = c = 1.2\text{mm})$

**Figure 9-77** - Variation of $K_{res}$ with phi along crack front under crack face pressure, Equation 9-1

$(m = 3$ and $n = 3. \ a/c = 1.0, \ a/t = 0.4, \ c/w = 0.24; \ a = c = 1.2\text{mm})$
9.4 Comments on the Raju-Newman reference solutions

The Raju-Newman results for the surface crack model PST1 and the corner crack model CRT1 discussed in sections 9.2 and 9.3 were taken from spreadsheet equation fits to FEA results reported in Ref. 13.

Further investigation using Ref. 19 showed that a Poisson Ratio, \( \nu \), of 0.3 was assumed for the surface crack. Assuming this was the case for corner cracks the effect of Poisson Ratio on the SIF distribution along the crack front in the initial configuration \( a_0 = c_0 = 0.75 \text{mm} \) is shown in Figure 9-78. It can be readily seen when running Afgrow that it takes no account of the actual Poisson Ratio specified by the user for these models.

All Zencrack analyses for these models were conducted using \( \nu = 0.33 \) (see Table 9-1). The inference from the results indicate that in both magnitude and distribution there is a significant dependency on Poisson Ratio and that Zencrack results using \( \nu = 0.3 \) are in closer agreement with Raju-Newman values.

The state of stress changes continuously along the crack front. It is important to remember, however, that the energy release rate approach adopted in Zencrack or in the nodal force method adopted by Raju-Newman are not troubled by state-of-stress issues since they are naturally taken into account in the finite element analysis. The important consideration is that the FEA results are only applicable to the material model adopted.
Corner Crack, a=0.75mm, c=0.75mm

Figure 9-78 Dependency of SIF distribution on Poisson Ratio in a corner crack (Raju-Newman results show ±2% error bars)
9.5 Single edge notched tension model (SENT1)

This section describes analyses of a single edge notched plate under the influence of residual stress and a cyclic tensile load. For this crack model the welded single edge notched specimens used by Kang et al (Ref. 15) were utilized, as this allowed comparison with some of the results of fatigue tests performed by the authors. The purpose of these tests was to investigate crack growth behavior in the presence of compressive residual stress under constant amplitude external tension. The structural steel used in the specimens had a yield stress of 330MPa and a tensile strength of 510MPa. The plate specimens were originally 200mm by 70mm by 10mm thick, welded along the longitudinal center line by electron beam welding which generated a symmetric distribution of residual stress about this line having a magnitude of about 330MPa (tension) at the center and about 40MPa (compression) at the two edges, the tensile region extending to about 10mm on both sides of the center line. After welding, the thickness of the welded specimen was reduced to 6mm by shaping and fine grinding both surfaces in order to minimize the variation of the residual stress through the thickness of the specimen. The model has been analysed with $\nu=0$ to allow comparison with Afgrow.

9.5.1 Residual stress distribution

The residual stress distribution for the plate half width of 35mm was obtained as digitized data from Figure 1 of Ref. 15. The original version of this data contained 36 points for the distribution across half the plate. This data has been used in all Zencrack runs. Afgrow has a limit of 25 points to define a residual stress distribution. Hence a reduced set of data was used in all Afgrow analyses. Both sets of data are shown in Figure 9-79. The differences are considered to be negligible.

The restriction of Afgrow to 25 data points means that Afgrow analyses with residual stress can only be carried out up to a crack length defined by these points i.e. 35mm. This crack length has therefore been used as a “stop” condition for all Afgrow and Zencrack analyses in which residual stress is included.
9.5.2 Crack growth and fracture data

The crack growth data for the unwelded specimen was digitized from figure 2 of Ref. 16. The published crack growth data are for R = 0.4, 0.2, 0.1, 0.0, -0.5, -1.0 and -2.0. An additional set, for R = 0.8, was adopted from NASGRO (ASTM A588). These data were approximated by Paris equations and arrived at a constant value for the parameter C equal to 2.0E-13 and values of exponent n as a function of R. The data and Paris fits are shown in Figure 9-80.

Since neither Zencrack nor Afgrow have input options for Paris equations as functions of R, we used the tabular data input option. Limiting values of da/dN of 1E-6 m/cycle and 1E-9 m/cycle are sufficient to describe the regime covered by the data from Ref. 16.

The final tabular data is shown in Table 9-16. A number of important points arise when this data is used in Afgrow:

- The Afgrow tabular material option does not allow input of the data exactly as it appears in Table 9-16. A problem arises with the value of Kmax=44.0283 at R=-0.2. This value is
greater than the value for $R=-0.1$ at the same $da/dN$ and so is not allowed. For Afgrow input this number has been changed to 43.8998. This only affects analyses in which the effective $R$ falls below -1. To keep the Zencrack data compatible the equivalent number in the Zencrack input is changed from to $\Delta K=4176.8879$ to $\Delta K=4164.7007$.

- For a similar reason to the above, it is not possible to enter a $(da/dn)_{high}$ value to extrapolate the data. Hence, an upper limit of $da/dn$ of 1e-6 m/cycle is prescribed for the analysis.
- The default behaviour for Zencrack when a point falls off the right-hand / top-edge of a tabular $R$ curve is to stop the analysis on the basis that there is no mandate to make any assumption beyond the data range supplied by the user. Afgrow on the other hand assumes that the crack growth rate remains constant for any higher value of $\Delta K$ than is defined by the supplied data i.e. the $da/dN$ vs $\Delta K$ curve becomes horizontal. This is clearly unrealistic. However, to allow a meaningful comparison between Zencrack and Afgrow with the supplied data, the Zencrack input has been modified to mimic this Afgrow behaviour by adding a horizontal segment to each crack growth curve. The behaviour may then be unrealistic, but at least it is consistent.
- Both Zencrack and Afgrow treat the lower end of the tabular data as a threshold condition.

In the absence of fracture toughness values, the analysis described in section 9.5.3 was used to determine that a value of $K_c$ of $125\text{MPa m}^{\frac{1}{2}}$ ($=3952 \text{ MPa mm}^{\frac{1}{2}}$) would give failure at just below 35mm. Hence the following conditions are used to stop the analyses reported in this section:

Without residual stress:
- Zencrack and Afgrow analyses limited by $K_c=125\text{MPa m}^{\frac{1}{2}}$.

With residual stress:
- Zencrack and Afgrow analyses limited by $K_c=125\text{MPa m}^{\frac{1}{2}}$ and maximum crack length=35mm. In the case of the Zencrack analyses, the limit of 35mm is not met precisely and the analysis stops at some value just greater than 35mm. This is controlled by forcing the Abaqus analysis to terminate in subroutine DLOAD if the crack length exceeds 35mm.
Paris constants as a function of $R$ (in m and MPa m units):

<table>
<thead>
<tr>
<th>$R$</th>
<th>0.8</th>
<th>0.4</th>
<th>0.2</th>
<th>0.1</th>
<th>0</th>
<th>-0.5</th>
<th>-1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
<td>2.0E-13</td>
</tr>
</tbody>
</table>

"Raw" 2-point tabular data in m and MPa m$^{1/2}$:

<table>
<thead>
<tr>
<th>$da/dN$ (m/cycle)</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E-09</td>
<td>6.3033</td>
<td>7.3908</td>
<td>7.7860</td>
<td>8.1075</td>
<td>8.3532</td>
<td>10.2957</td>
<td>11.8342</td>
</tr>
<tr>
<td>1.00E-06</td>
<td>28.0574</td>
<td>37.4309</td>
<td>41.1345</td>
<td>44.2617</td>
<td>46.7206</td>
<td>68.2270</td>
<td>87.7997</td>
</tr>
</tbody>
</table>

For Afgrow with $K_{max}$ when $R<0$:

<table>
<thead>
<tr>
<th>$da/dN$ (m/cycle)</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
<th>$\Delta K$ MPa m$^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E-06</td>
<td>28.0574</td>
<td>37.4309</td>
<td>41.1345</td>
<td>44.2617</td>
<td>46.7206</td>
<td>45.4847</td>
<td>43.8999</td>
<td>44.0283</td>
</tr>
</tbody>
</table>

For Zencrack in mm and MPa mm$^{1/2}$:

<table>
<thead>
<tr>
<th>$da/dN$ (mm/cycle)</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
<th>$\Delta K$ MPa mm$^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E-06</td>
<td>199.3280</td>
<td>233.7172</td>
<td>246.2159</td>
<td>256.3819</td>
<td>264.1510</td>
<td>325.5785</td>
<td>374.2299</td>
<td>468.8903</td>
</tr>
<tr>
<td>1.00E-03</td>
<td>887.2532</td>
<td>1183.6702</td>
<td>1300.7869</td>
<td>1399.6786</td>
<td>1477.4338</td>
<td>2157.5280</td>
<td>2776.4710</td>
<td>4176.8879</td>
</tr>
</tbody>
</table>

Table 9-16 - Tabular crack growth data
9.5.3 Determining K vs a

An uniform uncracked mesh consisting of a single layer of 200 elements was utilized for all analyses, there being ten 7mm x 10mm elements in the width direction. Due to the assumption of zero Poisson ratio to allow for comparison with Afgrow, the crack front remains straight during growth i.e. there are no through-thickness effects. Hence K values as a function of crack length can be determined by running a “dummy” crack growth analysis with fixed increments of da. This was done from an initial crack size of 1mm up to the half plate width of 35mm. The resulting K vs a plots for 150MPa external load and the residual stress distribution are shown in Figure 9-81 and Figure 9-82 respectively. The SEN equation method used by Kang is plotted in Figure 9-81. The Kres distribution calculated by Kang and a weight function curve are shown in Figure 9-82.

The agreement is close except for crack lengths larger than about 25mm when the Kang data for Kres appear to differ from the other three distributions. However, the mesh is clearly capable of producing good stress intensity results. This provides a sound basis for the following crack growth analyses.
Figure 9-81 - $K_{ext}$ vs crack length for applied load of 150MPa

Figure 9-82 - $K_{res}$ vs crack length
9.5.4 Solutions with and without residual stress

For the initial crack length of 10mm the first pair of elements on the two sides of the edge crack were treated as ‘split’ elements and the second pair from the edge were replaced by crack-blocks st111x5.sup with the crack tip occurring close to the middle of the element. Some details of the mesh before and after crack growth are shown in Figure 9-83, Figure 9-84 and Figure 9-85. Solutions were obtained invoking the ‘relax’ option in order to avoid possible mesh distortions. The full range of crack lengths is analysed by invoking the boundary shift option to allow the crack-blocks to transfer through the mesh.

The distance axis in the residual stress distribution of Figure 9-79 is measured in the x direction in the finite element model, starting at the lowest x value, through to the x-direction centerline of the model at a distance of 35mm from the edge.
Figure 9-84 - Detailed view of the mesh close to the edge crack (Model SENT1_s150_rxx_a1000)

Figure 9-85 - Detailed view of typical displaced mesh plot under external load
9.5.4.1 Analyses with maximum external load $S_{\text{max}} = 150$MPa

Four analyses were carried out at applied load ratios of $R = 0, 0.1, 0.2$ and $0.4$ with maximum external tension of 150MPa. These analyses were repeated with residual load applied. All analyses used the error control scheme in Zencrack with default tolerance values. The Zencrack results along with those given by Afgrow are shown in Table 9-17. All analyses terminated when $K_c$ was reached. Curves of a vs N for analyses without residual stress are shown in Figure 9-86 to Figure 9-89. The a vs N curves for analyses with residual stress are shown in Figure 9-90 to Figure 9-93. The following points are noted:

- There is extremely close agreement in the crack length at failure between Zencrack and Afgrow. Differences in the number of cycles at failure are less than 1.1%.
- The analyses with residual stress show a constant $da/dn$ slope towards the end of the analysis. This is due to the limitation in the materials data described in section 9.5.2 that forces the use of a constant maximum growth rate $da/dn$ of $10^{-6}$ m/cycle.

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>R</th>
<th>Zencrack 7.1 No. of FEA</th>
<th>Zencrack 7.1 $N_f$ (cycles)</th>
<th>Zencrack 7.1 ‘$a_f$’ (mm)</th>
<th>Afgrow $N_f$ (cycles)</th>
<th>Afgrow ‘$a_f$’ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-NoRes</td>
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<td>20</td>
<td>15781</td>
<td>33.64</td>
<td>15633</td>
<td>33.75</td>
</tr>
<tr>
<td>2-NoRes</td>
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<td>20</td>
<td>19540</td>
<td>33.66</td>
<td>19349</td>
<td>33.61</td>
</tr>
<tr>
<td>3-NoRes</td>
<td>0.20</td>
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<td>23767</td>
<td>33.65</td>
<td>23527</td>
<td>33.72</td>
</tr>
<tr>
<td>4-NoRes</td>
<td>0.40</td>
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<td>55257</td>
<td>33.65</td>
<td>54665</td>
<td>33.70</td>
</tr>
<tr>
<td>1-Res</td>
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<td>79174</td>
<td>33.94</td>
<td>79319</td>
<td>34.14</td>
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<td>91324</td>
<td>33.95</td>
<td>91580</td>
<td>33.14</td>
</tr>
<tr>
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<td>28</td>
<td>119869</td>
<td>33.95</td>
<td>118817</td>
<td>34.14</td>
</tr>
</tbody>
</table>

Table 9-17 - Results for $a_0 = 10.0$mm & $S_{\text{max}} = 150$MPa with different R & with and without res. stress. (Model SENT1_s150_rxx_a1000_noKres or KresKang)
Figure 9-86 - Crack length vs number of cycles (Model SENT1_s150_r00_a1000_noKres)

Figure 9-87 - Crack length vs number of cycles (Model SENT1_s150_r10_a1000_noKres)
Figure 9-88 - Crack length vs number of cycles (Model SENT1_s150_r20_a1000_noKres)

Figure 9-89 - Crack length vs number of cycles (Model SENT1_s150_r40_a1000_noKres)
Figure 9-90 - Crack length vs number of cycles (Model SENT1_s150_r00_a1000_KresKang)

Figure 9-91 - Crack length vs number of cycles (Model SENT1_s150_r10_a1000_KresKang)
Figure 9-92 - Crack length vs number of cycles (Model SENT1_s150_r20_a1000_KresKang)

Figure 9-93 - Crack length vs number of cycles (Model SENT1_s150_r40_a1000_KresKang)
9.5.4.2 Analyses using Test $S_{\text{max}}$ as estimated from Kang’s Plots for different $R$

One of the objectives of the analyses presented in the previous section was to compare the computed crack growth rates in the presence of residual stress with the test data given in Figure 3 of Kang’s 1990 paper (Ref. 15). The test data gives the $da/dN$ vs $\Delta K$ plots for plates with the analysed residual stress distribution. The test data for $da/dN$ have been shown as a function of the nominal values of $\Delta K$, defined as $(K_{\text{ext,max}}-K_{\text{ext,min}})$ or $K_{\text{ext,max}} (1- R_{\text{nom}})$, where $R_{\text{nom}}$ is the applied stress ratio. Hence, plotting the $da/dN$ vs $\Delta K$ curve from the Zencrack analyses will give an indication of how well the analyses compare with the test data.

The paper does not explicitly state the value of $S_{\text{max}}$ used for the tests, but from a closer examination of plots it became clear that it was not 150MPa as assumed, but actually varied with $R$. For $R = 0.1$, 0.2 and 0.4 the nominal $\Delta K$ values at the start of the tests were read from the figure to be 18.4, 17.9 and 14.1 respectively. Since the initial crack length was 10mm in all these cases, the use of the standard equation for $K$ for a SEN specimen gave the corresponding $S_{\text{max}}$ as 91.8, 100.5 and 105.5MPa. Accordingly, these three cases were rerun with the updated $S_{\text{max}}$, and the results are given in Table 9-18. The table also gives the results for one analysis carried out allowing contact at the crack face (Ser. No. 1-Res+contact).

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>$R$</th>
<th>$S_{\text{max}}$ (MPa)</th>
<th>Zencrack 7.1 No. of FEA</th>
<th>Zencrack 7.1 $N_f$ (cycles)</th>
<th>Zencrack 7.1 $a_f$ (mm)</th>
<th>Afgrow $N_f$ (cycles)</th>
<th>Afgrow $a_f$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Res</td>
<td>0.10</td>
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<td>36</td>
<td>1440844</td>
<td>35.17</td>
<td>1480079</td>
<td>35.00</td>
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<tr>
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<td>916025</td>
<td>35.00</td>
</tr>
<tr>
<td>3-Res</td>
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<td>105.5</td>
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<td>946043</td>
<td>35.56</td>
<td>960928</td>
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</tr>
<tr>
<td>1-Res+contact</td>
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<td>57</td>
<td>4632381</td>
<td>35.17</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 9-18 Results for $a_0 = 10.0\text{mm \& } S_{\text{max \ varying with R \& \ with res. stress.}}$ (Model SENT1_sxxx_rxz_a1000_KresKang)*

The crack growth curves for the three analyses without contact are shown in Figure 9-94, Figure 9-96 and Figure 9-98. In Figure 9-95, Figure 9-97 and Figure 9-99 the crack growth rates as computed by Zencrack are plotted against nominal $\Delta K$ and are compared with the test results. The figures also show predictions based on a MathCAD solution, which utilized the standard equations for $K_{\text{ext}}$ and $K_{\text{res}}$ for SEN specimens to calculate the combinations of $\Delta K$ and $K_{\text{eff}}$ for a range of crack lengths and then the corresponding $da/dN$ by interpolation from the relevant Paris equations. As expected, these results are in good agreement with the Zencrack solutions and are different from
the baseline curve. Apart from deviations at lower $\Delta K$ values, and taking into account the approximations in the materials data used in the analyses, the agreement between the Zencheck curves and the test data is generally good.

9.5.4.3 Analysis with crack face contact

Finally, we present the results of the analysis with contact for the $R = 0.1$ case. Due to the use of contact, the analysis must be carried out as a Zencheck “Method 2” combination (see section 3.1). In this method, the two steps in the finite element analysis are the total minimum and maximum conditions i.e. residual stress plus minimum cyclic load and residual stress plus maximum cyclic load. Other parameters are left the same as for the analysis without contact. Since the stress ratio must always be zero or positive, the crack growth data for negative $R$ values is not used.

The analysis was first carried out using the default option for the DIRECTION parameter on keyword GROWTH CONTROLS. This made the crack grow in the direction of the maximum energy release rate at the peak of the loading cycle. However, possibly because of small amounts of noise in the finite element solution as a result of introducing contact, the crack grew very slightly out-of-plane. The initial crack front coordinates are exactly 100.0 in the $y$ direction. At the end of the analysis they are 100.007. The final crack length was 35.1684mm after 4621034 cycles and 96 finite element analyses.

Since the crack is expected to grow in a planar fashion for this analysis, a further run was carried out in which the crack was forced to remain in the original crack plane. This was done by setting the parameter DIRECTION=INITIAL for the GROWTH CONTROLS keyword. The analysis then completes at a crack length of 35.1655mm in 4632381 cycles and 57 finite element analyses. The result is essentially the same as the first analysis but is achieved more efficiently because the small out-of-plane noise is not present and the steps between finite element analyses can be larger without exceeding the error tolerance check that controls the step sizes. Further comments are based on the second of these analyses.

Plots of $(K_{res}+K_{ext\min})$ and $(K_{res}+K_{ext\max})$ against crack length are shown in Figure 9-100 and Figure 9-101. The effective stress ratio is plotted in Figure 9-102. These figures include lines for the equivalent quantities from the method 1 analysis without contact. For the maximum load condition the $K$ plots are the same. This is because the crack is always fully open under the combined effect of
the residual stress and maximum external load - the inclusion of contact has no effect. At the minimum load plus residual stress there is a dramatic difference in the curves, as would be expected. The first 32 of the 56 completed FEAs with contact indicated a fully closed crack face under the minimum external and residual stress load. Up to this point, where the crack length is 24.87mm, the combined \((\text{Kres+Kext}_{\text{min}})\) values are zero (displacement based values are exactly zero as they are derived from displacements of zero, energy values are numerically close to zero). The crack then begins to open near the crack tip, with closure maintained further along the crack faces. As the crack grows, so the open amount of crack face increases and the \(K\) value increases above zero. The opening effect can be seen in the displaced mesh plots shown in Figure 9-103 and Figure 9-104 for FEA number 33 and 56 respectively.

Figure 9-105 shows the crack growth curve for the contact run along with those presented before with residual stress but no contact and without residual stress. For the case without residual stress the use of the contact option has no effect as with \(R_{\text{nom}} = 0.1\) the crack faces remain open at the minimum external load. Due to the incorporation of contact there is more than a threefold increase in the fatigue life of the specimen. This is in addition to an almost tenfold increase due to the residual stress itself. For the contact analysis it is not possible to extract the value of nominal \(\Delta K\). However, by taking the values of \(K_{\text{max}}\) for the external load from the analysis without contact and interpolating to the crack lengths for the contact run, the variation of nominal \(\Delta K\) with crack length can be found. This is shown in Figure 9-106. The line with contact falls below the test data. As a final plot from the contact analysis, the values of \(da/dn\) are plotted against actual \(\Delta K\) in Figure 9-107. Since the analysis with contact has \(R\) values of zero to about 0.4, as shown in Figure 9-102, it is expected that the curve should match the baseline curves. This is the case.
Figure 9-94 - Crack Length vs Number of Cycles (Model SENT1_s91.8_r10_a1000_KresKang)

Figure 9-95 - Growth rate vs nominal $\Delta K$ (Model SENT1_s91.8_r10_a1000_KresKang)
Figure 9-96 - Crack length vs number of cycles (Model SENT1_s100.5_r20_a1000_KresKang)

Figure 9-97 - Growth rate vs nominal $\Delta K$ (Model SENT1_s100.5_r20_a1000_KresKang)
\textbf{Figure 9-98 - Crack length vs number of cycles (Model SENT1_s105.5_r40_a1000_KresKang)}

\textbf{Figure 9-99 - Growth rate vs nominal $\Delta K$ (Model SENT1_s105.5_r40_a1000_KresKang)}
Figure 9-100 - \((K_{res}+K_{ext,min})\) vs crack length with and without contact

Figure 9-101 - \((K_{res}+K_{ext,max})\) vs crack length with and without contact
Reff vs. Crack Length, Smax=91.8MPa, 30% of 8A residual stress

Figure 9-102 - Reff vs crack length with and without contact
Figure 9-103 - Deflected plot (x5000) under minimum external load + residual stress at the 33rd analysed crack position

FEA=33, Crack Length= 25.1076mm, with contact, Smax=91.8MPa, R=0.1, 30% of 8A residual stress
Figure 9-104 - Deflections (x150) normal to crack face under minimum external load + residual stress at the 56th analysed crack position

FEA=56, Crack Length=34.7439mm, with contact, Smax=91.8MPa, R=0.1, 30% of 8A residual stress
Effect of including 30% 8A residual stress and contact in the analysis

Figure 9-105 - Effect of residual stress and contact on crack length vs number of cycles
(Model SENT1_s91.8_r10_a1000_noKres/KresKang, with & without contact)

Figure 9-106 - Growth rate vs nominal $\Delta K$, including contact analysis
Figure 9-107 - Fatigue crack growth rates as a function of actual $\Delta K$
10 Validation - Ansys Interface

Four test cases where considered in an Interim report to compare Ansys results with Abaqus and MSC.Marc results. The Ansys analyses used the PREDICTION=DISPLACEMENT option on the*ENERGY RELEASE RATE Zencrack keyword data.

10.1 Test case 1 - Double edge notched specimen with multiple load steps

This model was a quarter symmetry FEM with a constant pressure loading on the crack faces to simulate compressive residual stress together with a far-field tensile stress.

10.2 Test case 2 - CCT specimen using a large crack-block

This FEM was a quarter symmetry model using an ST28x1 large crack-block. Constant pressure on the crack faces and remote tensile loading were applied together in a single Ansys load step.

10.3 Test case 3 - Planar crack growth prediction

This test case was an 1/8th symmetry model of an initially elliptic embedded crack.

10.4 Test case 4 - Non-planar crack growth prediction

This test case modeled an initially elliptic corner crack in a rectangular bar subject to far-field tension and torsion.

10.5 Summary of Ansys test case results compared with Abaqus

Tests 1 and 2 showed only 0.625% maximum difference of stress intensity factors between Ansys and Abaqus and the results for Test Case 3 displacement based solutions were in close agreement. Some divergence in the Test 4 crack growth profiles between Abaqus and Ansys were observed after approximately 59% of the crack growth. This may be due to the severe limitations on the number of elements and degrees of freedom allowed in Ansys/Ed.
11 Discussion And Conclusions

A number of important features have been incorporated in Zencrack Version 7.1 under the SBIR Phase II contract. The following are major achievements in the project:

- improved generalized modeling of non-planar 3D crack growth
- incorporation of residual stress distributions
- combination of static load systems (e.g. residual stresses) with external cyclic stress using the LEFM principal of superposition
- user defined crack front shapes which can also address breakthrough of corner-type cracks to through crack fronts
- enhanced crack growth integration scheme with error control
- development of transition elements for large crack-blocks to correctly model stress gradients at the interfaces with surrounding elements
- boundary shifting routines to resize crack-blocks and minimize element distortion
- automatic “flipping” of through crack-blocks during crack growth
- a mesh relaxation algorithm to reduce distortion of elements surrounding crack-blocks
- implementation of spectrum loading
- implementation of time dependent crack growth (and combined time / fatigue growth)
- basic and generalized Willenborg retardation models (with or without Chang acceleration due to under-loads)
- fatigue crack growth data specified as a function of both stress ratio and temperature
- time dependent crack growth data specified as a function of temperature
- user subroutines for crack growth data and threshold
- a pre-processor utility to generate tabular data from TANH crack growth equations
- incorporation of crack-blocks in a tetrahedral finite element mesh
- 8-noded solid element crack-blocks
- crack growth in a preferred direction for orthotropic materials such as single crystals
- rainflow counting software of raw load spectrum data
- CTOD method to calculate stress intensity factors from displacements in LEFM
- modeling effects on crack growth of changing state of stress along crack fronts
- development of a Zencrack interface to Ansys
- development of a GUI pre-processor for Microsoft Windows XP or Windows 2000
• post-processing utilities to plot crack growth versus cycles and crack growth profiles

An important development to address the ERLE (Engine Rotor Life Extension) Program has been incorporated in Zencrack to analyze crack growth through non-uniform residual stress fields. The implementation has been achieved for the Abaqus FEA interface using the Abaqus user subroutine DLOAD. The interface to Ansys is implemented using the Zencrack user subroutine user_cbpressure.f.

Significant advances have been made to automatically analyse large crack growth for materials crack growth data dependent upon stress ratio, temperature and time. A variety of crack growth data formats have been implemented from a simple Paris equation to tabular data input and user equations via a special Zencrack user subroutine. This latter method ensures the security of client materials data.

“Large” crack-blocks have been developed to cater for more significant crack growth in a single Zencrack analysis with the capability to generate transition elements in Abaqus to ensure correct tying constraints between the crack-blocks and surrounding mesh as the crack front advances. Boundary shifting and crack-block flipping has also been implemented for extending the growth of normal crack-blocks. Furthermore, a relaxation algorithm has been developed to improve the condition of the elements surrounding the crack-blocks.

Non-planar crack-growth can be handled for any crack-block in a single Zencrack analysis including mapping of the crack plane history.

A comprehensive integration scheme has been developed to handle cycle-by-cycle integration during generalized Willenborg retardation with Chang acceleration due to compressive overloads and benchmarked against Afgrow software. During retardation intervals an iterative procedure has been developed to calculate the state of stress at each node along the crack front and the yield zone size or to use a user defined state of stress. The integration scheme has an error control scheme to obtain more accurate results for a reasonable number of FEA.

It is now possible to conduct a crack growth analysis from, say, a semi-elliptic or complicated crack front profile starter crack in a bore hole. Providing the crack is planar a restart can be performed for
a “through thickness” crack assuming the crack front breaks around a corner. The new through-crack profile is fitted to a cubic spline to user defined crack front nodal positions.
12 References

Ref. 1 Zencrack version 6.0, Zentech International Limited.
Ref. 2 Afgrow up to version 4.00008.12.11, 6/13/03, AFRL, WPAFB.
Ref. 3 Zencrack Version 7.1, Zentech International Limited.
Ref. 4 Zencrack Version 7.1, Examples manual.
Ref. 10 Abaqus is a trademark of Hibbitt, Karlsson & Sorensen Inc., Pawtucket, Rhode Island, U.S.A.
Ref. 11 MSC.Marc is a trademark of MSC.Software, California, U.S.A.
Ref. 12 Ansys/Ed version 6.1 is a trademark of Ansys Corporation, USA

Ref. 18  “FOD resistance and fatigue crack arrest in low plasticity burnished IN718”, P.S. Prevey et al, 5th Nat. Turbine Engine High Cycle Fatigue Conf, 2000

13 Appendix

This Appendix contains Ref. 8. The reference contains 18 pages.
UK Abaqus User Group Conference 2002

USING ABAQUS TO ANALYSE FATIGUE CRACK GROWTH UNDER THE COMBINED INFLUENCE OF RESIDUAL STRESS AND CYCLIC EXTERNAL LOAD

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Abstract

Many analysts are keen to investigate cracks in components under static or cyclic external loading. In the latter case fatigue crack growth prediction is also of importance. It is known that surface treatment effects such as shot peening have a beneficial effect on component life due to the compressive residual stresses introduced in the vicinity of the surface. Such treatments are used in maintenance programmes to extend the life of components in service. The effect on crack growth rate can be dramatic with significant changes in the crack growth profiles and increase in fatigue life to failure. The approach discussed in this paper applies fracture mechanics techniques to establish crack growth rates based on detectable defect sizes above the crack initiation stage established by non-destructive inspection. Thereby, extended fatigue life can be predicted, extended inspection periods calculated and the retirement of some components may be avoided.

This paper demonstrates how the Abaqus DLOAD user subroutine can be used to include the effect of residual stresses from shot peening in the analysis of crack growth in 3D finite element models. The inclusion of these effects may be so beneficial that a crack will change from a “growing” state without residual stress to a “non-growing” i.e. below threshold state, if residual stress is included. Initially the effect of residual stresses are presented using a linear fracture mechanics approach for a single edge notched specimen and a corner crack specimen. Subsequently, the non-linear effects of contact at the crack surfaces are modelled and the effects on the cyclic energy release rates are presented.

Examples are presented based on typical data for a titanium alloy.
Modelling the residual stress field

A typical residual stress distribution normal to the surface resulting from surface shot peening is shown in Figure 1. It is noted that the stress distribution is self-equilibrating in both force and moment so that the stresses remote from the surface must eventually become compressive to balance the near-surface compressive stress field.

![Titanium alloy shot peening](image)

1. Residual stress distributions due to shot peening

The residual stress field is in reality applied all the way along the peened surface. However the principle of superposition can be used to apply the residual stresses directly to the surfaces of a contained crack using the Abaqus DLOAD user subroutine. The principle of superposition was verified using an Abaqus half symmetry 2-D finite element model of a single edged notched (SEN) specimen with a crack depth of 0.11 mm. The true uni-directional stress distribution was introduced as a subsurface nodal temperature distribution using *EXPANSION, TYPE=ORTHO and specifying a coefficient of expansion equal to the reciprocal of the Young’s Modulus and setting Poisson Ratio to zero. The resulting pressure loads are shown in Figure 2 where each data point represents an integration point in the crack plane. In the legend, “var2” refers to thermal loading, “FE” refers to loading on the crack faces and the rest is the raw residual stress data. The average of the J-Integral second and third contour values for each loading method is shown in Table 1. Good agreement between the results verifies the principle of superposition.
2. Verification of the principle of superposition

<table>
<thead>
<tr>
<th>Shot size</th>
<th>Energy release rates for directly applied residual stress (MPa√mm)</th>
<th>Energy release rates for residual stress applied to crack face (MPa√mm)</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average of Contours 2 &amp; 3</td>
<td>Average of Contours 2 &amp; 3</td>
<td></td>
</tr>
<tr>
<td>4A</td>
<td>-2.1295</td>
<td>-2.1355</td>
<td>+0.281</td>
</tr>
<tr>
<td>8A</td>
<td>-3.638</td>
<td>-3.638</td>
<td>0.0</td>
</tr>
<tr>
<td>12A</td>
<td>-4.026</td>
<td>-4.018</td>
<td>-0.199</td>
</tr>
<tr>
<td>16A</td>
<td>-4.4085</td>
<td>-4.4045</td>
<td>-0.091</td>
</tr>
</tbody>
</table>

1. Comparison of averaged energy release rates to verify principle of superposition

Abaqus DLOAD user subroutine

The SEN crack is position along the x-axis with the x origin at the surface. The program uses a simple linear interpolation scheme between integration points. The subroutine uses a scale factor to examine the effects of stress relaxation on the initial residual stress field.
SUBROUTINE DLOAD

C
C    Shot peen residual stress distributions for a Titanium alloy
C
SUBROUTINE DLOAD(P,KSTEP,KINC,TIME,NOEL,NPT,LAYER,KSPT,COORDS,
1             JLTYP,SNAME)

C
C----------------------------------------------------------------------
C
C subroutine zinterp(c,x,sigma,p)

C
C----------------------------------------------------------------------
C
C
DIMENSION TIME(2),COORDS(3)
CHARACTER*80 SNAME
C
C
C     shot peen data
dimension sigma(19),x1(19),x2(19),x3(19),x4(19)
c
C     for all shot sizes
data sigma/-306.0,-450.7,-545.3,-598.4,-612.0,-611.4,
\ & -586.3,-546.9,-498.5,-445.0,-389.6,-282.2,
\ & -188.9,-114.9,-55.1,0.0,61.2,0.0,-61.4/
c
C     for 4A shot
data x1 /0.0,0.0063,0.0126,0.0189,0.0252,0.036225,
\ & 0.04347,0.050715,0.05796,0.065205,0.07245,0.08694,0.10143,
\ & 0.11592,0.13041,0.1449,0.25,1.811,100.0/
c
C     for 8A shot
data x2/0.0,0.009,0.018,0.027,0.036,0.05175,0.0621,0.07245,
\ & 0.0828,0.09315,0.1035,0.1242,0.1449,0.1656,0.1863,0.207,
\ & 0.3,2.597,100.0/
c
C     for 12A shot
data x3/0.0,0.01,0.02,0.03,0.04,0.0575,0.069,0.0805,0.092,
\ & 0.1035,0.115,0.138,0.161,0.184,0.207,0.23,0.345,2.89,
\ & 100.0/
c
C     for 16A shot
data x4/0.0,0.0115,0.023,0.0345,0.046,0.066125,0.07935,0.092575,
\ & 0.1058,0.11902,0.13225,0.1587,0.18515,0.2116,0.23805,
\ & 0.2645,0.345,3.3302,100.0/
data scale/1.0/
C
C
lunrep = 6
lunmsg = 7
C
select case (kstep)
case(1)
  call zinterp(COORDS(1),x1,sigma,p)
case(2)
  call zinterp(COORDS(1),x2,sigma,p)
case(3)
  call zinterp(COORDS(1),x3,sigma,p)
case(4)
  call zinterp(COORDS(1),x4,sigma,p)
case default
  write(lunmsg,*)'unsupported ABAQUS step'
end select

p = scale*p
write(lunmsg,10) kstep,coords(1),p,scale,noel,npt,jltyp
10 format('kstep,x,p,scale,element,jltyp=',i3,3e12.4,i8,2i3)
RETURN
END
C
C---------------------------------------------------------------
C
C subroutine zinterp(c,x,sigma,p)

C
C---------------------------------------------------------------
C
C
dimension sigma(19),x(19)
C
lunrep = 6
lunmsg = 7

c
    if((c.lt.x(1)).or.(c.gt.x(19))) then
        write(lunrep,10) c
        write(lunmsg,10) c
    10 format('coordinate ',e12.4,
            & ' is out of residual stress distribution range')
        stop
    endif
c
    do i= 2,19
        if(c.lt.x(i)) then
            p = sigma(i-1)+(c-x(i-1))*(sigma(i)-sigma(i-1))/(x(i)-x(i-1))
            return
        endif
    end do
    p = x(19)
    return
end

Sign of the energy release rate

Abaqus calculates a positive value of the J-Integral regardless of the sign of the applied pressure distribution on the crack faces.

In order to resolve the sign of the J-Integral is is necessary to determine an “open” or “closed” status for each corner node on the crack front by extracting and processing nodal displacements. Each crack front node and the associated crack face quarter point nodes in the collapsed crack front elements are examined. A local coordinate system is created to allow calculation of relative opening displacements of pairs of nodes on either side of the crack face in local mode I, II and III orientations. An example is shown in Figure 3. The local mode I opening displacement, if positive, indicates an open crack at that point and a positive J-Integral. If the displacement is negative the crack is closed indicating a negative J-Integral.

3. Local opening directions at a crack front node
Fatigue crack growth

Crack growth data

The crack growth data used in the 3-D FEA fatigue analyses was specified by the “TANH” equation commonly used for Titanium alloys which has the form:

\[
\log\left(\frac{da}{dn}\right) = C_i \left(\arctan h\left(C_3 \left[\log(\Delta K) + C_4]\right)\right) + C_4
\]

in which:

\[
C_i = A_{0i} + A_{1i} \times \log(1 - R) + A_{2i} \times [\log(1 - R)]^2, \quad i = 1, 2, ..., 4
\]

da/dn is the crack growth rate, R is the stress ratio and “A” values are material constants. The crack growth curves are plotted in Figure 4.

4. Crack growth data for 3-D FEA fatigue analyses

Crack growth integration schemes

The fatigue loading comprises of the superposition of the static residual stresses and external cyclic loading. The two systems are combined to give an effective stress ratio, \(R_{\text{effective}}\), that is a function of crack size, \(a\).
The integration scheme uses a numerical forward predictor method in which it is assumed that \( \frac{dG}{da} \) is constant over each integration step (i to f) where G is the energy release rate:

\[
\frac{da}{dn} = f(\Delta \sqrt{G})
\]

allowing a general integral to be written:

\[
\int_{i}^{f} dn = \int_{i}^{f} \frac{1}{f(\Delta \sqrt{G})} da
\]

where \( G_{\text{max}} = G_{\text{max} i} + \left( \frac{dG_{\text{max}}}{da} \right) da_{i} \)

and \( \Delta \sqrt{G} = \left( G_{\text{max}}^{\frac{1}{2}} - G_{\text{min}}^{\frac{1}{2}} \right) = \Delta K \left( \frac{1 - (\alpha v)^2}{E} \right) \)

\( \Delta K = \text{stress intensity factor range, } E = \text{Youngs Modulus, } v = \text{Poisson Ratio}, \)

\( \alpha \text{ ranges from 0 for plane stress to 1 for plane strain} \)

General response with residual stress and external cyclic loading

The typical stress intensity factor response at a crack front node in the presence of residual stresses is strongly influenced by contact (or partial contact) at the crack faces at lower external load levels and stress ratios as shown in Figure 5. In the figure:

- Point A represents the application of (compressive) residual stress in the absence of external loading.
- Line AB is the response to increasing external loading in the presence of the residual stress.
- Line FD is the response to external loading alone.
- Line CEB is a typical response to combined residual stress and external loading when contact at the crack faces is modelled in the FEA. This represents the “true” solution.
- Point E is located at the position when the external load is just sufficient to cause ALL the nodes on the crack faces to just become fully open.
5. Crack growth prediction methods in the presence of residual stress

Two methods were used to analyse the 3-D finite element models (FEA) in the presence of static load and cyclic external load.

Method 1

Method 1 is an LEFM approach and requires one linear FEA with two load steps. No crack face contact conditions were used in this method. The first Abaqus load step is with static load only (e.g. residual stress applied to the crack faces through the user subroutine DLOAD). This provides a solution for point A in Figure 5. The second load step is with the maximum value of the cyclic external load only. This provides a solution for point D. The external load results can be scaled and combined with the static load result to give a solution at any desired external load level. This is the required Method 1 response line AB. This method can be used with constant amplitude or spectrum external loading. It is noted that Method 1 causes a reduction in R but no reduction in $\Delta K$.

Method 2

Method 2 allows crack face contact to be incorporated in an analysis with combined static and cyclic loading. For crack growth prediction the method is currently limited to constant amplitude external loading in which results are generated at the minimum and maximum external loads in the cycle. For crack growth with spectrum loading it would be necessary to generate and use the complete non-linear curve of type CEB. However, it is possible to use Method 2 to generate the full curve CEB for each crack position.
In Method 2 the first Abaqus load step is with static load and minimum external load applied together. This minimum external load may be non-zero and provides a solution for the point of minimum external load in the load cycle. The second load step is with static load and maximum external load applied together and provides a solution for the point of maximum external load in the load cycle. It is noted that if the second load step is broken into a number of distinct points, the full curve (CEB) between minimum and maximum external load can be generated and the technique extended to analyze variable amplitude fatigue loading.

Without crack face contact this method produces a segment of the response line AB as generated from Method 1. If, however, crack face contact is included a non-linear “true solution” is obtained i.e. points between C and B. The stress intensity range for crack growth calculations can then be taken as the range between the two points.

It is important to note that the crack growth data in Figure 4 cannot be used directly in Method 2 with contact. Such experimental crack growth is usually constructed using standard LEFM 2-D stress intensity factors solutions for standard test specimens and $\Delta K$ calculated using the linear relationship $\Delta K = K_{\text{max}}(1-R)$ as in Method 1. Also, in the presence of contact, the response depends upon the external load level below point E in Figure 5.

These considerations are the subject of further research and all example fatigue crack growth analyses are restricted to the Method 1 LEFM approach.

Example 3-D finite element analyses

Three-dimension FEA have been conducted using the fracture mechanics Zencrack software (1) which is interfaced to Abaqus to analyses 3-D crack growth under generalized fatigue spectrum loading. With minimal user data, Zencrack can introduce crack front defects and advance the crack fronts under fatigue loading to user defined crack growth data. In these examples the loading was restricted to constant amplitude.

Single Edge notched specimen

The SEN specimen was used to generate the response curves typified in Figure 5 for a range of static external loads in the presence of residual stress with and without contact at the crack faces. The mesh density of the Zencrack “crack blocks” and was verified such that the DLOAD user subroutine generated a crack face pressure distribution that matched the specified residual stress distribution. From these results Methods 1 and 2 were developed and the mesh density of the Zencrack “crack blocks” were found to be adequate.

An important aspect of closure at the crack faces is the progressive opening of the crack faces leading to the curvilinear curve CE in Figure 5. The deformed shapes (with amplification x40) under increasing external loading of the SEN specimen are shown in the Figures 6 to 9.
6. Crack face displacement at 562.5 MPa far-field stress

7. Crack face displacement at 862.5 MPa far-field stress
8. Crack face displacement at 1012.5 MPa far-field stress

9. Crack face displacement at 1162.5 MPa far-field stress
Fatigue analysis of a semi-circular surface crack

The finite element model (FEM) is a quarter-symmetry model of a tensile specimen with a semi-circular surface crack.

Dimensions: x-section thickness = 5 mm, width = 10 mm, half length = 20 mm
Initial crack size = 0.12 mm

Boundary conditions: constant amplitude 2-direction remote stress of 621 MPa (free to rotate)
Stress ratio = 0.1
¼ symmetry FEM

10. Initial geometry of a FEM including a semi-elliptic surface crack

Zencrack has inserted a crack block into a regular mesh of the intact component. Notice the automatic shrinking of the crack block to maintain good aspect ratios in the crack block elements. Details of the crack block elements are shown in Figure 11.
11. Details of the semi-elliptic FEM crack block

Crack growth without residual stresses

The crack growth profiles are shown in Figures 12 and 13 where 11035 cycles to failure were obtained. Please note that the depth is in 1-direction in the Figures.

12. Crack growth profile with no residual stresses
13. Crack growth with no residual stresses

Crack growth with residual stresses

The finite element analyses applies 50% of the residual stress distribution shown in Figure 1 to investigate the effect of stress relaxation during early service after component refurbishment using shot peening.

Two analyses were conducted using different tolerances (TOLA) of incremental crack growth between automatic re-meshing and finite element analyses. The tolerances were set at 0.002 mm and 0.005 mm to test convergence of the solutions. The results are shown in Figures 14 to 16.

Table 2 shows the number of cycles to failure are significantly greater than those obtained without residual stress and clearly demonstrate the benefits of shot peening surface treatment.
14. Crack growth profile – forward predictor, TOLA=0.005 mm

15. Crack growth profile – forward predictor, TOLA=0.002 mm
16. Zencrack LEFM analysis using forward predictor integration with residual stresses

<table>
<thead>
<tr>
<th>Cycles to failure</th>
<th>Number of FEA</th>
<th>TOLA (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>485678</td>
<td>63</td>
<td>0.002</td>
</tr>
<tr>
<td>466231</td>
<td>32</td>
<td>0.005</td>
</tr>
</tbody>
</table>

2. Forward predictor life prediction with residual stresses

**Suggestions for new Abaqus capabilities for future development**

**Oblique cracks and cracks which grow non-planar**

The presented example analyses all deal with crack surfaces normal to the shot peened surface. If an initial oblique crack or crack growth becomes non-planar to the surface shear load components will be induced at the crack faces and will be resisted by friction forces on contacting areas of the crack surfaces.

It would be useful to pass the surface normal vectors at the integration points into the DLOAD user subroutine and to be able to specify surface shear tractions on element faces leading to mixed mode loading.
Accuracy of J-Integrals at near zero energy

It has been observed in developing the methods to superimpose residual stress and cyclic external load that Abaqus results are somewhat anomalous near zero strain energy. In the two point method the residual stress is applied in the first step and the “full” external load applied in a single step. In the detailed method the external cyclic loading is applied in increments (see Figure 17). This matter is being investigated by HKS.

17. Results for residual load magnitude varying with distance from crack front

Conclusions

The Abaqus DLOAD user subroutine provides sufficient functionality to model fatigue crack growth normal to the surface in the presence of residual stress distributions.

Such analyses are important in life extension programs to avoid early retirement of service components by surface treatments such as shot peening. Also inspection periods after refurbishment can be extended.

The presented fatigue growth analyses were limited to a conservative LEFM approach under constant amplitude loading which did not consider crack closure effects but provided a dramatic increase in service life.

Further research is being conducted to include crack closure to provide even more detailed enhancement to component fatigue life. This approach will require re-formulation of experimental fatigue crack growth data and introduction of the influence of external cyclic load magnitudes due to the non-linear response with crack closure. Current Zencrack capabilities can be used to conduct this re-formulation.
Suggestions have been proposed to extend the analysis capabilities in the DLOAD routine to analyse cracks which are not normal to the surface.

References

2. Further background references can be found on the web site http://www.zentech.co.uk/tech_pap.htm