NEW LIMITATION CHANGE

TO
Approved for public release, distribution unlimited

FROM
Distribution authorized to U.S. Gov’t. agencies only; Test and Evaluation; Dec 1979. Other requests shall be referred to U.S. Army Armament Research and Development Weapons Systems Lab., Dover, NJ 07801.

AUTHORITY
USARDC ltr, 10 Apr 1984
TECHNICAL REPORT ARLCD-TR-79030

FUZE GEAR-TRAIN ANALYSIS

G. G. LOWEN
CITY COLLEGE OF NEW YORK

F. R. TEPPER
ARRADCOM

DECEMBER 1979

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER
WEAPON SYSTEMS LABORATORY
DOVER, NEW JERSEY

Distribution limited to US Government agencies only because of test and evaluation, December 1979. Other requests for this document must be referred to Commander, ARRADCOM, ATTN: DRDAR-TSS, Dover, New Jersey 07801.
DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DDC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.
This report documents the development of the tools needed to compare the efficiency of fuze-related gear trains designed to operate in a spin environment. Computer models have been developed for two and three pass step-up designs with clock (ogival) and involute tooth shapes. Using appropriate moment input-output relationships, the computer programs develop point and cycle efficiency for each type of gear train. Pivot friction partially caused by centrifugal force on the gear and pinion combinations during spin, and
20. Abstract (continued)

- tooth-to-tooth contact friction are considered. All models allow a variety of parameter variations.
TABLE OF CONTENTS

| Introduction | 1 |
| Point Efficiency and Cycle Efficiency | 2 |
| Description of Study | 3 |

Appendixes

| A  | Step-up Gear Trains with Involute Teeth | A-1 |
| B  | Design of Unequal Addendum Involute Gear Sets with Standard Center Distances | B-1 |
| C  | Computer Models for Step-up Gear Trains with Involute Teeth | C-1 |
| D  | Geometry of General Clock Gear Tooth | D-1 |
| E  | Kinematics and Moment Input-Output Relationship for Single Step-up Gear Mesh with Clock Teeth | E-1 |
| F  | Computer Models for Single Step-up Gear Mesh with Clock Teeth | F-1 |
| G  | Kinematics of Two and Three Step-up Gear Trains with Clock Teeth | G-1 |
| H  | Moment Input-Output Relationships for Two and Three Step-up Gear Trains with Teeth Operating in a Spin Environment | H-1 |
| I  | Computer Models for Two and Three Step-up Gear Trains with Clock Teeth Operating in a Spin Environment | I-1 |

Distribution List
<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-1</td>
<td>Possible Combinations of Phases for Three Pass Step-up Gear Train as Shown in Figure G-1</td>
<td>H-2</td>
</tr>
<tr>
<td>H-2</td>
<td>Possible Combinations of Phases for Two Pass Step-up Gear Train as Shown in Figure A-10</td>
<td>H-3</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>A-1</td>
<td>Determination of Direction of Contact Friction Forces by Velocity Analysis</td>
<td>A-2</td>
</tr>
<tr>
<td>A-3</td>
<td>Moments due to Friction Components Always Oppose Motion</td>
<td>A-7</td>
</tr>
<tr>
<td>A-4</td>
<td>Free Body Diagram for Single Step-Up Involute Gear Mesh</td>
<td>A-10</td>
</tr>
<tr>
<td>A-5</td>
<td>Basic Configuration for Involute Three Step-Up Gear Train in Spin Environment</td>
<td>A-19</td>
</tr>
<tr>
<td>A-6</td>
<td>Free Body Diagram of Pinion 4</td>
<td>A-21</td>
</tr>
<tr>
<td>A-7</td>
<td>Equilibrium of Gear and Pinion Set No.3</td>
<td>A-28</td>
</tr>
<tr>
<td>A-8</td>
<td>Free Body Diagram of Gear and Pinion Set No.2</td>
<td>A-35</td>
</tr>
<tr>
<td>A-9</td>
<td>Free Body Diagram of Gear No.1</td>
<td>A-42</td>
</tr>
<tr>
<td>A-10</td>
<td>Basic Configuration for Involute Two Step-Up Gear Train in Spin Environment</td>
<td>A-48</td>
</tr>
<tr>
<td>A-11</td>
<td>Free Body Diagram of Pinion No.3</td>
<td>A-49</td>
</tr>
<tr>
<td>A-12</td>
<td>Free Body Diagram of Gear and Pinion Set No.2</td>
<td>A-56</td>
</tr>
<tr>
<td>A-13</td>
<td>Free Body Diagram of Gear No.1</td>
<td>A-64</td>
</tr>
<tr>
<td>A-14</td>
<td>Pivot Hole Relationships</td>
<td>A-70</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>A-15</td>
<td>Involute Mesh Geometry</td>
<td>A-75</td>
</tr>
<tr>
<td>B-1</td>
<td>Relationship between Pinion Pitch Radius $r_p$, Rack Cutter Addendum A and Resulting Pinion Root Radius $r_r$</td>
<td>B-4</td>
</tr>
<tr>
<td>B-2</td>
<td>Minimum Root Radius $r_{rm}$ for Rack Cutter with Sharp Corner</td>
<td>B-5</td>
</tr>
<tr>
<td>B-3</td>
<td>Rack Cutter with Corner Radius $r_c$ (Effective Addendum of Cutter is Decreased)</td>
<td>B-6</td>
</tr>
<tr>
<td>D-1</td>
<td>Geometry of Ogival Tooth</td>
<td>D-2</td>
</tr>
<tr>
<td>E-1</td>
<td>Round on Round Phase of Contact (Gear Drives Pinion)</td>
<td>E-2</td>
</tr>
<tr>
<td>E-2</td>
<td>Round on Flat Phase of Contact (Gear Drives Pinion)</td>
<td>E-11</td>
</tr>
<tr>
<td>E-3</td>
<td>Sensing Geometry for Contact of Subsequent Tooth Mesh</td>
<td>E-19</td>
</tr>
<tr>
<td>E-4</td>
<td>Flat of Gear Contacts Round of Pinion</td>
<td>E-23</td>
</tr>
<tr>
<td>E-5</td>
<td>Free Body Diagram for Round on Round Phase</td>
<td>E-27</td>
</tr>
<tr>
<td>E-6</td>
<td>Free Body Diagram for Round on Flat Phase</td>
<td>E-34</td>
</tr>
<tr>
<td>G-1</td>
<td>Basic Configuration for Ogival Three Step-Up Gear Train in Spin Environment</td>
<td>G-2</td>
</tr>
<tr>
<td>G-2</td>
<td>Round on Round Phase for Mesh No.1</td>
<td>G-5</td>
</tr>
<tr>
<td>G-3</td>
<td>Round on Flat Phase for Mesh No.1</td>
<td>G-13</td>
</tr>
<tr>
<td>G-4</td>
<td>Round on Round Phase for Mesh No.2</td>
<td>G-24</td>
</tr>
<tr>
<td>G-5</td>
<td>Round on Flat Phase for Mesh No.2</td>
<td>G-31</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>H-1</td>
<td>Free Body Diagram of Pinion No.4</td>
<td>H-7</td>
</tr>
<tr>
<td></td>
<td>Mesh No.3: Round on Round</td>
<td></td>
</tr>
<tr>
<td>H-2</td>
<td>Free Body Diagram of Gear and Pinion No.3</td>
<td>H-12</td>
</tr>
<tr>
<td></td>
<td>Mesh No.3: Round on Round</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Round</td>
<td></td>
</tr>
<tr>
<td>H-3</td>
<td>Free Body Diagram of Gear and Pinion No.2</td>
<td>H-19</td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Round</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.1: Round on Round</td>
<td></td>
</tr>
<tr>
<td>H-4</td>
<td>Free Body Diagram of Gear No.1</td>
<td>H-26</td>
</tr>
<tr>
<td></td>
<td>Mesh No.1: Round on Round</td>
<td></td>
</tr>
<tr>
<td>H-5</td>
<td>Free Body Diagram of Gear and Pinion No.2</td>
<td>H-32</td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Round</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.1: Round on Flat</td>
<td></td>
</tr>
<tr>
<td>H-6</td>
<td>Free Body Diagram of Gear No.1</td>
<td>H-39</td>
</tr>
<tr>
<td></td>
<td>Mesh No.1: Round on Flat</td>
<td></td>
</tr>
<tr>
<td>H-7</td>
<td>Free Body Diagram of Gear and Pinion No.3</td>
<td>H-45</td>
</tr>
<tr>
<td></td>
<td>Mesh No.3: Round on Round</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Flat</td>
<td></td>
</tr>
<tr>
<td>H-8</td>
<td>Free Body Diagram of Gear and Pinion No.2</td>
<td>H-52</td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Flat</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.1: Round on Flat</td>
<td></td>
</tr>
<tr>
<td>H-9</td>
<td>Free Body Diagram of Gear and Pinion No.2</td>
<td>H-60</td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Flat</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.1: Round on Round</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>H-10</td>
<td>Free Body Diagram of Pinion No.4</td>
<td>H-68</td>
</tr>
<tr>
<td></td>
<td>Mesh No.3: Round on Flat</td>
<td></td>
</tr>
<tr>
<td>H-11</td>
<td>Free Body Diagram of Gear and Pinion No.3</td>
<td>H-74</td>
</tr>
<tr>
<td></td>
<td>Mesh No.3: Round on Flat</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round On Flat</td>
<td></td>
</tr>
<tr>
<td>H-12</td>
<td>Free Body Diagram of Gear and Pinion No.3</td>
<td>H-83</td>
</tr>
<tr>
<td></td>
<td>Mesh No.3: Round on Flat</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Round</td>
<td></td>
</tr>
<tr>
<td>H-13</td>
<td>Free Body Diagram of Pinion No.3</td>
<td>H-94</td>
</tr>
<tr>
<td></td>
<td>Mesh No.2: Round on Round</td>
<td></td>
</tr>
<tr>
<td>H-14</td>
<td>Free Body Diagram of Pinion No.3</td>
<td>H-103</td>
</tr>
</tbody>
</table>
This project provides the computer programs needed to compare the efficiency of fuze-related gear trains operating in a spin environment. Specifically, two and three pass stop-up computer models with both involute and ogival (clock) tooth shapes were developed.

By using appropriate moment input-output relationships, the computer programs allow the determination of point and cycle efficiencies. Pivot friction, partly due to the centrifugal forces on the gear and pinion combinations, is considered in addition to tooth-to-tooth contact friction. The models derived allow a wide variety of parameter variations.

The main body of this report consists of nine appendixes, each of which contains a detailed analysis of each combination of tooth forms, the number of passes, and the spin environment. The derivation of moment relationships, pivot friction, gear tooth geometry, and the direct-contact mechanism kinematics are also included. The computer programs used are listed and instructions in their use and in interpreting the results are given.
The point efficiency $\varepsilon_P$ is defined as

$$\varepsilon_P = \frac{M_{oREF}}{M_{in}} \quad (1)$$

where $M_{in}$ represents the instantaneous input moment to the gear train. $M_{oREF}$ stands for the instantaneous equilibrant output moment $M_o$ after it has been referred to the input shaft by way of the instantaneous angular velocity ratio $K_{RATIO}$.

$$K_{RATIO} = \left| \begin{array}{c} \dot{\psi} \\ \dot{\phi} \end{array} \right| \quad (2)$$

In the above,

$\dot{\psi}$ = instantaneous angular velocity of the output gear

$\dot{\phi}$ = instantaneous angular velocity of the input gear

Equation 1 then becomes

$$\varepsilon_P = K_{RATIO} \frac{M_o}{M_{in}} \quad (3)$$

The cycle efficiency $\varepsilon_C$ represents the ratio of the work available at the output shaft to that done by the input moment during one tooth cycle of the input gear. Thus,

$$\varepsilon_C = \frac{\int_{M_o} M_o \, d\psi}{\int_{M_{in}} M_{in} \, d\phi} \quad (4)$$

The quantities $d\psi$ and $d\phi$ represent infinitesimal rotations of the output and input gears, respectively.
Appendix A furnishes the background, as well as the derivations, for the moment input-output expressions for two and three pass step-up gear trains (where in each mesh the gear is the driver) with involute teeth and unity contact ratio.

Section 1 shows the development of a sign convention for the direction of the contact point friction force. It is based on the direction of the relative velocity between the contact points on the gear and pinion teeth. Section 2 discusses how to deal with the normal and friction forces at the gear and pinion pivots of single and multiple mesh trains. Section 3 shows the application of the above results to the moment input-output analysis of a single mesh. The frame is assumed stationary for this case, and the external loads are confined to the driving input moment and the equilibrating output moment.

The basic geometry of the three pass step-up gear train, mounted on a rotating fuze body, is formulated in section 4 for use in the moment input-output analysis. Force and moment equilibria of the individual component gears, which also account for the centrifugal forces, lead to the desired expression. Section 5 includes a similar derivation for a two pass step-up gear train with
involute teeth. In order to be able to continuously compute the moment relationships for these trains, a method for determining the simultaneous locations of the contact points of all the meshes had to be worked out. Such a method is given in section 6 together with certain angular relationships of the pivot locations on the model fuze body.

The kinematic relationships in involute gear trains are relatively simple compared to those in ogival trains because of the constant transmission ratio and the invariant direction of the line-of-action in each individual mesh.

Appendix B

To avoid the severe undercutting of pinions which generally is associated with step-up gear meshes, it is necessary to use non-standard involute gearing. Appendix B shows both the theory and the necessary steps for the design of unequal addendum gears and pinions of unity contact ratio. In addition, a numerical example is given.

Appendix C

This appendix contains four computer programs which make it possible to determine the point and cycle efficiencies of three gear combinations containing unequal addendum involute meshes with
unity contact ratio. In each case, the structure of the program is thoroughly discussed and a sample run is used to interpret the results. The names of these programs and their relationship to work described in the other appendixes are given below:

1. Program INVOL 1: Design of unequal addendum involute gear and pinion set with unity contact ratio.
   This program is based on the work in Appendix B. Five sample computations, which are used in other programs, are shown.

2. Program INVOL 2: Point and cycle efficiencies for single pass involute step-up gear mesh with unity contact ratio.
   This program is based on the work in section 3 of Appendix A.

3. Program INVOL 3: Point and cycle efficiencies for three pass involute step-up gear train in spin environment. (All meshes have unity contact ratio).
   This program is based on the work in sections 4 and 6 of Appendix A.

4. Program INVOL 4: Point and cycle efficiencies for two pass involute step-up gear train in spin environment. (All meshes have unity contact ratio).
   This program is based on the work in sections 5 and 6 of Appendix A.
Appendix D

This appendix describes the geometry of an ogival tooth in which each side of the tooth profile has a circular arc blending tangentially into a radial straight line flank. The basic tooth nomenclature is defined and methods for determining the required tooth parameters are given.

Appendix E

Section 1 of Appendix E gives all necessary kinematic derivations for a single step-up mesh with ogival teeth. The motion of an ogival mesh consists of two phases. On first contact, the circular arc portion of the driving gear tooth makes contact with the circular arc portion of the driven pinion tooth. Later in the cycle, and up to the point of final disengagement, the circular arc portion of the gear tooth contacts the straight line portion of the pinion tooth. These phases of motion were named "round on round" and "round on flat," respectively. Equivalent four-link mechanism models were used for both regimes to obtain expressions for the pinion output angles, for transition angles, for output angular velocities and for contact-point relative velocities. In addition, a sensing expression was developed which allows the computer determination of that position of a given mesh at which the subsequent mesh comes into engagement. (Because of the variable transmission ratio of ogival meshes, there is only one set of teeth in contact at any one time).
Section 2 of this appendix shows derivations of moment input-output expressions for both phases of contact of a single ogival mesh. Again, while pivot friction is considered in addition to contact friction, the frame is assumed to be stationary for this single mesh.

Appendix F

Two computer programs which deal with the kinematics and the moment input-output relationships of a single pass step-up gear mesh with clock teeth are given in this appendix. The structure of each of these programs is again discussed in detail and sample runs are used to explain their input and output parameters. The names of these programs and their relationships to work in other sections are given below:

1. Program CLOCK 1: Kinematics of a single step-up gear mesh with clock teeth.
   
   This program is based on work in Appendix D as well as on work in section 1 of Appendix E. In addition, it has been used to check the geometry of the ogival meshes which were used in programs CLOCK 2, CLOCK 3 and CLOCK 4 (See Appendix I for the latter two).

2. Program CLOCK 2: Point and cycle efficiencies for single pass step-up gear mesh with clock teeth.
   
   This program is based on work in section 2 of Appendix E.
Appendix G

When one considers the kinematic relationships of ogival meshes which are mounted on a fuse body as parts of two or three pass step-up trains, it becomes necessary to account for the relative positions of the individual meshes on the fuse body. Appendix G gives the appropriate derivations for each of the three meshes of a three-pass train. The model of the fuse body is identical with that used for involute step-up trains.

Appendix H

This appendix shows the derivations of moment input-output expressions for two and three pass step-up gear trains with ogival teeth which must operate in a spin environment.

Because of the increase in rotational speed associated with each tooth mesh, increasingly more sets of teeth will come into engagement in the second and third meshes as one set of teeth moves through one complete contact cycle in the first (i.e., the input) mesh. With two phases of motion for each mesh, there will be eight contact combinations in a three-pass train. Section 1 of Appendix H gives a derivation for the moment input-output expression of each of these eight cases.

Section 2 shows similar work for the four contact combinations which are associated with two pass step-up gear trains with ogival teeth.

Both analyses account for the effects of the centrifugal forces.
Appendix I

This appendix contains two computer programs which allow the determination of point and cycle efficiencies for two and three pass step-up gear meshes with clock teeth. As for other programs, the origins of their mathematical formulations are thoroughly discussed. In addition, their input and output parameters are explained with the help of a sample run. The names of these programs as well as their relationships to work in other appendixes are given below:

1. Program CLOCK 3: Point and cycle efficiencies for three pass clock (ogival) step-up gear train in spin environment.
   This program is based on work given in Appendix G as well as in section 1 of Appendix H. The general fuze geometry is that described in section 6 of Appendix A.

2. Program CLOCK 4: Point and cycle efficiencies for two pass clock (ogival) step-up gear train in spin environment.
   The kinematics of this program is again based on work given in Appendix G. The moment input-output relationships are from section 2 of Appendix H.
APPENDIX A

STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

1. DIRECTION OF FRICTION FORCE AT TOOTH-TO-TOOTH CONTACT

Figure A-1 uses the base circle and the line-of-action configuration of an involute mesh, in which the gear drives the pinion, to determine the direction of the friction forces at contact point C before and after the contact point passes through pitch point P. Distance d represents the length of the line-of-action between base circle tangent points L and L'. The distance between contact point C and point L along the line-of-action is measured by length a.

Further:

\[
\begin{align*}
\theta &= \text{actual (rolling) pressure angle} \\
R_b &= \text{gear base radius} \\
r_b &= \text{pinion base radius}
\end{align*}
\]

The friction force of the pinion tooth on the gear tooth, for example, will have the direction of the relative velocity, \( \frac{V_C}{V_p} \), of the contact point on the pinion tooth, \( C_p \), with respect to the coincident contact point on the gear tooth, \( C_g \). (The friction force of the gear tooth on the pinion tooth has the opposite direction.) This relative velocity changes direction at the pitch point, where it becomes instantaneously zero.

A-1
Figure A-1 shows contact before the pitch point (during approach). To obtain the direction of the relative velocity, \( \vec{V}_{C_p}/\vec{C}_g \), by a graphical analysis, one makes use of the velocity equation

\[
\vec{V}_{C_p} = \vec{V}_{C_p}/\vec{C}_g + \vec{V}_{C_g}
\]

where

- \( \vec{V}_{C_p} \) = velocity of point \( C_p \) on pinion tooth with direction normal to line \( O_n - C \)
- \( \vec{V}_{C_p}/\vec{C}_g \) = relative velocity between point \( C_p \) and point \( C_g \). The direction of this velocity is normal to the line of action.
- \( \vec{V}_{C_g} \) = velocity of point \( C_g \) on gear tooth with direction normal to line \( O_N - C \). The magnitude of this velocity was arbitrarily chosen.

The graphical construction, according to Eq. (A1), shows that \( \vec{V}_{C_p}/\vec{C}_g \) has the direction opposite to that of the unit vector \( \vec{N}_g \) shown at point \( O_N \). As stated earlier, this represents
the friction force on the gear tooth during approach.

Figure A-1b shows the same graphical analysis for contact during recess. Once the pitch point is passed, both the relative velocity, $\frac{\bar{V}_{C_d}}{C_g}$, and the friction force of the pinion on the gear have the direction of the positive unit vector $\hat{n}_g$. 
2. **ASSUMPTIONS CONCERNING NORMAL AND FRICTION FORCES AT PIVOTS**

**FIGURE A-2 FREE-BODY DIAGRAMS OF PIVOT SHAFTS**

Figure A-2a shows a pivot shaft which is loaded by a known external force, $W$, and rotates in a clockwise direction. Due to friction between the shaft and the bearing, contact is made at an angle, $\phi = \tan^{-1} \mu$, where $\mu$ is the associated coefficient of friction. $N$ is the normal contact.
force. The friction force \( F_f = \mu N \) opposes the clockwise rotation by creating a counterclockwise moment.

\( N \) may be resolved into the components \( F_x \) and \( F_y \). The associated \( x \) and \( y \) components of the friction force are \( \mu F_y \) and \( \mu F_x \), respectively.

The directions of the components of \( N \) and \( F_f \) are drawn in the same manner in Fig. A-2b in a somewhat more convenient representation. When the direction of the resultant external force, \( W \), is not known, contact is possible anywhere on the periphery of the bearing and the components \( F_x \) and \( F_y \) of the normal contact force cannot be drawn with certainty in the free body diagram. The direction of the friction components must still oppose the motion.

Figure A-3 shows two general possibilities of drawing the free body diagram of a pivot which rotates in a clockwise direction. In either case, the moments of the friction components oppose the rotation while \( F_x \) and \( F_y \) may be positive or negative. Assume now, for example, that Fig. A-3a shows the wrong direction for \( F_x \) and that the solution of the applicable equilibrium equation will reverse the sign of \( F_x \). This will automatically reverse the sign of the friction component \( \mu F_x \) also. Since contact is now made on the opposite
side of the pivot, the correct sign of the friction component is automatically assured.

The total friction moment then is expressed by

$$M_f = \pm \mu \rho \sqrt{F_x^2 + F_y^2}$$  \hspace{1cm} (A-2)

where \( \rho \) is the pivot radius. The sign of the above is chosen so that the rotation is opposed. In case \( F_x \) and \( F_y \) contain required terms that cannot be factored out of the square root of equation (A-2), the friction moment is conservatively overstated by the use of the absolute values of \( F_x \) and \( F_y \).
If the expressions for $F_x$ and $F_y$ consist of sums of positive and negative terms, then $F_x$ and $F_y$ are presented as the sums of the absolute values of these terms. A conservative pivot friction moment becomes, similar to equation (A-3a),

$$M_f = \pm \mu \rho \left( |\vec{F}_x| + |\vec{F}_y| \right)$$ (A-3b)

The tildes represent the sums of the absolute values of the component terms.
3. MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP GEAR MESH WITH INVOLUTE TEETH

Figure A-4 shows free body diagrams of the gear and the pinion of a single mesh where the gear is driven by a counterclockwise input moment \( M_{in} \). It is desired to find the equilibrating output moment \( M_o \).

a. UNIT VECTORS

The unit vector directed from point \( O_N \) to point \( L \) is given by

\[
\vec{n}_{\theta} = \sin \theta \hat{i} + \cos \theta \hat{j} \quad (A-4)
\]

where \( \theta \) represents the actual pressure angle, regardless of tooth modification. The unit vector directed along the line of action from point \( L \) to point \( L' \) is given by

\[
\vec{n}_{\theta T} = -\cos \theta \hat{i} + \sin \theta \hat{j} \quad (A-5)
\]
FIGURE A-4. FREE BODY DIAGRAM FOR SINGLE STEP-UP INVOLUTE GEAR MESH
NOMENCLATURE AND SIGNUM CONVENTION

\[ F_{xN}, F_{yN} \]  = x and y components of normal force acting on
gear pivot

\[ \mu F_{xN}, \mu F_{yN} \]  = friction force components acting on gear
pivot. Directions chosen to result in friction
moments which oppose motion. (See part 2).

\[ F_{xn}, F_{yn} \]  = x and y components of normal force acting on
pinion pivot

\[ \mu F_{xn}, \mu F_{yn} \]  = friction force components acting on pinion pivot

\[ \mu \]  = coefficient of friction

\[ F_c \]  = normal contact force between gear and pinion.

The force of the pinion on the gear is \((-)F_c \bar{n}_\theta T\),
while the normal force of the gear on the pinion
becomes \(F_c \bar{n}_\theta T\).

\[ \mu F_c \]  = tooth contact friction force. The analysis in
Section 1 shows that the friction force of the
pinion on the gear, before the pitch point, acts
in the direction of \((-)\bar{n}_\theta\). Therefore

\[ -s \mu F_c \bar{n}_\theta \]  = friction force on gear with

\[ s = +1 \text{ for } a < \overline{LP} \text{ (approach)} \]  \( \text{(A-6)} \)
and

\[ s = -1 \text{ for } a > \overline{LP} \text{ (recess)} \quad (A-7) \]

while

\[ s = 0 \text{ for } a = \overline{LP} \text{ (at pitch point)} \quad (A-8) \]

Further,

\[ \rho_N, \rho_n = \text{gear and pinion pivot radii} \]

\[ R_b, r_b = \text{gear and pinion base circle radii} \]
c. **FORCE ANALYSIS OF THE GEAR**

By inspecting Figure A-4, one sees that force equilibrium of the gear is expressed by

\[-F_c \overline{a}_T - \mu F_c \overline{a}_T - F_{xN} - \mu F_{xN} - \mu F_{yN} = 0 \quad (A-9)\]

Similarly, moment equilibrium of the gear is given by

\[M_{in} - \mu N \overline{a} \sqrt{F_{xN}^2 + F_{yN}^2} = A \overline{a}_T + (R_b \overline{a}_T + a \overline{a}_T) \times (-)F_c \overline{a}_T = 0 \quad (A-10)\]

Note the use of equation (A-2) to express the friction moment at the gear pivot.

With the help of equations (A-4) and (A-5) one may write equations (A-9) and (A-10) in scalar form. Thus,

\[F_c \cos \theta - \mu a F_c \sin \theta - F_{xN} - \mu F_{yN} = 0 \quad (A-11)\]

\[-F_c \sin \theta - \mu a F_c \cos \theta + F_{yN} - \mu F_{xN} = 0 \quad (A-12)\]
Equation (A-10) becomes

$$M_{in} - \rho_N^\mu \sqrt{F_x^2 + F_y^2} - R_b F_c + \mu \sigma \alpha F_c = 0 \quad (A-13)$$

Simultaneous solution of equations (A-11) and (A-12) for $F_x N$ and $F_y N$ gives

$$F_x N = F_c \frac{(1 - \mu^2 s)\cos \theta - \mu (1 + s) \sin \theta}{1 + \mu^2} \quad (A-14)$$

and

$$F_y N = F_c \frac{(1 - \mu^2 s)\sin \theta + \mu (1 + s) \cos \theta}{1 + \mu^2} \quad (A-15)$$

When the above expressions are substituted into the moment equation (A-13) and if one notes that $s^2$ always equals $-1$, the following expression for $F_c$ is obtained:

$$F_c = \frac{M_{in}}{R_b + \mu (\rho_N - \alpha \sigma)} \quad (A-16)$$
d. FORCE ANALYSIS OF THE PINION

Force equilibrium of the pinion is assured by

\[ F_c \cos \theta + \mu sF_c \sin \theta + F_{xn} - \mu F_{yn} = 0 \]  
\[ F_c \sin \theta + \mu sF_c \cos \theta - \mu F_{xn} - F_{yn} = 0 \]

\[ M_o + \mu \rho_n \sqrt{F_{xn}^2 + F_{yn}^2} - r_b F_c + \mu s(d - a) F_c = 0 \]
Equations (A-19) and (A-20) are now solved simultaneously for $F_{xn}$ and $F_{yn}$. This gives

$$F_{xn} = F_c \frac{(1 + \mu^2 s) \cos \theta + \mu (1 - s) \sin \theta}{1 + \mu^2} \quad (A-22)$$

and

$$F_{yn} = F_c \frac{(1 + \mu^2 s) \sin \theta - \mu (1 - s) \cos \theta}{1 + \mu^2} \quad (A-23)$$

Equations (A-22) and (A-23) are then substituted into the moment equation (A-21). This furnishes the following expression for the normal contact force $F_c$. (Again, $s^2$ always equals 1):

$$F_c = \frac{M_0}{r_b - \mu [\rho_n + s(d - a)]} \quad (A-24)$$
e. MOMENT INPUT-OUTPUT RELATIONSHIP

The equilibrant moment, \( M_0 \), may be expressed as a function of the input moment, \( M_{in} \), after equations (A-16) and (A-24) have been set equal to each other. Thus,

\[
M_0 = M_{in} \left\{ \frac{r_b - \mu [\rho_n + s(d - a)]}{R_b + \mu [\rho_N - sa]} \right\} \quad (A-25)
\]

The input-output relationship may also then be expressed in terms of

\[
M_0 = M_{in} \frac{r_b}{R_b} E_2 \quad (A-26)
\]

\[
E_2 = \frac{1 - \mu [\rho_n + s(d - a)]}{1 + \frac{\mu (\rho_N - sa)}{R_b}} ,
\]

where \( E_2 \) represents the efficiency of moment transmission of a single step-up mesh with involute teeth.
4. Moment Input-Output Relationship for Three Step-Up Gear Train in Spin Environment

Figure A-5 shows the basic configuration of the three step-up gear train for which the relationship between the equivalent output moment $M_{04}$, acting on pinion 4, and the input moment $M_{1n}$, acting on gear 1, is to be found.

The body-fixed x-y coordinate system has its origin at the spin axis $C$, of the fuze body, and its x-axis coincides with the line $C-O_1$, where $O_1$ represents the pivot axis of the input gear-spin rotor combination. Points $O_2$, $O_3$ and $O_4$ represent the pivot axes of gear and pinion no. 2, gear and pinion no. 3, and pinion no. 4, respectively. Further,

\[
\begin{align*}
R_i &= \text{distance from the spin axis to the various pivot axes} \\
R_{bi} &= \text{base radii of gears} \\
r_{bi} &= \text{base radii of pinions} \\
\beta_1 &= \text{angle between positive x-axis and line of centers } O_1-O_2 \\
\beta_2 &= \text{angle between positive x-axis and line of centers } O_2-O_3 \\
\beta_3 &= \text{angle between positive x-axis and line of centers } O_3-O_4 \\
\gamma_1 &= \text{angle between positive x-axis and line } C-O_1
\end{align*}
\]
\[ \theta_1 = \text{pressure angle of mesh between gear no. 1 and pinion no. 2} \]
\[ \theta_2 = \text{pressure angle of mesh between gear no. 2 and pinion no. 3} \]
\[ \theta_3 = \text{pressure angle of mesh between gear no. 3 and pinion no. 4} \]

To obtain the moment input-output relationship of the total train, the input-output relationships of the individual components must first be obtained. The following equilibrium analyses include pivot as well as contact friction forces, in addition to loads due to the centrifugal forces on the individual components. The directions of the tooth-to-tooth friction forces are chosen according to the rules of Section 1 of this appendix, using an appropriate signum convention. The direction of the pivot friction forces is chosen according to Section 2 and, to avoid difficulties with the direction of the associated friction moment, equation (A-3b) will be used.

\[ a. \text{EQUILIBRIUM OF PINION 4} \]

Figure A-6 shows a free body diagram of pinion 4. The contact between gear 3 and pinion 4 is shown before contact point \( C_3 \) has passed through pitch point \( P_3 \). The unit vector \( e_{34} \) is along the line-of-action in the direction of the contact force of the gear on the pinion.

A-20
FIGURE A-6. FREE BODY DIAGRAM OF PINION 4
Thus,

\[ \bar{n}_{34} = \sin(\beta_3 + \theta_3) \hat{I} - \cos(\beta_3 + \theta_3) \hat{J} \]  \hspace{1cm} (A-27)

The unit vector normal to the line-of-action is given by

\[ \bar{n}_{N34} = \cos(\beta_3 + \theta_3) \hat{I} + \sin(\beta_3 + \theta_3) \hat{J} \]  \hspace{1cm} (A-28)

The contact force \( F_{34} \) then becomes

\[ F_{34} = F_{34} \bar{n}_{34} \]  \hspace{1cm} (A-29)

The friction force of gear 3 on pinion 4 is given by

\[ F_{f34} = \mu s_3 F_{34} \bar{n}_{N34} \]  \hspace{1cm} (A-30)

where \( s_3 = +1 \), for contact before the pitch point

\( s_3 = 0 \), for contact at the pitch point

\( s_3 = -1 \), for contact after the pitch point

(See also Section 1.)
The normal forces on the pivot shaft are given by

\[ \overline{F}_{x4} = F_{x4} \overline{I} \quad (A-31) \]

and

\[ \overline{F}_{y4} = -F_{y4} \overline{J} \quad (A-32) \]

The associated pivot friction forces are given by \( \mu F_{x4} \overline{J} \) and \( \mu F_{y4} \overline{I} \) for the indicated direction of rotation. The centrifugal force \( T_{4} \) on the pinion is represented by

\[ T_{4} = T_{4}(\cos \gamma_{4} \overline{I} + \sin \gamma_{4} \overline{J}) \quad (A-33) \]

where

\[ T_{4} = K_{4} \omega^{2} m_{4} \quad (A-34a) \]

with

\[ \omega = \text{spin angular velocity} \quad (A-34b) \]

and

\[ m_{4} = \text{mass of pinion 4} \quad (A-34c) \]

The force equilibrium equation is given by

\[ F_{34} \overline{H}_{34} + \mu s_{2} F_{34} H_{N34} + T_{4}(\cos \gamma_{4} \overline{I} + \sin \gamma_{4} \overline{J}) + F_{x4} \overline{I} \]

\[ + \mu F_{y4} \overline{I} + \mu F_{x4} \overline{J} - F_{y4} \overline{J} = 0 \quad (A-35) \]
Moment equilibrium is given by the following expression, in which the pivot friction moment is expressed according to Equation (A-3b):

\[-M_{04} = \rho_4'' (\vec{F}_{x4} + \vec{F}_{y4}) + r_{b4}F_{34} - \mu s_3(d_3 - a_3)F_{34} = 0 \quad (A-36)\]

where \( \rho_4 \) represents the pivot radius, \( d_3 \) is the length of the line-of-action of the mesh from points of tangency to the base circles, and \( a_3 \) is the distance on the line of action from the gear point of tangency to the contact point \( C_3 \).

Equation (A-35) gives the following component equations:

\[F_{34}\sin(\beta_3 + \theta_3) + \mu s_3F_{34}\cos(\beta_3 + \theta_3) + T_4\cos\gamma_4 + F_{x4} + \mu F_{y4} = 0 \quad (A-37)\]

\[-F_{34}\cos(\beta_3 + \theta_3) + \mu s_3F_{34}\sin(\beta_3 + \theta_3) + T_4\sin\gamma_4 - F_{y4} + \mu F_{x4} = 0 \quad (A-38)\]
Simultaneous solution of the above for $F_{x4}$ and $F_{y4}$ results in

$$F_{x4} = \frac{1}{1 + \mu^2} \left\{ -T_4 \left[ \cos \gamma_4 + \mu \sin \gamma_4 \right] - F_{34} \left[ (1 + \mu^2 s_3) \sin (\beta_3 + \theta_3) \right. \\
- \mu (1 - s_3) \cos (\beta_3 + \theta_3) \right\} \quad (A-39)$$

and

$$F_{y4} = \frac{1}{1 + \mu^2} \left\{ T_4 \left[ \sin \gamma_4 - \mu \cos \gamma_4 \right] - F_{34} \left[ (1 + \mu^2 s_3) \cos (\beta_3 + \theta_3) \right. \\
+ \mu (1 - s_3) \sin (\beta_3 + \theta_3) \right\} \quad (A-40)$$

To obtain conservative values for the pivot friction moment in equation (A-36) according to equation (A-3b), one substitutes the largest possible values for $F_{x4}$ and $F_{y4}$. This is accomplished by making the signs of $T_4$ and $F_{34}$ positive and by using the absolute values of their respective coefficients in Equations (A-39) and (A-40). Equation (A-36) then becomes

$$-M_{04} = \mu \rho_4 \left( T_4 A_1 + F_{34} A_2 + T_4 A_3 + F_{34} A_4 \right) + r_{b4} F_{34}$$

$$- \mu s_3 (d_3 - a_3) F_{34} = 0 \quad (A-41)$$

where

$$A_1 = \left| \frac{\sin \gamma_4 - \mu \cos \gamma_4}{1 + \mu^2} \right| \quad (A-42)$$
\[ A_2 = \left( \frac{1 + \mu^2 e_3 \cos(\beta_3 + \theta_3) + \mu(1 - e_3) \sin(\beta_3 + \theta_3)}{1 + \mu^2} \right) \quad (A-43) \]

\[ A_3 = \left( \frac{\cos \gamma_4 + \mu \sin \gamma_4}{1 + \mu^2} \right) \quad (A-44) \]

\[ A_4 = \left( \frac{(1 + \mu^2 e_3 \sin(\beta_3 + \theta_3) - \mu(1 - e_3) \cos(\beta_3 + \theta_3))}{1 + \mu^2} \right) \quad (A-45) \]

Finally equation (A-41) is solved for \( F_{34} \)

\[ F_{34} = \frac{M_{04}}{D_1} + \frac{T_{41}}{D_1} \quad (A-46) \]

where

\[ C_1 = \mu r_4 (A_1 + A_3) \quad (A-47) \]

\[ D_1 = r_{b4} - \mu \left[ e_3 (d_3 - a_3) + r_4 (A_2 + A_4) \right] \quad (A-48) \]
b. EQUILIBRIUM OF GEAR AND PINION SET NO. 3

Figure A-7 shows the free body diagram of gear and pinion set no. 3. Contact point C₃, between pinion 4 and gear 3, is, as shown previously in Figure A-6, before pitch point P₃. The normal force, along the line-of-action, is given [see equation (A-29)] by

\[ F_{43} = -F_{34}N_{34} \]  \hspace{1cm} (A-49)

and the associated friction force \( F_{f43} \) is given [see equation (A-30)] by

\[ F_{f43} = -\mu_{1}F_{34}N_{34} \]  \hspace{1cm} (A-50)

The unit vectors along (and perpendicular to) the line-of-action of gear 2 and pinion 3 are given by

\[ \hat{\Pi}_{23} = -\sin(\beta_2 - \phi_2) \mathbf{I} + \cos(\beta_2 - \phi_2) \mathbf{J} \]  \hspace{1cm} (A-51)

and

\[ \hat{\Pi}_{N23} = -\cos(\beta_2 - \phi_2) \mathbf{I} - \sin(\beta_2 - \phi_2) \mathbf{J} \]  \hspace{1cm} (A-52)

A-27
FIGURE A-7. EQUILIBRIUM OF GEAR AND PINION SET NO. 3
The contact point $C_2$ between gear 2 and pinion 3 is also shown before pitch point $P_2$ is passed.

The normal contact force between these teeth then becomes

$$F_{23} = F_{23}n_{23} \quad \text{(A-53)}$$

The associated friction force is given by

$$F_{f23} = -u_{2}F_{23}n_{23} \quad \text{(A-54)}$$

where $a_2 = +1$ for contact before pitch point $P_2$

$a_2 = 0$ for contact at pitch point $P_2$

$a_2 = -1$ for contact after pitch point $P_2$

The normal forces on the pivot shaft are given by

$$F_{x3} = F_{x3}^T \quad \text{(A-55)}$$

and

$$F_{y3} = -F_{y3}^T \quad \text{(A-56)}$$

The associated pivot friction forces are represented by $(-)F_{y3}$
and \((-\mu F_{x3})\) for the indicated direction of gear rotation.

The centrifugal force, \(T_3\), on the assembly is given by

\[
T_3 = T_3(\cos \gamma_3 + \sin \gamma_3) \quad (A-57)
\]

where

\[
T_3 = \omega^2 m_3 \quad (A-58a)
\]

with

\[
m_3 = \text{mass of pinion and gear 3} \quad (A-58b)
\]

Force equilibrium is given by

\[
F_{23N23} - \mu s F_{23N23} - F_{34N34} - \mu a F_{34N34} + T_3(\cos \gamma_3 + \sin \gamma_3) + F_{x3} = \mu F_{y3} - F_{y3} - \mu F_{x3} = 0 \quad (A-59)
\]

Moment equilibrium requires

\[
Rb_3F_{34} - \mu s \_F_{34} = rb_3F_{23} + \mu a (d_2 - a_2)F_{23} + \mu \rho_3(F_{x3} + F_{y3}) = 0 \quad (A-60)
\]

Note the use of equation \((A-3b)\) for the pivot friction moment.

\(\rho_3\) represents the pivot radius and length \(d_2\) is the length of the
line-of-action between the points of tangency to the base circles.

\( a_2 \) is the distance along the line-of-action from the gear point of tangency to the contact point \( C_2 \).

The component form of equation (A-59) is represented by the following two expressions:

\[
-F_{23} \sin(\theta_2) + \mu s_2 F_{23} \cos(\theta_2) - F_{34} \sin(\beta_3 + \theta_3) \\
- \mu s_3 F_{34} \cos(\beta_3 + \theta_3) - T_3 \cos \gamma_3 + F_x - \mu F_y = 0 \quad (A-61)
\]

and

\[
F_{23} \cos(\theta_2) + \mu s_2 F_{23} \sin(\theta_2) + F_{34} \cos(\beta_3 + \theta_3) \\
- \mu s_3 F_{34} \sin(\beta_3 + \theta_3) + T_3 \sin \gamma_3 - F_y - \mu F_x = 0 \quad (A-62)
\]

Simultaneous solution of the above for \( F_x \) and \( F_y \) leads to

\[
F_x = \frac{1}{1 + \mu^2} \left[ F_{23} \left[ (1 + \mu^2 s_2) \sin(\theta_2) + \mu (1 - s_2) \cos(\beta_2 - \theta_2) \right] \\
+ T_3 \left[ \mu \sin \gamma_3 - \cos \gamma_3 \right] + F_{34} \left[ (1 - \mu^2 s_3) \sin(\beta_3 + \theta_3) \right] \\
+ \mu (1 + s_3) \cos(\beta_3 + \theta_3) \right] \right] \quad (A-63)
\]
\[ F_{y3} = \frac{1}{1 + \mu^2} \left[ F_{23} \left[ \left(1 + \mu^2 a_2\right) \cos(\beta_2 - \theta_2) + \mu(a_2 - 1) \sin(\beta_2 - \theta_2) \right] \\
+ T_3 \left[ \sin \gamma_3 + \cos \gamma_3 \right] + F_{34} \left[ \left(1 - \mu^2 a_3\right) \cos(\beta_3 + \theta_3) \\
- \mu(a_3 + 1) \sin(\beta_3 + \theta_3) \right] \right] \]  
(A-64)

Now, equations (A-63) and (A-64) are substituted into the moment equation (A-60) with consideration of the pivot friction moment according to equation (A-3b). This gives

\[ R_{b3} F_{54} - \mu a_3 a_3 F_{34} - R_{b3} F_{23} + \mu a_3 (d_2 - a_2) F_{23} \]
\[ + \mu a_3 \left[ F_{23}(A_5 + A_8) + T_3 (A_6 + A_9) + F_{34}(A_7 + A_{10}) \right] \]
\[ = 0 \]  
(A-65)

where

\[ A_5 = \frac{(1 + \mu^2 a_2) \cos(\beta_2 - \theta_2) + \mu(a_2 - 1) \sin(\beta_2 - \theta_2)}{1 + \mu^2} \]  
(A-66)

\[ A_6 = \frac{\sin \gamma_3 + \mu \cos \gamma_3}{1 + \mu^2} \]  
(A-67)

A-32
\[ A_7 = \frac{(1 - \mu^2 a_3 \cos(\beta_5 + \theta_3) - \mu(1 + a_3) \sin(\beta_3 + \theta_3))}{1 + \mu^2} \]  (A-68)

\[ A_8 = \frac{(1 + \mu^2 a_2) \sin(\beta_2 - \theta_2) + \mu(1 - a_2) \cos(\beta_2 - \theta_2)}{1 + \mu^2} \]  (A-69)

\[ A_9 = \frac{\mu \sin \gamma_3 - \cos \gamma_3}{1 + \mu^2} \]  (A-70)

\[ A_{10} = \frac{(1 - \mu^2 a_3) \sin(\beta_3 + \theta_3) + \mu(1 + a_3) \cos(\beta_3 + \theta_3)}{1 + \mu^2} \]  (A-71)

Finally, equation (A-65) is solved for \( F_{23} \)

\[ F_{23} = \frac{F_{34} C_2 + T_3 C_3}{D_2} \]  (A-72)

where

\[ C_2 = R_{b3} - \mu \left[ a_3 a_3 - \rho_3 (A_7 + A_{10}) \right] \]  (A-73)

\[ C_3 = \mu \rho_3 (A_6 + A_9) \]  (A-74)

\[ D_2 = r_{b3} - \mu \left[ a_2 (d_2 - a_2) + \rho_3 (A_5 + A_8) \right] \]  (A-75)

A-33
Figure A-8 shows the free body diagram of gear and pinion set no. 2. The contact point O₂, between gear 2 and pinion 3, is again shown before the pitch point P₂ is passed. (See also Figure A-7.)

The normal force, along the line-of-action, becomes with equation (A-72)

\[ F_{32} = -F_{23} \hat{n}_{23} \]  \hspace{1cm} (A-76)

The associated friction force, \( F_{f32} \), is given by

\[ F_{f32} = \mu_s F_{23} \hat{n}_{N23} \]  \hspace{1cm} (A-77)

The unit vectors along (and perpendicular to) the line-of-action of gear 1 and pinion 2 are given by

\[ \hat{n}_{12} = \sin(\beta_1 + \theta_1) \hat{I} - \cos(\beta_1 + \theta_1) \hat{J} \]  \hspace{1cm} (A-78)

and

\[ \hat{n}_{N12} = \cos(\beta_1 + \theta_1) \hat{I} + \sin(\beta_1 + \theta_1) \hat{J} \]  \hspace{1cm} (A-79)

The contact point O₁ between gear 1 and pinion 2 is also shown before pitch point P₁ is passed.
FIGURE A8, FREE BODY DIAGRAM OF GEAR AND PINION SET NO. 2
The normal contact force between the teeth of this mesh becomes

\[ F_{12} = F_{12}n_{12}, \quad (A-80) \]

while the associated friction force is given by (See Section 1 of this Appendix.)

\[ F_{f12} = \mu s_1 F_{12} n_{12} \quad (A-81) \]

where \( s_1 = +1 \) for contact before pitch point \( P_1 \)
\[ s_1 = 0 \] for contact at pitch point \( P_1 \)
\[ s_1 = -1 \] for contact after pitch point \( P_1 \)

The normal forces on the pivot shaft are

\[ F_{x2} = -F_{x2}^T \quad (A-82) \]
\[ F_{y2} = -F_{y2}^T \quad (A-83) \]

The associated pivot friction forces are again chosen such that
their friction moments oppose the indicated rotation.

The centrifugal force, \( T_2 \), on this gear and pinion assembly is given by

\[
T_2 = T_2(\cos \gamma_2 \bar{I} + \sin \gamma_2 \bar{J})
\]  
(A-84)

where \( T_2 = \Omega_2^2 m_2 \)  
(A-85a)

with

\[
m_2 = \text{mass of gear and pinion set no. 2}
\]  
(A-85b)

Force equilibrium is given by

\[
-F_{23} \bar{R}_{23} + \mu_2 F_{23} R_{N23} + F_{12} \bar{R}_{12} + \mu_1 F_{12} R_{N12} - T_2(\cos \gamma_2 \bar{I} + \sin \gamma_2 \bar{J}) - F_{x2} \bar{I} - F_{y2} \bar{J} + \mu F_{x2} \bar{I} - \mu F_{x2} \bar{J} = 0
\]  
(A-86)

Moment equilibrium is given by

\[
-R_{b2} F_{23} + \mu_2 a_2 F_{23} + r_{b2} F_{12} - \mu_1 (d_1 - a_1) F_{12} - \mu r_2 (F_{x2} + F_{y2}) = 0
\]  
(A-87)
Again, equation (A-3b) is used to account for the pivot friction moment. \( r_2 \) represents the pivot radius. \( d_1 \) is the length of the line-of-action between the points of tangency to the base circles, and \( a_1 \) is the distance from the point of tangency of the gear to the contact point \( C_1 \).

The component form of equation (A-86) is given by

\[
F_{23}\sin(\beta_2 - \theta_2) - \mu s_2 F_{23}\cos(\beta_2 - \theta_2) + F_{12}\sin(\beta_1 + \theta_1)
+ \mu s_1 F_{12}\cos(\beta_1 + \theta_1) + T_2\cos\gamma_2 - F_{x2} + \mu F_{y2} = 0
\]

(A-88)

\[-F_{23}\cos(\beta_2 - \theta_2) - \mu s_2 F_{23}\sin(\beta_2 - \theta_2) - F_{12}\cos(\beta_1 + \theta_1)
+ \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + T_2\sin\gamma_2 - F_{y2} - \mu F_{x2} = 0
\]

(A-89)

Simultaneous solution of the above furnishes

\[
F_{x2} = \frac{1}{1 + \mu^2} \left\{ -F_{12} \left[ \mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1) \right]
+ T_2 \left[ \mu \sin\gamma_2 + \cos\gamma_2 \right]
+ F_{23} \left[ (1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2) \right] \right\}
\]

(A-90)
and

\[ F_{y2} = \frac{1}{1 + \mu^2} \left[ -F_{12} \left[ \mu(1 - a_1) \sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1) \cos(\beta_1 + \theta_1) \right] \right. \\
+ T_2 \left[ \sin \gamma_2 - \mu \cos \gamma_2 \right] \\
+ F_{23} \left[ -\mu(1 + a_2) \sin(\beta_2 - \theta_2) - (1 - \mu^2 s_2) \cos(\beta_2 - \theta_2) \right] \right] \\
(A-91) \]

Now, equations (A-90) and (A-91) are substituted into the moment equation (A-87). With consideration of the pivot friction moment according to equation (A-3b), this gives

\[-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + R_{b2}F_{12} - \mu s_1 (d_1 - a_1) F_{12} \\
- \mu \rho_2 \left[ F_{12}(A_{11} + A_{14}) + T_2(A_{12} + A_{15}) + F_{23}(A_{13} + A_{16}) \right] \]

\[ = 0 \]  \\
(A-92)

In the above

\[ A_{11} = \left| \frac{\mu(1 - a_1) \sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1) \cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \]  \\
(A-93)

\[ A_{12} = \left| \frac{\sin \gamma_2 - \mu \cos \gamma_2}{1 + \mu^2} \right| \]  \\
(A-94)

A-39
\[ A_{13} = \frac{\mu(1 + s_2)\sin(\beta_2 - \theta_2) + (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \] (A-95)

\[ A_{14} = \frac{\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)}{1 + \mu^2} \] (A-96)

\[ A_{15} = \frac{\mu \sin \gamma_2 + \cos \gamma_2}{1 + \mu^2} \] (A-97)

\[ A_{16} = \frac{(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \] (A-98)

Finally, equation (A-92) is solved for \( F_{12} \)

\[ F_{12} = \frac{F_{23}C_4 + T_2C_5}{D_3} \] (A-99)

where

\[ C_4 = R_{b2} - \mu \left[ s_2 a_2 - \rho_2(A_{13} + A_{16}) \right] \] (A-100)

\[ C_5 = \mu \rho_2(A_{12} + A_{15}) \] (A-101)

\[ D_3 = r_{b2} - \mu \left[ s_1(d_1 - a_1) + \rho_2(A_{11} + A_{14}) \right] \] (A-102)
d. EQUILIBRIUM OF GEAR NO. 1

Figure A-9 shows the free body diagram of gear no. 1, the input gear.

The contact point C₁ is identical with that shown in Figure A-8.

The normal force on gear no. 1 is given by (See equation (A-80).)

\[ F_{21} = -F_{12}N_{12} \]  \hspace{1cm} (A-103)

The associated friction force is

\[ F_{f21} = -\mu_{s1}F_{12}N_{12} \]  \hspace{1cm} (A-104)

The normal forces on the pivot shaft are given by

\[ F_{x1} = -F_{x1}J \]  \hspace{1cm} (A-105)

and

\[ F_{y1} = F_{y1}J \]  \hspace{1cm} (A-106)

The associated pivot friction forces are chosen in such a direction that their moments oppose the indicated rotation.
FIGURE A9. FREE BODY DIAGRAM OF GEAR 1
The centrifugal force $T_1$ on gear 1 is given by

$$T_1 = T_1 \overline{I}$$  

(A-107)

where

$$T_1 = \Phi_1 \omega^2 m_1$$  

(A-108a)

$$m_1 = \text{mass of gear 1}$$  

(A-108b)

Force equilibrium is given by

$$-F_{12} \overline{H}_{12} - \mu \overline{F}_{12} \overline{I}_{12} + T_1 \overline{I} - F_x \overline{I} + F_y \overline{J} + \mu F_y \overline{I} + \mu F_x \overline{J} = 0$$  

(A-109)

Moment equilibrium is found from

$$R_{b1} F_{12} - \mu \overline{a}_{12} F_{12} - M_{in} + \mu \rho_1 (\overline{F}_x + \overline{F}_y) = 0$$  

(A-110)

The form of equation (A-3b) is again utilized to obtain the pivot friction moment. $\rho_1$ represents the pivot radius of gear 1.
The component form of equation (A-109) becomes

\[-F_{12}\sin(\beta_1 + \theta_1) - \mu s_1 F_{12}\cos(\beta_1 + \theta_1) = F_x1 + \mu F_y1 + T_1\]

\[= 0 \quad (A-111)\]

and

\[F_{12}\cos(\beta_1 + \theta_1) - \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + F_y1 + \mu F_x1 = 0 \quad (A-112)\]

Simultaneous solution of equations (A-111) and (A-112) furnishes the forces on the pivot, i.e.,

\[F_{x1} = \frac{-F_{12}\left[(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)\right]}{1 + \mu^2} + T_1 \quad (A-113)\]

\[F_{y1} = \frac{F_{12}\left[\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\cos(\beta_1 + \theta_1)\right]}{1 + \mu^2} - \mu T_1 \quad (A-114)\]

Equations (A-113) and (A-114) are now substituted into the moment equation (A-110) in the following manner: (Again, the method of equation (A-3b) is applied.)
\[ R_{b1} F_{12} - \mu s_1 F_{12} - \xi_{in} + \mu \rho_1 \left[ F_{12} (A_{17} + A_{19}) + T_1 (A_{18} + A_{20}) \right] = 0 \]  

(A-115)

where

\[ A_{17} = \frac{(1 - \mu^2 s_1) \sin(\beta_1 + \theta_1) + \mu(1 + s_1) \cos(\beta_1 + \theta_1)}{1 + \mu^2} \]  

(A-116)

\[ A_{18} = \frac{1}{1 + \mu^2} \]  

(A-117)

\[ A_{19} = \frac{\mu(1 + s_1) \sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1) \cos(\beta_1 + \theta_1)}{1 + \mu^2} \]  

(A-118)

\[ A_{20} = \frac{\mu}{1 + \mu^2} \]  

(A-119)

Finally, equation (A-115) is solved for \( F_{12} \)

\[ F_{12} = \frac{M_{in}}{D_4} - \frac{T_1 C_6}{D_4} \]  

(A-120)

where

\[ C_6 = \mu \rho_1 (A_{18} + A_{20}) \]  

(A-121)

\[ D_4 = R_{b1} - \mu \left[ s_1 a_1 - \rho_1 (A_{17} + A_{19}) \right] \]  

(A-122)

A-45
e. INPUT-OUTPUT RELATIONSHIP

To obtain the input-output relationship for the complete gear train, equation (A-120) is now equated to equation (A-99). This furnishes

\[ F_{23} = \frac{D_3}{C_4D_4} (M_{in} - T_1C_6) - T_2 \frac{C_5}{C_4} \quad (A-123) \]

Further, the above is equated to equation (A-72). This results in the following expression for \( F_{34} \)

\[ F_{34} = \frac{D_2D_3}{C_4C_4D_4} (M_{in} - T_1C_6) - T_2 \frac{C_5D_2}{C_2C_4} - T_3 \frac{C_3}{C_2} \quad (A-124) \]

Finally, equation (A-124) is equated to equation (A-46). This establishes the input-output relationship

\[ M_{04} = \frac{D_1D_2D_3}{C_2C_4D_4} (M_{in} - T_1C_6) - T_2 \frac{C_5D_1D_2}{C_2C_4} - T_3 \frac{C_3D_1}{C_2} - T_4C_1 \quad (A-125) \]
5. **MOMENT INPUT-OUTPUT RELATIONSHIP FOR INVOLUTE TWO STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT**

Figure A-10 shows the basic configuration of a two step-up gear train with involute teeth for which the relationship between the equilibrant output moment, \( M_{o3} \), acting on pinion 3, and the input moment, \( M_{in} \), acting on gear 1 is to be found. All nomenclature is identical with that used in Section 4 in connection with the three step-up gear train.

Again, the general relationship between input and output is found by assembling the input-output relationships of the individual component gears.

a. **EQUILIBRIUM OF PINION 3**

Figure A-11 shows the free body diagram of pinion 3. The contact point \( C_2 \) between gear 2 and pinion 3 is shown before the pitch point \( P_2 \) is passed.
FIGURE A-11  FREE BODY DIAGRAM OF PINION 3
As in equation (A-53), the normal force between the teeth of gear 2 and pinion 3 is given by

\[ F_{23} = F_{23} N_{23} \]  \hspace{1cm} (A-126)

The associated friction force is given by equation (A-54), i.e.,

\[ F_{f23} = -\mu s_2 F_{23} N_{23} \]  \hspace{1cm} (A-127)

where \( s_2 = +1 \), for contact before pitch point \( P_2 \)

\[ s_2 = 0 \], for contact at pitch point \( P_2 \)

\[ s_2 = -1 \], for contact after pitch point \( P_2 \)

The unit vectors in equations (A-126) and (A-127) were defined by equations (A-51) and (A-52), respectively. The normal forces on the pivot shaft are given by

\[ F_{x3} = F_{x3} \hat{I} \]  \hspace{1cm} (A-128)

and

\[ F_{y3} = -F_{y3} \hat{J} \]  \hspace{1cm} (A-129)
The pivot friction forces become \((-)\mu F_{y3}\bar{I}\) and \((-)\mu F_{x3}\bar{J}\) for the indicated direction of rotation.

As in equation (A-57), the centrifugal force on the pinion is given by

\[
T_3 = T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) \tag{A-130}
\]

where

\[
T_3 = \mathcal{R}_3 \omega^2 m_3 \tag{A-131}
\]

with

\[
m_3 = \text{mass of pinion } 3 \tag{A-132}
\]

Force equilibrium is given by

\[
F_{23}\bar{r}_{23} - \mu s_2 F_{23}\bar{r}_{N23} + T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) + F_{x3}\bar{I} - \mu F_{y3}\bar{I} - F_{y3}\bar{J} - \mu F_{x3}\bar{J} = 0 \tag{A-133}
\]

Moment equilibrium is obtained from

\[
M_{03} = r_{b3} F_{23} + \mu s_2 (d_2 - a_2) F_{23} + \mu s_3 (\bar{F}_{x3} + \bar{F}_{y3}) = 0 \tag{A-134}
\]
Note the use of equation (A-3b) for the pivot friction moment. \( \nu_3 \) represents the pivot radius. \( d_2 \) is the length of the line of action between the points of tangency to the base circles of pinion 3 and gear 2. \( a_2 \) is the distance along the line-of-action from the gear point of tangency to contact point \( C_2 \).

The x and y components of equation (A-133) are given by

\[
-F_{23} \sin(\beta_2 - \theta_2) + \mu_3 F_{23} \cos(\beta_2 - \theta_2) + T_3 \cos \gamma_3 + F_{x3} \\
- \mu F_{y3} = 0 , \quad \text{(A-135)}
\]

and

\[
F_{23} \cos(\beta_2 - \theta_2) + \mu_3 F_{23} \sin(\beta_2 - \theta_2) + T_3 \sin \gamma_3 - F_{y3} \\
- \mu F_{x3} = 0 \quad \text{(A-136)}
\]

Simultaneous solution of these expressions for \( F_{x3} \) and \( F_{y3} \) yields

\[
F_{x3} = \frac{1}{1 + \mu^2} \left[ F_{23} \left( (1 + \mu^2 s_2) \sin(\beta_2 - \theta_2) + \mu (1 - s_2) \cos(\beta_2 - \theta_2) \right) \\
+ T_3 \left[ \sin \gamma_3 - \cos \gamma_3 \right] \right] \quad \text{(A-137)}
\]
and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (1 + \mu^2 s_2) \cos(\beta_2 - \theta_2) + \mu(s_2 - 1) \sin(\beta_2 - \theta_2) \right] \ight. + T_3 \left[ \sin \gamma_3 + \mu \cos \gamma_3 \right] \right\} \quad (A-138)$$

Now, equations (A-137) and (A-138) are substituted into the moment equation (A-134), with the pivot friction moment given according to the formulation of equation (A-3b):

$$M_{03} = r_{b3} F_{23} + \mu s_2 (d_2 - a_2 F_{23}) + \mu r_3 \left[ F_{23} (A_1 + A_3) + T_3 (A_2 + A_4) \right] = 0 \quad (A-139)$$

where

$$A_1 = \left| \begin{array}{c} (1 + \mu^2 s_2) \cos(\beta_2 - \theta_2) + \mu(s_2 - 1) \sin(\beta_2 - \theta_2) \\ 1 + \mu^2 \end{array} \right| \quad (A-140)$$

$$A_2 = \left| \begin{array}{c} \sin \gamma_3 + \mu \cos \gamma_3 \\ 1 + \mu^2 \end{array} \right| \quad (A-141)$$

$$A_3 = \left| \begin{array}{c} (1 + \mu^2 s_2) \sin(\beta_2 - \theta_2) + \mu(1 - s_2) \cos(\beta_2 - \theta_2) \\ 1 + \mu^2 \end{array} \right| \quad (A-142)$$
Asin

\[
A_4 = \frac{\mu \sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \quad (A-143)
\]

Finally, equation (A-139) is solved for \( F_{23} \)

\[
F_{23} = \frac{M_0}{D_1} + T_3 \frac{C_1}{D_1} \quad (A-144)
\]

where

\[
C_1 = \mu r_3 (A_2 + A_4) \quad (A-145)
\]

\[
D_1 = r_b - \mu \left[ a_2 (d_2 - a_2) + r_3 (A_1 + A_3) \right] \quad (A-146)
\]
b. EQUILIBRIUM OF GEAR AND PINION SET NO. 2

Figure A-12 shows the free body diagram of gear and pinion set no. 2. The contact point, $C_2$, between gear 2 and pinion 3 is again shown before the pitch point, $P_2$. With equation (A-126), the normal force between the teeth of this mesh becomes

$$F_{32} = -F_{23}\bar{n}_{23}$$  \hspace{1cm} (A-147)

The associated friction force is the negative of equation (A-127), i.e.,

$$F_{f32} = -\mu_{2}F_{23}\bar{n}_{23}$$  \hspace{1cm} (A-148)

Similar to equation (A-80), the normal force between gear 1 and pinion 2 is given by

$$F_{12} = F_{12}\bar{n}_{12}$$  \hspace{1cm} (A-149)

(See equation (A-78) for the definition of unit vector $\bar{n}_{12}$.)
FIGURE A-12  FREE BODY DIAGRAM OF GEAR AND PINION SET NO. 2
Figure A-12 shows the contact point $C_1$ before the pitch point $P_1$ is passed.

The associated friction force is given by

$$F_{f12} = \mu s_1 F_{12} \mathbf{W}_{N12} \quad (A-150)$$

where

- $s_1 = +1$, for contact before pitch point $P_1$
- $s_1 = 0$, for contact at pitch point $P_1$
- $s_1 = -1$, for contact after pitch point $P_1$

The unit vector $\mathbf{W}_{N12}$ is given by equation (A-79).

The normal forces on the pivot shaft are given by

$$F_{x2} = -F_{x2} \mathbf{i} \quad (A-151)$$

and

$$F_{y2} = -F_{y2} \mathbf{j} \quad (A-152)$$

The associated friction forces $\mu F_{y2} \mathbf{i}$ and $(-\mu) F_{x2} \mathbf{j}$ were chosen so that their moments oppose the indicated rotation.

The centrifugal force on this gear and pinion assembly is expressed by

$\mathbf{A-57}$
\[ T_2 = T_2(\cos \gamma_2 I + \sin \gamma_2 J) \quad (A-153) \]

where \( T_2 = \omega_2^2 m_2 \) \quad (A-154)

\[ m_2 = \text{mass of gear and pinion set no. 2} \quad (A-155) \]

Force equilibrium is given by

\[ -F_{23} F_{23} + \mu s_2^2 F_{23} R_{23} + F_{12} R_{12} + \mu s_1 F_{12} R_{N12} + T_2(\cos \gamma_2 I + \sin \gamma_2 J) - F_{x2} I + \mu F_{y2} I - F_{y2} J - \mu F_{x2} J = 0 \quad (A-156) \]

Moment equilibrium is given by

\[ -R_{b2} F_{23} + \mu s_2^2 F_{23} + R_{b2} F_{12} - \mu s_1 (d_1 - a_1) F_{12} - \mu \rho_2 (F_{x2} + F_{y2}) = 0 \quad (A-157) \]

Again, equation (A-3b) is used to obtain a conservative pivot friction moment. \( \rho_2 \) represents the pivot radius. \( d_1 \) and \( a_1 \) are similar to the previously used distances along the lines of action of the other meshes.
The component form of equation (A-156) becomes

\[ F_{23} \sin(\mu_2 - \theta_2) - \mu s_2 F_{23} \cos(\mu_2 - \theta_2) + F_{12} \sin(\beta_1 + \theta_1) \]
\[ + \mu s_1 F_{12} \cos(\beta_1 + \theta_1) + T_2 \cos \gamma_2 - F_{x2} + \mu F_{y2} = 0 \quad (A-158) \]

and

\[ -F_{23} \cos(\beta_2 - \theta_2) - \mu s_2 F_{23} \sin(\mu_2 - \theta_2) - F_{12} \cos(\beta_1 + \theta_1) \]
\[ + \mu s_1 F_{12} \sin(\beta_1 + \theta_1) + T_2 \sin \gamma_2 - F_{y2} - \mu F_{x2} = 0 \quad (A-159) \]

Simultaneous solution of the above expressions for \( F_{x2} \) and \( F_{y2} \) gives

\[ F_{x2} = \frac{1}{1 + \mu^2} \left\{ \frac{-F_{12} \left[ \mu(1 - s_1) \cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1) \sin(\beta_1 + \theta_1) \right]}{1 + \mu^2} \right\} 
\[ + \frac{T_2 \left[ \mu \sin \gamma_2 + \cos \gamma_2 \right]}{1 + \mu^2} \]
\[ + \frac{F_{23} \left[ (1 - \mu^2 s_2) \sin(\mu_2 - \theta_2) - \mu(1 + s_2) \cos(\beta_2 - \theta_2) \right]}{1 + \mu^2} \quad (A-160) \]

A-59
\[ F_{y2} = \frac{1}{1 + \mu^2} \left\{ - F_{12} \left[ \mu (1 - s_1) \sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1) \cos(\beta_1 + \theta_1) \right] \\
+ T_2 \left[ \sin \gamma_2 - \mu \cos \gamma_2 \right] \\
+ F_{23} \left[ -\mu (1 + s_2) \sin(\beta_2 - \theta_2) - (1 - \mu^2 s_2) \cos(\beta_2 - \theta_2) \right] \right\} \tag{A-161} \]
Now equations (A-160) and (A-161) are substituted into the moment equation (A-157) with the pivot friction moment formulated again according to equation (A-3b):

\[-r_{b2}F_{23} + \mu S_{a2}F_{23} + r_{b2}F_{12} - \mu S_{1}(d_1 - a_1)F_{12} \]

\[-\mu^2 \left[ F_{12}(A_5 + A_8) + T_2(A_6 + A_9) + F_{23}(A_7 + A_{10}) \right] = 0 \]  

(A-162)

where

\[A_5 = \frac{\mu(1 - S_{1})\sin(\beta_1 + \theta_1) + (1 + \mu^2 S_{1})\cos(\beta_1 + \theta_1)}{1 + \mu^2} \]  

(A-163)

\[A_6 = \left| \frac{\sin\gamma_2 - \mu\cos\gamma_2}{1 + \mu^2} \right| \]  

(A-164)

\[A_7 = \frac{\mu(1 + S_{2})\sin(\beta_2 - \theta_2) + (1 - \mu^2 S_{2})\cos(\beta_2 - \theta_2)}{1 + \mu^2} \]  

(A-165)

\[A_8 = \frac{\mu(1 - S_{1})\cos(\beta_1 + \theta_1) - (1 + \mu^2 S_{1})\sin(\beta_1 + \theta_1)}{1 + \mu^2} \]  

(A-166)

\[A_9 = \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \]  

(A-167)

A-61
Finally equation (A-162) is solved for $F_{12}$

$$F_{12} = \frac{F_{23}C_2}{D_2} + \frac{T_2C_3}{D_2}$$  \hspace{1cm} (A-169)

where

$$C_2 = R_{b2} - \mu \left[ a_2a_2 - \rho_2(A_7 + A_{10}) \right]$$  \hspace{1cm} (A-170)

$$C_3 = \mu \rho_2 (A_6 + A_9)$$  \hspace{1cm} (A-171)

$$D_2 = R_{b2} - \mu \left[ a_1(d_1 - a_1) + \rho_2(A_5 + A_8) \right]$$  \hspace{1cm} (A-172)
c. **EQUILIBRIUM OF GEAR NO. 1**

Figure A-13 shows the free body diagram of gear no. 1, the input gear.

The contact point, $C_1$, corresponds to that shown in Figure A-12. According to equation (A-149), the normal contact force between pinion 2 and gear 1 becomes

$$F_{21} = -F_{12}N_{12} \quad (A-173)$$

The associated friction force is the negative of equation (A-150), i.e.,

$$F_{f21} = -\mu_{21}F_{12}N_{12} \quad (A-174)$$

The normal forces on the pivot shaft are given by

$$F_{x1} = -F_{x1} \quad (A-175)$$

and

$$F_{y1} = F_{y1} \quad (A-176)$$
FIGURE A-13. FREE BODY DIAGRAM OF GEAR 1
The associated friction forces $\mu F_y \bar{I}$ and $\mu F_x \bar{J}$ were chosen with such directions that their moments oppose rotation due to input moment $M_{in}$.

Centrifugal force $\bar{T}_1$ on gear 1 is given by

$$\bar{T}_1 = T_1 \bar{I}$$

where, as with equation (A-107)

$$T_1 = \bar{K}_1 \omega^2 m_1$$

and

$$m_1 = \text{mass of gear 1}$$

Force equilibrium of gear 1 is given by

$$-F_{12} \bar{N}_{12} - \mu s_1 F_{12} \bar{N}_{12} + T_1 \bar{I} - F_{x1} \bar{J} + F_{y1} \bar{J} + \mu F_{y1} \bar{J} + \mu F_{x1} \bar{J} = 0$$

(A-180)

Moment equilibrium is given by

$$R_{b1} F_{12} - \mu s_1 a_1 F_{12} = M_{in} + \mu' (\bar{F}_{x1} + \bar{F}_{y1}) = 0$$

(A-181)
Note that equations (A-180) and (A-181) have the same forms as equations (A-109) and (A-110).

The force component expressions are the same as given by equations (A-111) and (A-112), and their simultaneous solution for the pivot forces is identical to that given by equations (A-113) and (A-114), i.e.,

\[
F_{x1} = \frac{-F_{12} \left[(1 - \mu^2 s_1) \sin(\beta_1 + \theta_1) + \mu(1 + s_1) \cos(\beta_1 + \theta_1)\right]}{1 + \mu^2} + T_1
\]  

(A-182)

and

\[
F_{y1} = \frac{F_{12} \left[\mu(1 + s_1) \sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1) \cos(\beta_1 + \theta_1)\right]}{1 + \mu^2} - \mu T_1
\]  

(A-183)

Equations (A-182) and (A-183) are now substituted into equation (A-181) according to the method of equation (A-3b)

\[
R_{b1} F_{12} - \mu s_1 a_1 F_{12} - M_{in} + \mu p_1 \left[F_{12}(A_{11} + A_{13}) + T_1(A_{12} + A_{14})\right] = 0
\]  

(A-184)
where

\[ A_{11} = \frac{(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \]  \hspace{1cm} (A-185)

\[ A_{12} = \frac{1}{1 + \mu^2} \]  \hspace{1cm} (A-186)

\[ A_{13} = \frac{\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \]  \hspace{1cm} (A-187)

\[ A_{14} = \frac{\mu}{1 + \mu^2} \]  \hspace{1cm} (A-188)

Finally, equation (A-184) is solved for \( F_{12} \)

\[ F_{12} = \frac{M_{in}}{D_3} - \frac{T_1 C_4}{D_3} \]  \hspace{1cm} (A-189)

where

\[ C_4 = \mu \rho_1 (A_{12} + A_{14}) \]  \hspace{1cm} (A-190)

\[ D_3 = R_{b1} - \mu [s_1 a_1 - \rho_1 (A_{11} + A_{13})] \]  \hspace{1cm} (A-191)
d. **INPUT-OUTPUT RELATIONSHIP**

To obtain the input-output relationship for the complete gear train, equation (A-189) is now set equal to equation (A-169).

This furnishes

\[ F_{23} = \frac{D_2}{C_2 D_3} \left[ M_{1n} - T_1 C_4 \right] - T_2 \frac{C_3}{C_2} \]  

(A-192)

The above is then set equal to equation (A-144). This results in the input-output moment relationship

\[ M_{03} = \frac{D_1 D_2}{C_2 D_3} \left[ M_{1n} - T_1 C_4 \right] - T_2 \frac{C_3 D_1}{C_2} - T_3 C_1 \]  

(A-193)
6. AUXILIARY GEOMETRIC AND KINEMATIC EXPRESSIONS FOR TWO AND THREE STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

a. NOMENCLATURE FOR INVOLUTE GEAR TEETH

\[ R_{pi}, r_{pi} = \text{pitch radii of gear and pinion of } i^{th} \text{ gear and pinion set} \]

\[ R_{bi}, r_{bi} = \text{base radii of gear and pinion of } i^{th} \text{ gear and pinion set} \]

\[ R_{oi}, r_{oi} = \text{outside radii of gear and pinion of } i^{th} \text{ gear and pinion set} \]

\[ \theta_j = \text{effective pressure angle of } j^{th} \text{ mesh} \]

\[ q_i = \text{distance from spin axis to pivot of } i^{th} \text{ gear and pinion set} \]

b. ANGULAR RELATIONSHIPS BETWEEN PIVOT HOLES

Figure A-14 shows the angular relationships between the lines connecting the pivot holes as well as the spin center. (See also Figures A-5 and A-10.) The following serves to determine the angles \( \gamma_i \) and \( \beta_i \) for certain combinations of gears and pinions as well as spin radii \( q_i \).
From

\[(R_{p1} + r_{p2})^2 = a_1^2 + r_2^2 - 2R_1R_2\cos\gamma_2\]

one obtains

\[\gamma_2 = \cos^{-1}\left[\frac{a_1^2 + r_2^2 - (R_{p1} + r_{p2})^2}{2R_1R_2}\right]\]  

(A-194)

Similarly, from

\[\gamma_3^1 = \cos^{-1}\left[\frac{a_2^2 + r_3^2 - (R_{p2} + r_{p3})^2}{2R_2R_3}\right]\]

one obtains

\[\gamma_3 = \gamma_2 + \gamma_3^1\]  

(A-195)
Also with

\[
\gamma_4 = \cos^{-1} \left[ \frac{\pi_3^2 + \pi_4^2 - (R_{p3}^2 + r_{p4}^2)}{2\pi_3\pi_4} \right]
\]

one obtains

\[
\gamma_4 = \gamma_3 + \gamma_4^1
\]  \hspace{1cm} (A-196)

ANGLES \( \delta_4 \)

Since

\[
\pi_2^2 = (R_{p1} + r_{p2})^2 + \pi_1^2 - 2(R_{p1} + r_{p2})\pi_1 \cos \delta_2
\]

\[
\delta_2 = \cos^{-1} \left[ \frac{(R_{p1} + r_{p2})^2 + \pi_1^2 - \pi_2^2}{2\pi_1(R_{p1} + r_{p2})} \right] \hspace{1cm} (A-197)
\]

Similarly,

\[
\delta_3 = \cos^{-1} \left[ \frac{(R_{p2} + r_{p3})^2 + \pi_2^2 - \pi_3^2}{2\pi_2(R_{p2} + r_{p3})} \right] \hspace{1cm} (A-198)
\]
\[ \beta_4 = \cos^{-1} \left[ \frac{(R_{p3} + r_{p4})^2 + \alpha_3^2 - \alpha_4^2}{2R_{p3}(R_{p3} + r_{p4})} \right] \]  

**(A-199)**

**ANGLES \( \beta_4 \)**

With Equation (A-197)

\[ \beta_1 = \pi - \delta_2 \]  

**(A-200)**

Further, with Equations (A-194) and (A-198)

\[ \beta_2 = \gamma_2 + (\pi - \delta_3) \]  

**(A-201)**

Finally, with Equations (A-195) and (A-199)

\[ \beta_3 = \gamma_3 + (\pi - \delta_4) \]  

**(A-202)**

A-73
c. DETERMINATION OF CONTACT POINT C FOR VARIOUS MESHES

Figure A-15 shows the points of interest along the line of action of an involute gear which drives an involute pinion.

Points L and L' are the points of tangency to the base circles of radius $R_b$ and $r_b$, respectively, and the distance $d = LL'$. Initial contact is made at point M, where the line of action intersects the pinion addendum circle of radius $r_o$. Final contact corresponds to point N. Here the line of action intersects the gear addendum circle of radius $R_o$.

The position of the instantaneous contact point C with respect to point L, i.e., the length $a$, is expressed with the help of instantaneous angle $\alpha$ which has its origin at the line $O,L$. Then,

$$a = LC = R_b \alpha$$

(A-203)

A computer procedure for the determination of instantaneous angle $\alpha$ of any mesh must first find the associated initial and final angles of contact $\alpha_{\text{in}}$ and $\alpha_{\text{fin}}$. In addition, it must contain a method for incrementing angle $\alpha$. The following shows such a procedure for each of the meshes of a two pass and a three pass step-up gear train, together with a means of obtaining the signs of the signum terms.
FIGURE A-15.
INVOLUTE MESH GEOMETRY
1.) MESH OF GEAR 1 AND PINION 2

The total length, \( d_1 = LL' \), is given by

\[
d_1 = (R_{b1} + r_{b1})\tan \theta_1
\]  

(A-204)

The initial angle of contact, \( \alpha_{1\text{IN}} \), is obtained from

\[
\alpha_{1\text{IN}} = \frac{ML}{R_{b1}} = \frac{(R_{b1} + r_{b2})\tan \theta_1}{R_{b1}} - \sqrt{\frac{r_{b2}^2 - r_{b2}}{R_{b1}}}
\]  

(A-205)

Similarly, the final angle of contact, \( \alpha_{1\text{FIN}} \), is given by

\[
\alpha_{1\text{FIN}} = \sqrt{\frac{R_{b1}^2 - R_{b1}}{R_{b1}}}
\]  

(A-206)

The magnitude of the increment \( \Delta \alpha_1 \) depends on whether one deals with a two step-up or a three step-up gear train.

Assuming that a two step-up train is involved and that one wishes to compute the length \( a_2 \) of mesh 2 \( K_2 \) times after the first contact, the angular increment, \( \Delta \alpha_{22} \), has the magnitude
\[ \Delta \alpha_{22} = \frac{\alpha_{2\text{in}} - \alpha_{2\text{in}}}{K_2} \]  

(The second subscript refers to a two step-up configuration.)

Because of the transmission ratio between gear sets 1 and 2, the associated angular increment of gear 1 will be smaller than \( \Delta \alpha_{22} \), i.e.,

\[ \Delta \alpha_{12} = \Delta \alpha_{22} \frac{r_{b2}}{R_{b1}} \]  

(A-208)

The instantaneous angle, \( \alpha_1 \), will then be given by

\[ \alpha_1 = \alpha_{1\text{in}} + j_{12} \Delta \alpha_{12} \]  

(A-209)

and, the instantaneous distance, \( a_1 \), becomes

\[ a_1 = R_{b1} ( \alpha_{1\text{in}} + j_{12} \Delta \alpha_{12} ) \]  

(A-210)

In the above, \( j_{12} \) represents the number of times the angle \( \alpha_1 \) has been incremented. While the total number of increments depends on the length of contact, the incrementing of \( \alpha_1 \) comes to an end when \( \alpha_1 \geq \alpha_{1\text{fin}} \). This also corresponds to a complete mechanism.
cycle. Since mesh 2 goes through $\frac{R_{b1}}{r_{b2}}$ times as many cycles as mesh no. 1, the angle $a_2$ has to be re-initialized to $a_{2\text{fin}}$ when $a_l \geq a_{2\text{fin}}$, i.e., after $K_2$ increments. (For simplicity it is assumed that the motion starts when all meshes are at their initial contact angles, $a_{1\text{in}}$.)

When a three step-up gear train is involved, one must make sure that enough computations are made for mesh 3. Assuming that $K_3$ increments are to be made, one obtains

$$\Delta a_{33} = \frac{a_{3\text{fin}} - a_{3\text{in}}}{K_3} \quad \text{(A-211)}$$

The associated angular increments for meshes 2 and 1 then become

$$\Delta a_{23} = \Delta a_{33} \frac{R_{b3}}{R_{b2}} \quad \text{(A-212)}$$

and

$$\Delta a_{13} = \Delta a_{33} \frac{R_{b3}}{R_{b2}} \times \frac{R_{b2}}{R_{b1}} \quad \text{(A-213)}$$

For this case the instantaneous distance $a_1$ becomes

$$a_1 = R_{b1}(a_{1\text{in}} + J_{13}\Delta a_{13}) \quad \text{(A-214)}$$
\[ j_{13} \text{ stands for the number of times } \alpha_1 \text{ has been incremented at any given instant. This incrementing again ends when } \alpha_1 \geq \alpha_1^{\text{fin}}. \]

Meshes 2 and 3 are re-initialized to \( \alpha_2^{\text{in}} \) and \( \alpha_3^{\text{in}} \) as often as is necessary to complete one cycle for mesh no. 1.

The sign of \( \alpha_1 \) is best obtained from the fact that at pitch point \( P_1 \)

\[ \alpha_{1p} = \frac{L_P}{R_{b1}} = \frac{R_{b1} \tan \theta_1}{R_{b1}} = \tan \theta_1 \quad (A-215) \]

then for

\[ \alpha_1 < \tan \theta_1 : \quad s_1 = +1 \]
\[ \alpha_1 = \tan \theta_1 : \quad s_1 = 0 \quad (A-216) \]
\[ \alpha_1 > \tan \theta_1 : \quad s_1 = -1 \]
2.) MESH OF GEAR 2 AND PINION 3

Similar to the previous section,

\[
d_2 = (R_{b2} + r_{b3})\tan\theta_2
\]

\[
\alpha_{2in} = \frac{(R_{b2} + r_{b3})\tan\theta_2 - \sqrt{r_{o3}^2 - r_{b3}^2}}{R_{b2}}
\]

\[
\alpha_{2fin} = \frac{\sqrt{r_{o2}^2 - R_{b2}^2}}{R_{b2}}
\]

For a two step-up gear train, the instantaneous length, \(a_2\), becomes

\[
a_2 = R_{b2}(a_{2in} + J_{22}\alpha_{22})
\]

where \(J_{22}\) is given by Equation (A-207) and

\[J_{22} = 1, 2, \ldots, k_2.\]

When \(a_2 \geq a_{2fin}\) it must be re-initialized to \(a_{2in}\) until mesh no. 1 has completed its full cycle.
For a three step-up gear train, the length, $a_2$, becomes

$$a_2 = R_{b2}(a_{2in} + j_{23} \Delta a_{23})$$

(A-221)

with $\Delta a_{23}$ given by Equation (A-212). $j_{23}$ stands for the number of times $a_2$ has been incremented and again depends on the length of contact. Re-initialization follows the same rule as given above.

The sign of $a_2$ is obtained as follows: (See Equation (A-215).)

For

$$a_2 < \tan \theta_2 : \quad a_2 = +1$$

$$a_2 = \tan \theta_2 : \quad a_2 = 0$$

(A-222)

$$a_2 > \tan \theta_2 : \quad a_2 = -1$$
3. MESH OF GEAR 3 AND PINION 4

Again, in the same vein as in Section 1,

\[ d_3 = (R_{b3} + r_{b4}) \tan \theta_3 \]  \hspace{1cm} (A-223)

\[ a_{3\text{in}} = \frac{(R_{b3} + r_{b4}) \tan \theta_3 - \sqrt{r_{04}^2 - r_{b4}^2}}{R_{b3}} \]  \hspace{1cm} (A-224)

\[ a_{3\text{fin}} = \frac{\sqrt{r_{03}^2 - R_{b3}^2}}{R_{b3}} \]  \hspace{1cm} (A-225)

The instantaneous length, \( a_3 \), is given by

\[ a_3 = R_{b3} (a_{3\text{in}} + j_{33} \Delta a_{33}) \]  \hspace{1cm} (A-226)

where \( \Delta a_{33} \) is given by Equation (A-211) and \( j_{33} = 1, 2, \ldots, K_3 \), whenever \( a_3 \geq a_{3\text{fin}} \) it must be re-initialized until mesh no. 1 completed its full cycle.

The sign of \( a_3 \) is again obtained with the help of an expression Equation (A-215):
\begin{align*}
\mathbf{\hat{a}_3} < \tan \theta_3 : \quad & \mathbf{\hat{a}_3} = +1 \\
\mathbf{\hat{a}_3} = \tan \theta_3 : \quad & \mathbf{\hat{a}_3} = 0 \\
\mathbf{\hat{a}_3} > \tan \theta_3 : \quad & \mathbf{\hat{a}_3} = -1
\end{align*}
APPENDIX B

DESIGN OF UNEQUAL ADDENDUM INVOLUTE GEAR SETS WITH STANDARD CENTER DISTANCES

One of the ways of preventing undercutting in pinions with small numbers of teeth, which must mesh with gears of unequal and larger numbers of teeth, is to decrease the pinion dedendum together with the gear addendum by the necessary amount. To maintain standard working depth for such a mesh, the addendum of the pinion as well as the dedendum of the gear are increased by the same amount. (This, of course, presupposes that the gear is not undercut by this modification.) This can be accomplished without any change in the standard base and pitch radii or the associated standard center distance by "withdrawing" the hob during the cutting of the pinion and feeding it "deeper" when the gear is cut. In this way the pinion, which has its outside radius increased by the hob withdrawal distance, will have a larger than standard circular tooth thickness at its standard pitch circle. The outside radius of the gear is decreased by the same amount. Because the cutter is fed to full depth, the gear tooth thickness at the standard pitch circle will be less than standard.

The following gives the design steps for this type of gearing and illustrates them by way of an example.
1. **STANDARD GEAR NOMENCLATURE**

\[ N = \text{number of teeth of gear} \]
\[ n = \text{number of teeth of pinion} \]
\[ \theta = \text{pressure angle} \]
\[ P_d = \frac{N}{2R_p} = \frac{n}{2r_p} = \text{diametral pitch} \]
\[ P_{cs} = \text{standard circular pitch at pitch circle, where also} \]
\[ P_{cs} = \frac{\pi}{P_d} \]
\[ R_p, r_p = \text{pitch radii of gear and pinion, respectively} \]
\[ R_b = R_p \cos \theta, \text{ base circle radius of gear} \]
\[ r_b = r_p \cos \theta, \text{ base circle radius of pinion} \]
\[ T_{cs}, t_{cs} = \frac{P_{cs}}{2}, \text{ circular tooth thickness of gear and pinion, respectively, at standard pitch circles} \]
\[ \frac{1}{P_d} = \text{standard gear addendum} \]
\[ \frac{1.157}{P_d} = \text{standard gear dedendum} \]
2. **DETERMINATION OF HOB WITHDRAWAL DISTANCE C**

Figure B-1 indicates the relationship between the pinion pitch radius, \( r_p \), its root radius, \( r_r \), and the rack cutter addendum \( A \). The root radius is formed by the addendum line of the cutter tooth, so that

\[
 r_r = r_p - A 
\]  
\( (B-1) \)

In order to avoid undercutting, the addendum line of a sharp cornered cutter must not pass below point \( L \), the tangent point of the line of action and the base circle (see Figure B-2). The minimum root radius, \( r_{rm} \), becomes for this case

\[
 r_{rm} = r_b \cos \theta = r_p \cos^2 \theta 
\]  
\( (B-2) \)

When the rack tooth corner is rounded off with a radius \( r_c \), as shown in Figure B-3, the effective addendum line of the cutter tooth moves up the distance \( r_c (1 - \sin \theta) \). To avoid undercutting with this type of cutter, the effective addendum line must not pass the base circle below point \( L \) in Figure B-2. This allows a reduction of the minimum allowable root radius to

\( B-3 \)
FIGURE B-1

RELATIONSHIP BETWEEN PINION PITCH RADIUS, $r_p$, RACK CUTTER ADDENDUM A AND RESULTING PINION ROOT RADIUS, $r_r$
FIGURE B-2
MINIMUM ROOT RADIUS, $r_{rm}$, FOR RACK CUTTER WITH SHARP CORNER
FIGURE B-3

RACK CUTTER WITH CORNER RADIUS, $r_c$, (EFFECTIVE ADDENDUM OF CUTTER IS DECREASED)
\[ r_{rmc} = r_p \cos^2 \theta - r_c (1 - \sin \theta) \]  

(B-3)

By common usage the corner radius may either be

\[ r_c = 0.1 \, t_s = \frac{0.05r}{P_d}, \]  

(B-4)

or it is chosen such that the second term in Equation (B-3) becomes

\[ r_c (1 - \sin \theta) \approx \frac{0.157}{P_d} \]  

(B-5)

(This makes for an effective cutter addendum of \(1/P_d\).)

Hob withdrawal becomes necessary if the root radius, obtained by setting the cutter to standard depth, is smaller than the minimum given by Equation (B-3). Thus, the hob withdrawal \(C\) is obtained from Equations (B-1) and (B-3), i.e.,

\[ C = r_{rmc} - r_r \]  

(B-6)

With the cutter addendum \(A = 1.157/P_d\), and using the expression of Equation (B-4) for the rack corner radius, Equation (B-6) becomes

B-7
If Equation (B-5) is used, with the same definition of the cutter addendum, one obtains for $C$

$$C = \frac{1}{P_d} (1 + .157\sin\phi) - r_p\sin^2\phi$$  \hspace{1cm} (B-7)

3. **OUTSIDE RADIOf PINION AND GEAR BLANKS**

The outside radius of the pinion blank becomes

$$r_o = r_p + \frac{1}{P_d} + C$$  \hspace{1cm} (B-8)

The outside radius of the gear blank becomes

$$R_o = R_p + \frac{1}{P_d} - C$$  \hspace{1cm} (B-9)
4. **TOOTH THICKNESS AT PITCH CIRCLES OF PINION AND GEAR**

Since the thickness of the hobtooth at the pinion pitch circle will be reduced by the amount $2C \tan \theta$, due to the withdrawal $C$, the circular thickness of the pinion tooth at this location will be increased by this amount, i.e.,

$$ t_c = t_{cs} + 2C \tan \theta $$  \hspace{1cm} (B-10)

The gear tooth thickness will be decreased to

$$ T_c = T_{cs} - 2C \tan \theta $$  \hspace{1cm} (B-11)

at its pitch circle.
5. TOOTH THICKNESS AT OUTSIDE AND BASE RADII BY INVOLUTOMETRY

The circular tooth thickness at an arbitrary radius of an involute tooth may be obtained if the tooth thickness, radius, and pressure angle of any other location, such as the pitch circle, are known together with the pressure angle at the arbitrary radius. [See Equation (5-5), pg. 80, E. Buckingham: Analytical Mechanics of Gears, McGraw-Hill Book Co., Inc. New York, 1949.]

Accordingly, the circular tooth thickness, $t_o$, at the outside radius of the pinion, may be obtained from

$$t_o = t_c \frac{r_0}{r_p} - 2r_0(INV\theta_{OP} - INV\theta)$$

(B-12)

where

$$INV\theta = \tan\theta - \theta$$, the involute function corresponding to the pressure angle $\theta$

$\theta = $ pressure angle of mesh. This is also the pressure angle at the pitch circle.

$$\theta_{OP} = \cos^{-1} \frac{r_b}{r_o}$$, the pressure angle associated with the outside radius $r_o$ of the pinion.
Similarly, the circular tooth thickness, $T_o$, at the outside radius of the gear is obtained from:

$$T_o = T_c \frac{R_o}{R_p} - 2R_o(INV \theta_{OG} - INV \theta) \tag{B-13}$$

where

$$\theta_{OG} = \cos^{-1} \frac{R_b}{R_o}$$

is the pressure angle associated with the outside radius of the gear.

The tooth thickness at the base circle radius of the modified pinion is given by

$$t_b = t_c \cos \theta + 2r_b INV \theta \tag{B-14}$$

This becomes for the gear

$$T_b = T_c \cos \theta + 2R_b INV \theta \tag{B-15}$$

(The above is only of theoretical interest in case the inter-tooth space is not cut below the base circle radius.)
6. **EXPRESSIONS FOR CONTACT RATIO AND PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO**

The expression for contact ratio [Equation 4-19, pg. 72, E. Buckingham: Analytical Mechanics of Gears] is given by

\[
m_p = \frac{\sqrt{R_o^2 - R_b^2} + \sqrt{r_o^2 - r_b^2} - (R_b + r_b)\tan \theta}{p_{ca}\cos \theta}
\]  \hspace{1cm} (B-16)

If the contact ratio of a certain mesh is larger than unity, and one wishes to reduce it to unity by reducing the outside radius of the pinion, one may find this new pinion outside radius with the help of

\[
r_o' = \sqrt{r_b^2 + \left[ p_{ca}\cos \theta + (R_b + r_b)\tan \theta - \sqrt{R_o^2 - R_b^2} \right]^2}
\]  \hspace{1cm} (B-17)
7. **EXAMPLE**

Design a pinion and a gear with the following specifications:

\[ P_d = 44, \; N = 42, \; n = 8, \; \theta = 20^\circ \]

This gives

\[ R_p = .47727 \; \text{in.} \; (1.212 \; \text{cm}) \quad r_p = .09091 \; \text{in.} \; (.231 \; \text{cm}) \]

\[ R_b = R_p \cos 20^\circ = .44848 \; \text{in.} \; (1.139 \; \text{cm}) \quad r_b = .08542 \; \text{in.} \; (.217 \; \text{cm}) \]

\[ P_{cs} = \frac{\pi}{44} = .07140 \; \text{in.} \; (.181 \; \text{cm}) \quad T_{cs} = t_{cs} = .03570 \; \text{in.} \; (.091 \; \text{cm}) \]

According to Equation (B-7), the hob withdrawal \( C \) is computed as

\[ C = \frac{1}{44} - .09091 \sin^2 20^\circ = .012093 \; \text{in.} \; (.031 \; \text{cm}) \]

Then, according to Equations (B-8) and (B-9)

\[ r_o = .09091 + .022727 + .012093 = .12573 \; \text{in.} \; (.319 \; \text{cm}) \]

\[ R_o = .47727 + .022727 - .012093 = .48790 \; \text{in.} \; (1.239 \; \text{cm}) \]
The new tooth thickness at the pitch radii is found with the help of Equations (B-10) and (B-11)

\[
t_c = 0.03570 + 2(0.012093)\tan 20^\circ = 0.04450 \text{ in. (0.113 cm)}
\]

\[
T_c = 0.03570 - 2(0.012093)\tan 20^\circ = 0.02689 \text{ in. (0.068 cm)}
\]

For the purposes of the present analysis, the pinion outside radius is now reduced to obtain a contact ratio of unity. With Equation (B-17)

\[
r'_0 = 0.110 \ldots (0.279 \text{ cm}) \text{ This is essentially the unmodified pinion radius of } 0.113 \text{ in. (0.287 cm).}
\]

Now compute the circular tooth thickness at the outside radii \(r'_0\) and \(R_0\) using the following data in Equations (B-12) and (B-13):

\[
s = 20^\circ \quad \text{INV } 20^\circ = 0.01490
\]

\[
\theta_{0p} = \cos^{-1} \frac{0.08542}{0.110} = 39.0509^\circ \quad \text{INV } 39.0509^\circ = 0.12969
\]

\[
\theta_{0g} = \cos^{-1} \frac{0.44848}{0.48790} = 23.1889^\circ \quad \text{INV } 23.1889^\circ = 0.02365
\]
Then

\[ t_o = 0.04450 \left( \frac{0.110}{0.0909} \right) - 2(0.110)(0.12969 - 0.0149) = 0.02860 \text{ in.} (0.073 \text{ cm}) \]

and

\[ T_o = 0.02689 \left( \frac{0.4879}{0.47727} \right) - 2(0.4879)(0.02365 - 0.0149) = 0.01896 \text{ in.} (0.048 \text{ cm}) \]

These are sufficient to allow for rounding off the teeth.

Finally, check that the gear is not undercut. The actual root radius of the gear is

\[ R_o = \frac{2.157}{P_d} = 0.4879 - 0.04902 = 0.4389 \text{ in.} (1.115 \text{ cm}) \]

The minimum permissible root radius without undercutting, according to Equations (B-2) and (B-4), is computed from

\[ R_{rm} = R_P \cos^2 \theta - \frac{0.025}{P_d} = 0.47727(0.883) - 0.0023 \]

\[ = 0.419 \text{ in.} (1.064 \text{ cm}) \]

Thus, the gear is not undercut.
APPENDIX C

COMPUTER MODELS FOR STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

The present appendix contains descriptions, listings, and sample outputs of the following involute gear-train-related computer programs:

1. Program INVOL 1: Design of unequal addendum involute gear and pinion set with unity contact ratio.

2. Program INVOL 2: Point and cycle efficiencies for single pass involute step-up gear mesh with unity contact ratio.

3. Program INVOL 3: Point and cycle efficiencies for three pass involute step-up gear train in spin environment. (All meshes have unity contact ratio).

4. Program INVOL 4: Point and cycle efficiencies for two pass involute step-up gear train in spin environment. (All meshes have unity contact ratio).

The relevant background, the input parameters, the manner of the computations and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.
Program INVOL 1: Design of Unequal Addendum Involute Gear and Pinion Set with Unity Contact Ratio.

The program INVOL 1 is based on Appendix B which shows the design equations for unequal addendum involute gear sets with standard center distances.

The nomenclature of the program is chosen to coincide as much as possible with that of Appendix B.

a. Input Parameters (See also Program INVOL 1, below)

The following parameters represent the input data of the program:

PUBD = Pd, the diametral pitch
NG = Ng, the number of teeth of the gear
NP = Np, the number of teeth of the pinion
THETAD = θ, the pressure angle of the mesh as well as of the hob (in degrees)
ISTOP = arbitrary single-digit integer for multiple data sets. Must be zero for last set of data.

b. Computations

The program computes the following quantities (where not otherwise indicated, consult Section 1 of Appendix B for nomenclature):

CAPRD = Rp
RP = rp
CAPRB = Rb
RB = rb
- PSUBC - $P_c$
- TSTAND - $t_{cs}$
- C - hob withdrawal distance. See eq (B-7)
- RO - $r_o$, See eq (B-8). This is the original pinion blank radius.
- CAPRO - $R_o$, see eq (B-9)
- TC - $t_c$, see eq (B-10)
- CAPTC - $T_c$, see eq (B-11)
- TB - $t_b$, see eq (B-14). Note also the computation of the involute function at the end of the program.
- CAPTB - $T_b$, see eq (B-15)
- ROFIN - $r_o'$, see eq (B-17). This is the pinion outside radius for unity contact ratio.
- THETOG - $\theta_{OG}$, according to expression associated with eq (B-13)
- THETOP - $\theta_{OP}$, according to expression associated with eq (B-12)
- CAPTO - $T_o$, see eq (B-13)
- TO - $t_o$, see eq (B-12). Computed with ROFIN.
- CRATIO - $m_p$, see eq (B-16). This is the original contact ratio, which uses the unmodified pinion blank radius RO.
- CRFIN - represents the contact ratio when computed with the final pinion blank radius ROFIN.
- ROOT - actual root radius of pinion
- CAPROOT - actual root radius of gear
- CAPRMIN - $R_{rm}$, see equations (B-3) and (B-4)
c. Output of Program

The output of the program is best explained by means of the first sample computations which are shown at the end of the program. This example is identical with that given in section 7 of Appendix B. The output lists the following:

I. The input parameters, $PSUBD$, $NG$, $NP$, and pressure angle, $\Theta_{D}$, are printed out.

II. The above is followed by computational results for

CAPRP
RP
CAPRB
TSTAND
C
CAPRO

$RO$, this is the original pinion blank radius before the unity contact ratio modification.

$CRATIO$, note that for the given case, this original contact ratio equals 1.3

$ROFIN$, the pinion outside radius which corresponds to unity contact ratio.

$CRFIN$, this, of course, has to be unity because of the use of $ROFIN$. This computation serves as a check.

$CAPTC$, this tooth thickness, as well as the following one, is useful for strength computations.
THEOGD, this pressure angle, as well as the following one, corresponds to the final outside radii of the gear and pinion, respectively, and is needed for the computations of the tooth thicknesses at the outside radii.

CAPTO, this tooth thickness at the outside radius of the gear, as well as the following one for the pinion, must be sufficiently large to allow for the presence of a tip radius.

TO, CAPTB, TB, CAPROOT, ROOT, CAPRMIN

The values of the last five parameters are to some extent interconnected. First, it is important to see whether the gear is undercut. This does not occur as long as CAPROOT is larger than CAPRMIN, i.e., the actual root radius is larger than the minimum allowable one. The gear tooth thickness at the base circle is of interest for strength purposes. Since, for the given case, CAPRB = .44849 in. (1.139 cm) and CAPROOT = .4388 in. (1.115 cm) the base circle lies above the root circle, and the greatest cross-section of the tooth is approximately equal to CAPTB. TB is called the theoretical
pinion tooth thickness at the base circle since it is possible that the base circle lies below the minimum allowable root circle of the pinion. In such a case, the actual tooth may be weaker than indicated by this dimension. For the present case, \( RB = 0.08543 \text{ in.} \) (.217 cm) and \( \text{ROOT} = 0.07671 \text{ in.} \) (.195 cm), and therefore, the base circle, which lies above the root circle, gives a good indication of the actual tooth cross-section at the root.
Program INVOL 1

This program contains five different sets of gears. This will be of use in programs INVOL 2, INVOL 3 and INVOL 4.
PROGRAM INVL

C DESIGN OF UNEQUAL ADDENDUM INVOLUTE GEAR AND PINION SET
C WITH UNITY CONTACT RATIO

REAL INV

1 READ(5,2)PSUBD,NG,NP,THEA,STOP
2 FORMAT(*10,3.2,1510,F10.4,10.1,10.1)

PI = 3.14159

Z = PI/180

THETA = THEA/2

CAPRD = NG/(2.*PSUBD)

CPRB = CAPRD*COS(THETA)

PSUBC = PI/PSUBD

TSTAND = PSUBC/2

C = 1./PSUBD - NP*Sin(THETA)*Sin(THETA)

CAPRO = CAPRD + 1./PSUBD - C

RD = RP - 1./PSUBD + C

CAPTC = TSTAND - 2.*C*Tan(THETA)

TC = TSTAND - 2.*C*Tan(THETA)

CAPRB = CAPTC*COS(THETA) - 2.*CAPRD*INV(THETA)

TB = TC*COS(THETA) - 2.*RB*INV(THETA)

RFIN = SORT(RB+RB+.PSUBC*COS(THETA) + (CAPRB + RB)*TAN(THETA))

I = SORT(CAPRO+CAPRO - CAPRB+CAPRA)**2

THEG = ACOS(CAPRO/CAPR)

THEPG = THEG/2

THEPD = THEPG/2

CAPTO = CAPTC+CAPRO/CAPRD - 2.*CAPRO*INV(THETO) - INV(THETO))

TG = TC+ROFIN/RP - 2.*ROFIN*INV(THETO) - INV(THETO))

CRATIO = (CAPRO+CAPRO - CAPRB+CAPRB) + SORT(ROFIN+ROFIN)

I = RB+RB+(CAPRO+CAPRO)*TAN(THETA)/PSUBC*COS(THETA)

RST = RO = 2.*PSUBD

CAPPN = CAPPD+CAPRD*COS(THETA)*COS(THETA) + 0.5*pi*(1. - SIN(THETA))/PSUBD

WRITE 6,3)PSUBD,NG,NP,THEA

J FORMAT(*10,3.2,1510,F10.4,10.1,10.1)

WRITE 6,4)CAPRD,RP

A FORMAT(*10,3.2,1510,F10.4,10.1,10.1)

WRITE 6,5)CAPRO,RB

B FORMAT(*10,3.2,1510,F10.4,10.1,10.1)

WRITE 6,6)STAND

D FORMAT(*10,3.2,1510,F10.4,10.1,10.1)

WRITE 6,7)

WRITE 6,8)
FUNCTION INV
    74/74 - OPT=1
    REAL FUNCTION INV(TETA)
    INV = TAN(TETA) - TETA
    RETURN
    END

PROGRAM INVOL
55 WRITE(6,8)CAPRO=NO
8 FORMAT(*GEPGEANA RADIUS (CAPRO) =*F9.5,3X,ORIGINAL PINION BLA
11X.RADIUS (RO) =*F9.5)
WRITE(6,9)CRATIO
9 FORMAT(*GEPGINAL CONTACT RATIO (CRATIO) =*F6.3)
WRITE(6,10)ROFIN
10 FORMAT(*GEPINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) =*F9.5)
WRITE(6,11)CFIN
11 FORMAT(*GEPINAL CONTACT RATIO (CFIN) =*F6.3)
WRITE(6,12)CPRC=TC
12 FORMAT(*GEPGEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPIC) =*F9.5,3X
1X,PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) =*F9.5)
WRITE(6,13)THEOGR THEOPD
13 FORMAT(*GEPGEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGR) =*F9.5
1X,PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPD) =*F9.5)
WRITE(6,14)CAPO=TO
14 FORMAT(*GEPGEAR TOOTH THICKNESS AT OUTSIDE RADII (CAPO) =*F9.5
1X,PINION TOOTH THICKNESS AT FINAL OUTSIDE RADII (TO) =*F9.5)
WRITE(6,15)CAPB=TO
15 FORMAT(*GEPGEAR TOOTH THICKNESS AT BASE RADIUS (CAPB) =*F9.5,3X
1X,PINION TOOTH THICKNESS AT BASE CIRCLE (TB) =*F9.5)
WRITE(6,16)CAPR=CAPRMIN
16 FORMAT(*GEPGEAR TOOTH THICKNESS AT ROOT CIRCLE OF GEAR (CAPR) =*F9.5,3X
1X,PINION TOOTH THICKNESS AT ROOT CIRCLE OF PINION (CAPRMIN) =*F9.5
2 IF(CAPR>GE. CAPRMIN)WRITE(*,17)

C
17 IF(CAPR<LT. CAPRMIN)WRITE(*,18)
18 WRITE(*,19)ROOT

D
19 IF(CAPR=GE. ROOT)WRITE(*,19)
IF(CAPR=LT. ROOT)GO TO 9999
GO TO 1
9999 STOP
<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demetal Pitch (mod)</td>
<td>44.00</td>
</tr>
<tr>
<td>Gear Number of Teeth (NG)</td>
<td>42</td>
</tr>
<tr>
<td>Pinion Number of Teeth (NP)</td>
<td>8</td>
</tr>
<tr>
<td>Pressure Angle ((\theta))</td>
<td>20.00</td>
</tr>
<tr>
<td>Gear Pitch Radius (CAPMIN)</td>
<td>0.1772</td>
</tr>
<tr>
<td>Pinion Pitch Radius (NP)</td>
<td>0.9906</td>
</tr>
<tr>
<td>Gear Base Radius (CAPMIN)</td>
<td>0.4299</td>
</tr>
<tr>
<td>Pinion Base Radius (NP)</td>
<td>0.8643</td>
</tr>
<tr>
<td>Standard Tooth Thickness at Pitch Radii (ISAND)</td>
<td>0.03470</td>
</tr>
<tr>
<td>Non-Withdrawal Distance (CT)</td>
<td>0.01209</td>
</tr>
<tr>
<td>Gear Blank Radius (CAPMIN)</td>
<td>0.1671</td>
</tr>
<tr>
<td>Original Pinion Blank Radius (NG)</td>
<td>1.2573</td>
</tr>
<tr>
<td>Original Contact Ratio (CAPTION)</td>
<td>1.342</td>
</tr>
<tr>
<td>Pinion Outside Radius for Unity Contact Ratio (TRPIN)</td>
<td>11.000</td>
</tr>
<tr>
<td>Final Contact Ratio (CAPFIN)</td>
<td>1.0099</td>
</tr>
<tr>
<td>Gear Tooth Thickness at Pitch Circle (CAPC)</td>
<td>0.02400</td>
</tr>
<tr>
<td>Pinion Tooth Thickness at Pitch Circle (IC)</td>
<td>0.04450</td>
</tr>
<tr>
<td>Gear Pressure Angle at Outside Radii (THEOR)</td>
<td>23.17894</td>
</tr>
<tr>
<td>Pinion Pressure Angle at Final Outside Radii (THEOR)</td>
<td>39.95092</td>
</tr>
<tr>
<td>Gear Tooth Thickness at Outside Radii (CAPO)</td>
<td>0.1693</td>
</tr>
<tr>
<td>Pinion Tooth Thickness at Final Outside Radii (IOI)</td>
<td>0.2843</td>
</tr>
<tr>
<td>Gear Tooth Thickness at Base Circle (CAPB)</td>
<td>0.03664</td>
</tr>
<tr>
<td>Theoretical Pinion Tooth Thickness at Base Circle (IB)</td>
<td>0.04437</td>
</tr>
<tr>
<td>Radius of Root Circle of Gear (CAPROOT)</td>
<td>0.53865</td>
</tr>
<tr>
<td>Min Allowable Radius of Root Circle of Gear (CAPMIN)</td>
<td>0.4109</td>
</tr>
<tr>
<td>The Gear is Not Hardened</td>
<td></td>
</tr>
<tr>
<td>Radius of Root Circle of Pinion (ROOT)</td>
<td>0.79671</td>
</tr>
</tbody>
</table>
DIAMETRAL PITCH (1-INCH) = 65.00

GEAR NUMBER OF THIN (NO) = 27

PINION NUMBER OF TEETH (NP) = 9

PRESSURE ANGLE (T-FIA) = 20.00

GEAR PITCH RADIUS (CAPH) = .20709

PINION PITCH RADIUS (NP) = .06923

GEAR BASE RADIUS (CAPB) = .19717

PINION BASE RADIUS (NPB) = .05586

STANDARD TOOTH THICKNESS AT PITCH RADIUS (1-INCH) = .02447

MOD. INTEGRAL DISTANCE (IC) = .89729

GEAR BEAK RADIUS (CAPB) = .21579

ORIGINAL PINION BEAK RADIUS (NPB) = .05199

CENTRAL CONTACT POINTS \textsuperscript{2} (CP) = 1.371

PINION OUTSIDE RADIUS FOR UNIT CONTACT RATIO (ROF) = .09889

FINAL CONTACT RATIO (ICF) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPC) = .01686

PINION TOOTH THICKNESS AT PITCH CIRCLE (IC) = .02047

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEO) = 25.25.52

PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEO) = 38.65782

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPC) = .01297

PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (I) = .02024

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPB) = .02354

THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (I) = .02363

RADIUS OF ROOT CIRCLE OF GEAR (CAPR00) = .19261

MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPMIN) = .19161

THE GEAR IS NOT HOEY
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diametral Pitch (PS1/2D)</td>
<td>3.7700</td>
</tr>
<tr>
<td>Gear Number of Teeth (NG)</td>
<td>27</td>
</tr>
<tr>
<td>Pinion Number of Teeth (NP)</td>
<td>9</td>
</tr>
<tr>
<td>Pressure Angle (θ)</td>
<td>20.00</td>
</tr>
<tr>
<td>Gear Pitch Radius (CAPP)</td>
<td>0.17532</td>
</tr>
<tr>
<td>Pinion Pitch Radius (NP)</td>
<td>0.05944</td>
</tr>
<tr>
<td>Gear Base Radius (CABA)</td>
<td>0.10419</td>
</tr>
<tr>
<td>Pinion Base Radius (NPB)</td>
<td>0.05492</td>
</tr>
<tr>
<td>Standard Tooth Thickness at Pitch Radio (1+1/4D)</td>
<td>0.02940</td>
</tr>
<tr>
<td>Horn Ultimate Distance (C)</td>
<td>0.00615</td>
</tr>
<tr>
<td>Gear Blank Radius (CAPP)</td>
<td>0.15218</td>
</tr>
<tr>
<td>Original Pinion Blank Radius (NP)</td>
<td>0.07750</td>
</tr>
<tr>
<td>Original Contact Ratio (CRatio)</td>
<td>1.371</td>
</tr>
<tr>
<td>Pinion Outside Radius for Unity Contact Ratio (1/CRatio)</td>
<td>0.94284</td>
</tr>
<tr>
<td>Final Contact Ratio (CRatio)</td>
<td>1.000</td>
</tr>
<tr>
<td>Gear Tooth Thickness at Pitch Circle (CAPP)</td>
<td>0.01592</td>
</tr>
<tr>
<td>Pinion Tooth Thickness at Pitch Circle (1/C)</td>
<td>0.02488</td>
</tr>
<tr>
<td>Gear Pressure Angle at Outside Radius (θ)</td>
<td>25.25782</td>
</tr>
<tr>
<td>Pinion Pressure Angle at Final Outside Radius (θ)</td>
<td>36.05942</td>
</tr>
<tr>
<td>Gear Tooth Thickness at Outside Radius (CAPP)</td>
<td>0.01710</td>
</tr>
<tr>
<td>Pinion Tooth Thickness at Final Outside Radius (1/CRatio)</td>
<td>0.01710</td>
</tr>
<tr>
<td>Gear Tooth Thickness at Base Circle (CAPP)</td>
<td>0.1967</td>
</tr>
<tr>
<td>Theoretical Pinion Tooth Thickness at Base Circle (1/CRatio)</td>
<td>0.2581</td>
</tr>
<tr>
<td>Radius of Root Circle of Gear (CAPROOT)</td>
<td>0.19415</td>
</tr>
<tr>
<td>Minimum Allowable Radius of Root Circle of Gear (CAPMIN)</td>
<td>0.15347</td>
</tr>
<tr>
<td>The Gear is not Interchangeable</td>
<td></td>
</tr>
<tr>
<td>Radius of Root Circle of Pinion (NP)</td>
<td>0.06475</td>
</tr>
</tbody>
</table>
DIAMETRAL PITCH (P<sub>UNO</sub>) = 44.00
GEAR NUMBER OF TOOTH (NG) = 56
PINION NUMBER OF TOOTH (NP) = 8
PRESSURE ANGLE (Theta) = 20.00
GEAR PITCH RADIUS (CAPRI) = 0.3636
PINION PITCH RADIUS (RP) = 0.0989
GEAR BASE RADIUS (CBASI) = 0.59798
PINION BASE RADIUS (RB) = 0.08543
STANDARD TOOTH THICKNESS AT PITCH RADIUS (tSIA) = 0.03578
MOUTH AXIAL DISTANCE (C) = 0.91209
GEAR BLANK RADIUS (CBAPNO) = 0.04708
ORIGINAL PINION BLANK RADIUS (RO) = 0.12573
ORIGINIAL CONTACT RATIO (CRA) = 1.349
PINION OUTSIDE DIAMETER FOR UNITY CONTACT RATIO (ROFIN) = 0.10970
FINAL CONTACT RATIO (CFIN) = 1.000
GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPIC) = 0.02690
PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = 0.04450
GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEDO) = 22.44445
PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEPO) = 38.85337
GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPIO) = 0.01991
PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = 0.02401
GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPBI) = 0.0310
THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = 0.04437
RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = 0.59798
MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPPRI2) = 0.55957
THE GEAR IS NOT DESIGN....
RADIUS OF ROOT CIRCLE OF PINION (ROOT) = 0.07671
DIAMETRAL PITCH (PHEAD) = 65.00

GEAR NUMBER OF TEETH (NG) = 56

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (theta) = 20.00

GEAR PITCH RADIUS (CAPR) = .43677
PINION PITCH RADIUS (RP) = .06156

GEAR BASE RADIUS (CABR) = .40479
PINION BASE RADIUS (RB) = .05783

STANDARD TOOTH THICKNESS AT PITCH RADIUS (ISLAND) = .02417

HOE WILDCARD DISTANCE (IC) = .00889

GEAR BLANK RADIUS (CAPBH) = .43797
ORIGINAL PINION BLANK RADIUS (RO) = .98511

ORIGINAL CONTACT RATIO (CRATIO) = 1.349

PINION OUTSIDE RADIUS FOR UNIFORMITY CONTACT RATIO (ROFIN) = .07426

FINAL CONTACT RATIO (FCRFIN) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPIC) = .01212
PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .03012

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEO) = 22.4448
PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOD) = 38.85337

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPIC) = .1287
PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .01464

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPBH) = .02918
THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .03003

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .44478
MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .37879

THE GEAR IS NOT UNDERMOUNTED

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .049192
2. **Program INVOL 2: Point and Cycle Efficiencies for Single Pass Involute Step-Up Gear Mesh With Unity Contact Ratio**

The program INVOL 2 is based on section 3 of Appendix A, which gives the moment input-output relationship for a single step-up gear mesh with involute teeth. The mesh has unity contact ratio. Again, the nomenclature of the program is chosen to coincide as much as possible with that of the original derivation. The contact geometry is adapted from section 6c of Appendix A.

a. **Input Parameters (See also program in section d below)**

The following parameters represent the input data of the program. Most of these are taken from the results of INVOL 1 since the moment expressions are for unity contact ratio only.

- **CAPRP** = $R_p$, the pitch radius of the gear
- **RP** = $r_p$, the pitch radius of the pinion
- **CAPRO** = $R_o$, the outside radius of the gear
- **ROPIN** = $R'_o$, see eq. (B-17). This is the pinion outside radius for unity contact ratio.
- **RHOCAPN** = $r'_N$, the pivot radius of the gear
- **RHON** = $r'_N$, the pivot radius of the pinion
- **MU** = $\mu$, the coefficient of friction at both pivots as well as at the gear and pinion contact point
- **K** = range divisor, i.e., it represents the number of times the output moment and the efficiency are computed between initial and final contact of the gear and the pinion
ISTOP, arbitrary single digit for multiple data sets. It must be zero for the last set of data.

b. **Computations**

Both point and cycle efficiencies are based on eq. (A-25).

With the help of eq. (3),

\[
\eta_p = \frac{r_b - \mu [r_n + s(d - a)]}{R_b + \mu [r_n - sa]} \phi
\]

\[(C-1)\]

Since the angular velocity is constant for involute gears, and may be expressed in terms of the base circle radii \( R_b \) and \( r_b \),

\[K_{ratio} = \frac{\phi}{\phi} = \frac{R_b}{r_b}\]

\[(C-2)\]

Therefore,

\[\eta_p = E_2 \text{ (POINTEP)} \]

\[(C-3)\]

as given by eq. (A-26).

The cycle efficiency expression is based on eq. (4). If one replaces both integrals by summations, one obtains

\[\eta_c = \frac{\Sigma I_0 \Delta \phi}{\Sigma I_{Min} \Delta \phi}\]

\[(C-4)\]

where \( \Delta \phi \) and \( \Delta \phi \) now represent incremental changes in the input and output angles, respectively. The input moment is constant
over the total interval. Also

\[ \Sigma \Delta \phi = \alpha_{FIN} - \alpha_{IN} \]  

(C-5)

(See eqs. (A205) and (A206).)

The increment \( \Delta \psi \) is also constant and may therefore be
taken outside the summation sign. It is expressed with the
help of \( \Delta \phi \)

\[ \Delta \phi = \Delta \alpha = \frac{\alpha_{FIN} - \alpha_{IN}}{K} = \Delta \alpha_{LPH} \]  

(C-6)

(This is an adaptation of eq. (A-207) to the single step-up
mesh.)

With the above

\[ \Delta \psi = \text{kratio} \Delta \alpha \]  

(C-7)

Substitution into eq. (C-4) gives

\[ \rho_0 = \frac{\text{kratio} \Delta \alpha \Sigma M_o}{\text{Min} (\alpha_{FIN} - \alpha_{IN})} \]  

(C-8)

Since

\[ \rho_p = \text{kratio} \frac{M_o}{\text{Min}} \]  

(C-9)

one obtains for the cycle efficiency

\[ \rho_0 = \frac{\Lambda \alpha \Sigma \rho_p}{(\alpha_{FIN} - \alpha_{IN})} \] \hspace{1cm} \text{(CYCLEEFF)} 

(C-10)

C-18
To arrive at expressions for both \texttt{POINT} and \texttt{CYCLEF} in the program, the following other important computations are necessary:

\begin{align*}
\text{ALPHIN} &= \alpha_{IN}, \text{ see eq. (A-205)} \\
\text{ALPHFIN} &= \alpha_{FIN}, \text{ see eq. (A-206)} \\
D &= d, \text{ see eq. (A-204)} \\
A &= a, \text{ see eq. (A-203)}
\end{align*}

The signum value \( s \) is obtained according to eqs. (A-215) and (A-216), which are alternate ways of expressing eqs. (A-6) to (A-8).

c. Output of Program (see Program INVOL 2, below)

The output of the program is best explained by means of the single sample computation which is shown at the end of the program. This example uses the data of the first sample output of INVOL 1. The output lists the following:

I. Input Parameter:

The input parameters \texttt{CAPRP}, \texttt{RP}, \texttt{CAPRO}, \texttt{RO} and \texttt{THETAD} are reproduced. In addition, the following dimensions, which were selected from a practical viewpoint, are shown:

\begin{align*}
\text{RHOCAPN} &= \rho_N = .060 \text{ in. (}.152 \text{ cm)} \\
\text{RHON} &= \rho_n = .030 \text{ in. (}.076 \text{ cm)} \\
\text{MU} &= \mu = .2 \\
K &= 25 \quad \text{(no substantial changes in cycle efficiency were encountered for larger values of } K \text{)}
\end{align*}
II. Computed Values

The point efficiency is listed as a function of the angle $\alpha$, while the cycle efficiency is computed for the interval from $\alpha_{IN}$ to $\alpha_{FIN}$. The signum parameter, $s$, is listed for checking purposes.
PROGRAM INVOL2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C POINT AND CYCLE EFFICIENCY FOR SINGLE PASS INVOLUTE STEP-UP
C
C COEFFICIENT OF FRIC
C REAL MI,MOT
1 PEAT =CARRP,CP,CARR,RFIN,THETA,ROFCAPN,P0,
2 FORMAT(6F16.4/TF16.4/TF16.4/TF16.4)
WRITE(6,3) CARRP,CP,CARRP,ROFCAPN,PHON,K
3 FORMAT(10FAR PITCH RADIUS (CARRP) =*F7.5,1X,ROFIN PITCH RADIUS
1 (CP) =*F7.5,0GEAR OUTSIDE RADIUS (CARRP) =*F7.5,2X,ROFIN OUT
2 RADIUS FOR UNITY CONTACT RATIO (ROFIN) =*F7.5,0PRESSURE ANG
3 LE IN PEGGES (THETA) =*F7.5,0GEAR PIVOT RADIUS (ROFCAPN) =
1 F5.3,1X,ROFIN PIVOT RADIUS (ROFCAPN)*F5.3,0COEFFICIENT OF FRIC
4 TON (ALPHA) =*F6.2,0FRANC DISordination (R) =*F6.2/
5 PI = 3.14159
7 = PI/190.
THETA = 3IFIN
CABB = CARRP/COS(THETA)
CARRP = CARRP*COS(THETA)
KP = K*COS(THETA)
ADP = ((CARRP-CP)*TAN(THETA) - ROFIN*ROFIN - RA*R)/CARRP
ALPH = ROFIN*CARRP - CARRP*ALPH/CARRP
DELPH = (ALPHIN-ALPHIN)/K
N = CARRP*CP)*TAN(THETA)
CARRP = CARRP+ (I-1)*DELPH
IF (ALPHA+LT. TAN(THETA))/5 = 1.
IF (ALPHA+LT. TAN(THETA))/5 = 0.
IF (ALPHA+LT. TAN(THETA))/5 = 1.
A = CARRP*ALPHA
AA = 1. - M/N*(R/M + 0.5)/P
P0 = 1. - M/N*(R/M + 0.5)/CARRP
POINT = AA/P0
MTOT = MTOT + PTENS
ALPHAD = ALPHAD*Z
WRITE(4,4) ALPHAD,5,POINTEN
4 FORMAT(A4,4) ALPHAD =*F6.2,0,3X,*F6.4,0
5 CONTINUE
C Cycles = M/ALPHAD*MTOT/(ALPHAD-ALPHAD)
WRITE(4,4) CYCLEFF
4 FORMAT(///) CYCLE EFFICENCY =*F6.4)
IF (ISTOP =*F0. 0) GO TO 0000
GO TO :
40 STOP 0000
FNM
GEAR PITCH RADIUS (CAPRO) = .47727  PINION PITCH RADIUS (RP) = .09091
GEAR OUTSIDE RADIUS (CAPRO) = .49791  PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROSIN) = .11000
PRESSURE ANGLE IN DEGREES (THETA) = 20.00
GEAR PIVOT RADIUS (PHOCADN) = .060  PINION PIVOT RADIUS (PHON) = .070
COEFFICIENT OF FRICTION (MU) = .20
RANGE DIVIDEND (k) = 25

<table>
<thead>
<tr>
<th>Alpha</th>
<th>S</th>
<th>Pointf</th>
<th>S</th>
<th>Pointf</th>
<th>S</th>
<th>Pointf</th>
<th>S</th>
<th>Pointf</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.97</td>
<td>1</td>
<td>7.905</td>
<td>1</td>
<td>7.905</td>
<td>1</td>
<td>7.905</td>
<td>1</td>
<td>7.905</td>
</tr>
<tr>
<td>15.92</td>
<td>1</td>
<td>7.979</td>
<td>1</td>
<td>7.979</td>
<td>1</td>
<td>7.979</td>
<td>1</td>
<td>7.979</td>
</tr>
<tr>
<td>15.96</td>
<td>1</td>
<td>8.054</td>
<td>1</td>
<td>8.054</td>
<td>1</td>
<td>8.054</td>
<td>1</td>
<td>8.054</td>
</tr>
<tr>
<td>17.94</td>
<td>1</td>
<td>8.129</td>
<td>1</td>
<td>8.129</td>
<td>1</td>
<td>8.129</td>
<td>1</td>
<td>8.129</td>
</tr>
<tr>
<td>17.79</td>
<td>1</td>
<td>8.204</td>
<td>1</td>
<td>8.204</td>
<td>1</td>
<td>8.204</td>
<td>1</td>
<td>8.204</td>
</tr>
<tr>
<td>17.69</td>
<td>1</td>
<td>8.279</td>
<td>1</td>
<td>8.279</td>
<td>1</td>
<td>8.279</td>
<td>1</td>
<td>8.279</td>
</tr>
<tr>
<td>14.81</td>
<td>1</td>
<td>8.355</td>
<td>1</td>
<td>8.355</td>
<td>1</td>
<td>8.355</td>
<td>1</td>
<td>8.355</td>
</tr>
<tr>
<td>14.37</td>
<td>1</td>
<td>8.430</td>
<td>1</td>
<td>8.430</td>
<td>1</td>
<td>8.430</td>
<td>1</td>
<td>8.430</td>
</tr>
<tr>
<td>14.72</td>
<td>1</td>
<td>8.506</td>
<td>1</td>
<td>8.506</td>
<td>1</td>
<td>8.506</td>
<td>1</td>
<td>8.506</td>
</tr>
<tr>
<td>19.86</td>
<td>1</td>
<td>8.582</td>
<td>1</td>
<td>8.582</td>
<td>1</td>
<td>8.582</td>
<td>1</td>
<td>8.582</td>
</tr>
<tr>
<td>19.48</td>
<td>1</td>
<td>8.658</td>
<td>1</td>
<td>8.658</td>
<td>1</td>
<td>8.658</td>
<td>1</td>
<td>8.658</td>
</tr>
<tr>
<td>19.75</td>
<td>1</td>
<td>8.735</td>
<td>1</td>
<td>8.735</td>
<td>1</td>
<td>8.735</td>
<td>1</td>
<td>8.735</td>
</tr>
<tr>
<td>24.99</td>
<td>1</td>
<td>8.811</td>
<td>1</td>
<td>8.811</td>
<td>1</td>
<td>8.811</td>
<td>1</td>
<td>8.811</td>
</tr>
<tr>
<td>20.43</td>
<td>1</td>
<td>8.887</td>
<td>1</td>
<td>8.887</td>
<td>1</td>
<td>8.887</td>
<td>1</td>
<td>8.887</td>
</tr>
<tr>
<td>20.77</td>
<td>1</td>
<td>8.963</td>
<td>1</td>
<td>8.963</td>
<td>1</td>
<td>8.963</td>
<td>1</td>
<td>8.963</td>
</tr>
<tr>
<td>21.46</td>
<td>1</td>
<td>9.115</td>
<td>1</td>
<td>9.115</td>
<td>1</td>
<td>9.115</td>
<td>1</td>
<td>9.115</td>
</tr>
<tr>
<td>21.80</td>
<td>1</td>
<td>9.191</td>
<td>1</td>
<td>9.191</td>
<td>1</td>
<td>9.191</td>
<td>1</td>
<td>9.191</td>
</tr>
<tr>
<td>22.15</td>
<td>1</td>
<td>9.267</td>
<td>1</td>
<td>9.267</td>
<td>1</td>
<td>9.267</td>
<td>1</td>
<td>9.267</td>
</tr>
<tr>
<td>22.49</td>
<td>1</td>
<td>9.343</td>
<td>1</td>
<td>9.343</td>
<td>1</td>
<td>9.343</td>
<td>1</td>
<td>9.343</td>
</tr>
<tr>
<td>22.83</td>
<td>1</td>
<td>9.419</td>
<td>1</td>
<td>9.419</td>
<td>1</td>
<td>9.419</td>
<td>1</td>
<td>9.419</td>
</tr>
<tr>
<td>23.17</td>
<td>1</td>
<td>9.495</td>
<td>1</td>
<td>9.495</td>
<td>1</td>
<td>9.495</td>
<td>1</td>
<td>9.495</td>
</tr>
<tr>
<td>23.52</td>
<td>1</td>
<td>9.571</td>
<td>1</td>
<td>9.571</td>
<td>1</td>
<td>9.571</td>
<td>1</td>
<td>9.571</td>
</tr>
<tr>
<td>24.20</td>
<td>1</td>
<td>9.723</td>
<td>1</td>
<td>9.723</td>
<td>1</td>
<td>9.723</td>
<td>1</td>
<td>9.723</td>
</tr>
</tbody>
</table>

CYCLE EFFICIENCY = .8564
3. **Program INVOL 3: Point and Cycle Efficiencies for Three Pass Involute Step-Up Gear Train in Spin Environment (All Meshes Have Unity Contact Ratio)**

The program INVOL 3 is based on section 4 of Appendix A, which derives the moment input-output relationship for a three pass step-up gear train operating in a spin environment. Again, all meshes have unity contact ratio. As previously, the nomenclature of the program is chosen to coincide as closely as possible with that of the original derivations. The expressions for the contact geometry and other auxiliary geometric terms may be found in section 6 of Appendix A.

a. **Input Parameters (see Program INVOL 3, below)**

The following parameters represent the input data for the program. Those which involve gear dimensions only must be obtained from the results of INVOL 1 since the moment expressions are derived for unity contact ratio only.

- $\mu$, the coefficient of friction at all pivots and at all tooth contact points
- $\text{RPM}$, revolutions per minute of the fuze body
- $\text{CAPRP}_1 = R_{p1}$
- $\text{CAPRP}_2 = R_{p2}$
- $\text{CAPRP}_3 = R_{p3}$
- $\text{RP}_2 = r_{p2}$
- $\text{RP}_3 = r_{p3}$
- $\text{RP}_4 = r_{p4}$
\[ \begin{align*}
\theta_1 &= \theta_1 \\
\theta_2 &= \theta_2 \\
\theta_3 &= \theta_3 \\
\text{ISTOP, arbitrary single digit integer for multiple data set.} & \\
\text{It must be zero for last set of data.} & \\
R_1 &= R_1 \\
R_2 &= R_2 \\
R_3 &= R_3 \\
R_4 &= R_4 \\
\rho_1 &= \rho_1 \\
\rho_2 &= \rho_2 \\
\rho_3 &= \rho_3 \\
\rho_4 &= \rho_4 \\
R_{b1} &= R_{b1} \\
R_{b2} &= R_{b2} \\
R_{b3} &= R_{b3} \\
R_{b4} &= R_{b4} \\
R_{o1} &= R_{o1} \\
R_{o2} &= R_{o2} \\
R_{o3} &= R_{o3} \\
R'_{o2} &= R'_{o2} \\
R'_{o3} &= R'_{o3} \\
R'_{o4} &= R'_{o4} \\
M_1 &= m_1, \text{ mass of input gear 1} \\
M_2 &= m_2, \text{ mass of gear and pinion 2}
\end{align*} \]
M3 = m3, mass of gear and pinion 3
M4 = m4, mass of pinion 4
MD = md^2, the "mass-distance" product contained in the expression for the input moment M_in
K = K3, the range divisor which is associated with gear 3, the driving gear of the last mesh (see eq. (A-211))

b. Computations (see COMMENT cards in program)

I. Computation of MIN, GAMMAS and BETAS

To start with, the program computes the input moment

\[ M_{\text{IN}} = M_{\text{IN}} = md^2 \cdot 2 \]  \hspace{1cm} (C-11)

Subsequently, the angles \( \gamma_2, \gamma_3, \gamma_4 \) and \( \beta_1, \beta_2, \beta_3 \) are established according to the expressions given in section 6b of Appendix A.

II. Determination of Gear Train Constants

The determination of the gear train constants consists of the following:

\[ \text{RATIO} = K_{\text{RATIO}} \]  \hspace{1cm} (see eq. (2)). Since the angular velocity is constant, this parameter may be expressed in terms of the applicable base radii, i.e.,

\[ \frac{R_{b1} \times R_{b2} \times R_{b3}}{R_{b2} \times R_{b3} \times R_{b4}} \]

TEST1, TEST2 and TEST3 represent the tangent functions of the mesh pressure angles, which are used in conjunction with the values of the signum functions, a.
D1, D2 and D3 are given by eqs. (A-204), (A-217) and (A-223), respectively, and represent the distances between the points of tangency to the base circles along the lines-of-action of the three meshes.

MT0T = 0 represents the initialization of the sum of the output moments. This is used for the determination of the cycle efficiency.

III. Determination of Initial and Final Values of ALPHAS

Initialization of ALPHAS and Centrifugal Forces

The determination of the initial and final angles of rotation is accomplished with the help of subroutine ALPHA, at the end of the program, which makes use of eqs. (A-205), (A-206), (A-218), (A-219), (A-224) and (A-225). Thus, the initial values of the individual angles of rotation, ALPHAl, ALPHA2 and ALPHA3 are represented by ALIIN, AL2IN and AL3IN, while the final angles are given by ALFIN, AL2FIN and AL3FIN.

The angular increments of gears 3, 2 and 1, i.e., DELAL3, DELAL2 and DELAL1, are determined with the help of eqs. (A-211) - (A-213), respectively.

The centrifugal forces, which act on the pivots of the various gear and/or pinion assemblies, are obtained by way of eqs. (A-33), (A-57), (A-84) and (A-107).

IV. Point and Cycle Efficiencies (See "output moment"
in program)

Both point and cycle efficiencies are based on eq. (A-125) for the output moment \( M_{04} = M_{04} \).

C-27
The point efficiency is computed directly in the manner of eq. (3), i.e.,

\[ p = K_{\text{ratio}} \frac{M_0^4}{M_{\text{in}}} = \text{POINTEF} \]  
(C-12)

The cycle efficiency is treated in the manner of eq. (C-8), i.e.,

\[ p = K_{\text{ratio}} \frac{\Delta \alpha_1 \Sigma M_0^4}{M_{\text{in}} (\alpha_{1\text{FIN}} - \alpha_{1\text{IN}})} = \text{CYCLEFF} \]  
(C-13)

The program gives the summation as

\[ M_{\text{TOT}} = \Sigma M_0^4 \]  
(C-14)

V. Gear Train Motion Model

The simulation of the gear train motion, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop which starts with statement label no. 14 (card no. 116) and ends with card no. 199. As discussed earlier, the motions of the individual driving gears are initialized at their respective angles, \(a_{1\text{IN}}, a_{2\text{IN}}, a_{3\text{IN}}\). (This starting of the total train is arbitrary and is done only for convenience. There is an infinite number of other starting combinations each of which produces a different starting point efficiency.) The position of each mesh is subsequently incremented by the appropriate \(\Delta \alpha_1, \Delta \alpha_2, \text{or } \Delta \alpha_3\). When the angle \(\alpha_{1\text{FIN}}\) reaches the magnitude \(\alpha_{1\text{FIN}}\), CYCLEFF is determined, and the computation is ended. Since meshes 2 and 3 go through numerous cycles while mesh 1 goes through one cycle, they have
to be reset to their initial angles of rotation once their respective final angles have been reached. This is accomplished by the conditional statements on cards 116 and 117.

The values of the signum functions \( s_1, s_2 \) and \( s_3 \) are determined continuously according to eqs. (A-216), (A-222) and (A-227).

The instantaneous positions of the contact \( A_1 = a_1 \), \( A_2 = a_2 \), and \( A_3 = a_3 \) are determined for each of the meshes by an appropriate adaptation of eq. (A-203). (See also eqs. (A-214), (A-220) and (A-226).)

The determination of the instantaneous output moment, \( M_0^4 = N_0^4 \), requires the continuous computation of the variable quantities \( A_1 \) to \( A_{20} \), \( C_1 \) to \( C_6 \) and \( D_1 \) to \( D_4 \), which are given originally in conjunction with the various equilibrium conditions in section 4 of Appendix A. The program uses the following nomenclature for these variables:

- \( AA_1 \) to \( AA_{20} \)
- \( CC_1 \) to \( CC_6 \)
- \( DD_1 \) to \( DD_4 \)

0. Output (see Program INVOL 3, below)

Again, the output of the program is best explained by means of the sample computation which is shown at the end of the program. This example uses the gear data of the first three sample computations of program INVOL 1. The output lists the following:
I. **Input Parameters**

**Mesh No. 1**

<table>
<thead>
<tr>
<th>CAPRP1</th>
<th>Rp1</th>
<th>.47727 in. (1.212 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRB1</td>
<td>Rb1</td>
<td>.44849 in. (1.139 cm)</td>
</tr>
<tr>
<td>CAPRO1</td>
<td>Ro1</td>
<td>.48791 in. (1.239 cm)</td>
</tr>
<tr>
<td>RP2</td>
<td>rp2</td>
<td>.09091 in. (.231 cm)</td>
</tr>
<tr>
<td>RB2</td>
<td>rb2</td>
<td>.08543 in. (.217 cm)</td>
</tr>
<tr>
<td>RO2</td>
<td>ro2</td>
<td>.11000 in. (.279 cm)</td>
</tr>
</tbody>
</table>

(This is an ROFIN as given by INVOL 1.)

Also,

| THETA1 | θ1   | 20° |

**Mesh No. 2**

<table>
<thead>
<tr>
<th>CAPRP2</th>
<th>Rp2</th>
<th>.20769 in. (.527 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRB2</td>
<td>Rb2</td>
<td>.19517 in. (.496 cm)</td>
</tr>
<tr>
<td>CAPRO2</td>
<td>Ro2</td>
<td>.21579 in. (.548 cm)</td>
</tr>
<tr>
<td>RP3</td>
<td>rp3</td>
<td>.06923 in. (.176 cm)</td>
</tr>
<tr>
<td>RB3</td>
<td>rb3</td>
<td>.06506 in. (.165 cm)</td>
</tr>
<tr>
<td>RO3</td>
<td>ro3</td>
<td>.08089 in. (.205 cm)</td>
</tr>
</tbody>
</table>

Also,

| THETA2 | θ2   | 20° |

**Mesh No. 3**

<table>
<thead>
<tr>
<th>CAPRP3</th>
<th>Rp3</th>
<th>.17532 in. (.445 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRB3</td>
<td>Rb3</td>
<td>.16475 in. (.418 cm)</td>
</tr>
<tr>
<td>CAPRO3</td>
<td>Ro3</td>
<td>.18216 in. (.463 cm)</td>
</tr>
<tr>
<td>RP4</td>
<td>rp4</td>
<td>.05844 in. (.148 cm)</td>
</tr>
<tr>
<td>RB4</td>
<td>rb4</td>
<td>.05492 in. (.139 cm)</td>
</tr>
<tr>
<td>RO4</td>
<td>ro4</td>
<td>.06828 in. (.173 cm)</td>
</tr>
</tbody>
</table>

C-30
Also,

\[ \theta_3 = 20^\circ \]

In addition,

\[ \mu = 0.2 \]

\[ \text{RPM} = 1000 \]

\[ M_1 = \frac{m_1}{200} \times 10^{-4} \text{ lb-sec}^2/\text{in.} \quad (8.943 \text{ slug}) \]

\[ M_2 = \frac{m_2}{200} \times 10^{-4} \text{ lb-sec}^2/\text{in.} \quad (3.049 \text{ slug}) \]

\[ M_3 = \frac{m_3}{200} \times 10^{-5} \text{ lb-sec}^2/\text{in.} \quad (1.222 \text{ slug}) \]

\[ M_4 = \frac{m_4}{200} \times 10^{-6} \text{ lb-sec}^2/\text{in.} \quad (0.136 \text{ slug}) \]

\[ R_1 = \frac{r_1}{200} \times 0.75 \text{ in.} \quad (1.905 \text{ cm}) \]

\[ R_2 = \frac{r_2}{200} \times 0.75 \text{ in.} \quad (1.905 \text{ cm}) \]

\[ R_3 = \frac{r_3}{200} \times 0.75 \text{ in.} \quad (1.905 \text{ cm}) \]

\[ R_4 = \frac{r_4}{200} \times 0.75 \text{ in.} \quad (1.905 \text{ cm}) \]

\[ \rho_1 = \rho_1 \times 0.060 \text{ in.} \quad (0.152 \text{ cm}) \]

\[ \rho_2 = \rho_2 \times 0.030 \text{ in.} \quad (0.076 \text{ cm}) \]

\[ \rho_3 = \rho_3 \times 0.025 \text{ in.} \quad (0.064 \text{ cm}) \]

\[ \rho_4 = \rho_4 \times 0.020 \text{ in.} \quad (0.051 \text{ cm}) \]

\[ md^2 = \frac{15}{200} \times 10^{-4} \text{ lb-sec}^2/\text{in.} \quad (16.944 \text{ g-cm}^2) \]

\[ K = 25 \]

II. Computed Values

The point efficiency is given as a function of the angle \( \alpha \), together with the signum parameters \( s_1, s_2 \) and \( s_3 \) (given for checking purposes). The cycle efficiency is shown at the end of the output. In addition, the input moment, \( \text{MIN} \), is printed out.
Program INVOL 3
PROGRAM INVP3    74/74  OPT=1      FTN 4.6-420  07/06/78  18:01.55  PAGE 1

      1 PROGRAM INVP3 (INPUT*INPUT*TAPES=INPUT*TAPE6=OUTPUT)
      2      PROGRAM INVP3 (INPUT*INPUT*TAPES=INPUT*TAPE6=OUTPUT)
      3      POINT AND CYCLE EFFICIENCIES FOR THREE PASS "INVPOLUTE" STEP-UP
      4      IN SPIN ENVIRONMENT (ALL WFSM6'S HAVE UNITY CONTACT RATIO)
      5      REAL MIN+MU,M1,M2,M3,M4,M5,M6,M7,M8,M9
      6      READ AND WRITE INPUT DATA
      7
      8      100 READ(S+1)MU,RPH+CAPR1, CAPR2, CAPR3,RP2, RP3, RP4, THETA1,
      9           1THETA2,THETA3,1STIP
      10           READ(S+1)RH01,RH02,RH03,RH04
      11           READ(S+2)CAPR1, CAPR2, CAPR3, RP2, RP3, RP4
      12           READ(S+3)M1, M2, M3, M4
      13           READ(S+4)M5, M6, M7, M8
      14           PI = 3.14159
      15           OMEGA = RPH + PI/180
      16           OM2 = OMEGA*OMEGA
      17           1 FORMAT(F16.3,F10.0,F16.5/3F10.4/1)
      18           2 FORMAT(4F10.4)
      19           3 FORMAT(4F10.4)
      20           4 FORMAT(6F10.5)
      21           5 FORMAT(6F10.5)
      22           6 FORMAT(6F10.5)
      23           7 FORMAT(10.2/1)
      24
      25       C COMPUTATION OF MIN+ GAMMA, AND RETAS
      26
      27       30 MIN = W0*OM2
      28       GAMMA2 = ACOS((R1*R1 + R2*R2 - (CAPR1+PP2)*(CAPR1+RP2))/
      29           1(R1*R1+R2*R2))
      30       GAMMA3 = ACOS((R2*R2 + R3*R3 - (CAPR2+PP3)*(CAPR2+RP3))/
      31           1(R2*R2+R3*R3))
      32       GAMMA4 = GAMMA2 + GAMMA3
      33       GAMMA5 = ACOS((R3*R3 + R4*R4 - (CAPR3+PP4)*(CAPR3+RP4))/
      34           1(R3*R3+R4*R4))
      35       DELTA2 = ACOS((CAPR1+RP2)*(CAPR1+PP2) + R1*R1 - R2*R2)/
      36           1(R1*R1+R2*R2)
      37       DELTA3 = ACOS((CAPR2+RP3)*(CAPR2+PP3) + R2*R2 - R3*R3)/
      38           1(R2*R2+R3*R3)
      39       DELTA4 = ACOS((CAPR3+RP4)*(CAPR3+PP4) + R3*R3 - R4*R4)/
      40           1(R3*R3+R4*R4)
      41       DELTA5 = ACOS((CAPR4+RP5)*(CAPR4+PP5) + R4*R4 - R5*R5)/
      42           1(R4*R4+R5*R5)
      43       RETA1 = PI - DELTA2
      44       RETA2 = GAMMA2 + PI - NFRA3
      45       RETA3 = GAMMA3 + PI - NFRA4
      46       RETA4 = GAMMA4 + PI - NFRA5
      47       RETA5 = GAMMA5 + PI - NFRA6
      48       WRITE(6)MIN+MU,RPH+CAPR1, CAPR2, CAPR3, RP2, RP3, RP4, THETA1,
      49           1THETA2, THETA3
      50           WRITE(6,10)RH01, RH02, RH03, RH04
      51           WRITE(6,11)CAPR3, CAPR4, CAPR5, RP2, RP3, RP4, THETA1,
      52           1THETA2, THETA3
      53           WRITE(6,12)RH01, RH02, RH03, RH04
      54           WRITE(6,13)CAPR3, CAPR4, CAPR5, RP2, RP3, RP4, THETA1,
      55           1THETA2, THETA3
      56           WRITE(6,14)RH01, RH02, RH03, RH04
      57           WRITE(6,15)CAPR3, CAPR4, CAPR5, RP2, RP3, RP4, THETA1,
      58           1THETA2, THETA3
      59           WRITE(6,16)RH01, RH02, RH03, RH04
      60           WRITE(6,17)CAPR3, CAPR4, CAPR5, RP2, RP3, RP4, THETA1,
55 WRITE(6+12)CAPR1,CAPR2,CAPR3,HO2,RO2,RO4
56 WRITE(6+13)K,X
57 FORMAT(1*5X,PHI,F12.6,2X,HU,F8.3,3X,PRM,F6.0)
58 14X+CAPR1 = +F8.5,3X+CAPR3 = +F8.5,3X+CAPR2 = +F8.5,3X
59 14X+RO2 = +F8.5,3X+RO4 = +F8.5,5/6X
60 3X+THETA1 = +F8.3,3X+THETA2 = +F8.3,3X+THETA3 = +F8.3,3/
61 9 FORMAT(6X+GHL) = +F7.5,3X+RH02 = +F7.5,3X+RH03 = +F7.5,3X
62 +RH04 = +F7.5/)
63 11 FORMAT(6X+CAPR1 = +F7.5,3X+CAPR2 = +F7.5,3X+CAPR3 = +F7.5,3X
64 13X+RO2 = +F7.5,3X+RO4 = +F7.5,3X)
65 12 FORMAT(6X+CAPR1 = +F7.5,3X+CAPR2 = +F7.5,3X+CAPR3 = +F7.5,3X
66 13X+RO2 = +F7.5,3X+RO4 = +F7.5,3X)
67 13 FORMAT(6X+HO = +E10.3,6X+RH03 = +E15.3,3X)
68 84 Z = PHI/180.0
69 THETA1 = THESTA/2
70 THETA2 = THETA2*2
70 THETA3 = THETA3*2
70 C DETERMINATION OF GEAR TRAIN CONSTANTS
70 C
70 C
70 Q RATIO = CAPR3/CAPR1/2*(PB2+RB3]*R84)
71 TEST1 = TAN(THEA)
72 TEST2 = TAN(THEA)
73 TEST3 = TAN(THEA)
74 D1 = (CAPR1 + PB2)TAN(THEA)
75 D2 = (CAPR2 + RB3)TAN(THEA)
76 D3 = (CAPR3 + R84)TAN(THEA)
77 DCT = 0.
78 C DETERMINATION OF INITIAL AND FINAL VALUES OF ALPHAS
79 C CALL ALPHA(CAPR1,RB2,THETA1,CAPR1,RO2,AL11N,AL11F)
80 CALL ALPHA(CAPR2,RB2,THETA2,CAPR2,RO2,AL21N,AL21F)
81 CALL ALPHA(CAPR3,RB2,THETA3,CAPR3,RO4,AL31N,AL31F)
82 C
83 DELAL1 = (AL11F - AL11N)/3
84 DELAL2 = (AL13N - AL13F)/3
85 C
86 C INITIALIZATION OF ALPHAS
87 C ALPHAN = AL11N
88 ALPHAN = AL21N
89 ALPHAN = AL31N
90 C
91 C CENTRIFUGAL FORCE
PROGRAM INVOLV 76/74 FNL=1

T1 = M1*X1+Y1
T2 = M2*X2+Y2
T3 = M3*X3+Y3
Tn = Mn*Xn+Yn

DENOM = 1 + MU*NU

UPDATE VALUES OF ALPHAS

IF T1 > T2 IF T2 > T3 THEN T3 = AL1

IF T1 > T3 IF T3 > T2 THEN T2 = AL1

IF T2 > T1 IF T1 > T3 THEN T3 = AL2

IF T2 > T3 IF T3 > T1 THEN T1 = AL2

IF T3 > T1 IF T1 > T2 THEN T2 = AL2

IF T3 > T2 IF T2 > T1 THEN T1 = AL2

TEST TO DETERMINE IF CONTACT POINT IS IN APPROACH OR RECESS

IF S > 1.
IF S = -1.
AT PITCH POINT S = n.

IF(ALPHA1 .LT. TEST1)*1.
IF(ALPHA2 .LT. TEST2)*1.
IF(ALPHA3 .LT. TEST3)*1.
IF(ALPHA1 .GT. TEST1)*1.
IF(ALPHA2 .GT. TEST2)*1.
IF(ALPHA3 .GT. TEST3)*1.

DETERMINATION OF INPUT FOR MOMENT EXPRESSIONS

A1 = ALPHA1*CAPR1
A2 = ALPHA2*CAPR2
A3 = ALPHA3*CAPR3

AA1 = ABS(SIN(GAMMA1) - MU*COS(GAMMA1)/DENOM)
AA2 = ABS((1. - 5.8*MINUS2)*COS(THETA3-THETA3) + MU*(5.8 - 5.3)
1*MIN(THETA3-THETA3)/DENOM)
AA3 = ABS((COS(GAMMA3) + MU*MIN(GAMMA3)/DENOM)
AA4 = ABS((COS(GAMMA3) + MU*MIN(GAMMA3)/DENOM)
AA5 = ABS((1. - 5.8*MINUS2)*COS(THETA2-THETA2) - MU*(5.8 - 5.3)
1*MIN(THETA2-THETA2)/DENOM)
AA6 = ABS((COS(GAMMA2) + MU*COS(GAMMA2)/DENOM)
AA7 = ABS((1. - 5.8*MINUS2)*COS(THETA3-THETA3) + MU*(5.8 - 5.3)
1*MIN(THETA3-THETA3)/DENOM)
AA8 = ABS((1. - 5.8*MINUS2)*COS(THETA2-THETA2) + MU*(5.8 - 5.3)
1*MIN(THETA2-THETA2)/DENOM)
AA9 = ABS((1. - 5.8*MINUS2)*COS(THETA3-THETA3) + MU*(5.8 - 5.3)
1*MIN(THETA3-THETA3)/DENOM)
AA10 = ABS((1. - 5.8*MINUS2)*COS(THETA2-THETA2) + MU*(5.8 - 5.3)
1*MIN(THETA2-THETA2)/DENOM)
AA11 = ABS((1. - 5.8*MINUS2)*COS(THETA1+THETA1) + (1. - 5.8*MINUS2)
1*MIN(THETA1+THETA1)/DENOM)
AA12 = ABS((1. - 5.8*MINUS2)*COS(THETA2-THETA2) + (1. - 5.8*MINUS2)
1*MIN(THETA2-THETA2)/DENOM)
PROGRAM INVOL3 74/76  _OPT=1

160 AA14 = ABS((MU*(1.-S1)+COS(BETA1+THEITAL) - (1.*MU+MU*S1))
1*SIN(BETA1+THEITAL))/DENOM
AA15 = ABS((MU*SIGMA2 - CO(SIGMA2)/DENOM)
AA16 = ABS((1.-MU+MU*S1)*SIN(BETA2-THETA2) - MU*(1.+S2))
1*COS(BETA2-THETA2)/DENOM
AA17 = ABS((1.-MU+MU*S1)*SIN(BETA1+THEITAL) + MU*(1.+S1))
1*COS(BETA1+THEITAL)/DENOM
AA18 = ABS(1./DENOM)
AA19 = ABS((MU*(1.+S1)+SIN(BETA1+THEITAL) - (1.*MU+MU*S1))
1*COS(BETA1+THEITAL)/DENOM

170 AA20 = ABS(MU/DENOM)
DD1 = RHA - MU*(S3+0.03-0.31) - RHO9*/AA2+AA6)
DD2 = -MU*(R0.03-0.0A5+AA9) + S2*(0.2-0.21) + RHA
DD3 = -M2 - MU*(S1+0.04-0.41) + RHO2*(AA11+AA14)
DD4 = CAPPA1 - MU*(S1+0.41 - RHO1*(A91+AA19))

CC1 = MUPRHO4*(AA1-0.1A)
CC2 = CAPPA3 - MUPRHO4*(AA1-0.1A)
CC3 = MUPRHO4*(AA1-0.1A)
CC4 = CAPPA2 - MUPRHO4*(AA1-0.1A)
CC5 = CAPPA2 - MUPRHO4*(AA1-0.1A)
CC6 = MUPRHO4*(AA1-0.1A)

C OUTPUT MOUNT

180 ALPHAT = ALPHAT-180./P1
HD4 = DD1*DD2*DD03*(CC2+CC4+004) - (MIN-T1+CC6) - T2+CC5*DD1*DD2
T/(CC2+CC4) = T3*CC9*DD1*CC2 - T4*CC4
POINTER = GATETROPO/M4
WRITE(6,15)ALPHAT+51.5*POINTER

15 FORMAT(6X,ALPHAT,=F4.2,0 (DEG)*360,0.5)+=F5.1-3X,=F5.1-3X,=F5.1-3X,=F5.1-3X
13X=53 =F5.1-3X,=POINTER,EFFICIENCY =*F7.5:
HTOT = HTOT + HD4

C ADVANCE SEAR TO NEXT POSITION
C

190 ALPHA1 = ALPHA1 + DELTA1
IF(ALPHA1 .GT. ALFIN) GO TO 16
ALPHA2 = ALPHA2 + DELTA2
ALPHA3 = ALPHA3 + DELTA3
GO TO 14

200 CYCLEFT = RHA+DELTA1+HTOT/(MIN+ALFIN-ALFIN)
WRITE(6,17)CYCLEFT
FORMA1:=(.09,52)*CYCLE EFFICIENCY =*F5.3)
IF(STOP .LT. 0.160 TO 100
STOP
END
SUBROUTINE ALPHA 76/74 OPT=1

SUBROUTINE ALPHA (CAPRA, RR, THETA, CAPRO, RO, ALIN, ALF(N))

THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS

ALIN = (1*CAPRA + RR)*TAN(THETA) - SQRT((PO*PO - RR*RR))/CAPRB
ALF(N) = SQRT(CAPRO*CAPR) - (CAPRA*CAPRB)/CAPRB
RETURN
END
<table>
<thead>
<tr>
<th>ALPHA1</th>
<th>Angle (Deg)</th>
<th>Si = 1.0</th>
<th>X = 1.0</th>
<th>S3 = 1.0</th>
<th>POINT EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.10</td>
<td>(Deg)</td>
<td>-1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1163</td>
</tr>
<tr>
<td>24.17</td>
<td>(Deg)</td>
<td>-1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1676</td>
</tr>
<tr>
<td>24.26</td>
<td>(Deg)</td>
<td>-1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1676</td>
</tr>
<tr>
<td>24.34</td>
<td>(Deg)</td>
<td>-1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1676</td>
</tr>
<tr>
<td>24.43</td>
<td>(Deg)</td>
<td>-1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1676</td>
</tr>
<tr>
<td>24.51</td>
<td>(Deg)</td>
<td>-1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1676</td>
</tr>
</tbody>
</table>

CYCLE EFFICIENCY = 0.1676
4. **Program INVOL 4**: Point and Cycle Efficiencies for Two
   Pass Involute Step-Up Gear Train in
   Spin Environment (All Meshes Have
   Unity Contact Ratio)

The program INVOL 4 is based on section 5 of Appendix A, which derives the moment input-output relationship for a two pass step-up gear train, operating in a spin environment. Here again, all meshes have unity contact ratio. INVOL 4 is very similar to INVOL 3 in its construction. Again, the expressions for the contact geometry and other auxiliary geometric terms may be found in section 6 of Appendix A.

a. **Input Parameters (see Program INVOL 4, below)**

The following parameters represent the input data for the program. Those which involve gear dimensions only must be obtained from the results of INVOL 1 since the moment expressions are again derived for unity contact ratio only.

- **MU** = \( \mu \), coefficient of friction at all pivots and at all tooth contact points
- **RPM**, revolutions per minute of the fuze body
- **CAPRP1** = \( R_p^1 \)
- **CAPRP2** = \( R_p^2 \)
- **RF2** = \( r_p^2 \)
- **RP3** = \( r_p^3 \)
- **THKTA1** = \( \theta_1 \)
- **THETA2** = \( \theta_2 \)
ISTOP, arbitrary single-digit integer for multiple data sets. It must be zero for last set of data.

\[ R_1 = R_1 \]
\[ R_2 = R_2 \]
\[ R_3 = R_3 \]
\[ RHO_1 = \rho_1 \]
\[ RHO_2 = \rho_2 \]
\[ RHO_3 = \rho_3 \]
\[ CAPRB_1 = R_{b1} \]
\[ CAPRB_2 = R_{b2} \]
\[ RB_2 = r_{b2} \]
\[ RB_3 = r_{b3} \]
\[ CAPR_1 = R_{o1} \]
\[ CAPR_2 = R_{o2} \]
\[ RO_2 = r_{o2} \]
\[ RO_3 = r_{o3} \]

\[ M_1 = m_1, \text{ mass of input gear 1} \]
\[ M_2 = m_2, \text{ mass of gear and pinion 2} \]
\[ M_3 = m_3, \text{ mass of pinion 3} \]
\[ MD = m d^2, \text{ "mass-distance" product contained in the following expression for the input moment } M_{in} \]

\[ K = K_2, \text{ the range divisor which is associated with gear 2, the driving gear of the last mesh for this case (see eq. (A-207))} \]
b. Computations (see COMMENT cards in program)

I. Computation of MIN, Gammas and Betas

To start with, the program computes the input moment

\[ \text{MIN} = \text{Min} = \text{md}^2 \]  

(C-15)

The program computes the angles \( \gamma_2, \gamma_3 \) and \( \beta_1, \beta_2 \) according to the expressions given in section 6b of Appendix A.

II. Determination of the Gear Train Constants

The determination of the gear train constants consists of the following:

\[ \text{RATIO} = K_{\text{ratio}} \]  

(see eq. (2)). Since the angular velocity is constant, this parameter may be expressed in terms of the applicable base radii, i.e.,

\[ \frac{R_{b1} \times R_{b2}}{R_{b2} \times R_{b3}} \]

\( \text{TEST1} \) and \( \text{TEST2} \) represent the tangent functions of the mesh pressure angles, which are used in conjunction with the values of the signum functions \( s \).

\( \text{D1}, \text{and D2} \) are given by eqs. (A-204) and (A-217), respectively, and represent the distances between the points of tangency to the base circles along the lines-of-action of the two meshes.

\( \text{MTOT} = 0 \) represents the initialization of the sum of the output moments. This is used for the determination of the cycle efficiency.

C-45
III. Determination of Initial and Final Values of ALPHAS, 
Initialization of ALPHAS and Centrifugal Forces

The determination of the initial and final angles of rotation is accomplished with the help of subroutine ALPHA, at the end of the program, which makes use of eqs. (A-205), (A-206) as well as (A-218) and (A-219). Thus, the initial values of the individual angles of rotation, ALPHAI and ALPHA2, are represented by AL1IN and AL2IN, respectively, while the final ones are given by AL1FIN and AL2FIN.

The angular increments of gears 2 and 1, i.e., DELAL2 and DELAL1, are determined with the help of eqs. (A-207) and (A-208), respectively.

The centrifugal forces, which act on the pivots of the various gear and/or pinion assemblies, are obtained by way of eqs. (A-131), (A-154) and (A-178).

IV. Point and Cycle Efficiencies (See "output moment" in program)

Both point and cycle efficiencies are based on eq. (A-193) for the output moment $M_0 = M_0^3$.

The point efficiency is computed directly in the manner of eq. (3), i.e.,

$$\eta_p = \frac{M_{02}}{M_{01}} = \text{POINTEF}$$ (C-16)

The cycle efficiency is treated in the manner of eq. (C-13), i.e.,
The program gives the summation as

\[ \text{MTOT} = \sum \Sigma \Delta \]  

V. Gear Train Motion Model

The simulation of the gear train motion, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop which begins with statement label no. 14 (card no. 98) and ends with card no. 161.

As discussed earlier, the motions of the individual driving gears are initialized at their respective angles, \( a_{1\text{IN}} \) and \( a_{2\text{IN}} \). (This starting of the total train is arbitrary and is done only for convenience. There is an infinite number of other starting combinations, each of which produces a different starting point efficiency.) The position of each mesh is subsequently incremented by the appropriate \( \Delta a_1 \) or \( \Delta a_2 \). When the angle \( \alpha_{1\text{FIN}} \) reaches the magnitude \( a_{1\text{FIN}} \), CYCLEFF is determined, and the computation is ended. Since mesh 2 goes through a number of cycles while mesh 1 goes through one cycle, mesh 2 has to be reset to its starting position, \( a_{2\text{IN}} \), once the angle \( a_{2\text{FIN}} \) has been reached. This is accomplished by the conditional statement on card no. 98.

The values of the signum functions \( s_1 \) and \( s_2 \) are determined continuously according to eqs. (A-216) and (A-222).

The instantaneous positions of the contact, \( A_1 = a_1 \) and \( A_2 = a_2 \), are determined for each of the meshes by appropriate
adaptations of eq. (A-203). (See also eqns. (A-214) and (A-220).)

The determination of the instantaneous output moment, \( M_{03} = M_{03} \), requires the continuous computation of the variable quantities \( A_1 \) to \( A_{14} \), \( C_1 \) to \( C_4 \) and \( D_1 \) to \( D_3 \), which are given originally in conjunction with the various equilibrium conditions in section 5 of Appendix A. The program uses the following nomenclature for these variables:

\[
\begin{align*}
A_{A1} & \text{ to } A_{A14} \\
C_{C1} & \text{ to } C_{C4} \\
D_{D1} & \text{ to } D_{D3}
\end{align*}
\]

**c. Output (see Program INVOL 4, below)**

The output of the program is again best explained with the help of the sample computation shown at the end of the program. This example uses the gear data of the fourth and fifth sample computations of program INVOL 1. The output lists the following:

**I. Input Parameters**

<table>
<thead>
<tr>
<th>Mesh No. 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{CAPRPI} = R_{p1} )</td>
<td>( .63636 \text{ in.} ) (( 1.616 \text{ cm} ))</td>
</tr>
<tr>
<td>( \text{CAPRBI} = R_{b1} )</td>
<td>( .59799 \text{ in.} ) (( 1.519 \text{ cm} ))</td>
</tr>
<tr>
<td>( \text{CAPROI} = R_{o1} )</td>
<td>( .64700 \text{ in.} ) (( 1.643 \text{ cm} ))</td>
</tr>
<tr>
<td>( R_{p2} = r_{p2} )</td>
<td>( .09091 \text{ in.} ) (( 0.231 \text{ cm} ))</td>
</tr>
<tr>
<td>( R_{b2} = r_{b2} )</td>
<td>( .08543 \text{ in.} ) (( 0.217 \text{ cm} ))</td>
</tr>
<tr>
<td>( R_{o2} = r_{o2} )</td>
<td>( .1097 \text{ in.} ) (( 0.279 \text{ cm} )) (This is a ROFIN as given by INVOL 1.)</td>
</tr>
<tr>
<td>Also ( \theta_1 )</td>
<td>( 20^\circ )</td>
</tr>
</tbody>
</table>

C-48
Mesh No. 2

CAPR2 = \( R_p2 \) = 0.43077 in. (1.094 cm)
CAPRB2 = \( R_b2 \) = 0.40479 in. (1.028 cm)
CAPRO2 = \( R_o2 \) = 0.43977 in. (1.117 cm)
RP3 = \( r_p3 \) = 0.06154 in. (0.156 cm)
RB3 = \( r_b3 \) = 0.05783 in. (0.147 cm)
RO3 = \( r_o3 \) = 0.07426 in. (0.189 cm)

Also

\( \Theta_2 = 20^\circ \)

In addition,

\( \mu = 0.2 \)
RPM = 1000

\( M_1 = m_1 = \frac{0.51079}{10^{-4}} \text{ lb-sec}^2/\text{in.} \) (8.943 g)
\( M_2 = m_2 = \frac{0.17413}{10^{-4}} \text{ lb-sec}^2/\text{in.} \) (3.049 g)
\( M_3 = m_3 = \frac{0.69788}{10^{-5}} \text{ lb-sec}^2/\text{in.} \) (1.222 g)

\( R_1 = \mathcal{R}_1 = 0.75 \text{ in.} \) (1.905 cm)
\( R_2 = \mathcal{R}_2 = 0.75 \text{ in.} \) (1.905 cm)
\( R_3 = \mathcal{R}_3 = 0.75 \text{ in.} \) (1.905 cm)

\( \rho_1 = \rho_{10} = 0.060 \text{ in.} \) (0.152 cm)
\( \rho_2 = \rho_{10} = 0.030 \text{ in.} \) (0.076 cm)
\( \rho_3 = \rho_{10} = 0.025 \text{ in.} \) (0.064 cm)

\( M_D = m_d^2 = \frac{0.15}{10^{-4}} \text{ lb-sec}^2 \text{ in.} \) (16.944 g-cm^2)

\( K = 25 \)
II. Computed Values

The point efficiency is given as a function of the angle $\alpha_1$, together with the signum parameters $\alpha_1$ and $\alpha_2$ (given for checking purposes). The cycle efficiency is shown at the end of the output. In addition, the input moment $MIN$ is printed out.
Program INVOL 4
PROGRAM INVOL

C POINT AND CYCLE EFFICIENCIES FOR TWO PASS INVOLUTE STEP-UP GEAR TRAIN
C IN SPIN ENVIRONMENT (ALL WHEELS HAVE UNITY CONTACT RATIO)
C
C REAL NIN,M1,R1,R2,N2,M2,M3,MTOT,M0
C
C READ AND WRITE INPUT DATA

100 READ(5,11)NU,RP,M1,CAPPR1,CAPPR2,RP2,RP3,THETA1,THETA2,ISTOP
    READ(5,21)R1,R2,R3
    READ(5,31)RHO1,RHO2,RHO3
    READ(5,41)CAPPR1,CAPPR2,R22,R33
    READ(5,51)CAPPR1,CAPPR2,R22,R33
    READ(5,61)M1,M2,M3
    READ(15,71)NU,K
    1 FORMAT(F10.5,F10.5/F10.5,F10.5/F10.5/F10.5/F10.5/F10.5/F10.5)
    2 FORMAT(3F10.5)
    3 FORMAT(3F10.5)
    4 FORMAT(4F10.5)
    5 FORMAT(4F10.5)
    6 FORMAT(3F10.5)
    7 FORMAT(F10.4/I3)
    PI = 3.14159
    OMEGA = RP*M1*2.*PI/60.
    OM2 = OMEGA*OMEGA

C COMPUTATION OF MIN. GAMMAS AND BETAS

30 KIN = W*OM2
    GAMMA2 = ACOS((R1*R1 + R2*R2 - (CAPPR1+CAPPR2)*(CAPPR1+CAPPR2))/
                (R1*R2*R2))
    GAMMA3 = ACOS((R2*R2 + R3*R3 - (CAPPR2+CAPPR3)*(CAPPR2+CAPPR3))/
                (R2*R3*R3))
    GAMMA4 = GAMMA2 + GAMMA3

35 DELTA2 = ACOS(((CAPPR1+CAPPR2)*R1*R2 + R1*R1 - R2*R2)/
                 (R1*R2*R2*R2))
    DELTA3 = ACOS(((CAPPR2+CAPPR3)*R2*R3 + R2*R2 - R3*R3)/
                 (R2*R3*R3*R3))
    DELTA4 = DELTA2 + DELTA3

40 BETA2 = PI - DELTA2
    BETAM2 = GAMMA2 - PI - DELTA3
    WRITE(6,8)KIN,NU,RP,M1,CAPPR1,CAPPR2,RP2,RP3,THETA1,THETA2
    WRITE(6,9)R1,R2,R3,M1,KIN
    WRITE(6,10)RHO1,RHO2,RHO3
    WRITE(6,11)CAPPR1,CAPPR2,R22,R33
    WRITE(6,12)CAPPR1,CAPPR2,R22,R33
    WRITE(6,13)KIN
    8 FORMAT(F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4)
    9 FORMAT(F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4)
    10 FORMAT(F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4,F10.4)
PROGRAM INVOLV

74/74 OPT=1

55 11 FORMAT(6X*CAPR8) = +F7.5+3X*CAPR82 = +F7.5+3X*R82 = +F7.5+3X,
1* R83 = +F7.5/1
12 FORMAT(6X*CAPR01 = +F7.5+3X*CAPR82 = +F7.5+3X*R02 = +F7.5+3X,
1* R03 = +F7.5/1
13 FORMAT(6X*MD) = +E18.3//6X RANGE DIVISOR = +I4/1

60 C CONVERSION TO RADIANS
.
C
Z = PI/180.
THETA1 = THETA1*Z
THETA2 = THETA2*Z

65 C DETERMINATION OF GEAR TRAIN CONSTANTS
.
C
RATIO = (CAPR02*CAPR81)/(RB2*RB3)
TEST1 = TAN(THETA1)
TEST2 = TAN(THETA2)
D1 = (CAPR81 + RB2)*TAN(THETA1)
D2 = (CAPR82 + RB3)*TAN(THETA2)
M01 = 6.

70 C DETERMINATION OF INITIAL AND FINAL VALUES OF ALPHAS
.
C
CALL ALPHA(CAPR81,R02,THETA1,CAPR01,R02,AL1IN,AL1FIN)
CALL ALPHA(CAPR82,R03,THETA2,CAPR02,R03,AL2IN,AL2FIN)

C
C
DELAL2 = (AL2FIN - AL2IN) / K
DELAL1 = DELAL2*RB2/CAPR81

80 C INITIALIZATION OF ALPHAS
.
C
ALPHA1 = AL1IN
ALPHA2 = AL2IN

C
C CENTRIFUGAL FORCES
.
C
T1 = N1*R1*OM2
T2 = N2*R2*OM2
T3 = N3*R3*OM2

C
C
DENOM = 1. + NUM1R

90 C UPDATE VALUES OF ALPHAS
.
C
16 IF(ALPHA2 .GT. AL2FIN) ALPHA2 = AL2IN

100 C TEST TO DETERMINE IF CONTACT POINT IS IN APPROACH OR Recess
.
C
IF APPROACH, S = 1.
IF RECESS, S = -1.
S = PITCH POINT, S = 0.

105 C
C
IF(ALPHA1 .LT. TEST15) = 1.
IF(ALPHA2 .LT. TEST215) = 1.
Determination of Input for Moment Expressions

A1 = ALPHA1*CAPPA1
A2 = ALPHA2*CAPPA2
AA1 = ABS((1.0+MU*MU*52)-COS(BETA2-THE2)+MU*(5.0-1.0)*
      SIN(BETA2-THE2)/DENOM)
AA2 = ABS((1.0+MU*MU*52)-MU*COS(GAMMA2))/DENOM)
AA3 = ABS((1.0+MU*MU*52)-SIN(BETA2-THE2)+(1.0+MU*MU*52)*
      COS(BETA2-THE2))/DENOM)
AA4 = ABS((1.0+MU*MU*52)-SIN(GAMMA2))/DENOM)
AA5 = ABS((1.0+MU*MU*52)-SIN(BETA1-THE1)+(1.0+MU*MU*52)*
      COS(BETA1-THE1))/DENOM)
AA6 = ABS((1.0+MU*MU*52)-SIN(BETA1-THE1)+(1.0+MU*MU*52)*
      COS(BETA1-THE1))/DENOM)
AA7 = ABS((1.0+MU*MU*52)-SIN(BETA2-THE2)+(1.0+MU*MU*52)*
      COS(BETA2-THE2))/DENOM)
AA8 = ABS((1.0+MU*MU*52)-SIN(GAMMA2))/DENOM)

Output Moment

MO3 = (MO1+MO2)/CC1-CC1*CC1-CC1

Output Gear Train 10. Next Position

ALPHA1 = ALPHA1 + DELTA1
IF(ALPHA1 GT. AL1) INGO TO 16
160       ALPHA2 = ALPHA2 + DELAL1*CAPR01/R02
        GO TO 14
165       CYCLEFT = (RATIO*DELAL1+MT01)/(MIN*(ALIFIN-ALI1H))
        WRITE(6,17) CYCLEFT
17       FORMAT('**+6I**+CYCLE EFFICIENCY =**+FS**+3)
        IF(J .NE. 0) GO TO 100
        STOP
        END
SUBROUTINE ALPHA (CAPRB, RB, THETA, CAPRO, RD, ALIN, ALFIN)

C THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS

1

5

ALIN = ((CAPRB + RD) * TAN(THETA) - SQRT(RD*RD - RB*RB)) / CAPRB
ALFIN = SQRT(CAPRO*CAPRO - CAPRH*CAPRH) / CAPRH
RETURN
END
<table>
<thead>
<tr>
<th>ALPHA</th>
<th>Si</th>
<th>S2</th>
<th>POINT EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.24</td>
<td>1.0</td>
<td>1.0</td>
<td>0.41055</td>
</tr>
<tr>
<td>11.26</td>
<td>1.2</td>
<td>1.2</td>
<td>0.41630</td>
</tr>
<tr>
<td>12.31</td>
<td>1.0</td>
<td>1.0</td>
<td>0.42287</td>
</tr>
<tr>
<td>12.94</td>
<td>1.0</td>
<td>1.0</td>
<td>0.42960</td>
</tr>
<tr>
<td>17.04</td>
<td>1.0</td>
<td>1.0</td>
<td>0.45239</td>
</tr>
<tr>
<td>17.54</td>
<td>1.0</td>
<td>1.0</td>
<td>0.45128</td>
</tr>
<tr>
<td>14.03</td>
<td>1.0</td>
<td>1.0</td>
<td>0.45728</td>
</tr>
<tr>
<td>11.57</td>
<td>1.0</td>
<td>1.0</td>
<td>0.46314</td>
</tr>
<tr>
<td>11.01</td>
<td>1.0</td>
<td>1.0</td>
<td>0.46910</td>
</tr>
<tr>
<td>10.06</td>
<td>1.0</td>
<td>1.0</td>
<td>0.47509</td>
</tr>
<tr>
<td>12.69</td>
<td>1.0</td>
<td>1.0</td>
<td>0.48099</td>
</tr>
<tr>
<td>17.72</td>
<td>1.0</td>
<td>1.0</td>
<td>0.48714</td>
</tr>
<tr>
<td>17.75</td>
<td>1.0</td>
<td>1.0</td>
<td>0.49321</td>
</tr>
<tr>
<td>17.70</td>
<td>1.0</td>
<td>1.0</td>
<td>0.49925</td>
</tr>
<tr>
<td>11.83</td>
<td>1.0</td>
<td>1.0</td>
<td>0.51250</td>
</tr>
<tr>
<td>11.66</td>
<td>1.0</td>
<td>1.0</td>
<td>0.51859</td>
</tr>
<tr>
<td>17.90</td>
<td>1.0</td>
<td>1.0</td>
<td>0.53652</td>
</tr>
<tr>
<td>17.94</td>
<td>1.0</td>
<td>1.0</td>
<td>0.54155</td>
</tr>
<tr>
<td>17.97</td>
<td>1.0</td>
<td>1.0</td>
<td>0.54760</td>
</tr>
<tr>
<td>18.01</td>
<td>1.0</td>
<td>1.0</td>
<td>0.55145</td>
</tr>
<tr>
<td>18.84</td>
<td>1.0</td>
<td>1.0</td>
<td>0.55444</td>
</tr>
<tr>
<td>18.04</td>
<td>1.0</td>
<td>1.0</td>
<td>0.56047</td>
</tr>
<tr>
<td>18.17</td>
<td>1.0</td>
<td>1.0</td>
<td>0.56422</td>
</tr>
<tr>
<td>18.18</td>
<td>1.0</td>
<td>1.0</td>
<td>0.56930</td>
</tr>
<tr>
<td>18.19</td>
<td>1.0</td>
<td>1.0</td>
<td>0.57300</td>
</tr>
<tr>
<td>18.23</td>
<td>1.0</td>
<td>1.0</td>
<td>0.58034</td>
</tr>
<tr>
<td>18.27</td>
<td>1.0</td>
<td>1.0</td>
<td>0.58539</td>
</tr>
<tr>
<td>18.31</td>
<td>1.0</td>
<td>1.0</td>
<td>0.59191</td>
</tr>
<tr>
<td>18.34</td>
<td>1.0</td>
<td>1.0</td>
<td>0.59799</td>
</tr>
<tr>
<td>18.32</td>
<td>1.0</td>
<td>1.0</td>
<td>0.60595</td>
</tr>
<tr>
<td>18.35</td>
<td>1.0</td>
<td>1.0</td>
<td>0.61210</td>
</tr>
<tr>
<td>18.32</td>
<td>1.0</td>
<td>1.0</td>
<td>0.61821</td>
</tr>
<tr>
<td>18.34</td>
<td>1.0</td>
<td>1.0</td>
<td>0.62521</td>
</tr>
<tr>
<td>18.35</td>
<td>1.0</td>
<td>1.0</td>
<td>0.63219</td>
</tr>
<tr>
<td>18.34</td>
<td>1.0</td>
<td>1.0</td>
<td>0.63915</td>
</tr>
<tr>
<td>18.35</td>
<td>1.0</td>
<td>1.0</td>
<td>0.64543</td>
</tr>
</tbody>
</table>
The general ogival tooth of thickness $t$ and outside radius $r_0$ consists of a circular arc of radius $\rho$ which blends tangentially into a radial tooth flank, as shown in Figure D-1 (only one center of curvature is indicated). The center of curvature $C$ is located at a distance $a$ from the center of the gear or pinion. Frequently this distance $a$ equals the pitch radius $r_p$. The tooth geometry may either be described with the help of the parameters $t$, $\rho$ and $a$, or with the combination $t$, $\rho$ and $r_0$. Both approaches are shown below.

1. **TOOTH GEOMETRY WITH HELP OF PARAMETERS $t$, $\rho$ AND $a$**

$C_x$ and $C_y$ represent the coordinates of the center of curvature $C$. $C_x$ is defined by

$$C_x = \rho - \frac{t}{2} \quad (D-1)$$
FIGURE D-1

GEOMETRY OF OGIVAL TOOTH (ONLY ONE CENTER OF CURVATURE SHOWN)
The angle $\theta$ is given by

$$\theta = \sin^{-1} \frac{C_x}{a} \quad (D-2)$$

With the above,

$$C_y = a \cos \theta \quad (D-3)$$

Further, the outside radius $r_o$ may be computed from

$$r_o = C_y + \sqrt{\rho^2 - C_x^2} \quad (D-4)$$

The angle $\gamma$ is obtained from

$$\gamma = \sin^{-1} \frac{\rho}{a} \quad (D-5)$$

and the flank angle $\alpha$ from

$$\alpha = \gamma - \theta \quad (D-6)$$
The distance, $f$, from the center of rotation, $O$, to the blend point, $F$, of the flank of the tooth, is given by

$$f = a \cos \gamma$$  \hspace{1cm} (D-7)

The angle $\gamma$ defines the tooth contact point $S$ on the ogival, i.e., circular, portion of the tooth with the lines $OC$ and $CS$. The minimum and maximum angles $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ are, respectively,

$$\gamma_{\text{min}} = \frac{\pi}{2} - \gamma$$  \hspace{1cm} (D-8)

and

$$\gamma_{\text{max}} = \sin^{-1} \left( \frac{r_0 \sin \delta}{p} \right)$$  \hspace{1cm} (This angle extends into the second quadrant)  \hspace{1cm} (D-9)

D-4
2. **TOOTH GEOMETRY FROM PARAMETERS \( p, t \) AND \( r_o \)**

If the outside radius \( r_o \) is given, distance \( a \) must be computed. The length \( C_x \) is still given by Equation (D-1), while

\[
a = \sqrt{(r_o - h)^2 + C_x^2} \quad \text{(D-10)}
\]

where, according to Figure D-1,

\[
h = \sqrt{\rho^2 - C_x^2} \quad \text{(D-11)}
\]

All other quantities of interest remain as before, i.e.,

\[
\delta = \sin^{-1} \frac{C_x}{a} \quad \text{See Equation (D-2)}
\]

\[
\gamma = \sin^{-1} \frac{\rho}{a} \quad \text{See Equation (D-5)}
\]

\[
\sigma = \gamma - \delta \quad \text{See Equation (D-6)}
\]

\[
f = a \cos \gamma \quad \text{See Equation (D-7)}
\]

\[
\eta_{\min} = \frac{\pi}{2} - \gamma \quad \text{See Equation (D-8)}
\]

\[
\eta_{\max} = \sin^{-1} \frac{r_o \sin \delta}{\rho} \quad \text{See Equation (D-9)}
\]
This appendix shows the kinematics of a single step-up gear mesh with ogival teeth and derives the moment input-output relationship. Both contact and pivot friction are included.

1. **KINEMATICS OF AN OGIVE MESH**

   Figure E-1 indicates the condition of initial contact in ogival meshes when the circular arc portion of the gear tooth drives the circular arc portion of the pinion tooth.\(^1\) This type of direct contact will be called "round on round". As the motion progresses, the circular arc of the gear tooth moves into contact with the straight flank of the pinion tooth (Figure E-2). During this "round on flat" phase, distance \(g\) at first decreases and then increases again. Before round on round contact can again occur for a given mesh, i.e., \(g\) has become equal to \(f_p\) again, the subsequent mesh comes into engagement with round on round contact. (See below.)

\(^1\)See section b-VII of this appendix for a check that indicates whether the initial contact is round on round or whether possibly the round portion of the pinion touches the flat portion of the gear.
The kinematics of both round on round and round on flat phases will be given first. Further it will be shown how to determine those gear angles, \( \phi \), at which the regime changes from round on round to round on flat, and at which contact is taken over by the subsequent set of teeth. Since the ratio of angular velocities between gear and pinion varies with contact point, and the pinion moves faster for a given gear speed for round on round than for round on flat, the original set of teeth disengages as soon as the subsequent one makes contact. Thus, the contact ratio is unity for this type of gearing.

The gear tooth nomenclature given in Appendix D is used throughout. The additional subscripts G and P refer to gear and pinion, respectively.

a. ROUND ON ROUND PHASE OF MOTION

Figure E-1 shows the input angle \( \phi \) as the angle between the x-axis and the center line, \( O_GW_G \), of the gear tooth. The output angle, \( \psi \), is the angle between the x-axis and the centerline \( O_PW_P \). During this phase of motion, the line connecting the centers of curvature \( C_G \) and \( C_P \) is of constant length \( L = r_G + r_P \) and passes
through the instantaneous contact points S on the gear and T on
the pinion. Because of this constant length one may obtain both
the output angle $\phi$ and the "coupler" angle $\alpha$ from the equivalent
four-bar linkage $O_g-C_c-C_p-O_p$, with ground pivot distance $b$.

I. UNIT VECTORS AND ANGLES $\gamma_g$ AND $\gamma_p$

The unit vectors associated with round on round phase are
given now.

In direction $O_g-C_g$

$$\vec{\eta}_g = \cos(\phi - s_g) \hat{I} + \sin(\phi - s_g) \hat{J}$$  \hspace{1cm} (E-1)

In direction $C_g-C_p$:

$$\vec{\eta}_\alpha = \cos \alpha \hat{I} + \sin \alpha \hat{J}$$  \hspace{1cm} (E-2)

The unit vector normal to $\vec{\eta}_\alpha$ (in the right hand sense) becomes

$$\vec{\eta}_{N\alpha} = -\sin \alpha \hat{I} + \cos \alpha \hat{J}$$  \hspace{1cm} (E-3)

E-4
In direction $O_P-C_P$,

$$\overrightarrow{p} = \cos(\psi - \delta_P)I + \sin(\psi - \delta_P)J \quad (E-4)$$

Since $\lambda - (\phi - \delta_G) = \psi - \eta_G$

$$\eta_G = \psi + \phi - \lambda - \delta_G \quad (E-5)$$

The angle $\eta_p$ is obtained from

$$\eta_p = (\psi - \delta_P) - \lambda \quad \text{for} \quad \psi - \delta_P > 90^\circ \quad (E-6a)$$

and

$$\eta_p = 2\pi + (\psi - \delta_P) - \lambda \quad \text{for} \quad \psi - \delta_P < 90^\circ \quad (E-6b)$$

The above values for $\eta_G$ and $\eta_p$ are only good for the round on round phase of the motion. For the round on flat phase the angle $\eta_{GF}$ of the gear, may be of interest. (See Figure E-2).

This angle is given by:

$$\eta_{GF} = (\phi - \delta_G) - (\psi + \alpha_P) + \frac{3\pi}{2} \quad \text{for} \quad \psi + \alpha_P > 90^\circ \quad (E-6c)$$

[Pinion flank in 4th quadrant]

and

$$\eta_{GF} = (\phi - \delta_G) - (\psi + \alpha_P) - \frac{\pi}{2} \quad \text{for} \quad \psi + \alpha_P < 90^\circ \quad (E-6d)$$

[Pinion flank in first quadrant]
II. DETERMINATION OF OUTPUT ANGLE $\phi$ AND "COUPLER" ANGLE $\lambda$

The loop equation of the equivalent four-bar linkage is given by

$$a_G \bar{n}_G + (\rho_G + \rho_P) \bar{n}_\lambda - a_P \bar{n}_P + b \mathbf{i} = 0 \quad (E-7)$$

With appropriate substitutions for the unit vectors, according to Equations (E-1), (E-2), and (E-4), one obtains the following component equations

$$a_G \cos(\phi - \phi_G) + L \cos \lambda + b - a_P \cos(\psi - \phi_P) = 0 \quad (E-8)$$

$$a_G \sin(\phi - \phi_G) + L \sin \lambda - a_P \sin(\psi - \phi_P) = 0 \quad (E-9)$$

where

$$L = \rho_G + \rho_P \quad (E-10)$$

To eliminate $\lambda$, let

$$\sin^2 \lambda + \cos^2 \lambda = 1 \quad (E-11)$$
Substitution of expressions for $\sin \lambda$ and $\cos \lambda$ from Equations (E-9) and (E-8), respectively, leads to

$$A_R \sin \psi + B_R \cos \psi = C_R \tag{E-12}$$

where

$$A_R = \sin(\phi - \theta) + \cos(\phi - \theta) \tan \gamma + \frac{b}{a} \tan \gamma$$

$$B_R = \cos(\phi - \theta) + \frac{b}{a} - \sin(\phi - \theta) \tan \gamma$$

$$C_R = \frac{a^2 + a^2 + b^2 - L^2}{2a_p a_g \cos \gamma} + \frac{b \cos(\phi - \theta)}{a_p \cos \gamma}$$

Equation (E-12) is solved for $\psi$ by a method similar to the one given on pg. 296 of R. S. Hartenberg and J. Denavit; Kinematic Synthesis of Linkages, McGraw-Hill Book Co., New York, 1964. Thus,

$$\psi = 2 \tan^{-1} \left( \frac{A_R \pm \sqrt{A_R^2 + B_R^2 - C_R^2}}{B_R + C_R} \right) \tag{E-13}$$

The correct sign in Equation (E-13) must be found from the geometric conditions of the equivalent four-bar linkage.
The "coupler" angle $\lambda$ may now be determined either from Equation (E-8) or from Equation (E-9), i.e.,

$$\lambda = \cos^{-1} \left[ \frac{a_p \cos(\psi - \delta_p) - a_G \cos(\phi - \delta_G) - b}{L} \right]$$ (E-14)

or

$$\lambda = \sin^{-1} \left[ \frac{a_p \sin(\psi - \delta_p) - a_G \sin(\phi - \delta_G)}{L} \right]$$ (E-15)

III. DETERMINATION OF OUTPUT PINION ANGULAR VELOCITY $\dot{\phi}$

Differentiation of Equation (E-12) with respect to time leads to

$$\dot{\phi} = \dot{\phi} \left[ \frac{A_R \sin \phi + B_R \cos \phi - C_R}{A_R \cos \phi - B_R \sin \phi} \right]$$ (E-16)

where $A_R$ and $B_R$ are given with Equation (E-12), and where

$$A_R = \tan \delta_p \sin(\phi - \delta_G) - \cos(\phi - \delta_G)$$

$$B_R = \sin(\phi - \delta_G) + \tan \delta_p \cos(\phi - \delta_G)$$

$$C_R = \frac{b}{a_p \cos \delta_p} \sin(\phi - \delta_G)$$
IV. RELATIVE VELOCITY AT CONTACT POINT BETWEEN GEAR AND PINION

With point S on the gear and point T on the pinion, as shown by Figure E-1, the relative velocity between gear and pinion at the contact point is given by

$$\vec{v}_{s/t} = \vec{v}_{s/o_g} - \vec{v}_{t/o_p} \tag{E-17}$$

This relative velocity is tangent to the contacting surfaces and can therefore be expressed in terms of the unit vector $\vec{n}_{N_A}$ (see Figure E-1). Then, the above becomes

$$\vec{v}_{s/t} = \left[ (\vec{v}_{s/o_g} \cdot \vec{n}_{N_A}) - (\vec{v}_{t/o_p} \cdot \vec{n}_{N_A}) \right] \cdot \vec{n}_{N_A} \tag{E-18}$$

or

$$\vec{v}_{s/t} = \left\{ \dot{\vec{r}} k \times (a_G \vec{r}_G + \rho_G \vec{r}_G) \cdot \vec{n}_{N_A} - \left[ \dot{\vec{r}} k \times (a_p \vec{r}_P - \rho_P \vec{r}_P) \right] \cdot \vec{n}_{N_A} \right\} \cdot \vec{n}_{N_A} \tag{E-19}$$

Appropriate substitution of unit vectors and simplification results in

$$\vec{v}_{s/t} = \left\{ \dot{x} [a_G \cos(\phi - \delta - \lambda) + \rho_G] - \dot{\psi} [a_p \cos(\psi - \delta - \lambda) - \rho_P] \right\} \cdot \vec{n}_{N_A} \tag{E-20}$$
b. ROUND ON FLAT PHASE OF MOTION

Figure E-2 gives the details of the round on flat contact between the driving gear and the driven pinion. The input angle $\phi$ and the output angle $\psi$ are again defined counterclockwise between the x-axis and the respective tooth center lines $O_GW_G$ and $O_PW_P$. Since contact is always made on the straight radial flank of the pinion, the line $SC_G$ of the gear is always normal to the flank of the pinion. The contact point is at a distance $g$ from the pinion center, $O_p$, and this distance is always smaller than, or equal to the distance $f_p$ (which is defined by Equation (D-7) in Appendix D). Again, the subscripts G and P are used for gear and pinion tooth parameters, respectively.

I. UNIT VECTORS

As before, the unit vector in direction $O_GC_G$ is given by

$$\vec{n}_G = \cos(\phi - \gamma_g)\hat{I} + \sin(\phi - \gamma_g)\hat{J} \quad (E-21)$$

The unit vector in direction $O_PT$, along the flank of the pinion is given by

$E-10$
\[ \mathbf{F} = \cos(\psi + \alpha_p) \mathbf{i} + \sin(\psi + \alpha_p) \mathbf{j} \]  \hspace{1cm} (E-22)

The unit vector in direction SCG is normal to \( \mathbf{F} \) in the right hand sense

\[ \mathbf{n}_{NF} = -\sin(\psi + \alpha_p) \mathbf{i} + \cos(\psi + \alpha_p) \mathbf{j} \]  \hspace{1cm} (E-23)

II. DETERMINATION OF OUTPUT ANGLE \( \psi \) AND DISTANCE \( g \)

The vector equation for the mechanism loop \( O_o-C_o-S-T-O_p \), which forms the basis of the desired solution, has the following form:

\[ a \mathbf{r}_G - \rho \mathbf{n}_{NF} - g \mathbf{n}_F + b \mathbf{i} = 0 \]  \hspace{1cm} (E-24)

Substitution of Equations (E-21) to (E-23) furnishes the component equations

\[ a \cos(\psi - \delta_o) + \rho \sin(\psi + \alpha_p) - g \cos(\psi + \alpha_p) + b = 0 \]  \hspace{1cm} (E25)

\[ a \sin(\psi - \delta_o) - \rho \cos(\psi + \alpha_p) - g \sin(\psi + \alpha_p) = 0 \]  \hspace{1cm} (E-26)
From Equation (E-26) one obtains for \( g \)

\[
g = \frac{a_G \sin(\phi - \delta_G) - \rho_G \cos(\psi + \alpha_p)}{\sin(\psi + \alpha_p)} \tag{E-27}
\]

This expression for \( g \) is now substituted into Equation (E-25).

Rearrangement and simplification lead to

\[
A_F \sin \psi + B_F \cos \psi = C_F \tag{E-28}
\]

where

\[
A_F = b \cos \alpha_p + a_G \cos(\phi - \delta_G - \alpha_p)
\]

\[
B_F = b \sin \alpha_p - a_G \sin(\phi - \delta_G - \alpha_p)
\]

\[
C_F = -\rho_G
\]

The solution of Equation (E-28) for \( \psi \) is obtained in the same way as that of Equation (E-13), i.e.,

\[
\psi = 2 \tan^{-1} \frac{A_F \pm \sqrt{A_F^2 + B_F^2 - C_F^2}}{B_F + C_F} \tag{E-29}
\]

The correct sign in Equation (E-29) depends on the geometric conditions of the mechanism position as in all four-bar linkage solutions of this type.

E-13
III. DETERMINATION OF PINION ANGULAR VELOCITY $\dot{\phi}$

Differentiation of Equation (E-28) with respect to time gives

$$
\dot{\phi} = \dot{\phi} \left[ \frac{A_{FD} \sin \phi + B_{FD} \cos \phi}{A_F \cos \phi - B_F \sin \phi} \right]
$$

where $A_F$ and $B_F$ are given with Equation (E-28) and where

$$
A_{FD} = a_G \sin (\phi - \delta_G - \alpha_P)
$$

$$
B_{FD} = a_G \cos (\phi - \delta_G - \alpha_P)
$$

IV. RELATIVE VELOCITY AT CONTACT POINT BETWEEN GEAR AND PINION

As for round on round contact, the relative velocity between point $S$ on the gear tooth and point $T$ on the pinion tooth is tangent to the contacting surfaces. In this case it will have the direction of unit vector $\mathbf{n}_F$ (see Figure E-2).

Then,
\[ \vec{V}_{S/T} = \vec{V}_{S/O} - \vec{V}_{T/O_P} = \left[ (\vec{V}_{S/O} \cdot \vec{n}_F) - (\vec{V}_{T/O_P} \cdot \vec{n}_F) \right] \vec{n}_F \]  

(E-31)

Since, because of the radial flank of the pinion, the velocity of the contact point \( T \) is normal to unit vector \( \vec{n}_F \).

\[ \vec{V}_{T/O_P} \cdot \vec{n}_F = 0 \]  

(E-32)

Therefore,

\[ \vec{V}_{S/T} = \left[ \vec{V}_{S/O} \cdot \vec{n}_F \right] \vec{n}_F \]  

(E-33)

or

\[ \vec{V}_{S/T} = \left\{ \sqrt{\dot{k}} \times (a_G \vec{n}_P - \rho_G \vec{n}_{NP}) \right\} \vec{n}_F \]  

(E-34)

Appropriate substitution of unit vectors gives:

\[ \vec{V}_{S/T} = \dot{\phi} \left[ \rho_G + a_G \sin(\psi - \phi + a_P + \delta_G) \right] \vec{n}_F \]  

(E-35)
V. DETERMINATION OF TRANSITION ANGLES FROM ROUND ON ROUND TO ROUND ON FLAT MOTION

The transition angles $\phi_T$ and $\psi_T$, which occur when the round on flat phase follows the round on round one, may be determined with the help of the modified component equations (E-25) and (E-26), i.e., one lets $\phi = \phi_T$, $\psi = \psi_T$ and $\beta = f_p$. This results in

\begin{align*}
a_G \cos(\phi_T - \delta_G) + r_G \sin(\psi_T + \alpha_p) - f_p \cos(\psi_T + \alpha_p) + b &= 0 \\
\text{(E-36)}
\end{align*}

and

\begin{align*}
a_G \sin(\phi_T - \delta_G) - r_G \cos(\psi_T + \alpha_p) - f_p \sin(\psi_T + \alpha_p) &= 0 \\
\text{(E-37)}
\end{align*}

From the above,

\begin{align*}
\cos(\phi_T - \delta_G) &= \frac{1}{a_G} \left[ f_p \cos(\psi_T + \alpha_p) - b - r_G \sin(\psi_T + \alpha_p) \right] \\
\text{(E-38)}
\end{align*}

and

\begin{align*}
\sin(\phi_T - \delta_G) &= \frac{1}{a_G} \left[ f_p \sin(\psi_T + \alpha_p) + r_G \cos(\psi_T + \alpha_p) \right] \\
\text{(E-39)}
\end{align*}
The angle $\psi_T$ may now be obtained by first eliminating $\phi_T$ from Equations (E-38) and (E-39). This is accomplished with

$$\sin^2(\phi_T - \delta_G) + \cos^2(\phi_T - \delta_G) = 1 \quad (E-40)$$

Substitution into the above leads to the following expression in $\psi_T$:

$$A_T \sin \psi_T + B_T \cos \psi_T = C_T \quad (E-41)$$

where

$$A_T = r_G \cos \alpha_P + f_P \sin \alpha_P$$
$$B_T = r_G \sin \alpha_P - f_P \cos \alpha_P$$
$$C_T = \frac{a_G^2 - f_P^2 - b^2 - f_G^2}{2b}$$

Equation (E-41) is again solved in the manner of Equation (E-13)

$$\psi_T = 2 \tan^{-1} \left( \frac{A_T \pm \sqrt{A_T^2 + B_T^2 - C_T^2}}{B_T + C_T} \right) \quad (E-42)$$
The correct sign must again be determined from the geometric conditions.

The associated transition angle $\psi_T$ can now be obtained either with the help of Equation (E-38) or Equation (E-39):

$$\psi_T = \cos^{-1}\left[\frac{f_p \cos(\psi_T + \alpha_p) - \rho_G \sin(\psi_T + \alpha_p) - b}{a_G}\right] + \delta_G$$

(E-43)

or

$$\psi_T = \sin^{-1}\left[\frac{f_p \sin(\psi_T + \alpha_p) + \rho_G \cos(\psi_T + \alpha_p)}{a_G}\right] + \delta_G$$

(E-44)
VI. SENSING GEOMETRY FOR THE DETERMINATION OF CONTACT OF

SUBSEQUENT TOOTH MESH

The following derives a computer sensing equation which indicates when contact is transferred from one tooth mesh to the succeeding one. Figure E-3 illustrates the case. The active mesh is in the round on flat mode and the subsequent mesh will make its initial contact in the round on round mode. This

![Diagram of sensing geometry for contact of subsequent tooth mesh]

Figure E3
Sensing Geometry for Contact of Subsequent Tooth Mesh

E-19
assertion is based on experience with the three gear and pinion combinations of the M125A1 booster. In each of these instances, the subsequent mesh makes contact before the round of the gear has left the flat of the pinion in the active mesh, i.e., \( g < f_p \).

(See work in section V.) It has also been found that initial contact of the subsequent mesh is always in the round on round mode. Generally, contact between the flat of the gear and the round of the pinion does not occur, and it has not been considered in this report. Section VII gives a criterion for the existence of this inverted round on flat mode of contact.

Once contact has been made by the subsequent mesh it becomes the new active mesh. This can be proven by the fact that, for a given angular velocity \( \phi \) of the gear, the angular velocity of the pinion is always larger in the initial stages of the round on round mode than in the final stages of the round on flat one. Thus, once the new mesh has made contact, the old one separates rapidly and the "contact ratio" is always unity.

The above may be shown theoretically by the position of the instant center of rotation between the gear and the pinion on line \( O_g O_p \).

If \( \omega_g \) and \( \omega_p \) represent the angles between the individual tooth center lines of the gear and pinion, respectively, (see Figure E-3), the
closure equation for the subsequent mesh may be written in terms of the active mesh as follows:

\[ a_p \left[ \cos(\psi - \delta_p + \Delta\psi)I + \sin(\psi - \delta_p + \Delta\psi)J \right] + L_xI + L_yJ \]

\[ = bI + a_G \left[ \cos(\phi - \delta_G - \Delta\phi)I + \sin(\phi - \delta_G - \Delta\phi)J \right] \]

(E-45)

where \( L_x, L_y \) = components of the distance \( L = C_pC_G \)

\( \psi = \) pinion angle determined from round on flat mode according to Equation (E-29)

The components \( L_x \) and \( L_y \) may be obtained from Equation (E-45):

\[ L_x = b + a_G \cos(\phi - \delta_G - \Delta\phi) - a_p \cos(\psi - \delta_p + \Delta\psi) \]  

(E-46)

and

\[ L_y = a_G \sin(\phi - \delta_G - \Delta\phi) - a_p \sin(\psi - \delta_p + \Delta\psi) \]  

(E-47)
Contact will have occurred when the distance $L$ becomes equal to, or slightly smaller than, the sum of the two radii of curvature $\rho_g$ and $\rho_p$. Thus, the criterion of contact becomes

$$\sqrt{L_x^2 + L_y^2} \leq \rho_g + \rho_p$$

(E-48)
VII. POSSIBILITY OF FLAT OF GEAR MAKING CONTACT WITH THE ROUND PORTION OF THE PINION

Figure E-4 shows a transition configuration in which the flat part of the gear tooth makes contact with the circular arc of the pinion, i.e., $\overline{OGS} = f_G$. The radius of curvature $\rho_p$ of the pinion will only then be normal to the flat of the gear.
when $G_0 \leq f_G$. Accordingly, in order to avoid this type of contact, the following criterion must be met:

$$b > \left[ a_p + \sqrt{f_G^2 + r_p^2} \right]$$

(E-49)

The three gear meshes of the M125A1 booster always satisfy equation (E-49), and therefore the initial contact is made in the round on round mode.
2. MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP GEAR
MESH WITH OGIVAL TEETH

The present section concerns itself with the determination of the equilibrant moment $M_o$ acting on the output pinion of a single ogival mesh, when the input moment $M_{in}$, which acts on the gear, is given. This relationship must be derived both for the round on round and for the round on flat phases of the motion. The directions of the pivot friction forces are again chosen such that the resulting moments oppose the motion of the respective component. (See Appendix A.) The magnitude of these friction moments is expressed in the manner of equation (A-2) of the aforementioned appendix. The direction of the friction force of the gear on the pinion is the same as the direction of the relative velocity $\nabla S/T$, of the gear contact point $S$ with respect to the pinion contact point $T$. This will be expressed by the appropriate signum convention. It must also be remembered that the kinematic expressions must conform to the motion phase under consideration.
a. INPUT-OUTPUT RELATIONSHIP FOR THE ROUND ON ROUND PHASE OF MOTION

Figure E-5 shows the free body diagrams for the round on round phase of motion with the gear considered to be component no. 1 and the pinion defined as component no. 2. The contact forces $F_{12}$ and $F_{21}$ are expressed with the help of the unit vector $\vec{H}_1$ [see equation (E-2)]. This unit vector is always normal to both contacting surfaces at points S and T. The unit vector $\vec{R}_{N1}$ is used to describe the direction of the friction forces at the contact point. The sense of these friction forces is determined with the help of

$$s = \frac{V_{S/T}}{|V_{S/T}|}$$  \hspace{1cm} (E-50)

(See equation (E-20) for $V_{S/T}$.) Since the friction force $F_{f12}$ of the gear on the pinion has the same direction as the relative velocity $V_{S/T}$, one may write

$$F_{f12} = \mu e F_{12} \vec{R}_{N1}$$  \hspace{1cm} (E-51)

where $\mu$ represents the coefficient of friction at the contact point.
Figure E-5
Free Body Diagram for Round on Round Phase

Note: $\phi$ is positive in ccw direction
Since

\[
\bar{F}_{21} = -F_{12}\bar{n}_A
\]  \hspace{1cm} (E-52)

the friction force of the pinion on the gear becomes

\[
\bar{F}_{f21} = -\mu sF_{12}\bar{n}_A
\]  \hspace{1cm} (E-53)

I. FORCE AND MOMENT EQUILIBRIUM OF THE GEAR

The force equilibrium of the gear is assured when

\[
-F_{12}\bar{n}_A - \mu sF_{12}\bar{n}_A - F_{x1}\bar{I} + F_{y1}\bar{J} - \mu F_{y1}\bar{I} - \mu F_{x1}\bar{J} = 0
\]  \hspace{1cm} (E-54)

In the above \(\mu\) also stands for the pivot coefficient of friction.

Moment equilibrium is given by

\[
M_{in} = \mu \rho_1 \sqrt{F_{x1}^2 + F_{y1}^2} - \left[ a_{g}\bar{n}_g + p_{g}\bar{n}_A \right] \times \left[ -F_{12}\bar{n}_A - \mu sF_{12}\bar{n}_A \right] = 0
\]  \hspace{1cm} (E-55)
The unit vector $\mathbf{R}_g$ is defined by equation (E-1).

The following component equations may be obtained from equation (E-54):

\[-F_{12}\cos\lambda + \mu sF_{12}\sin\lambda - F_x - \mu F_y = 0 \quad (E-56)\]

and

\[-F_{12}\sin\lambda - \mu sF_{12}\cos\lambda + F_y - \mu F_x = 0 \quad (E-57)\]

After substitution and cross-multiplication, one obtains the moment expression, equation (E-55), in scalar form:

\[M_{in} = \frac{\mu^2}{\mu^2 + 1} \frac{F_{12}^2}{F_x + F_y} + F_{12}\left[-\frac{\mu s\Upsilon + s_g\sin(\phi - \delta - \lambda)}{-\mu s\Upsilon + s_g\cos(\phi - \delta - \lambda)}\right] = 0 \quad (E-58)\]

Simultaneous solution of equations (E-56) and (E-57) for $F_x$ and $F_y$ results in

\[F_x = -F_{12}\left[\frac{(1 + \mu^2 s)\cos\lambda + \mu(1 - s)\sin\lambda}{1 + \mu^2}\right] \quad (E-59)\]

and

\[F_y = F_{12}\left[\frac{(1 + \mu^2 s)\sin\lambda - \mu(1 - s)\cos\lambda}{1 + \mu^2}\right] \quad (E-60)\]
These results are now substituted in equation (E-58). Since $s^2$ is unity and always positive, the resulting expression for $F_{12}$ has the following form:

$$F_{12} = \frac{\text{Min}}{\mu(p_1 + s_\theta) + s_\theta[\mu \cos(\phi - s_\theta - \lambda) - \sin(\phi - s_\theta - \lambda)]}$$

(E-61)

II. FORCE AND MOMENT EQUILIBRIUM OF THE PINION

Force equilibrium of the pinion is assured by

$$F_{12}R_\lambda + \mu sF_{12}N_\lambda + F_{x_2}I + F_{y_2}J + \mu F_{y_2}I - \mu F_{x_2}J = 0$$

(E-62)

Moment equilibrium must satisfy

$$M_0k + \mu s \sqrt{F_{x_2}^2 + F_{y_2}^2} k + [a_pR_p - \rho_pR_\lambda] \times [F_{12}R_\lambda + \mu sF_{12}N_\lambda] = 0$$

(E-63)
The unit vector \( \vec{n}_p \) is defined by equation (E-4).

Equation (E-62) gives rise to the following component equations:

\[
F_{12} \cos \lambda - \mu s F_{12} \sin \lambda + F_2 + \mu F_y = 0 \quad (E-64)
\]

and

\[
F_{12} \sin \lambda + \mu s F_{12} \cos \lambda + F_y - \mu F_x = 0 \quad (E-65)
\]

The moment equation (E-63) becomes, in scalar form,

\[
M_0 + \mu \rho \sqrt{F_{x2}^2 + F_{y2}^2} + F_{12} \left[ - \mu s \rho - \mu \rho \sin(\psi - \delta_p - \lambda) 
\right.
\left. + \mu \rho \cos(\psi - \delta_p - \lambda) \right] = 0
\quad (E-66)
\]

Simultaneous solution of equations (E-64) and (E-65) for \( F_{x2} \)
and \( F_{y2} \) results in

\[
F_{x2} = F_{12} \left[ \frac{\mu (1 + s) \sin \lambda - (1 - \mu^2 s) \cos \lambda}{1 + \mu^2} \right] \quad (E-67)
\]

and

\[
F_{y2} = -F_{12} \left[ \frac{\mu (1 + s) \cos \lambda + (1 + \mu^2 s) \sin \lambda}{1 + \mu^2} \right] \quad (E-68)
\]

E-31
The above expressions are now substituted into equation (E-66). Again, $s^2$ is unity and always positive, and the following expression may be obtained for the contact force $F_{12}$ in terms of the equilibrant moment $M_o$.

$$F_{12} = \frac{M_o}{\mu(s_P - r_2) - a_P[\mu \cos(\psi - \delta_P - \lambda) - \sin(\psi - \delta_P - \lambda)]}$$  

(E-69)

III. MOMENT INPUT-OUTPUT RELATIONSHIP

When equations (E-61) and (E-69) are set equal to each other, one obtains the following input-output relationship:

$$M_o = \min \left[ \frac{\mu(s_P r_P - r_2) - a_P[\mu s_R \cos(\psi - \delta_P - \lambda) - \sin(\psi - \delta_P - \lambda)]}{\mu(r_1 + s_R r_G) + a_G[\mu s_R \cos(\psi - \delta_G - \lambda) - \sin(\psi - \delta_G - \lambda)]} \right]$$  

(E-70)

To account for the fact that equ. (E-70) is only valid for the round on round phase of motion, the signum symbol has now been changed to $s_R$.

E-32
b. **INPUT-OUTPUT RELATIONSHIP FOR ROUND ON FLAT PHASE OF MOTION**

Figure E-6 gives the free body diagrams for the round on flat phase of the motion. Again, the gear is considered to be component no. 1, while the pinion is component no. 2. Using the unit vector $\mathbf{n}_{NF}$ of equation (E-22), which is normal to the flat side of the pinion to express the force $\mathbf{F}_{12}$ of the gear on the pinion, one obtains:

$$\mathbf{F}_{12} = -F_{12}\mathbf{n}_{NF} \quad (E-71)$$

The friction force of the gear on the pinion again has the same direction as the now applicable relative velocity $\mathbf{V}_{S/T}$ of equation (E-35). With

$$s = \frac{V_{S/T}}{|V_{S/T}|} \quad (E-72)$$

as the applicable signum convention, the friction force $\mathbf{F}_{f12}$ becomes

$$\mathbf{F}_{f12} = \mu s F_{12}\mathbf{n}_{F} \quad (E-73)$$
FIGURE E-6
FREE BODY DIAGRAM FOR ROUND ON FLAT PHASE

Note: \( \phi \) is positive in ccw direction
The contact force $F_{21}$ of the pinion on the gear and the associated friction force are equal and opposite to the forces given by equations (E-71) and (E-73) respectively. Thus,

$$F_{21} = F_{12}$$ \hspace{1cm} (E-74)

and

$$F_{f21} = -\mu F_{12}$$ \hspace{1cm} (E-75)

Note that the kinematics of the round on flat phase must now be used.

I. FORCE AND MOMENT EQUILIBRIUM OF THE GEAR

Force equilibrium of the gear is given by

$$F_{12} - \mu F_{12} F_{f21} = F_{x1} + F_{y1} - \mu F_{y1} F_{x1} = 0$$ \hspace{1cm} (E-76)

Moment equilibrium requires

$$M_{nk} = \mu \sqrt{F_{x1}^2 + F_{y1}^2} + \left[G_{\bar{G}} - \mu G_{\bar{N}F}\right] \times \left[F_{12} - \mu F_{12} F_{f21}\right] = 0$$ \hspace{1cm} (E-77)
Equation (E-76) furnishes the following component expressions

\[-F_{12}\sin(\psi + \alpha_p) = \mu sF_{12}\cos(\psi + \alpha_p) - F_{x1} - \mu F_{y1} = 0\]

(E-78)

and

\[F_{12}\cos(\psi + \alpha_p) = \mu sF_{12}\sin(\psi + \alpha_p) + F_{y1} - \mu F_{x1} = 0\]

(E-79)

The scalar form of equation (E-77) is given by

\[M_{\text{in}} = \mu sF_{12}\sqrt{F_{x1}^2 + F_{y1}^2} + F_{12}\left[ -\mu s\phi + s\cos(\phi - \delta_p - \psi - \alpha_p) \right.
\]
\[\left. + \mu s\phi\sin(\phi - \delta_p - \psi - \alpha_p) \right] = 0\]

(E-80)

Simultaneous solution of equations (E-78) and (E-79) for $F_{x1}$ and $F_{y1}$ furnishes

\[F_{x1} = \frac{\mu(1 - s)\cos(\psi + \alpha_p) - (1 + \mu^2)s\sin(\psi + \alpha_p)}{1 + \mu^2}\]

(E-81)

and

\[F_{y1} = -\frac{\mu(1 - s)\sin(\psi + \alpha_p) + (1 + \mu^2)s\cos(\psi + \alpha_p)}{1 + \mu^2}\]

(E-82)
The above results are now substituted into equation (E-80).

Since \( s^2 \) is again unity and positive at all times, the resulting expression for \( F_{12} \) becomes, in terms of \( M_{in} \),

\[
F_{12} = \frac{M_{in}}{\mu (\rho_1 + s \rho_3) - \rho_3 [\cos(\phi - \delta_G - \psi - \alpha_p) + \mu \sin(\phi - \delta_G - \psi - \alpha_p)]}
\]

(E-83)

II. FORCE AND MOMENT EQUILIBRIUM OF THE PINION

Force equilibrium of the pinion is expressed by

\[
-F_{12} \bar{h}_{NF} + \mu s F_{12} \bar{h}_F + F_{x2} \bar{l} + F_{y2} \bar{j} + \mu F_{y2} \bar{l} - \mu F_{x2} \bar{j} = 0
\]

(E-84)

Moment equilibrium requires that

\[
M_0 \bar{k} + \mu^2 \sqrt{F_{x2}^2 + F_{y2}^2} \bar{k} + \bar{g}_{\bar{h}_F} \times (-F_{12} \bar{h}_{NF}) = 0
\]

(E-85)

E-37
Equation (E-84) furnishes the following component equations

\[ F_{12}\sin(\psi + \alpha_p) + \mu F_{12}\cos(\psi + \alpha_p) + F_{x2} + \mu F_{y2} = 0 \]

and

\[ -F_{12}\cos(\psi + \alpha_p) + \mu F_{12}\sin(\psi + \alpha_p) + F_{y2} - \mu F_{x2} = 0 \]  \hspace{1cm} (E-86)

\[ (E-87) \]

The scalar form of the moment equation (E-85) becomes

\[ M_o + \mu F^2_{x2} + \mu F^2_{y2} - gF_{12} = 0 \] \hspace{1cm} (E-88)

Simultaneous solution of equations (E-86) and (E-87) for \( F_{x2} \) and \( F_{y2} \) leads to

\[ F_{x2} = -F_{12} \left[ \frac{(1 - \mu^2 s)\sin(\psi + \alpha_p) + \mu(1 + s)\cos(\psi + \alpha_p)}{1 + \mu^2} \right] \] \hspace{1cm} (E-89)

and

\[ F_{y2} = F_{12} \left[ \frac{(1 - \mu^2 s)\cos(\psi + \alpha_p) - \mu(1 + s)\sin(\psi + \alpha_p)}{1 + \mu^2} \right] \] \hspace{1cm} (E-90)
The above expressions are now substituted into equation (E-88). Again, $e^2$ is unity and positive. The following expression for $F_{12}$ is now obtained

$$F_{12} = \frac{M_0}{e - \mu P_2}$$  \hfill (E-91)

III. MOMENT INPUT-OUTPUT RELATIONSHIP

When equations (E-83) and (E-91) are set equal to each other, one obtains the following input-output relationship:

$$M_0 = M_{\text{in}} \left[ \frac{e - \mu P_2}{\mu (\rho_1 + s_f \rho_g) - s_g [ \cos(\phi - \delta_g - \psi - \alpha_p) + \mu s_f \sin(\phi - \delta_g - \psi - \alpha_p)]} \right]$$  \hfill (E-92)

Note that the signum symbol has been changed to $s_f$ in the above expression to account for the fact that the expression is only valid in the round on flat phase of motion.

E-39
The present appendix contains descriptions, listings, and sample outputs of the following computer models relating to single step-up gear meshes containing clock (ogival) teeth:

1. Program CLOCK 1: Kinematics of single pass step-up gear mesh with clock (ogival) teeth
2. Program CLOCK 2: Point and cycle efficiencies for single pass step-up gear mesh with clock (ogival) teeth

The relevant background, the input parameters, the manner of the computations, and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.
1. **Program CLOCK 1: Kinematics of Single Pass Step-Up Gear Mesh With Clock (Ogival) Teeth**

The program CLOCK 1 is based upon that portion of Appendix E which deals with the kinematics of single mesh step-up clock gearing. It may be used to check on the geometric performance of clock type meshes.

The nomenclature of the program is chosen to coincide as much as possible with that of Appendix D as well as that of Appendix E.

a. **Input Parameters (see Program CLOCK 1, below)**

The following parameters represent the input data for the program:

- $\text{CAPRP} = R_p$, the pitch radius of the gear
- $\text{RP} = r_p$, the pitch radius of the pinion
- $\text{AG} = a_G$, the distance from the center of rotation of the gear to the center of curvature of the circular arc portion of the gear tooth
- $\text{AP} = a_p$, the distance from the center of rotation of the pinion to the center of curvature of the circular arc portion of the pinion tooth
- $\text{RHOG} = \rho_G$, the radius of curvature of the circular arc portion of the gear tooth
- $\text{RHOP} = \rho_p$, the radius of curvature of the circular arc portion of the pinion tooth
- $\text{TG} = t_G$, the maximum thickness of the gear tooth
TP = t_p, the maximum thickness of the pinion tooth
NG = n_G, the number of teeth of the gear
NP = n_p, the number of teeth of the pinion
K = range divisor
PHIDOT = ϕ = 1, all velocity computations are based on this value

b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The following tooth parameters of the gear as well as of the pinion are first computed:

For the gear

CXG = c_XG (see eq. (D-1))
DELG = δ_G (see eq. (D-2))
CYG = c_YG (see eq. (D-3))
ROG = r_OG (see eq. (D-4))
GAMMG = γ_G (see eq. (D-5))
ETMING = γ_ming (see eq. (D-8))
ETMAXG = γ_maxG (see eq. (D-9))

For the pinion

CXP = c_XP (see eq. (D-1))
DELP = δ_P (see eq. (D-2))
CYP = c_YP (see eq. (D-3))
ROP = r_OP (see eq. (D-4))
GAMMP = γ_P (see eq. (D-5))
ALPHP = α_P (see eq. (D-6))
ETMINP = \( \eta_{\text{min}} \) (see eq. (D-8))
ETMAXP = \( \eta_{\text{max}} \) (see eq. (D-9))
FP = \( f_p \) (see eq. (D-7))

In general

\[ B = b \], the center distance between the gear and the pinion

\[ D_{\text{PHI}} = \Delta \phi \], the angle between the center lines of adjacent
gear teeth

\[ D_{\text{PSI}} = \Delta \psi \], the angle between the center lines of adjacent
pinion teeth

\[ L \] (see eq. (E-10))

II. Determination of Transition Angle

In order to define the ranges for the round on round and
the round on flat phases of motion, it is first necessary to
locate the transition angles \( \phi_T \) and \( \psi_T \). This is accomplished
with the help of the expression contained in section 1b-V.
Appendix E. The transition angle \( \phi_T \) is first solved according
to eq. (E-42). Since the solution furnishes two answers, i.e.,
the angles \( \phi_{T1} \) and \( \phi_{T2} \), it is necessary to decide which of
these is the desired one. The principal criterion for selecting
the correct value of \( \phi_T \) is based on the motion of the contact
point T with respect to the pinion. The first transition
angles \( \phi_{T1} \) and \( \phi_{T1} \) occur as the contact point moves from the
round portion of the pinion to the flat one (for increasing
values of \( \phi \)). As this motion is continued, the distance \( g \)
becomes smaller than the transition parameter \( f_p \). Once \( g \) has
reached its minimum, the contact point moves outward on the
pinion flat until $g$ theoretically equals $f_p$ once again. This part of the motion is never completed in actuality since the subsequent set of teeth takes over before $g$ reaches this transition value. It is this second transition situation which corresponds to the transition angles $\psi_{T_2}$ and $\varphi_{T_2}$. Since the round on flat equation does not recognize any limitation on the length of the pinion flat, an increase of $\varphi$ over the value of $\varphi_{T_2}$ will have associated with it a theoretical value for $g$ which is larger than $f_p$. Thus, one may identify the correct value of $\varphi$ and $\psi_T$ by noting that an increase in the angle $\varphi$ must lead to an associated value of $g$ which is smaller than $f_p$.

The program uses this criterion in the following manner after $\psi_{T_1}$ and $\psi_{T_2}$ are known:

A. Subroutine TRANS is called and the angle $\psi_{T_1}$, which is associated with $\psi_{T_1}$, is computed with the help of eqs. (E-43) and (E-44).

B. The angle $\varphi$ is made slightly larger than $\psi_{T_1}$ to produce PHINEXT, and eq. (E-29) is used to find the associated angle PSINEX. Since there are two such angles, the subroutine selects the one which is closest to the value of transition angle $\psi_{T_1}$. Subsequently, the associated value of $g_1$ is computed according to eq. (E-27).

C. Steps A and B are then repeated identically for the second transition angle $\psi_{T_2}$. This results in the determination of $g_2$.

D. Control remains with the main program, and that value of $\psi_T$ is chosen for which the associated value of $g$ is smaller.
than $f_p$.

For checking purposes, a subsidiary test for the determination of the transition angle was added to the program. It is based on the idea that, for the correct transition angle $\psi_T$, the line representing the flat portion of the pinion will make a smaller angle with the center line $O_0O_p$ than will be the case for the incorrect one. To this end, TEST1 and TEST2 find both angles with the help of

\[ \text{TEST} = 360^\circ - (\psi_T + \alpha_p) \]  

\[(F-1)\]

III. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (E-29), for the round on flat regime, is determined with the help of the angle $\psi_T$. The condition which yields that angle $\psi$ which is closest to $\psi_T$ governs. The variable SIGNF is used for the sign under consideration.

IV. Computation of Final and Initial Values of $\psi$ and $\psi$

The final and initial values of the gear and pinion angles are found by continuously evaluating the round on flat equation (E-29), using the previously determined value of SIGNF, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (E-48). This loop is initiated at the transition angle and terminated once the condition of eq. (E-48) is met. This allows the simultaneous determination
of both the angles at which the first set of teeth loses contact and at which the second set of teeth comes into engagement. The latter is accomplished by subtracting \( \Delta \psi \) from the "loss of contact" angle \( \phi_F \) and adding \( \Delta \psi \) to the "loss of contact" angle \( \phi_T \) (see computations following statement label no. 70). PHIF and PSIIF represent the angles when contact is lost for a given mesh, while PHI and PSII stand for the angles when initial contact is made.

V. **Determination of Correct Sign for Round on Round Regime**

Eq. (E-13) is used to determine the angle \( \psi \) while the gear and pinion are in the round on round regime. The correct sign in eq. (E-13) is obtained by comparing the value of the angle \( \psi \), as computed with PHI, with the value of PSII. SIGNR is the variable which furnishes this sign.

VI. **Kinematics**

Since the limits as well as the correct signs for both the round on round and the round on flat regimes are known, the various kinematic properties of interest may now be computed.

The angular increment DDPHI of the input angle is found by dividing the range from PHI to PHIF into \( K \) parts. (\( K \) is the range divisor.) The computational loop begins with statement label no. 110 and ends with the third card from the end of the main program. The computation is terminated when the angle \( \psi \) exceeds PHIF.
A. Round on Round Regime

As long as PHI is smaller than the transition angle PHIT, the kinematics of the round on round phase of motion is computed. This results in the following:

\[ \psi = \psi \] (see eq. (E-13))

\[ \lambda = \lambda \] (see eqs. (E-14) and (E-15))

\[ \dot{\psi} = \dot{\psi} \] (see eq. (E-16))

\[ V_{STR} = \bar{V}_{G/T} \text{ for round on round} \] (see eq. (E-20))

\[ S_R = S \] (see eq. (E-50) for later use)

\[ \eta_P = \eta_P \] (see eqs. (E-6a) and (E-6b))

\[ \eta_G = \eta_G \] (see eq. (E-5). For discussion of the usefulness of \( \eta_P \) and \( \eta_G \), see output in section c below.)

B. Round on Flat Regime

When PHI is larger than the transition angle PHIT, the kinematics of the round on flat phase is computed. This computation includes the following:

\[ \psi = \psi \] (see eq. (E-29))

\[ G = g \] (see eq. (E-27))

\[ \dot{\psi} = \dot{\psi} \] (see eq. (E-30))

\[ V_{STF} = \bar{V}_{G/T} \text{ for round on flat} \] (see eq. (E-35))

\[ S_F = S \] (see eq. (E-72) for later use)

\[ \eta_{GP} = \eta_{GP} \] (see eqs. (E-6c) and (E-6d))

F-8
c. **Output (see Program CLOCK 1, below)**

As previously, the output of the program is best explained with the help of the sample problem at the end of the program. It contains the following:

**I. Input Parameters**

- CAPRP = \( R_p = 0.47725 \text{ in. (1.212 cm)} \)
- RP = \( r_p = 0.09085 \text{ in. (0.231 cm)} \)
- AG = \( a_G = 0.47725 \text{ in. (1.212 cm)} \)
- AP = \( a_P = 0.09085 \text{ in. (0.231 cm)} \)
- RHOG = \( \rho_G = 0.03870 \text{ in. (0.098 cm)} \)
- RHOP = \( \rho_P = 0.01740 \text{ in. (0.044 cm)} \)
- TG = \( t_G = 0.03480 \text{ in. (0.088 cm)} \)
- TP = \( t_P = 0.02800 \text{ in. (0.071 cm)} \)
- NG = \( n_G = 42 \)
- NP = \( n_P = 8 \)
- K = 50

**II. Computed Values**

The following parameters are printed out:

- DELGD = \( \delta_G = 2.5880^\circ \)
- DELPD = \( \delta_P = 2.1448^\circ \)
- GAMMPD = \( \gamma_P = 11.0418^\circ \)
- ALPHPD = \( \alpha_P = 8.8970^\circ \)
- ETMINGD = \( \eta_{\text{ming}} = 85.3488^\circ \)
- ETMAXGD = \( \eta_{\text{maxg}} = 144.0484^\circ \)
ETMINPD = \( \gamma_{\text{min}} = 78.9582^\circ \)
ETMAXPD = \( \gamma_{\text{max}} = 166.5870^\circ \)
FP = \( f_p = .08917 \) in. (0.226 cm)

The printout concerning the transition angles consists of two lines. While the program automatically picks that transition configuration which leads to a decreasing value of \( \gamma \) as \( \varphi \) is increased, both transition angles are printed out for checking purposes. Thus, one finds

\[
\begin{align*}
\varphi_{T_1} &= 186.36^\circ \\
\psi_{T_1} &= 308.63^\circ \\
\gamma_1 &= .0895 \text{ in. (0.227 cm)} \quad \text{TEST1} = 42.47^\circ \\
\varphi_{T_2} &= 178.75^\circ \\
\psi_{T_2} &= 346.65^\circ \\
\gamma_2 &= .0888 \text{ in. (0.226 cm)} \quad \text{TEST2} = 4.45^\circ
\end{align*}
\]

Clearly, the second line represents the solution since \( \gamma_2 = .0888 \) for a slight increase in the angle \( \varphi \) over \( \varphi_T \). This is less than the value of \( f_p = .08917 \). In addition, TEST2 furnishes the smaller number of degrees. Recall that TEST represents the angle between the flat of the pinion and the line connecting the pivots.

Furthermore, the program shows the initial and final angles of contact:

\[
\begin{align*}
\text{PHIID} &= \text{the angle } \varphi \text{ at initial contact} \\
\text{PSIID} &= \text{the angle } \psi \text{ at initial contact} \\
\text{PHIFD} &= \text{the angle } \varphi \text{ at final contact} \\
\text{PSIFD} &= \text{the angle } \psi \text{ at final contact}
\end{align*}
\]
Note that angle $\phi$ increases while the angle $\psi$ decreases from the beginning to the end of the contact.

The computational loop begins with PHIID and ends with PHIFD. Further, when PHIID reaches the approximate value of PHITD, the output shifts from round on round parameters to round on flat ones. The purpose of the multiple output throughout the motion of the mesh is to gain insight concerning the behavior of the mesh as well as to be able to check geometric values.

The following conclusions may be drawn for either of the phases as well as in general for the gears under consideration.

A. Round on Round Phase ($176.2623^\circ < \phi < 178.6623^\circ$)

PSIDOT, the angular velocity of the pinion, is negative, and at all times near the "gear ratio" of $42/8 = 5.25$.

SR is printed here only for checking purposes. It becomes important in the moment input-output analyses of program CLOCK 2.

The angles ETAGRD and ETAPRD are of interest because one needs to make sure that the contact between gear and pinion does not occur too close to the respective tooth tips. This is necessary since the present mathematical model has a pointed tip while in a real situation the tips are rounded. Thus, if contact is sufficiently far from the tips of the teeth, the present model will give valid answers.

The range of ETAGRD is approximately between $86^\circ$ and $90^\circ$. This is considerably smaller than the ETMAXGD of approximately $144^\circ$ and larger than the ETMINGD of approximately $85^\circ$. (The
latter shows that the round of the pinion tooth does not touch the flat of the gear tooth.

The angle ETAPRD of the pinion tooth is always considerably smaller than ETMAXPD. Since contact is transferred to the flat portion of the pinion at the end of this phase of motion, ETAPRD almost equals ETMINPD at that point.

LAMDAD is given for general checking purposes only.

B. Round on Flat Phase ($178.6623 ^\circ < \varphi < 184.8340 ^\circ$)

The angular velocity $\psi$ continues relatively smoothly after the transition. The distance $g$ remains smaller than $f_p$, as expected, and it reaches a minimum of 0.0822 in. at $\varphi \approx 182.5^\circ$. It is further to be noted that $g$ never reaches the value of $f_p$ again since the subsequent set of teeth takes over when $g = 0.08439$ in. (0.214 cm). For the present program to be valid, it is necessary that contact ends before the round on flat phase is completed.

As before, $s_p$ is given for checking purposes only, and its value has been confirmed, just as was done for $s_R$, by graphical analysis (not shown).

The angle ETAGFD of the gear reaches a maximum of approximately $130^\circ$ at the end of this phase. Since this is well below the maximum value ETMAXGD = $144^\circ$, there is enough margin for a tip radius on the gear tooth.
C. General Considerations

As expected for a direct contact mechanism of this type, the angular velocity ratio, as represented by PSIDOT, is not constant and has a greater absolute value at initial contact than at final contact. This indicates that the pinion will speed up somewhat as the subsequent set of teeth comes into contact, and that therefore, the original set of teeth loses contact at that instant. This means that the "effective contact ratio" is unity.
Program CLOCK 1
PROGRAM CLOCK

C KINEMATICS OF SINGLE PASS STEP-UP GEAR MESH WITH CLOCK (ODIVAL) TEETH

REAL XH, L, X, YT, LAMBD, LAMDA
READ(5, 1) CAPP, RP
1 FORMAT(2F10.1)
READ(4, 1) A
READ(5, 1) H
READ(5, 1) T
READ(5, 1) NP
2 FORMAT(2F10.1)
READ(5, 1) K
3 FORMAT(2F10.1)
4 FORMAT(2F10.1)
5 FORMAT(2F10.1)
6 FORMAT(2F10.1)
7 FORMAT(2F10.1)
8 FORMAT(2F10.1)
P1 = 3.1415
2 = P/180
P1D001 = 1.

WRITE(6, 10) CAPP, RP, A, A

10 FORMAT(2F10.1)

WRITE(6, 10) T, RP
WRITE(6, 10) K
WRITE(6, 10) P1
WRITE(6, 10) P1D001
WRITE(6, 10)

11 C COMPUTATION OF GEAR TOOTH PARAMETER

C CAC = RHOG - TC/2
35 DELG = ASIN(CAC/AG)
DELG0 = DELG/2
CAG = AG*COS(DELG)

RUG = EAP - SORTN(RHOG, CAC, CAG)
GAMMG = ASIN(RHOG/AG)
GAMMG = GAMMG/2
ETHMGD = 00. - GAMMG
ETHMGD = 100. - ASIN(108.5*ETHMGD/RHOG)/2
CAP = RHOP - TP/2.

DELP = ASIN(ETHMGD)
DELPO = DELP/2
DEL = DELG/2
DELPO = DELG/2

RUP = CTP - SORTN(RHOP, RHOP - CTP, CTP)
GAMMP = ASIN(RHOP/AG)
GAMMP = GAMMP/2
ALPHP = GAMMP - DELP
ALPHP = ALPHP/2

ETHAPD = 100. - ASIN(RHP*ETHMGD/RHOP)/2

PAGE 1
PROGRAM LOCAL 7 3 7 4  IP #1

FP = AP+CS(ALPHP)
B = CAMP - 1.0

DPHI = 360./NP-Z
PSI1 = 360./NP-Z
L = MM0G - RHP

*RIE(0+1)DELDG+DELPH+GAMMP+ALPHP+EING+ETEAXG+ETENPO+

C FORMAT(8X,2(DELPH,F9.4,J3),*GAMMP,F9.4,J3,*ETEAXG,F9.4,J3)*ETENPO+

C C DETERMINATION OF TRANSITION ANGLE

AT = RHO0+CS(ALPHP) - FP+8IN(ALPHP)
BT = RHO0+8IN(ALPHP) - FP+CS(ALPHP)
CT = (ATMP - FP+8IN) - MB - RHO0+MM0G)/12.*61

MOIT1 = AT+AT + BT+BT - CT+CT

Y1 = AT - SQRT(MOIT1) -
Y2 = AT - SQRT(MOIT1)

AT = BT - CT

PSI1 = 2.+ATN2(Y1,AT)
PSI2 = 2.+ATN2(Y2,AT)

IF(ISIT1 .LT. 0.) PSIT1 = PSI1 + Z.*P1

IF(ISIT2 .LT. 0.) PSIT2 = PSI2 + Z.*P1

PSIT1D = PSIT1/Z
PSIT2D = PSIT2/Z

CALL TRANS(RHO0+ALPHP,FP+AG+BT+DELPH,PSIT1D+PH11D+61)

CALL TRANS(RHO0+ALPHP,FP+AG+B+DELPH,PSIT1D+PH11D+62)

PH11D = PSIT1D/Z

PH12D = PSIT2D/Z

TEST1 = 360. - ((PSIT1D + ALPHP))

TEST2 = 360. - ((PSIT2D + ALPHP))

IF(TE1(6,11)+PSIT1D+61+TEST1

IF(TE1(6,12)+PSIT2D+62+TEST2

IF FORMAT(8X,2(DELPH,F9.4,J3),*GAMMP,F9.4,J3,*ETEAXG,F9.4,J3)

IF(PS1D .LT. FF100G TO Z)

PH11 = PS11D

PS12 = PS12

C C DETERMINATION OF CORRECT SIGN FOR FOUR-POINT ON FL+1 REGIME

C IF = B+COAT(ALPHP) - AB+COAT(ALPHP)
BF = B+8IN(ALPHP) - AB+8IN(ALPHP)

C IF = MM0G

HOUTF = IF*IF + BF*BF - CF*CF
III

I

II

1

L

K

LI

16

If

LMM

IP11

t 4

N 1 A I A 4

407

SINF = 1

GO TO 50

48 SIGN = 1

C COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI

120

DO 60 I = 1, 1000

PHI0 = PHI0 + (I-1)/100

PHI = PHI0 + 1

125

AF = 2*(COS(PI/2))

BF = 3*SIN(ALPHA) = AGSIN(SIN(DELG-ALPHA))

C = 1000

ROOTIF = AF*AF + BF*BF - C*C

TF = AF - SIGN(RootIF)*ROOTIF

AF = BF + C

130

PSI1 = 2*(ATAN2(TF, TF))

IF PSI1 LT 0.1, PSI1 = PSI1 + 2*PI

LT = AGSIN(SIN(DELG-DFI)) - 0.1*PSI1*DELP + DFI

IF (SOLFT = LT + LT) LT = 30 TO 10

135

CONTINUE

IF PHI = PHI1

PSIFF = PSI1

PSII = PSI1 + DPHI

PSII = PSI1 + DPHI

PHTO = PHI1 + 1

140

IF PSI1 LT 2*PI + PI

PHI1 = PHI1 + 1

PHT1 = PHI1 + 1

PHF1 = PHI + 1

PSIFF = PSI1

145

WRITE (6, 81) PHI1 + PHI1 + PHI1 + PHI1 + PHI1 + PHI1 + PHI1

8 FORMATTED (phi11 = ff9.432*0.0094*df11 = ff9.4*3x = phi11 = ff9.4*3x = phi11 = ff9.4*3x = phi11 = ff9.4*3x)

C DETERMINATION OF CORRECT SIGN FOR indifference or indifference

150

AR = SIN(PHI1-DELP) = COS(PHI1-DELP)*TAN(DELP) = IMAGINARY(DELP)/AG

BR = COS(PHI1-DELP) = 0.0 = SIN(PHI1-DELP)*TAN(DELP) = CR

CT = (AP = AP = AP = AP = 0.0 = LML/12 = AP = AP = COS(DELP))

TF1 = AR + SQRT(MU10)

TH2 = AR - SQRT(MO10)

TF1 = 2*(ATAN2(TF1, TF1))
110 100 C001 = (1-F)1/2

114 PMI = PMII - 90PMI

115 PMI = PHI + COPH1

120 IF (PMI < 20) PMII = PHI + (100 - 200)

125 PM10 = PHI/2

130 IF (PM11 < 20) PHI = PHI - 100

136 ANA = SIN(PHI - DELP) * COS(PHI - DELG) * TAN(DEL) / TAN(DELG) * AN(SIN(PHI - DELG) * TAN(DEL) / TAN(DELG))

141 CH1 = (APA * APA + APA + APA) / (2 + APA * APA * COS(DEL))

146 10 = COST(PHI - DELG) / (APA * COST(DEL))

151 ROOTH = AN30 + 0.006 - COPH1

155 VV = AN * SGNPOS + 0.001 + ROOTH

160 AN = AN + PHI + COPH1

165 PSI = AN * SGNPOS + 0.001

170 AN = AN + PHI + COPH1

175 IF (PSI < 20) PSI = PSI - 20

180 SLAN = SLA(SIN(PHI - DELG) - ACA / SIN(PHI - DELG)) / TAN(DELG)

185 SLAN = SLAN(DELG) / TAN(DELG)

190 LAMDA = ATAN(5, SLAN)

195 LAMDAO = LAMDAO

200 IF LSTM0 = LAMDAO

205 IF LSTM0 = LAMDAO

210 IF LSTM0 = LAMDAO

215 IF LSTM0 = LAMDAO

220 GO TO 110

225 AF = A * COS(ALPHA) - B * COS(ALPHA)

230 DF = B * SIN(ALPHA) - A * SIN(ALPHA)

235 DF = DF - DMA

240 UT = UT A * COS(ALPHA) - B * COS(ALPHA)

245 UT = UT A * COS(ALPHA) - B * COS(ALPHA)

250 UT = UT A * COS(ALPHA) - B * COS(ALPHA)
PSI = 2.*ATAN2(YF*XF)
IF (PSI < 0.0) PSI = PSI + 2.*PI

PSID = PSI/7
0 = 1.0 + SIGMA(PSI-DELG) = 4.0+G*(COS(PSI*ALPH)))/SIN(PSI*ALPH)
AFD = (SIGMA-PSI)*COS(PSI-DELG-ALPH)

PSIDOT = PHIDOT*(AFD*SIGMA(PSI)+AFD*COS(PSI))/SIN(PSI*ALPH)

SIM(PSI)

PSIT = PHIDOT*(SIGMA+AFD*SIGMA(PSI-PMI+ALPH-DELG))
SF = PSF/ABS(PST)

IF (PSID*DELALPH>9.0) ETAFO = PMI-DELE-PSI-ALPH-P+3.0/E2+/2

IF (PSID+ALPHAL+LT+8*E+2*E+2=PMI-DELE-PSI-ALPH-P+3.0/E2+/2

IF (E+13.0/E1.E+10:PSIDOT=6.5F+ETAFO

166 =*F+5.3*E+SF =0.015.3*E+ETAFO =*F+0.4

GO TO 110

201ST00

210END
SUMOOLTINE TRANS(MOG+ALPHP+FP+AG+DEL6+Z0+PSI+PHI+G)

I = 3.14159
ST = (FP+PSI+ALPHP) + AMPSG+COS(PSI+ALPHP))/AG
CS = (FP+PSI+ALPHP) - AMPSG+SIN(PSI+ALPHP) - 8)/AG

PHI = ATAN2(ST,CL) - DEL6
IF(PHI<LT, B11, PHI = PHI + 2.*PI)
PHIEXT = PHI + 1.*Z

AF = B*COS(ALPHP) - 60*COS(PHIEXT) + DEL6 - ALPHP
NF = C*SIN(ALPHP) - 60*SIN(PHIEXT) - DEL6 - ALPHP

CF = 10000.

HoTF = AF*AF + 8.*NF - CF*CF

YF1 = AF - SQRT(HoTF)
YF2 = AF + SQRT(HoTF)

CF = 0.0 - CF

PSINE1 = 2.*ATAN(YF1+YF)
PSINE2 = 2.*ATAN(YF2+YF)

IF(PSINE1 < LT, 0., PSINE1 = PI - PSINE1 + 2.*PI)
IF(PSINE2 < LT, 0., PSINE2 = PI - PSINE2 + 2.*PI)

IFS = 14000.

PSINEXT = PSINE
GO TO 2

IF(PSINEXT = PSINE)
C = G = (8*SIN(PHIEXT-DEL6) - AMPSG*COS(PSI+ALPHP))/SIN(PSI+EXT)

RETURN
END
<table>
<thead>
<tr>
<th>pi10</th>
<th>176.2423</th>
<th>ps10</th>
<th>359.8276</th>
<th>ps10dot</th>
<th>-5.0465</th>
<th>sf</th>
<th>1.0</th>
<th>ETA060 = 64.7219</th>
<th>ETA060 = 91.6293</th>
<th>LOADAD = 256.9824</th>
</tr>
</thead>
<tbody>
<tr>
<td>pi10</td>
<td>177.4377</td>
<td>ps10</td>
<td>359.9264</td>
<td>ps10dot</td>
<td>-5.0496</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 64.8999</td>
<td>ETA060 = 91.7858</td>
<td>LOADAD = 256.9098</td>
</tr>
<tr>
<td>pi10</td>
<td>176.6652</td>
<td>ps10</td>
<td>359.8211</td>
<td>ps10dot</td>
<td>-5.0446</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 64.9772</td>
<td>ETA060 = 91.8464</td>
<td>LOADAD = 256.9329</td>
</tr>
<tr>
<td>pi10</td>
<td>176.7789</td>
<td>ps10</td>
<td>357.1214</td>
<td>ps10dot</td>
<td>-5.2501</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 87.6557</td>
<td>ETA060 = 91.8378</td>
<td>LOADAD = 267.1629</td>
</tr>
<tr>
<td>pi10</td>
<td>176.9466</td>
<td>ps10</td>
<td>354.2138</td>
<td>ps10dot</td>
<td>-5.3662</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.9977</td>
<td>ETA060 = 91.8486</td>
<td>LOADAD = 267.1846</td>
</tr>
<tr>
<td>pi10</td>
<td>177.1104</td>
<td>ps10</td>
<td>356.3136</td>
<td>ps10dot</td>
<td>-5.2646</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.6145</td>
<td>ETA060 = 80.8271</td>
<td>LOADAD = 267.1449</td>
</tr>
<tr>
<td>pi10</td>
<td>177.2919</td>
<td>ps10</td>
<td>354.4174</td>
<td>ps10dot</td>
<td>-5.0876</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 87.6557</td>
<td>ETA060 = 91.8378</td>
<td>LOADAD = 267.1629</td>
</tr>
<tr>
<td>pi10</td>
<td>177.4423</td>
<td>ps10</td>
<td>353.5138</td>
<td>ps10dot</td>
<td>-5.2752</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 87.7931</td>
<td>ETA060 = 91.8486</td>
<td>LOADAD = 267.1846</td>
</tr>
<tr>
<td>pi10</td>
<td>177.6337</td>
<td>ps10</td>
<td>352.6087</td>
<td>ps10dot</td>
<td>-5.3056</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.9028</td>
<td>ETA060 = 80.8271</td>
<td>LOADAD = 267.1449</td>
</tr>
<tr>
<td>pi10</td>
<td>177.7522</td>
<td>ps10</td>
<td>351.7027</td>
<td>ps10dot</td>
<td>-5.3503</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.9028</td>
<td>ETA060 = 80.8271</td>
<td>LOADAD = 267.1449</td>
</tr>
<tr>
<td>pi10</td>
<td>177.9706</td>
<td>ps10</td>
<td>350.7938</td>
<td>ps10dot</td>
<td>-5.3833</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.7931</td>
<td>ETA060 = 82.6235</td>
<td>LOADAD = 267.1450</td>
</tr>
<tr>
<td>pi10</td>
<td>178.1408</td>
<td>ps10</td>
<td>349.8838</td>
<td>ps10dot</td>
<td>-5.3187</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.1521</td>
<td>ETA060 = 81.9011</td>
<td>LOADAD = 266.4350</td>
</tr>
<tr>
<td>pi10</td>
<td>178.3199</td>
<td>ps10</td>
<td>348.9720</td>
<td>ps10dot</td>
<td>-5.2444</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.1521</td>
<td>ETA060 = 81.9011</td>
<td>LOADAD = 266.4350</td>
</tr>
<tr>
<td>pi10</td>
<td>178.4999</td>
<td>ps10</td>
<td>348.0633</td>
<td>ps10dot</td>
<td>-5.3393</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.0549</td>
<td>ETA060 = 79.9347</td>
<td>LOADAD = 255.9780</td>
</tr>
<tr>
<td>pi10</td>
<td>178.6663</td>
<td>ps10</td>
<td>347.1417</td>
<td>ps10dot</td>
<td>-5.3862</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 86.1521</td>
<td>ETA060 = 80.8271</td>
<td>LOADAD = 267.1450</td>
</tr>
<tr>
<td>pi10</td>
<td>178.8337</td>
<td>ps10</td>
<td>346.2252</td>
<td>ps10dot</td>
<td>-5.3460</td>
<td>g</td>
<td>0.00129</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 01.1536</td>
</tr>
<tr>
<td>pi10</td>
<td>178.9996</td>
<td>ps10</td>
<td>345.3121</td>
<td>ps10dot</td>
<td>-5.4892</td>
<td>g</td>
<td>0.00031</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 02.2480</td>
</tr>
<tr>
<td>pi10</td>
<td>178.1664</td>
<td>ps10</td>
<td>344.4044</td>
<td>ps10dot</td>
<td>-5.5294</td>
<td>g</td>
<td>0.000727</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 03.3472</td>
</tr>
<tr>
<td>pi10</td>
<td>178.3468</td>
<td>ps10</td>
<td>343.5066</td>
<td>ps10dot</td>
<td>-5.5596</td>
<td>g</td>
<td>0.001272</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 04.4552</td>
</tr>
<tr>
<td>pi10</td>
<td>178.5166</td>
<td>ps10</td>
<td>342.6088</td>
<td>ps10dot</td>
<td>-5.6781</td>
<td>g</td>
<td>0.002717</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 05.5648</td>
</tr>
<tr>
<td>pi10</td>
<td>178.6964</td>
<td>ps10</td>
<td>341.7110</td>
<td>ps10dot</td>
<td>-5.7377</td>
<td>g</td>
<td>0.004523</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 06.6731</td>
</tr>
<tr>
<td>pi10</td>
<td>178.8663</td>
<td>ps10</td>
<td>340.8136</td>
<td>ps10dot</td>
<td>-5.7949</td>
<td>g</td>
<td>0.006729</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 07.7831</td>
</tr>
<tr>
<td>pi10</td>
<td>179.0337</td>
<td>ps10</td>
<td>340.0132</td>
<td>ps10dot</td>
<td>-5.8505</td>
<td>g</td>
<td>0.009654</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 08.8959</td>
</tr>
<tr>
<td>pi10</td>
<td>179.2006</td>
<td>ps10</td>
<td>339.2128</td>
<td>ps10dot</td>
<td>-5.9052</td>
<td>g</td>
<td>0.013699</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 09.0169</td>
</tr>
<tr>
<td>pi10</td>
<td>179.3799</td>
<td>ps10</td>
<td>338.4126</td>
<td>ps10dot</td>
<td>-5.9606</td>
<td>g</td>
<td>0.018629</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 10.1279</td>
</tr>
<tr>
<td>pi10</td>
<td>179.5488</td>
<td>ps10</td>
<td>337.6124</td>
<td>ps10dot</td>
<td>-5.9160</td>
<td>g</td>
<td>0.021370</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 11.2389</td>
</tr>
<tr>
<td>pi10</td>
<td>179.7171</td>
<td>ps10</td>
<td>336.8122</td>
<td>ps10dot</td>
<td>-5.9714</td>
<td>g</td>
<td>0.023115</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 12.3499</td>
</tr>
<tr>
<td>pi10</td>
<td>179.8859</td>
<td>ps10</td>
<td>336.0120</td>
<td>ps10dot</td>
<td>-5.9268</td>
<td>g</td>
<td>0.024060</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 13.4599</td>
</tr>
<tr>
<td>pi10</td>
<td>180.0543</td>
<td>ps10</td>
<td>335.2118</td>
<td>ps10dot</td>
<td>-5.9822</td>
<td>g</td>
<td>0.024060</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 14.5709</td>
</tr>
<tr>
<td>pi10</td>
<td>180.2223</td>
<td>ps10</td>
<td>334.4116</td>
<td>ps10dot</td>
<td>-5.0376</td>
<td>g</td>
<td>0.024060</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 15.6819</td>
</tr>
<tr>
<td>pi10</td>
<td>180.3903</td>
<td>ps10</td>
<td>333.6114</td>
<td>ps10dot</td>
<td>-5.0920</td>
<td>g</td>
<td>0.024060</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 16.7929</td>
</tr>
<tr>
<td>pi10</td>
<td>180.5583</td>
<td>ps10</td>
<td>332.8112</td>
<td>ps10dot</td>
<td>-5.1464</td>
<td>g</td>
<td>0.024060</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 17.9039</td>
</tr>
<tr>
<td>pi10</td>
<td>180.7263</td>
<td>ps10</td>
<td>332.0110</td>
<td>ps10dot</td>
<td>-5.1998</td>
<td>g</td>
<td>0.024060</td>
<td>sf</td>
<td>1.0</td>
<td>ETA060 = 18.0149</td>
</tr>
</tbody>
</table>

The program CLOCK 2 furnishes the moment input-output relationship for a single step-up gear mesh with clock (ogival) teeth as derived in section 2 of Appendix E. This program uses the same kinematics as program CLOCK 1, and it differs from that program only in that it also determines both point and cycle efficiencies. Because of the above, sections a to c below will only contain discussions on those portions of the program which are different from CLOCK 1. The last section shows the complete program CLOCK 2.

a. Input Parameters (see Program CLOCK 2, below)

In addition to the input parameters of CLOCK 1, the following data must be supplied to the program:

\[ \mu \] the coefficient of friction at the gear and pinion pivots as well as at the contact point between the gear and the pinion

\[ \rho_1 \] the pivot radius of the gear

\[ \rho_2 \] the pivot radius of the pinion

b. Computations (See also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The computation of the gear tooth parameters is identical
with that given in program CLOCK 1.

II. Determination of Transition Angle

The transition angle is determined in the same manner as described in program CLOCK 1.

III. Determination of Correct Sign for Round on Flat Regime

This computation is also identical with that given in program CLOCK 1.

IV. Computation of Final and Initial Values of φ and θ

This computation is also identical with that given in program CLOCK 1.

V. Determination of Correct Sign for Round on Round Regime

This computation is also identical with that given in program CLOCK 1.

VI. Kinematics, Point and Cycle Efficiencies

The angular increment DDPHI of the input angle is found in the same manner as shown in program CLOCK 1. While the initial, transition and final angles of the mesh are obtained in the same manner as shown in CLOCK 1, the computational loop of the program ranges from the initial angle PHI to one increment before the final angle PHIF. This is necessary in
order to accommodate the numerical integration for the cycle efficiency.

A. Round on Round Regime

As before for CLOCK 1, as long as PHI is smaller than the transition angle PHIT, the kinematics of the round on round phase of the motion is computed for each increment. These kinematic computations are identical with those given in CLOCK 1. The point efficiency in this phase of motion is obtained with the help of eq. (E-70) and takes the form of eq. (3), i.e.,

\[ \eta_P = \text{Kratio} \frac{M_o}{M_{\text{in}}} \]  

(F-2)

Since the transmission ratio depends on the angular velocity ratio, and the input angular velocity is unity, eq. (F-2) becomes

\[ \eta_P = \frac{M_o}{M_{\text{in}}} \cdot |\dot{r}| \quad \text{(POINTEF)} \]  

(F-3)

B. Round on Flat Regime

When PHI is larger than the transition angle PHIT, the kinematics of the round on flat phase becomes applicable. While the computed kinematic quantities are identical with those of CLOCK 1, the present program contains an expression for the point efficiency for the round on flat phase of the motion. This expression is obtained with the help of eq. (E-92), and it
is computed in the manner of eq. (F-3) with the now current values of the output angular velocity PSIDOT.

C. Computation of Cycle Efficiency

Once the computational loop is terminated, the cycle efficiency is computed in the manner of eq. (C-10) of Appendix C, i.e.,

\[
\epsilon_C = \frac{\Delta \varphi \sum P}{(\Phi_{FIN} - \Phi_{IN})} \quad \text{(CYCLEFF)} \quad \text{(F-4)}
\]

The summation was accomplished inside the loop in terms of

\[
MTOT = MTOT + POINTEF \quad \text{(F-5)}
\]

Further,

\[
\Delta \varphi = \text{DDPHI in the program, and similarly} \\
\Phi_{FIN} = \text{PHIP and} \\
\Phi_{IN} = \text{PHII}
\]

c. Output (See section d below)

The output of CLOCK 2 is identical with that of CLOCK 1 with the exception that the point and cycle efficiencies are printed out. The identical geometric parameters are used, and therefore, the same kinematic output is obtained.
Program CLOCK 2
PROGRAM CLOCK 2 (INPUT, OUTPUT, TAPE=INPUT, TAPE=OUTPUT)

C POINT AND CYCLE EFFICIENCIES FOR SINGLE PASS STEP-UP
C GEAR-MESH WITH CLOCK (LOGICAL) - TELM

C

18
C
C
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
C.
AF = B*COS(ALPHAP) - B*COS(PHI-DELG-ALPHAP)
BF = B*SIN(ALPHAP) - B*SIN(PHI-DELG-ALPHAP)
CF = -H/MOG
HF1 = AF*AF + BF*BF - CF*CF
YF1 = AF + SQRT(HF1)
YF2 = AF - SQRT(HF1)
AF = BF + CF
PSIF1 = 2*A*TAN2(YF1,AF)
PSIF2 = 2*A*TAN2(YF2,AF)
IF(IFPSIF1 < 9.0, PSIF1) = PSIF1 + 2.*PI
IF(IFPSIF2 < 9.0, PSIF2) = PSIF2 + 2.*PI
SF = ABS(PSIF1-PSIF2) + LT. ABS(PSIF2-PSIF1) > 0.0
SIGN = -1.
60 TO 50

C
SIGN = 1.

C COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI
C
30 DO 60 I=1,1800
PHI = PHI0 + I/180.*PI
AF = B*COS(ALPHAP) - B*COS(PHI-DELG-ALPHAP)
BF = B*SIN(ALPHAP) - B*SIN(PHI-DELG-ALPHAP)
CF = -H/MOG
HF1 = AF*AF + BF*BF - CF*CF
YF1 = AF + SQRT(HF1)
YF2 = AF - SQRT(HF1)
AF = BF + CF
PSIF1 = 2*A*TAN2(YF1,AF)
PSIF2 = 2*A*TAN2(YF2,AF)
IF(IFPSIF1 < 9.0, PSIF1) = PSIF1 + 2.*PI
IF(IFPSIF2 < 9.0, PSIF2) = PSIF2 + 2.*PI
LF = B + A*COS(PHI-DELG-DPHI) - B*COS(PSIF1+DPSI-DELPS)
LT = B + A*SIN(PHI-DELG-DPHI) - B*SIN(PSIF1+DPSI-DELPS)
IF(IFSORTCFL0 > LT) LT = LT60 TO 70
C
C C CONTINUE
C
PSIFF = PSIF
PHII = PHII + PHI1
PSII = PSII + PSI1
IF = IF1T + PSIFF + PSI1
PHI00 = PHI1/I2
PS110 = PSI1/I2
PHII0 = PHII/I2
PSIFF = PSIFF
WHITE(I080) = PI10 + PSI10 + PHI0 + PSI0
C
C \textbf{C} DETERMINATION OF CORRECT SIGN FOR \textit{H} ON \textit{H} REDGE
AF = SINC(PHI1-DELG) - SINC(PHI1-DELG)+TAN(DELG)/AC
BF = COS(PHI1-DELG) + B/AC - SIN(PHI1-DELG)+TAN(DELG)
CF = -ATAN2(AF + BF, BF - ATAN2(AF + BF))
1 = A*COS(PHI1-DELG)/(A*COS(DELG))
NORTH = NORT + PHI1 + SIGN
THI = AN + SQRT(HF1)
GO TO 150

120 AF = RCOS(PHI) + AGOSIN(PHI-DELF-ALPH)
       UG = 0 + SIN(PHI) - AGOSIN(PHI-DELF-ALPH)
       CF = -RHUG
       HUGIF = AF*AF - AF*UG - CF*UG
       WIF = AF + SGN*SWIT(HUGIF)
       AF = WU + CF

220 PSI = 2.466466118

210 IF(PSI.LT.0) PSI = PSI + 2.012

230 IF(PSI+ALPHP.EQ.90.ETAGFD=PHI-DELF-PSI-ALPHP) G = (AGOSIN(PHI-DELF) - RHUG*COS(PHI+ALPHP))/SIN(PHI+ALPHP)
       AFD = AGOSIN(PHI-DELF-ALPHP)

240 UFD = AG*COS(PHI-DELF-ALPHP)
       PSIDOT = PHIDOT*(AFD*SIN(PHI) / AFD*COS(PHI+ALPHP))

VSTF = PHIDOT*(RHUG + AGOSIN(PHI+ALPHP-DELF))

250 IF(PSID+ALPHP.EQ.90.ETAGFD=PHI-DELF-PSI-ALPHP+1.012)
       IF(PSID+ALPHP.EQ.90.ETAGFD=PHI-DELF-PSI-ALPHP-1.012)
       EFFX = UFD*(RHUG + SFD*RHUG) - AG*COS(PHI-DELF-PSI-ALPHP)

260 UFD = UFD*(SFD*UFD) + SMSIDOT(PSID+ALPHP)

270 PSI = PSIDOT + 0.5*PSD*ETAGFD*PSIDOT

280 AOT = AOT + 0.5*PSD*PSD*ETAGFD*PSIDOT

290 CYCLEFF = AOT+SMDPHI/(PMIF-PHI)

STOP

END
APPENDIX G

KINEMATICS OF TWO AND THREE STEP-UP GEAR TRAINS WITH CLOCK TEETH

Figure G-1 shows the basic configuration of a three step-up gear train used in certain fuze applications. The general layout is identical to that shown in Figure A-5 of Appendix A. Now, ogival type gear teeth are used instead of involute type ones. Again, it is required to find the equilibrant moment $M_{04}$, acting on pinion no. 4 which holds the input moment $M_{in}$, acting on gear no. 1, in equilibrium when both pivot and contact friction are taken into account, and when the fuze body spins. Appendix H gives the force and moment analyses for the determination of this moment input-output relationship. The same appendix also shows the derivation of such an input-output relationship for a two step-up gear train with ogival gear teeth which must operate in a spin environment. (Figure A-10 of Appendix A shows this type of configuration with involute teeth.) The present appendix lays the groundwork for the moment relationships of Appendix H by providing the kinematics of the three ogival gear meshes involved.

In each case both round on round and round on flat phases
FIGURE G-1

BASIC CONFIGURATION FOR OGIVAL THREE STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT
of motion have to be considered. All derivations follow the pattern set in Section 1 of Appendix E. The derivations must take into account that the driving gears of meshes no. 1 and no. 3, i.e., between gear no. 1 and pinion no. 2 and between gear no. 3 and pinion no. 4, respectively, have clockwise rotations. The driving gear of mesh no. 2, i.e., between gear no. 2 and pinion no. 3, has counterclockwise rotation.

Finally, the inclinations of the various pivot to pivot centerlines with respect to the body-fixed X-axes, as represented by the angles \( \beta_1, \beta_2 \) and \( \beta_3 \), must be considered.

For the sake of simplicity, the notation will in most cases not differentiate between round on round and round on flat phases of motion. For example, the output angle, \( \psi_1 \), and its derivatives will have the same symbol for both phases.

For definitions of angles \( \beta_1 \) and \( \gamma_1 \) as well as the distances \( \Omega_1 \), see Appendix A-6.
1. KINEMATICS OF MESH NO. 1 (GEAR NO. 1 AND PINION NO. 2)

a. ROUND ON ROUND PHASE OF MOTION

Figure G-2 shows the round on round phase of the motion of mesh no. 1 in a schematic manner. Only the contacting faces of the gear and the pinion are indicated.

1. UNIT VECTORS

The unit vector in the direction $O_1C_{G_1}$ of the gear is given by

$$\vec{n}_{G_1} = \cos(\phi_1 + \delta_{G_1})\vec{I} + \sin(\phi_1 + \delta_{G_1})\vec{J}$$  \hspace{1cm} (G-1)

The unit vector in the direction $C_{G_1}C_{P_1}$ is given by

$$\vec{n}_{A_1} = \cos\lambda_1\vec{I} + \sin\lambda_1\vec{J}$$  \hspace{1cm} (G-2)

The unit vector normal to $\vec{n}_{A_1}$ (in the right hand sense) becomes

$$\vec{n}_{N_{A_1}} = -\sin\lambda_1\vec{I} + \cos\lambda_1\vec{J}$$  \hspace{1cm} (G-3)

G-4
FIGURE G-2
ROUND ON ROUND PHASE FOR MESH NO. 1
The unit vector in the direction $O_2C_{P_1}$ is given by

$$
\vec{h}_{P_1} = \cos(\psi + \delta_{P_1}) \hat{I} + \sin(\psi + \delta_{P_1}) \hat{J}
$$

Finally, the unit vector along the centerline, $O_1O_2$, is given by

$$
\vec{h}_{\beta_1} = \cos\beta \hat{I} + \sin\beta \hat{J}
$$

II. DETERMINATION OF OUTPUT ANGLE $\psi_1$ AND "COUPLER" ANGLE $\lambda_1$

The loop equation of the equivalent four-bar linkage is given by

$$
a_{G1} \vec{h}_{G_1} + L_1 \vec{h}_{\lambda_1} - a_{P_1} \vec{h}_{P_1} - b_1 \vec{h}_{\beta_1} = 0
$$

where $L_1 = r_{G1} + r_{P1}$

With the appropriate substitution for the unit vectors, one obtains the following component equations:
\[ a_{G1}\cos(\phi_1 + \delta_{G1}) + L_1\cos\lambda_1 - b_1\cos \beta_1 - a_{P1}\cos(\psi_1 + \delta_{P1}) = 0 \]  

(G-8)

and

\[ a_{G1}\sin(\phi_1 + \delta_{G1}) + L_1\sin\lambda_1 - b_1\sin \beta_1 - a_{P1}\sin(\psi_1 + \delta_{P1}) = 0 \]  

(G-9)

To eliminate \( \lambda_1 \), let

\[ \sin^2 \lambda_1 + \cos^2 \lambda_1 = 1 \]  

(G-10)

The above trigonometric functions are obtained from equations (G-8) and (G-9), respectively. Substitution into equation (G-10) furnishes

\[ A_{1R}\sin\psi_1 + B_{1R}\cos\psi_1 = C_{1R} \]  

(G-11)

where

\[ A_{1R} = a_{G1}\sin(\phi_1 + \delta_{G1} - \delta_{P1}) - b_1\sin(\beta_1 - \delta_{P1}) \]

\[ B_{1R} = a_{G1}\cos(\phi_1 + \delta_{G1} - \delta_{P1}) - b_1\cos(\beta_1 - \delta_{P1}) \]

\[ C_{1R} = \frac{a_{P1}^2 + a_{G1}^2 + b_1^2 - L_1^2 - 2a_{G1}b_1\cos(\phi_1 + \delta_{G1} - \beta_1)}{2a_{P1}} \]  

(G-7)
Equation (G-11) is now solved for $\psi_1$ in the manner described in Appendix E in connection with equation (E-12), i.e.,

$$\psi_1 = 2 \tan^{-1} \frac{A_{1R} \pm \sqrt{A_{1R}^2 + B_{1R}^2 - C_{1R}^2}}{B_{1R} + C_{1R}} \quad (G-12)$$

The correct sign on equation (G-12) must be found from geometric considerations.

The coupler angle $\lambda_1$ may now be determined either from equation (G-8) or from equation (G-9). Thus,

$$\lambda_1 = \cos^{-1} \left[ \frac{b_1 \cos \beta_1 + a_{p1} \cos(\psi_1 + \delta_{p1}) - a_{G1} \cos(\phi_1 + \delta_{G1})}{L_1} \right] \quad (G-13)$$

or

$$\lambda_1 = \sin^{-1} \left[ \frac{b_1 \sin \beta_1 + a_{p1} \sin(\psi_1 + \delta_{p1}) - a_{G1} \sin(\phi_1 + \delta_{G1})}{L_1} \right] \quad (G-14)$$
III. DETERMINATION OF ANGULAR VELOCITY $\dot{\psi}_1$ OF PINION NO. 2

Differentiation of equation (G-11) with respect to time gives

$$\dot{\psi}_1 = \dot{\phi}_1 \left[ \frac{B_{RD}\cos\psi_1 - A_{RD}\sin\psi_1 + C_{RD}}{A_{R}\cos\psi_1 - B_{R}\sin\psi_1} \right]$$  \hspace{1cm} (G-15)

where

$$A_{RD} = a_{g1}\cos(\phi_1 + s_{g1} - s_{p1})$$

$$B_{RD} = a_{g1}\sin(\phi_1 + s_{g1} - s_{p1})$$

$$C_{RD} = \frac{a_{g1}\beta_{1}\sin(\phi_1 + s_{g1} - \beta_1)}{a_{p1}}$$
IV. RELATIVE VELOCITY AT THE CONTACT POINT

The relative velocity \( \vec{v}_{S1/T1R} \) of the contact point \( S_1 \) on gear no. 1 with respect to point \( T_1 \) on pinion no. 2, represents the vectorial difference between the absolute velocities of these points. Thus,

\[
\vec{v}_{S1/T1R} = \vec{v}_{S1/C} - \vec{v}_{T1/C}
\]

where \( C \) represents the spin center of the fuze body. If \( \vec{\omega} \) stands for the angular velocity of the fuze body, then appropriate substitution into equation (G-16) gives (See also Figures G-1 and G-2)

\[
\vec{v}_{S1/T1R} = \left[ \vec{\omega} \times \vec{r}_1 + (\vec{\omega} + \vec{\omega}_1) \times (\vec{r}_g1 + \vec{r}_g1) \right] \\
- \left[ \vec{\omega} \times \vec{r}_2 + (\vec{\omega} + \vec{\omega}_1) \times (\vec{r}_p1 + \vec{r}_p1) \right]
\]

(S-17)

Since

\[
\vec{\omega} \times \left[ \vec{r}_1 + \vec{r}_g1 + \vec{r}_g1 \right] = \vec{\omega} \times \left[ \vec{r}_2 + \vec{r}_p1 + \vec{r}_p1 \right]
\]

(S-18)
because the position vectors in brackets are equal, equation (G-17) may be written

\[ \vec{V}_{S1/T1 R} = \vec{V}_{S1/O1} - \vec{V}_{T1/O2} \]

\[ = \vec{v}_1 \times (\vec{r}_{G1} + \vec{r}_{Q1}) - \vec{v}_1 \times (\vec{r}_{P1} + \vec{r}_{P1}) \quad (G-19) \]

Note that \( \vec{V}_{S1/T1 R} \) becomes the vectorial difference of the contact point velocities with respect to the fuze body.

Since this relative velocity is tangent to the contacting surfaces, it may be written as the vectorial difference of the velocity components along these surfaces. Accordingly, equation (G-19) becomes, with the help of the unit vector \( \vec{n}_{N\Lambda1} \),

\[ \vec{V}_{S1/T1 R} = \left\{ \left[ \vec{v}_1 \times (\vec{r}_{G1} \vec{n}_{G1} + \vec{r}_{Q1} \vec{n}_{\Lambda1}) \right] \cdot \vec{n}_{N\Lambda1} \right\} \vec{n}_{N\Lambda1} \]

\[ - \left[ \vec{v}_1 \times (\vec{r}_{P1} \vec{n}_{P1} + \vec{r}_{P1} \vec{n}_{\Lambda1}) \right] \cdot \vec{n}_{N\Lambda1} \}

\[ \vec{n}_{N\Lambda1} \quad (G-20) \]

Appropriate substitution of unit vectors given earlier in Section I and simplification results in:

G-11
\[ \nabla_{S1/T1R} = \left\{ \hat{\psi}_1 \left[ a_{G1} \cos(\psi_1 + \delta_{G1} - \lambda_1) + \rho_{G1} \right] \right. \\
\left. - \hat{\psi}_1 \left[ a_{P1} \cos(\psi_1 + \delta_{P1} - \lambda_1) - \rho_{P1} \right] \right\} \vec{R}_{N1} \] (G-21)

b. ROUNDS ON FLAT PHASE OF MOTION

Figure G-3 shows a schematic view of mesh no. 1 in the round on flat phase of the motion.

I. UNIT VECTORS

The unit vector in the direction \( O_2T_1 \) is given by

\[ \vec{r}_{F1} = \cos(\psi_1 - \alpha_{P1}) \vec{I} + \sin(\psi_1 - \alpha_{P1}) \vec{J} \] (G-22)

The unit vector \( \vec{r}_{NF1} \), in the direction \( C_{G1}S_1 \), is always normal to \( \vec{r}_{F1} \)

\[ \vec{r}_{NF1} = -\sin(\psi_1 - \alpha_{P1}) \vec{I} + \cos(\psi_1 - \alpha_{P1}) \vec{J} \] (G-23)
FIGURE G-3

ROUND ON FLAT PHASE FOR MESH NO. 1
II. DETERMINATION OF OUTPUT ANGLE $\phi_1$ AND DISTANCE $s_1$

The vector equation for the mechanism loop $0_1-G_1-S_1-T_1-O_2$ has the form:

$$a_{G1}\vec{r}_{G1} + r_{G1}\vec{r}_{NF1} - s_1\vec{x}_{F1} - b_1\vec{x}_{\beta1} = 0 \quad (G-24)$$

Substitution of equations (G-1), (G-5), (G-22) and (G-23) leads to the following component equations:

$$a_{G1}\cos(\phi_1 + s_{G1}) - r_{G1}\sin(\psi_1 - \alpha_{P1}) - b_1\cos\beta_1 - s_1\cos(\psi_1 - \alpha_{P1}) = 0 \quad (G-25)$$

and

$$a_{G1}\sin(\phi_1 + s_{G1}) + r_{G1}\cos(\psi_1 - \alpha_{P1}) - b_1\sin\beta_1 - s_1\sin(\psi_1 - \alpha_{P1}) = 0 \quad (G-26)$$

From equation (G-26) one obtains for the quantity $s_1$

$$s_1 = \frac{a_{G1}\sin(\phi_1 + s_{G1}) + r_{G1}\cos(\psi_1 - \alpha_{P1}) - b_1\sin\beta_1}{\sin(\psi_1 - \alpha_{P1})} \quad (G-27)$$
This expression is now substituted in equation (G-25). This leads to the following:

\[ A_{1F} \sin \psi_1 + B_{1F} \cos \psi_1 = C_{1F} \]  

(G-28)

where

\[ A_{1F} = a_{G1} \cos(\phi_1 + \delta_{G1} + \alpha_{p1}) - b_{1} \cos(\beta_1 + \alpha_{p1}) \]

\[ B_{1F} = -a_{G1} \sin(\phi_1 + \delta_{G1} + \alpha_{p1}) + b_{1} \sin(\beta_1 + \alpha_{p1}) \]

\[ C_{1F} = \rho_{G1} \]

Equation (G-28) is solved for \( \psi_1 \) in the now customary manner:

\[ \psi_1 = 2 \tan^{-1} \frac{A_{1F} \pm \sqrt{A_{1F}^2 + B_{1F}^2 - C_{1F}^2}}{B_{1F} + C_{1F}} \]  

(G-29)

The appropriate sign is again found from geometric considerations.
III. **DETERMINATION OF ANGULAR VELOCITY \( \dot{\psi}_1 \) DURING ROUND ON FLAT PHASE OF MOTION**

Implicit differentiation of equation (G-28) with respect to time gives, for \( \dot{\psi}_1 \):

\[
\ddot{\psi}_1 = \dot{\psi}_1 \left[ \frac{A_{1FD} \sin \dot{\psi}_1 + B_{1FD} \cos \dot{\psi}_1}{A_{1F} \cos \dot{\psi}_1 - B_{1F} \sin \dot{\psi}_1} \right] \quad \text{(G-30)}
\]

where

\[
A_{1FD} = a_{G1} \sin (\dot{\psi}_1 + \delta_{G1} + \alpha_{P1})
\]

\[
B_{1FD} = a_{G1} \cos (\dot{\psi}_1 + \delta_{G1} + \alpha_{P1})
\]
IV. RELATIVE VELOCITY $\mathbf{V}_{S1/T1_f}$ AT CONTACT POINT DURING ROUND ON FLAT PHASE OF MOTION

For the round on flat phase, the relative velocity $\mathbf{V}_{S1/T1_f}$ may be expressed by

$$\mathbf{V}_{S1/T1_f} = \mathbf{V}_{S1/O_1} - \mathbf{V}_{T1/O_2}$$  \hspace{1cm} (G-31)

Now, this velocity has the direction of the unit vector $\mathbf{r}_{F1}$. Since there is no velocity component along the pinion flank, equation (G-31) becomes

$$\mathbf{V}_{S1/T1_f} = \mathbf{V}_{S1/O_1} \cdot \mathbf{r}_{F1}$$

$$= \left\{ \left[ \dot{\alpha}_{1f} \left( \alpha_{G1} \mathbf{E}_{G1} + \rho_{G1} \mathbf{N}_{NF1} \right) \right] \cdot \mathbf{r}_{F1} \right\} \mathbf{r}_{F1}$$  \hspace{1cm} (G-32)

Appropriate substitution of unit vectors furnishes

$$\mathbf{V}_{S1/T1_f} = \dot{\alpha}_{1} \left[ \alpha_{G1} \sin(\psi_{1} - \alpha_{P1} - \phi_{1} - \delta_{G1}) - \rho_{G1} \right] \mathbf{r}_{F1}$$  \hspace{1cm} (G-33)
V. DETERMINATION OF TRANSITION ANGLES

The transition angle, $\phi_{1T}$, and the corresponding angle, $\psi_{1T}$, are reached when the round or round phase is followed by the round on flat one. They are obtained by letting $g_1 = f_{p1}$ in the component equations (G-25) and (G-26). This gives

$$a_{g1}\cos(\phi_{1T} + \delta_{g1}) - \rho_{g1}\sin(\psi_{1T} - \alpha_{P1}) - b_1\cos\beta_1 - f_{P1}\cos(\psi_{1T} - \alpha_{P1}) = 0 \quad (G-34)$$

and

$$a_{g1}\sin(\phi_{1T} + \delta_{g1}) + \rho_{g1}\cos(\psi_{1T} - \alpha_{P1}) - b_1\sin\beta_1 - f_{P1}\sin(\psi_{1T} - \alpha_{P1}) = 0 \quad (G-35)$$

From the above, one obtains

$$\cos(\phi_{1T} + \delta_{g1}) = \frac{1}{a_{g1}}\left[\rho_{g1}\sin(\psi_{1T} - \alpha_{P1}) + b_1\cos\beta_1 + f_{P1}\cos(\psi_{1T} - \alpha_{P1})\right] \quad (G-36)$$

$$\sin(\phi_{1T} + \delta_{g1}) = \frac{1}{a_{g1}}\left[-\rho_{g1}\cos(\psi_{1T} - \alpha_{P1}) + b_1\sin\beta_1 + f_{P1}\sin(\psi_{1T} - \alpha_{P1})\right] \quad (G-37)$$

G-18
The angle $\psi_{1T}$ is now obtained by letting

$$\sin^2(\phi_{1T} + \delta_{G1}) + \cos^2(\phi_{1T} + \delta_{G1}) = 1$$

This results in

$$A_{1T}\sin\psi_{1T} + B_{1T}\cos\psi_{1T} = C_{1T} \quad \text{(G-38)}$$

where

$$A_{1T} = \rho_{G1}\cos(\beta_1 + \alpha_{p1}) + f_{p1}\sin(\beta_1 + \alpha_{p1})$$

$$B_{1T} = -\rho_{G1}\sin(\beta_1 + \alpha_{p1}) + f_{p1}\cos(\beta_1 + \alpha_{p1})$$

$$C_{1T} = \frac{a_{G1}^2 - \rho_{G1}^2 - b_1^2 - f_{p1}^2}{2b_1}$$

Finally,

$$\psi_{1T} = 2 \tan^{-1} \frac{A_{1T} \pm \sqrt{A_{1T}^2 + B_{1T}^2 - C_{1T}^2}}{B_{1T} + C_{1T}} \quad \text{(G-39)}$$

The appropriate sign must be found from geometric considerations.

The associated angle $\phi_{1T}$ may be found with the help of either equation (G-36) or equation (G-37):

$\text{G-19}$
\[
\phi_1 T = \cos^{-1} \left[ \frac{\rho_{G1} \sin(\psi_1 T - \alpha_{P1}) + f_{P1} \cos(\psi_1 T - \alpha_{P1}) + b_1 \cos \beta_1}{a_{G1}} \right] - \delta_{G1} \quad (G-40)
\]

or

\[
\phi_1 T = \sin^{-1} \left[ \frac{-\rho_{G1} \cos(\psi_1 T - \alpha_{P1}) + f_{P1} \sin(\psi_1 T - \alpha_{P1}) + b_1 \sin \beta_1}{a_{G1}} \right] - \delta_{G1} \quad (G-41)
\]

VI. SENSING EQUATION FOR THE DETERMINATION OF CONTACT ON SUBSEQUENT TOOTH MESH

The following contact sensing equation is derived with the assumption that subsequent contact is made in the round on round phase of the motion, in the manner shown in Section VI of Appendix E. Now, the configuration is that of Figure G-2 where gear no. 1 rotates in a clockwise direction.

Before contact is made, the distance between the centers of curvature \( C_{G1} \) and \( C_{P1} \) is given by
\[ \overline{C_{G1}C_{P1}} = L_{x1} \overline{i} + L_{y1} \overline{j} \]  \hspace{1cm} (G-42)

If \( \Delta \phi_1 \) and \( \Delta \psi_1 \) represent the tooth spacing angles of gear no. 1 and pinion no. 2, respectively, the associated loop equation becomes (see Figures E-3 and G-2).

\[
\begin{align*}
\overline{a_{G1}} & \left[ \cos(\phi_1 + \Delta \phi_1 + \delta_{G1}) \overline{i} + \sin(\psi_1 + \Delta \psi_1 + \delta_{G1}) \overline{j} \right] + L_{x1} \overline{i} + L_{y1} \overline{j} \\
& = b_1 (\cos \beta_1 \overline{i} + \sin \beta_1 \overline{j}) + a_{P1} \left[ \cos(\psi_1 - \Delta \psi_1 + \delta_{P1}) \overline{i} + \sin(\psi_1 - \Delta \psi_1 + \delta_{P1}) \overline{j} \right]
\end{align*}
\]

where

\[
\varphi_1 = \text{angle of pinion no. 2 as determined for the round on flat mode with equation (G-29)}
\]

The magnitudes of \( L_{x1} \) and \( L_{y1} \) are determined from the component form of equation (G-43), i.e.,

\[
\begin{align*}
L_{x1} &= b_1 \cos \beta_1 + a_{P1} \cos(\psi_1 - \Delta \psi_1 + \delta_{P1}) - a_{G1} \cos(\phi_1 + \Delta \phi_1 + \delta_{G1}) \\
& \hspace{1cm} (G-44)
\end{align*}
\]

and

\[
\begin{align*}
L_{y1} &= b_1 \sin \beta_1 + a_{P1} \sin(\psi_1 - \Delta \psi_1 + \delta_{P1}) - a_{G1} \sin(\phi_1 + \Delta \phi_1 + \delta_{G1}) \\
& \hspace{1cm} (G-45)
\end{align*}
\]

G-21
Contact will have occurred as soon as

\[ \sqrt{L_{x_1}^2 + L_{y_1}^2} \leq \rho_{G_1} + \rho_{P_1} \]  

(G-46)
2. KINEMATICS OF MESH NO. 2 (GEAR NO. 2 AND PINION NO. 3)

a. ROUND ON ROUND PHASE OF MOTION

Figure G-4 gives a schematic representation of the round on round phase of the motion. Only the contacting faces of the gear and the pinion are shown. It is to be noted that the input gear rotates in the counter-clockwise direction, and that the output angle $\psi_1$ of mesh no. 1 is identical to the input angle $\phi_2$ of mesh no. 2.

I. UNIT VECTORS

The unit vector in the direction $G_2C_2$ of the gear is given by

$$\vec{n}_{G2} = \cos(\phi_2 - \delta_{G2}) \hat{I} + \sin(\phi_2 - \delta_{G2}) \hat{J} \quad (G-47)$$

The unit vector in the direction $C_2C_2$ is given by

$$\vec{n}_{\lambda 2} = \cos \lambda_2 \hat{I} + \sin \lambda_2 \hat{J} \quad (G-48)$$
FIGURE G-4
ROUND ON ROUND PHASE FOR MESH NO. 2
The unit vector normal to $\vec{n}_{A2}$ in the right hand sense becomes

$$\vec{n}_{A2} = -\sin \lambda_2 \vec{I} + \cos \lambda_2 \vec{J} \quad (G-49)$$

The pinion unit vector $\vec{n}_{P2}$, in the direction $O_3C_{P2}$, is represented by

$$\vec{n}_{P2} = \cos(\psi_2 - \delta_{P2}) \vec{I} + \sin(\psi_2 - \delta_{P2}) \vec{J} \quad (G-50)$$

Finally, the unit vector along the centerline $O_2O_3$ is given by

$$\vec{n}_{02} = \cos \beta_2 \vec{I} + \sin \beta_2 \vec{J} \quad (G-51)$$

II. DETERMINATION OF OUTPUT ANGLE $\psi_2$ AND "COUPLER" ANGLE $\lambda_2$

The loop equation of the equivalent four-bar linkage is given by

$$a_{G2} \vec{n}_{G2} + L_2 \vec{n}_{A2} - a_{P2} \vec{n}_{P2} - b_{2} \vec{n}_{02} = 0 \quad (G-52)$$
where

\[ L_2 = r_{G2} + r_{p2} \]  \hspace{1cm} (G-53)

After substitution of the unit vector, as given earlier, one obtains the following component equations

\[ a_{G2} \cos(\phi_2 - \delta_{G2}) + L_2 \cos \lambda_2 - a_{p2} \cos(\psi_2 - \delta_{p2}) - b_2 \cos \beta_2 = 0 \]  \hspace{1cm} (G-54)

and

\[ a_{G2} \sin(\phi_2 - \delta_{G2}) + L_2 \sin \lambda_2 - a_{p2} \sin(\psi_2 - \delta_{p2}) - b_2 \sin \beta_2 = 0 \]  \hspace{1cm} (G-55)

To solve for the output angle \( \psi_2 \) in terms of the input angle \( \phi_2 \), substitute the expressions for \( \sin \lambda_2 \) and \( \cos \lambda_2 \), as obtained from the component equations (G-54) and (G-55), into

\[ \sin^2 \lambda_2 + \cos^2 \lambda_2 = 1 \]  \hspace{1cm} (G-56)

This leads to

\[ A_2 R \sin \psi_2 + B_2 R \cos \psi_2 = C_2 R \]  \hspace{1cm} (G-57)
where

\[ A_{2R} = b_2 \sin(\beta_2 + \delta_2) - a_2 \sin(\phi_2 - \delta_2 + \delta_2) \]

\[ B_{2R} = b_2 \cos(\beta_2 + \delta_2) - a_2 \cos(\phi_2 - \delta_2 + \delta_2) \]

\[ C_{2R} = \frac{L_2 - b_2^2 - a_2^2 + a_2^2 + 2a_2 b_2 \cos(\phi_2 - \delta_2 - \beta_2)}{2a_2} \]

Equation (G-57) is then solved for \( \psi_2 \) in the manner discussed in Appendix E

\[ \psi_2 = 2 \tan^{-1} \left( \frac{A_{2R} \pm \sqrt{A_{2R}^2 + B_{2R}^2 - C_{2R}^2}}{B_{2R} + C_{2R}} \right) \] (G-58)

The correct sign must again be determined from geometric considerations.

The angle \( \lambda_2 \) may now be determined either from equation (G-54) or equation (G-55)

\[ \lambda_2 = \cos^{-1} \left[ \frac{b_2 \cos \beta_2 + a_2 \cos(\psi_2 - \delta_2) - a_2 \cos(\phi_2 - \delta_2)}{L_2} \right] \] (G-59)

or

\[ \lambda_2 = \sin^{-1} \left[ \frac{b_2 \sin \beta_2 + a_2 \sin(\psi_2 - \delta_2) - a_2 \sin(\phi_2 - \delta_2)}{L_2} \right] \] (G-60)
III. DETERMINATION OF OUTPUT ANGULAR VELOCITY $\dot{\psi}_2$

Implicit differentiation of equation (G-57) with respect to time leads to

$$
\dot{\psi}_2 = \dot{\phi}_2 \left[ \frac{A_{2RD} \sin \psi_2 - B_{2RD} \cos \psi_2 - C_{2RD}}{A_{2R} \cos \psi_2 - B_{2R} \sin \psi_2} \right] 
$$

where

$$
A_{2RD} = a_{G2} \cos (\phi_2 - \delta G_2 + \delta P_2) \\
B_{2RD} = a_{G2} \sin (\phi_2 - \delta G_2 + \delta P_2) \\
C_{2RD} = \frac{a_{G2} b_2 \sin (\phi_2 - \delta G_2 - \beta_2)}{a_{P2}}
$$
IV. RELATIVE VELOCITY AT THE CONTACT POINT

The relative velocity \( \bar{V}_{S2/T2} \), of point \( S_2 \) on gear no. 2 with respect to point \( T_2 \) on pinion no. 3 has the direction of the unit vector \( \bar{N}_{NA2} \). Thus, in the manner of equation (G-20),

\[
\bar{V}_{S2/T2} = \left\{ \left[ \hat{\psi}_2 \times (a_{g2} \bar{N}_{g2} + \rho_{g2} \bar{N}_{\lambda2}) \right] \cdot \bar{N}_{NA2} \right. \\
- \left. \left[ \hat{\psi}_2 \times (a_{p2} \bar{N}_{p2} - \rho_{p2} \bar{N}_{\lambda2}) \right] \cdot \bar{N}_{NA2} \right\} \bar{N}_{NA2}
\]  

(G-62)

Substitution of unit vectors yields

\[
\bar{V}_{S2/T2} = \left\{ \hat{\psi}_2 \left[ a_{g2} \cos(\psi_2 - \delta_{g2} - \lambda_2) + \rho_{g2} \right] \right. \\
- \left. \hat{\psi}_2 \left[ a_{p2} \cos(\psi_2 - \delta_{p2} - \lambda_2) - \rho_{p2} \right] \right\} \bar{N}_{NA2}
\]  

(G-63)
b. ROUND ON FLAT PHASE OF MOTION

Figure G-5 shows a schematic view of mesh no. 2 in the round on flat phase of the motion. Again, only the contacting sides of the gear teeth are indicated.

I. UNIT VECTORS

The unit vector in the direction $O_2T_2$, along the flank of pinion no. 3, is given by

$$
\vec{r}_{F2} = \cos(\psi_2 + a_{p2}) \vec{I} + \sin(\psi_2 + a_{p2}) \vec{J}
$$

(G-64)

The unit vector $\vec{r}_{NF2}$, in the direction $S_2C_2$, is normal to $\vec{r}_{F2}$ in the right hand sense

$$
\vec{r}_{NF2} = -\sin(\psi_2 + a_{p2}) \vec{I} + \cos(\psi_2 + a_{p2}) \vec{J}
$$

(G-65)
FIGURE G-5

ROUND ON FLAT PHASE OF MOTION OF MESH NO. 2
II. DETERMINATION OF OUTPUT ANGLE \( \psi_2 \) AND DISTANCE \( s_2 \)

The vector equation for the mechanism loop \( O_2-C_2-S_2-T_2-O_3 \) has the form

\[
a_{G2} \hat{n}_{G2} - \rho_{G2} \hat{n}_{NF2} - s_2 \hat{n}_{F2} - b_2 \hat{n}_{\beta_2} = 0 \quad (G-66)
\]

Appropriate substitutions for the unit vectors furnish the following component equations.

\[
a_{G2} \cos(\phi_2 - \gamma_{G2}) + \rho_{G2} \sin(\psi_2 + \alpha_{P2}) - b_2 \cos \beta_2 - s_2 \cos(\psi_2 + \alpha_{P2}) = 0 \quad (G-67)
\]

and

\[
a_{G2} \sin(\phi_2 - \gamma_{G2}) - \rho_{G2} \cos(\psi_2 + \alpha_{P2}) - b_2 \sin \beta_2 - s_2 \sin(\psi_2 + \alpha_{P2}) = 0 \quad (G-68)
\]

From equation (G-68) one obtains the following expression for \( s_2 \)

\[
s_2 = \frac{b_2 \sin \beta_2}{\sin(\psi_2 + \alpha_{P2})} \quad (G-69)
\]
This expression is now substituted into equation (G-67), and one obtains the following:

\[ A_{2F} \sin \psi_2 + B_{2F} \cos \psi_2 = C_{2F} \]  \hfill (G-70)

where

\[ A_{2F} = a_{G2} \cos(\phi_2 - \delta_{G2} - \alpha_{p2}) - b_{2} \cos(\beta_2 - \alpha_{p2}) \]

\[ B_{2F} = -a_{G2} \sin(\phi_2 - \delta_{G2} - \alpha_{p2}) + b_{2} \sin(\beta_2 - \alpha_{p2}) \]

\[ C_{2F} = -\rho_{G2} \]

Equation (G-70) is solved in the customary manner:

\[ \psi_2 = 2 \tan^{-1} \frac{A_{2F} \pm \sqrt{A_{2F}^2 + B_{2F}^2 - C_{2F}^2}}{B_{2F} + C_{2F}} \]  \hfill (G-71)

The appropriate sign is again found from geometric considerations.

G-33
III. DETERMINATION OF ANGULAR VELOCITY \( \dot{\theta}_2 \) DURING ROUND ON FLAT PHASE OF MOTION

Implicit differentiation of equation (G-70) with respect to time gives the following expression for \( \dot{\theta}_2 \):

\[
\dot{\theta}_2 = \phi_2 \left[ \frac{A_{2FD} \sin \phi_2 + B_{2FD} \cos \phi_2}{A_{2F} \cos \phi_2 - B_{2F} \sin \phi_2} \right] 
\]

where

\[
A_{2FD} = a_2 \sin(\phi_2 - \delta_2 - \alpha_2)
\]
\[
B_{2FD} = a_2 \cos(\phi_2 - \delta_2 - \alpha_2)
\]

IV. RELATIVE VELOCITY \( \bar{V}_{S2/T2_F} \) AT CONTACT POINT DURING ROUND ON FLAT PHASE OF MOTION

Again, the relative velocity \( \bar{V}_{S2/T2_F} \) consists only of that component of \( \bar{V}_{S2/02} \) which is directed along the pinion flank. Thus,

G-34
\[
\begin{align*}
\bar{V}_{S2/T_2} &= \left[ \bar{V}_{S2/0_2} \cdot \bar{n}_{F2} \right] \bar{n}_{F2} \\
&= \left\{ \left[ \phi_2 \bar{\kappa} \times (a_{g2} \bar{n}_{0_2} - \rho_{g2} \bar{n}_{NF2}) \right] \cdot \bar{n}_{F2} \right\} \bar{n}_{F2}
\end{align*}
\]

V. DETERMINATION OF TRANSITION ANGLES

The transition angle \( \phi_{2T} \) and the angle \( \psi_{2T} \), which correspond to the transition from the round on round to the round on flat phase of motion, are obtained by letting \( g_2 = f_{p2} \) in the component equations (G-67) and (G-68). From this one finds the following with \( \phi_2 = \phi_{2T} \) and \( \psi_2 = \psi_{2T} \):

\[
\cos(\phi_{2T} - \phi_{g2}) = \frac{1}{a_{g2}} \left[ - \rho_{g2} \sin(\phi_{2T} + \phi_{p2}) + b_2 \cos \beta_2 + f_{p2} \cos(\phi_{2T} + \phi_{p2}) \right]
\]

(G-75)
and

\[
\sin(\phi_2 T - \delta_{g2}) = \frac{1}{a_{g2}} \left[ p_{g2} \cos(\phi_2 T + \alpha_{p2}) + b_2 \sin \delta_{g2} + f_{p2} \sin(\phi_2 T + \alpha_{p2}) \right]
\]

\[
(\text{a-76})
\]

The angle \(\phi_2 T\) is now found by substituting the above expressions into

\[
\sin^2(\phi_2 T - \delta_{g2}) + \cos^2(\phi_2 T - \delta_{g2}) = 1
\]

\[
(\text{a-77})
\]

This results in

\[
A_{2T} \sin \phi_2 T + B_{2T} \cos \phi_2 T = C_{2T}
\]

\[
(\text{a-78})
\]

where

\[
A_{2T} = -p_{g2} \cos(\beta_2 - \alpha_{p2}) + f_{p2} \sin(\beta_2 - \alpha_{p2})
\]

\[
B_{2T} = p_{g2} \sin(\beta_2 - \alpha_{p2}) + f_{p2} \cos(\beta_2 - \alpha_{p2})
\]

\[
C_{2T} = \frac{a_{g2}^2 - p_{g2}^2 - b_2^2 - f_{p2}^2}{2 b_2}
\]

G-36
Finally, in the usual way

\[ \psi_{2T} = 2 \tan^{-1} \frac{A_{2T} \pm \sqrt{A_{2T}^2 + B_{2T}^2 - C_{2T}^2}}{B_{2T} + C_{2T}} \]  

\hspace{1cm} (G-79)

Again, the sign must be decided from geometric considerations.

The associated angle \( \phi_{2T} \) may be found with the help of either equation (G-75) or equation (G-76), i.e.,

\[ \phi_{2T} = \cos^{-1} \left[ \frac{-g_2 \sin(\psi_{2T} + \alpha_{P2}) + b_2 \cos\alpha_2 + f_{P2} \cos(\psi_{2T} + \alpha_{P2})}{a_g} \right] + \delta_{G2} \]  

\hspace{1cm} (G-80)

or

\[ \phi_{2T} = \sin^{-1} \left[ \frac{g_2 \cos(\psi_{2T} + \alpha_{P2}) + b_2 \sin\alpha_2 + f_{P2} \sin(\psi_{2T} + \alpha_{P2})}{a_g} \right] + \delta_{G2} \]  

\hspace{1cm} (G-81)

G-37
VI. SENSING EQUATION FOR THE DETERMINATION OF CONTACT ON

SUBSEQUENT TOOTH MESH

The contact sensing equation for mesh no. 2 is derived similarly to that for mesh no. 1 before contact is made in the round on round mode. The distance between the centers of curvature $C_{G2}$ and $C_{P2}$ is given by

$$\overrightarrow{C_{G2}C_{P2}} = L_{x2}\bar{i} + L_{y2}\bar{j} \quad (G-82)$$

If $\Delta \phi_2$ and $\Delta \psi_2$ represent the tooth spacing angles of gear no. 2 and pinion no. 3 respectively, the associated loop equation becomes (see Figures E-3 and G-4)

$$a_{g2} [\cos(\phi_2 - \Delta \phi_2 - \delta_{g2})\bar{i} + \sin(\phi_2 - \Delta \phi_2 - \delta_{g2})\bar{j}] + L_{x2}\bar{i} + L_{y2}\bar{j}$$

$$- a_{p2} [\cos(\psi_2 + \Delta \psi_2 - \delta_{p2})\bar{i} + \sin(\psi_2 + \Delta \psi_2 - \delta_{p2})\bar{j}]$$

$$- b_{2} [\cos\beta_2\bar{i} + \sin\beta_2\bar{j}] = 0 \quad (G-83)$$

Note that for mesh no. 2, the angular increment $\Delta \phi_2$ is negative while $\Delta \psi_2$ is positive. Further, as before, the angle $\psi_2$ must be
determined for the round on flat phase of the motion.

The magnitudes of \( L_{x2} \) and \( L_{y2} \) are determined from the components of equation (G-83), i.e.,

\[
L_{x2} = b_2 \cos \beta_2 + a_p \cos(\phi_2 + \Delta \phi_2 - \delta_{p2}) - a_{g2} \cos(\phi_2 - \Delta \phi_2 - \delta_{g2})
\]

while

\[
L_{y2} = b_2 \sin \beta_2 + a_p \sin(\phi_2 + \Delta \phi_2 - \delta_{p2}) - a_{g2} \sin(\phi_2 - \Delta \phi_2 - \delta_{g2})
\]

Contact will occur as soon as

\[
\sqrt{L_{x2}^2 + L_{y2}^2} \leq \rho_{g2} + \rho_{p2}
\]

(G-86)
3. KINEMATICS OF MESH NO. 3 (GEAR NO. 3 AND PINION NO. 4)

Since mesh no. 3 is kinematically equivalent to mesh no. 1, the kinematic equations for mesh no. 3 may be obtained from those for mesh no. 1. The angle $\beta_3$ must replace the angle $\beta_1$ and the center distance $b_3$ is used instead of $b_1$. All parameters of gear no. 1 are replaced by those of gear no. 3 and the pinion parameters of pinion no. 4 are substituted for those of pinion no. 2.

It is to be noted that the input angle $\psi_3$ of mesh no. 3 is identical to the output angle $\psi_2$ of mesh no. 2.

a. ROUND ON ROUND PHASE OF MOTION

The output angle $\psi_3$ is obtained with the help of equation (G-12)

$$\psi_3 = 2 \tan^{-1} \left( \frac{A_{3R} \pm \sqrt{A_{3R}^2 + B_{3R}^2 - C_{3R}^2}}{B_{3R} + C_{3R}} \right)$$  

(G-87)

where

$$A_{3R} = a_3 \sin(\phi_3 + \theta_{33} - \theta_{p3}) - b_3 \sin(\mu_3 - \theta_{p3})$$

G-40
\[ B_{3R} = a_3 \cos(\phi_3 + \varphi_3 - \delta_{p3}) - b_3 \cos(\beta_3 - \delta_{p3}) \]

\[ C_{3R} = \frac{a_p^2 + a_3^2 + b_3^2 - L_3^2 - 2a_3b_3 \cos(\phi_3 + \varphi_3 - \delta_{p3})}{2a_p} \]

and

\[ L_3 = \rho_3 + \rho_{p3} \quad (G-88) \]

The angle \( \lambda_3 \) may be found with the help of equation (G-13) or equation (G-14)

\[ \lambda_3 = \cos^{-1} \left[ \frac{b_3 \cos \beta_3 + a_p \cos(\phi_3 + \delta_{p3}) - a_3 \cos(\phi_3 + \delta_{p3})}{L_3} \right] \quad (G-89) \]

or

\[ \lambda_3 = \sin^{-1} \left[ \frac{b_3 \sin \beta_3 + a_p \sin(\phi_3 + \delta_{p3}) - a_3 \sin(\phi_3 + \delta_{p3})}{L_3} \right] \quad (G-90) \]

The angular velocity \( \dot{\lambda}_3 \) is obtained from equation (G-15)

\[ \dot{\lambda}_3 = \dot{\lambda}_3 \left[ \frac{B_{3RD} \cos \lambda_3 - A_{3RD} \sin \lambda_3 + C_{3RD}}{A_{3R} \cos \lambda_3 - B_{3R} \sin \lambda_3} \right] \quad (G-91) \]

G-41
where

\[ A_{3RD} = a_3 \cos(\phi_3 + \delta_3 - \delta_3) \]
\[ B_{3RD} = a_3 \sin(\phi_3 + \delta_3 - \delta_3) \]
\[ C_{3RD} = \frac{a_3 b_3 \sin(\phi_3 + \delta_3 - \beta_3)}{a_p_3} \]

The relative velocity \( \vec{v}_{S3/T3R} \) becomes, according to Equation (G-21),

\[ \vec{v}_{S3/T3R} = \left\{ \begin{array}{c} \dot{\phi}_3 [a_3 \cos(\phi_3 + \delta_3 - \lambda_3) + \rho_3] \\ - \dot{\psi}_3 [a_p_3 \cos(\psi_3 + \delta_p - \lambda_3) - \rho_p_3] \end{array} \right\} \hat{n}_{N\lambda_3} \tag{G-92} \]

where, according to equation (G-3),

\[ \hat{n}_{N\lambda_3} = - \sin \lambda_3 \hat{i} + \cos \lambda_3 \hat{j} \tag{G-93} \]
b. ROUND ON FLAT PHASE OF MOTION

The output angle $\psi_3$ is obtained from equation (G-29)

$$
\psi_3 = 2 \tan^{-1} \frac{A_{3F} \pm \sqrt{A_{3F}^2 + B_{3F}^2 - C_{3F}^2}}{B_{3F} + C_{3F}}
$$

(G-94)

where

$$
A_{3F} = a_3 \cos(\phi_3 + \delta_3 + \alpha_3) - b_3 \cos(\beta_3 + \alpha_3)
$$

$$
B_{3F} = -a_3 \sin(\phi_3 + \delta_3 + \alpha_3) + b_3 \sin(\beta_3 + \alpha_3)
$$

$$
C_{3F} = \rho_3
$$

The distance $s_3$ becomes, according to equation (G-27),

$$
s_3 = \frac{a_3 \sin(\phi_3 + \delta_3) + \rho_3 \cos(\psi_3 - \alpha_3) - b_3 \sin \beta_3}{\sin(\psi_3 - \alpha_3)}
$$

(G-95)
The angular velocity $\psi_3$ for the round on flat phase of the motion is found from equation (G-30)

$$\psi_3 = \begin{vmatrix} A_{3FD} \sin \psi_3 + B_{3FD} \cos \psi_3 \\ A_{3FD} \cos \psi_3 - B_{3FD} \sin \psi_3 \end{vmatrix}$$

(G-96)

where

$$A_{3FD} = a_{G3} \sin (\phi_3 + \delta_3 + \alpha_{P3})$$

$$B_{3FD} = a_{G3} \cos (\phi_3 + \delta_3 + \alpha_{P3})$$

The relative velocity $\overline{V}_{S3/T3_F}$ for the round on flat phase comes from equation (G-33)

$$\overline{V}_{S3/T3_F} = \begin{vmatrix} a_{G3} \sin (\psi_3 - \alpha_{P3} - \phi_3 + \delta_3) - \rho_{G3} \end{vmatrix} \overline{\eta}_{F3}$$

(G-97)

where

$$\overline{\eta}_{F3} = \cos (\psi_3 - \alpha_{P3}) \overline{I} + \sin (\psi_3 - \alpha_{P3}) \overline{J}$$

(G-98)

according to equation (G-22).

G-44
The transition angle $\psi_{3T}$ is obtained by way of equation (G-39)

$$\psi_{3T} = 2 \tan^{-1} \frac{A_{3T} \pm \sqrt{A_{3T}^2 + B_{3T}^2 - C_{3T}^2}}{B_{3T} + C_{3T}} \quad (G-99)$$

where

$$A_{3T} = \rho_{G3} \cos(\beta_3 + \alpha_{P3}) + f_{P3} \sin(\beta_3 + \alpha_{P3})$$

$$B_{3T} = -\rho_{G3} \sin(\beta_3 + \alpha_{P3}) + f_{P3} \cos(\beta_3 + \alpha_{P3})$$

$$C_{3T} = \frac{a_{G3}^2 - \rho_{G3}^2 - b_3^2 - f_{P3}^2}{2 b_3}$$

The associated angle $\phi_{3T}$ may be obtained from equation (G-40)

or from equation (G-41)

$$\phi_{3T} = \cos^{-1} \left[ \frac{\rho_{G3} \sin(\psi_{3T} - \alpha_{P3}) + f_{P3} \cos(\psi_{3T} - \alpha_{P3}) + b_3 \cos \beta_3}{a_{G3}} \right] - \delta_{G3} \quad (G-100)$$

or

$$\phi_{3T} = \sin^{-1} \left[ \frac{-\rho_{G3} \cos(\psi_{3T} - \alpha_{P3}) + f_{P3} \sin(\psi_{3T} - \alpha_{P3}) + b_3 \sin \beta_3}{a_{G3}} \right] - \delta_{G3} \quad (G-101)$$

G-45
Finally, the contact sensing equation is based on equations (G-44) - (G-46). Contact will occur, when

\[ \sqrt{L_x^2 + L_y^2} \leq \rho_{G3} + \rho_{P3} \]  

\[(G-102)\]

where with the tooth spacing angles \( \Delta \phi_3 \) and \( \Delta \psi_3 \),

\[ L_x = b_3 \cos \beta_3 + a_p \cos(\psi_3 - \Delta \psi_3 + \delta_{P3}) - a_{G3} \cos(\phi_3 + \Delta \phi_3 + \delta_{G3}) \]

\[(G-103)\]

and

\[ L_y = b_3 \sin \beta_3 + a_p \sin(\psi_3 - \Delta \psi_3 + \delta_{P3}) - a_{G3} \sin(\phi_3 + \Delta \phi_3 + \delta_{G3}) \]

\[(G-104)\]
The following gives the derivations for the moment input-output relationships of two and three step-up gear trains which operate in a spin environment.

Figure G-1 of Appendix G shows the basic configuration of a three step-up gear train. The input moment $M_{in}$, which acts on gear no. 1, is held in equilibrium by the moment $M_{o4}$, which acts on pinion no. 4.

Since in all three meshes there may either be round on round or round on flat type of contact, the force and moment analyses must account for various contact combinations. Table H-1 shows the eight different phase combinations which may occur in a three step-up gear train, and for which input-output relationships must be found. The two step-up gear train, which is shown in Figure A-10 of Appendix A for involute gearing, does not contain pinion 4 and gear no. 3. Here, the input moment $M_{in}$, which acts on gear no. 1, is held in equilibrium by moment $M_{o3}$, which acts on pinion no. 3.
<table>
<thead>
<tr>
<th>Case no.</th>
<th>Mesh No. 3 (Gear 3 &amp; Pinion 4)</th>
<th>Mesh No. 2 (Gear 2 &amp; Pinion 3)</th>
<th>Mesh No. 1 (Gear 1 &amp; Pinion 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>R</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>R</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>4</td>
<td>R</td>
<td>F</td>
<td>R</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>F</td>
<td>R</td>
</tr>
<tr>
<td>7</td>
<td>F</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>8</td>
<td>F</td>
<td>R</td>
<td>F</td>
</tr>
</tbody>
</table>

**TABLE H-1**

POSSIBLE COMBINATIONS OF PHASES FOR THREE STEP-UP GEAR TRAIN AS SHOWN IN FIGURE G-1

R = Round on Round
F = Round on Flat

When ogival teeth are involved, there are four possible phase combinations of the two remaining meshes. These are shown in Table H-2. Again input-output relationships must be obtained for each of them.

H-2
TABLE H-2

POSSIBLE COMBINATIONS OF PHASES FOR TWO STEP-UP GEAR TRAIN AS SHOWN IN FIGURE A-10
R = Round on Round
F = Round on Flat

The unit vectors, mechanism angles and kinematic terms necessary for the following analyses were derived in Appendix G. (See also Appendix D for a description of the geometry of ogival teeth.) Certain terms used in connection with mesh no. 3 may be obtained from expressions derived for mesh no. 1 in Appendix G by the replacement of the appropriate subscript numbers, since the kinematics of these meshes are identical. The following additional nomenclature is used:
\( S_i \) = distance from spin axis C to pivot points \( O_i \) of individual gears. \((i = 1, 2, 3, 4 \text{ as applicable})\)

\( \gamma_i \) = angle of lines \( S_i = CO_i \) with respect to the body-fixed X-axis

\( \omega \) = spin velocity of fuze body

\( m_i \) = mass of various gears, pinions and gear-pinion combinations

\( Q_i = m_i \omega^2 \), centrifugal force acting on individual gear components. (Now called \( q_i \) to differentiate it from the pinion contact point \( T_i \).)

\( \rho_i \) = pivot radius

\( \rho_{Pi} \) = radius of curvature of pinion tooth (ogival)

\( \rho_{Gi} \) = radius of curvature of gear tooth (ogival)

\( \mu \) = coefficient of friction at pivots as well as at contact point between gears and pinions

The pivot friction moments are obtained according to equation (A-3b) of Appendix A. They always oppose motion regardless of the assumption of direction of the pivot reactions \( F_{x1} \) and \( F_{y1} \). To this end the pivot forces \( \tilde{F}_{x1} \) and \( \tilde{F}_{y1} \), which represent the sums of the absolute values of their component parts, are added.

H-4
algebraically. The algebraic addition of such modified reactions provides a conservative, i.e., a somewhat overstated friction moment.

The directions of the friction forces of the gears on the pinions are always those of the relative velocities \( \overline{V_{S_i/T_i}} \), where points \( S_i \) and \( T_i \) are located at the contact points of the gears and pinions, respectively. This allows the introduction of a signum convention. For the round on round phases,

\[
\bar{s}_R = \frac{V_{S_i/T_iR}}{|V_{S_i/T_iR}|} \quad (H-1)
\]

For the round on flat phase, the convention becomes

\[
\bar{s}_F = \frac{V_{S_i/T_iF}}{|V_{S_i/T_iF}|} \quad (H-2)
\]

The expressions for the above relative velocities, which are different for round on round and for round on flat, are given in Appendix G.
I. INPUT-OUTPUT ANALYSIS OF THREE STEP-UP GEAR TRAIN

a. CASE NO. 1: RRR

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 4

Figure H-1 shows a schematic free body diagram of pinion no. 4 in the round on round mode of contact. The equivalent four-bar linkage associated with mesh no. 3 is also indicated. The pinion is acted upon by the equilibrant moment $M_{o4}$ in the direction opposite to its counterclockwise rotation. The contact force of gear no. 3 on the pinion is given by

$$ F_{34} = F_{34} \hat{n}_{A3} \quad (H-3) $$

The associated friction force exerted by the gear on the pinion has the direction of the relative velocity $\overrightarrow{V_{s3/T3_R}}$, as given by equation (G-92). With the use of the signum convention of equation (H-1) the friction force $F_{f34}$ becomes

$$ F_{f34} = \mu_{s3} F_{34} \hat{n}_{A3} \quad (H-4) $$

H-6
Note: $\phi$ is positive in counterclockwise direction.

FIGURE H-1

FREE BODY DIAGRAM OF PINION NO. 4
MESH NO. 3: ROUND ON ROUND
The centrifugal force, due to the pinion mass, is given by

\[ \overline{Q}_4 = Q_4 (\cos \gamma_4 \bar{i} + \sin \gamma_4 \bar{j}) \]  \hfill (H-5)

where

\[ Q_4 = R_4 m_4 \omega^2 \]  \hfill (H-6)

Force equilibrium is given by

\[ F_{34} \bar{R}_3 + \mu s_3 R F_{34} \bar{R} \lambda_3 + F_{x4} \bar{i} + \mu F_{y4} \bar{i} - F_{y4} \bar{j} + \mu F_{x4} \bar{j} + \overline{Q}_4 = 0 \]  \hfill (H-7)

Moment equilibrium requires the following:

\[ -M_{04} \bar{R} = \mu s_4 (F_{x4} + F_{y4}) \bar{R} + (a_{p3} \bar{R} \lambda_3 - \rho_{p3} \bar{R} \lambda_3) \times (F_{34} \bar{R} \lambda_3 + \mu s_3 R F_{34} \bar{R} \lambda_3) \]

\[ = 0 \]  \hfill (H-8)

Equation (H-7) furnishes the following component equations:

\[ F_{34} \cos \lambda_3 - \mu s_3 R F_{34} \sin \lambda_3 + F_{x4} + \mu F_{y4} + Q_4 \cos \gamma_4 = 0 \]  \hfill (H-9)

and

\[ F_{34} \sin \lambda_3 + \mu s_3 R F_{34} \cos \lambda_3 - F_{y4} + \mu F_{x4} + Q_4 \sin \gamma_4 = 0 \]  \hfill (H-10)
The scalar form of the moment equation becomes

\[-M_0 - \mu \rho_4 (F_{x4} + F_{y4}) - F_{34} \left\{ \begin{array}{l} \omega_3 [\sin(\psi_3 + \delta_3 - \lambda_3) - \mu \omega_3 \cos(\psi_3 + \delta_3 - \lambda_3)] \\ + \mu \omega_3 \rho \delta_3 \end{array} \right\} = 0 \quad (H-11)\]

Simultaneous solution of equations (H-9) and (H-10) for \( F_{x4} \) and \( F_{y4} \) gives

\[ F_{x4} = \frac{F_{34} [\alpha (s_3 - 1) \sin \lambda_3 - (1 + \mu^2 s_3 R) \cos \lambda_3]}{1 + \mu^2} - Q_4 (\mu \sin \gamma_4 + \cos \gamma_4) \quad (H-12) \]

and

\[ F_{y4} = \frac{F_{34} [(1 + \mu^2 s_3 R) \sin \lambda_3 + \mu (s_3 R - 1) \cos \lambda_3]}{1 + \mu^2} + Q_4 (\sin \gamma_4 - \mu \cos \gamma_4) \quad (H-13) \]

The sum \( F_{x4} + F_{y4} \) of equation (H-11) is now made up of equations (H-12) and (H-13) in the sense of equation (A-3b) of Appendix A

\[ F_{x4} + F_{y4} = Q_4 A_1 + F_{34} A_2 + Q_4 A_3 + F_{34} A_4 \quad (H-14) \]

R-9
\[ A_1 = \frac{\mu \sin \gamma_4 + \cos \gamma_4}{1 + \mu^2} \]  
\[ A_2 = \frac{\mu (s_{3R} - 1) \sin \lambda_3 - (1 + \mu^2 s_{3R}) \cos \lambda_3}{1 + \mu^2} \]  
\[ A_3 = \frac{\sin \gamma_4 - \mu \cos \gamma_4}{1 + \mu^2} \]  
\[ A_4 = \frac{(1 + \mu^2 s_{3R}) \sin \lambda_3 + \mu (s_{3R} - 1) \cos \lambda_3}{1 + \mu^2} \]

Equation (H-14) is now substituted into the moment equation (H-11) and the latter is solved for the contact force \( F_{34} \)

\[ F_{34} = \frac{M_{40}}{C_2} + \frac{Q_4 C_1}{C_2} \]  
\[ \text{where} \]
\[ C_1 = \mu \rho_4 (A_1 + A_3) \]
\[ C_2 = \rho_3 \left[ \mu s_{3R} \cos (\phi_3 + \delta_{3P} - \lambda_3) - \sin (\phi_3 + \delta_{3P} - \lambda_3) \right] \]
\[ - \mu \left[ \rho_3 s_{3R} + \rho_4 (A_2 + A_4) \right] \]
II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-2 gives a schematic free body diagram of the gear and pinion combination no. 3. Mesh no. 2 is also indicated to obtain the directions of the forces of gear no. 2 on pinion no. 3. The forces of pinion no. 4 on gear no. 3 are opposite to those given by equations (H-3) and (H-4) respectively. Thus, the contact force becomes

\[ F_{43} = -F_{34} = -F_{34} \bar{\lambda}_3 \]  \hspace{1cm} (H-20)

The friction force of pinion no. 4 on gear no. 3 becomes:

\[ F_{f43} = -F_{f34} = -\mu s_{3R} F_{34} \bar{\lambda}_3 \]  \hspace{1cm} (H-21)

The contact force of gear no. 2 on pinion no. 3 is given by

\[ F_{23} = F_{23} \bar{\lambda}_2 \]  \hspace{1cm} (H-22)

while the associated friction force of gear no. 2 on pinion no. 3 becomes

H-11
FIGURE H-2
FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON ROUND
MESH NO. 2: ROUND ON ROUND

H-12
The pivot reactions $F_{x3}$ and $F_{y3}$ as well as the associated friction forces are drawn in a separate diagram in Figure H-2. As was the case earlier, the friction moment due to the friction forces again opposes rotation.

The centrifugal force, due to the mass of the combined gear and pinion no. 3, is given by

$$
\bar{Q}_3 = \bar{Q}_3 (\cos \gamma_3 \bar{l} + \sin \gamma_3 \bar{j}) \tag{H-24}
$$

where

$$
\bar{Q}_3 = \bar{\alpha}_3 \bar{m}_3 \bar{\omega}^2 \tag{H-25}
$$

The force equilibrium of the combination is given by

$$
-F_{34} \bar{\lambda}_3 - \bar{\mu}_3 \bar{R}_{34} \bar{\lambda}_3 + \bar{F}_{23} \bar{\lambda}_2 + \bar{\mu}_2 \bar{R}_{23} \bar{\lambda}_2 + F_{x3} \bar{l} + \bar{\mu}_y \bar{y}_3 \bar{j} + \bar{F}_{y3} \bar{j} = \bar{Q}_3 + \bar{Q}_3 = 0 \tag{H-26}
$$
The moment equation is given by

$$
\mu \rho_3 (\tilde{F}_x + \tilde{F}_y) + [a_{G3} \bar{G}_3 + \rho_{G3} \bar{R}_3] \times [-F_{34} \bar{R}_3 - \mu s_{3R} R_{34} \bar{R}_N \lambda_3] \\
+ [a_{p2} \bar{r}_{p2} - \rho_{p2} \bar{r}_{p2}] \times [F_{23} \bar{R}_2 + \mu s_{2R} R_{23} \bar{R}_N \lambda_2] = 0
$$

Equation (H-26) gives the following component expressions:

$$
-F_{34} \cos \lambda_3 + \mu s_{3R} F_{34} \sin \lambda_3 + Q_3 \cos \gamma_3 + F_{x3} + \mu F_y + F_{23} \cos \lambda_2 \\
- \mu s_{2R} F_{23} \sin \lambda_2 = 0
$$

and

$$
-F_{34} \sin \lambda_3 - \mu s_{3R} F_{34} \cos \lambda_3 + Q_3 \sin \gamma_3 + F_{y3} - \mu F_{x3} + F_{23} \sin \lambda_2 \\
+ \mu s_{2R} F_{23} \cos \lambda_2 = 0
$$

The scalar form of the moment equation (H-27) becomes

$$
\mu \rho_3 (\tilde{F}_x + \tilde{F}_y) - \mu s_{3R} F_3 + \bar{F}_{23} v_{23} + a_{G3} F_{34} \sin (\phi_3 + \delta_{G3} - \lambda_3) \\
- \mu s_{3R} \cos (\phi_3 + \delta_{G3} - \lambda_3)] + a_{p2} F_{23} \sin (\psi_2 - \delta_{p2} - \lambda_2) \\
+ \mu s_{2R} \cos (\psi_2 - \delta_{p2} - \lambda_2)] = 0
$$

(H-30)
Simultaneous solution of equations (H-28) and (H-29) for the pivot reactions $F_{x3}$ and $F_{y3}$ gives

$$F_{x3} = \frac{1}{1 + \mu^2} \left\{ F_{34} \left[ (1 - \mu^2 s_{3R}) \cos \alpha_3 - \mu (1 + s_{3R}) \sin \alpha_3 \right] + F_{23} \left[ \mu (1 + s_{2R}) \sin \alpha_2 - (1 - \mu^2 s_{2R}) \cos \alpha_2 \right] + Q_3 \left[ \mu \sin \gamma_3 - \cos \gamma_3 \right] \right\}$$

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{34} \left[ (1 - \mu^2 s_{3R}) \sin \alpha_3 + \mu (1 + s_{3R}) \cos \alpha_3 \right] + F_{23} \left[ \mu^2 s_{2R} - 1 \right] \sin \alpha_2 - \mu (1 + s_{2R}) \cos \alpha_2 \right] - Q_3 \left[ \sin \gamma_3 + \mu \cos \gamma_3 \right] \right\}$$

The sum $\bar{F}_{x3} + \bar{F}_{y3}$ of equation (H-30) is now made up of equations (H-31) and (H-32) in the sense of equation (A-3b).

$$\bar{F}_{x3} + \bar{F}_{y3} = F_{34} A_5 + F_{23} A_6 + Q_3 A_7 + F_{34} A_8 + F_{23} A_9 + Q_3 A_{10}$$

(H-33)
where

\[ A_5 = \frac{(1 - \mu^2 s_{3R}) \cos \gamma_3 - \mu(1 + s_{3R}) \sin \gamma_3}{1 + \mu^2} \]  \hspace{1cm} (H-34)

\[ A_6 = \frac{\mu(1 + s_{2R}) \sin \gamma_2 - (1 - \mu^2 s_{2R}) \cos \gamma_2}{1 + \mu^2} \]  \hspace{1cm} (H-35)

\[ A_7 = \frac{\mu \sin \gamma_3 - \cos \gamma_3}{1 + \mu^2} \]  \hspace{1cm} (H-36)

\[ A_8 = \frac{(1 - \mu^2 s_{3R}) \sin \gamma_3 + \mu(1 + s_{3R}) \cos \gamma_3}{1 + \mu^2} \]  \hspace{1cm} (H-37)

\[ A_9 = \frac{(\mu^2 s_{2R} - 1) \sin \gamma_2 - \mu(1 + s_{2R}) \cos \gamma_2}{1 + \mu^2} \]  \hspace{1cm} (H-38)

\[ A_{10} = \frac{\sin \gamma_3 + \mu \cos \gamma_3}{1 + \mu^2} \]  \hspace{1cm} (H-39)

Equation (H-33) is now substituted into equation (H-30) and the resulting expression is solved for the contact force \( F_{23} \). Thus,

\[ F_{23} = -\frac{F_{34} C_3 - Q_3 C_4}{C_5} \]  \hspace{1cm} (H-40)
where

\[ C_3 = \mu p_3(A_5 + A_8) - \mu R_p R G_3 + a G_3 \left[ \sin(\phi_3 + \delta G_3 - \lambda_3) \right. \]
\[ \left. - \mu^2 R \cos(\phi_3 + \delta G_3 - \lambda_3) \right] \]

\[ C_4 = \mu p_3(A_7 + A_{10}) \]

\[ C_5 = \mu p_3(A_6 + A_9) - \mu R_p R R_2 + a R_2 \left[ \mu^2 R \cos(\psi_2 - \delta R_2 - \lambda_2) \right. \]
\[ \left. - \sin(\psi_2 - \delta R_2 - \lambda_2) \right] \]
III. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-3 gives the free body diagram of the gear and pinion combination no. 2. In addition, mesh no. 1 is indicated to obtain the directions of the forces of gear no. 1 on pinion no. 2.

The forces of pinion no. 3 on gear no. 2 have directions opposite to those given by equation (H-22) and (H-23) respectively. Thus, the normal force is given by

\[ \bar{F}_{32} = -F_{23} \bar{N}_{\lambda 2} \]  \hspace{1cm} (H-41)

The friction force of pinion no. 3 on gear no. 2 becomes

\[ \bar{F}_{f32} = -\mu S_{23} F_{23} \bar{N}_{\lambda 2} \]  \hspace{1cm} (H-42)

The contact force of gear no. 1 on pinion no. 2 is given by

\[ \bar{F}_{12} = F_{12} \bar{N}_{\lambda 1} \]  \hspace{1cm} (H-43)

while the associated friction force of gear no. 1 on pinion no. 2 becomes

\[ \bar{F}_{f12} = \bar{F}_{12} \]  

H-18
FIGURE H-3
FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2: ROUND ON ROUND
MESH NO. 1: ROUND ON ROUND

Gear & Pinion no. 2
Mesh No. 2: ROUND ON ROUND
Mesh No. 1: ROUND ON ROUND

H-19
The pivot reactions of the fuse body on the pivot shaft, together with the pivot friction forces, are shown in a separate diagram in Figure H-3. The centrifugal force, due to the mass of gear and pinion assembly no. 2, is given by

\[ \bar{Q}_2 = Q_2 (\cos \gamma_2 \bar{I} + \sin \gamma_2 \bar{J}) \quad (H-45) \]

where

\[ Q_2 = m_2 \omega^2 \quad (H-46) \]

Force equilibrium is assured by

\[-F_{23} R_{\lambda 2} - \mu_2 F_{23} R_{\lambda 2} + F_{12} R_{\lambda 1} + \mu_1 R_{12} R_{N \lambda 1} + \bar{Q}_2 \]

\[ + F_{x2} \bar{I} - \mu F_{y2} \bar{I} + F_{y2} \bar{J} + \mu F_{x2} \bar{J} = 0 \quad (H-47) \]
Moment equilibrium must satisfy

\[-\mu_2 (\dot{F}_x + \dot{F}_y) + [e_2 \vec{q}_2 + \rho_2 \vec{\alpha}_2] \times [-\vec{F}_{23} \vec{\alpha}_2 - \mu_2 R^2 \vec{F}_{23} \vec{\alpha}_2] + [e_1 \vec{q}_1 - \rho_1 \vec{\alpha}_1] \times [\vec{F}_{12} \vec{\alpha}_1 + \mu_1 R \vec{F}_{12} \vec{\alpha}_1] = 0 \quad (H-48)\]

Equation \((H-47)\) gives the following component expressions

\[-F_{23} \cos \lambda_2 - \mu_2 R F_{23} \sin \lambda_2 + F_{12} \cos \lambda_1 + \mu_1 R F_{12} \sin \lambda_1 + 2Q_2 \cos \gamma_2 + F_{x2} - \mu F_{y2} = 0 \quad (H-49)\]

and

\[-F_{23} \sin \lambda_2 - \mu_2 R F_{23} \cos \lambda_2 + F_{12} \sin \lambda_1 + \mu_1 R F_{12} \cos \lambda_1 + 2Q_2 \sin \gamma_2 + F_{y2} + \mu F_{x2} = 0 \quad (H-50)\]
The scalar form of the moment equation (H-48) becomes

\[
-\mu^2 (\ddot{x}_2 + \ddot{y}_2) + a_{23} F_{23} \left[ \sin(\phi_2 - \phi_2 - \lambda_2) - \mu a_2 \cos(\phi_2 - \phi_2 - \lambda_2) \right] \\
- \mu a_2 \ddot{r}_2 F_{23} + a_{12} F_{12} \left[ -\sin(\psi_1 + \phi_1 - \lambda_1) + \mu a_1 \cos(\psi_1 + \phi_1 - \lambda_1) \right] \\
- \mu a_1 \ddot{r}_1 F_{12} = 0
\]  

(H-51)

Simultaneous solution of equations (H-49) and (H-50) for \( F_{x2} \) and \( F_{y2} \) leads to

\[
F_{x2} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (1 + \mu^2 a_2) \cos \lambda_2 - \mu (a_2 - 1) \sin \lambda_2 \right] \\
+ F_{12} \left[ (a_1 - 1) \sin \lambda_1 - (1 + \mu^2 a_1) \cos \lambda_1 \right] \\
+ Q_2 \left[ -\cos \phi_2 - \mu \sin \phi_2 \right] \right\}
\]  

(H-52)

and

\[
F_{y2} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (1 + \mu^2 a_2) \sin \lambda_2 - \mu (a_2 - 1) \cos \lambda_2 \right] \\
+ F_{12} \left[ (1 - a_1) \cos \lambda_1 - (1 + \mu^2 a_1) \sin \lambda_1 \right] \\
+ Q_2 \left[ \mu \cos \phi_2 - \sin \phi_2 \right] \right\}
\]  

(H-53)
The sum of $\tilde{F}_{x2}$ and $\tilde{F}_{y2}$ of equation (H-51) is now made up of equations (H-52) and (H-53) in the sense of equation (A-3b) of Appendix A

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23A_{11}} + F_{12A_{12}} + Q_{2A_{13}} + F_{23A_{14}} + F_{12A_{15}} + Q_{2A_{16}}$$  \(\text{ (H-54)}\)

where

$$A_{11} = \begin{vmatrix} (1 + \mu^2 s_{2R}) \cos \lambda_2 - \mu(s_{2R} - 1) \sin \lambda_2 \\ 1 + \mu^2 \end{vmatrix}$$  \(\text{ (H-55)}\)

$$A_{12} = \begin{vmatrix} \mu(s_{1R} - 1) \sin \lambda_1 - (1 + \mu^2 s_{1R}) \cos \lambda_1 \\ 1 + \mu^2 \end{vmatrix}$$  \(\text{ (H-56)}\)

$$A_{13} = \begin{vmatrix} - \cos \gamma_2 - \mu \sin \gamma_2 \\ 1 + \mu^2 \end{vmatrix}$$  \(\text{ (H-57)}\)

$$A_{14} = \begin{vmatrix} (1 + \mu^2 s_{2R}) \sin \lambda_2 - \mu(1 - s_{2R}) \cos \lambda_2 \\ 1 + \mu^2 \end{vmatrix}$$  \(\text{ (H-58)}\)

$$A_{15} = \begin{vmatrix} \mu(1 - s_{1R}) \cos \lambda_1 - (1 + \mu^2 s_{1R}) \sin \lambda_1 \\ 1 + \mu^2 \end{vmatrix}$$  \(\text{ (H-59)}\)
Equation (H-54) is now substituted into the moment equation (H-51). The resulting expression is then solved for the contact force $F_{12}$

$$F_{12} = \frac{-F_{23}C_6 + Q_2C_7}{C_8} \quad (H-61)$$

where

$$C_6 = 8_2 \left[ \sin(\phi_2 - \delta_2 - \lambda) - \mu_2 R \cos(\phi_2 - \delta_2 - \lambda) \right] - \mu_2^2 (A_{11} + A_{14}) - \mu_2^2 R^2 \delta_2$$

$$C_7 = \mu_2 (A_{13} + A_{16})$$

$$C_8 = - \left\{ 8_1 \left[ \sin(\psi_1 - \delta_1 - \lambda_1) - \mu_1 R \cos(\psi_1 + \delta_1 - \lambda_1) \right] + \mu_2 (A_{12} + A_{15}) + \mu_1 R \psi_1 \right\}$$

$A_{16} = \begin{vmatrix} \mu \cos \gamma_2 - \sin \gamma_2 \\ 1 + \mu^2 \end{vmatrix} \quad (H-60)$
Figure H-4 represents the free body diagram of the input gear no. 1 which has the input moment $M_{in}$ acting on it.

The forces of pinion no. 2 on gear no. 1 are given according to equations (H-43) and (H-44),

$$ F_{21} = -F_{12} \hat{E} \lambda_1 $$  \hspace{1cm} (H-62)

and

$$ F_{12} = -\mu_{11}R F_{12} \tilde{N} \lambda_1 $$  \hspace{1cm} (H-63)

The moments due to the friction forces on the pivot oppose rotation as indicated.

The centrifugal force, due to the mass of gear no. 1, is given by

$$ \bar{Q}_1 = Q_1 \ddot{1} $$  \hspace{1cm} (H-64)

where

$$ \bar{Q}_1 = \bar{X} \dot{m}_1 \omega^2 $$  \hspace{1cm} (H-65)
FIGURE H-4

FREE BODY DIAGRAM OF GEAR NO. 1
MESH NO. 1: ROUND ON ROUND
Force equilibrium requires, that

\[-F_{12}N_{1} - \mu_{1} R_{12} F_{12} N_{1} + Q_{1} + F_{x1} \bar{I} + \mu_{F_{y1}} \bar{I} + F_{y1} \bar{J} - \mu F_{x1} \bar{J} = 0 \] (H-66)

Moment equilibrium is given by.

\[\mu F_{1}(\bar{F}_{x1} + \bar{F}_{y1}) \bar{k} - M_{1} \bar{K} + \left[ a_{G1} \bar{F}_{1} + \rho_{G1} \bar{N}_{1} \right] \times \]

\[\left[ -F_{12} N_{1} - \mu_{1} R_{12} F_{12} N_{1} \right] = 0 \] (H-67)

Equation (H-66) furnishes the following component equations:

\[-F_{12} \cos \lambda_{1} + \mu_{1} R_{12} \sin \lambda_{1} + Q_{1} + F_{x1} + \mu_{F_{y1}} = 0 \] (H-68)

and

\[-F_{12} \sin \lambda_{1} - \mu_{1} R_{12} \cos \lambda_{1} + F_{y1} - \mu F_{x1} = 0 \] (H-69)

The scalar form of equation (H-67) becomes

\[\mu F_{1}(\bar{F}_{x1} + \bar{F}_{y1}) - M_{1} + a_{G1} F_{12} \left[ \sin(\phi_{1} + \delta_{G1} - \lambda_{1}) \right] - \mu_{1} R \cos(\phi_{1} + \delta_{G1} - \lambda_{1}) - \mu_{1} R \rho_{G1} F_{12} = 0 \] (H-70)
The simultaneous solution of equations (H-68) and (H-69) furnishes

\[ F_{x1} = \frac{1}{1 + \mu^2} \left\{ F_{12} \left[ (1 - \mu^2 s_{1R}) \cos \lambda_1 - \mu (1 + s_{1R}) \sin \lambda_1 \right] - Q_1 \right\} \]  

(H-71)

and

\[ F_{y1} = \frac{1}{1 + \mu^2} \left\{ F_{12} \left[ (1 - \mu^2 s_{1R}) \sin \lambda_1 + \mu (1 + s_{1R}) \cos \lambda_1 \right] - \mu Q_1 \right\} \]  

(H-72)

Then, with the same reasoning as before,

\[ \tilde{F}_{x1} + \tilde{F}_{y1} = F_{12} A_{17} + Q_1 A_{18} + F_{12} A_{19} + Q_1 A_{20} \]  

(H-73)

where

\[ A_{17} = \frac{(1 - \mu^2 s_{1R}) \cos \lambda_1 - \mu (1 + s_{1R}) \sin \lambda_1}{1 + \mu^2} \]  

(H-74)

\[ A_{18} = \frac{1}{1 + \mu^2} \]  

(H-75)
\[ A_{19} = \frac{(1 - \mu^2 s_{1R}) \sin \lambda_1 + \mu(1 + s_{1R}) \cos \lambda_1}{1 + \mu^2} \]  \hspace{1cm} (H-76)

\[ A_{20} = \frac{\mu}{1 + \mu^2} \]  \hspace{1cm} (H-77)

Equation (H-73) is now substituted into equation (H-70) and the result is solved for the contact force \( F_{12} \)

\[ F_{12} = \frac{M_{in} - Q_1 C_9}{C_{10}} \]  \hspace{1cm} (H-78)

where

\[ C_9 = \mu p_1 (A_{18} + A_{20}) \]

\[ C_{10} = \mu p_1 (A_{17} + A_{19}) + a_{G1} \left[ \sin(\phi_1 + \delta_{G1} - \lambda_1) - \mu s_{1R} \cos(\phi_1 + \delta_{G1} - \lambda_1) \right] - \mu s_{1R} p_{G1} \]
V. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-61) and (H-78), which are both expressions in $F_{12}$, are now set equal to each other and the result is solved for $F_{23}$

$$F_{23} = \frac{-c_8}{c_6 c_{10}} (M_{in} - Q_1 C_9) + \frac{c_7}{c_6} Q_2 \quad (H-79)$$

The above is now equated to equation (H-40) and the result is solved for $F_{34}$

$$F_{34} = \frac{c_5 c_8 (M_{in} - Q_1 C_9) - c_5 c_{10} c_7 Q_2 - c_4 c_6 c_{10} Q_3}{c_5 c_6 c_{10}} \quad (H-80)$$

Finally equation (H-80) is equated to equation (H-19). This permits the determination of the equilibrant moment $M_{041}$

(for case no. 1: RRR)

$$M_{041} = M_{in} \frac{c_2 c_5 c_8}{c_3 c_6 c_{10}} - Q_1 \frac{c_2 c_5 c_8 c_9}{c_3 c_6 c_{10}} - Q_2 \frac{c_2 c_5 c_7}{c_3 c_6} - Q_3 \frac{c_2 c_4}{c_3} - Q_4 C_1 \quad (H-81)$$

H-30
b. CASE NO. 2: RRF

Since only mesh 1 is now assumed to be in the round on flat phase of motion, the forces \( F_{34} \) and \( F_{23} \) remain as given by equations (H-19) and (H-40), respectively.

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-5 shows the free body diagram of the gear and pinion set with the necessary portions of mesh no. 1.

The forces of pinion no. 3 on gear no. 2 are given by equations (H-41) and (H-42), i.e.,

\[
F_{32} = -F_{23}N_{\lambda 2} \quad (H-82)
\]

and

\[
F_{f32} = -R_{2}F_{23}N_{\lambda 2} \quad (H-83)
\]
FIGURE H-5
FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2: ROUND ON ROUND
MESH NO. 1: ROUND ON FLAT
H-32
The contact force of gear no. 1 on pinion no. 2 is now given by

\[ F_{12F} = F_{12F}^N F_1 \quad \text{(H-84)} \]

(Note that the additional subscript F is introduced to distinguish round on flat from round on round contact.)

The associated friction force is given by

\[ F_{f12F} = \mu_{1F} F_{12F} F_1 \quad \text{(H-85)} \]

(See equation (H-2) for \( \mu_{1F} \).)

The pivot reactions, together with the pivot friction forces, are shown in a separate diagram in Figure H-5. As before, the pivot friction moments oppose rotation.

The centrifugal force \( \bar{Q}_2 \) is again given by equations (H-45) and (H-46).

Force equilibrium is given by

\[ -F_{23} \bar{\alpha}_2 - \mu_{2R} F_{23} \bar{\alpha}_{N2} + F_{12F} \bar{F}_N F_1 + \mu_{1F} F_{12F} F_1 + \bar{Q}_2 \]
\[ + F_{x2} \bar{I} - \mu F_{y2} \bar{I} + F_{y2} \bar{J} + \mu F_{x2} \bar{J} = 0 \quad \text{(H-86)} \]
Moment equilibrium about point $O_2$ requires

$$-\mu_2^2 (F_{x2} + F_{y2}) \hat{e} + \left[ a_{02} \vec{e}_{02} + p_{02} \vec{e}_{\lambda 2} \right] \times \left[ -F_{23} \vec{e}_{\lambda 2} - \mu_2^2 \cdot R_{23} \vec{e}_{\lambda 2} \right]$$

$$+ g_1 \vec{e}_{p1} \times F_{12} \vec{e}_{NP1} = 0 \quad (H-87)$$

Note that since the line of action of the friction force $F_{f12}$ passes through point $O_2$, this friction force exerts no moment about point $O_2$.

Equation (H-86) furnishes the following component equations:

$$-F_{23} \cos \lambda_2 + \mu_2 R F_{23} \sin \lambda_2 = F_{12} \sin (\psi_1 - \alpha_1) + \mu_1 R F_{12} \cos (\psi_1 - \alpha_1)$$

$$+ F_{x2} - \mu F_{y2} + Q_2 \cos \gamma_2 = 0 \quad (H-88)$$

and

$$-F_{23} \sin \lambda_2 - \mu_2 R F_{23} \cos \lambda_2 = F_{12} \cos (\psi_1 - \alpha_1) + \mu_1 R F_{12} \sin (\psi_1 - \alpha_1)$$

$$+ F_{y2} + \mu F_{x2} + Q_2 \sin \gamma_2 = 0 \quad (H-89)$$

H-34
The scalar form of the moment equation (H-87) becomes

\[- \mu F_{x2}(F_{x2} + F_{y2}) + \nu_2 F_{23} \left[ \sin(\phi_2 - \theta_{22} - \lambda_2) - \mu_2 \cos(\phi_2 - \theta_{22} - \lambda_2) \right] \]

\[- \mu_2 F_{123} F_{23} + F_{123} \delta_1 = 0 \quad \text{(H-90)} \]

Simultaneous solution of these component equations for \( F_{x2} \) and \( F_{y2} \) leads to

\[ F_{x2} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (1 - \mu_2) \sin \lambda_2 + (1 + \mu^2 \mu_2) \cos \lambda_2 \right] \right. \]

\[ + F_{123} \left[ (1 - \mu^2 \nu_2) \sin \left( \phi_1 - \phi_{p1} \right) - \mu \left( 1 + \nu_2 \right) \cos \left( \phi_1 - \phi_{p1} \right) \right] \]

\[ - Q_2 \left[ \mu \sin \gamma_2 + \cos \gamma_2 \right] \} \right. \quad \text{(H-91)} \]

and

\[ F_{y2} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (1 + \mu^2 \mu_2) \sin \lambda_2 - \mu \left( 1 - \mu_2 \right) \cos \lambda_2 \right] \right. \]

\[ + F_{123} \left[ \mu \left( 1 + \nu_2 \right) \sin \left( \phi_1 - \phi_{p1} \right) + \left( \mu^2 \nu_2 - 1 \right) \cos \left( \phi_1 - \phi_{p1} \right) \right] \]

\[ + Q_2 \left[ - \sin \gamma_2 + \mu \cos \gamma_2 \right] \} \right. \quad \text{(H-92)} \]
The sum ($F_{x2} + F_{y2}$) of equation (H-90) is now made up of equations (H-91) and (H-92) in the sense of equation (A-3b) of Appendix A.

$$F_{x2} + F_{y2} = F_{23}A_{21} + F_{12}A_{22} + Q_2A_{23} + F_{23}A_{24} + F_{12}A_{25} + Q_2A_{26}$$  \hspace{1cm} (H-93)

where

$$A_{21} = \frac{\mu(1 - s_{2R})\sin \alpha_2 + (1 + \mu^2 s_{2R})\cos \alpha_2}{\lambda(1 + \mu^2)}$$  \hspace{1cm} (H-94)

$$A_{22} = \frac{(1 - \mu^2 s_{1P})\sin(\psi_1 - \alpha P_1) - \mu(1 + s_{1P})\cos(\psi_1 - \alpha P_1)}{1 + \mu^2}$$  \hspace{1cm} (H-95)

$$A_{23} = \frac{\mu \sin \gamma_2 + \cos \gamma_2}{1 + \mu^2}$$  \hspace{1cm} (H-96)

$$A_{24} = \frac{(1 + \mu^2 s_{2R})\sin \alpha_2 - \mu(1 - s_{2R})\cos \alpha_2}{1 + \mu^2}$$  \hspace{1cm} (H-97)

$$A_{25} = \frac{-\mu(1 + s_{1P})\sin(\psi_1 - \alpha P_1) + (\mu^2 s_{1P} - 1)\cos(\psi_1 - \alpha P_1)}{1 + \mu^2}$$  \hspace{1cm} (H-98)
\[ A_{26} = \begin{vmatrix} -\sin\gamma_2 + \mu\cos\gamma_2 \\ 1 + \mu^2 \end{vmatrix} \quad (H-99) \]

Equation (H-93) is now substituted into equation (H-90), and the resulting expression is solved for the contact force \( F_{12f} \):

\[ F_{12f} = \frac{-f_{21}c_{11} + q_{21}c_{12}}{c_{13}} \quad (H-100) \]

where

\[ c_{11} = a_{g2}[\sin(\phi_2 - \delta_2 - \lambda_2) - \mu^2 R\cos(\phi_2 - \delta_2 - \lambda_2)] - \mu^2 (A_{21} + A_{24}) - \mu^2 R\delta_2 \]

\[ c_{12} = \mu^2 (A_{23} + A_{26}) \]

\[ c_{13} = \delta_1 - \mu^2 (A_{22} + A_{25}) \]
II. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

Figure H-6 represents the free body diagram of input gear no. 1.

The forces of pinion no. 2 on this gear are given, according to equations (H-84) (H-85).

\[ F_{21} = -F_{12} H_{NP1} \]  \hspace{1cm} (H-101)

and

\[ F_{f21} = -s_{1} F_{12} H_{FP1} \]  \hspace{1cm} (H-102)

The moments due to the pivot friction forces oppose the indicated rotation due to the moment \( M_{in} \).

The centrifugal force \( Q_{i} \) has been defined by equations (H-64) and (H-65).

The force equilibrium equation is given by

\[ -F_{12} H_{NP1} - s_{1} F_{12} H_{FP1} + Q_{i} I + F_{x1} I + F_{y1} J + \mu F_{y1} J - \mu F_{x1} J = 0 \]  \hspace{1cm} (H-103)
**FIGURE H-6**

FREE BODY DIAGRAM OF GEAR NO. 1

MELH NO. 1: ROUND ON FLAT

H-39
The moment equilibrium equation becomes

\[-M_{in} + \mu F_{12F} \left( \tilde{F}_{x1} + \tilde{F}_{y1} \right) + \left[ \sigma_{1F} \gamma_{1} + \rho_{G1} \overline{\gamma}_{MF} \right] \cdot \left[ \overline{F}_{12F} \overline{\gamma}_{MF} \right] = 0 \]

Equation (H-103) furnishes the following component expressions

\[F_{12F} \sin(\phi_1 - \alpha_{p1}) - \mu s_{1F} F_{12F} \cos(\phi_1 - \alpha_{p1}) + Q_{1} + F_{x1} + \mu F_{y1} = 0 \]  

and

\[-F_{12F} \cos(\phi_1 - \alpha_{p1}) - \mu s_{1F} F_{12F} \sin(\phi_1 - \alpha_{p1}) + F_{y1} - \mu F_{x1} = 0 \]

The scalar form of the moment equation (H-104) becomes

\[-M_{in} + \mu F_{12F} \left( \tilde{F}_{x1} + \tilde{F}_{y1} \right) + \mu s_{1F} \sigma_{1} F_{12F} + \sigma_{1F} F_{12F} \left[ -\cos(\phi_1 + \delta_{G1} - \phi_1 + \alpha_{p1}) \right]
+ \mu s_{1F} \sin(\phi_1 + \delta_{G1} - \phi_1 + \alpha_{p1}) = 0 \]

Simultaneous solution of equations (H-105) and (H-106) for \(F_{x1}\) and \(F_{y1}\) furnishes

\[F_{x1} = \frac{F_{12F} \left[ -\left( 1 + \mu^2 s_{1F} \right) \sin(\phi_1 - \alpha_{p1}) + \mu (s_{1F} - 1) \cos(\phi_1 - \alpha_{p1}) \right] - Q_{1}}{1 + \mu^2} \]

H-40
and

\[ F_{y1} = \frac{F_{12F}}{1 + \mu^2} \left[ (1 + \mu^2 s_1) \cos(\psi_1 - \alpha_{p1}) + \mu (s_1 - 1) \sin(\psi_1 - \alpha_{p1}) \right] - \mu Q_1 \]

Now let

\[ \tilde{F}_{x1} + \tilde{F}_{y1} = F_{12F} A_{27} + Q_1 A_{28} + F_{12F} A_{29} + Q_1 A_{30} \]

where

\[ A_{27} = \left| \frac{-(1 + \mu^2 s_1 \sin(\psi_1 - \alpha_{p1}) + \mu (s_1 - 1) \cos(\psi_1 - \alpha_{p1})}{1 + \mu^2} \right| \]

\[ A_{28} = \left| \frac{\mu (s_1 - 1) \sin(\psi_1 - \alpha_{p1}) + (1 + \mu^2 s_1) \cos(\psi_1 - \alpha_{p1})}{1 + \mu^2} \right| \]

\[ A_{29} = \left| \frac{\mu}{1 + \mu^2} \right| \]

\[ A_{30} = \left| \frac{\mu}{1 + \mu^2} \right| \]
Equation (H-110) is now substituted into the moment equation (H-107).

This furnishes

\[ F_{12F} = \frac{M_{in} - Q_1 C_{14}}{C_{15}} \]  

(H-115)

where

\[ C_{14} = \mu_{p1}(A_{28} + A_{30}) \]

\[ C_{15} = \mu_{p1}(A_{27} + A_{29}) + \mu_{s1} \rho G_1 \]

\[ + a_{g1}[\mu_{s1} \sin(\phi_1 + \delta G_1 - \psi_1 + \alpha_{P1}) - \cos(\phi_1 + \delta G_1 - \psi_1 + \alpha_{P1})] \]
III. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-100) and (H-115), which are both expressions in \( F_{12} \), are now set equal to each other and the result is solved for \( F_{23} \)

\[
F_{23} = \frac{c_{13}}{c_{11}c_{15}} (M_n - Q_1 c_{14}) + q_2 \frac{c_{12}}{c_{11}}
\]

(H-116)

The above expression is now equated to equation (H-40) and solved for \( F_{34} \)

\[
F_{34} = \frac{c_{2}c_{13}}{c_{2}c_{11}c_{15}} (M_n - Q_1 c_{14}) - q_2 \frac{c_{2}c_{12}}{c_{2}c_{11}} - q_3 \frac{c_{4}}{c_{3}}
\]

(H-117)

Finally, this expression is set equal to equation (H-19) and the result is solved for the equilibrant moment \( M_{042} \) (for case 2: RRF)

\[
M_{042} = M_n \frac{c_{2}c_{2}c_{13}}{c_{2}c_{11}c_{15}} - Q_1 \frac{c_{2}c_{2}c_{13}c_{14}}{c_{2}c_{11}c_{15}} - q_2 \frac{c_{2}c_{2}c_{12}}{c_{2}c_{11}} - q_3 \frac{c_{2}c_{4}}{c_{3}} - q_4 c_{1}
\]

(H-118)
With both meshes no. 1 and no. 2 in the round on flat phase of motion, only force $F_{34}$ of the round on round phase can be incorporated for the present case. The equilibrium equations for gear and pinion set no. 3, gear and pinion set no. 2 and the input gear no. 1 must be newly derived.

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-7 shows the free body diagram of gear and pinion set no. 3, together with the necessary outline of mesh no. 2.

The forces of pinion no. 4 on gear no. 3 are given by equations (H-20) and (H-21)

\[
F_{43} = -F_{34}H_{33} \quad \text{(H-119)}
\]

and

\[
F_{f43} = -\mu s_{3R} F_{34} H_{N3} \quad \text{(H-120)}
\]
FIGURE H-7
FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON ROUND
MESH NO. 2: ROUND ON FLAT

H-45
The normal contact force of gear no. 2 on pinion no. 3 is given by

\[ F_{23} = -F_{23}N_{23} \]  \hspace{1cm} (H-121)

The associated friction force is given by

\[ F_{f23} = \mu_{f}F_{23}N_{23} \]  \hspace{1cm} (H-122)

The pivot friction forces are chosen so that the resulting friction moments oppose the indicated rotation. The centrifugal force \( \bar{Q}_3 \) is that of equations (H-24) and (H-25).

Force equilibrium is given by

\[ -F_{34}H_{3} + \mu_{s}3F_{34}N_{3} + F_{23}N_{23} + \mu_{f}F_{23}N_{23} + \bar{Q}_3 + F_{x3}i + \mu F_{y3}i + F_{y3}j - \mu F_{x3}j = 0 \]  \hspace{1cm} (H-123)

Moment equilibrium about point \( O_3 \) is given by

\[ \mu F_{x3} + F_{y3}i + \mu F_{y3}i + F_{y3}j - \mu F_{x3}j = 0 \]  \hspace{1cm} (H-124)
Note that the friction force \( F_{23f} \) exerts no moment about point \( O_3 \).

Equation (H-123) furnishes the following component equations:

\[
-F_{34} \cos \alpha_3 + \mu_3 \beta_{34} F_{34} \sin \alpha_3 + Q_3 \cos \gamma_3 + F_{x3} + \mu F_{y3} + F_{23f} \sin (\dot{\gamma}_2 + \alpha_{p2})
\]

\[
+ \mu_2 \beta_{23} F_{23f} \cos (\dot{\gamma}_2 + \alpha_{p2}) = 0
\]  

(H-125)

and

\[
-F_{34} \sin \alpha_3 - \mu_3 \beta_{34} F_{34} \cos \alpha_3 + Q_3 \sin \gamma_3 + F_{y3} - \mu F_{x3} - F_{23f} \cos (\dot{\gamma}_2 + \alpha_{p2})
\]

\[
+ \mu_2 \beta_{23} F_{23f} \sin (\dot{\gamma}_2 + \alpha_{p2}) = 0
\]  

(H-126)

The scalar form of the moment equation (H-124) becomes

\[
\mu_3 (F_{x3} + F_{y3}) + a_{03} F_{34} [\sin (\dot{\gamma}_2 + \dot{\gamma}_3 - \lambda_3) - \mu_3 \rho \cos (\dot{\gamma}_2 + \dot{\gamma}_3 - \lambda_3)]
\]

\[
- \mu_2 \rho_0 F_{34} = 0
\]  

(H-127)
Simultaneous solution of equations (H-125) and (H-126) for $F_{x3}$ and $F_{y3}$ leads to

$$F_{x3} = \frac{1}{1 + \mu^2} \left\{ F_{x4} \left[ (1 - \mu^2 a_{3R}) \cos \gamma_3 - \mu(1 + a_{3R}) \sin \gamma_3 \right] \\
+ F_{23} \left[ -\mu(1 + a_{2P}) \cos (\gamma_2 - \phi_2) - (1 - \mu^2 a_{2P}) \sin (\gamma_2 - \phi_2) \right] \\
+ Q_3 \left[ \mu \sin \gamma_3 - \cos \gamma_3 \right] \right\} \quad (H-128)$$

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{y4} \left[ (1 - \mu^2 a_{3R}) \sin \gamma_3 + \mu(1 + a_{3R}) \cos \gamma_3 \right] \\
+ F_{23} \left[ (1 - \mu^2 a_{2P}) \cos (\gamma_2 - \phi_2) - \mu(1 + a_{2P}) \sin (\gamma_2 - \phi_2) \right] \\
- Q_3 \left[ \sin \gamma_3 + \mu \cos \gamma_3 \right] \right\} \quad (H-129)$$

The sum $F_{x3} + F_{y3}$ of equation (H-127) is now made up of equations (H-128) and (H-129) in the sense of equation (A-3b)

$$F_{x3} + F_{y3} = F_{x4} A_{31} + F_{23} A_{32} + Q_3 A_{33} + F_{x4} A_{34} + F_{23} A_{35} + Q_3 A_{36} \quad (H-130)$$

H-48
where

\[ A_{31} = \left( 1 - \mu^2 a_{3R} \right) \cos \lambda_3 - \mu (1 + a_{3R}) \sin \lambda_3 \right) \frac{1}{1 + \mu^2} \]  \hspace{1cm} (H-131)

\[ A_{32} = \left( -\mu (1 + a_{2R}) \cos (\psi_2 + \alpha_2) - (1 - \mu^2 a_{2R}) \sin (\psi_2 + \alpha_2) \right) \frac{1}{1 + \mu^2} \]  \hspace{1cm} (H-132)

\[ A_{33} = \left( \mu \sin \gamma_3 - \cos \gamma_3 \right) \frac{1}{1 + \mu^2} \]  \hspace{1cm} (H-133)

\[ A_{34} = \left( (1 - \mu^2 a_{3R}) \sin \lambda_3 + \mu (1 + a_{3R}) \cos \lambda_3 \right) \frac{1}{1 + \mu^2} \]  \hspace{1cm} (H-134)

\[ A_{35} = \left( (1 - \mu^2 a_{3R}) \cos (\psi_2 + \alpha_2) - (1 + \mu^2 a_{3R}) \sin (\psi_2 + \alpha_2) \right) \frac{1}{1 + \mu^2} \]  \hspace{1cm} (H-135)

\[ A_{36} = \left( \sin \gamma_3 + \mu \cos \gamma_3 \right) \frac{1}{1 + \mu^2} \]  \hspace{1cm} (H-136)

Equation (H-130) is now substituted into equation (H-127) and the result is solved for \( F_{23p} \)
\[ F_{23r} = -\frac{F_{34}c_{16} - q_{3}c_{17}}{c_{18}} \]  

where

\[ c_{16} = \mu_{3}(A_{31} + A_{34}) + a_{33}[\sin(\phi_{3} + \delta_{33} - \phi_{3}) - \mu_{3r}\cos(\phi_{3} + \delta_{33} - \phi_{3}) - \mu_{3r}\phi_{3}] \]

\[ c_{17} = \mu_{3}(A_{33} + A_{36}) \]

\[ c_{18} = \mu_{3}(A_{32} + A_{35}) - \delta_{2} \]
II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-8 shows the free body diagram of the gear and pinion set no. 2.

The forces of pinion no. 3 on gear no. 2 are equal and opposite to those given by equations (H-121) and (H-122), i.e.,

\[ F_{32F} = F_{23F} F_{NF2} \quad \text{(H-138)} \]

and

\[ F_{f32F} = -F_{23F} F_{23F} F_{F2} \quad \text{(H-139)} \]

The forces of gear no. 1 on pinion no. 2 are those of equations (H-84) and (H-85).

The pivot friction is accounted for in the usual manner and the centrifugal force \( \bar{Q}_2 \) is given by equation (H-45).

Force equilibrium is given by

\[ F_{23F} F_{NF2} - F_{NF1} F_{12F} + F_{12F} F_{NF1} + \bar{Q}_2 + F_{x2} \bar{J} - \mu F_{y2} \bar{J} + F_{y2} \bar{J} + \mu F_{x2} \bar{J} = 0 \quad \text{(H-140)} \]
FIGURE H-8
FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2: ROUND ON FLAT
MESH NO. 1: ROUND ON FLAT

H-52
Moment equilibrium about point $O_2$ requires

$$-\mu p_2 (\tilde{F}_{x2} + \tilde{F}_{y2}) \tilde{r} + \left[ a_{G2} \tilde{g}_{G2} - \dot{p}_{G2} \tilde{r}_{NF2} \right] \times \left[ \tilde{F}_{23} \tilde{r}_{NF2} - \mu s_{2F} \tilde{F}_{23} \tilde{r}_{F2} \right]$$

$$+ \varepsilon_1 \tilde{r}_{F1} \times F_{12} \tilde{r}_{NF1} = 0 \quad (H-141)$$

Equation (H-140) gives the component equations

$$-F_{23} \sin(\psi_2 + \alpha_{p2}) - \mu s_{2F} \tilde{F}_{23} \cos(\psi_2 + \alpha_{p2}) + Q_{2} \cos \gamma_2 + F_{x2} - \mu F_{y2}$$

$$- F_{12} \sin(\psi_1 - \alpha_{P1}) + \mu s_{1F} \tilde{F}_{12} \cos(\psi_1 - \alpha_{P1}) = 0 \quad (H-142)$$

and

$$F_{23} \cos(\psi_2 + \alpha_{p2}) - \mu s_{2F} \tilde{F}_{23} \sin(\psi_2 + \alpha_{p2}) + Q_{2} \sin \gamma_2 + F_{y2} + \mu F_{x2}$$

$$+ F_{12} \cos(\psi_1 - \alpha_{P1}) + \mu s_{1F} \tilde{F}_{12} \sin(\psi_1 - \alpha_{P1}) = 0 \quad (H-143)$$

H-53
The scalar form of the moment equation (H-141) becomes

\[-\mu_2 (F_{x2} + F_{y2}) + a g_2 f_{23F} [\cos(\psi_2 - \delta_2 - \phi_2 - \alpha_{P2})]
+ \mu s_{2F}^2 \sin(\phi_2 - \delta_2 - \phi_2 - \alpha_{P2}) - \mu s_{2F}^2 g_2 f_{23F} + g_1 f_{12F} = 0\]  
\hspace{10cm} (H-144)

Simultaneous solution of the component equations (H-142) and (H-143)
for $F_{x2}$ and $F_{y2}$ leads to

\[F_{x2} = \frac{1}{1 + \mu^2} \left\{ f_{23F} [(1 + \mu^2 s_{2F}) \sin(\phi_2 + \alpha_{P2}) + \mu (s_{2F} - 1) \cos(\phi_2 + \alpha_{P2})]
+ f_{12F} [(1 - \mu^2 s_{1F}) \sin(\phi_1 - \alpha_{P1}) - \mu (s_{1F}) \cos(\phi_1 - \alpha_{P1})]
- Q_2 \left[ \mu \sin \gamma_2 + \cos \gamma_2 \right] \right\} \]  
\hspace{10cm} (H-145)

and

\[F_{y2} = \frac{1}{1 + \mu^2} \left\{ f_{23F} \left[ \mu (s_{2F} - 1) \sin(\phi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F}) \cos(\phi_2 + \alpha_{P2}) \right]
- f_{12F} \left[ \mu (1 + s_{1F}) \sin(\phi_1 - \alpha_{P1}) + (1 - \mu^2 s_{1F}) \cos(\phi_1 - \alpha_{P1}) \right]
+ Q_2 \left[ \mu \cos \gamma_2 - \sin \gamma_2 \right] \right\} \]  
\hspace{10cm} (H-146)
The sum $\bar{F}_{x2} + \bar{F}_{y2}$ of equation (H-144) is now made up of equations (H-145) and (H-146) in the sense of equation (A-3b)

\[
\bar{F}_{x2} + \bar{F}_{y2} = F_{23}F_{37} + F_{12}F_{38} + Q_2A_{39} + F_{23}F_{40} + F_{12}F_{41} + Q_2A_{42}
\]

(H-147)

where

\[
A_{37} = \frac{(1 + \mu^2 s_2 F)\sin(\psi_2 + \alpha_2) + \mu(s_2 F - 1)\cos(\psi_2 + \alpha_2)}{1 + \mu^2}
\]

(H-148)

\[
A_{38} = \frac{(1 - \mu^2 s_1 F)\sin(\psi_1 - \alpha_1) - \mu(1 + s_1 F)\cos(\psi_1 - \alpha_1)}{1 + \mu^2}
\]

(H-149)

\[
A_{39} = \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2}
\]

(H-150)

\[
A_{40} = \frac{\mu(s_2 F - 1)\sin(\psi_2 + \alpha_2) - (1 + \mu^2 s_2 F)\cos(\psi_2 + \alpha_2)}{1 + \mu^2}
\]

(H-151)

\[
A_{41} = \frac{\mu(1 + s_1 F)\sin(\psi_1 - \alpha_1) + (1 - \mu^2 s_1 F)\cos(\psi_1 - \alpha_1)}{1 + \mu^2}
\]

(H-152)

\[
A_{42} = \frac{\mu\cos\gamma_2 - \sin\gamma_2}{1 + \mu^2}
\]

(H-153)
Equation (H-147) is now substituted into equation (H-144) and the result
is solved for \( F_{12F} \)

\[
F_{12F} = \frac{-F_{23F}c_{19} + Q_{2}c_{20}}{c_{21}}
\]

(H-154)

where

\[
c_{19} = -\mu_{2}(A_{37} + A_{40}) + a_{2}a_{2}^{s}[\cos(\phi_{2} - \theta_{2} - \phi_{2} - \theta_{2})
+ \mu_{2}F_{2F}\sin(\phi_{2} - \theta_{2} - \phi_{2} - \theta_{2}) - \mu_{2}F_{2F}g_{2}]
\]

\[
c_{20} = \mu_{2}(A_{39} + A_{42})
\]

\[
c_{21} = -\mu_{2}(A_{38} + A_{41}) + g_{1}
\]

H-56
III. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

While the numerical values of the force $F_{21F}$ and its associated friction force $F_{f12F}$, both acting on gear no. 1, are peculiar to the present combination of contact phases, its functional relationship to the input moment $M_{in}$ and the centrifugal force $Q_1$ is identical to that derived in section 1b-II of this appendix. (See also Figure H-6.)

According to equation (H-115), one obtains for $F_{12F}$

$$F_{12F} = \frac{M_{in} - Q_1C_{14}}{C_{15}}$$  \hspace{1cm} (H-155)
IV. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-154) and (H-155), both in \( F_{12} \), are set equal to each other and the result is solved for \( F_{23} \)

\[
F_{23} = \frac{-c_{21}}{c_{15}c_{19}} (M_{in} - Q_1c_{14}) + Q_2 \frac{c_{20}}{c_{19}} \tag{H-156}
\]

The above is now equated to equation (H-137) and the result is solved for \( F_{34} \)

\[
F_{34} = \frac{c_{18}c_{21}}{c_{15}c_{16}c_{19}} (M_{in} - Q_1c_{14}) - Q_2 \frac{c_{18}c_{20}}{c_{16}c_{19}} - Q_3 \frac{c_{17}}{c_{16}} \tag{H-157}
\]

Finally, equation (H-157) is equated to equation (H-19), which corresponds to the round on round phase of mesh no. 3. The result is solved for the equilibrant moment \( M_{043} \) (for case 3: RFF)

\[
M_{043} = M_{in} \frac{c_{2}c_{18}c_{21}}{c_{15}c_{16}c_{19}} - Q_1 \frac{c_{2}c_{14}c_{18}c_{21}}{c_{15}c_{16}c_{19}} - Q_2 \frac{c_{2}c_{18}c_{20}}{c_{16}c_{19}} - Q_3 \frac{c_{2}c_{17}}{c_{16}} - Q_4c_1 \tag{H-158}
\]

H-58
d. CASE NO. 4: RFR

For this contact combination force $F_{34}$ may be taken from the results of case no. 1 [see equation (H-19)], since mesh no. 3 is in the round on round phase of motion. The force $F_{23}$ of case no. 3, i.e., equation (H-137), also is incorporated.

The input-output relationship of the gear and pinion combination no. 2 must be newly derived, i.e., the force $F_{12}$ must be expressed in terms of the contact force $F_{32}$ and the centrifugal force $Q_2$. Finally, the results of the equilibrium equations for the input gear no. 1 of case no. 1 are used. For this case of contact, the force $F_{12}$ is given by equation (H-78).

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-9 shows the free body diagram of gear and pinion set no. 2 with the necessary portions of mesh no. 1.

The forces of pinion no. 3 on gear no. 2 were given by equations (H-138) and (H-139)
FIGURE H-9
FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2: ROUND ON FLAT
MESH NO. 1: ROUND ON ROUND

H-60
\[ F_{32F} = F_{23F}\bar{R}_{NF2} \]  \hspace{1cm} (H-159)

and

\[ F_{f32F} = -\mu s_2 F_{23F}\bar{R}_{F2} \]  \hspace{1cm} (H-160)

The forces of gear no. 1 on pinion no. 2 are given by equations (H-43) and (H-44).

\[ F_{12} = F_{12}\bar{R}_{\lambda 1} \]  \hspace{1cm} (H-161)

and

\[ F_{f12} = \mu s_1 R_{12}\bar{R}_{N\lambda 1} \]  \hspace{1cm} (H-162)

The centrifugal force \( \bar{Q}_2 \) is given by equation (H-45). The pivot reactions and friction forces are handled as before.

Force equilibrium is given by

\[ F_{23F}\bar{R}_{NF2} - \mu s_2 F_{23F}\bar{R}_{F2} + F_{12}\bar{R}_{\lambda 1} + \mu s_1 R_{12}\bar{R}_{N\lambda 1} + \bar{Q}_2 + F_{x2}\bar{J} - \mu F_{y2}\bar{J} + F_{y2}\bar{J} + \mu F_{x2}\bar{J} = 0 \]  \hspace{1cm} (H-163)
Moment equilibrium about point $O_2$ is given by

$$-\mu_{e2}(\vec{F}_{x2} + \vec{F}_{y2}) + [a_{q2} - \rho_{q2}] \times [F_{23F} + \mu_{s2}F_{23F}]$$

$$+ [a_{p1} - \rho_{p1}] \times [F_{12} + \mu_{s1}F_{12}] = 0 \quad (H-164)$$

Equation (H-163) gives the following component equations

$$-F_{23F}\sin(\psi_2 + \alpha_{2}) - \mu_{s2}F_{23F}\cos(\psi_2 + \alpha_{2}) + F_{x2} - \mu_{y2} + F_{12}\cos\lambda_1$$

$$- \mu_{s1}F_{12}\sin\lambda_1 + Q_2\cos\gamma_2 = 0 \quad (H-165)$$

and

$$F_{23F}\cos(\psi_2 + \alpha_{2}) - \mu_{s2}F_{23F}\sin(\psi_2 + \alpha_{2}) + F_{y2} + \mu_{x2} + F_{12}\sin\lambda_1$$

$$+ \mu_{s1}F_{12}\cos\lambda_1 + Q_2\sin\gamma_2 = 0 \quad (H-166)$$

H-62
The scalar form of the moment equation (H-164) becomes

\[- p_2 (\tilde{F}_{x2} + \tilde{F}_{y2}) + a_{g2} F_{23f} [\cos(\phi_2 - \delta_2 - \psi_2 - \alpha_{p2})
\quad + \mu s_{2g} \sin(\phi_2 - \delta_2 - \psi_2 - \alpha_{p2})] - \mu a_{2g} F_{23F} - \mu a_{R} F_{12}
\quad + a_{p1} F_{12} [\mu s_{1R} \cos(\psi_1 + \delta_{p1} - \lambda_1) - \sin(\psi_1 + \delta_{p1} - \lambda_1)] = 0 \quad (H-167)\]

Simultaneous solution of the force component equations (H-165) and (H-166) for \( F_{x2} \) and \( F_{y2} \) results in

\[ F_{x2} = \frac{1}{1 + \mu^2} \left\{ F_{23f} [(1 + \mu^2 s_{2f}) \sin(\psi_2 + \alpha_{p2}) - \mu (1 + s_{2f}) \cos(\psi_2 + \alpha_{p2})]
\quad + F_{12} [\mu (s_{1R} - 1) \sin \lambda_1 - (1 + \mu^2 s_{1R}) \cos \lambda_1]
\quad - Q_2 [\mu \sin \gamma_2 + \cos \gamma_2] \right\} \quad (H-168) \]

and

\[ F_{y2} = \frac{1}{1 + \mu^2} \left\{ F_{23f} [\mu (s_{2f} - 1) \sin(\psi_2 + \alpha_{p2}) - (1 + \mu^2 s_{2f}) \cos(\psi_2 + \alpha_{p2})]
\quad + F_{12} [- (1 + \mu^2 s_{1R}) \sin \lambda_1 + \mu (1 - s_{1R}) \cos \lambda_1]
\quad + Q_2 [- \sin \gamma_2 + \mu \cos \gamma_2] \right\} \quad (H-169) \]
The sum $\vec{F}_{x2} + \vec{F}_{y2}$ of equation (H-167) is now made up of equations (H-168) and (H-169) in the usual manner

$$\vec{F}_{x2} + \vec{F}_{y2} = F_{23}A_{43} + F_{12}A_{44} + Q_{2}A_{45} + F_{23}A_{46} + F_{12}A_{47} + Q_{2}A_{48}$$  \hspace{1cm} (H-170)

where

$$A_{43} = \left| \frac{(1 + \mu^2 s_{2p})\sin(\psi_2 + \sigma_p) - \mu(1 - s_{2p})\cos(\psi_2 + \sigma_p)}{1 + \mu^2} \right|$$ \hspace{1cm} (H-171)

$$A_{44} = \left| \frac{\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1}{1 + \mu^2} \right|$$ \hspace{1cm} (H-172)

$$A_{45} = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right|$$ \hspace{1cm} (H-173)

$$A_{46} = \left| \frac{\mu(s_{2F} - 1)\sin(\psi_2 + \sigma_p) - (1 + \mu^2 s_{2p})\cos(\psi_2 + \sigma_p)}{1 + \mu^2} \right|$$ \hspace{1cm} (H-174)

$$A_{47} = \left| \frac{-(1 + \mu^2 s_{1R})\sin\lambda_1 + \mu(1 - s_{1R})\cos\lambda_1}{1 + \mu^2} \right|$$ \hspace{1cm} (H-175)

$$A_{48} = \left| \frac{-\sin\gamma_2 + \mu\cos\gamma_2}{1 + \mu^2} \right|$$ \hspace{1cm} (H-176)

H-64
Equation (H-170) is now substituted into the moment equation (H-167) and the result is solved for $F_{12}$

$$F_{12} = - \frac{F_{23}C_{22} + Q_2C_{23}}{C_{24}}$$  \hspace{1cm} (H-177)

where

$$C_{22} = -\mu_p^2(A_{43} + A_{46}) + a_q^2[\cos(\phi_2 - \phi_2 - \phi_2 - \phi_2)] - \mu_2^2g_2g_2$$

$$C_{23} = \mu_p^2(A_{45} + A_{47})$$

$$C_{24} = -\mu_p^2(A_{44} + A_{47}) + a_p^2[\mu_1R\cos(\psi_1 + \delta_p - \lambda_1)] - \sin(\psi_1 + \delta_p - \lambda_1)] - \mu_1R\rho_p$$

H-65
II. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-177) and (H-78) are now set equal to each other and the result is solved for $F_{234}$

$$F_{234} = \frac{-c_{24}}{c_{10}c_{22}} (M_{in} - Q_1C_9) + Q_2 \frac{c_{23}}{c_{22}}$$

(H-178)

The above is now equated to equation (H-137) to obtain $F_{34}$

$$F_{34} = \frac{c_{18}c_{24}}{c_{10}c_{16}c_{22}} (M_{in} - Q_1C_9) - Q_2 \frac{c_{18}c_{23}}{c_{16}c_{22}} - Q_3 \frac{c_{17}}{c_{16}}$$

(H-179)

Finally equations (H-179) and (H-19) are set equal to each other and the equilibrant moment $M_{o44}$ (For case 4: RFR) is determined

$$M_{o44} = M_{in} \frac{c_{2}c_{18}c_{24}}{c_{10}c_{16}c_{22}} - Q_1 \frac{c_{2}c_{9}c_{18}c_{24}}{c_{10}c_{16}c_{22}} - Q_2 \frac{c_{2}c_{18}c_{23}}{c_{16}c_{22}} - Q_3 \frac{c_{2}c_{17}}{c_{16}}$$

$$- Q_4C_1$$

(H-180)
For this contact combination it is necessary to determine new expressions for the force $F_{34F}$ of gear no. 3 on pinion no. 4, and for force $F_{23F}$ of gear no. 2 on pinion no. 3.

Equation (H-156), derived for case 3, and which relates force $F_{23F}$ to the input moment $M_{in}$, may be used for the determination of the final input-output relationship.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 4

Figure H-10 gives the free body diagram of pinion no. 4 in the round on flat phase of motion with gear no. 3.

The equilibrant moment $M_{64}$ acts in a clockwise direction and opposes the counter-clockwise rotation of the pinion.

The normal contact force $F_{34F}$ is given by

$$F_{34F} = F_{34F}^n F_{34F}$$

(H-181)
Figure H-10
Free Body Diagram of Pinion No. 4
Mesh No. 3: Round on Flat
The associated friction force becomes

\[ F_{34F} = \mu S_{3F} F_{34F} \times F_3 \]  \hspace{1cm} (H-182)

The centrifugal force \( \ddot{Q}_4 \) is given by equation (H-5) and the pivot friction forces are chosen such that they oppose rotation.

Force equilibrium is given by

\[ F_{34F} \times F_3 + \mu S_{3F} F_{34F} \times F_4 \times (\dot{F}_{y4} - \mu F_{y4}) + \mu F_{x4} \times \dot{F}_{y4} + \mu F_{x4} \times \dot{F}_{x4} \]

\[ + Q_4 (\cos y_4 \times \dot{F}_{y4} + \sin y_4 \times \dot{F}_{x4}) = 0 \]  \hspace{1cm} (H-183)

Moment equilibrium about point \( O_4 \) requires that

\[ -M_{x4} \times (\dot{F}_{x4} + \dot{F}_{y4}) + \mu S_{3F} \times F_{34F} \times F_3 = 0 \]  \hspace{1cm} (H-184)

Note that the friction force \( F_{34F} \) does not exert a moment about point \( O_4 \), since its line of action passes through it.

Equation (H-183) furnishes the following component equations

\[ -F_{34F} \sin (\psi_3 - \alpha_3) + \mu S_{3F} F_{34F} \cos (\psi_3 - \alpha_3) + F_{x4} = \mu F_{y4} \]

\[ + Q_4 \cos y_4 = 0 \]  \hspace{1cm} (H-185)
and

\[ F_{34F} \cos(\phi_3 - a_{p3}) + \mu s_{3F} F_{34F} \sin(\phi_3 - a_{p3}) + F_{y4} + \mu F_{x4} + Q_4 \sin \gamma_4 = 0 \]  

(H-186)

The scalar form of the moment equation (H-184) becomes

\[ -M_{04} = \mu^4 (F_{x4} + F_{y4}) + \varepsilon_3 F_{34F} = 0 \]  

(H-187)

Simultaneous solution of equations (H-185) and (H-186) for \( F_{x4} \) and \( F_{y4} \) results in

\[ F_{x4} = \frac{1}{1 + \mu^2} \left\{ F_{34F} \left[ (1 - \mu^2 s_{3F}) \sin(\phi_3 - a_{p3}) - \mu (1 + s_{3F}) \cos(\phi_3 - a_{p3}) \right] 
+ Q_4 \left[ -\mu \sin \gamma_4 - \cos \gamma_4 \right] \right\} \]  

(H-188)

and

\[ F_{y4} = \frac{1}{1 + \mu^2} \left\{ F_{34F} \left[ -\mu (1 + s_{3F}) \sin(\phi_3 - a_{p3}) - (1 - \mu^2 s_{3F}) \cos(\phi_3 - a_{p3}) \right] 
+ Q_4 \left[ -\sin \gamma_4 + \mu \cos \gamma_4 \right] \right\} \]  

(H-189)
The sum \( \tilde{F}_{x4} + \tilde{F}_{y4} \) of equation (H-187) is now made up of equations (H-188) and (H-189) in the sense of equation (H-3b)

\[
\tilde{F}_{x4} + \tilde{F}_{y4} = F_{34F}A_{49} + Q_{4}A_{50} + F_{34F}A_{51} + Q_{4}A_{52}
\]

(H-190)

where

\[
A_{49} = \frac{(1 - \mu^2 \alpha_3) \sin(\psi_3 - \alpha_p^3) - \mu(1 + \alpha_3) \cos(\psi_3 - \alpha_p^3)}{1 + \mu^2}
\]

(H-191)

\[
A_{50} = \frac{-\sin \gamma_4 - \cos \gamma_4}{1 + \mu^2}
\]

(H-192)

\[
A_{51} = \frac{-\mu(1 + \alpha_3) \sin(\psi_3 - \alpha_p^3) - (1 - \mu^2 \alpha_3) \cos(\psi_3 - \alpha_p^3)}{1 + \mu^2}
\]

(H-193)

\[
A_{52} = \frac{-\sin \gamma_4 + \mu \cos \gamma_4}{1 + \mu^2}
\]

(H-194)

Equation (H-190) is now substituted into equation (H-187) and the result is solved for \( F_{34F} \)

\[
F_{34F} = \frac{M_{04} + Q_{4} \omega_{25}}{\zeta_{26}}
\]

(H-195)

H-71
where

\[ c_{25} = \mu_4 (A_{50} + A_{52}) \]

\[ c_{26} = B_3 - \mu_4 (A_{49} + A_{51}) \]
II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-11 gives the free body diagram of gear and pinion combination no. 3. Both mesh 3 and mesh 2 are in their round on flat phase of contact.

The forces of pinion 4 on gear 3 are equal to, but opposite in direction to, those given by equations (H-181) and (H-182)

\[
F_{43F} = - F_{34F} \quad (H-196)
\]

and

\[
F_{f43F} = - \mu_{3F} F_{34F} \quad (H-197)
\]

The forces of gear 2 on pinion 3 are given by

\[
F_{23F} = - F_{23F} \quad (H-198)
\]

and

\[
F_{f23F} = - \mu_{2F} F_{23F} \quad (H-199)
\]
FIGURE H-11
FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON FLAT
MESH NO. 2: ROUND ON FLAT
The pivot forces and moments are chosen in the usual manner and the centrifugal force $\bar{Q}_3$ is defined by equation (H-24).

Force equilibrium is given by

$$-F_{34F_3} - \mu_3 F_{34F_3} - F_{23F_3} + \mu_2 F_{23F_3} + \bar{Q}_3 + F_{x3}$$

$$+ \mu F_{y3} + F_{y3} - \mu F_{x3} = 0$$  \hspace{1cm} (H-200)

Moment equilibrium about point $O_3$ requires that

$$\mu_3 (F_{x3} + F_{y3}) + (a_3 \bar{a}_3 + \rho a_3 \bar{a}_3) \times (-F_{34F_3} - \mu a_3 F_{34F_3})$$

$$+ s_2 \bar{a}_2 + (-)F_{23F_3} = 0$$  \hspace{1cm} (H-201)

Note that the friction force $F_{23F}$ does not exert a moment about point $O_3$.

Equation (H-200) furnishes, after all necessary substitutions, the following component equations:

$$F_{34F} \sin(\psi_3 - \alpha_{p3}) - \mu_3 F_{34F} \cos(\psi_3 - \alpha_{p3}) + Q_3 \cos \gamma_3 + F_{x3} + \mu F_{y3}$$

$$+ F_{x3} \sin(\psi_2 + \alpha_{p2}) + \mu_2 F_{23F} \cos(\psi_2 + \alpha_{p2}) = 0$$  \hspace{1cm} (H-202)
and

\[-F_{34F}\cos(\psi_3 - a_3) - \mu s_{3F}F_{34F}\sin(\psi_3 - a_3) + Q_3 \sin \gamma_3 + F_y - \mu F_x \]

\[-F_{23F}\cos(\psi_2 + a_2) + \mu s_{2F}F_{23F}\sin(\psi_2 + a_2) = 0 \quad (H-203)\]

The scalar form of the moment equation (H-201) becomes

\[\mu \rho_3(F_{x3} + F_{y3}) + \mu s_{3F}F_{34F}[-\cos(\psi_3 + a_3 - \psi_3 + a_3)] + \mu s_{3F}F_{34F} - \mu s_{2F}F_{23F} = 0 \quad (H-204)\]

Simultaneous solution of the component equations (H-202) and (H-203)

for \(F_{x3}\) and \(F_{y3}\) gives

\[F_{x3} = \frac{1}{1 + \mu^2} \left\{ F_{34F}[-(1 + \mu^2 s_{3F})\sin(\psi_3 - a_3) + \mu(s_{3F} - 1)\cos(\psi_3 - a_3)] \right\}

+ \frac{F_{23F}[(\mu^2 s_{2F} - 1)\sin(\psi_2 + a_2) - \mu(1 + s_{2F})\cos(\psi_2 + a_2)] + Q_3 [\mu \sin \gamma_3 - \cos \gamma_3]}{\mu s_{2F}F_{23F} - \mu s_{2F}F_{23F}} \quad (H-205)\]

H-76
and

\[ F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{34F} \left[ \mu(s_{3F} - 1)\sin(\psi_3 - \alpha_3) + (1 + \mu^2 s_{3F})\cos(\psi_3 - \alpha_3) \right] 
+ F_{23F} \left[ -\mu(1 + s_{2F})\sin(\psi_2 + \alpha_2) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_2) \right] 
+ Q_3 \left[ -\sin\gamma_3 - \mu\cos\gamma_3 \right] \right\} \]  

(H-206)

The sum \( F_{x3} + F_{y3} \) is now made up from equations (H-205) and (H-206)

\[ F_{x3} + F_{y3} = F_{34F}A_{53} + F_{23F}A_{54} + Q_3A_{55} + F_{34F}A_{56} + F_{23F}A_{57} + Q_3A_{58} \]  

(H-207)

where

\[ A_{53} = \left| \frac{-(1 + \mu^2 s_{2F})\sin(\psi_3 - \alpha_3) + \mu(s_{3F} - 1)\cos(\psi_3 - \alpha_3)}{1 + \mu^2} \right| \]  

(H-208)

\[ A_{54} = \left| \frac{(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_2) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_2)}{1 + \mu^2} \right| \]  

(H-209)

\[ A_{55} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \]  

(H-210)
\[
A_{56} = \frac{\mu(a_{3F} - 1)\sin(\gamma_3 - a_{P3}) + (1 + \mu^2 a_{3F})\cos(\gamma_3 - a_{P3})}{1 + \mu^2} \quad (H-211)
\]

\[
A_{57} = \frac{-\mu(1 + a_{2F})\sin(\gamma_2 + a_{P2}) + (1 - \mu^2 a_{2F})\cos(\gamma_2 + a_{P2})}{1 + \mu^2} \quad (H-212)
\]

\[
A_{58} = \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \quad (H-213)
\]

Equation (H-207) is now substituted into equation (H-204).

The result is solved for \( F_{23F} \)

\[
F_{23F} = \frac{-F_{34F}C_{27} - Q_{3028}}{C_{29}} \quad (H-214)
\]

where

\[
C_{27} = \mu_F(a_{53} + A_{56}) + a_{G3}[\cos(\gamma_3 + \delta_{G3} - \gamma_3 + a_{P3})
+ \mu a_{3F}\sin(\gamma_3 + \delta_{G3} - \gamma_3 + a_{P3})] + \mu a_{3F}^2 \delta_{G3}
\]

\[
C_{28} = \mu_F(a_{53} + A_{58})
\]

\[
C_{29} = \mu_F(a_{54} + A_{57}) - \delta_2
\]

H-78
III. MOMENT INPUT-OUTPUT RELATIONSHIP

Equation (H-156) is an expression for $F_{23F}$ as a function of the moment $M_{in}$ and the centrifugal forces $Q_1$ and $Q_2$, when both meshes no. 1 and no. 2 are in the round on flat phase of motion. This expression for $F_{23F}$ is now equated to equation (H-214). The result is solved for $F_{34F}$

$$F_{34F} = \frac{C_{21}C_{29}}{C_{15}C_{19}C_{27}} (M_{in} - Q_1C_{14}) - \frac{C_{20}C_{29}}{C_{19}C_{27}} - \frac{C_{28}}{C_{27}}$$  \hspace{1cm} (H-215)

The above is now equated to equation (H-195) and the resulting expression is used to determine the equilibrant moment $M_{045}$ (for Case 5: FFF)

$$M_{045} = M_{in} \frac{C_{21}C_{26}C_{29}}{C_{15}C_{19}C_{27}} - Q_1 \frac{C_{14}C_{21}C_{26}C_{29}}{C_{19}C_{27}} - Q_2 \frac{C_{20}C_{26}C_{29}}{C_{19}C_{27}}$$

$$- Q_3 \frac{C_{26}C_{28}}{C_{27}} - Q_4C_{25}$$  \hspace{1cm} (H-216)

H-79
MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for this contact combination can be assembled entirely from previously derived component relationships. As for case no. 4, mesh no. 1 is in the round on round phase while mesh no. 2 is in the round on flat phase. Therefore, equation (H-178), which relates the force $F_{23F}$ to the input moment $M_{in}$, may be used. The input-output relationship of the gear and pinion set no. 3, i.e. the relationship between the forces $F_{34F}$ and $F_{23F}$, is given by equation (H-214) of case no. 5. The force $F_{34F}$ may be obtained from equation (H-195). This expression was also derived for a round on flat contact in case no. 5.

Thus, equation (H-178) is first set equal to equation (H-214) and the result is solved for the force $F_{34F}$

$$F_{34F} = \frac{C_{24}C_{29}}{C_{10}C_{22}C_{27}} (M_{in} - Q_{1}C_{9}) - Q_{2} \frac{C_{23}C_{29}}{C_{22}C_{27}} - Q_{3} \frac{C_{28}}{C_{27}}$$  \hspace{1cm} (H-217)
The above expression is now set equal to equation (H-195).

This then allows the determination of the equilibrant moment $M_{046}$ for the present contact combination.

\[
M_{046} = M_{1n} \frac{c_{24}c_{26}c_{29}}{c_{10}c_{22}c_{27}} - q_1 \frac{c_{9}c_{24}c_{26}c_{29}}{c_{10}c_{22}c_{27}} - q_2 \frac{c_{23}c_{26}c_{29}}{c_{22}c_{27}} - q_3 \frac{c_{26}c_{28}}{c_{27}} - q_4 c_{25}
\]  

(H-218)
For the present contact combination the expression for force $F_{34F}$ may be taken over from equation (H-195) of case no. 5. The input-output relationship of gear and pinion set no. 3, which relates the forces $F_{34F}$ and $F_{23}$, must be newly derived. The relationship between force $F_{23}$ and the input moment $M_{in}$ is taken from case no. 1 in the form of equation (H-79).

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-12 gives the free body diagram of gear and pinion set no. 3. Mesh no. 3 is in the round on flat phase of contact, while mesh no. 2 is in the round on round one.

The forces of pinion no. 4 on gear no. 3 are equal to, but opposite in direction to those given by equations (H-181) and (H-182)

$$\bar{F}_{43F} = -F_{34F} \bar{F}_{NF3} \quad (H-219)$$
FIGURE H-12
FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON FLAT
MESH NO. 2: ROUND ON ROUND

H-83
and

\[ \bar{F}_{f43F} = -\mu_3F_3F_{34F}F_3 \]  

(H-220)

The forces of gear no. 2 on pinion no. 3 are given by

\[ \bar{F}_{23} = F_{23\overline{\lambda}2} \]  

(H-221)

and

\[ \bar{F}_{f23} = \mu_{2R}F_{23\overline{\lambda}2} \]  

(H-222)

The pivot reactions are chosen in the same manner as before. The centrifugal force \( \overline{Q}_3 \) was defined by equation (H-24).

Force equilibrium of the gear set requires

\[ -F_{34F}F_{NF3} - \mu_3F_3F_{34F}F_{T3} + F_{23\overline{\lambda}2} + \mu_{2R}F_{23\overline{\lambda}2} + \overline{Q}_3 + F_{x3I} + \mu F_{y3I} + F_{y3J} - \mu F_{x3J} = 0 \]  

(H-223)
Moment equilibrium about point $O_3$ is given by

\[ \mu_3 (\mathbf{F}_x + \mathbf{F}_y) \mathbf{R} + [a_{3R}G_3 + \rho G_3 NF_3] \times [-F_{34F}NF_3 - \mu_3 F_{34F}F_3] + [a_{p2}p_2 - \rho_{p2}R_{\lambda2}] \times [F_{23}R_{\lambda2} + \mu_2 R_{23}N_{\lambda2}] = 0 \quad (H-224) \]

Equation (H-223) gives the following component expressions

\[ F_{34F}\sin(\psi_3 - \alpha_3) - \mu_3 F_{34F}\cos(\psi_3 - \alpha_3) + Q_3 \cos \gamma_3 + F_x + \mu F_y + F_{23}\cos \lambda_2 - \mu_2 R_{23}\sin \lambda_2 = 0 \quad (H-225) \]

and

\[ -F_{34F}\cos(\psi_3 - \alpha_3) - \mu_3 F_{34F}\sin(\psi_3 - \alpha_3) + Q_3 \sin \gamma_3 + F_y - \mu F_x + F_{23}\sin \lambda_2 + \mu_2 R_{23}\cos \lambda_2 = 0 \quad (H-226) \]
The scalar form of the moment equation (H-224) becomes:

\[ \mu_3 (\bar{F}_{x3} + \bar{F}_{y3}) + a_{03} F_{34F} [-\cos(\phi_3 + \theta_3 - \phi_3 + a_3) + \mu a_{3F} F_{34F} + a_{p2} F_{23} [-\sin(\phi_2 - \lambda_2) + \mu a_{2R} \cos(\phi_2 - \lambda_2)] - \mu a_{2R} F_{23} = 0 \] (H-227)

Simultaneous solution of the component equations (H-225) and (H-226) for \( F_{x3} \) and \( F_{y3} \) results in:

\[ F_{x3} = \frac{1}{1 + \mu^2} \left\{ F_{34F} \left[-(1 + \mu^2 a_{3F}) \sin(\phi_3 - a_3) + \mu (a_{3F} - 1) \cos(\phi_3 - a_3) \right] + F_{23} \left[ \mu (1 + a_{2R}) \sin \lambda_2 + (\mu^2 a_{2R} - 1) \cos \lambda_2 \right] + Q_3 \left[ \delta \sin \gamma_3 - \cos \gamma_3 \right] \right\} \] (H-228)
and

\[ F_{y3} = \frac{1}{1 + \mu^2} \left( F_{34F} \left[ \mu (\varepsilon_{3F} - 1) \sin(\psi_3 - \phi_3) + (1 + \mu^2 \varepsilon_{3F}) \cos(\psi_3 - \phi_3) \right] 
+ F_{23} \left[ (\mu^2 \varepsilon_{2R} - 1) \sin \lambda_2 - \mu (1 + \varepsilon_{2R}) \cos \lambda_2 \right] 
+ Q_3 \left[ -\sin \gamma_3 - \mu \cos \gamma_3 \right] \right) \]  

(H-229)

The sum \( F_{x3} + F_{y3} \) in equation (H-227) is now made up from

equations (H-228) and (H-229) in the sense of equation (A-3b)

\[ F_{x3} + F_{y3} = F_{34F} A_{59} + F_{23} A_{60} + Q_3 A_{61} + F_{34F} A_{62} + F_{23} A_{63} + Q_3 A_{64} \]  

(H-230)

where

\[ A_{59} = \left| \frac{(1 + \mu^2 \varepsilon_{3F}) \sin(\psi_3 - \phi_3) + \mu (\varepsilon_{3F} - 1) \cos(\psi_3 - \phi_3)}{1 + \mu^2} \right| \]  

(H-231)

\[ A_{60} = \left| \frac{\mu (1 + \varepsilon_{2R}) \sin \lambda_2 + (\mu^2 \varepsilon_{2R} - 1) \cos \lambda_2}{1 + \mu^2} \right| \]  

(H-232)

\[ A_{61} = \left| \frac{\mu \sin \gamma_3 - \cos \gamma_3}{1 + \mu^2} \right| \]  

(H-233)

H-87
Equation (H-230) is now substituted into the moment equation (H-227) and the resulting expression is solved for $F_{23}$

$$F_{23} = \frac{F_{34}c_{30} - Q_3c_{31}}{c_{32}}$$

where

$$c_{30} = \mu s_3(A_{39} + A_{62}) - a_3\left[\cos(\phi_3 + \theta_0 - \phi_3 + \phi_3)\right] - \mu s_3\left[\sin(\phi_3 + \theta_0 - \phi_3 + \phi_3)\right] + \mu s_3\phi_3$$

$$c_{31} = \mu s_3(A_{61} + A_{64})$$
\[ c_{32} = \mu p_3(a_{60} + a_{63}) - a_{p2} \left[ \sin(\nu_2 - \theta_2) - \mu a_{2R} \cos(\nu_2 - \theta_2) \right] - \mu a_{2R} p_2 \]

II. MOMENT INPUT-OUTPUT RELATIONSHIP

Equation (H-79), which gives the force \( F_{23} \) in terms of \( M_{in}, Q_1 \), and \( Q_2 \) for the appropriate contact combinations, is now set equal to equation (H-237). Subsequently, one finds the following formulation for \( F_{34F} \):

\[ F_{34F} = \frac{c_8 c_{32}}{c_6 c_{10 c_{30}}} (M_{in} - Q_1 c_{9}) - Q_2 \frac{c_7 c_{32}}{c_6 c_{30}} - Q_3 \frac{c_{31}}{c_{30}} \quad (H-238) \]

The above expression is now set equal to equation (H-195) which gives \( F_{34F} \) in terms of \( M_{04} \) and \( Q_4 \). The determination of the equilibrant moment \( M_{047} \) (for case 7: FRR) is now possible.

Thus,

H-89
\[ M_{047} = M_{1n} \frac{c_8c_{26}c_{32}}{c_6c_{10}c_{30}} - Q_1 \frac{c_8c_9c_{26}c_{32}}{c_6c_{10}c_{30}} - Q_2 \frac{c_7c_{26}c_{32}}{c_6c_{30}} \]

\[-Q_3 \frac{c_{26}c_{31}}{c_{30}} - Q_4 c_{25} \]  

(H-239)
The input-output relationship for this contact combination can also be assembled entirely from existing expressions. With mesh no. 2 in the round on round phase of contact and mesh no. 1 in the round on flat one, the relationship between force $F_{23}$ and the moment $M_{in}$ is that of case no. 2. Equation (H-116) is applicable. The input-output relationship of gear and pinion set no. 3, which relates $F_{34F}$ to $F_{23}$, was derived for case no. 7 and is given by equation (H-237). Finally, with mesh no. 3 in the round on flat phase, one uses equation (H-195) for the relationship between force $F_{34F}$ and moment $M_{04}$. Thus, equation (H-116) is first set equal to equation (H-237) to obtain an expression for force $F_{34F}$

$$F_{34F} = \frac{c_{13}c_{32}}{c_{11}c_{15}c_{30}} (M_{in} - q_1c_{14}) - q_2 \frac{c_{12}c_{32}}{c_{11}c_{30}} - q_3 \frac{c_{31}}{c_{30}} \quad (H-240)$$
The above expression is now set equal to equation (H-195). This allows the determination of $M_{o48}$ (for case 8: FRF)

$$M_{o48} = \min \frac{c_{13}c_{26}c_{32}}{c_{11}c_{15}c_{30}} - q_1 \frac{c_{13}c_{14}c_{26}c_{32}}{c_{11}c_{15}c_{30}} - q_2 \frac{c_{12}c_{26}c_{32}}{c_{11}c_{30}}$$

$$- q_3 \frac{c_{26}c_{31}}{c_{30}} - q_4 c_{25} \quad (H-241)$$
2. INPUT-OUTPUT ANALYSIS OF TWO STEP-UP GEAR TRAIN

The following gives derivations for the moment input-output relationships associated with the four possible contact combinations of a two step-up gear train. (See Table H-2).

a. CASE NO. 1: RR

For this contact combination, the relationship between the equilibrant moment $M_{03}$ and force $F_{23}$, both acting on pinion no. 3, must first be newly obtained. The relationships between forces $F_{23}$ and $F_{12}$ of gear and pinion set no. 2, as well as between force $F_{12}$ and input moment $M_{in}$ of gear no. 1, may be taken from case no. 1 of the three step-up gear train analysis. Equations (H-61) and (H-78), respectively, are applicable.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 3

Figure H-13 shows a schematic free body diagram of pinion no. 3 in the round on round phase of contact. The equilibrant
FIGURE H-13
FREE BODY DIAGRAM OF PINION NO. 3
MESH NO. 2: ROUND ON ROUND

H-94
The normal contact force of gear no. 2 on pinion no. 3 is given by

\[ F_{23} = F_{23} n_2 \]  \hspace{1cm} (H-242)

The associated friction force becomes

\[ F_{f23} = \mu s_{2n} F_{23} n_2 \]  \hspace{1cm} (H-243)

The pivot reactions are chosen in the usual manner.

The centrifugal force \( Q_{3P} \) is now due to the mass \( m_{3P} \) of the pinion alone, i.e.,

\[ Q_{3P} = m_{3P} (\cos \gamma_3 T + \sin \gamma_3 J) \]  \hspace{1cm} (H-244)

where

\[ Q_{3P} = m_{3P} \omega^2 \]  \hspace{1cm} (H-245)
Force equilibrium of the pinion is assured by

\[ F_{23}^\lambda + \mu s_{2R}F_{23}^\lambda \hat{n}_{\lambda 2} + F_{x3}^3 + \mu F_{y3}^3 + \hat{F}_{y3}^3 - \mu F_{x3}^3 + \hat{Q}_{3p} = 0 \]  

\( \text{(H-246)} \)

Moment equilibrium about point \( O_3 \) is given by

\[ \mu \hat{F}_3 (F_{x3}^3 + F_{y3}^3) + M_{o3}^3 + [a_{p2}^2 - p_{p2}^2] \times [F_{23}^\lambda \hat{n}_{\lambda 2} + \mu s_{2R}F_{23}^\lambda \hat{n}_{\lambda 2}] = 0 \]  

\( \text{(H-247)} \)

Equation (H-246) gives the following component expressions

\[ F_{23}^\cos \lambda_2 - \mu s_{2R}F_{23}^\sin \lambda_2 + F_{x3} + \mu F_{y3}^3 + Q_{3p} \cos \gamma^3 = 0 \]  

\( \text{(H-248)} \)

and

\[ F_{23}^\sin \lambda_2 + \mu s_{2R}F_{23}^\cos \lambda_2 + F_{y3} - \mu F_{x3} + Q_{3p} \sin \gamma^3 = 0 \]  

\( \text{(H-249)} \)

H-96
The scalar form of the moment equation (H-247) becomes

\[ \mu \rho (\mathbf{F}_x \mathbf{3} + \mathbf{F}_y \mathbf{3}) + M_0 \mathbf{3} - \mu s_{2R}^p \mathbf{p}_2 \mathbf{F}_2 \mathbf{3} + a_{2R}^p \mathbf{F}_2 \mathbf{3} \left[ -\sin(\psi_2 - \delta_2 - \lambda_2) \right] + \mu s_{2R} \cos(\psi_2 - \delta_2 - \lambda_2) \right] = 0 \]  

(H-250)

Simultaneous solution of equations (H-248) and (H-249) for the forces \( F_x \mathbf{3} \) and \( F_y \mathbf{3} \) gives

\[ F_x \mathbf{3} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ \mu (1 + s_{2R}) \sin \lambda_2 + (\mu^2 s_{2R} - 1) \cos \lambda_2 \right] + Q_{3p} \left[ \mu \sin \gamma_3 - \cos \gamma_3 \right] \right\} \]  

(H-251)

and

\[ F_y \mathbf{3} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (\mu^2 s_{2R} - 1) \sin \lambda_2 + (1 + s_{2R}) \cos \lambda_2 \right] \right. \\
\left. + Q_{3p} \left[ -\sin \gamma_3 - \mu \cos \gamma_3 \right] \right\} \]  

(H-252)
The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ of equation (H-250) is now made up from equations (H-251) and (H-252) in the sense of equation (A-3b).

\[ \tilde{F}_{x3} + \tilde{F}_{y3} = F_{23}A_{65} + Q_{3}pA_{66} + F_{23}A_{67} + Q_{3}pA_{68} \]  

(H-253)

where

\[ A_{65} = \frac{\mu(1 + s_{2R})\sin\lambda_{2} + (\mu^2 s_{2R} - 1)\cos\lambda_{2}}{1 + \mu^2} \]  

(H-254)

\[ A_{66} = \frac{\mu\sin\gamma_{3} - \cos\gamma_{3}}{1 + \mu^2} \]  

(H-255)

\[ A_{67} = \frac{(\mu^2 s_{2R} - 1)\sin\lambda_{2} - \mu(1 + s_{2R})\cos\lambda_{2}}{1 + \mu^2} \]  

(H-256)

\[ A_{68} = \frac{-\sin\gamma_{3} - \mu\cos\gamma_{3}}{1 + \mu^2} \]  

(H-257)

Equation (H-253) is now substituted into the moment equation (H-250) and the result is solved for $F_{23}$

\[ F_{23} = \frac{-M_{o3} - Q_{3}pC_{33}}{C_{34}} \]  

(H-258)
where

\[ C_{33} = \mu_{3} \rho_{3} (A_{65} + A_{66}) \]

\[ C_{34} = \mu_{3} \rho_{3} (A_{65} + A_{67}) - \mu_{2} \rho_{2} \rho_{2} \]

\[ + \ a_{2} \left[ \mu_{2} \rho_{2} \cos(\psi_{2} - \delta_{2} - \lambda_{2}) - \sin(\psi_{2} - \delta_{2} - \lambda_{2}) \right] \]
II. MOMENT INPUT-OUTPUT RELATIONSHIP

In the derivation for case 1 of the three step-up gear train it was shown that if one sets equations (H-61) and (H-78) equal to each other, one obtains the following relationship [i.e., equation (H-79)] between force $F_{23}$ and the input moment $M_{\text{in}}$:

$$F_{23} = -\frac{C_8}{C_6 C_{10}} (M_{\text{in}} - Q_1 C_9) + Q_2 \frac{C_7}{C_6}$$  \hspace{1cm} (H-259)

The above expression is now set equal to equation (H-258), and the result is solved for the equilibrant moment $M_{031}$ (for case 1: RR):

$$M_{031} = M_{\text{in}} \frac{C_6 C_{34}}{C_6 C_{10}} - Q_1 \frac{C_8 C_9 C_{34}}{C_6 C_{10}} - Q_2 \frac{C_7 C_{34}}{C_6} - Q_3 P_{033}$$  \hspace{1cm} (H-260)
The moment equation for the present case, in which mesh no. 1 is in the round on flat phase and mesh no. 2 in the round on round one, may be derived entirely from existing relationships. Equation (H-116) gives an expression for the force $F_{23}$ in terms of the input moment $M_{\text{in}}$ and the centrifugal forces $Q_1$ and $Q_2$ for the present combination of contacts in both meshes. When this expression, from case no. 2 of the three step-up gear train, is set equal to equation (H-259) of case no. 1 of the two step-up gear train, one obtains for $M_{032}$ (case no. 2: RF)

$$M_{032} = M_{\text{in}} \frac{c_{13}c_{34}}{c_{11}c_{15}} - q_1 \frac{c_{13}c_{14}c_{34}}{c_{11}c_{15}} - q_2 \frac{c_{12}c_{34}}{c_{11}} - q_3c_{33}$$

(H-261)
c. CASE NO. 3: FF

For this contact combination, where both meshes no. 1 and no. 2 are in the round on flat phase, the relationship between the equilibrant moment $M_{03}$ and the normal contact force $F_{23F}$, both acting on pinion no. 3, must first be determined. The resulting expression in $F_{23F}$ can then be set equal to the relationship between $F_{23F}$ and the input moment $M_{in}$, which is given by equation (H-156), and which was derived in conjunction with case no. 3 of the three step-up gear train.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 3

Figure H-14 shows the free body diagram of pinion no. 3 in the round on flat phase of contact. Again, the equilibrant moment $M_{03}$ acts in a counter-clockwise direction. The normal force $F_{23F}$ of gear no. 2 on pinion no. 3 is given by

$$F_{23F} = -F_{23F}N_{NF2}$$

(H-262)
FIGURE H-14
FREE BODY DIAGRAM OF PINION NO. 3
MESH NO. 2: ROUND ON FLAT
The associated friction force of gear no. 2 on pinion no. 3 becomes

\[ \vec{F}_{f23F} = \mu s_{2F} \bar{F}_{23F} \bar{F}_2 \]  \hspace{1cm} (H-263)

The pivot reactions and pivot friction forces are chosen in the usual manner. The centrifugal force \( \bar{Q}_{3p} \) was given previously by equation (H-244).

The force equilibrium expression becomes

\[ -\bar{F}_{23F} \bar{F}_{NF2} + \mu s_{2F} \bar{F}_{23F} \bar{F}_2 + F_{x3} \bar{I} + \mu F_{y3} \bar{I} + F_{y3} \bar{J} - \mu F_{x3} \bar{J} + Q_{3p} \]

\[ = 0 \]  \hspace{1cm} (H-264)

Moment equilibrium about point 03 is assured by

\[ \mu \rho_3 (F_{x3} + F_{y3}) \bar{k} + M_{03} \bar{k} + \delta_{2F} \bar{F}_2 \times (-) \bar{F}_{23F} \bar{F}_{NF2} = 0 \]  \hspace{1cm} (H-265)

As always before, the friction force \( \vec{F}_{f23F} \) exerts no moment about point 03.
Equation (H-264) gives the following component expressions

\[ F_{23p}\sin(\psi_2 + \alpha_p) + \mu s_2 F_{23p}\cos(\psi_2 + \alpha_p) + F_{x3} + \mu F_{y3} + Q_p\cos\gamma_3 \]

\[ = 0 \quad \text{(H-266)} \]

and

\[ -F_{23p}\cos(\psi_2 + \alpha_p) + \mu s_2 F_{23p}\sin(\psi_2 + \alpha_p) + F_{y3} - \mu F_{x3} + Q_p\sin\gamma_3 \]

\[ = 0 \quad \text{(H-267)} \]

The scalar form of the moment equation (H-265) is given by

\[ \mu_0^3(F_{x3} + F_{y3}) + M_{03} - \varepsilon_2 F_{23p} = 0 \quad \text{(H-268)} \]

Simultaneous solution of equations (H-266) and (H-267) for \( F_{x3} \) and \( F_{y3} \) leads to

\[ F_{x3} = \frac{1}{1 + \mu^2} \left\{ F_{23p}\left[\left(\mu^2 s_2 - 1\right)\sin(\psi_2 + \alpha_p) - \mu(1 + s_2)\cos(\psi_2 + \alpha_p)\right] \right. \\
\left. + Q_p\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \quad \text{(H-269)} \]

H-105
\[ F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{23F} \left[ -\mu(1 + s_2F)\sin(\psi_2 + \alpha_{p2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{p2}) \right] 
+ Q_{3P} \left[ -\sin \gamma_3 - \mu \cos \gamma_3 \right] \right\} \]  
\[ \text{(H-270)} \]

The sum \( \bar{F}_{x3} + \bar{F}_{y3} \) of equation (H-268) is now made up of equations (H-269) and (H-270) in the sense of equation (A-3b)

\[ \bar{F}_{x3} + \bar{F}_{y3} = F_{23F} A_{69} + Q_{3P} A_{70} + F_{23F} A_{71} + Q_{3P} A_{72} \]  
\[ \text{(H-271)} \]

where

\[ A_{69} = \frac{(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_{p2}) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_{p2})}{1 + \mu^2} \]  
\[ \text{(H-272)} \]

\[ A_{70} = \frac{\mu \sin \gamma_3 - \cos \gamma_3}{1 + \mu^2} \]  
\[ \text{(H-273)} \]

\[ A_{71} = \frac{-\mu(1 + s_{2F})\sin(\psi_2 + \alpha_{p2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{p2})}{1 + \mu^2} \]  
\[ \text{(H-274)} \]

\[ A_{72} = \frac{-\sin \gamma_3 - \mu \cos \gamma_3}{1 + \mu^2} \]  
\[ \text{(H-275)} \]
Equation (H-271) is now substituted into the moment equation (H-268), and the result is solved for \( F_{23F} \)

\[
F_{23F} = \frac{-M_03 - Q_3pC_{35}}{C_{36}}
\]  

(H-276)

where

\[
C_{35} = \mu p_3(A_{70} + A_{72})
\]

\[
C_{36} = \mu p_3(A_{69} + A_{71}) - \sigma_2
\]

II. MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for the present case is obtained, as stated earlier, by setting equation (H-276) equal to equation (H-156) and solving the resulting expression for the equilibrant moment \( M_{033} \) (case no. 3: FF)

\[
M_{033} = M_{in} \frac{C_{21}C_{36}}{C_{15}C_{19}} - q_1 \frac{C_{14}C_{21}C_{36}}{C_{15}C_{19}} - q_2 \frac{C_{20}C_{36}}{C_{19}} - q_3pC_{35}
\]

(H-277)

H-107
d. CASE NO. 4: FR

MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for this case, where mesh no. 1 is in round on round contact while mesh no. 2 is in the round on flat phase, may also be derived entirely by assembling existing relationships.

Equation (H-276), of the previous section, gives force $F_{23F}$ in terms of the equilibrant moment $M_{03}$ when mesh no. 2 is in the round on flat phase. Equation (H-178), derived for case no. 4 (RFR) of the three step-up gear train, relates this force $F_{23F}$ to the input moment $M_{in}$.

Thus, one first sets equations (H-178) and (H-276) equal to each other and then solves the result for $M_{034}$ (case 4: FR)

\[
M_{034} = M_{in} \frac{C_{24}C_{36}}{C_{10}C_{22}} - Q_1 \frac{C_9C_{24}C_{36}}{C_{10}C_{22}} - Q_2 \frac{C_{23}C_{36}}{C_{22}} - Q_3pC_{35}
\]

(H-278)
APPENDIX I

COMPUTER MODELS FOR THREE AND TWO STEP-UP GEAR TRAINS
WITH CLOCK TEETH OPERATING IN A SPIN ENVIRONMENT

The following appendix contains descriptions, listings and sample outputs of the computer models relating to step-up gear trains containing clock (ogival) gear teeth:

1. Program CLOCK 3: Point and cycle efficiencies for three pass clock (ogival) step-up gear train in spin environment.

2. Program CLOCK 4: Point and cycle efficiencies for two pass clock (ogival) step-up gear train in spin environment.

The relevant background, the input parameters, the manner of the computations and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.
1. **Program CLOCK 3: Point and Cycle Efficiencies for Three Pass Clock (Ogival) Step-Up Gear Train in Spin Environment**

The kinematics of program CLOCK 3 is based on the work in Appendix G, while the moment input-output relationships are derived in section 1 of Appendix H. The nomenclature of the program is chosen to coincide as much as possible with the above appendices. It is to be noted that, even though the fuze related geometry produces different expressions for the various meshes, the kinematic computations of the individual meshes are very similar to those shown in CLOCK 1 in Appendix F for the single mesh in the standard position. It is also assumed that all three meshes will have been tested by program CLOCK 1 for their geometric suitability, i.e., whether there is enough room for tip radii.

a. **Input Parameters (see Program CLOCK 3, below)**

The following parameters represent the input data for the program (for explanations and nomenclature, see sections 1 and 2 of Appendix F, as well as section 3 of Appendix C):

- $\mu$, coefficient of friction, as before
- RPM, spin velocity
- $\text{CAPRP}_1, \text{CAPRP}_2, \text{CAPRP}_3, \text{RP}_2, \text{RP}_3, \text{RP}_4$, pitch radii of gears and pinions with nomenclature of Fig. G-1
- $\text{RHOG}_1, \text{RHOG}_2, \text{RHOG}_3, \text{RHOP}_1, \text{RHOP}_2, \text{RHOP}_3$, radii of curvature of circular arc portion of gear and pinion teeth
ACG1, ACG2, ACG3* = aCG
distance from the center of rotation of the gear of the i\textsuperscript{th} mesh to the center of curvature of the circular arc portion of the gear tooth. (Unless otherwise noted, this and all following numbering schemes refer to those associated with the mesh mechanics as given in the text of Appendices G and H.)

ACP1, ACP2, ACP3 = aCP
distance from the center of rotation of the pinion of the i\textsuperscript{th} mesh to the center of curvature of the circular arc portion of the pinion tooth

R1, R2, R3, R4 = R\textsubscript{1} (nomenclature of Fig. G-1)

TG1, TG2, TG3, TP1, TP2, TP3, maximum thickness of gear and pinion teeth (mesh nomenclature)

NG1, NG2, NG3, NP2, NP3, NP4, numbers of teeth in various gears and pinions (nomenclature of Fig. G-1)

RHO1, RHO2, RHO3, RHO4, gear and/or pinion pivot radii (nomenclature of Fig. G-1)

M1, M2, M3, M4, masses of gear and/or pinion combinations

MD, see program INVOL 3

*Since many parts of the computer program were written before the nomenclature for these distances was changed in the report from \( a_{CG} \) and \( a_{CP} \) to \( a_{G} \) and \( a_{P} \), there is a certain discrepancy between the program and the report.
K. range divisor

PHDOT1 = -1, all velocity computations are based on this value. The input motion in the fuze gearing model is negative (see Fig. (G-1)).

b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The tooth parameters of the gears and pinions of all three meshes are first computed. These computations are essentially the same as those shown in program CLOCK 1 for a single mesh. Certain parameters are omitted because they have been checked separately by using CLOCK 1 and are not required for the kinematics of CLOCK 3.

In addition, the pivot to pivot distances B1, B2 and B3 are obtained.

II. Computation of MIN, GAMMAS and BETAS

To begin with, the program computes the input moment

\[ MIN = M_{in} = md^2 \]  \hspace{1cm} (I-1)

Subsequently, the angles \( \gamma_2, \gamma_3, \gamma_4 \) and \( \beta_1, \beta_2, \beta_3 \) are established according to the expressions of section 6b of Appendix A.

III. Computation of Other Parameters

The angles \( \Delta \psi_1 \) and \( \Delta \psi_1 \) between the centerlines of adjacent gear and pinion teeth, respectively, are determined in this
section of the computations. In addition, the lengths $L_1$ are found (see eqs. (G-7), (G-53) and (G-88)). Finally, the centrifugal forces $Q_1$, $Q_2$, $Q_3$ and $Q_4$ are computed according to eqs. (H-65), (H-46), (H-25) and (H-6), respectively.

IV. Preliminary Computations for Mesh 1

A. Determination of Transition Angle

The primary consideration for determining the transition angles in the fuze related clock gear meshes is identical with that used in Appendix F. The transition angle $\psi_T$ is established as that angle for which, depending upon whether the input angle $\psi$ has counterclockwise or clockwise motion, a small increase or decrease in $\psi$, respectively, will cause the associated value of $g$ to become smaller than its transition value $f_p$. Since the gear of mesh 1 turns in a clockwise direction, the above increment of $\psi$ will be negative.

The program uses this criterion in the following manner:

(a) Transition angles $\psi_{1T1}$ and $\psi_{1T2}$ are computed according to eq. (G-39).

(b) The subroutine TRANS1 (which is valid for meshes in which the input gear has clockwise rotation, as is the case also for mesh 3) is called, and the angle $\phi_{1T1}$ ($\Phi_{1T}$), which is associated with $\psi_{1T1}$, is computed with the help of eqs. (G-40) and (G-41).

(c) The angle $\phi$ is made slightly smaller than $\psi_{1T1}$ to produce the angle $\Phi_{NEXT}$, and eq. (G-29) is used to find the associated angle $\Phi_{SINEX}$. Since there are two such angles, the
subroutine selects the one which is closest in value to the transition angle \( \psi_{1T_1} \). Subsequently, the associated value of \( g_{11} \) is computed according to eq. (G-27).

(d) Steps (b) and (c) are then repeated identically for the second transition angle \( \psi_{1T_2} \). This results in the determination of \( g_{12} \).

(e) Control returns to the main program, and that value of \( \psi_{1T} \) is chosen for which the associated value of \( g_1 \) is smaller than \( f_{p1} \).

For checking, a subsidiary test, which is similar to the one shown in Appendix F, is added to the program. It is based on the idea that for the correct transition angle \( \psi_{1T} \), the line representing the flat portion of the pinion will make a smaller angle with the centerline \( O_1O_2 \) than will be the case for the incorrect one. TEST11 and TEST12 find these angles with the help of the expressions shown below. These expressions hold for all values of \( \beta_1 \) and make use of a new variable \( \psi_{test} \), which had to be introduced since the tests require that the transition angles be expressed in a range between \(-180^\circ\) and \(+180^\circ\). Thus,

\[
\text{For } 0^\circ < \psi_{test} < 180^\circ \\
\text{TEST11} = |\pi - \beta_1 + \psi_{test} - \alpha P_1| \quad (I-2)
\]

\[
\text{For } -180^\circ < \psi_{test} < 0^\circ \\
\text{TEST12} = |\pi + \beta_1 - (\psi_{test} + 2\pi - \alpha P_1)| \quad (I-3)
\]
To determine the angle $\psi_{\text{test}}$, let

$$\psi_{\text{test}} = \psi_1T \text{ if } -180^\circ < \psi_1T < 180^\circ \quad (I-4)$$

$$\psi_{\text{test}} = \psi_1T + 2\pi \text{ if } \psi_1T < -180^\circ \quad (I-5)$$

$$\psi_{\text{test}} = \psi_1T - 2\pi \text{ if } \psi_1T > 180^\circ \quad (I-6)$$

B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-29), for the round on flat regime, is determined with the help of $\varphi_1T$. The condition yielding that angle $\psi_1F$ which is closest to the angle $\psi_1T$ governs. The variable $\text{SIGNIF}$ is used for the sign in question.

C. Computation of Final and Initial Values of $\varphi_1$ and $\psi_1$

The final and initial values of the gear and pinion angles $\varphi_1$ and $\psi_1$, respectively, are found by continuously evaluating the round on flat regime eq. (G-29), using the previously determined value of $\text{SIGNIF}$, and simultaneously checking the contact condition for the subsequent set of teeth as given by eq. (G-46). This loop is initiated at the transition angle $\varphi_1T$, and it is terminated when the condition of eq. (G-46) is met. This allows the determination of the angles $\varphi_{1\text{I}}F$ and $\psi_{1\text{I}}F$, at which the first set of teeth loses contact, as well as of the angles $\varphi_{1\text{II}}$ and $\psi_{1\text{II}}$ at which the second set of teeth simultaneously comes into engagement. The initial engagement angles $\varphi_{1\text{I}}$ and $\psi_{1\text{I}}$ are obtained by adding $\varphi_1$ to the "loss of contact" angle $\varphi_{1\text{I}}F$. 

I-7
and by subtracting $\Delta\psi_1$ from the "loss of contact" angle PSIIFF.

D. Determination of Correct Sign for Round on Round Regime

Eq. (G-12) is used to determine the angle $\psi_1$, while the gear and pinion are in the round on round regime. The correct sign for this expression is obtained by comparing the value $\psi_1$, as computed with PHII, with the value for PSII. SIGN1R is the variable used for the desired sign.

V. Preliminary Computations for Mesh 2

A. Determination of Transition Angle

The primary criterion for determining the transition angle is again similar to that used in Appendix F and described earlier for mesh 1.

(a) Transition angles $\psi_{2T1}$ and $\psi_{2T2}$ are computed according to eq. (G-79).

(b) The subroutine TRANS2, which is valid for meshes in which the input gear has counterclockwise rotation, is called, and the angle $\psi_{2T1}$, which is associated with $\psi_{2T1}$ is computed, with the help of eqs. (G-80) and (G-81).

(c) The angle $\varphi_2$ is made slightly larger than $\varphi_{2T1}$ to produce the angle PHINEXT, and eq. (G-71) is used to find the associated output angle PSINEX. Since there are two such angles, the subroutine selects the one which is closest to the transition value $\psi_{2T1}$. Subsequently, the associated value of $\varphi_{21}$ is computed according to eq. (G-69).
(d) Steps (b) and (c) are then repeated identically for the second transition angle $\phi_{2T2}$. This results in the determination of $g_{22}$.

(e) Control returns to the main program, and that value of $\phi_{2T}$ is chosen for which the associated value of $g_{2}$ is smaller than $f_p$. The procedure for the associated subsidiary test for the transition angle is similar to that for mesh 1 and is given by

For $0^\circ < \phi_{test} < 180^\circ$

$$TEST21 = |\pi - \beta_2 + \phi_{test} + \alpha_{p2}|$$  \hspace{1cm} (I-7)

For $-180^\circ < \phi_{test} < 0^\circ$

$$TEST22 = |\beta_2 + \pi - (\phi_{test} + 2\pi + \alpha_{p2})|$$  \hspace{1cm} (I-8)

To determine the angle $\phi_{test}$, let

$\phi_{test} = \phi_{2T}$ if $-180^\circ < \phi_{2T} < 180^\circ$  \hspace{1cm} (I-9)

$\phi_{test} = \phi_{2T} + 2\pi$ if $\phi_{2T} < -180^\circ$  \hspace{1cm} (I-10)

$\phi_{test} = \phi_{2T} - 2\pi$ if $\phi_{2T} > 180^\circ$  \hspace{1cm} (I-11)

B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-71), for the round on flat regime, is determined with the help of $\phi_{2T}$. The condition yielding that angle $\phi_{2F}$ which is closest to the angle $\phi_{2T}$ governs. The variable SIGN2F is used for the sign in question.
C. Computation of Final and Initial Values of $\varphi_2$ and $\varphi_2$

The final and initial values of the gear and pinion angles $\varphi_2$ and $\varphi_2$, respectively, are found by continuously evaluating the round on flat eq. (G-7i), using the previously determined value of SIGN2F, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (G-86). This loop is initiated at the transition angle $\varphi_2T$ and is terminated when the condition of eq. (G-86) is met. (Recall that in meshes 1 and 3 the driving gear turns clockwise, while in mesh 2 it turns in a counterclockwise direction.) This allows the determination of the two angles PHI2F and PSI2FF at which the first set of teeth loses contact as well as of the angles PHI2I and PSI2I at which the second set of teeth simultaneously comes into contact. The initial engagement angles PHI2I and PSI2I are obtained by subtracting $\varphi_2$ from the "loss of contact" angle PHI2F and by adding $\varphi_2$ to the "loss of contact" angle PSI2FF.

D. Determination of Correct Sign for Round on Round Regime

Eq. (G-58) is used to determine the angle $\varphi_2$ while the gear and pinion are in the round on round phase of motion. The correct sign for this expression is obtained by comparing the value of $\varphi_2$, as computed with PHI2I, with the previously obtained value for PSI2I. SIGN2R is the variable used for the desired sign.
VI. Preliminary Computations for Mesh 3

A. Determination of Transition Angle

The determination of the transition angles for mesh 3 runs along parallel lines to the one shown for mesh 1 since the driving gear also rotates in a clockwise direction. In all cases, the parameters of section 3 of Appendix C are used.

(a) Transition angles $\varphi_{3T1}$ and $\varphi_{3T2}$ are computed with the help of eq. (G-99).

(b) The subroutine TRANS1 determines the angle $\varphi_{3T1}$, associated with $\varphi_{3T1}$, according to eqs. (G-100) and (G-101).

(c) PHINEXT, which is now obtained by a decrease of the angle $\varphi_{3}$ from $\varphi_{3T1}$, serves as the input variable of eq. (G-94), and is used to determine PSINEX. Appropriate controls, as described before, determine the angle $\varphi_{3T1}$. In addition, the associated value of $g_{31}$ is computed with the help of eq. (G-95).

(d) Steps (b) and (c) are again repeated for the second transition angle $\varphi_{3T2}$ and $g_{22}$ is determined.

(e) After control is returned to the main program, that value of $\varphi_{3T}$ is chosen for which the associated value of $g_{3}$ is smaller than $f_{p3}$.

The subsidiary test for the transition angles runs parallel to that described for mesh 1, i.e.,

For $0^\circ < \phi_{test} < 180^\circ$

\[
\text{TEST31} = |\pi - \beta_{3} + \phi_{test} - \alpha_{p3}|
\]  

(I-12)
For $-180^° < \phi_{test} < 0^°$

$$TEST32 = |\pi + \beta_3 - (\phi_{test} + 2\pi - \alpha p_3)| \quad (I-13)$$

To determine the angle $\phi_{test}$ let

$$\phi_{test} = \phi_3T \text{ if } -180^° < \phi_3T < 180^° \quad (I-14)$$

$$\phi_{test} = \phi_3T + 2\pi \text{ if } \phi_3T < -180^° \quad (I-15)$$

$$\phi_{test} = \phi_3T - 2\pi \text{ if } \phi_3T > 180^° \quad (I-16)$$

B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-94), for the round on flat regime, is determined with the help of the angle $\phi_3T$. The condition yielding that angle $\phi_3F$ which is closest to the angle $\phi_3T$ will govern. The variable SIGN3F is used for the sign in question.

C. Computations of Final and Initial Values of $\phi_3$ and $\phi_3$

The final and initial values of the gear and pinion angles $\phi_3$ and $\phi_3$, respectively, are found by continuously evaluating the round on flat regime eq. (G-94), using the previously determined value of SIGN3F, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (G-102). This loop is initiated at the transition angle $\phi_3T$, and it is terminated when the condition of eq. (G-102) is met. This allows the determination of the two angles PHI3F and PSI3FF at which the first set of teeth loses contact as well.
as the angles PHI3I and PSI3I at which the second set of teeth simultaneously comes into contact. The initial engagement angles PHI3I and PSI3I are obtained by adding $\Delta \varphi_3$ to the "loss of contact" angle PHI3F and by subtracting $\Delta \varphi_3$ from the "loss of contact" angle PSI3FF.

D. Determination of Correct Sign for Round on Round Regime

Eq. (G-87) is used to determine the angle $\varphi_3$ while the gear and pinion are in the round on round phase of motion. The correct sign for this expression is obtained by comparing the value of $\varphi_3$, as computed with PHI3I, with the previously obtained value for PSI3I. SIGN3R is the variable used for the desired sign.

VII. Gear Train Motion Model: Kinematics, Point and Cycle Efficiency

The simulation of the gear train model, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop, starting with statement label no. 29 (card no. 458) and ending with card no. 812. The motions of the individual driving gears are initialized at their respective angles PHI1I, PHI2I and PHI3I. (This again is arbitrary.) The meshes will be in round on round contact until they reach their respective transition angles PHI1T, PHI2T and PHI3T. Once the transition angles are passed, the meshes will be in round on flat contact. These regimes continue until the final angles PHI1F, PHI2F and PHI3F are reached.

The increment DDPHI1 of the angle PHI1 of the input gear 1
is obtained from an adaptation of eqs. (A-211) and (A-213), in which tooth numbers, rather than base circle radii, are used. The increment $DD\Phi I_2$ of gear 2 is related to the increment of the pinion angle $PSI_1$. Similarly, the increment $DD\Phi I_3$ is obtained with the help of the pinion angle $PSI_2$.

While the motion of gear 1 is terminated when the angle $PHI_1$ reaches the angle $PHI_{1F}$ (or rather $PHI_{1F} + DDPHI_1$ for moment summation purposes), both gears 2 and 3 must be reset to their respective starting angles whenever their final angles $PHI_{2F}$ and $PHI_{3F}$ are reached.

The appropriate choice of moment equation depends upon which of the eight possible combinations of contact conditions, as indicated by Table H-1, is applicable.

The following discusses the kinematics of the individual meshes as well as the determination of the point and cycle efficiencies in greater detail.

A. Kinematics

(1) Mesh 1

Depending on whether $PHI_1$ is larger or smaller than $PHI_{1T}$, the parameters of the round on round or the round on flat regime are computed. (Recall that gear 1 turns in a clockwise direction.)

For the round on round phase, the following calculations are made:
\[ \phi_1, \text{ according to eq. (G-11), and with the help of the previously determined SIGN1R} \]
\[ \lambda_1, \text{ according to eqs. (G-13) and (G-14)} \]
\[ \dot{\phi}_1, \text{ according to eq. (G-15)} \]
\[ \frac{V_{S1}}{T1R}, \text{ according to eq. (G-20)} \]
\[ s_{1R}, \text{ according to eq. (H-1) as adapted to mesh 1} \]

For the round on flat phase, the following calculations are made:

\[ \phi_1, \text{ according to eq. (G-29), and with the help of the previously determined SIGN1F} \]
\[ s_1, \text{ according to eq. (G-27)} \]
\[ \dot{\phi}_1, \text{ according to eq. (G-30)} \]
\[ \frac{V_{S1}}{T1F}, \text{ according to eq. (G-33)} \]
\[ s_{1F}, \text{ according to eq. (H-2) as adapted to mesh 1} \]

(2) **Mesh 2**

The increment \( \Delta \phi_{H2} \) for each round of computation is obtained with the help of the change in the angle \( \phi_1 \) between the present and the previous computation, i.e., as shown at statement label no. 31

\[ \Delta \phi_{H2} = \psi_1 - \psi_{1P} \quad \text{(I-17)} \]

For the first round of computations, the "previous" \( \psi_1 \), i.e., \( \psi_{1P} \), is equal to \( \psi_{1I} \).

It must be recalled that gear 2 rotates in a positive direction, and therefore, the angle \( \phi_2 \) increases with continued
motion. The angle PHI2 is re-indexed to PHI2I once it becomes larger than PHI2F.

As for mesh 1, comparison with the transition angle decides whether the mesh is in the round on round or in the round on flat regime.

The following round on round parameters are calculated:

$\phi_2$, according to eq. (G-58), and with the help of the previously determined SIGN2R

$\lambda_2$, according to eqs. (G-59) and (G-60)

Note that the "input angular velocity" for mesh 2, i.e., $\dot{\phi}_2$, equals the momentary value of $\psi_1$.

$\dot{\phi}_2$, according to eq. (G-61)

$V_{S2}/T_{2R}$, according to eq. (G-63)

$s_{2R}$, according to eq. (H-1) as adapted to mesh 2

For the round on flat phase, the following calculations are made:

$\phi_2$, according to eq. (G-71), and with the help of the previously determined SIGN2F

$s_2$, according to eq. (G-69)

Again, $\phi_2$ equals the momentary value of $\psi_1$

$\dot{\phi}_2$, according to eq. (G-72)

$V_{S2}/T_{2F}$, according to eq. (G-74)

$s_{2F}$, according to eq. (H-2) as adapted to mesh 2
The increment $\Delta \phi_3$, for each round of computation, is obtained with the help of the change in the angle $\psi_2$ between the present and the previous computation, i.e., as shown at statement label no. 33:

$$
\Delta \phi_3 = \psi_{21} - \psi_{2P}
$$

(18) For the first round of computations, the "previous" $\psi_2$, i.e., $\psi_{2P}$, is equal to $\psi_{21}$.

Gear 3 rotates in a negative (clockwise) direction, and therefore, the angle $\phi_3$ decreases with continued rotation. The angle $\phi_{HI3}$, which represents this angle, is re-indexed to $\phi_{HI31}$ once it becomes smaller than $\phi_{HI3P}$.

As for meshes 1 and 2, comparison with the applicable transition angle decides whether the mesh is in the round on round or in the round on flat regime.

The following round on round parameters are calculated:

- $\psi_3$, according to eq. (G-87), and with the help of the previously determined $\text{SIGN}_3R$
- $\lambda_3$, according to eqs. (G-89) and (G-90)

Note that the "input angular velocity" for mesh 3, i.e., $\dot{\psi}_3$, equals the momentary value of $\psi_2$.

- $\dot{\psi}_3$, according to eq. (G-91)
- $V_{3S}/T_{3R}$, according to eq. (G-92)
- $s_{3R}$, according to eq. (H-1) as adapted to mesh 3
For the round on flat phase, the following calculations are made:

\[ \psi_3, \text{ according to eq. (G-94), and with the help of the previously determined SIGN3F} \]

\[ \delta_3, \text{ according to eq. (G-95)} \]

Again, \( \dot{\psi}_3 \) is equal to the momentary value of \( \dot{\psi}_2 \).

\[ \psi_3, \text{ according to eq. (G-96)} \]

\[ V_{S3/T3P}, \text{ according to eq. (G-97)} \]

\[ s_{3P}, \text{ according to eq. (H-2) as adapted to mesh 3} \]

B. Moment Computations, Point and Cycle Efficiencies

Regardless of the combination of contact conditions, the point efficiency is computed according to eq. (3), i.e.,

\[ \eta_p = \text{POINTEF} = K_{\text{ratio}} \frac{N_{041}}{N_{\text{in}}} \quad \text{(I-19)} \]

where, with \( \psi_1 = -1 \),

\[ K_{\text{ratio}} = |\psi_3| \quad \text{(I-20)} \]

The cycle efficiency determination is based on eq. (C-10) in Appendix C, which represents an adaptation of eq. (4):

\[ \psi_C = \frac{\psi_{1P} \Delta \psi_{1P}}{\psi_{1\text{FIN}} - \psi_{1\text{IN}}} \quad \text{(I-21)} \]

(See page C-18.) The associated expression in the program, at statement label no. 45 becomes

I-18
\[ CYCLEFF = -MTOT \times DDPHI1/(PHI1P-PHI1I) \quad (I-22) \]

where

\[ MTOT = MTOT + POINTEF \quad (I-23) \]

The moment computations begin with the statement label no. 35, and initially consist of the determination of the variables A1 to A64 and C1 to C32 of section 1 of Appendix H. The governing contact combination (see also Table H-1) is determined with the help of the 8 moment control statements, which start with card no. 737. Once the appropriate combination is established, the program is directed to one of the 8 associated moment expressions. These expressions for \( M_{o41} \) coincide in nomenclature with those given by eqs. (H-81), (H-118), (H-158), (H-180), (H-216), (H-218), (H-239) and (H-241). They are listed in the above order, beginning with statement label no. 36 and ending with statement label no. 43.

In devising the control statements, the manner of rotation of the individual mesh input gears had to be taken into account. Thus:

For mesh 3:

Round on round (R) corresponds to
\[ PHI31 > PHI3 > PHI3T \]
Round on flat (F) corresponds to
\[ PHI3T > PHI3 > PHI3F \]
For mesh 2:

Round on round (R) corresponds to
\[ \text{PHI2I} < \text{PHI2} < \text{PHI2T} \]
Round on flat (F) corresponds to
\[ \text{PHI2T} < \text{PHI2} < \text{PHI2F} \]

For mesh 1:

Round on round (R) corresponds to
\[ \text{PHI1I} > \text{PHI1} > \text{PHI1T} \]
Round on flat (F) corresponds to
\[ \text{PHI1T} > \text{PHI1} > \text{PHI1F} \]

c. **Output (see Program CLOCK 3, below)**

The output of the program is best explained with the help of the sample problem at the end of the program.

I. **Input Parameters**

**Mesh 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{P1} )</td>
<td>( 0.47725 ) in. (1.212 cm)</td>
</tr>
<tr>
<td>( R_{P2} )</td>
<td>( 0.09085 ) in. (0.231 cm)</td>
</tr>
<tr>
<td>( a_{G1} )</td>
<td>( 0.47725 ) in. (1.212 cm)</td>
</tr>
<tr>
<td>( a_{P1} )</td>
<td>( 0.09085 ) in. (0.231 cm)</td>
</tr>
<tr>
<td>( \rho_{G1} )</td>
<td>( 0.03870 ) in. (0.098 cm)</td>
</tr>
<tr>
<td>( \rho_{P1} )</td>
<td>( 0.01740 ) in. (0.044 cm)</td>
</tr>
<tr>
<td>( t_{G1} )</td>
<td>( 0.03480 ) in. (0.088 cm)</td>
</tr>
<tr>
<td>( t_{P1} )</td>
<td>( 0.02800 ) in. (0.071 cm)</td>
</tr>
<tr>
<td>( n_{G1} )</td>
<td>( 42 )</td>
</tr>
<tr>
<td>( n_{P2} )</td>
<td>( 8 )</td>
</tr>
</tbody>
</table>
Mesh 2

$\text{CAPRP}_2 = R_{P2} = .20670 \text{ in.} \quad (0.525 \text{ cm})$

$\text{RP}_3 = r_{P3} = .06890 \text{ in.} \quad (0.175 \text{ cm})$

$\text{ACG}_2 = a_{G2} = .20670 \text{ in.} \quad (0.525 \text{ cm})$

$\text{ACP}_2 = a_{P2} = .06890 \text{ in.} \quad (0.175 \text{ cm})$

$\text{RHOG}_2 = r_{G2} = .02070 \text{ in.} \quad (0.053 \text{ cm})$

$\text{RHOP}_2 = r_{P2} = .01040 \text{ in.} \quad (0.026 \text{ cm})$

$\text{TG}_2 = t_{G2} = .02520 \text{ in.} \quad (0.064 \text{ cm})$

$\text{TP}_2 = t_{P2} = .02080 \text{ in.} \quad (0.053 \text{ cm})$

$\text{NG}_2 = n_{G2} = 27$

$\text{NP}_3 = n_{P3} = 9$

Mesh 3

$\text{CAPRP}_3 = R_{P3} = .17560 \text{ in.} \quad (0.446 \text{ cm})$

$\text{RP}_4 = r_{P4} = .05905 \text{ in.} \quad (0.150 \text{ cm})$

$\text{ACG}_3 = a_{G3} = .17560 \text{ in.} \quad (0.446 \text{ cm})$

$\text{ACP}_3 = a_{P3} = .05905 \text{ in.} \quad (0.150 \text{ cm})$

$\text{RHOG}_3 = r_{G3} = .01910 \text{ in.} \quad (0.049 \text{ cm})$

$\text{RHOP}_3 = r_{P3} = .00875 \text{ in.} \quad (0.022 \text{ cm})$

$\text{TG}_3 = t_{G3} = .02170 \text{ in.} \quad (0.055 \text{ cm})$

$\text{TP}_3 = t_{P3} = .01750 \text{ in.} \quad (0.044 \text{ cm})$

$\text{NG}_3 = n_{G3} = 27$

$\text{NP}_4 = n_{P4} = 9$

In addition

$\mu = \mu = .2$

$\text{RPM} = 1000$

$M_1 = m_1 = .69515 \times 10^{-4} \text{ lb-sec}^2/\text{in.} \quad (12.171 \text{ g})$
II. Computed Values

At the beginning of the output, one finds $M_{IN} = M_{IN}$.

Subsequently, the following are listed for each mesh:

- $f_{p1}$: the length of the pinion flats
- $\beta_1$: the fuse body pivot to pivot line angles
- $\psi_{T1}$ and $\varphi_{T1}$: the transition angles as well as the associated subsidiary tests
- $\psi_{IN1}$ and $\varphi_{IN1}$: the initial angles
- $\psi_{FIN1}$ and $\varphi_{FIN1}$: the final angles

Finally, for the full range of the input angle $\varphi_1$, the point efficiency $POINTERF$ is listed, in addition to other parameters which are useful for checking purposes. Note that $DPS11, DPS12$.
and DPS13 represent $\psi_1$, $\psi_2$ and $\psi_3$, respectively. The cycle efficiency CYCLEFF is found at the end of the output.
COMPUTATION OF GEAR TOOTH PARAMETERS

\[ \text{CXLG1 = RHOG1 - TG1/2.} \]
\[ \text{DELG1 = ASIN(CXLG1/CAPRP1)} \]
\[ \text{CXP1 = RHOP1 - TP1/2.} \]
\[ \text{DELP1 = ASIN(CXP1/RP2)} \]
\[ \text{GAMMP1 = ASIN(RHOP1/RP2)} \]
\[ \text{ALPHP1 = GAMMP1 - DELP1} \]
\[ \text{FP1 = ACPSiley(GAMMP1)} \]
\[ \text{B1 = LARMPS1 + RP2} \]
\[ \text{CXLG2 = RHOG2 - TG2/2.} \]
\[ \text{DELG2 = ASIN(CXLG2/CAPRP2)} \]
\[ \text{CXP2 = RHOP2 - TP2/2.} \]
\[ \text{DELP2 = ASIN(CXP2/RP3)} \]
\[ \text{GAMMP2 = ASIN(RHOP2/RP3)} \]
\[ \text{ALPHP2 = GAMMP2 - DELP2} \]
\[ \text{FP2 = ACPSiley(GAMMP2)} \]
\[ \text{B2 = LARMPS2 + RP3} \]
\[ \text{CXLG3 = RHOG3 - TG3/2.} \]
\[ \text{DELG3 = ASIN(CXLG3/CAPRP3)} \]
\[ \text{CXP3 = RHOP3 - TP3/2.} \]
\[ \text{DELP3 = ASIN(CXP3/RP4)} \]
\[ \text{GAMMP3 = ASIN(RHOP3/RP4)} \]
\[ \text{ALPHP3 = GAMMP3 - DELP3} \]
\[ \text{FP3 = ACPSiley(GAMMP3)} \]
\[ \text{B3 = LARMPS3 + RP4} \]

COMPUTATION OF MIN, GAMMAS AND BETAS

\[ \text{MIN = MD = OM2} \]
\[ \text{DELTA2 = ACOS((CAPRP1 + RP2) * (CAPRP1 + RP2) + R1 + R2 - R2) / (2.0 * R1)} \]

CAPRP1 = CAPRP2 = CAPRP3
PROGRAM CLOCK3  74/74  OPT=1  FTN 4.6+420  08/01/79  14.52.13  PAGE 2

1+RP2))
55 DELTA4=ACOS(((CAPRP2+RP3)*(CAPRP2+RP3)+R2+R2-R3+R3)/(2.*R2*(CAPRP2 A 55
1+RP2))
60 DELTA4=ACOS(((CAPRP3+RP4)*(CAPRP3+RP4)+R3+R3-R4+R4)/(2.*R3*(CAPRP3 A 57
1+RP4))
65 GAMMA4=ACOS(((R1*R1+R2*R2-(CAPRP1+RP2)*(CAPRP1+RP2))/(2.*R1*R2)) A 59
70 GAMMA3=ACOS(((R2*R2+R3*R3-(CAPRP2+RP3)*(CAPRP2+RP3))/(2.*R2*R3)) A 60
GAMMA2=GAMMA2+GAMMA3
75 GAMMA1=ACOS(((R3*R3+R4*R4-(CAPRP3+RP4)*(CAPRP3+RP4))/(2.*R3*R4)) A 62
GAMMA1=GAMMA1+GAMMA4
80 BETAS1=PI-DELT4
85 BETAS2=GAMMA2+PI-DELT4
90 BETAS3=GAMMA3+PI-DELT4
95 BETAS4=BETAS2/Z
100 WRITE (6,66) M1,M2,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,ACG1,ACG2 A 70
105 WRITE (6,66) M1,M2,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,ACG1,ACG2 A 71
110 WRITE (6,63) R1,R2,R3,R4 A 72
115 WRITE (6,67) RHOG1,RHOG2,RHOG3,RHOP1,RHOP2,RHOP3 A 73
120 WRITE (6,68) TG1,TG2,TG3,TP1,TP2,TP3 A 74
125 WRITE (6,69) NG1,NG2,NG3,WP2,WP3,WP4 A 75
130 WRITE (6,66) M1,M2,R3,B4 A 76
135 WRITE (6,70) RH01,RH02,RH03,RH04,RD,K,RH001 A 77
140 WRITE (6,71) FP1,FP2,FP3 A 78
145 WRITE (6,71) BETAS1,BETAS2,BETAS3 A 79
150 WRITE (6,71) BETAS4,BETAS4 A 80
155 COMPUTATION OF OTHER PARAMETERS A .81
160 DPHI1=360./MG1+Z A 82
165 DP51=360./NP2+Z A 83
170 DPHI2=360./MG2+Z A 84
175 DPHI3=360./MG3+Z A 85
180 DPS1=360./MP3+Z A 86
185 DPHI=360./MP4+Z A 87
190 L1=RHOG1+RHOP1 A 88
195 L2=RHOG2+RHOP2 A 89
200 L3=RHOG3+RHOP3 A 90
205 Q1=M1+R1+OM2 A 91
210 Q2=M2+R2+OM2 A 92
215 Q3=M3+R3+OM2 A 93
220 Q4=M4+R4+OM2 A 94
225 Q5=RH01+RH02 A 95
235 COMPREHENSIVE COMPUTATIONS FOR MESH 1 A .96
240 A11=RHOG1+RACG1+RACG1 A 97
245 B11=RHOG1+RACG1+RACG1 A 98
300 A10=RHOG1+RACG1+RACG1 A 99
305 B10=RHOG1+RACG1+RACG1 A 100
310 C10=(ACG1+ACG1+RHO1-RH01-FP1+FP1)/(2.*B1) A 101
315 ROOT1=A11+B11-B1-B1*C11+C11 A 102
320 A11=ROOT1+C11 A 103
325 A11=ROOT1+C11 A 104
330
PROGRAM CLOK3 74/74 OPT=+ FTN 4.6-420 08/01/79 14:52:13 PAGE 3

XIT=B1T+C1T
PSI1T1=2.*ATAN2(Y1T1,X1T)
PSI1T2=2.*ATAN2(Y1T2,X1T)
PSITE11=PSI1T1
PSITE12=PSI1T2
110 IF (PSI1T1.GT.PI) PSITE11=PSI1T1-2.*PI
IF (PSI1T1.LT.PI) PSITE11=PSI1T1+2.*PI
IF (PSI1T2.GT.PI) PSITE12=PSI1T2-2.*PI
IF (PSI1T2.LT.PI) PSITE12=PSI1T2+2.*PI
115 IF (PSITE1GE.0.) TEST1=ABS(F1-BETA1+PSITE1-ALPH1)/Z
IF (PSITE1LT.0.) TEST1=ABS(F1-BETA1-PSITE1+ALPH1)/Z
IF (PSITE2GE.0.) TEST2=ABS(F1-BETA1+PSITE2-ALPH1)/Z
IF (PSITE2LT.0.) TEST2=ABS(F1-BETA1-PSITE2+ALPH1)/Z
120 IF (PSI1T1.LT.0.) PSI1T1=PSI1T1+2.*PI
IF (PSI1T2.LT.0.) PSI1T2=PSI1T2+2.*PI
PSIT1D=PSI1T1/Z
PSIT2D=PSI1T2/Z
125 WRITE (6,46) PSIT1D,TEST1
WRITE (6,47) PSIT2D,TEST2
CALL TRANS1 (PHG1,ALPH1,BETA1,FP1,ACG1,BL,DELG1,Z,PSI1T1,PHIT1, A126
127 IF (GI2G.T.PF1) GO TO 2
PHIT=PHIT1
PSIT=PSIT1
130 GO TO 4
132 CALL TRANS1 (PHG1,ALPH1,BETA1,FP1,ACG1,BL,DELG1,Z,PSI1T2,PHIT2, A133
135 IF (GI2L.LT.PF1) GO TO 3
WRITE (6,72)
138 SIGF
140 PHIT=PHIT2
PSIT=PSIT2
4 IF (PHIT.LT.0.) PHIT=PHIT+2.*PI
IF (PHIT.LT.0.) PSI1T=PSIT+2.*PI
PHITD=PHIT1/Z
PSITD=PSIT1/Z
145 WRITE (6,73) PHITD,PSITD
148 C DETERMINATION OF CORRECT SIGN FOR BOUNDARY REGIME OF MESH 1
149 C
150 A1F=A1G1+cos(PH1T+DELG1+ALPH1)-B1*cos(BETA1+ALPH1)
B1F=A1G1+sin(PH1T+DELG1+ALPH1)+B1*sin(BETA1+ALPH1)
C1F=PHG1
155 ROOT=AF=AF+B1F-C1F*C1F
Y1F=AF-SQRT(ROOT1F)
Y1F2=AF-SQRT(ROOT1F)
X1F=B1F+C1F
159 PSI1F1=2.*ATAN2(Y1F1,X1F)
PSI1F2=2.*ATAN2(Y1F2,X1F)
160 IF (PSI1F1.LT.0.) PSI1F1=PSI1F1+2.*PI
IF (PSI1F2.LT.0.) PSI1F2=PSI1F2+2.*PI
165 IF (ABS(PSI1F1+PSI1T).LT.ABS(PSI1F2-PSI1T)) GO TO 5
168 SIGNF=-1.
160  GO TO 6
5  SIG1F=1.

C

COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 1

165  8  DO 7 1=1,2000
PHI1=PHI1D-(I-1)/100.
PHI2=PHI2T
ACGI=ACGI+ACG1+ALPHP1-B1*CSS(BETA1+ALPHP1)
170  C1F=RHO1
ROOT1F=AF=AF=AF=AF+BI1*BI1*AF=AF+AF=AF+AF=AF
Y1F=AF*SIG1F+SIG1F=SIG1F

7  CONTINUE

IF (DELE1.LE.0.) GO TO 8

8  PHI1F=PHI1
PSI1FF=PSI1F
PSI1D=PSI1D
PX1D=PSI1D

190  IF (PS1F,LT,0.) PSI1F=PS1F+2.*PI
PS1F=PS1F
LX1=BI1*CSS(BETA1)*ACGI-COS(PS1F-DS1F+DELPI)-ACGI*CSS(PS1F+DELPI)

195  DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 1

1 A1=ACGI*CSS(PS1F-DL1F)-B1*SIN(BETA1-DL1F)
B1=ACGI*CSS(PS1F-DL1F)-B1*SIN(BETA1-DL1F)
C1=(ACGI1*ACGI1*ACGI1*BI1*LI1+LI1-2.+ACGI1*BI1*CSS(PS1F1+DELPI)-BE

200  11A1))+(2.+ACGI1)
ROOT1=AF=AF=AF=AF+BI1*BI1*AF=AF=AF=AF=AF
Y1A=AF*SIG1F+SIG1F=SIG1F

205  PS1R1=2.*ATAN2(Y1A1,Y1A1)
PS1R2=2.*ATAN2(Y1A2,Y1A2)
IF (PS1R1,LT,0.) PS1R1=PS1R1+2.*PI
IF (PS1R2,LT,0.) PS1R2=PS1R2+2.*PI
IF (ABS(PS1R1),PS1R2),LT,ABS(PS1R1,PS1R2)) GO TO 9

210  SIGNR=1.
GO TO 10
9  SIGNR=1.
PROGRAM CLOCK3 74/74 OPT=1 08/01/79 14:52:13 PAGE 5

C PRELIMINARY COMPUTATIONS FOR MESH 2 A 213
C
C DETERMINATION OF TRANSITION ANGLE OF MESH 2 A 215

10 A2T=-RHOG2*COS(BETA2-ALPHP2)+FP2+SIN(BETA2-ALPHP2) A 216
B2T=RHOG2*SIN(BETA2-ALPHP2)+FP2+COS(BETA2-ALPHP2) A 217
ROOT2T=A2T+A2T+B2T+B2T-C2T=C2T A 219
Y2T1=A2T+SQRT(ROOT2T) A 220
Y2T2=A2T-SQRT(ROOT2T) A 221
X2T=B2T+C2T A 222

225 PS12T1=2.*ATAN2(Y2T1,X2T) A 223
PS12T2=2.*ATAN2(Y2T2,X2T) A 224
PSITE21=PS12T1 A 225
PSITE22=PS12T2 A 226
IF (PS12T1.GT.PI) PSITE21=PS12T1-2.*PI A 227
IF (PS12T2.GT.PI) PSITE22=PS12T2-2.*PI A 228
IF (PS12T1.LE.PI) PSITE21=PS12T1+2.*PI A 229
IF (PS12T2.LE.PI) PSITE22=PS12T2+2.*PI A 230

235 IF (PSITE21.GE.0.) TEST21=ABS(PI-BETA2+PSITE21+ALPHP2)/Z A 231
IF (PSITE21.LT.0.) TEST21=ABS(BETA2+PI-(PSITE21+2.*PI+ALPHP2))/Z A 232
IF (PSITE22.GE.0.) TEST22=ABS(PI-BETA2+PSITE22+ALPHP2)/Z A 233
IF (PSITE22.LT.0.) TEST22=ABS(BETA2+PI-(PSITE22+2.*PI+ALPHP2))/Z A 234
IF (PS12T1.LT.0.) PS12T1=PS12T1+2.*PI A 235
IF (PS12T2.LT.0.) PS12T2=PS12T2+2.*PI A 236

240 PS12T1D=PS12T1/Z A 237
PS12T2D=PS12T2/Z A 238
WRITE (6,48) PS12T1D,TEST21 A 239
WRITE (6,49) PS12T2D,TEST22 A 240
CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PS12T1,PHI2T1,1G21) A 241

245 IF (G21.GT.FP2) GO TO 11 A 242
PHI2T=PHI2T1 A 243
PS12T=PS12T1 A 244
GO TO 13 A 245

11 CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PS12T1,PHI2T1,1G22) A 246

12 PH12T=PHI2T2 A 247
PS12T=PS12T2 A 248
IF (PH12T.GT.PI) PH12T=PHI2T2-2.*PI A 249
IF (PH12T.LT.PI) PH12T=PHI2T2+2.*PI A 250
PH12TD=PH12T/Z A 251
PS12TD=PS12T/Z A 252
WRITE (6,76) PH12TD,PS12TD A 253

C DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 2 A 260

365 A2F=-ACG2*COS(PHI12T-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2) A 261
A2R = B2 * SIN(BETA2 + DELP2) - ACG2 * SIN(PHI2I - DELG2 + DELP2)
B5F = B2 * COS(BETA2 + DELP2) - ACG2 * COS(PHI2I - DELG2 + DELP2)

C150

C

COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 2

C

15 DO 16 I = 1 , 1000

PHI2D = PHI2D + (I-1.) / 100.

PHI2 = PHI2D + Z

A2F = ACG2 + COS(PHI2 - DELG2 + DELP2) - B2 * COS(BETA2 - ALPHP2)

B2F = - ACG2 + SIN(PHI2 - DELG2 + DELP2) + B2 * SIN(BETA2 - ALPHP2)

C2F = - RHO2G2

290

RHO2F = A2F/2 + A2F - B2F - C2F

Y2F = A2F + SIGNY2F * SQRT(RHO2F)

X2F = B2F + C2F

PS12F = A2F - B2F + C2F

PS12F = A2F + C2F

300

IF (PS12F .LT. 0.) PS12F = PS12F + 2. * PI

LY2 = B2 * SIN(BETA2) + ACP2 + SIN(PS12F + DELG2 - DELP2) - ACG2 * SIN(PHI2 - PHI2D + DELG2)

LL2 = SQRT(LX2 + LX2 + LY2)

DELEL2 = LL2 - L2

IF (DELEL2 .LE. 0.) GO TO 17

16 CONTINUE

17 PHI2F = PHI2F

PS12FF = PS12F

PHI2 = PHI2D - DPHI2

PS12I = PS12F + DPS12


PHI2I = PHI2I + Z

PS12ID = PS12I

PHI2ID = PHI2I + Z

PS12FD = PS12FF

WRITE (6,77) PHI2ID, PSI2ID, PHI2FD, PSI2FD

C

DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 2

C

315
A preliminary computation for mesh 3

Determination of transition angle of mesh 3

320 ITA21/(12.-ACP2)
    ROOT2R=A2R+AR+AR+AR+C2R
    Y3R1=A2R+SORT(ROOT2R)
    Y3R2=A2R+SORT(ROOT2R)
    Z2R-2=2*R2+2R+C2R
    PS12R1=2.*ATAN(Y3R1,Z2R)
    PS12R2=2.*ATAN(Y3R2,Z2R)
    IF (PS12R1.LT.0.) PS12R1=PS12R1+2.*PI
    IF (PS12R2.LT.0.) PS12R2=PS12R2+2.*PI
    IF (ABS(PS12R1-PS12R2).LT.ABS(PS12R1-PS12R2)) GO TO 18
    SIG2R=-1.
    GO TO 19

18 SIG2R=1.

19 A3T=RHOG3+SQRT(BETA3+ALPHA3)+FP3=SQRT(BETA3+ALPHA3)
    B3T=RHOG3+BETA3+ALPHA3+FP3=SQRT(BETA3+ALPHA3)
    C3T=(ACG3+RHOG3)*RHOG3+BETA3+ALPHA3+FP3=SQRT(BETA3+ALPHA3)
    ROOT3T=A3T+A3T+B3T+C3T+CT
    Y3T1=A3T+SORT(ROOT3T)
    Y3T2=A3T+SORT(ROOT3T)
    X3T=B3T+CT

340 PS31T1=2.*ATAN(Y3T1,X3T)
    PS31T2=2.*ATAN(Y3T2,X3T)
    PSITE31=PSITE32=PSITE32
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    IF (PSITE31.LT.PI) PSITE31=PSITE31+2.*PI
    PS31T0=PSITE31/Z
    PS31T2=PS31T0/Z

350 WRITE (6.50) PS31T0,TEST31
    WRITE (6.51) PS31T2,TEST32
    CALL TRANS1(RHOG3,ALPHA3,BETA3,FP3,ACG3,B3,DELG3,Z,PS31T1,PHI31T1,1G31)

360 IF (G31.GT.FP3) GO TO 20
    PHI31T=PHI31T
    PSITE3=PSITE3
    GO TO 22

20 CALL TRANS1(RHOG3,ALPHA3,BETA3,FP3,ACG3,B3,DELG3,Z,PS31T2,PHI31T,1G32)

370 IF (G32.LT.FP3) GO TO 21
    WRITE (6.78) PSITE3

371
PROGRAM CLOC3 74/74 OPT=1

STOP
21 PHI3=PHI3+2
PSI3=PSI3+2
375 IF (PHI3LT.0.) PHI3=PHI3+2.*PI
IF (PSI3LT.0.) PSI3=PSI3+2.*PI
PHI3=PHI3/*PI
PSI3=PSI3/*PI
WRITE (6,79) PHI3,PSI3
380 DETERMINATION OF CORRECT SIGN FOR ROUGH ON FLAT REGIME OF MESH 3
A3F=ACOS(COS(PHI3+DELG3+ALPH3)-B3*COS(BETA3+ALPH3))
B3F=ACOS(SIN(PHI3+DELG3+ALPH3)+B3*SIN(BETA3+ALPH3))
C3F=RHOG3
ROOT3F=A3F+B3F+C3F
Y3F1=ASQRT(ROOT3F)
Y3F2=A3F=ASQRT(ROOT3F)
X3F=B3F+C3F
PSI3F=2.*ATAN2(X3F,Y3F)
PSI3F2=2.*ATAN2(Y3F2,X3F)
IF (PSI3FLT.0.) PSI3F=PSI3F+2.*PI
IF (PSI3FLT.0.) PSI3F2=PSI3F2+2.*PI
IF (ABS(PSI3F-PSI3)+LT.ABS(PSI3F2-PSI3)) GO TO 23
395 SIGN3=1.
GO TO 24
23 SIGMA3=1.
COMPUTATION OF FINAL AND INITIAL VALUES OF PHII AND PSI FOR MESH 3
400 DO 25 I=1,1206
PHI3=PHI3*(1-1./100.
PHI3=PHI3+2
A3F=ACOS(COS(PHI3+DELG3+ALPH3)-B3*COS(BETA3+ALPH3))
B3F=ACOS(SIN(PHI3+DELG3+ALPH3)+B3*SIN(BETA3+ALPH3))
C3F=RHOG3
ROOT3F=A3F+B3F+C3F
Y3F1=A3F=ASQRT(ROOT3F)
Y3F2=A3F=ASQRT(ROOT3F)
X3F=B3F+C3F
PSI3F=2.*ATAN2(Y3F,X3F)
PSI3F2=2.*ATAN2(Y3F2,X3F)
IF (PSI3FLT.0.) PSI3F=PSI3F+2.*PI
IF (PSI3FLT.0.) PSI3F2=PSI3F2+2.*PI
LY3=3*SIN(BETA3)+ACPSIN(PHI3-DELPHI3)
LX3=3*COS(BETA3)+ACPSIN(PHI3-DELPHI3)
1DELP3)
LL3=ASQRT(LX3*LX3+LY3*LY3)
DELEL3=LL3/L3
25 CONTINUE
26 PHII=PHII
PSI3FF=PSI3F
PHII=PHII+PHII3+2.*PI
425 IF (PS131.LT.0.) PS131=PS131+2.*PI
PH131D=PH131/Z
PH131D=PS131/Z
PH13FD=PH13FD/Z
PS13FD=PS13FD/Z
WRITE (6,60) PH131D,PS131D,PH13FD,PS13FD

DETERMINATION OF CORRECT SIGN OF ROUND ON ROUND REGIME FOR MESH 3

435 A3R=ACG3* SIN (PH131+DELG3-DELP3)-B3* SIN (BETA3-DELP3)
B3R=ACG3*COS (PH131+DELG3-DELP3)-B3*COS (BETA3-DELP3)
C3R=(ACP3+ACP3+ACG3+ACG3+B3+B3-L3+L3-2.*ACG3+B3*COS (PH131+DELG3-DELP3))
ROOT3R=A3R*A3R+B3R+B3R+C3R+C3R
Y3R1=A3R+SQRT (ROOT3R)
Y3R2=A3R-SQRT (ROOT3R)
X3R=B3R+C3R
PS13R1=2.*ATAN2 (X3R1,X3R)
PS13R2=2.*ATAN2 (X3R2,X3R)
IF (PS13R1.LT.0.) PS13R1=PS13R1+2.* PI
IF (PS13R2.LT.0.) PS13R2=PS13R2+2.* PI
IF (ABS (PS131-PS131L).LT. ABS (PS131-PS131L)) GO TO 27
SIGN3R=1.
GO TO 28

27 SIGN3R=1.

450 IF (PH11.LE.PHI1F+DOPHI1) GO TO 45

GEAR TRAIN MOTION MODEL, KINEMATICS

28 DOPHI1=M2P2+M3P3-(PHI131-PHI13F)/(K*NG1*NG2)
PHI1=PHI11+DOPHI1
WRITE (6,52)

29 PH11=PHI11-DOPHI1
PH1D=PHI11/Z
IF (PHI11.LE.PHI1F+DOPHI1) GO TO 45

460 IF (PHI11.LE.PHI1F) GO TO 30
A1R=ACG1+ SIN (PHI1+DELG1-DELP1)-B1+ SIN (BETA1-DELP1)
B1R=ACG1+COS (PHI1+DELG1-DELP1)-B1*COS (BETA1-DELP1)
C1R=ACP1+ACP1+ACG1+ACG1+B1+B1-L1-1.-ACG1+B1*COS (PH111+DELG11-BET
111))/ (2.*ACP1)
ROOT1R=A1R*A1R+B1R+B1R+C1R+C1R
Y1R=A1R+SIGN1R* SQRT (ROOT1R)
X1R=B1R+C1R
PS11=2.* ATAN2 (Y1R,X1R)
IF (PS11.LT.0.) PS11=PS11+2.* PI
PS11D=PS11/Z
IF (ABS (PHI11-PS11L).LT.0.0001) PS111=PS11
IF (ABS (PH111-PS11L).LT.0.0001) PS111=PS11
SLAM1= (B1+ SIN (BETA1)+ACP1+ACP1+ACG1+ACG1+ACG1+ACG1+B1+B1-L1)/L1
CLAM1= (B1+COS (BETA1)+ACP1+ACP1+ACG1+ACG1+ACG1+ACG1+B1+B1-L1)/L1
LAMDA1=ATAN2 (SLAM1,CLAM1)
A gas is

\[ G^3 = \sin(P13 + DEL3) + RH0G3 + \cos(P13 - ALPHP3) - B3 + \sin(BETA3) \] / \sin(P13)

\[ P13 = -ALPHP3 \]

\[ P3D13 = P3D2 \]

\[ P3G13 = P3D13 + G3 - \sin(P13 + DEL3 + ALPHP3) / (A3F + \cos(P13) - B3F - \sin(P13)) \]

\[ P3G13 = P3D13 + G3 - \sin(P13 + DEL3 + ALPHP3) / (A3F + \cos(P13) - B3F - \sin(P13)) \]

\[ VST3F = P3D13 + G3 - \sin(P13 + ALPHP3 + DEL3 - RH0G3) \]

\[ S3F = VST3F / ABS(VST3F) \]

C

C

MOMENT COMPUTATIONS

C

C

25

D = 1 + \mu + \mu

A1 = \operatorname{ABS}(\mu + \sin(\text{GAMMA4}) + \cos(\text{GAMMA4}) / \mu)

A2 = \operatorname{ABS}(\mu + (S3 + 1) \sin(\text{LAMA}3) + (1 + \mu + \mu) + 3 \sin(\text{LAMA}3) / \mu)

A3 = \operatorname{ABS}(\mu + \sin(\text{GAMMA4}) + \mu) + \cos(\text{GAMMA4}) / \mu)

A4 = \operatorname{ABS}((1 + \mu + \mu) + 3 \sin(\text{LAMA}3) + \mu + (S3 + 1) + \cos(\text{LAMA}3) / \mu)

A5 = \operatorname{ABS}((1 - \mu + \mu + S3) + \cos(\text{LAMA}3) + \mu + (1 + S3) + \sin(\text{LAMA}3) / \mu)

A6 = \operatorname{ABS}(\mu + (1 + S2R + \sin(\text{LAMA}2) + (1 + \mu + \mu) + 2 \sin(\text{LAMA}2) / \mu)

A7 = \operatorname{ABS}(\mu + \sin(\text{LAMA}2) + \cos(\text{GAMMA3}) / \mu)

A8 = \operatorname{ABS}(\mu + \sin(\text{LAMA}2) + \cos(\text{LAMA}2) + \mu + (1 + S3) + \cos(\text{LAMA}2) / \mu)

A9 = \operatorname{ABS}(\mu + \sin(\text{LAMA}2) + \cos(\text{LAMA}2) + \mu + (1 + S2R) + \cos(\text{LAMA}2) / \mu)

A10 = \operatorname{ABS}(\mu + \sin(\text{GAMMA3}) + \cos(\text{GAMMA3}) / \mu)

A11 = \operatorname{ABS}((1 + \mu + \mu + S3) + \cos(\text{LAMA}2) + \mu + (S3 + 1) + \sin(\text{LAMA}2) / \mu)

A12 = \operatorname{ABS}(\mu + (S1 + 1) + \sin(\text{LAMA}1) + (1 + \mu + \mu) + 2 \sin(\text{LAMA}1) / \mu)

A13 = \operatorname{ABS}((1 + \mu + \mu + S3) + \cos(\text{LAMA}2) + \mu + (S1 + 1) + \sin(\text{LAMA}1) / \mu)

A14 = \operatorname{ABS}(\mu + (1 + \mu + \mu + S2R) + \sin(\text{LAMA}2) + \mu + (1 + S2R) + \cos(\text{LAMA}2) / \mu)

A15 = \operatorname{ABS}(\mu + (1 + 1) + \sin(\text{LAMA}1) + (1 + \mu + \mu) + 2 \sin(\text{LAMA}1) / \mu)

A16 = \operatorname{ABS}(\mu + \cos(\text{GAMMA2}) + \sin(\text{GAMMA2}) / \mu)

A17 = \operatorname{ABS}(\mu + (1 + \mu + \mu) + \sin(\text{LAMA}1) + (1 + S1) + \sin(\text{LAMA}1) / \mu)

A18 = \operatorname{ABS}(\mu + (1 + \mu + \mu) + \sin(\text{LAMA}1) + (1 + S1) + \sin(\text{LAMA}1) / \mu)

A19 = \operatorname{ABS}(\mu + (1 - \mu - 1) + \sin(\text{LAMA}1) + (1 + S1) + \sin(\text{LAMA}1) / \mu)

A20 = \operatorname{ABS}(\mu / \mu)

A21 = \operatorname{ABS}(\mu + (1 - \mu) + \sin(\text{LAMA}2) + (1 + \mu - \mu) + \cos(\text{LAMA}2) / \mu)

A22 = \operatorname{ABS}(\mu + (1 - \mu) + \sin(\text{LAMA}1) + (1 + \mu - \mu) + \cos(\text{LAMA}1) / \mu)

A23 = \operatorname{ABS}(\mu + \sin(\text{GAMMA2}) + \cos(\text{GAMMA2}) / \mu)

A24 = \operatorname{ABS}(\mu + (1 - \mu + \mu + S2R) + \sin(\text{LAMA}2) + (1 - S2R) + \cos(\text{LAMA}2) / \mu)

A25 = \operatorname{ABS}(\mu + (1 - \mu + \mu + S1F) + \sin(\text{P11} + \text{ALPH1}) + (1 - \mu + \mu + S1F) + \cos(\text{P11} + \text{ALPH1}) / \mu)

A26 = \operatorname{ABS}(\mu + \cos(\text{GAMMA2}) + \mu + \sin(\text{GAMMA2}) / \mu)

A27 = \operatorname{ABS}(\mu + (1 - \mu + \mu + S1F) + \sin(\text{P11} + \text{ALPH1}) + (1 - \mu + \mu + S1F) + \cos(\text{P11} + \text{ALPH1}) / \mu)

A28 = \operatorname{ABS}(\mu / \mu)

A29 = \operatorname{ABS}(\mu + (1 + S1F) + \sin(\text{P11} + \text{ALPH1}) + (1 + \mu + \mu + S1F) + \cos(\text{P11} + \text{ALPH1}) / \mu)

A30 = \operatorname{ABS}(\mu / \mu)

A31 = \operatorname{ABS}(\mu + (1 - \mu + \mu) + \cos(\text{LAMA}1) + (1 + S3) + \sin(\text{LAMA}1) / \mu)

A32 = \operatorname{ABS}(\mu + (1 + S2F) + \cos(\text{P12} + \text{ALPH2}) + (1 + \mu + S2F) + \sin(\text{P12} + \text{ALPH2}) / \mu)}
690  C4=NU=RHO3*(A7+A10)          A 690
C5=NU=RHO3*(A6+A9)-MU=S2R+RHO2+AQP2*(MU+S2R+COS(P12-DELP2-LAMDA2)  A 691
1=SIGN(P12-DELP2-LAMDA2)         A 692
C6=ACG2*(SIGN(PHI2-DELP2-LAMDA2)-MU+S2R+COS(PHI2-DELP2-LAMDA2)-MU)+
RHO2*(A14+A14)-MU=RHO2+520     A 694
C7=NU=RHO2*(A13+A16)             A 695
C8=ACP1*(SIGN(PHI2-DELP1-LAMDA1)-MU+SIGN(COS(PHI2-DELP1-LAMDA1))-MU) A 696
1=SIGN(PHI2-DELP1-LAMDA1)       A 697
C9=NU=RHO1*(A18+A20)             A 698
C10=MU=RHO1*(A17+A19)+ACGI*(SIGN(PHI1+DELP1-LAMDA1)-MU+SIN+COS(PHI1 A 699
1+DELP1-LAMDA1))-MU=S1R+RHO01
C11=ACG2*(SIGN(PHI2-DELP2-LAMDA2)-MU+S2R+COS(PHI2-DELP2-LAMDA2))-MU A 700
1=RHO2*(A21+A24)-MU=RHO2+520    A 701
C12=MU=RHO2*(A23+A26)            A 702
C13=GI-MU=RHO2*(A23+A26)         A 703
C14=NU=RHO1*(A26+A30)            A 704
C15=NU=RHO1*(A27+A29)+MU+S1F+RHO01+ACGI*(MU+S1F+SIGN(PHI1+DELP1-LAMDA1) A 705
1+ALPHPI)-COS(PHI1+DELP1-PSI1+ALPHPI))         A 706
C16=NU=RHO3*(A31+A34)+ACG3*(SIGN(PHI3+DELP3-LAMDA3)-MU+S3R+COS(PHI3 A 707
1+DELP3-LAMDA3))-RHO03+MU=S3R
C17=NU=RHO3*(A33+A36)            A 709
C18=NU=RHO3*(A32+A35)-G2         A 710
C19=NU=RHO2*(A37+A40)+ACG3*(COS(PHI2-DELP2-PSI2-ALPHI2)-MU+S2F+SI A 711
1=SIGN(PSI2-DELP2-PSI2-ALPHI2)-MU=S2F+RHO02
C20=NU=RHO2*(A39+A42)            A 713
C21=NU=RHO2*(A38+A41)+G1         A 714
C22=NU=RHO3*(A42+A46)+ACG2*(COS(PHI2-DELP2-PSI2-ALPHI2)-MU+S2F+SI A 715
1=SIGN(PSI2-DELP2-PSI2-ALPHI2)-MU=S2F+RHO02
C23=NU=RHO2*(A45+A48)            A 717
C24=NU=RHO2*(A44+A47)+ACP1*(MU+S1F+SIGN(PHI1+DELP1-LAMDA1)-SIGN(PSI1 A 718
1+DELP1-LAMDA1))-MU+S1R+RHO01
C25=NU=RHO4*(A50+A52)            A 719
C26=NU=RHO4*(A49+A51)            A 720
C27=NU=RHO3*(A53+A56)+ACG3*(COS(PHI3+DELP3-PSI3+ALPHI3)-MU+S3F+SI A 721
1=SIGN(PSI3+DELP3-PSI3+ALPHI3)-MU=S3F+RHO03
C28=NU=RHO3*(A55+A58)            A 724
C29=NU=RHO3*(A54+A57)-G2         A 725
C30=NU=RHO3*(A59+A62)+ACG3*(COS(PHI3+DELP3-PSI3+ALPHI3)-MU+S3F+SI A 726
1=SIGN(PSI3+DELP3-PSI3+ALPHI3)-MU=S3F+RHO03
C31=NU=RHO3*(A61+A64)            A 727
720
725
730

735
IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO 735 A 734
1 36
IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO 737 A 736
1 37
IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO 739 A 740
1 38
IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO 741 A 742
PROGRAM CLOCK3

IF ((PHI1.LT.PHI1L) .AND. (PHI2.GE.PHI2T) .AND. (PHI3.LT.PHI3L)) GO TO 743
1 40
745 IF ((PHI1.LT.PHI1L) .AND. (PHI2.GE.PHI2T) .AND. (PHI3.LT.PHI3L)) GO TO 745
1 41
746 IF ((PHI1.LT.PHI1L) .AND. (PHI2.GE.PHI2T) .AND. (PHI3.LT.PHI3L)) GO TO 747
1 42
748 IF ((PHI1.LT.PHI1L) .AND. (PHI2.GE.PHI2T) .AND. (PHI3.LT.PHI3L)) GO TO 749
1 43
750

MOMENT EXPRESSIONS

36 N41=MIN(C2+C5+C8/(C3+C6+C10)-Q4+C2+C5+C7/(C3+C6)-Q3+C2+C4/C
13-Q1+C2+C5+C8/(C3+C6+C10)
754
N44=N41
755
POINTER=ABS(PSD0T3)+N04/MIN
756
WRITE (6,53) PHI10.PHI20.PHI30.PSI10.PSI20.PSI30.PSD0T1.PSD0T2.PSD0T3
757
1035.S1R.S2R.S3R.POINTER
758
GO TO 760
759
760

37 N42=MIN(C2+C5+C13/(C3+C11+C15)-Q4+C2+C5+C13+C14/(C3+C11+C15)-Q2+C
12+C5+C12/(C3+C11)-Q3+C2+C4/C
762
N44=N44
763
POINTER=ABS(PSD0T3)+N04/MIN
764
WRITE (6,54) PHI10.PHI20.PHI30.PSI10.PSI20.PSI30.PSD0T1.PSD0T2.PSD0T3
765
1035.S1R.S2R.S3R.POINTER
766
GO TO 760
767
768

39 N44=MIN(C2+C18+C21/(C15+C16+C19)-Q1+C2+C14+C18+C21/(C15+C16+C19)-
12+C18+C16+C21/(C15+C16+C19)-Q2+C18+C16/C
769
N44=N44
770
POINTER=ABS(PSD0T3)+N04/MIN
771
WRITE (6,55) PHI10.PHI20.PHI30.PSI10.PSI20.PSI30.PSD0T1.PSD0T2.PSD0T3
772
1035.S1R.S2R.S3R.POINTER
773
GO TO 775
774
775

40 N44=MIN(C2+C18+C21/C15+C16+C19)-Q1+C2+C18+C21/(C15+C16+C19)-Q2+C18+C21/C
775
12+C2+C18+C21/C15+C16+C19)-Q3+C2+C17/C16+Q4+C1
776
N44=N44
777
POINTER=ABS(PSD0T3)+N04/MIN
778
WRITE (6,56) PHI10.PHI20.PHI30.PSI10.PSI20.PSI30.PSD0T1.PSD0T2.PSD0T3
779
1035.S1R.S2R.S3R.POINTER
780
GO TO 785
781
785

41 N44=MIN(C2+C18+C21/(C15+C16+C19)-Q1+C2+C18+C21/(C15+C16+C19)-Q2+C18+C21/C
782
12+C2+C18+C21/(C15+C16+C19)-Q3+C2+C17/C16+Q4+C1
783
N44=N44
784
POINTER=ABS(PSD0T3)+N04/MIN
785
WRITE (6,57) PHI10.PHI20.PHI30.PSI10.PSI20.PSI30.PSD0T1.PSD0T2.PSD0T3
786
1035.S1R.S2R.S3R.POINTER
787
GO TO 785
788
785

GO TO 44
789
790

GO TO 44
791
792

GO TO 44
793
794

GO TO 44
795
PROGRAM CLOCK3
74/74 OPT-1

FIN 4.6420 08/01/79 14.52.13 PAGE 18

42 MOV7:MIN+C8+C26+C32/(C6+C19+C30)O1+C9+C26+C32/(C6+C19+C30)O2 A 796
1C7+C26+C32/(C6+C30)O3+C26+C31+C30+O4+C25 A 797
MOV4=MOV4 A 798
POINTER=AP5(PSDDT3)+MOV4/MIN A 799

800
WHITE (6,59) PH'10.PH'12.PH'30.PS110.PS120.PS130.PSD001.PSD002.PSD A 800
1013.S1R.S26.S3F.G3.POINTER A 801
GO TO 44 A 802

43 MOV4=MIN+C13+C26+C32/(C11+C15+C30)O1+C13+C14+C26+C32/(C11+C15+C30) A 803
1)Q1+C12+C26+C32/(C11+C30)O3+C30+O4+C25 A 804
MOV4=MOV4 A 805
POINTER=AP5(PSDDT3)+MOV4/MIN A 806
WHITE (6,60) PH'10.PH'12.PH'30.PS110.PS120.PS130.PSD001.PSD002.PSD A 807
1013.S2R.S1F.C1.S3F.G3.POINTER A 808
44)WT01+WT0+POINTER A 809
GO TO 29 A 810

45 CYCLEFF=WT0+IDPHII/(PHII–PHII) A 811
WHITE (6,81) CYCLEFF A 812
WT0=0 A 813
IF (ISTOP.NE.0) GO TO 1 A 814
STOP A 815

C

46 FORMAT (6X,SHPS1T1D,=,F9.4,3X,OHTEST11 =,F9.4) A 816
47 FORMAT (6X,SHPS1T2D,=,F9.4,3X,OHTEST12 =,F9.4/) A 817
48 FORMAT (6X,SHPS2T1D,=,F9.4,3X,OHTEST21 =,F9.4) A 818
49 FORMAT (6X,SHPS2T2D,=,F9.4,3X,OHTEST22 =,F9.4/) A 819
50 FORMAT (6X,SHPS3T1D,=,F9.4,3X,OHTEST31 =,F9.4) A 820
51 FORMAT (6X,SHPS3T2D,=,F9.4,3X,OHTEST32 =,F9.4/) A 821
825 1 DPS13 S1R S2R S3R S1F G1 S2F G2 S3F G3 POINTER A 823
2/1 A 824
53 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),3(F3.0,2X),3X,F5.3) A 825
54 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),5X3(F3.0,2X),F3.363,F5.3) A 826
55 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),1X,F3.0,2X,F3.0,2X,F5.3,2X,F3.0,2X,F5.3) A 827
56 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,1X,F3.0,1X,F3.0,2X,F5.3,1X,F5.3) A 828
57 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,2X,F3.0,2X,F3.0,2X,F3.0,2X,F3.0,2X) A 829
58 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,2X,F3.0,2X,F3.0,2X,F3.0,2X,F3.0,2X) A 830
59 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),2X,F3.0,2X,F3.0,2X,F5.3,2X,F5.3) A 831
60 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),5X3(F3.0,2X),F3.0,2X,F5.3,1X,F3.0, A 832
12X,F5.3,1F5.3) A 833
61 FORMAT (F10.3,F10.0/6F10.5/6F10.5/11) A 834
62 FORMAT (6F10.4) A 835
63 FORMAT (6F10.4) A 836
64 FORMAT (6F10.4) A 837
65 FORMAT (4F10.4/F4.0/F10.6) A 838
66 FORMAT (11H1,5X,5MIN =,F8.4,3X,4MINU =,F6.3,3X,SHRP1 =,F6.0/6X,8MC A 839
1APR1 =,F8.5,3X,GNACP1F =,F8.5,3X,GNACP2F =,F8.5,3X,GNACP3F =,F8.5,6X,8MC A 840
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 841
3.5,3X,8MC A 842
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 843
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 844
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 845
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 846
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 847
25.3X,5MIN =,F8.5,3X,SHRP1 =,F8.5,4X,SHRP2 =,F8.5,6X,8MC A 848
PROGRAM CLOCK3
74/74 OPT=1

4, F8.5/)

850
67 FORMAT (6X, 7HRHOG1 =, F8.5, 3X, 7HRHOG2 =, F8.5, 3X, 7HRHOG3 =, F8.5, 3X, 7)
A 850
1HRHDP1 =, F8.5, 3X, 7HRHDP2 =, F8.5, 3X, 7HRHDP3 =, F8.5/)
A 851
68 FORMAT (6X, SHG01 =, F8.5, 3X, SHG02 =, F8.5, 3X, SHG03 =, F8.5, 3X, SHG01 =
A 852
1, F8.5, 3X, SHP2 =, F8.5, 3X, SHP3 =, F8.5/)
A 853
69 FORMAT (6X, SHH01 =, F5.0, 3X, SHH02 =, F5.0, 3X, SHH03 =, F5.0, 3X, SHH01 =
A 854
1, F5.0, 3X, SHP2 =, F5.0, 3X, SHP3 =, F5.0/)
A 855
70 FORMAT (6X, BHRH01 =, F6.3, 3X, BHRH02 =, F6.3, 3X, BHRH03 =, F6.3, 3X, BHRH
A 856
104 =, F6.3//6X, 4H3D =, E12.4//6X, 3H4K =, F6.1//6X, BHRH01 =, F5.1/)
A 857
71 FORMAT (6X, BHM1A1D =, F8.4, 3X, BHBETA2D =, F8.4, 3X, BHBETA3D =, F8.4//
A 858
1)
A 859
72 FORMAT (6X, 3DHSOMETHING IS WRONG WITH MESH 1)
A 860
73 FORMAT (6X, BHRH11D =, F8.4, 3X, BHP11D =, F8.4)
A 861
74 FORMAT (6X, BHRH11D =, F8.4, 3X, BHP11D =, F8.4, 3X, BHRH11D =, F8.4, 3X
A 862
1X, BHP11D =, F8.4//)
A 863
75 FORMAT (6X, 3DHSOMETHING IS WRONG WITH MESH 2)
A 864
76 FORMAT (6X, BHRH11D =, F8.4, 3X, BHP11D =, F8.4, 3X, BHRH11D =, F8.4, 3X
A 865
1X, BHP11D =, F8.4//)
A 866
77 FORMAT (6X, BHRH11D =, F8.4, 3X, BHP11D =, F8.4, 3X, BHRH11D =, F8.4, 3X
A 867
1X, BHP11D =, F8.4//)
A 868
78 FORMAT (6X, 3DHSOMETHING IS WRONG WITH MESH 3)
A 869
79 FORMAT (6X, BHRH13D =, F8.4, 3X, BHP13D =, F8.4)
A 870
80 FORMAT (6X, BHRH13D =, F8.4, 3X, BHP13D =, F8.4, 3X, BHRH13D =, F8.4, 3X
A 871
1X, BHP13D =, F8.4//)
A 872
81: FORMAT (1H0, 5X, 10BCYCLE EFFICIENCY =, F8.3)
A 873
82 FORMAT (4F10.5)
A 874
83 FORMAT (6X, 4HR1 =, F8.5, 3X, 4HR2 =, F8.5, 3X, 4HR3 =, F8.5, 3X, 4HR4 =, F8.
A 874
1X/)
A 875
84 FORMAT (6X, SHP1 =, F8.5, 3X, SHP2 =, F8.5, 3X, SHP3 =, F8.5/)
A 876
85 FORMAT (4E15.5)
A 877
86 FORMAT (6X, 4HR1 =, E15.5, 3X, 4HR2 =, E15.5, 3X, 4HR3 =, E15.5, 3X, 4HR4 =, A 878
1E15.5/)
A 879
880 END
A 880-
<table>
<thead>
<tr>
<th>SUBROUTINE TRANS1</th>
<th>74/74</th>
<th>OPT=1</th>
<th>FTN 4.6+420</th>
<th>08/01/79 14.52.15</th>
<th>PAGE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SUBROUTINE TRANS1 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)</td>
<td>B</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>PI=3.14159</td>
<td>B</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>ST1=-RHOG<em>COS(PSIT+ALPHP)+B</em>SIN(BETA)+FP*SIN(PSIT+ALPHP)/ACG</td>
<td>B</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>CT1=(RHOG<em>SIN(PSIT+ALPHP)+B</em>COS(BETA)+FP*COS(PSIT+ALPHP))/ACG</td>
<td>B</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>PHIT=ATAN2(ST,CT)-DELG</td>
<td>B</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>PHINEXT=PHIT+1.1*Z</td>
<td>B</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>AF=ACG<em>COS(PHINEXT+DELG+ALPHP)-B</em>COS(BETA+ALPHP)</td>
<td>B</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>BF=ACG<em>SIN(PHINEXT+DELG+ALPHP)+B</em>SIN(BETA+ALPHP)</td>
<td>B</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>CF=RHOG</td>
<td>B</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>ROOTF=AF+BF+CF*CF</td>
<td>B</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>YF1=AF-SQRT(ROOTF)</td>
<td>B</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>YF2=AF-SQRT(ROOTF)</td>
<td>B</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>XF=BF*CF</td>
<td>B</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>PSINEX1=2.*ATAN2(YF1,XF)</td>
<td>B</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>PSINEX2=2.*ATAN2(YF2,XF)</td>
<td>B</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI</td>
<td>B</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI</td>
<td>B</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1</td>
<td>B</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>PSINEXT=PSINEX2</td>
<td>B</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>GO TO 2</td>
<td>B</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>PSINEXT+PSINEX1</td>
<td>B</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>G=(ACG<em>SIN(PHINEXT+DELG)+RHOG</em>COS(PSINEXT+ALPHP)-B*SIN(BETA))/SIN(IPSINEXT-ALPHP)</td>
<td>B</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>RETURN</td>
<td>B</td>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>END</td>
<td>B</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>B</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SUBROUTINE TRANS2 (RHOG, ALPHB, BETA, FP, ACG, B, DELG, Z, PSIT, PHIT, G) C 1
PI=3.14159 C 2
ST=(RHOG*COS(PSIT+ALPHB)+B*SIN(BETA)+FP*SIN(PSIT+ALPHB))/ACG C 3
CT=(-RHOG*SIN(PSIT+ALPHB)+B*COS(BETA)+FP*COS(PSIT+ALPHB))/ACG C 4
PHIT=ATAN2(ST,CT)+DELG C 5
PHINEXT=PHIT+.1*Z C 6
AF=ACG*COS(PHINEXT-DELG-ALPHB)-B*COS(BETA-ALPHB) C 7
BF=ACG*SIN(PHINEXT-DELG-ALPHB)+B*SIN(BETA-ALPHB) C 8
CF=-RHOG C 9
ROOTF=AF*AF+BF*BF-CF*CF C 10
YF1=AF+SQRT(ROOTF) C 11
YF2=AF-SQRT(ROOTF) C 12
XF=BF+CF C 13
PSINEX1=2.*ATAN2(YF1, XF) C 14
PSINEX2=2.*ATAN2(YF2, XF) C 15
IF (PSINEX1.LT.0.) PSINEX=PSINEX1+2.*PI C 16
IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI C 17
IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1 C 18
PSINEX=PSINEX2 C 19
GO TO 2 C 20
1 PSINEX=PSINEX1 C 21
2 G=(ACG+SIN(PHINEXT-DELG)-RHOG*COS(PSINEX+ALPHB)-B*SIN(BETA))/SIN(1*PSINEX+ALPHB) C 22
RETURN C 23
END C 24
<table>
<thead>
<tr>
<th>MIN</th>
<th>.1645</th>
<th>MU</th>
<th>.200</th>
<th>RPM</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRP1</td>
<td>.47725</td>
<td>CAPRP2</td>
<td>.20670</td>
<td>CAPRP3</td>
<td>.17560</td>
</tr>
<tr>
<td>RP2</td>
<td>.09085</td>
<td>RP3</td>
<td>.06890</td>
<td>RP4</td>
<td>.05905</td>
</tr>
<tr>
<td>ACG1</td>
<td>.47725</td>
<td>ACG2</td>
<td>.20670</td>
<td>ACG3</td>
<td>.17560</td>
</tr>
<tr>
<td>ACP1</td>
<td>.09085</td>
<td>ACP2</td>
<td>.06890</td>
<td>ACP3</td>
<td>.05905</td>
</tr>
<tr>
<td>R1</td>
<td>.75000</td>
<td>R2</td>
<td>.75000</td>
<td>R3</td>
<td>.75000</td>
</tr>
<tr>
<td>RHOG1</td>
<td>.03870</td>
<td>RHOG2</td>
<td>.02670</td>
<td>RHOG3</td>
<td>.01910</td>
</tr>
<tr>
<td>TG1</td>
<td>.03480</td>
<td>TG2</td>
<td>.02520</td>
<td>TG3</td>
<td>.02170</td>
</tr>
<tr>
<td>MG1</td>
<td>42.</td>
<td>MG2</td>
<td>27.</td>
<td>MG3</td>
<td>27.</td>
</tr>
<tr>
<td>N1</td>
<td>.69515E-04</td>
<td>N2</td>
<td>.97028E-05</td>
<td>N3</td>
<td>.70027E-05</td>
</tr>
<tr>
<td>MD</td>
<td>.15000E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>25.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHMODT</td>
<td>-1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FP1</td>
<td>.09317</td>
<td>FP2</td>
<td>.06811</td>
<td>FP3</td>
<td>.05840</td>
</tr>
<tr>
<td>BET10</td>
<td>112.2552</td>
<td>BET20</td>
<td>145.0978</td>
<td>BET30</td>
<td>164.6050</td>
</tr>
<tr>
<td>PSI1TD</td>
<td>305.6017</td>
<td>TEST11</td>
<td>4.4495</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSI1TD</td>
<td>343.6259</td>
<td>TEST12</td>
<td>42.4736</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHI1TD</td>
<td>113.5016</td>
<td>PSI1TD</td>
<td>305.6017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHI1ID</td>
<td>115.9930</td>
<td>PSI1ID</td>
<td>292.4283</td>
<td>PHI1FD</td>
<td>107.4216</td>
</tr>
<tr>
<td>PSI2TD</td>
<td>286.9442</td>
<td>TEST21</td>
<td>29.4719</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSI2TD</td>
<td>312.9765</td>
<td>TEST22</td>
<td>4.2377</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHI2TD</td>
<td>142.0461</td>
<td>PSI2TD</td>
<td>312.0725</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHI2ID</td>
<td>136.7628</td>
<td>PSI2ID</td>
<td>330.9521</td>
<td>PHI2FD</td>
<td>150.1181</td>
</tr>
<tr>
<td>PSI3TD</td>
<td>357.5003</td>
<td>TEST31</td>
<td>4.2938</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSI3TD</td>
<td>25.1346</td>
<td>TEST32</td>
<td>31.0282</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHI3TD</td>
<td>166.7857</td>
<td>PSI3TD</td>
<td>357.5003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PHI3ID</td>
<td>173.6791</td>
<td>PSI3ID</td>
<td>336.0610</td>
<td>PHI3FD</td>
<td>160.3457</td>
</tr>
<tr>
<td>PHI1</td>
<td>PHI2</td>
<td>PHI3</td>
<td>PSI1</td>
<td>PSI2</td>
<td>PSI3</td>
</tr>
<tr>
<td>116.</td>
<td>137.</td>
<td>174.</td>
<td>292.</td>
<td>331.</td>
<td>337.</td>
</tr>
<tr>
<td>108</td>
<td>140</td>
<td>167</td>
<td>336</td>
<td>320</td>
<td>357</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>108</td>
<td>140</td>
<td>167</td>
<td>336</td>
<td>320</td>
<td>358</td>
</tr>
<tr>
<td>108</td>
<td>141</td>
<td>166</td>
<td>337</td>
<td>320</td>
<td>359</td>
</tr>
<tr>
<td>108</td>
<td>141</td>
<td>166</td>
<td>337</td>
<td>319</td>
<td>1</td>
</tr>
<tr>
<td>108</td>
<td>141</td>
<td>165</td>
<td>337</td>
<td>319</td>
<td>2</td>
</tr>
<tr>
<td>108</td>
<td>141</td>
<td>165</td>
<td>337</td>
<td>318</td>
<td>3</td>
</tr>
<tr>
<td>107</td>
<td>141</td>
<td>164</td>
<td>337</td>
<td>318</td>
<td>5</td>
</tr>
<tr>
<td>107</td>
<td>141</td>
<td>164</td>
<td>337</td>
<td>317</td>
<td>6</td>
</tr>
</tbody>
</table>

CYCLE EFFICIENCY = .491
2. **Program CLOCK 4: Point and Cycle Efficiencies for Two Pass Clock (Oxival) Step-Up Gear Train in Spin Environment**

The kinematics of program CLOCK 4 is again based on the work in Appendix G. The moment input-output relationships are derived in section 2 of Appendix H. This program is in many ways very similar to CLOCK 3 with the exception that only two meshes are involved, and therefore, wherever possible, reference will be made to CLOCK 3. Again, it is assumed that the two meshes will have been tested by program CLOCK 1 for their geometric suitability. The format of the following is identical to that used in section 1 of this appendix. For the sake of clarity, it will be helpful to refer to these parallel descriptions.

a. **Input Parameters (see Program CLOCK 4, below)**

The following parameters represent the input data for the program (for explanation, refer to section 1a of this appendix):

- $\mu$ 
- RPM 
- $\text{CAPRP}_1$, $\text{CAPRP}_2$, $RP_2$, $RP_3$ 
- $\text{RHO}_2$, $\text{RHO}_2$, $\text{RHOP}_1$, $\text{RHOP}_2$ 
- $\text{ACG}_1$, $\text{ACG}_2$, $\text{ACP}_1$, $\text{ACP}_2$ 
- $R_1$, $R_2$, $R_3$ 
- $TG_1$, $TG_2$, $TP_1$, $TP_2$ 
- $NG_1$, $NG_2$, $NP_2$, $NP_3$ 
- $\text{RHO}_1$, $\text{RHO}_2$, $\text{RHO}_3$
b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters
The required computations are identical to those in CLOCK 3.

II. Computation of MIN, GAMMAS and BETAS
The input moment is computed in the manner of eq. (I-1). In addition, the angles $\gamma_2$, $\gamma_3$, $\beta_1$ and $\beta_2$ are found according to the expressions given in section 6b of Appendix A.

III. Computation of Other Parameters
The computation of the angles $\Delta \varphi_1$ and $\Delta \psi_1$, the length $L_1$ as well as the centrifugal forces $Q_1$, $Q_2$ and $Q_3$ (called $Q_{3p}$ by eq. (H-245)) are identical to those described in the parallel section dealing with CLOCK 3.

IV. Preliminary Computations for Mesh 1
The preliminary computations for mesh 1 are identical to those given in section 1-IV of this appendix.

V. Preliminary Computations for Mesh 2
The preliminary computations for mesh 2 are identical to those given in section 1-V of this appendix.
VI. Gear Train Motion Model: Kinematics, Point and Cycle Efficiencies

The simulation of the gear train model, which is necessary for the determination of both POINTEF and CYCLEFF, is found in a loop starting with statement label no. 20 and ending with card no. 531. The motions of the individual driving gears are initialized at their respective angles PHI1I and PHI2I. The meshes will be in round on round contact until they reach their respective transition angles PHI1T and PHI2T. After the transition angles are passed, the meshes will be in round on flat contact. These regimes continue until the final angles PHI1F and PHI2F are reached.

The increment DDPHI1 of the input gear 1 is obtained from an adaptation of eqs. (A-207) and (A-208), in which tooth numbers, rather than base circle radii are used. The increment DDPHI2 of gear 2 is related to the increment of the pinion angle PSII.

While the motion of gear 1 is terminated when the angle PHI1 reaches the magnitude PHI1F (or rather PHI1F + DDPHI1 for moment summation purposes), gear 2 must be reset to its starting angle PHI2I whenever its final angle PHI2F has been reached.

The appropriate choice of moment equation depends upon which of the four possible combinations of contact conditions, as indicated by Table H-2, is applicable.

The following discusses the kinematics of the individual meshes as well as the determination of the point and cycle efficiencies where they differ from the description in section 1 of this appendix.
A. Kinematics

The program only utilizes the kinematics of meshes 1 and 2. These are identical with those for program CLOCK 3, as given in section 1.

B. Moment Computations, Point and Cycle Efficiencies

Regardless of the combination of contact conditions, the point efficiency is computed according to eq. (3), i.e.,

$$\eta_p = \text{POINTEF} = \frac{M_{031}}{M_{\text{in}}}$$  \hspace{1cm} (I-24)

where, with $\eta_1 = -1$

$$\frac{M_{031}}{M_{\text{in}}} \hspace{1cm} (I-25)$$

The cycle efficiency determination is based on eqs. (I-21) to (I-23).

The moment computations begin with the statement label no. 24, and initially consist of the determination of selected variables between A11 and A72 and selected variables between C6 and C36, as applicable to the analyses of section 2 of Appendix H. The governing contact combination (see also Table H-2) is determined with the help of the four moment control statements, which start with card no. 498. Once the appropriate combination is established, the program is directed to one of the four associated moment expressions. These expressions for $M_{031}$ coincide with those given by eqs. (H-260), (H-261), (H-277) and (H-278). They are listed in the above...
order beginning with statement label no. 25 and ending with
statement label no. 28.

The rationale of the control statements for meshes 1 and 2
is identical to that given for program CLOCK 3 (see section
1-VIIB of this appendix).

c. **Output (see Program CLOCK 4, below)**

The output of the program is best explained with the help
of the sample problem at the end of the program.

### I. Input Parameters

#### Mesh 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRP1</td>
<td>0.47725</td>
<td>in. (1.212 cm)</td>
</tr>
<tr>
<td>RP2</td>
<td>0.09085</td>
<td>in. (0.231 cm)</td>
</tr>
<tr>
<td>GCG1</td>
<td>0.47725</td>
<td>in. (1.212 cm)</td>
</tr>
<tr>
<td>ACP1</td>
<td>0.09085</td>
<td>in. (0.231 cm)</td>
</tr>
<tr>
<td>RHOG1</td>
<td>0.03870</td>
<td>in. (0.098 cm)</td>
</tr>
<tr>
<td>RHOPI</td>
<td>0.01740</td>
<td>in. (0.044 cm)</td>
</tr>
<tr>
<td>TG1</td>
<td>0.03480</td>
<td>in. (0.088 cm)</td>
</tr>
<tr>
<td>TP1</td>
<td>0.02800</td>
<td>in. (0.071 cm)</td>
</tr>
<tr>
<td>N1</td>
<td>n1 = 42</td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>n2 = 8</td>
<td></td>
</tr>
</tbody>
</table>

#### Mesh 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRP2</td>
<td>0.20670</td>
<td>in. (0.525 cm)</td>
</tr>
<tr>
<td>RP3</td>
<td>0.06890</td>
<td>in. (0.175 cm)</td>
</tr>
<tr>
<td>GCG2</td>
<td>0.20670</td>
<td>in. (0.525 cm)</td>
</tr>
<tr>
<td>ACP2</td>
<td>0.06890</td>
<td>in. (0.175 cm)</td>
</tr>
</tbody>
</table>

I-54
RHOG2 = ρG2 = .02070 in. (0.053 cm)
RHOP2 = ρP2 = .01040 in. (0.026 cm)
TG2 = tG2 = .02520 in. (0.064 cm)
TP2 = tP2 = .02080 in. (0.053 cm)
NG2 = nG2 = 27
NP3 = nP3 = 9

In addition
MU = .2
RPM = 1000
M1 = m1 = .69515 \times 10^{-4} \text{ lb-sec}^2/\text{in.} (12.171 g)
M2 = m2 = .97028 \times 10^{-5} \text{ lb-sec}^2/\text{in.} (1.699 g)
M3 = m3 = .10780 \times 10^{-5} \text{ lb-sec}^2/\text{in.} (0.189 g)
R1 = R_1 = .750 in. (1.905 cm)
R2 = R_2 = .750 in. (1.905 cm)
R3 = R_3 = .750 in. (1.905 cm)
RHO1 = ρ_1 = .060 in. (0.152 cm)
RHO2 = ρ_2 = .030 in. (0.076 cm)
RHO3 = ρ_3 = .025 in. (0.051 cm)
MD = md^2 = .15 \times 10^{-4} \text{ lb-sec}^2 \text{ in.} (16.944 g - \text{cm}^2)
K = 25

II. Computed Values

At the beginning of the output, one finds MIN = M_n. Subsequently, the following are listed for each mesh:

f_p1, the length of the pinion flats
β_1, the fuse body pivot to pivot line angles
\( \psi_{T_1} \) and \( \varphi_{T_1} \), the transition angles as well as the associated subsidiary tests

\( \varphi_{INI} \) and \( \psi_{INI} \), the initial angles

\( \varphi_{FINi} \) and \( \psi_{FINi} \), the final angles

Finally, for the full range of the input angle \( \varphi_1 \), the point efficiency \( \text{POINTEF} \) is listed, in addition to other parameters which are useful for checking purposes. Note that \( \text{DPSI}_1 \) and \( \text{DPSI}_2 \) represent \( \psi_1 \) and \( \psi_2 \), respectively. The cycle efficiency \( \text{CYCLEFF} \) is found at the end of the output.
Program CLOCK 4
1  C  PROGRAM CLOCK4 (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)  A  1

2  C  POLY1: AND CYCLE EFFICIENCIES FOR TWO PASS CLOCK (QGIVAL) STEP-UP  A  2

3  C  GEAR TRAIN IN SPIN ENVIRONMENT  A  3

4  C  REAL MU, LAMDA1, LAMDA2, LX1, LY1, LX2, LY2, LP, NP2, NP3, NG1, NG2, MIN, M1
   1, J2, M3, MO31, MO32, MO33, MO34, MT0, MO, LL1, LL2, K  A  4

5  C  READ (5.40) MU, RPM, CAPRP1, CAPRP2, RP2, RP3, AG1, AG2, ACM1, ACM2, ACM3, ACM4
   1, 3, 14, 15  A  5

6  C  READ (5.58) R1, R2, R3  A  6

7  C  READ (5.41) RHOG1, RHOG2, RHOG3, RHOP2  A  10

8  C  READ (5.42) TG1, TG2, TP1, TP2  A  11

9  C  READ (5.43) NG1, NG2, NP2, NP3  A  12

10  C  READ (5.61) M1, M2, M3  A  13

11  C  READ (5.44) RH01, RH02, RH03, M0, K, J1, J2  A  14

12  C  F1=3.14159  A  15

13  C  Z=PI/180.  A  16

14  C  OMEGA=RH0M2+PI/60.  A  17

15  C  DM2=OMEGA*OMEGA  A  18

16  C  PHOCT1=1.  A  19

17  C  COMPUTATION OF GEAR TOOTH PARAMETERS FOR BOTH MESHES  A  21

18  C  CXG1=RHOG1-TG1/2.  A  22

19  C  DELG1=ASIN(CXG1/CAPRP1)  A  23

20  C  CXP1=RHOP1-TG1/2.  A  24

21  C  DELP1=ASIN(CXP1/RP2)  A  25

22  C  GAMMA1=ASIN(RHOG1/RP2)  A  26

23  C  ALPH1=GMMP1-DELMP1  A  27

24  C  FP1=ACMP1*COS(GAMMA1)  A  28

25  C  BX=CAPRP1+RP2  A  29

26  C  CM2=RHOG2-TG2/2.  A  30

27  C  DEP1=ASIN(CXG2/CAPRP2)  A  31

28  C  CXP2=RHOP2-TG2/2.  A  32

29  C  DELP2=ASIN(CXP2/RP3)  A  33

30  C  GAMMA2=ASIN(RHOP2/RP3)  A  34

31  C  ALPH2=GMMP2-DELMP2  A  35

32  C  FP2=ACMP2*COS(GAMMA2)  A  36

33  C  B2=CAPRP2+RP3  A  37

34  C  COMPUTATION OF MIN, GAMMAS AND BETAS  A  38

35  C  MIN=V0*OM2  A  39

36  C  DELTA1=ACOS(((CAPRP1+RP2)+(CAPRP1+RP2)+R1+R1-R2+R2)/(2.*R1+(CAPRP1
   1+RP2)))  A  40

37  C  DELTA2=ACOS(((CAPRP2+RP3)+(CAPRP2+RP3)+R2+R2-R3+R3)/(2.*R2+(CAPRP2
   1+RP3)))  A  41

38  C  GAMMA2=ACOS((R1+R1+R2-R2-R2+R2-R2+R2)/(2.*R1+R1))  A  42

39  C  GAMMA3=GMMP2+GMMP3  A  43

40  C  BETA1=PI-DELTA1  A  44

41  C  BETA2=GMMA2+PI-DELTA2  A  45

42  C  BETA2=GMMA2+PI-DELTA3  A  46

43  C  BETA1+Z  A  47

44  C  BETA2+Z  A  48

45  C  BETA2D=BETA1+Z  A  49

46  C  BETA2D=BETA2D+BETA2  A  50

47  C  BETA2D=BETA2D+BETA2  A  51

48  C  BETA2D=BETA2D+BETA2  A  52

49  C  BETA2D=BETA2D+BETA2  A  53

50  C  BETA2D=BETA2D+BETA2  A  54
PROGRAM CLOCK4  74/74  OPT=1  FTN 4.6+420  07/31/79  11.35.22  PAGE 2

55  WRITE (6,45) MIN,NU,RPM,CAPR1,CPAPR2,RP2,RP3,ACG1,ACG2,ACP1,ACP2  A 54
WRITE (6,59) R1,R2,R3  A 55
WRITE (6,46) RHG1,RHG2,RHDP1,RHDP2  A 56
WRITE (6,47) T1,TG2,FP1,FP2  A 57
WRITE (6,48) NG1,NG2,MP3  A 58
WRITE (6,62) M1,M2,M3  A 59
WRITE (6,49) RHG1,RHG2,RH3,WD,K,PHDOT1  A 60
WRITE (6,60) FP1,FP2  A 61
WRITE (6,50) BETAI0,BETA2D  A 62

60  COMPUTATION OF OTHER PARAMETERS  A 63

65  DPHI1=360./NG1+2  A 64
DPHI2=360./MP3+2  A 65

70  L1=RHG1+RHD1  A 66
L2=RHG2+RHD2  A 67
Q1=M1+R1-OM2  A 68
Q2=M2+R2-OM2  A 69
Q3=M3+R3-OM2  A 70

75  PRELIMINARY COMPUTATIONS FOR MESH 1  A 71

DETERMINATION OF TRANSITION ANGLE OF MESH 1  A 72

80  A1=RHG1-COS(BETA1+ALPH1)+FP1*INCOSBETA1+ALPH1  A 73
B1=RHG1+SIN(BETA1+ALPH1)+FP1*INCSBETA1+ALPH1  A 74
C1=(ACG1+ACG1-RHGO1-RHDP1-B1*FP1+FP1)/(2.*B1)  A 75
ROOT1=A1+B1*+B1-B1*CT1-C1  A 76
Y11=A1+SORT(ROOT1)  A 77
Y12=A1+SORT(ROOT1)  A 78
X11=B1+CT1  A 79
PS1112.2=ATAN2(Y111,X11)  A 80
PS1122.2=ATAN2(Y122,X11)  A 81

90  PSITE11=PSITE11+PSITE1  A 82
PSITE12=PSITE1  A 83
PSITE12=PSITE12+2.-PI  A 84
IF (PSITE12.GT.PI) PSITE11=PSITE11+2.-PI  A 85
IF (PSITE11.LT.PI) PSITE11=PSITE11+2.-PI  A 86
IF (PSITE12.LT.PI) PSITE12=PSITE12+2.-PI  A 87

100  IF (PSITE11.GE.O.) TEST11=ABS(PI-BETA1+PSITE11-ALPH1)/Z  A 88
IF (PSITE11.LT.O.) TEST11=ABS(PSITE11-ALPH1)/Z  A 89
IF (PSITE11.GE.O.) TEST12=ABS(PI-BETA1-PSITE12+ALPH1)/Z  A 90
IF (PSITE11.LT.O.) TEST12=ABS(PSITE12+ALPH1)/Z  A 91
IF (PSITE11.LT.O.) PSITE11=PS1112.2=PI  A 92
PS1112=PS1112+2.-PI  A 93
WRITE (6,31) PS1112,TEST11  A 94
WRITE (6,32) PS1122,TEST12  A 95

105  CALL TRANS1 (RHGO1,ALPH1,BETA1,FP1,ACG1,B1,DELG1,Z,PS111,PH111,  A 96
1G111)  A 97
IF (G11.GT.FP1) GO TO 2
PHI1=PHI1+1
PSI1=PSI1+1
GO TO 4

2 CALL TRANS1 (RHOG1,ALPH1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T2,PHI1T2, B1G12) A 112
IF (G12.LT.FP1) GO TO 3
WRITE (6,51) A 113
STOP

3 PHI1=PHI1+2
PSI1=PSI1+2

4 IF (PHI1.LT.0.) PHI1=PHI1+2.*PI A 118
IF (PSI1.LT.0.) PSI1=PSI1+2.*PI A 119
PHI1T=PHI1/Z A 120
PSI1T=PSI1/Z A 121
WRITE (6,52) PHI1T,PSI1T A 122

C DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 1 A 124
C
A1F=ACG1+COS(PHI1+DELG1+ALPH1)-B1*COS(BETA1+ALPH1) A 125
B1F=ACG1+SIN(PHI1+DELG1+ALPH1)+B1*5.*SIN(BETA1+ALPH1) A 127
CIF=RHOG1 A 128
ROOT1F=A1F=A1F+B1F*B1F-CIF*CIF A 129
Y1F=A1F+SQRT(ROOT1F) A 130
Y1F2=A1F-SQRT(ROOT1F) A 131
X1F=B1F+CIF A 132
PS1IF1=F.ATAN(Y1F,X1F) A 133
PS1IF2=F.ATAN(Y1F2,X1F) A 134

IF A1F.LT.0. PS1IF1=PS1IF1+2.*PI A 135
IF A1F.2. PS1IF2=PS1IF2+2.*PI A 136
IF ABS(PS1IF1-PSI1T).LT.ABS(PS1IF2-PSI1T) GO TO 5 A 137
SIGN1=1. A 138
GO TO 6 A 139

5 SIGN1=-1. A 140

C COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 1 A 142.

6 DO 7 I=1,2000 A 143
PHI1=PHI1T-(I-1.)/100. A 144
PHI1=PHI1+1 A 145
A1F=ACG1+COS(PHI1+DELG1+ALPH1)-B1*COS(BETA1+ALPH1) A 147
B1F=ACG1+SIN(PHI1+DELG1+ALPH1)+B1*SIN(BETA1+ALPH1) A 148
CIF=RHOG1 A 149
ROOT1F=A1F=A1F+B1F*CIF C1F A 150
Y1F=A1F+SIGN1*SQRT(ROOT1F) A 151
X1F=B1F*CIF A 152
PS1IF1=F.ATAN(Y1F,X1F) A 153
IF PS1IF1.LT.0. PS1IF1=PS1IF1+2.*PI A 154
LX1=1.0+ACG1+ACG1+0.0+DELG1-Z A 155
DLG1 A 156
LY1=1.0+B1*LX1+LY1 A 157
LX1=SQRT(LX1+LY1) A 158

PAGE 3
DELEL = LL1 = L1
IF (DELE1 .LE. 0.) GO TO 8
7 CONTINUE
8 PHIFI = PHI1
PSI1FF = PSI1F
PHI1F = PHI1F + DPHI1
PSI1F = PSI1FF - DPSI1
IF (PSI1F .LT. 0.) PSI1F = PSI1F + 2. * PI
PHI1F = PHI1F / Z
PSI1F = PSI1F / Z
PHIFI = PHI1F / Z
PSI1FF = PSI1FF / Z
WRITE (6,53) PHI1ID, PSI1ID, PHI1FD, PSI1FD

C DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 1
A1R = ACG1 + SIN(PI1I + DELGI - DELP1) - B1 * SIN(BETA1 - DELP1)
B1R = ACG1 + COS(PI1I + DELGI - DELP1) - B1 * COS(BETA1 - DELP1)
C1R = (ACP1 + ACP1 + ACG1 + ACG1 + B1 - L1 * L1 - 2. * ACG1 + B1 * COS(PI1I + DELGI - BE)
1TA1I) / (2. * ACP1)

ROOTR = A1R + A1R + B1R + B1R - C1R + C1R
Y1R = A1R + SQRT(ROOTR)
Y1R2 = B1R - SQRT(ROOTR)
X1R = B1R + C1R
PSI1RI = PI1I + ATAN2(Y1R, X1R)
PSI1RI2 = PSI1RI + 2.

IF (PSI1RI - PSI1RI2) .LT. 0.

IF (ABS(PSI1RI - PSI1RI2)) .LT. ABS(PSI1RI - PSI1RI2)
GO TO 1
SIGNR = -1.

GO TO 10

9 SIGNR = 1.

C PRELIMINARY COMPUTATIONS FOR MESH 2

C DETERMINATION OF TRANSITION ANGLE OF MESH 2
A2T = - RHOG2 + COS(BETA2 - ALPHP2) + FP2 * SIN(BETA2 - ALPHP2)
B2T = RHOG2 + SIN(BETA2 - ALPHP2) + FP2 * COS(BETA2 - ALPHP2)

K0012T = A2T + B2T + B2T + C2T + C2T
Y2T1 = A2T + SQRT(ROOT12T)
Y2T1 = A2T + SQRT(ROOT12T)
Y2T2 = B2T + C2T
PSI21T = ATAN2(Y2T1, X2T)
PSI22T = PSI21T + 2.

IF (PSI21T .GT. PI) PSI21T = PSI21T - 2.
IF (PSI22T .LT. PI) PSI22T = PSI22T + 2.

IF (PSI21T .LE. PI) PSI21T = PSI21T + 2.
IF (PSI22T .GE. PI) PSI22T = PSI22T - 2.
IF (PS1E21.LT.0.) TEST21=ABS(PI+BETA2-(PS1E21+2.*PI+ALPHAP2))/Z A 213
IF (PS1E22.GE.0.) TEST22=ABS(PI-BETA2+PS1E22+ALPHAP2)/Z A 214
IF (PS1E22.LT.0.) TEST22=ABS(PI+BETA2-(PS1E22+2.*PI+ALPHAP2))/Z A 215
IF (PS1I2T.LT.0.) PSI12T=PSI12T+2.*PI A 216
IF (PS1I2T.LT.0.) PSI12T=PSI12T+2.*PI A 217
PSI12TD=PSI12T/Z A 218
PSI12TD=PSI12T/Z A 219
WRITE (6,33) PSI12TD,TEST21 A 220
WRITE (6,34) PSI12TD,TEST22 A 221
CALL TRANS2 (RHOG2,ALPHAP2,BETA2,FP2,ACG2,B2,DELG2,Z,PS1I2T,PHI2T1, A 222
1G21) A 223
1G21
IF (G21.GT.FP2) GO TO 11 A 224
PHI12=PHI12T1 A 225
PSI12T=PSI12T1 A 226
GO TO 12 A 227
A 228
11 CALL TRANS2 (RHOG2,ALPHAP2,BETA2,FP2,ACG2,B2,DELG2,Z,PS1I2T,PHI2T2, A 229
1G22) A 230
IF (G22.LT.FP2) GO TO 12 A 230
WRITE (6,54) A 231
STOP A 232
A 233
12 PHI12=PHI12T2 A 234
PSI12T=PSI12T2 A 235
A 236
12 IF (PHI12T.LT.0.) PHI12=PHI12T+2.*PI A 237
IF (PS1I2T.LT.0.) PSI12T=PSI12T+2.*PI A 238
GO TO 12 A 239
A 240
A 241
Determination of correct sign for round on flat regime of mesh 2
A 242
A 243
A 244
A 245
A 246
A 247
A 248
A 249
A 250
A 251
A 252
A 253
A 254
A 255
A 256
A 257
A 258
A 259
A 260
Computation of final and initial values of phi and psi for mesh 2
A 261
A 262
A 263
A 264
A 265
DO 16 I=1,1000 A 266
PHI12=PHI12TD+(I-1.)/100. A 267
PHI12=PHI12T I=1. A 268
A2F=ACG2+COS(PHI12-DELG2-ALPHA2)-B2+COS(BETA2-ALPHA2)
B2F=ACG2+SIGN2+SORT(ROOT2F)
X2F=B2F+C2F
PS12F=2.ATAN2(Y2F,X2F)
IF (PS12F.LT.0.) PS12F=PS12F+2.*PI
LX2=2.*COS(BETA2)+ACP2+COS(PS12F+DPS12-DELP2)-ACG2+COS(PHI22-DELP2)
LY2=2.*COS(BETA2)+ACP2+COS(PS12F+DPS12-DELP2)-ACG2+COS(PHI22-DELP2)
L2=SORT(LX2+LY2+LY2)
DELEL2=L2-L2
IF (DELEL2.LE.0.) GO TO 17
CONTINUE

A2F=ACG2+COS(PHI12-DELG2-ALPHA2)-B2+COS(BETA2-ALPHA2)
B2F=ACG2+SIGN2+SORT(ROOT2F)
X2F=B2F+C2F
PS12F=2.ATAN2(Y2F,X2F)
IF (PS12F.LT.0.) PS12F=PS12F+2.*PI
LX2=2.*COS(BETA2)+ACP2+COS(PS12F+DPS12-DELP2)-ACG2+COS(PHI2-DELP2)
LY2=2.*COS(BETA2)+ACP2+COS(PS12F+DPS12-DELP2)-ACG2+COS(PHI2-DELP2)
L2=SORT(LX2+LY2+LY2)
DELEL2=L2-L2
IF (DELEL2.LE.0.) GO TO 17
CONTINUE
IF (PHI1.LE.PHI1F+DDPHI1) GO TO 30
A 319
A 320
MESH 1
A 321
A 322
A 323
A 324
A 325
A 326
A 327
A 328
A 329
A 330
IF (PHI1.LE.PHI1T) GO TO 21
A 331
AIR=ACG1+SIGN(PI1)+DELG1+DELPI)-B1+SIGN(BETA1-DELPI)
A 332
A 333
A 334
A 335
A 336
A 337
A 338
A 339
A 340
A 341
A 342
A 343
A 344
A 345
A 346
A 347
A 348
A 349
A 350
A 351
A 352
A 353
A 354
A 355
A 356
A 357
A 358
A 359
A 360
A 361
A 362
A 363
A 364
A 365
A 366
A 367
A 368
A 369
A 370
A 371
IF (PHI1.LE.PHI11) GO TO 30
A 372
IF (PHI1.LE.PHI11F+DDPHI11) GO TO 30
A 373
A 374
A 375
A 376
A 377
A 378
A 379
A 380
A 381
A 382
A 383
A 384
A 385
A 386
A 387
A 388
A 389
A 390
A 391
A 392
A 393
A 394
A 395
A 396
A 397
A 398
A 399
A 400
A 401
A 402
A 403
A 404
A 405
A 406
A 407
A 408
A 409
A 410
A 411
A 412
A 413
A 414
A 415
A 416
A 417
A 418
A 419
A 420
A 421
A 422
A 423
A 424
A 425
A 426
A 427
A 428
A 429
A 430
A 431
A 432
A 433
A 434
A 435
A 436
A 437
A 438
A 439
A 440
A 441
A 442
A 443
A 444
A 445
A 446
A 447
A 448
A 449
A 450
A 451
A 452
A 453
A 454
A 455
A 456
A 457
A 458
A 459
A 460
A 461
A 462
A 463
A 464
A 465
A 466
A 467
A 468
A 469
A 470
A 471
A 472
A 473
A 474
A 475
A 476
A 477
A 478
A 479
A 480
A 481
A 482
A 483
A 484
A 485
A 486
A 487
A 488
A 489
A 490
A 491
A 492
A 493
A 494
A 495
A 496
A 497
A 498
A 499
A 500
A 501
A 502
A 503
A 504
A 505
A 506
A 507
A 508
A 509
A 510
A 511
A 512
A 513
A 514
A 515
A 516
A 517
A 518
A 519
A 520
A 521
A 522
A 523
A 524
A 525
A 526
A 527
A 528
A 529
A 530
A 531
A 532
A 533
A 534
A 535
A 536
A 537
A 538
A 539
A 540
A 541
A 542
A 543
A 544
A 545
A 546
A 547
A 548
A 549
A 550
A 551
A 552
A 553
A 554
A 555
A 556
A 557
A 558
A 559
A 560
A 561
A 562
A 563
A 564
A 565
A 566
A 567
A 568
A 569
A 570
A 571
A 572
A 573
A 574
A 575
A 576
A 577
A 578
A 579
A 580
A 581
A 582
A 583
A 584
A 585
A 586
A 587
A 588
A 589
A 590
A 591
A 592
A 593
A 594
A 595
A 596
A 597
A 598
A 599
A 600
A 601
A 602
A 603
A 604
A 605
A 606
A 607
A 608
A 609
A 610
A 611
A 612
A 613
A 614
A 615
A 616
A 617
A 618
A 619
A 620
A 621
A 622
A 623
A 624
A 625
A 626
A 627
A 628
A 629
A 630
A 631
A 632
A 633
A 634
A 635
A 636
A 637
A 638
A 639
A 640
A 641
A 642
A 643
A 644
A 645
A 646
A 647
A 648
A 649
A 650
A 651
A 652
A 653
A 654
A 655
A 656
A 657
A 658
A 659
A 660
A 661
A 662
A 663
A 664
A 665
A 666
A 667
A 668
A 669
A 670
A 671
A 672
A 673
A 674
A 675
A 676
A 677
A 678
A 679
A 680
A 681
A 682
A 683
A 684
A 685
A 686
A 687
A 688
A 689
A 690
A 691
A 692
A 693
A 694
A 695
A 696
A 697
A 698
A 699
A 700
A 701
A 702
A 703
A 704
A 705
A 706
A 707
A 708
A 709
A 710
A 711
PROGRAM CLOCK4 74/74 OPT=1 FTK 4.6+420 07/31/79 11.35.22 PAGE 10

C14=MU*RH01*(A28+A30) C15=MU*RH01*(A27+A29)+MU*S1F*RH01+ACG1+(MU*SF+SM)PH11+DELG1=PSI A478
A479
11+1PH11)=COS(PH11+DELG1=PSI+ALPHA1) A480
C19=MU*RH22+(A37+440)+ACG2+COS(PH11-DELG2-PSI12-ALPHA2)+MU+S2F*SI A481
1N(PH11-DELG2-PSI12-ALPHA2)+MU+S2F*RH02 A482
C20=MU*RH02+(A39+A42) A483
C21=MU*RH02+(A38+A41)+G1 A484
C22=MU*RH02+(A44+A46)+ACG2+COS(PH11-DELG2-PSI12-ALPHA2)+MU+S2F*SI A485
1N(PH11-DELG2-PSI12-ALPHA2)+MU+S2F*RH02 A486
C23=MU*RH02+(A45+A48) A487
C24=MU*RH02+(A44+A47)+ACP1+(MU*SF+SM)(PSI11+DELPI-LAMDA1)-SIN(PHI1) A488
11+1DELPI-LAMDA1))=MU+S1R=RH01 A489
C33=MU*RH03+(A66+A68) A490
C34=MU*RH03+(A65+A67)-MU+S2R=RH02+ACP2+(MU+S2R+COS(PSI2=DEL2-PSI2-DEL2-LAMDA2)) A491
102=1SIN(PS12-DEL2=PSI12-LAMDA2)) A492
C35=MU*RH03+(A70+A72) A493
C36=MU*RH03+(A69+A71)-G3 A494
C495
C496
C497
C498
C499
C500
C501
C502
C503
C504
C505
C506
C507
C508
C509
C510
C511
C512
C513
C514
C515
C516
C517
C518
C519
C520
C521
C522
C523
C524
C525
C526
C527
C528

I.7

IF ((PH11.GE.PHI11).AND.(PH12.LE.PHI12)) GO TO 25
IF ((PH11.LE.PHI11).AND.(PH12.GE.PHI12)) GO TO 26
IF ((PH11.LE.PHI11).AND.(PH12.LE.PHI12)) GO TO 27
IF ((PH11.GE.PHI11).AND.(PH12.LE.PHI12)) GO TO 28

I.8

M03=MIN+(C34/(C6+C10))=Q1+C8+C9+C34/(C6+C10)-Q2+C7+C34/C6-Q=3+C3 A505
M03=M031
POINTEF=ABS(P50DT2)=M03/MIN
WRITE (6,36) PHI10,PH120,PS110,PS010,PS001,PS012,PS12,PS12,S1R,S2R,POINTEF
GO TO 29

M03=M032
POINTEF=ABS(P50DT2)=M03/MIN
WRITE (6,37) PHI10,PH120,PS110,PS010,PS001,PS012,PS2R,PS1F,G1,POINTEF
GO TO 29

M03=M031
POINTEF=ABS(P50DT2)=M03/MIN
WRITE (6,38) PHI10,PH120,PS110,PS010,PS001,PS012,PS1F,G1,POINTEF
GO TO 29

M03=M032
POINTEF=ABS(P50DT2)=M03/MIN
WRITE (6,39) PHI10,PH120,PS110,PS010,PS001,PS012,PS1F,G1,POINTEF
GO TO 29
PROGRAM CLOCE4        74/74  OPT=1

GO TO 20
30 CYCLEFF=1DIT=OPH1111/PHI1F-PM111)
WRITE(6,57) CYCLEFF
1DIT=0.
525 IF (ISTOP.NE.0) GO TO 1
STOP

540 C

31 FORMAT (6X,9HPSI1ID =,F9.4,3X,BMTEST11 =,F9.4)
32 FORMAT (6X,9HPSI112D =,F9.4,3X,BMTEST12 =,F9.4/)  
33 FORMAT (6X,9HPSI22D =,F9.4,3X,BMTEST21 =,F9.4)
34 FORMAT (6X,9HPSI22D =,F9.4,3X,BMTEST12 =,F9.4/)  
35 FORMAT (8SM PHI1 PHI2 PS11 P512 DPS12 S1R S2R $  
11F G1 S2F G2 POINTEF)
545 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),2(F3.0,2X),24X,F5.3)
546 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),5X,F3.0,2X,F3.0,2X,F5.3,14X,F5.3)
547 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),10X,2(F3.0,2X,F3.0,2X,F5.3,2X,F5.3)
548 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),F3.0,19X,F3.0,2X,F5.3,2X,F5.3)
549 FORMAT (F10.3,F10.0/F10.5/F10.5/)
550 FORMAT (4F10.4)
551 FORMAT (4F10.4)
552 FORMAT (4F10.4)
553 FORMAT (3F10.4/F10.6/)
554 FORMAT (1H1,5X,SHMIN =,F8.4,3X,4HMIN =,F6.3,3X,SHRPM =,F6.0//EX,8HC
555 TAPPP =,F8.5,3X,OMCAPPP2 =,F8.5//EX,SHRPP2 =,F8.5,3X,SHRPP3 =,F8.5//  
556 26X,OMHAP1 =,F8.5,3X,OMHAP2 =,F8.5,3X,OMHAP1 =,F8.5,3X,OMHAP2 =,F8.  
557 35/)
558 FORMAT (6X,7HMRHG1 =,F8.5,3X,7HMRHG2 =,F8.5,3X,7HMRHG1 =,F8.5,3X,  
559 7HMRHG2 =,F8.5/)
560 FORMAT (6X,SHTG1 =,F8.5,3X,SHTG2 =,F8.5,3X,SHTP1 =,F8.5,3X,SHTP2 =  
561 1,F3.5/)
562 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
563 1,F5.0/)
564 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
565 1,F5.0/)
566 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
567 1,F5.0/)
568 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
569 1,F5.0/)
570 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
571 1,F5.0/)
572 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
573 1,F5.0/)
574 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
575 1,F5.0/)
576 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
577 1,F5.0/)
578 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
579 1,F5.0/)
580 FORMAT (6X,5HSHG1 =,F3.0,3X,5HSHG2 =,F3.0,3X,5HSHP2 =,F3.0,3X,5HSP3 =  
581-
**SUBROUTINE TRANS1** 74/74 OPT=1

```
1
SUBROUTINE TRANS1 (RHOG,ALHPH,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)  B 1
PI=3.14159  B 2
ST=(-RHOG+COS(PSIT-ALPH)+B*SIN(PSIT-ALPH))/ACG  B 3
CT=(RHOG*SIN(PSIT-ALPH)+B*COS(PSIT-ALPH))/ACG  B 4
PHI=ATAN2(ST,CT)-DELG  B 5
PHI=EXT-PHI-.1+Z  B 6
AF=ACG*COS(PHINEXT+DELG+ALPH)-B*COS(BETA+ALPH)  B 7
BF=-ACG*SIN(PHINEXT+DELG+ALPH)+B*SIN(BETA+ALPH)  B 8
CF=RHOG  B 9
10
ROOTF=AF+BF+B*CF  B 10
YF1=AF+SQR{(ROOTF)  B 11
YF2=AF-SQR{(ROOTF)  B 12
XF=BF+CF  B 13
PSINEX2=2.*ATAN2(YF1, XF)  B 14
PSINEX=PSINEX2+YF1  B 15
IF (ABS(PSINEX-PSIT) LT. ABS(PSINEX-PSIT)) GO TO 1  B 16
IF (PSINEX1 LT. 0.) PSINEX1=PSINEX1+2.*PI  B 17
IF (PSINEX2 LT. 0.) PSINEX2=PSINEX2+2.*PI  B 18
PSINEX1=PSINEX1  B 19
20
GO TO 2  B 20
1 PSINEX=PSINEX1  B 21
2 G=(ACG*SIN(PHINEXT+DELG)+RHOG*COS(PSINEX-ALPH)-B*SIN(BETA))/SIN(1*PHINEXT-ALPH)  B 22
RETURN  B 23
END  B 24
```
SUBROUTINE TRANS2

1  SUBROUTINE TRANS2 (RHOG, ALPHP, BETA, FP, ACG, B, DELG, Z, PSIT, PHIT, G) C 1
2  PI=3.14159 C 2
3  ST=(RHOG+COS(ALPHP)+B+ SIN(BETA)+FP+SIN(PSIT+ALPHP))/ACG C 3
4  CT=-RHOG* SIN(PSIT+ALPHP)+B+COS(BETA)+FP+COS(PSIT+ALPHP))/ACG C 4
5  PHIT=ATAN2(ST, CT)+DELG C 5
6  PHINEXT=PHIT+.1*Z C 6
7  AF=ACG+COS(PHINEXT-DELG-ALPHP)-B+COS(BETA-ALPHP) C 7
8  BF=ACG+SIN(PHINEXT-DELG-ALPHP)+B+SIN(BETA-ALPHP) C 8
9  CF=-RHOG C 9
10  ROOTF(AF+BF)=BF-CF*CF C 10
11  YF1=AF+SQRT(ROOTF) C 11
12  YF2=AF-SQRT(ROOTF) C 12
13  XF=BF+CF C 13
14  PSINEX1=2.*ATAN2(YF1, XF) C 14
15  PSINEX2=2.*ATAN2(YF2, XF) C 15
16  IF (PSINEX1.LT.0.) PSINEXI=PSINEXI+2.*PI C 16
17  IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI C 17
18  IF (ABS(PSINEX1-PSIT).LT.1.*ABS(PSINEX2-PSIT)) GO TO 1 C 18
19  PSINEX1=PSINEX2 C 19
20  GO TO 2 C 20
1  PSINEX1=PSINEX1 C 21
2  G=(ACG+SIN(PHINEXT-DELG)-RHOG+COS(PSINEX1+ALPHP)-B+SIN(BETA))/SIN( C 22
3  PSINEX1+ALPHP) C 23
4  RETURN C 24
5  END C 25
MIN = 0.1645  MJ = 0.200  RPM = 1000
CAPRP1 = 0.47725  CAPRP2 = 0.20670
RP2 = 0.09095  RP3 = 0.06890
ACG1 = 0.47725  ACG2 = 0.20670  ACP1 = 0.09085  ACP2 = 0.06890
R1 = 0.75000  R2 = 0.75600  R3 = 0.75000
RHOG1 = 0.03870  RHOG2 = 0.03070  RHOF1 = 0.01740  RHOF2 = 0.01040
TG1 = 0.03400  TG2 = 0.02520  TP1 = 0.02800  TP2 = 0.02060
MG1 = 42.  MG2 = 27.  NP1 = 8.  NP2 = 9.
M1 = 0.69514E-04  M2 = 0.97028E-05  M3 = 0.10780E-05
RHO1 = 0.060  RHO2 = 0.030  RHO3 = 0.025
MD = 0.00015
K = 25.0
PHDOT1 = -1.0
FP1 = 0.08917  FP2 = 0.06811
BETA10 = 112.552  BETA20 = 145.0978
PSI11TD = 305.6017  TEST11 = 4.4485
PSI112D = 343.6259  TEST12 = 42.4736
PHI11TD = 113.5016  PHI11TD = 305.6017
PHI11FD = 115.9930  PHI11FD = 292.4283  PHI11FD = 107.4216  PHI11FD = 337.4283
PSI12TD = 206.9442  TEST21 = 29.4719
PSI122D = 312.0785  TEST22 = 4.3377
PHI12TD = 143.0461  PHI12TD = 312.0785
PHI12FD = 136.7828  PHI12FD = 330.9521  PHI12FD = 150.1164  PHI12FD = 290.9521
PHI1 Philip  PSI1  PSI2  DPSI1  DPSI2  S1R  S2R  S1F  G1  S2F  G2  PHIEF
115.  137.  292.  331.  5.  -16.  1.  -1.  .617
115.  137.  293.  329.  5.  -16.  1.  -1.  .626
115.  138.  294.  328.  5.  -16.  1.  -1.  .634
115.  139.  294.  326.  5.  -16.  1.  -1.  .642
115.  139.  295.  324.  5.  -16.  1.  -1.  .650
115.  139.  295.  323.  5.  -16.  1.  -1.  .657
115.  140.  296.  321.  5.  -16.  1.  -1.  .665
115.  141.  296.  320.  5.  -16.  1.  -1.  .672
115.  141.  297.  318.  5.  -16.  1.  -1.  .679
115.  142.  297.  316.  5.  -16.  1.  -1.  .686
115.  142.  298.  315.  5.  -16.  1.  -1.  .693
115.  143.  298.  313.  5.  -16.  1.  1.  .689
115.  143.  299.  312.  5.  -16.  1.  1.  .685
DISTRIBUTION LIST

Commander
U.S. Army Armament Research and Development Command
ATTN: DRDAR-LCN, F. Tepper (30)
       DRDAR-TSS (5)
Dover, NJ 07801

Commander
Harry Diamond Laboratories
ATTN: Library
       DRXDO-DAB, D. Overman
Washington, DC 20418

Defense Technical Information Center (2)
Cameron Station
Alexandria, VA 22314

Weapon System Concept Team/CSL
ATTN: DRDAR-ACW
Aberdeen Proving Ground, MD 21010

Technical Library
ATTN: DRDAR-CLJ-L
Aberdeen Proving Ground, MD 21010

Director
U.S. Army Ballistic Research Laboratory
ARRADCOM
ATTN: DRDAR-TSB-S (STINFO)
Aberdeen Proving Ground, MD 21005

Benet Weapons Laboratory
Technical Library
ATTN: DRDAR-LCB-TL
Watervliet, NY 12189

Commander
U.S. Army Armament Materiel Readiness Command
ATTN: DRSAR-LEP-L
Rock Island, IL 61299

Director
U.S. Army TRADOC Systems Analysis Activity
ATTN: ATAA-SL (Tech Lib)
White Sands Missile Range, NM 88002
This report has been delimited and cleared for public release under DOD Directive 5200.20 and no restrictions are imposed upon its use and disclosure.

Distribution Statement A

Approved for public release; distribution unlimited.