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DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE;
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A NEW FINITE ELEMENT SUPERSONIC KERNEL FUNCTION METHOD IN LIFTING SURFACE THEORY
USER'S MANUAL

LOCKHEED MISSILES & SPACE COMPANY, INC.
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APRIL 1976

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This technical report has been reviewed and is approved for publication.

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Chief, Structures Division

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20. ABSTRACT (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)
A new computational method based on the finite element approximation is applied to the kernel function formulation of the supersonic planar lifting surface theory. A computer program developed for the new method is described. The program is applicable to any general planform undergoing harmonic oscillation in a supersonic flow. The input data are the Mach number, reduced frequency and mesh information. The output consists of the lift distributions and a table of the generalized force coefficients.
FOREWORD

This report was prepared by personnel in the Engineering Sciences Section of the Lockheed Missiles & Space Company, Inc., Huntsville Research & Engineering Center, Huntsville, Alabama, for the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. The research study was performed under Contract F33615-75-C-3001. Capt. Gerald Van Keuren, AFFDL/FBR was the Air Force Project Engineer.

V. Y. C. Young was the principal investigator under the supervision of M. R. Brashears.

The theory for the method used in this computer program is documented as AFFDL-TR-76-3, Vol. I.
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LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^{(e)}$</td>
<td>weighted kernel coefficients</td>
</tr>
<tr>
<td>$f_i$</td>
<td>dimensionless modal functions</td>
</tr>
<tr>
<td>$i$</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>$K$</td>
<td>kernel function</td>
</tr>
<tr>
<td>$k$</td>
<td>reduced frequency, $\omega s/V$</td>
</tr>
<tr>
<td>$M$</td>
<td>freestream Mach number</td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>shape functions</td>
</tr>
<tr>
<td>$Q_{ij}$</td>
<td>generalized force coefficients</td>
</tr>
<tr>
<td>$Q'_{ij}$</td>
<td>real part of generalized force coefficients in AGARD notation</td>
</tr>
<tr>
<td>$Q''_{ij}$</td>
<td>imaginary part of generalized force coefficients in AGARD notion</td>
</tr>
<tr>
<td>$x$</td>
<td>nondimensional coordinate</td>
</tr>
<tr>
<td>$x_o$</td>
<td>running coordinate in $x$-direction</td>
</tr>
<tr>
<td>$y$</td>
<td>nondimensional coordinate</td>
</tr>
<tr>
<td>$y_o$</td>
<td>running coordinate in $y$-direction</td>
</tr>
<tr>
<td>$\lambda^{(e)}_i$</td>
<td>nodal lift vector at element level</td>
</tr>
</tbody>
</table>
SECTION I
INTRODUCTION

A new finite element supersonic kernel function method in lifting surface theory was presented in Ref. 1. This manual contains the Finite Element Supersonic Kernel Analysis Program (FESKAP), developed for the new method. Descriptions of the main program are presented as well as on the preparation of input necessary to execute the program. A sample run is included to illustrate the usage of the program. Descriptions of each subroutine are presented in Appendix A, and the program listing is contained in Appendix B.

The purpose of the computer program is to generate the generalized force coefficients at one specified Mach number and reduced frequency for a given planform and a given set of modal deflections. The program is applicable to any isolated arbitrary planform in supersonic flow with subsonic/supersonic leading/trailing edges. No thickness effect is accounted for. The unsteady motion is assumed to be harmonic for the analysis.

SECTION II
PROBLEM DESCRIPTION

According to Ref. 1, the finite element formulation of the integral equation in the lifting surface theory is given as

\[
\left( \frac{\partial}{\partial x} + ik \right) f_i(x, y) = \frac{1}{4\pi} \sum_{(e)} C_i^{(e)} \lambda_i^{(e)}
\]

(1)

where \( k \) is the reduced frequency, \( f_i(x, y) \) is the \( i^{\text{th}} \) modal function and \( \lambda_i^{(e)} \) is the column vector containing the nodal lift values of an element, due to a unit displacement in the \( i^{\text{th}} \) mode. \( \sum_{(e)} \) denotes summation over the elements within the forward Mach cone.

The row vector containing the integrated kernel coefficients is defined as

\[
C_i^{(e)} = \int_A N_i^t(x, y) \cdot K(x - x_o, y - y_o) \, dA
\]

(2)

where \( N_i^t(x, y) \) is the row vector of shape functions and \( K \) is the kernel function.

The generalized force coefficients are

\[
Q_{ij} = \sum_{i=1}^{N} \int_A f_i(x, y) \cdot \bar{N}_i^t(x, y) \, dx \, dy \cdot \bar{\lambda}_j^{(e)}
\]

(3)

where \( \sum_{i=1}^{N} \) denotes summation over all elements.

Equations (1), (2) and (3) form the framework for the computer program development.
SECTION III
PROGRAM DESCRIPTION

The program as presently set up is extremely compact. For example, for a case of 239 nodes constituting 222 elements, the program size is slightly under 20K (decimal) words. Variable dimensions are used in all the subroutines, so that the user needs only to change the first dimension statement in the main program to fit in a new planform. No overlay nor auxiliary file is used in this program.

For convenience, the mode shapes are built into the program. These are represented by the set of $1, x, x^2, y^2, x^2y^2, y, xy$ or the set of $1, x, x^2, y^2, y, xy$. The sets are identified by the number of modes they contain. The user has the option of specifying the first set (NMODE = 7) or the second set (NMODE = 6).

For a given planform and Mach number, the user must first define the characteristic mesh that best fits the planform. The nodes are then numbered starting from left to right with the foremost points and proceeding downstream. The elements are numbered in a similar manner. Fill-in triangular elements with horizontal sides at the supersonic trailing edge are numbered last since they cannot be an influencing element to any collocation point.

Influencing elements within the forward Mach cone of a collocation point are determined automatically by the program. The element containing the collocation point as its most downstream node is defined as the pivotal element. All elements with element number less than the pivotal element number are scanned. Thus all candidates are either forward of or on the same level as the pivotal element. Each in turn is further tested by a logic statement to see if it is within the Mach cone.
As explained in Ref. 1, a table of weighted kernel function coefficients is first tabulated for later table look-up during the solution process. The size of this table is governed by two parameters IMAX and JMAX. IMAX is the maximum number of characteristic elements in the chordwise direction as determined from the mesh. For most planforms, this is given by the number of elements on the centerline. To find JMAX, locate the most extreme collocation point, which is usually the one close to the tip of the trailing edge. JMAX is the number of layers of characteristic elements necessary to cover all the elements within the forward Mach cone.

To take advantage of the symmetry and anti-symmetry, the lift calculation is performed only on the right half of the planform. Each node has its associated mirror image with respect to the centerline. Lift values, with positive or negative sign depending on whether the mode is symmetric or anti-symmetric, are simply substituted in for the mirror node. For consistency, a node on the centerline has itself as the mirror node.

A schematic flow chart of the program is shown in Fig. 1.

**Description of Variables**

- **BETA** \( \beta = \sqrt{M^2 - 1} \)
- **COEF** Array of integrated kernel function coefficients
- **IANGLE** Number of angle of element; \( \text{IANGLE} = 3 \) for triangle and \( \text{IANGLE} = 4 \) for quadrilateral
- **IBUF** Buffer array for printing out the list of influencing element number
- **ICHECK** Option parameter for quick mesh check run
- **IMAX** Maximum number of regular characteristic elements in the chordwise direction
- **INFO** Array of element nodal information
- **JMAX** Maximum number of regular characteristic elements in the Mach line direction
- **KMAX** Number of element information data cards to be read in
Fig. 1 - Flow Chart for Main Program
Fig. 1 - (Concluded)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LENODE</td>
<td>Array containing the leading edge nodes</td>
</tr>
<tr>
<td>LIST1</td>
<td>Input namelist name</td>
</tr>
<tr>
<td>LMAX</td>
<td>Number of entries to the coefficient table; also the number of nodal information data card to be read in</td>
</tr>
<tr>
<td>MAXINT</td>
<td>Number of integration points for the modified Gauss-Chebyshev quadrature</td>
</tr>
<tr>
<td>MIRROR</td>
<td>Node number of the mirror image point with respect to the centerline</td>
</tr>
<tr>
<td>NEL</td>
<td>Pivotal element number</td>
</tr>
<tr>
<td>NELEM</td>
<td>Number of elements</td>
</tr>
<tr>
<td>NLE</td>
<td>Number of leading edge nodes</td>
</tr>
<tr>
<td>NMODE</td>
<td>Number of mode shapes</td>
</tr>
<tr>
<td>NP</td>
<td>Number of nodes</td>
</tr>
<tr>
<td>QIMAG</td>
<td>Array of $Q_{ij}$</td>
</tr>
<tr>
<td>QREAL</td>
<td>Array of $Q_{ij}$</td>
</tr>
<tr>
<td>SWPBK</td>
<td>Sweepback factor</td>
</tr>
<tr>
<td>TITLE</td>
<td>Array containing alphanumeric information for identification purposes</td>
</tr>
<tr>
<td>TOL</td>
<td>Tolerance, set at $10^{-5}$ for this program</td>
</tr>
<tr>
<td>UPWASH</td>
<td>Array of the upwash</td>
</tr>
<tr>
<td>W</td>
<td>Array of the weights of the modified Gauss-Chebyshev quadrature</td>
</tr>
<tr>
<td>X</td>
<td>Array of the $x$-ordinates of the nodes</td>
</tr>
<tr>
<td>XEL</td>
<td>Array of the $x$-ordinates of the elemental nodes</td>
</tr>
<tr>
<td>XK</td>
<td>Reduced frequency</td>
</tr>
<tr>
<td>XLAMDA</td>
<td>Sweptback angle of the leading edge, in degrees</td>
</tr>
<tr>
<td>XLIFT</td>
<td>Array of the lifts at the nodes</td>
</tr>
<tr>
<td>XM</td>
<td>Mach number</td>
</tr>
<tr>
<td>XO</td>
<td>Relative position in $x$ direction</td>
</tr>
<tr>
<td>Y</td>
<td>Array of the $y$-ordinates of the nodes</td>
</tr>
<tr>
<td>YEL</td>
<td>Array of the $y$-ordinates of the elemental nodes</td>
</tr>
<tr>
<td>YO</td>
<td>Relative position in $y$ direction</td>
</tr>
</tbody>
</table>
SECTION IV
INPUT DESCRIPTION

Input cards to this program should be prepared and arranged in the order described below.

A. TITLE CARD (8A10)

   Col. 1-80 Information on planform, Mach number, reduced frequency, modes and mesh spacing, for identification purposes

B. NAMELIST Input

$LIST1
XM Mach number
XK Reduced frequency
DELTAM Mesh spacing as measured by the length of the side of the characteristic element
MAXINT Number of integration points for the modified Gauss-Chebyshev quadrature
DEL Ratio of the singular strip half width to the element half width
IMAX Maximum number of characteristic elements in the chordwise direction for the stencil
JMAX Maximum number of characteristic elements in the Mach line direction for the stencil
XLAMDA Sweptback angle of leading edge in degrees
NMODE Number of modes in the set of mode shapes (either 6 or 7)
NP Number of nodes
NELEM Number of elements
ICHECK Option parameter used to check the mesh correctness. For ICHECK = 1, a quick run is performed to print out the element information list, as well as a list of the collocation points with its associated influencing elements. For normal run, this card is to be omitted.
NLE  Number of leading edge nodes
LENODE  Array of the leading edge node numbers
X  Array of x-ordinates of the nodes
Y  Array of y-ordinates of the nodes
$END

C. Card for Total Number of Element Information Cards to Follow (15)

D. Element Information Cards (615)

These are cards to generate the element number and its nodal numbers in a consecutive manner. Each card begins a new sequence.

<table>
<thead>
<tr>
<th>Col.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of elements to be generated in this sequence</td>
</tr>
<tr>
<td>1-5</td>
<td>Element number of the first element in this sequence</td>
</tr>
<tr>
<td>6-10</td>
<td>First nodal number of the first element in this sequence</td>
</tr>
<tr>
<td>11-15</td>
<td>Second nodal number of the first element in this sequence</td>
</tr>
<tr>
<td>16-20</td>
<td>Third nodal number of the first element in this sequence</td>
</tr>
<tr>
<td>21-25</td>
<td>Fourth nodal number of the first element in this sequence (leave blank for triangles).</td>
</tr>
</tbody>
</table>

For example, the card

| 5 | 9 | 22 | 13 | 6 | 12 |

generates the following information

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Node Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>22 13 6 12</td>
</tr>
<tr>
<td>10</td>
<td>23 14 7 13</td>
</tr>
<tr>
<td>11</td>
<td>24 15 8 14</td>
</tr>
<tr>
<td>12</td>
<td>25 16 9 15</td>
</tr>
<tr>
<td>13</td>
<td>26 17 10 16</td>
</tr>
</tbody>
</table>
while the card

\[ \begin{array}{cccc}
  1 & 14 & 27 & 18 & 17
\end{array} \]

generates the information on a single triangle

\begin{center}
\begin{tabular}{cc}
\textbf{Element No.} & \textbf{Node Numbers} \\
14 & 27 18 17
\end{tabular}
\end{center}

Element nodes are ordered in a counterclockwise direction, starting with the most downstream node.

E. Card for Total Number of Node Information Cards to Follow (15)

F. Collocation Point Information Cards (415)

These are cards to generate the collocation point, pivotal element number and mirror image node number in a consecutive manner. Each card begins a new sequence.

\begin{center}
\begin{tabular}{c|l}
\textbf{Col.} & \textbf{Description} \\
1-5 & Node number of the first collocation point in this sequence \\
6-10 & Pivotal element number containing the collocation point \\
10-15 & Node number of the mirror image point \\
16-20 & Number of collocation points to be generated
\end{tabular}
\end{center}

For example, the card

\[ \begin{array}{cccc}
  49 & 30 & 49 & 6
\end{array} \]

generates the following information.

\begin{center}
\begin{tabular}{ccc}
\textbf{Node} & \textbf{Pivot Element} & \textbf{Mirror} \\
49 & 30 & 49 \\
50 & 31 & 48 \\
51 & 32 & 47 \\
52 & 33 & 46 \\
53 & 34 & 45 \\
54 & 35 & 44 \\
\end{tabular}
\end{center}
while the card generates the following:

<table>
<thead>
<tr>
<th>Node</th>
<th>Pivot Element</th>
<th>Mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>37</td>
<td>21</td>
<td>34</td>
</tr>
<tr>
<td>38</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>39</td>
<td>23</td>
<td>32</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>31</td>
</tr>
</tbody>
</table>
SECTION V
SAMPLE RUN

The case of a rectangular planform with $A = 2$, $M = 1.2$ and $k = 0.3$ is used to illustrate the use of this program. The planform with the node numbers and element numbers is set up as in Fig. 2. For a production run, a much finer mesh can be used, and the main program can be dimensioned accordingly. The input deck for this problem is listed on page 17. Some suggested values for the parameters are: $\text{MAXINT} \geq 12$ and $\text{DEL} = 0.7$. These were determined through an accuracy study.
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
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<tr>
<td>11</td>
<td>12</td>
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<td>13</td>
<td>14</td>
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<td>15</td>
<td>16</td>
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<td>17</td>
<td>18</td>
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<td>21</td>
<td>22</td>
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<td>50</td>
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<td>51</td>
<td>52</td>
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<td>53</td>
<td>54</td>
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<td>55</td>
<td>56</td>
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<tr>
<td>57</td>
<td>58</td>
</tr>
<tr>
<td>59</td>
<td>60</td>
</tr>
<tr>
<td>61</td>
<td>62</td>
</tr>
<tr>
<td>63</td>
<td>64</td>
</tr>
</tbody>
</table>

*RECTANGULAR A=24. M=102. K=3. MODE=1+ X X+ Y Y+ X+Y Y+ Y+ Y+*
For the initial run, it is desirable to check the correctness of the mesh first. For this purpose, a card containing ICHECK = 1 can be included within the input namelist. This option will bypass all the time consuming computations and output only the mesh information. The user can compare this information against the mesh pattern to eliminate any input error. The output for ICHECK = 1 for this sample run is listed in the following pages.
<table>
<thead>
<tr>
<th>NODE</th>
<th>X-ORDINATE</th>
<th>Y-ORDINATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.000000</td>
<td>-1.000001</td>
</tr>
<tr>
<td>2</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>3</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>4</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>5</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>6</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>7</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>8</td>
<td>-2.000000</td>
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<tr>
<td>9</td>
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<tr>
<td>11</td>
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<td>-3.000000</td>
</tr>
<tr>
<td>12</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>13</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>14</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>15</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>16</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>17</td>
<td>-2.000000</td>
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</tr>
<tr>
<td>18</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>19</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
<tr>
<td>20</td>
<td>-2.000000</td>
<td>-3.000000</td>
</tr>
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After the mesh has been verified, the program can be run without the ICHECK = 1 option card. The output consists of the arrays of complex lifts due to a unit displacement in the respective mode shapes. The output is formatted such that each line contains four nodal lift values, with the real and imaginary part given in pairs (pages 24 through 27). Following these are the tables of generalized force coefficients (page 28). The upper table represents $Q_{ij}$, while the lower table represents $Q''_{ij}$. The output for the sample run is listed in the following pages.
RECTANGULAR $A=2$, $M=1$, $K=3$; Modes: 1, $x$, $x+y$, $y$: $x+y+y$, $y$, $xy$

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Appendix A

SUBROUTINE DESCRIPTIONS
Appendix A

This appendix contains a brief outline on the purpose, method and use of each of the eight subroutines. The principal input and output variables are described. The subroutines are arranged in alphabetical order as follows:

KERNEL
LGSPAN
POLYGN
QIJ
SGRHBS
SGTRGL
SINGUL
TABLE
SUBROUTINE KERNEL

PURPOSE: To evaluate the reduced kernel function \( \bar{K} = \gamma^2 K/2 \), where \( K \) is the supersonic kernel function.

METHOD: The nonplanar form of the oscillatory supersonic kernel function was derived by Harder and Rodden and was reduced to the planar form by A.M. Cunningham in the appendix, J. Aircraft, Vol. 11, No. 10, October 1974, pp. 615.

USE: CALL KERNEL (XO, YO, XKREAL, XKIMAG)

Input:

XO, YO Coordinates of the point of evaluation relative to the collocation point.

Output:

XKREAL, Real and imaginary parts of the reduced XKIMAG kernel function.
SUBROUTINE LGSPAN

PURPOSE:
To integrate a function with inverse square singularity over half of the singular strip by extracting the Cauchy's principal value.

METHOD:
Since a singular element contains either the complete strip or half of the strip, it is more convenient to treat only half a strip at a time. As the inverse square singularity occurs only in the spanwise integration, the chordwise integration can be performed first as

$$F(\eta) = \int_{\xi_a(\eta)}^{\xi_b(\eta)} f(\xi, \eta) \, d\xi$$

where

$$f_i = f(\xi_i, \eta)$$

and

$$\xi_i = \frac{\xi_b(\eta) - \xi_a(\eta)}{2} \xi_i + \frac{\xi_b(\eta) + \xi_a(\eta)}{2}$$

with

$$w_i$$ and $$\xi_i$$ are some Gaussian weights and abscissas over (-1, 1).

A sixth degree quadrature based on Lagrangian interpolation in conjunction with the Cauchy's principal value was devised by Watkins as

$$\int_{y-\epsilon}^{y+\epsilon} \frac{F(\eta)}{(y-\eta)^{\nu}} \, dy$$

$$= \frac{1}{100\epsilon} \left[ 13(F_1 + F_7) + 72(F_2 + F_6) 
+ 495(F_3 + F_5) + (-1360) F_4 \right]$$

For use in this subroutine, the above quadrature is modified to
\[ \int_{y-\epsilon}^{y} \frac{F(n)}{(y - \eta)^2} \, dy = \frac{1}{100\epsilon} \left[ 13F_1 + 72F_2 + 496F_3 - 680F_4 \right] \]

for the left half, and to

\[ \int_{y}^{y+\epsilon} \frac{F(n)}{(y - \eta)^2} \, dy = \frac{1}{100\epsilon} \left[ 13F_7 + 72F_6 + 495F_5 - 680F_4 \right] \]

for the right half.

**USE:**

CALL LGSPAN(X, Y, XEL, YEL, EINT, W, MAXINT, F)

**Input:**

- \(X, Y\): Coordinates of the collocation point.
- \(XEL, YEL\): Arrays of nodal coordinates of the half strip.
- \(EINT\): Array of Gauss-Chebyshev quadrature abscissas.
- \(W\): Array of Gauss-Chebyshev quadrature weights.
- \(MAXINT\): Number of integration points.

**Output:**

- \(F\): Array of integrated values at nodal points.

**SUBROUTINES CALLED:**

KERNEL

**ERROR RETURNS:**

None
SUBROUTINE POLYGN

PURPOSE: To perform the integration over a regular triangular or quadrilateral element.

METHOD: The arbitrary triangular or quadrilateral element is mapped into a square region \(-1 \leq \xi \leq 1\) and \(-1 \leq \eta \leq 1\). Integration is accomplished by a repeated application of the Gauss-Chebyshev quadrature in both directions. Since the mapping is different for a triangle and a quadrilateral, different computational routines are employed.

USE: CALL POLYGN(X, Y, XEL, YEL, EINT, W, MAXINT, F, IANGLE)

Input:
X, Y Coordinates of the collocation point.
XEL, YEL Arrays of nodal coordinates of the element.
EINT Array of Gauss-Chebyshev quadrature abscissas.
W Array of Gauss-Chebyshev quadrature weights.
MAXINT Number of integration points.
IANGLE Number of angles in the polygon element.

Output:
F Array of integrated values at nodal points.

SUBROUTINE CALLED: KERNEL

ERROR RETURNS: None
SUBROUTINE QIJ

PURPOSE: To compute the generalized force coefficients for a given load distribution.

METHOD: With the finite element approximation, the generalized force coefficients are

\[ Q_{ij} = \sum_{n=1}^{N} \int \int_{A(e)} f_i(x, y) \bar{N}^t(x, y) \, dx \, dy \cdot \lambda_j^{(e)} \]

where \( \sum \) denotes summation over all elements,

\( f_i \) are the modes, \( \bar{N}^t \) are the shape functions and

\( \lambda_j^{(e)} \) is the elemental lift vector. The real and imaginary parts are defined as

\[ Q'_{ij} = \text{Re}(Q_{ij}) \]

and

\[ Q''_{ij} = \text{Im}(Q_{ij}) / k \]

where \( k \) is the reduced frequency. The integrations are performed using the Gaussian quadrature (2 points as set up in the subroutine).

USE: CALL QIJ(X, Y, INFO, XLIFT, FF, QREAL, QIMAG, NP, NELEM, NMODE, MX)

Input:

NP Number of nodal points.
NELEM Number of elements.
NMODE Number of modes.
MX Maximum number of nodes the the elements.
X, Y Arrays of nodal coordinates.
INFO Array of element information.
XLIFT Array of modal lift distribution.
FF Array for temporary storage.
Output:

QREAL  Real part of the generalized force coefficient, $Q_{ij}'$.

QIMAG  Imaginary part of the generalized force coefficient, $Q_{ij}''$.

SUBROUTINES CALLED: None

ERROR RETURNS: None
SUBROUTINE SGRHBS

PURPOSE: To perform the integration of a rhombic element with the singular strip passing through either the left, middle or right node.

METHOD: The rhombic element is divided into the right half triangle and the left half triangle. These triangular elements can be either regular or singular. The integrations are accomplished by calling another subroutine and the results are re-assembled.

USE:

CALL SGRHBS(XNODE, YNODE, XEL, YEL, EINT, W, MAXINT, F, DEL)

Input:

XNODE, YNODE Coordinates of the collocation point.
XEL, YEL Arrays of nodal coordinates of the element.
EINT Array of Gauss-Chebyshev quadrature abscissas.
W Array of Gauss-Chebyshev quadrature weights.
MAXINT Number of integration points.
DEL Ratio of the singular strip half width to the element half width.

Output:

F Array of integrated values at nodal points.

SUBROUTINES CALLED:

SINGUL

ERROR RETURNS:

None
SUBROUTINE SGTRGL

PURPOSE: To integrate a general triangular element with the singular strip passing through either one of the three vertices.

METHOD: A vertical line through the lower vertex divides the element into two sub-triangles, which can be either regular or singular. The integrations are accomplished by calling another subroutine and the results are re-assembled.

USE: CALL SGTRGL(XNODE, YNODE, XEL, YEL, EINT, W, MAXINT, F, DEL)

Input:
XNODE, YNODE Coordinates of the collocation point.
XEL, YEL Arrays of nodal coordinates of the element.
EINT Array of Gauss-Chebyshev quadrature abscissas.
W Array of Gauss-Chebyshev quadrature weights.
MAXINT Number of integration points.
DEL Ratio of the singular strip half width to the element half width.

Output:
F Array of integrated values at nodal points.

SUBROUTINES CALLED: SINGUL

ERROR RETURNS: None
SUBROUTINE SINGUL

PURPOSE:
To integrate the special type of triangular element bounded by a vertical line and two other straight lines. The triangular element may be singular or regular.

METHOD:
The triangular element is first tested to see if it is singular or regular. For singular element, further test is conducted to locate the singularity with respect to the element such that the element can be divided into a singular strip and a regular polygon. Integrations are accomplished by calling other subroutines and the results are re-assembled.

USE:
CALL SINGUL(XNODE, YNODE, XEL, YEL, EINT, W, MAXINT, F, DEL)

Input:
XNODE, YNODE Coordinates of the collocation point.
XEL, YEL Arrays of nodal coordinates of the element.
EINT Array of Gauss-Chebyshev quadrature abscissas.
W Array of Gauss-Chebyshev quadrature weights.
MAXINT Number of integration points.
DEL Ratio of the singular strip half width to the element half width.

Output:
F Array of integrated values at nodal points.

SUBROUTINES CALLED:
POLYGN, LGSPAN

ERROR RETURNS:
None
SUBROUTINE TABLE

PURPOSE:
To create a table of the weighted kernel function coefficients for a given uniform characteristic mesh. This table is stored for later table look-up in the solution process.

METHOD:
A stencil of uniform characteristic mesh large enough to cover the most extreme case for the planform is set up. Because of the symmetry in the spanwise direction, only the rhombic elements on one side need to be evaluated and stored. Each element is uniquely defined by a pair of relative indices based on its relative location from the collocation point.

USE:
CALL TABLE(COEF, DELTA, EINT, W, MAXINT, DEL, IMAX, JMAX, LMAX)

Input:
DELTA  Length of the side of the characteristic element.
EINT  Array of Gauss-Chebyshev quadrature abscissas.
W  Array of Gauss-Chebyshev quadrature weights.
MAXINT  Number of integration points.
DEL  Ratio of the singular strip half width to the element half width.
IMAX  Number of elements in the chordwise direction for the stencil.
JMAX  Number of elements in the Mach line direction for the stencil.
LMAX  Number of entries to the table.

Output:
COEF  Array of the weighted kernel function coefficients

SUBROUTINES CALLED:
SGRHBS, POLYGN

ERROR RETURNS:
None
Appendix B

PROGRAM LISTING OF FESKAP
PROGRAM FESKAP (INPUT, OUTPUT, TAPED=INPUT, TAPED=OUTPUT)
C
C A NEW FINITE ELEMENT KERNEL FUNCTION METHOD IN SUPERSONIC
C LIFTING SURFACE THEORY
C DEVELOPED BY V. Y. L. YOUNG AT LOCKHEED - MUNCIE
C
C COMPLEX * SUM*UPWASH*CCEF*XLIFT
C DIMENSION X(67), Y(67), IN(65), N(4)*XLIFT(67+7)*COLF(4+26)
C LENODE(7), BLBUF(65), IF(67+7)
C DIMENSION F(65), NLE(6), R(4), SUM(7)*UPWASH(7)*TIL(8)
C URLAX(7+7), IMAG(7+7), W(30), INT(30)
C COMMON XI2*XI2*XI2*XI2*XI2/BLT(BETA)
C EQUVALENCE(IP(11),COLF(11))
C
C THE FIRST DIMENSION STATEMENT IS DIMENSIONED AS X(MP+Y(NP-1))
C INPUT Nếu tên) = XLIFT(1)*COLF(1)*LENODE(NL)*IF(N ELEMENT)
C FF(NP*NLE)
C CAN BE ALTERED BY THE USER TO FIT THE PROBLEM
C
C IF1 AND FF ARE TEMPORARY STORAGE ARRAYS
C
C DELL = WIDTH OF THE SINGULAR STRIP RELATIVE TO THE WIDTH OF
C THE ELEMENT
C DELL = LENGTH OF THE SIDE OF THE CHARACTERISTIC ELEMENT
C ICHECK = RUN TO CHECK THE MESH CONNECTIVITY
C IF ICHECK IS SET TO 1
C OTHERWISE ICHECK IS SET TO 0
C IMAX = MAXIMUM NUMBER OF CHARACTERISTIC ELEMENT IN THE
C CHOWNF DIRECTION
C UMAX = MAXIMUM NUMBER OF CHARACTERISTIC ELEMENT IN THE
C MAIN LINE DIRECTION
C LNODE = ARRAY CONTAINING THE LEADING EDGE NODES
C MAXINT = NUMBER OF INTEGRATION POINTS FOR THE MODIFIED
C GAUSS-CHEBYSHEV QUADRATURE (MAXIMUM SET AT 30)
C LMAX = IMAX*UAX*(IMAX+1)/2
C NLENE = NUMBER OF ELEMENTS
C NLE = NUMBER OF LEADING EDGE NODES
C NLE = NUMBER OF EDGE NODES
C NLE = NUMBER OF NODES
C XX = REDUCED FREQUENCY
C XLAMDA = SPLITBACK ANGLE OF LEADING EDGE IN DEGREES
C XM = MAIN NUMBER
C
C DATA TOL/1E-3/7)
C NAMENLIST /LIST1/XI2*XI2*XI2*XI2*XI2/DELTA*MAXINT*DEL
C MAXINT*DELMAX*XLAMDA*PP
C 1
C NLENE, ICHECK, NLE, LNODE, X*Y
C INODE(N); IF I*LIST1(A+BETA#)+5+1
C LMAX= (IMAX+1)*(MAX+1)/2+1
C
C READ TITLE CARD AND NAMENLIST INPUT
C
C READ(S+200) TITLE
C READ(SLIST1)
C BETASW=MM*XM-1.
C BETA=SGT(BETASW)
C CSTI=5*XM/(BETA+DELTA)
C IF (ICHECK=2+1) WRITE(6+9610) TITLE
C IF (ICHECK=2+1) WRITE(6+9610) (X*I1+Y(I)+1=1, NP))
C GENERATE ELEMENT INFORMATION FROM DATA CARDS
C
READ(15,9400) KMAX
IF (ICHECK.EQ.1) WRITE(6,9620)
DO 12 K=1,KMAX
READ(15,9400) IREPT, I1+1*N1+2*N2+N3+N4
DO 10 I=1, IREPT
IM1=I-1
12=11+IM1
INFO(12:2)=N1+IM1
INFO(12:3)=N2+IM1
INFO(12:4)=N3+IM1
INFO(12:4)=N4+IM1
IF (NA.EQ.0) INFO(12:4)=0
IF (ICHECK.EQ.1) WRITE(6,9600) 12, INFO(12:1)+INFO(12:2)+
INFO(12:3)+INFO(12:4)
10 CONTINUE
12 CONTINUE
IF (ICHECK.EQ.1) GO TO 5001
C GENERATE THE MODIFIED GAUSS-CHEBYSHEV WEIGHTS AND ABSCISSAS
C
CST=.14159265359/MAXINT
DO 20 I=1,MAXINT
ARG=(I-5)*CST
EINT(I)=COS(ARG)
20 W(I)=CST*SIN(ARG)
C CLEAR THE NODAL LIFT VALUES
C
DO 30 MODE=1,NMODE
DO 30 I=1,NP
30 XLIFT(I,MODE)=CMPLX(0.0,0.0)
C GENERATE THE WEIGHTED KERNEL FUNCTION COEFFICIENTS FOR THE STENCIL
C OF UNIFORM CHARACTERISTIC MESH
C
LMAX=LINEAR(MAX,2*M)
CALL TABLE(COEFF,DELTA+EINT+W*MAXINT+DELM+MAX,M,LMAX)
C COMPUTE THE SWEEPBACK FACTOR
C FOR SUBSONIC LEADING EDGE: SET SWEEPBACK FACTOR TO 1.
C
CST=TAN(-17.4532925E-1*X/LAMBDA)/BETA
SWBK=1.
CST=1.-CST*CST
IF (CST.GT.1.e-10) SWBK=1./Sqrt(CST)
CST=-2.*SWBK/BETA
C COMPUTE THE STARTING SOLUTION AT THE LEADING EDGE
C
DO 40 I=1,NLE
40 XNODE(I)=YNODE(I)
XISQ=XNODE*XRCL
YISQ=YNODE*YRCL
C LOOP THROUGH THE UNKNOWN NODAL POINTS TO COMPUTE THE LIFT

C
READ(5,Y000) LMAX
IF (ICHECK.EQ.1) WRITE(6,9910)
DO 1000 LNODE=1,LMAX
READ(5,Y000) NL,N2,N3,IREPT
DO 1000 I=1,IREPT
IM1=I-1
NODE=NI+1IM1
NEL=N2+1IM1
MIRROR=N3-1IM1
XNODE=X(NODE)
YNODE=Y(NODE)
IF (ICHECK.EQ.1) GO TO 5002
C COMPUTE THE MODAL UAWASH AT THE COLLOCATION POINT
XISU=XNODE*YNODE
YISU=YNODE*YNODE
UAWASH(1)=CMPLX(0+XX)
UAWASH(2)=CMPLX(1+XX*XNODE)
UAWASH(3)=CMPLX(XNODE+YNODE+XX*XISU)
UAWASH(4)=CMPLX(0+XX*YISU)
UAWASH(5)=CMPLX(2*XNODE+YISU+XX*YISU+YISU)
UAWASH(NODE-1)=CMPLX(0+XX*YNODE)
UAWASH(NODE)=CMPLX(YYNODE+XX*YNODE)*YNODE)
C
C CLEAN THE SUMS BEFORE ACCUMULATION
C
DO 1000 MODE=1,NODE
SUM(MODE)=CMPLX(0+0I)
1000 CONTINUE
5002 CONTINUE
C
C LOOP THROUGH THE FORWARD ELEMENTS
C
KOUNT=O
DO 800 L=1,NEL
ANGLE=4
IF (INFO(L+4).EQ.0) IANGLE=3
C
C ASSIGN THE ELEMENT NODAL POSITION
C
DO 200 K=1,1444
XX=INFO(L+4)
XEL(K)=X(KK)
200 YEL(K)=Y(KK)
XO=XNODE-XEL(1)
YO=YNODE-YEL(1)
C
C SKIP THE TEST FOR THE PIVOTAL ELEMENT
C IF (L=EL+NEL) GO TO 201
C C SKIP THE ELEMENT IF IT IS NOT WITHIN THE FORWARD MACH CONE
C IF ((X0-BETA*ABS(Y0))LT(-TOL)) GO TO 800
201 CONTINUE
       KOUNT=KOUNT+1
       IDEF(KOUNT)=L
       IF (I CHECK=EU+1) GO TO 800
       IF (I ANGLE=EU+3) GO TO 400
C C REGULAR CHARACTERISTIC ELEMENTS
C COMPUTE THE RELATIVE INDICES
C LL=INOREL(X0,Y0)
MM=INOREL(X0,-Y0)
       IF (LL*GT*MM) GO TO 300
C C LOWER TRIANGLE OF TABLE
C LL=L INEAR(LL,MM)
F(1)=COEF(1,L1)
F(2)=COEF(2,L1)
F(3)=COEF(3,L1)
F(4)=COEF(4,L1)
GO TO 600
300 CONTINUE
C C UPP ER TRIANGLE OF TABLE
C MM=L INEAR(MM,LL)
F(1)=COEF(1,L1)
F(2)=COEF(4,L1)
F(3)=COEF(3,L1)
F(4)=COEF(2,L1)
GO TO 600
C C TRIANGULAR FILL-IN ELEMENTS
C 400 CONTINUE
C C TEST FOR SINGULARITY
C DO 500 K=1,I ANGLE
       IF (ABS(YNO DE-YEL(K))LT(TOL)) GO TO 502
500 CONTINUE
C C NON-SINGULAR POLYGON
C CALL POLY GN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,I ANGLE)
GO TO 600
C C SINGULAR TRIANGLE
C 502 CALL S GTRL G(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
600 CONTINUE
COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION
9800 FORMAT(3X,1H1,8X,SHJ = 1,10X,SHJ = 2,10X,SHJ = 3,10X,SHJ = 4)
  A 291
  1
  10X,SHJ = 5,10X,SHJ = 6,10X,SHJ = 7/)
  A 292
9810 FORMAT(3X,1H1,8X,SHJ = 1,10X,SHJ = 2,10X,SHJ = 3,10X,SHJ = 4)
  A 293
  1
  10X,SHJ = 5,10X,SHJ = 6/)
  A 294
9900 FORMAT(32X,2514)
  A 295
9910 FORMAT(/ /////// 3X,4HNODE,6X,3HNL,4X,6HMINRUK,6X,)
  A 296
  1
  20HINFLUENCING ELEMENTS///)
  A 297
9920 FORMAT(1X,15,2110)
  A 298
END
SUBROUTINE KERNEL(XO,YO,XKAYO,XK,XKREAL,XKIMAG)  
C SUPERSONIC KERNEL FUNCTION DERIVED BY HARDEN AND HODDEN  
C AS GIVEN BY A M CUNNINGHAM IN APPENDIX OF J. AIRCRAFT,  
C VOL. 11, NO. 10, 1976  
C  
DIMENSION A(11)  
COMMON XM,XK,BETASU,BETA  
DATA A/-2.4180190,5E-7,46027.5,4.91079,111.5,1.96,2.71,4.3549/  
1.0D+75<88*41.163830,+-5.82*9537.644*78152.3*8*72755/  
<6*27931/>  
DATA C/1.727/  
M=SQRIT(XO*X0-BETASU*YO*YO)  
IF (XM+LT+1,E-5) GO TO 400  
AYO=ABS(YO)  
ANG=XK*YO  
CS=COS(ANG)  
SN=SIN(ANG)  
IF (AYO+LT+1,E-5) GO TO 500  
C  
C GENERAL FORM OF KERNEL FUNCTION  
C  
XKAYO=XK*AYO  
XXYOSU=XKAYO*XXAYO  
B2Y01=1/(BETASU*AYO)  
XM=XM+H  
U1=B2Y01*(XO-XM)  
U2=B2Y01*(X0+XM)  
E1=EXP(-C*ABS(U1))  
E2=EXP(-C*U2)  
CST=0.  
CST1=1.  
CST2=1.  
SUM=0.  
SUM1RL=0.  
SUM1LM=0.  
SUM2RL=0.  
SUM2LM=0.  
DO 100 I=1,11  
CST=CST+C  
CST1=CST1*E1  
CST2=CST2*E2  
COEF=A(11)/(CST*CST+XXYOSU)  
SUM=SUM+COEF  
COEFL=CST*COEF  
COEFML=-XKAYO*COEFL  
SUM1RL=SUM1RL+COEFL*CST1  
SUM1LM=SUM1LM+COEFL*CST1  
SUM2RL=SUM2RL+COEFL*CST2  
SUM2LM=SUM2LM+COEFL*CST2  
SUM2LM=SUM2LM+CST1*SUM2LM  
100 CONTINUE  
ANG*XKAYO*U1  
CS1=COS(ARG)  
SN1=SIN(ARG)  
ANG*XKAYO*U2  
CS2=COS(ARG)  
SN2=SIN(ARG)  
XI12RL=XKAYO*(SN2*SUM2RL-CS2*SUM2LM)
XI121*XXAYO*(CST2*SUM2+SN2*SUM2M)
CST1=SN1*SUM1RL
CST2=CST1*SUM1M
CST3=CST1*SUM1M
CST4=SN1*SUM1M
IF (1+LT+0.) GO TO 200
XI11RL=XXAYO*(CST1+CST2)
XI11IM=XXAYO*(CST3+CST4)
GO TO 300
200 XI11RL=XXAYO*(CST1+CST4)+2*(CS1-1+XY050*SUM)
XI11IM=XXAYO*(CST3-CST4)-SN1-SN1
300 CONTINUE
CST1=(XO/R)+10
CST2=CST1-20
XI11RL=CST1*CS1-XI11RL
XI11IM=-CST1*SN1-XI11IM
XI12RL=CST2*CS2+XI12RL
XI12IM=-CST2*SN2+XI12IM
SUM1RL=XX11RL+XX12RL
SUM1IM=XX11IM+XX12IM
XXREAL=8*(CS1*SUM1RL+SN1*SUM1M)
XXIMAG=8*(CS1*SUM1M-SN1*SUM1RL)
RETURN
C C STANLEY FORM OF KERNEL FUNCTION
C 400 XXREAL=XO/R
XXIMAG=0.
RETURN
C C SPECIAL FORM OF KERNEL FUNCTION AT Y0=O
C 500 XXREAL=CS
XXIMAG=-SN
RETURN
END
SUBROUTINE LGSPAN(X,Y,XEL,YEL,IEINT,IMAXINT,F)
C INTEGRATION OVER HALF OF THE SINGULAR STRIP
C WITH LAUHY'S PRINCIPAL VALUE
C USING A SIXTH DEGREE LAGRANGIAN INTERPOLATION
C
C HEAL N
    COMPLEX XKBAR,F,SUM
    DIMENSION F(1),SUM(1),N(4)
    DIMENSION COEF(1),S(1),XEL(1),YEL(1),IEINT(MAXINT),IMAXINT
    DATA COEF(/13372495003/)
    DATA S(1),XEL(1),YEL(1),IEINT(1),IMAXINT(1)
C TEST TO SEE IF SINGULARITY IS TO LEFT OR RIGHT
C
C IFILIP=0
    IF ((Y-YEL(1)-YEL(2)+GT+0)) IFILIP=1
    EPS=ABS(YEL(2)-YEL(1))
    DO 10 II=1,4
    10 F(II)=CMPLX(00,00)
C BEGIN SPANWISE INTEGRATION
C
C DO 1000 II=1,4
C
C COMPUTE THE UPPER AND LOWER LIMITS FOR THE CHORDWISE INTEGRATION
C REVERSE SIGN IF SINGULARITY IS TO THE RIGHT
C
C Z1=S(1)
    IF (IFILIP.EQ.1) Z1=-Z1
    CST1=500(1-Z1)
    CST2=500(1+Z1)
    A=CST1*XEL(1)+CST2*XEL(3)
    B=CST1*XEL(1)+CST2*XEL(2)
    C1=500(B-A)
    C2=500(B+A)
    Y0=Y-CST1*YEL(1)-CST2*YEL(2)
    DO 20 II=1,4
    20 SUM(II)=CMPLX(00,00)
C BEGIN CHORDWISE INTEGRATION
C
C DO 100 L=1,IMAXINT
    ZZ=IEINT(L)
    XK=X-(C1+Z2+C2)
    CALL KERNEL(A,YU*XXREAL+XXIMAG)
    XXBAR=CMPLX(XXREAL+XXIMAG)
    N(1)=SCFN(-Z1+L2)
    N(2)=SCFN(Z1+L2)
    N(3)=SCFN(Z1+L2)
    N(4)=SCFN(-Z1+L2)
    DO 30 II=1,4
    30 SUM(II)=SUM(II)+N(II)*XXBAR
    100 CONTINUE
C END CHORDWISE INTEGRATION
C
C 000
CST=C1*COEF(1)
D0 40 II=1,4
40 F(II)=F(II)*CST*SUM(II)
1000 CONTINUE
D0 50 II=1,4
50 F(II)=F(II)/EPS
C
C END SPANWISE INTEGRATION
C
RETURN
END
SUBROUTINE POLYGON \(X, Y, XE1, YE1, XE2, YE2, MAXINT, IANGLE\)

C INTEGRATION OVER A REGULAR TRIANGULAR QUADRILATERAL ELEMENT
C AS INDICATED BY THE VALUE OF IANGLE

C

REAL N
COMPLEX XBAR, F, SUM
DIMENSION F(4), SUM(4), N(4), XE1(4), YE1(4)
DIMENSION EINT(MAXINT), B(MAXINT)
SFUN(A,B) = A**B - (1 + A) * (1 + B)
IFLAG = IANGLE - 2
DO 100 II = 1, 4
100 F(II) = CMPLX(0.0, 0.0)
XE1 = XE1(3) - XE1(1)
YE1 = YE1(3) - YE1(1)
GO TO (200, 300, 400) IFLAG

C IANGLE

C

200 X23 = XE1(2) - XE1(3)
Y23 = YE1(2) - YE1(3)
C JACOBIAN / 4

C

XJACB/N = 25* (Y1*Y3 - X1*Y23)
DO 250 J = 1, MAXINT
XJ = EINT(J)
N(2) = N(1) + XJ
C1 = C1 + XJ
WJ = CI*W(J)
DO 210 II = 1, 3
210 SUM(II) = CMPLX(0.0, 0.0)
DO 230 I = 1, MAXINT
XI = EINT(I)
N(1) = N(1) + XI
N(3) = C1 - N(1)
LTA = N(1) * XE1(1) + N(2) * XE1(2) + N(3) * XE1(3)
ETA = N(1) * YE1(1) + N(2) * YE1(2) + N(3) * YE1(3)
XQ = X - ZETA
YQ = Y - ETA
CALL KERNEL(XQ, YQ, XKREAL, XKIMAG)
XKBAR = CMPLX(XKREAL, XKIMAG)
W = W(1) / (YQ*Y)
DO 220 II = 1, 3
220 SUM(II) = SUM(II) + W * N(II) * XKBAR
230 CONTINUE
DO 240 II = 1, 3
240 F(II) = F(II) + J * SUM(II)
250 CONTINUE
GO TO 400

C QUADRILATERAL

C

300 X42 = XE1(4) - XE1(2)
Y42 = YE1(4) - YE1(2)
C JACOBIAN / 8

C
XJCN=1.25*(X31*Y42-Y31*X42)
00 350 I=1:MAXINT
DU 310 K=1:4
310 SUM(K)=CMPLX(0.0,0.0)
   Z1=EINT(I)
   DO 330 J=1:MAXINT
      Z2=EINT(J)
      N(1)=SFCN(Z1-Z2)
      N(2)=SFCN(Z1+Z2)
      N(3)=SFCN(Z1+Z2)
      N(4)=SFCN(-Z1+Z2)
      THETA=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)+N(4)*XEL(4)
   END
   ETA=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)+N(4)*YEL(4)
   X0=X-ZETA
   Y0=Y-ETA
   CALL KERNEL(X0*Y0*XXREAL+XXIMAG)
   XKBAR=CMPLX(XXREAL+XXIMAG)
   WJ=W(J)/Y0*Y0)
   DO 320 K=1:4
      320 SUM(K)+WJ*N(K)*XKBAR
330 CONTINUE
   DU 340 K=1:4
   340 F(K)+W(1)*SUM(K)
350 CONTINUE
400 CONTINUE
   DU 500 K=1:4
   500 F(K)*XJCN=F(K)
RETURN
END
SUBROUTINE OJIX(X,Y,INFO,XLIFT,FF,REAL,IMAG,N,ELEM,NMODE,NX)
C
C THIS SUBROUTINE COMPUTES THE GENERALIZED FORCE COEFFICIENTS
C FOR A GIVEN LOAD DISTRIBUTION
C
REAL N
COMPLEX XLIFT
DIMENSION X(NP),Y(NP),INFO(ELEM),XLIFT(NP,NMODE),FF(NP,NMODE)
DIMENSION XEL(4),YEL(4),SUM(4),F(7),N(4),EINT(2),#(2)
COMMON XM,XK,ETA,H,T,W
DATA MAXINT/5/
DATA K,INT/577735026918766/}
DATA W/24/
SFCN(A)=+25*(1+A)*(1+B)
C
C INITIALIZE
C
DO 2000 I=1,NMODE
DO 2200 J=1,NMODE
REAL(I,J)=0
2200 CONTINUE
2000 CONTINUE
C
C LOOP THROUGH THE ELEMENTS
C
DO 4000 L=1,NLEM
ANGLE=4
IF(INFO(L)=1) ANGLE=3
DO 3000 K=1,ANGLE
K=INFO(L+K)
XEL(K)=X(K)
YEL(K)=Y(K)
3000 CONTINUE
IFLAG=ANGLE-2
C
C CLEAR F BEFORE ACCUMULATION
C
DO 100 I=1,4
DO 100 J=1,NMODE
100 F(J,J)=0
X31=XEL(3)-XEL(1)
Y31=YEL(3)-YEL(1)
C
C BEGIN INTEGRATION -- OUTER LOOP
C
DO 250 J=1,MAXINT
GO TO (251,252),IFLAG
C
C TRIANGULAR ELEMENT
C
251 XJ=EINT(J)
N2=5*(1+XJ)
C1=5*(1-XJ)
WJ=C1*KW(J)
GO TO 253
C
COPY AVAILABLE TO DDG DOES NOT PERMIT FULLY LEGIBLE PRODUCTION
C QUADRILATERAL ELEMENT
C
252 ZI=EINT(J)
     WJ=W(J)
253 DO 210 I=1,N
     DO 210 MODE+1,NMODE
     SUM(I+MODE)=0.
C BEGIN INTEGRATION -- INNER LOOP
C
210 SUM(I+MODE)=SUM(I+MODE)+XITENT(I)
     DO 220 I=1,N
     DO 220 MODE+1,NMODE
     DO 220 ANGLE=1,MAXMT
     CST=W(I)*SUM(I)
     DO 220 MODE+1,NMODE
     SUM(I+MODE)=SUM(I+MODE)*CST
     DO 220 ANGLE=1,MAXMT
     CONTINUE
C END INNER LOOP
C
C TRIANGULAR ELEMENT
C
231 XI=EINT(1)
     N(1)=SCFNC(-Z1-Z2)
     N(2)=SCFNC(Z1-Z2)
     N(3)=SCFNC(Z1+Z2)
     N(4)=SCFNC(21+Z2)
     XP=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)+N(4)*XEL(4)
     YP=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)+N(4)*YEL(4)
     GO TO 233
C QUADRILATERAL ELEMENT
C
232 ZI=EINT(1)
     N(1)=SCFNC(-Z1-Z2)
     N(2)=SCFNC(Z1-Z2)
     N(3)=SCFNC(Z1+Z2)
     N(4)=SCFNC(-Z1+Z2)
     XP=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)+N(4)*XEL(4)
     YP=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)+N(4)*YEL(4)
C COMPUTE THE MODAL DEFLECTION
C
233 FI(I)=1.
     FI(I+1)=FI(I)*XP
     FI(I+2)=FI(I)*XP
     FI(I+3)=FI(I)*XP
     FI(I+4)=FI(I)*XP
     FI(I+5)=FI(I+1)-FI(I+4)
     FI(I+6)=FI(I+2)-FI(I+5)
     FI(I+7)=FI(I+3)-FI(I+6)
     FI(I+8)=FI(I+4)-FI(I+7)
     DO 250 ANGLE=1,MAXMT
     CST=W(I+1)*SUM(I+1)
     DO 250 MODE+1,NMODE
     SUM(I+1+MODE)=SUM(I+1+MODE)+CST*FI(I+1+MODE)
     DO 250 ANGLE=1,MAXMT
     CONTINUE
C END INNER LOOP
C
DO 240 ANGLE=1,MAXMT
     DO 240 MODE+1,NMODE
     DO 240 FI(I+MODE)=FI(I+MODE)+WJ*SUM(I+MODE)
     CONTINUE
C END OUTER LOOP
C
GO TO (200,300), IFLAG
C TRIANGULAR ELEMENT
C
200 X23=XEL(2)-XEL(1)
Y23=YEL(2)-YEL(1)
AREA=.5*(Y31*X23-X31*Y23)
GO TO 400
C
C QUADRILATERAL ELEMENT
C
300 X42=XEL(4)-XEL(2)
Y42=YEL(4)-YEL(2)
AREA=+.25*(X31*Y42-Y31*X42)
C
C GLOBALIZE THE ELEMENTAL INTEGRATED COEFFICIENT VECTOR TO
C FORM THE INTEGRATED COEFFICIENT VECTOR
C
400 DO 500 J=1,NANGLE
K=INFO(J+1)
DO 500 MODE=1,NMODE
500 FF(K,MODE)=FF(K,MODE)*AREA*F(J,MODE)
4000 CONTINUE
C
C COMPUTE THE GENERALIZED FORCE COEFFICIENTS G(I,J)
C
DO 5000 I=1,NMODE
DO 5000 J=1,NMODE
DO 5000 K=1,NP
UXEAL(I,J)=UXEAL(I,J)-FF(K,1)*UXEAL(K+J)
UYEAL(I,J)=UYEAL(I,J)-FF(K,1)*UYEAL(K+J)
UIMAG(I,J)=UIMAG(I,J)-FF(K,1)*UIMAG(K+J)
5000 CONTINUE
C
C DIVIDE IMAGINARY PART OF G BY REDUCED FREQUENCY
C SKIP THE CONVERSION FOR ZERO FREQUENCY
C
IF (XX*LT+1.E-5) GO TO 9999
DO 6000 I=1,NMODE
DO 6000 J=1,NMODE
6000 UIMAG(I,J)=UIMAG(I,J)/XX
9999 RETURN
END
SUBROUTINE SGHMS(XNODE, YNODE, XEL, YEL, EINT, MAXINT, F, DEL)  C
C THIS SUBROUTINE PERFORMS THE INTEGRATION OF A RHOMBIC ELEMENT
C WITH THE SINGULAR STRIP PASSING THROUGH EITHER THE LEFT:
C MIDDLE OR RIGHT NODE
C
COMPLEX F*F1
DIMENSION XEL(4), YEL(4), XEL1(4), YEL1(4), F(4), F1(4)
DIMENSION EINT(MAXINT, MAXINT)

I.
C INTEGRATE THE RIGHT HALF TRIANGULAR REGION
C
CALL SINGUL(XNODE, YNODE, XEL, YEL, EINT, MAXINT, F, DEL)
C
C INTEGRATE THE LEFT HALF TRIANGULAR REGION
C
XEL1(1) = XEL(1)
YEL1(1) = YEL(1)
XEL1(2) = XEL(3)
YEL1(2) = YEL(3)
XEL1(3) = XEL(4)
YEL1(3) = YEL(4)
CALL SINGUL(XNODE, YNODE, XEL1, YEL1, EINT, MAXINT, F, DEL)
F(1) = F(1) + F1(1)
F(3) = F(3) + F1(2)
F(4) = F1(3)
RETURN
END
SUBROUTINE SITRGL(XNODE,YNODE,XEL,YEL,EINT,MAXINT,F,DEL)

C INTEGRATION OVER A GENERAL TRIANGULAR ELEMENT WITH THE
C SINGULAR STRIP PASSING THROUGH EITHER ONE OF THE VERTICES

C COMPLEX F
DIMENSION XEL(4),YEL(4),EINT(MAXINT),F(MAXINT)
DEL1=(YEL(1)-YEL(3))/(YEL(2)-YEL(3))
IF (DEL1.0E-5) GO TO 1000
DM1=1.-DEL1
IF (DM1.LT.1.E-5) GO TO 1000

C INTEGRATE THE LEFT SIDE OF THE TRIANGLE

XEL1(1)XEL(1)
YEL1(1)YEL(1)
XEL1(2)DEL1*XEL(2)+DM*XEL(3)
YEL1(2)DEL1*YEL(2)+DM*YEL(3)
XEL1(3)XEL(3)
YEL1(3)YEL(3)
CALL SINGUL(XNODE,YNODE,XEL1,YEL1,EINT,MAXINT,F,DEL)
F(1)=F(1)
F(2)=DEL1*F(1)
F(3)=F(3)+DM*F(1)
RETURN

C INTEGRATE THE RIGHT SIDE OF THE TRIANGLE

XEL1(3)XEL(3)
YEL1(3)YEL(3)
XEL1(2)XEL(2)
YEL1(2)YEL(2)
CALL SINGUL(XNODE,YNODE,XEL1,YEL1,EINT,MAXINT,F,DEL)
F(1)=F(1)+F(1)
F(2)=F(2)+F(1)*F(1)*DEL1
F(3)=F(3)+DM*F(1)
RETURN

C DEGENERATE CASE WHERE THE TRIANGULAR ELEMENT HAS ONE SIDE
C COINCIDE WITH THE SINGULAR STRIP

1000 CALL SINGUL(XNODE,YNODE,XEL,YEL,EINT,MAXINT,F,DEL)
RETURN
END

COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION
SUBROUTINE SING(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,JEL)
C
C INTEGRATION OVER THE SPECIAL TYPE OF TRIANGULAR ELEMENT BOUNDED
C BY A VERTICAL LINE AND TWO OTHER STRAIGHT LINES
C THE TRIANGLE MAY BE SINGULAR OR REGULAR
C
C DIMENSION F(4),F(1)
C DIMENSION XEL(4),YEL(4),XEL(4),YEL(4),EINT(MAXINT)+W(MAXINT)
C X1,Y1,DEL+F[C
C IF (ABS(YL1(1)-Y1L(1)) .GT. 1.E-5) GO TO 10
C IF (ABS(YL1(2)-Y1L(2)) .LT. 1.E-5) GO TO 1
C IF (ABS(YL1(3)-Y1L(3)) .LT. 1.E-5) GO TO 3
C IF (ABS(YL1(4)-Y1L(4)) .LT. 1.E-5) GO TO 4
C
C C NON-SINGULAR TRIANGLE
C CALL POLYGN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,J)
C RETURN
C
C TRIANGLE POINTING LEFT WITH SINGULAR STRIP PASSING THROUGH
C THE VERTICAL SIDE
C
1 XEL(1)=XEL(3)
YEL(1)=YEL(3)
XEL(2)=XEL(1)+XEL(3))
YEL(2)=YEL(1)+YEL(3))
YEL(3)=YEL(2)+YEL(3))
YEL(4)=Y4L(2)+YEL(3)
XEL(4)=XEL(3)
YEL(4)=Y4L(3)
CALL L=SPAN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F(1)
F(1)=DEL+F(1)
F(2)=DEL+F(3)
F(3)=F(1)+F(4)+CM*(F(2)+F(3))
IF (ABS(F-DEL)+LT.1.E-5) RETURN
XEL(1)=XEL(2)
YEL(1)=YEL(2)
XEL(3)=XEL(1)
YEL(4)=YEL(3)
XEL(2)=XEL(4)
YEL(2)=YEL(4)
YEL(1)=YEL(1)
XEL(1)=XEL(1)
YEL(1)=YEL(1)
YEL(3)=YEL(3)
CALL POLYGN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F(4)
F(1)=F(1)+F(2)+DEL+F(1)
F(2)=F(2)+F(3)+DEL+F(4)
F(3)=F(3)+CM*(F(1)+F(4))
RETURN
C
C TRIANGLE POINTING RIGHT WITH SINGULAR STRIP PASSING THROUGH
C THE VERTICAL SIDE
C
2 XEL(1)=XEL(1)+XEL(2)
YEL(1)=YEL(1)+YEL(2)
56
COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION
XEL1(2)=XEL(2)+XEL(1)
YEL1(2)=YEL(2)+YEL(1)
XEL1(3)=XEL(2)+XEL(3)
YEL1(3)=YEL(2)+YEL(3)
XEL1(4)=XEL(3)
YEL1(4)=YEL(3)
CALL LGSUPAN(XNODE+YNODE+XEL1+YEL1+E.INT+W+MAXINT+F1)
F(1)=F1(1)+CM*F1(2)
F(2)=DEL*(F1(2)+F1(3))
F(3)=F1(4)+CM*F1(3)
IF (ABS1-DEL)*LT+1*E-5) RETURN
XEL1(1)=XEL1(2)
YEL1(1)=YEL1(2)
XEL1(2)=XEL1(2)
YEL1(2)=YEL1(2)
CALL POLYGN(XNODE+YNODE+XEL1+YEL1+E.INT+W+MAXINT+F1+3)
F(1)=F1(1)+CM*F1(1)
F(2)=F1(2)+F1(3)
F(3)=F3(3)+CM*F1(3)
RETURN
END
SUBROUTINE TABLE(COEFF,DELTA,INT,W,MAXINT,DEL,IMAX,JMAX,LMAX)

C THIS SUBROUTINE CREATES A TABLE OF THE WEIGHTED KERNEL
C COEFFICIENTS FOR A REGULAR CHARACTERISTIC MESH
C EACH ELEMENT IS UNIQUELY DEFINED BY A PAIR OF RELATIVE INDICES
C DELTA IS THE LENGTH IF THE SIDE OF THE CHARACTERISTIC ELEMENT
C IMAX IS THE MAXIMUM POSSIBLE NUMBER OF ELEMENTS IN THE COLUMNWISE DIRECT
C JMAX IS THE MAXIMUM POSSIBLE NUMBER OF ELEMENTS IN THE COLUMNWISE DIRECT
C THE TABLE IS COMPACTLY STORED INTO A RECTANGULAR ARRAY
C
C COMPLEX F+COEFF
C DIMENSION XEL(4),YEL(4),INT(MAXINT),W(MAXINT)
C COMMON XM,XK,BETAS,BETA
C LINEAR(1,J)={1-1*(JMAX+JMAX-1)/2+J}
C YEL,JDEL=DELTA/XM
C XDEL=BETA*YDEL
C DO 20 LL=1,IMAX
C DO 20 MM=1,JMAX
C
C DEFINE THE ELEMENT LOCATION ACCORDING TO ITS RELATIVE INDICES
C
C L=LINEAR(LL,MM)
C XEL(1)=(2-MM-MM)*XDEL
C XEL(2)=XEL(1)+XDEL
C XEL(3)=XEL(2)+XDEL
C XEL(4)=XEL(3)
C YEL(1)=(MM-MM)*YDEL
C YEL(2)=YEL(1)+YDEL
C YEL(3)=YEL(2)
C YEL(4)=YEL(3)
C
C CHARACTERISTIC ELEMENT CUT BY THE SINGULAR STRIP
C
C IF ((MM-LL).LE.1) CALL SGWEB(XNODE,YNODE,XEL,YEL,INT,W,MAXINT,F+DEI)
C
C REGULAR CHARACTERISTIC ELEMENT
C
C IF ((MM-LL).GT.1) CALL POLYONIXNODE,YNODE,XEL,YEL,INT,W,MAXINT,F+4)
C DO 20 LI*4
C COEFF(1,L)=F(1)
C 20 CONTINUE
C RETURN
C END

COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION
SUPPLEMENTARY

INFORMATION
SUBROUTINE KERNEL(XO,YO,XX,REAL,XX,IMAG)

C SUPersonic KERNEL FUNCTION DERIVED BY HANDLER AND HODDEN
C AS GIVEN BY A. CUNNINGHAM IN APPENDIX OF J. AERONAV.
C VOl. 11, NO. 10, 1974 WITH CORRECTIONS AT LINES PRECEDED BY **

DIMENSION A(11)
COMMON XM,XX,DELTA0,BETA
DATA A(-249919.95,2,75,105,2,74,499,2,75,105,2,74,499),
1 73289,1,183,30,1,98339,75,105,326,72755,
2 12793171

DATA C#372/ K=SUMRT(XO*XO-BETA**Y0*Y0)
IF (XX<LT1.E-5) GO TO 400
AYO=ABS(Y0)
ARG=XXX0
C=COS(Ang)
SN=SIN(Ang)
IF (AY0+LT1.E-5) GO TO 500

C GENReIAL FORM OF KERNEL FUNCTION

XXAYO=XXAYO
XXY0=XXY0+XXAYO
BXY0=1.1(I-BETAUAY0)
XXM=XXM
U1=BY01((XO-XXM)
U2=BY01((XO+XXM)
E1=EXP(-C*ABS(U1))
E2=EXP(-C*U2)
CST=00
CST1=10
CST2=10
SUMO=0
SUMR1=0
SUM1M=0
SUM2R=0
SUM2M=0
DO 100 I=1+11
CST=CST+C
CST1=CST1+EI
CST2=CST2+EI
COEF=00/(CST*Y0)*CST+Y0)
SUM=SUM-CUEF
CUEF=11(CST-CUEF)
COEF1M=XXAYO*XUEF
SUM1R=SUM1R+CUEF1*CST1
SUM1M=SUM1M+CUEF1M*CST1
SUM2R=SUM2R+CUEF2*CST2
SUM2M=SUM2M+CUEF2M*CST2
100 CONTINUE

ARG=XXAYO*U1
CS=105(CS)
SN=SIN(CS)
ARG=XXAYO*U2
C2=105(C2)
SN2=SIN(C2)
X112RL=XXAYO*(542*SUM2R-C2*SUM2M)

B 001
B 002
B 003
B 004
B 005
B 006
B 007
B 008
** B 009
B 010
B 011
B 012
B 013
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B 058
XI121M*XXAYO*(CS2*SUMZRL+SNZ*SUM2IM)
CST1=SN1=SUM1RL
*SST2*CS1=SUM1IM
/CST3=CS1*SUM1RL
CST4=SN1=SUM1IM
IF (UL+LT=0.) GO TO 200
 XI11RL*XXAYO*(CST1+CST2)
 XI11IM*XXAYO*(CST3+CST4)
GO TO 300
200 XI11RL*XXAYO*(CST1+CST2)+2.*(CS1-1.)*XXYOS6=SUM1
 XI11IM*XXAYO*(CST3-CST4)-SN1-SN1
300 CONTINUE
CST1=(XOL/R)+1.
CST2=CST1-2.
XI11RL=CST1+XI11RL
XI11IM=-CST1*SN1=XI11IM
XI12RL=CST2*CS2+XI12RL
XI12IM=-CST2*SNZ+XI12IM
SUM1RL=XI11RL*XX12RL
SUM1IM=XI11IM*XX12IM
XXREAL=+9.*(CS*SUM1RL+SN*SUM1IM)
XXIMG=-3.*(CS*SUM1IM-SN*SUM1RL)
RETURN
C
C STEADY FORM OF KERNEL FUNCTION
C
400 XXREAL=XO/X
XXIMG=0
RETURN
C
C SPECIAL FORM OF KERNEL FUNCTION AT YO=0
C
500 XXREAL=CS
XXIMG=SN
RETURN
END