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EFFECT OF LARGE HIGH-FREQUENCY ANGULAR MOTION OF A SHELL ON THE ANALYSIS OF ITS YAWSONDE RECORDS

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February 1976

USA BALLISTIC RESEARCH LABORATORIES
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The usual data analysis of yawsonde records assumes that the sun angle is constant during one revolution of the shell and uses three successive sightings of the sun to yield sun angle and $\dot{\phi}$, the roll rate with respect to the plane containing the sun and the missile's axis. An improved analysis technique is described which assumes a linear variation of the sun angle and uses five successive sun sights to obtain sun angle and $\dot{\phi}$. For large high-frequency yawing motion, $\dot{\phi}$ has a periodic component which is usually at the frequency of the yawing motion. An expression for this periodic component is derived and it is

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Block 20. ABSTRACT (continued)

shown that the average of $\dot{\phi}$ is very close to the spin rate of the shell.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

A yawsonde is a device developed to give a continuous record of the angle between a shell's axis of rotation and the line of sight to the sun[1-6]. It consists of two light sensors located behind two inclined slits on the shell body and a telemetry unit. The instant each sensor sees the sun, a discrete pulse is transmitted to a ground station and recorded versus time. The resulting pulse train can then be analyzed to yield sun angles to an accuracy of 0.1° and the spin rate. The usual reduction, however, assumes the sun angle to be constant for a single revolution of the shell. If the shell is performing large high-frequency angular motion, this assumption can be invalid and erroneous data can be produced. It is the purpose of this report to develop a valid data analysis system to handle yawsonde data without assuming the constancy of the sun angle during a single revolution of the shell.

II. CONSTANT SUN ANGLE ANALYSIS

We will make use of a solar fixed-plane coordinate system which can be described in terms of three unit vectors. $\vec{e}_1$ is along the missile's axis, $\vec{e}_{3S}$ is in the plane determined by the sun and the

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1. *I. Amery, H. Henning, K. Lawrie, and E. Wlatnig, "A Telemetry System for the Measurement of the Yaw of a Projectile Throughout the Major Part of Its Trajectory (U)," RARDE Report 1/65, March 1965. AD 359250. (Confidential Report)*

2. *W. R. Haseltine, "Yawing Motion of 5.0" MK 41 Projectile Studied by Means of Yaw Sondes," Naval Weapons Center Report NWC TP 4779, August 1969. AD 862065.*

3. *W. H. Mermagen, "Measurements of the Dynamical Behavior of Projectiles over Long Flight Paths," Journal of Spacecraft and Rockets, Vol. 8, April 1971, pp. 380-385. (See also BRL MR 2079. AD 717002)*

4. *W. H. Clay, "A Precision Yawsonde Calibration Technique," BRL MR 2263, January 1973. AD 758158.*

5. *R. H. Whyte and W. H. Mermagen, "A Method for Obtaining Aerodynamic Coefficients from Yawsonde and Radar Data," Journal of Spacecraft and Rockets, Vol. 10, June 1973, pp. 384-388. (See also BRL MR 2280. AD 759482)*

6. *W. H. Mermagen and W. H. Clay, "The Design of a Second Generation Yawsonde," BRL MR 2368, April 1974. AD 780064.*

missile's axis and points toward the sun's side of the missile's axis, while $\vec{e}_{2s}$ is $\vec{e}_{3s} \times \vec{e}_1$.

Each sun sensor unit emits a pulse at the instant that the sun is in the plane of its inclined slit. Let γ_a be the inclination of the first slit with respect to the missile's axis and let ϕ_a be the missile's roll angle with respect to the vector $\vec{e}_{2s}$ when the line-of-sight from the first sensor to the sun lies in the $\vec{e}_1\vec{e}_{3s}$ plane. **Then the vector** normal to the plane of the first sun sensor is

$$\vec{N}_a = \vec{e}_1 \sin \gamma_a + [\, \vec{e}_{2s} \cos (\phi-\phi_a) + \vec{e}_{3s} \sin (\phi-\phi_a) \,] \cos \gamma_a \qquad (1)$$

where ϕ is the roll angle of the missile. The sun line-of-sight can be specified by the unit vector

$$\vec{s} = \vec{e}_1 \cos \sigma + \vec{e}_{3s} \sin \sigma \qquad (2)$$

where σ is the angle between the sun's rays and the missile's axis. When the sun is in the plane of the first sensor

$$\vec{N}_a \cdot \vec{s} = 0 \qquad (3)$$

or

$$\sin (\phi-\phi_a) = - \tan \gamma_a \operatorname{ctn} \sigma \qquad (4)$$

A similar relation applies to the second sun sensor:

$$\sin (\phi-\phi_b) = - \tan \gamma_b \operatorname{ctn} \sigma \qquad (5)$$

We now consider a triple of time pulses consisting of two sightings of the sun by the first sensor at $t = t_1, t_3$ and one by the second sensor at $t = t_2$. The pairs of ϕ_i, σ_i's for these three times satisfy the relations:

$$\sin (\phi_i-\phi_a) = - \tan \gamma_a \operatorname{ctn} \sigma_i \qquad i = 1,3 \qquad (6)$$

$$\sin (\phi_2-\phi_b) = - \tan \gamma_b \operatorname{ctn} \sigma_2 \qquad (7)$$

We now assume that $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ and that $\dot{\phi}$ is a constant. Then

$$\sin (\phi_1-\phi_a) = \sin (\phi_3-\phi_a) \qquad (8)$$

so that*

$$\phi_3 - \phi_1 = 2\pi \tag{9}$$

or

$$\dot{\phi} = \frac{2\pi}{t_3 - t_1} \tag{10}$$

Now

$$\begin{aligned} \phi_2 - \phi_1 &= \dot{\phi}\ (t_2 - t_1) \\ &= 2\pi\ (t_2 - t_1)\ (t_3 - t_1)^{-1} \end{aligned} \tag{11}$$

Let

$$\delta = \phi_2 - \phi_1 - (\phi_b - \phi_a) \tag{12}$$

Then

$$\begin{aligned} \sin(\phi_2 - \phi_b) &= \sin(\phi_1 - \phi_a + \delta) \\ &= \sin(\phi_1 - \phi_a) \cos\delta + \sin\delta \cos(\phi_1 - \phi_a) \end{aligned} \tag{13}$$

Equations (6-7) can now be substituted in Equation (13) and the result simplified to yield

$$\tan\sigma = [\tan^2\gamma_a - 2\tan\gamma_a \tan\gamma_b \cos\delta + \tan^2\gamma_b]^{\frac{1}{2}} (\sin\delta)^{-1} \tag{14}$$

This equation gives the sun angle in terms of the three times, (t_1, t_2, t_3) and three parameters of the sensors (γ_a, γ_b, ϕ_b-ϕ_a) and is valid for idealized sun sensors. Actual sun sensors will not exactly satisfy Equations (4-5) but roll angles for each sun sensor can be measured as functions of the sun angle[4].

$$\phi_i = f_a(\sigma_i) + (i-1)\pi \qquad i = 1, 3 \tag{15}$$

$$\phi_2 = f_b(\sigma_2) + 2\pi\ell \tag{16}$$

* Since the sun sensor plane is really a half plane, the root $\phi_3 - \phi_1 = \pi$ is not appropriate.

where ℓ is an integer selected so that

$$\phi_1 < \phi_2 < \phi_3$$

Once again we assume that $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ and that $\dot{\phi}$ is a constant. Equations (10-11) are still valid and

$$\phi_2 - \phi_1 = f_b(\sigma) - f_a(\sigma) + 2\pi j \equiv g(\sigma) \qquad (17)$$

or

$$\sigma = g^{-1}\ (\phi_2 - \phi_1) \qquad (18)$$

where $\phi_2 - \phi_1$ is given by Equation (11).

III. VARYING SUN ANGLE ANALYSIS

We assume that the sun angle is a linear function of time and that the roll angle is a quadratic function of time and hence we need five successive sightings of the sun to determine the five parameters:

$$\sigma_i = \dot{\sigma}_3\ (t_i - t_3) + \sigma_3 \qquad i = 1,2,3,4,5 \qquad (19)$$

$$\phi_i = \ddot{\phi}_3\ (t_i - t_2)^2/2 + \dot{\phi}_3\ (t_i - t_2) + \phi_3 \qquad (20)$$

If three of the sightings are by the first sensor and two by the second, then

$$\phi_i = f_a\ (\sigma_i) + (i-1)\pi \qquad i = 1,3,5 \qquad (21)$$

$$\phi_i = f_b\ (\sigma_i) + 2\pi\ell + (i-2)\pi \qquad i = 2,4 \qquad (22)$$

The five parameters $(\sigma_3, \dot{\sigma}_3, \phi_3, \dot{\phi}_3, \ddot{\phi}_3)$ can be obtained from the five times $(t_1, t_2, t_3, t_4, t_5)$ by substituting Equations (19-20) in Equations (21-22) and solving the resulting nonlinear algebraic equations by an iterative differential correction technique[7]. The first set of values of $(\sigma_3, \dot{\sigma}_3)$, i.e. $(\sigma_{31}, \dot{\sigma}_{31})$ can be obtained by using the constant-sun-angle solutions for (t_1, t_2, t_3) and (t_3, t_4, t_5). Since these

7. *P. R. Bevington, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, New York, 1969.*

approximately correspond to the sun angles for t_2 and t_4, we will denote them by σ_2, σ_4.

$$\therefore \quad \sigma_{31} = (\sigma_2 + \sigma_4)/2 \tag{23}$$

$$\dot{\sigma}_{31} = (\sigma_4 - \sigma_2)/(t_4 - t_2) \tag{24}$$

Equations (19-20) for the (j-1) iteration can be written in the form:

$$\ddot{\phi}_{3j}\,(t_i - t_3)^2/2 + \dot{\phi}_{3j}\,(t_i - t_3) + \phi_{3j}$$

$$= f_K\,(\sigma_{ij-1}) + \left(\frac{\partial\, f_K\,(\sigma_{ij-1})}{\partial \sigma_3}\right)(\sigma_{3j} - \sigma_{3j-1})$$

$$+ \frac{\partial\, f_K\,(\sigma_{ij-1})}{\partial \dot{\sigma}_3}\,(\dot{\sigma}_{3j} - \dot{\sigma}_{3j-1}) + m\pi \tag{25}$$

where

$$K = a,\ m = i-1 \text{ for } i = 1,3,5$$

$$K = b,\ m = 2(\ell-1) + i \text{ for } i = 2,4$$

Equations (25) are linear in the unknowns $(\sigma_{3j}, \dot{\sigma}_{3j}, \phi_{3j}, \dot{\phi}_{3j}, \ddot{\phi}_{3j})$ and can be easily solved; usually only three or four iterations are required.

A second version of a varying sun angle analysis using five times can be used for constant $\dot{\phi}$. In this case, Equations (19-20) are replaced by:

$$\sigma_i = \ddot{\sigma}_3\,(t_i - t_3)^2/2 + \dot{\sigma}_3\,(t_i - t_3) + \sigma_3 \tag{26}$$

$$\phi_i = \dot{\phi}_3\,(t_i - t_3) + \phi_3 \tag{27}$$

Equations similar to Equations (25) can be easily derived for this variant.

IV. EFFECT OF MOVING COORDINATE SYSTEM

ϕ is the roll angle measured with respect to $\vec{e}_{2S}$. $\dot{\phi}$ is a roll rate measured in this solar fixed-plane coordinate system and is not necessarily equal to the roll rate, p, measured in an inertia coordinate system since the solar coordinate system can itself have a non-zero roll rate.

For our inertia system, we will use an earth-fixed coordinate system X_1, X_2, X_3, so aligned that the X_3 axis is vertical and the trajectory is initially in the X_1-X_3 plane. Since the actual trajectory can be reasonably well approximated by a planar trajectory, we will approximate the velocity vector along the trajectory by $(V \cos \theta_T, 0, V \sin \theta_T)$.

In the appendix, the roll component (ω_{1s}) of the angular velocity of the sun fixed-plane coordinates is computed in terms of the horizontal and vertical components of the complex yaw in the aeroballistic fixed-plane coordinates[8]:

$$\omega_{1s} = (b_1 \dot{\hat{\xi}}_H + b_2 \dot{\hat{\xi}}_V) \cos \sigma_T + b_3 \hat{\xi}_H \dot{\hat{\xi}}_H + b_4 \hat{\xi}_V \dot{\hat{\xi}}_V + b_5 \hat{\xi}_H \dot{\hat{\xi}}_V + b_6 \dot{\hat{\xi}}_H \hat{\xi}_V \qquad (28)$$

where

$\hat{\xi} = \hat{\xi}_H + i \hat{\xi}_V = \sin \hat{\beta} + i \cos \hat{\beta} \sin \hat{\alpha}$

$\hat{\alpha}$, $\hat{\beta}$ are the angles of attack and sideslip

σ_T is the angle between the sun vector and the tangent to the trajectory;

b_i's are defined in Table 1.

TABLE 1. COEFFICIENTS IN EQUATIONS (28) AND (A17)
$b_1 = (s_3 \cos \theta_T - s_1 \sin \theta_T) \csc^2 \sigma_T$
$b_2 = -s_2 \csc^2 \sigma_T$
$b_3 = b_2 [2 \cos \sigma_T \tan \theta_T - (1 + \cos^2 \sigma_T) b_1]$
$b_4 = b_2 [2 \cos \sigma_T \tan \theta_T + (1 + \cos^2 \sigma_T) b_1]$
$b_5 = \cot^2 \sigma_T - b_1 \cos \sigma_T \tan \theta_T - b_2^2 (1 + \cos^2 \sigma_T)$
$b_6 = -\cot^2 \sigma_T - b_1 \cos \sigma_T \tan \theta_T + b_1^2 (1 + \cos^2 \sigma_T)$

8. *C. H. Murphy, "Free Flight Motion of Symmetric Missiles", BRL Report 1216, July 1963, AD 442757.*

The variation of the complex yaw can be described by the usual epicycle

$$\hat{\xi} = K_1 e^{i\phi_1} + K_2 e^{i\phi_2}$$

$$\phi_j = \phi_{jo} + \dot{\phi}_j t \tag{29}$$

The presence of derivatives in all terms of Equations (28) suggests that the high frequency mode is the more important mode in Equation (29). If we neglect the low frequency mode (K_2=0), Equation (28) becomes

$$\omega_{1s} = (-b_1 \sin \phi_1 + b_2 \cos \phi_1) \dot{\phi}_1 K_1 \cos \sigma_T$$
$$+ [(b_4 - b_3) \sin \phi_1 \cos \phi_1 + b_5 \cos^2 \phi_1 - b_6 \sin^2 \phi_1] \dot{\phi}_1 K_1^2 \tag{30}$$

Since $\dot{\phi}$ is the difference between the projectile spin and the coordinate system spin:

$$\dot{\phi} = p - \omega_{1s}, \tag{31}$$

it will be a periodic function with frequency $\dot{\phi}_1$ except when $\sigma_T = \pi/2$. For this special case where the sun is perpendicular to the projectile, the frequency will be $2\dot{\phi}_1$. The average value of $\dot{\phi}$ is, however, quite close to the projectile spin:

$$\dot{\phi}_{av} = p + (b_5 - b_6) \dot{\phi}_1 K_1^2/2 \tag{32}$$

V. DISCUSSION

In Figure 1, the sun angle complement ($\sigma_n = \pi/2 - \sigma$) and $\dot{\phi}$ as obtained from the three-time-measurements, constant-sun-angle reduction are plotted for an eight-inch projectile. These data are then reduced by the five-time-measurements analysis of this report and plotted in Figure 2. The sun angles are changed by very little but the oscillations in $\dot{\phi}$ are reduced by 50%. The remainder is clearly periodic with frequency $\dot{\phi}_1$. Later in the flight, the sun angle went through $\pi/2$; Figure 3 shows σ_n and $\dot{\phi}$ for this portion of the trajectory. We note the $\dot{\phi}$ oscillations are much smaller, with frequency $2\dot{\phi}_1$.

It is interesting to note that the need for the varying σ data reduction can be eliminated by a modified yawsonde. If the first sun sensor is oriented so that $\gamma_a = 0$, that sensor is insensitive to sun angle (see Equation (4)), so that

$$f_a(\sigma) \doteq 0 \tag{33}$$

This sensor directly measures $\dot{\phi}$. The second sensor, then, yields a σ_2 at $t=t_2$ from the standard three-time-measurements procedure.

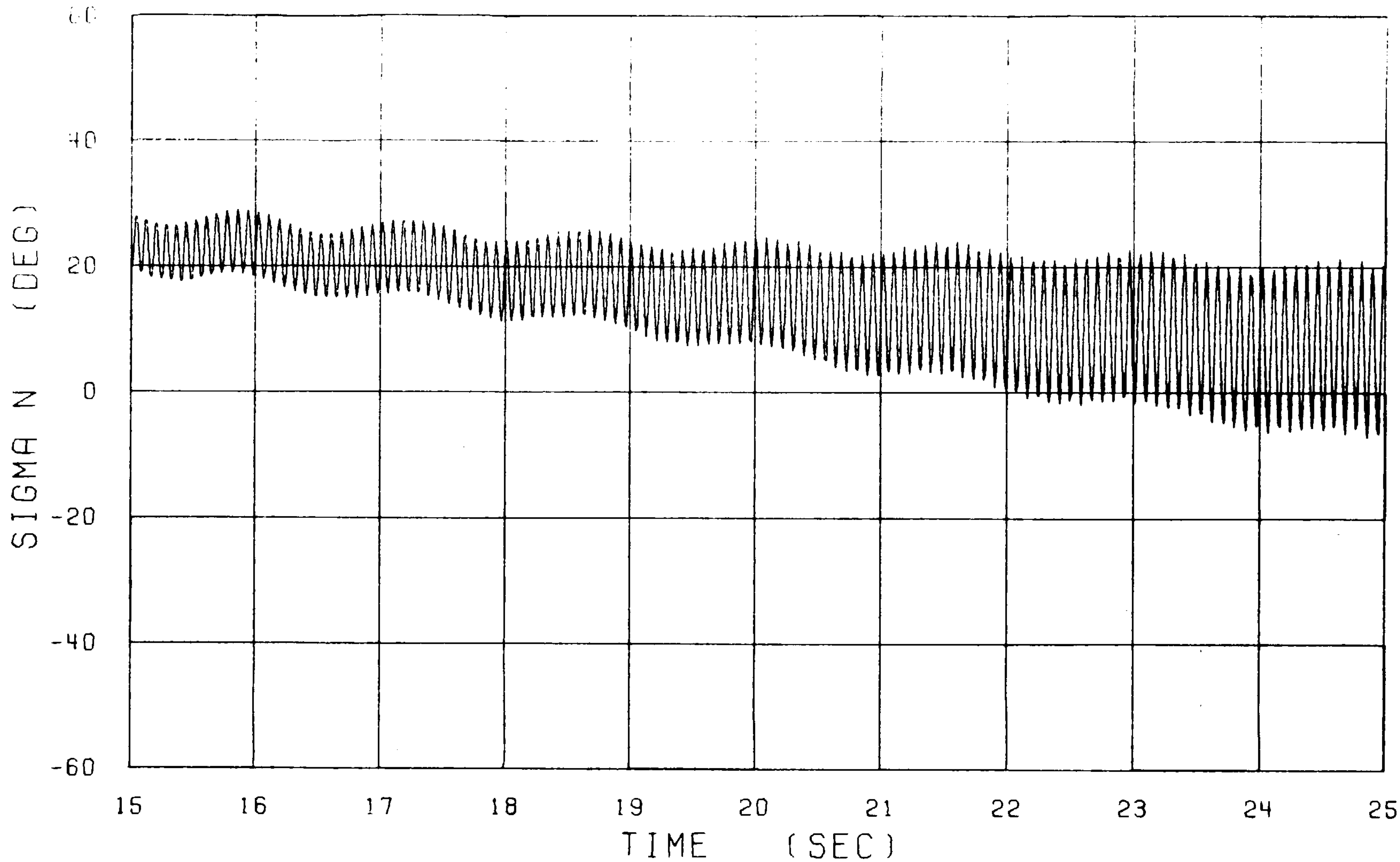

Figure 1A. Sun Angle versus Time for Eight-Inch Shell Computed from Sets of Three Sun Sights

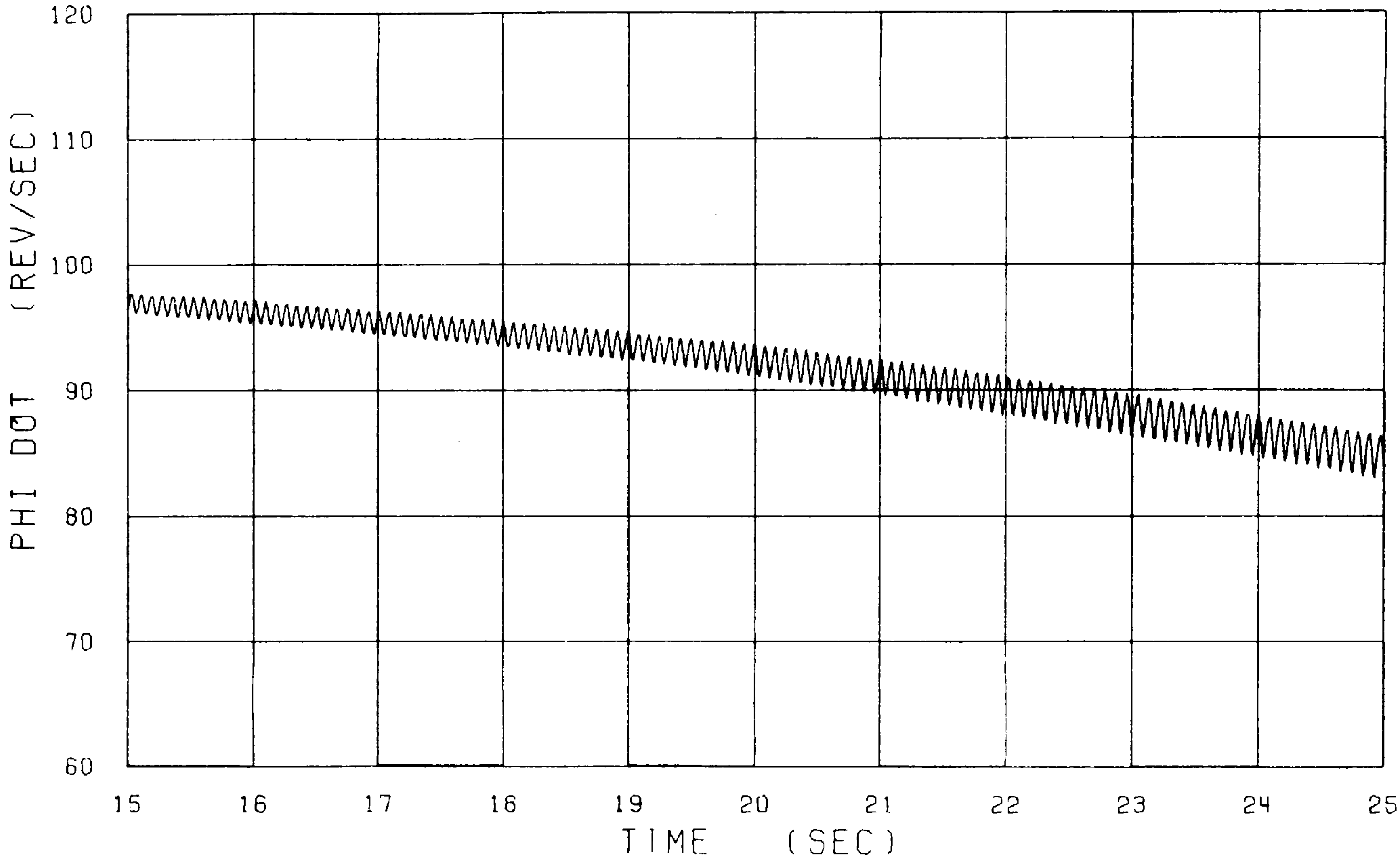

Figure 1B. $\dot{\phi}$ versus Time for Eight-Inch Shell Computed from Sets of Three Sun Sights

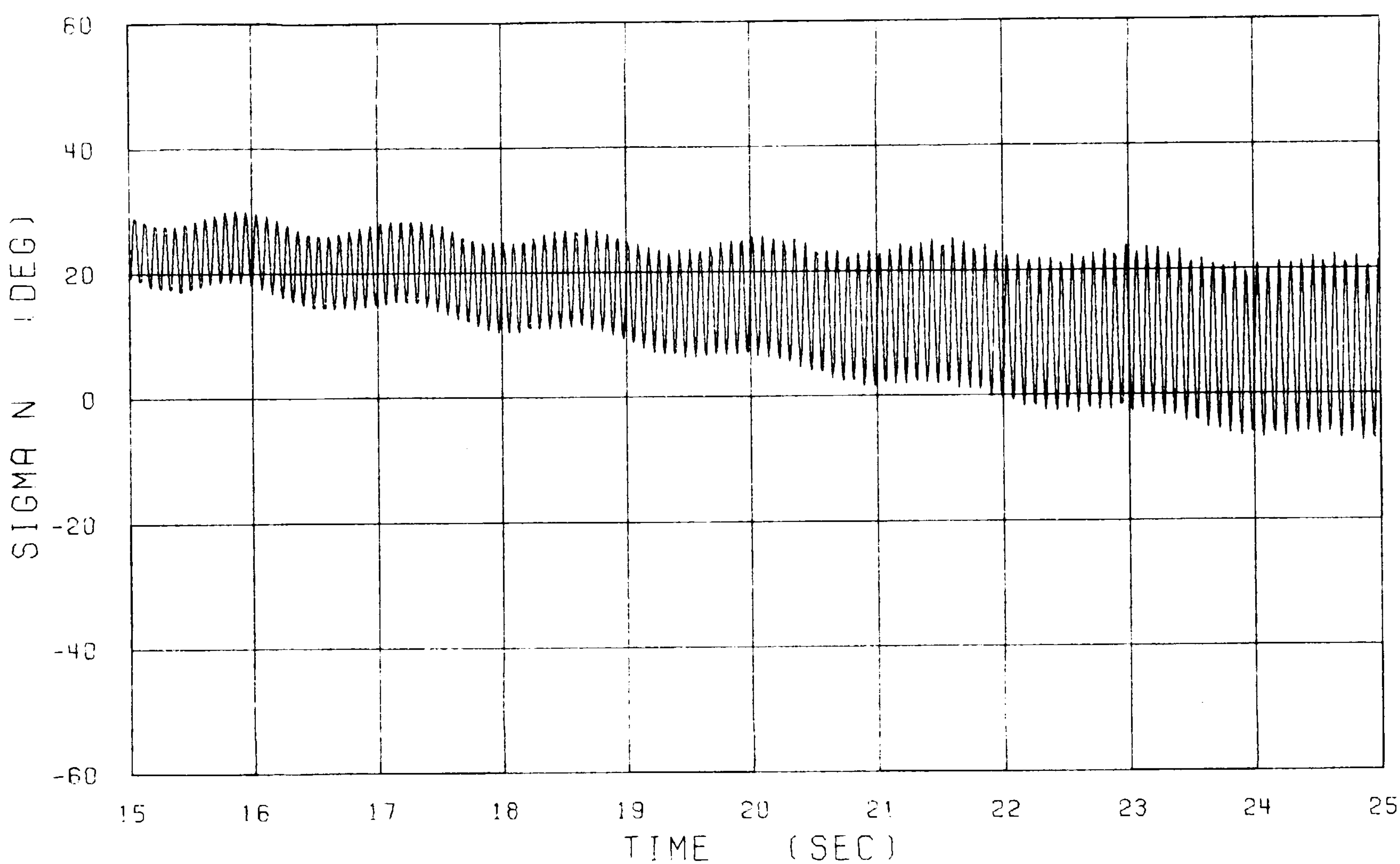

Figure 2A. Sun Angle versus Time for Eight-Inch Shell Computed from Sets of Five Sun Sights ($\sigma_T \neq 90^0$; $\sigma_n \neq 0$)

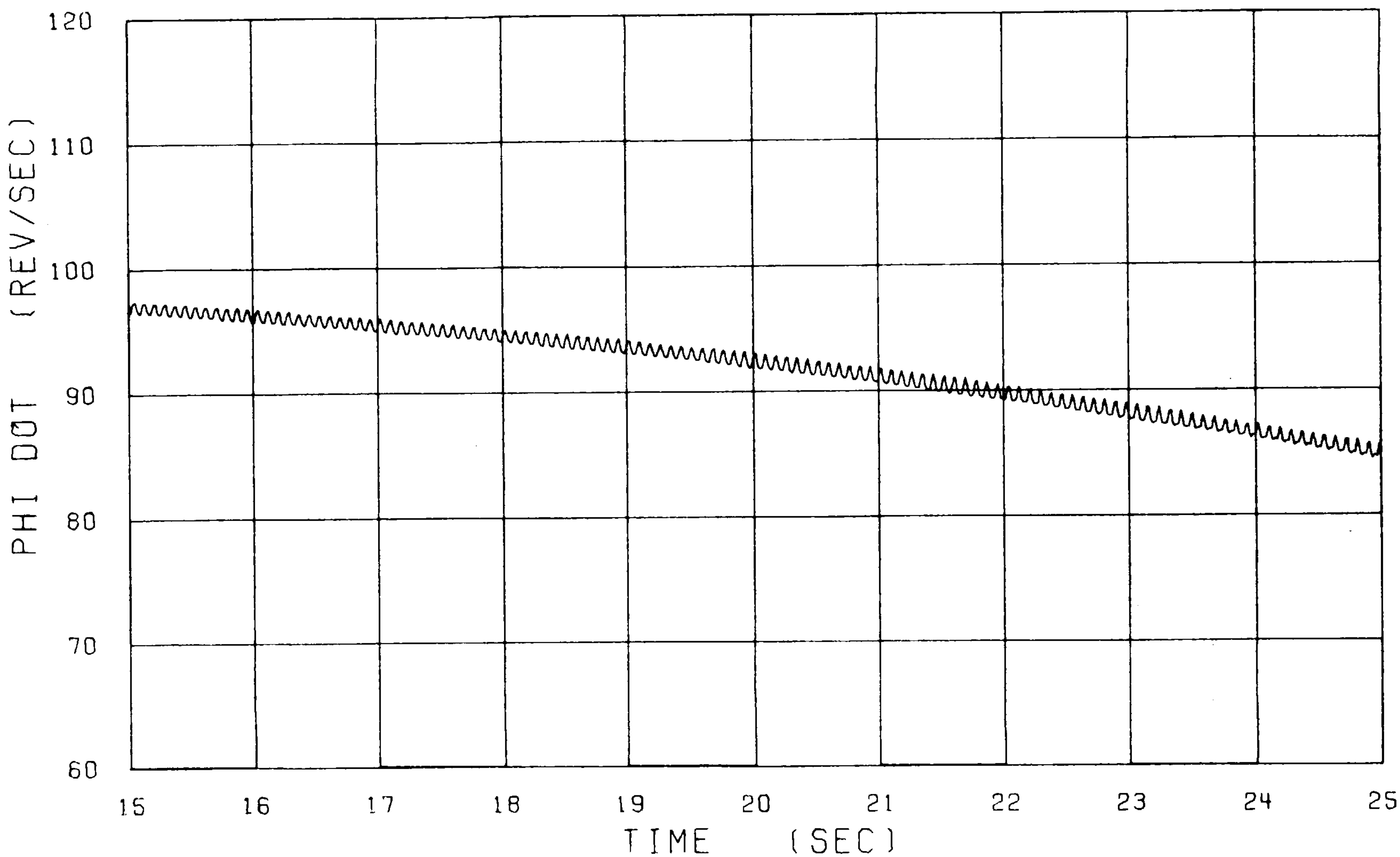

Figure 2B. $\dot{\phi}$ versus Time for Eight-Inch Shell Computed from Sets of Five Sun Sights ($\sigma_T \neq 90^0$; $\sigma_n \neq 0$)

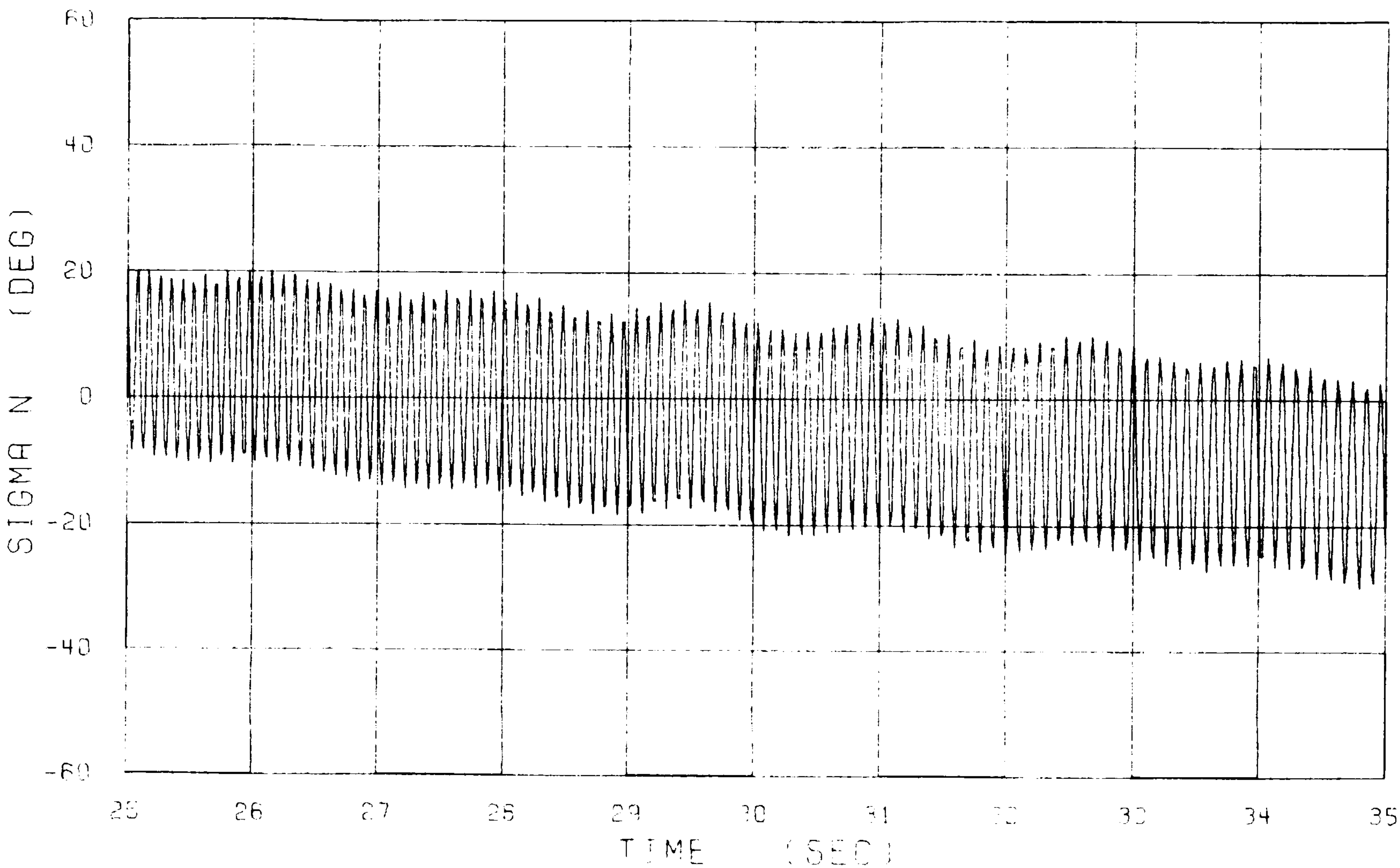

Figure 3A. Sun Angle versus Time for Eight-Inch Shell Computed from Sets of Five Sun Sights ($\sigma_T = 90^0$; $\sigma_n = 0$)

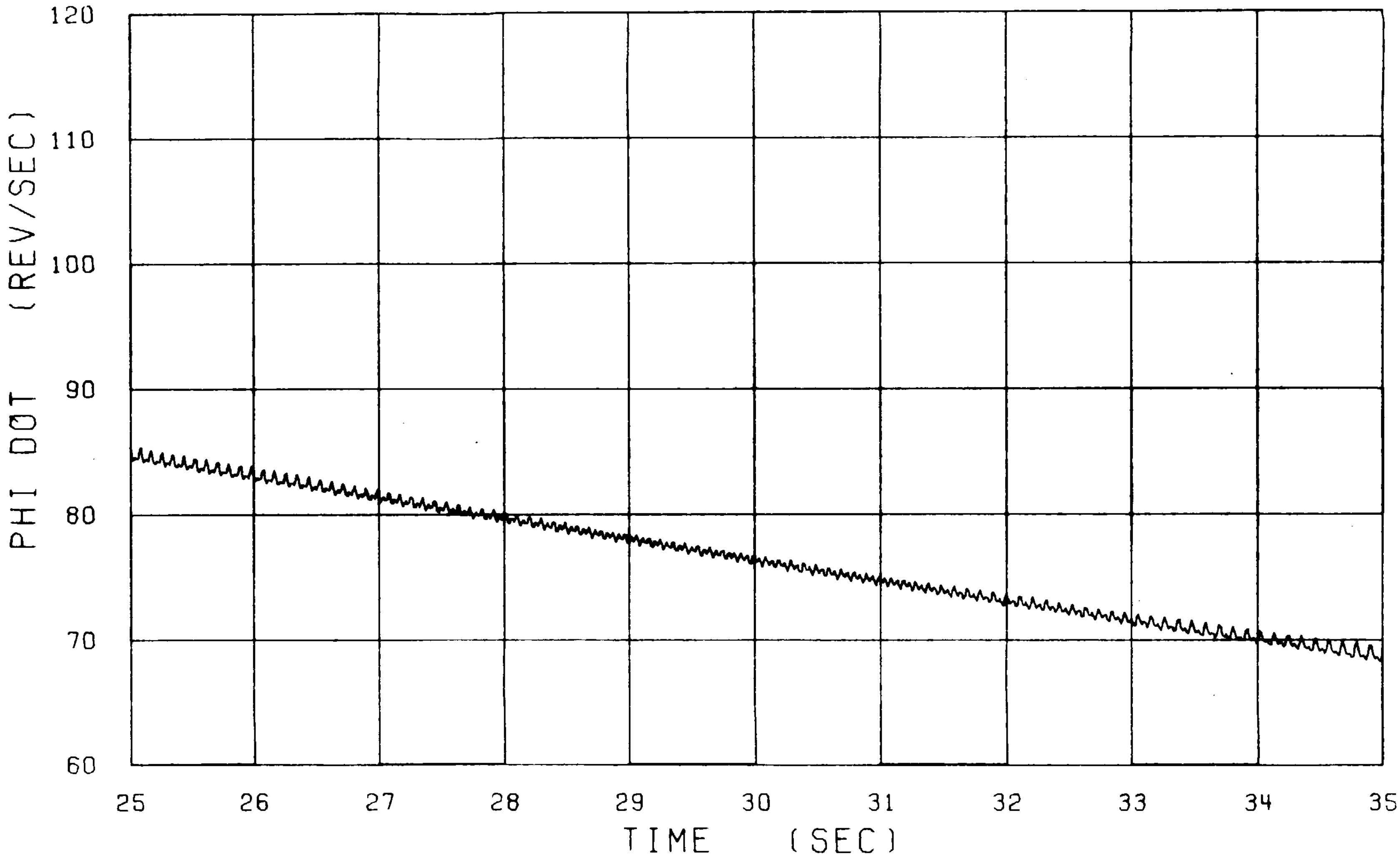

Figure 3B. $\dot{\phi}$ versus Time for Eight-Inch Shell Computed from Sets of Five Sun Sights
(σ_T = 90^o; σ_n = 0)

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1. I. Amery, H. Henning, K. Lawrie and E. Wlatnig, "A Telemetry System for the Measurement of the Yaw of a Projectile Throughout the Major Part of Its Trajectory (U)," RARDE Report 1/65, March 1965, AD 359250, (CONFIDENTIAL).

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8. C. H. Murphy, "Free Flight Motion of Symmetric Missiles," U.S. Army Ballistic Research Laboratories Report No. 1216, July 1963, AD 442757.

LIST OF SYMBOLS

$b_1, b_2, \ldots b_6$	coefficients in the expression for ω_{1s}, Equation 28; defined in Table 1
$\vec{e}_1$	unit vector along the missile's axis
$\vec{e}_1, \vec{e}_{2a}, \vec{e}_{3a}$	unit vectors in the aeroballistic fixed-plane system, Equations (A4-A5)
$\vec{e}_1, \vec{e}_{2s}, \vec{e}_{3s}$	unit vectors in the solar fixed-plane system, Equations (A7-A8)
f_a, f_b	sun angle functions obtained from roll angle versus sun angle data provided by sun sensor (a) or (b)
f_K	f_a or f_b, Equation (25)
g	a function relating the difference in successive roll angle values to the sun angle, Equation (17)
g^{-1}	the inverse of function g, Equation (18)
K_1, K_2	magnitude of the high (1) or low (2) frequency yaw mode, Equation (29)
ℓ	an integer in Equations (16, 22, 25) providing the proper multiple of 2π
m	$i - 1$ $i = 1, 3, 5$ $2(\ell - 1) + i$ $i = 2, 4$
$\vec{N}_a$	vector normal to the plane of sun sensor (a)
n_1, n_2, n_3	direction cosines of the missile's axis in the earth-fixed X_1, X_2, X_3 system

LIST OF SYMBOLS (CONTINUED)

p	roll rate in the earth-fixed X_1, X_2, X_3 system
$\vec{s}$	the unit vector from the missile to the sun
s_1, s_2, s_3	components of s in the earth-fixed X_1, X_2, X_3 system
t	time
t_i	time at which a sensor sees the sun, i = 1, 2, 3 ...
V	magnitude of the velocity vector
$\vec{V}$	velocity vector
X_1, X_2, X_3	axes in an earth-fixed coordinate system: X_3 is vertical and the trajectory is initially in the $X_1 - X_3$ plane
$\hat{\alpha}$	angle of attack
$\hat{\beta}$	angle of sideslip
γ_a, γ_b	the inclination of slit (a) or (b) with respect to the missile's axis
δ	$\phi_2 - \phi_1 - (\phi_b - \phi_a)$
η_1, η_2, η_3	components of the yaw vector $(\vec{e}_1 - \vec{V}/V)$ in the earth-fixed X_1, X_2, X_3 system
θ_T	the angle between the missile's axis and the tangent to the trajectory

LIST OF SYMBOLS (CONTINUED)

$\hat{\xi}$	$\hat{\xi}_H + i\,\hat{\xi}_V$, the complex yaw in the aeroballistic fixed-plane system
$\hat{\xi}_H$	$\sin\hat{\beta}$, the horizontal component of $\hat{\xi}$
$\hat{\xi}_V$	$\cos\hat{\beta}\,\sin\hat{\alpha}$, the vertical component of $\hat{\xi}$
σ	the sun angle: the angle between the sun vector and the missile's axis
σ_i	$\sigma(t_i)$, $i = 1, 2, 3, \ldots$
σ_{ik}	the value of σ_i computed at the k-th iteration of the data analysis
σ_n	$\frac{\pi}{2} - \sigma$, the sun angle complement
σ_T	the angle between the sun vector and the tangent to the trajectory
ϕ	the missile's roll angle in the solar fixed-plane system, that is, with respect to the vector $\vec{e}_{2s}$
ϕ_a, ϕ_b	the value of ϕ when the line-of-sight from sun sensor (a) or (b) to the sun lies in the $\vec{e}_1\vec{e}_{3s}$ plane
ϕ_i	(a) $\phi(t_i)$, $i = 1, 2, 3 \ldots$ Equations (6-27)
	(b) the orientation angle of the high- (i = 1) or low- (i = 2) frequency yaw mode, Equation (29, on)

ϕ_{ik}	the value of $\phi(t_i)$ computed at the k-th iteration of the data analysis
$\dot{\phi}$	roll rate in the solar fixed-plane system
ω_{1a}	roll component of the angular velocity of the aeroballistic fixed-plane system
ω_{1s}	roll component of the angular velocity of the solar fixed-plane system
$\dot{(\ )}$	derivative with respect to time
$\hat{(\ )}$, $(\ )_a$	value in the aeroballistic fixed-plane system
$(\ )_s$	value in the solar fixed-plane system

APPENDIX A. DERIVATION OF EQUATION (28)

Our earth-fixed coordinate system X_1, X_2, X_3 will be so oriented that the X_1 - X_3 plane is the vertical plane containing the initial velocity vector. If we make the very reasonable approximation that the velocity vector stays in this plane, it can be written in the form

$$\vec{V} = V\ (\cos\ \theta_T,\ 0,\ \sin\ \theta_T). \qquad \text{(A1)}$$

The unit vector along the missile's axis then takes the form

$$\vec{e}_1 = (n_1,\ n_2,\ n_3) = (\cos\ \theta_T + \eta_1,\ \eta_2,\ \sin\ \theta_T + \eta_3) \qquad \text{(A2)}$$

$$n_1^{\ 2} + n_2^{\ 2} + n_3^{\ 2} = 1 \qquad \text{(A3)}$$

where the η_j's become zero for zero-amplitude yawing motion.

We will be using two coordinate systems which use two different fixed planes:

1. The usual aeroballistic fixed plane which contains the missile's axis and the vertical vector;

2. A sun fixed plane which contains the missile's axis and the sun vector and is the reference plane for the yaw sonde's ϕ.

The other two unit vectors for the aeroballistic fixed-plane system are

$$\vec{e}_{2a} = \frac{(0,0,1)\ X\ \vec{e}_1}{\cos\ \theta} = \frac{(-\eta_2,\ \cos\ \theta_T + \eta_1,\ 0)}{\cos\ \theta} \qquad \text{(A4)}$$

$$\vec{e}_{3a} = \frac{\vec{e}_1\ X\ [(0,0,1)\ X\ \vec{e}_1]}{\cos\ \theta} = \frac{(0,0,1) - (\sin\ \theta_T + \eta_3)\ \vec{e}_1}{\cos\ \theta} \qquad \text{(A5)}$$

$$\cos\ \theta = [\cos^2\ \theta_T + 2\eta_1\ \cos\ \theta_T + \eta_1^{\ 2} + \eta_2^{\ 2}]^{\frac{1}{2}} \qquad \text{(A6)}$$

Similarly, the other two units vectors for the sun fixed-plane system are:

$$\vec{e}_{2s} = \frac{\vec{s} \times \vec{e}_1}{\sin \sigma} \tag{A7}$$

$$\vec{e}_{3s} = \frac{\vec{e}_1 \times (\vec{s} \times \vec{e}_1)}{\sin \sigma} = \frac{\vec{s} - (\cos \sigma)\, \vec{e}_1}{\sin \sigma} \tag{A8}$$

where

$\vec{s} = (s_1, s_2, s_3)$ is the unit vector pointing to the sun,

$$\cos \sigma = \vec{s} \cdot \vec{e}_1,$$

and σ is in the first quadrant when the sun's rays illuminate the missile's nose and is in the second quadrant when the sun's rays illuminate the missile's base.

The horizontal and vertical components of the complex yaw in the aeroballistic fixed-plane coordinates can be computed from their definitions:

$$\hat{\xi}_H = \frac{\vec{e}_{2a} \cdot \vec{V}}{V} = \frac{-\eta_2 \cos \theta_T}{\cos \theta} \tag{A9}$$

$$\hat{\xi}_V = \frac{\vec{e}_{3a} \cdot \vec{V}}{V}$$

$$= \frac{\eta_1 \sin \theta_T \cos \theta_T - \eta_3 (1 + \sin^2 \theta_T + \eta_1 \cos \theta_T - \eta_3 \sin \theta_T)}{\cos \theta_T} \tag{A10}$$

where

$$\hat{\xi} = \hat{\xi}_H + i\hat{\xi}_V = \sin \hat{\beta} + i \cos \hat{\beta} \sin \hat{\alpha}$$

and $\hat{\beta}$, $\hat{\alpha}$ are the angles of attack and sideslip respectively. A quadratic approximation for η_1 and a linear approximation for $\cos \theta$ can be obtained from Equations (A3, A6, A9-10).

$$\eta_1 = -\eta_3 \tan \theta - \frac{\eta_2^2 \cos^2 \theta_T + \eta_3^2}{2 \cos^3 \theta_T} \qquad \text{(A11)}$$

$$\frac{\cos \theta_T}{\cos \theta} = 1 + \frac{\eta_3 \sin \theta_T}{\cos^2 \theta_T} \qquad \text{(A12)}$$

$$\eta_2 = -\hat{\xi}_H + \hat{\xi}_V \hat{\xi}_H \tan \theta_T \qquad \text{(A13)}$$

$$\eta_3 = -\hat{\xi}_V \cos \theta_T + (\hat{\xi}_V^2 - \hat{\xi}_H^2)(\sin \theta_T)/2 \qquad \text{(A14)}$$

The roll component of the angular velocity of the sun fixed-plane coordinates can now be computed in terms of the η_j's:

$$\omega_{1s} = -\vec{e}_{2s} \cdot \dot{\vec{e}}_{3s}$$

$$= -\vec{e}_{2s} \cdot [(\csc \sigma)^{\bullet} \vec{s} - (\text{ctn } \sigma)^{\bullet} \vec{e}_1 - \text{ctn } \sigma \dot{\vec{e}}_1]$$

$$= (\text{ctn } \sigma) \vec{e}_{2s} \cdot \dot{\vec{e}}_1$$

$$= \text{ctn } \sigma \csc \sigma [\dot{\eta}_1 (-s_2 \sin \theta_T + \eta_2 s_3 - \eta_3 s_2)$$

$$+ \dot{\eta}_2 (s_1 \sin \theta_T - s_3 \cos \theta_T + s_1 \eta_3 - s_3 \eta_1)$$

$$+ \dot{\eta}_3 (s_2 \cos \theta_T + s_2 \eta_1 - s_1 \eta_2) \qquad \text{(A15)}$$

$$\cos \sigma = \vec{s} \cdot \vec{e}_1$$

$$= \cos \sigma_T + s_1 \eta_1 + s_2 \eta_2 + s_3 \eta_3 \qquad \text{(A16)}$$

where

$$\cos \sigma_T = s_1 \cos \theta_T + s_3 \sin \theta_T$$

We can now use Equations (A11-13, A16) to express ω_{1s} as a quadratic expansion in $\hat{\xi}_H$ and $\hat{\xi}_V$

$$\omega_{1s} = (b_1 \dot{\hat{\xi}}_H + b_2 \dot{\hat{\xi}}_V) \cos \sigma_T$$

$$+ b_3 \hat{\xi}_H \dot{\hat{\xi}}_H + b_4 \hat{\xi}_V \dot{\hat{\xi}}_V + b_5 \hat{\xi}_H \dot{\hat{\xi}}_V + b_6 \dot{\hat{\xi}}_H \hat{\xi}_V \qquad \text{(A17)}$$

where the b_j's are defined in Table 1.

Equation (A17) is precisely Equation (28) of the text. It is interesting to note that for the special case of the sun directly overhead, the two fixed-plane coordinate systems are the same. For this case,

$$s_1 = s_2 = 0 \qquad s_3 = 1$$

$$\theta_T = \pi/2 - \sigma_T. \qquad \text{(A18)}$$

The b_j's of Table 1 become

$$b_1 = \sec \theta_T;\ b_2 = b_3 = b_4 = b_5 = 0;\ b_6 = 1 \qquad \text{(A19)}$$

and

$$\hat{\omega}_1 = \omega_{1a} = \omega_{1s} = \dot{\hat{\xi}}_H \tan \theta_T + \dot{\hat{\xi}}_H \hat{\xi}_V \qquad \text{(A20)}$$

Equation (A20) for $\theta_T = 0$ is equivalent to Equation (4.3) on page 150 of Reference 8.

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