PRELIMINARY INVESTIGATION OF MULTIPLANE INTERFERENCE APPLIED TO PROPELLER THEORY.
By R. MCK. WOOD and H. GLAUBERT, OF THE ROYAL AIRCRAFT ESTABLISHMENT.—Presented by the Controller of the Technical Department, Aircraft Production.

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SUMMARY.—(a) Reasons for enquiry.—The method of calculating the characteristics of a propeller in use at present depends to a considerable extent on an empirical inflow factor which is assumed to represent the mutual interference of the propeller blades. The experiments and analysis described in the present report are an attempt to examine the inflow to be expected by regarding the interference effects as equivalent to those occurring in a multiplane structure of large negative stagger.

(b) Range of investigation.—A series of five aerofoils (to represent an infinite series) were tested in an arrangement chosen to represent a section of a propeller at a radial distance of one-third of the diameter. The experiments consisted of measurements of lift and drag on one of the aerofoils with and without the interference of the other four aerofoils. The results were analysed to derive the inflow and slipstream velocity. The wind channel experiment was rough and incomplete, and the report is brought forward principally as suggesting a new method of attacking propeller theory.

(c) Conclusions.—The results derived from the analysis are that the translational or axial inflow velocity is not proportional to the corresponding slipstream velocity. The rotational inflow is small.

(d) Applications and developments.—The method of experiment and analysis developed in the present report could be carried out with advantage with a series of propeller sections so as to give comparative results with a propeller whose characteristics are known. To form any general theorem different pitch diameter ratios, blade widths and numbers of blades should be represented by similar series of aerofoils and similarly investigated.

1. The method of calculating the characteristics of a propeller at the present time is based on Drzewiecki's analysis of the blade into a series of elements or strips, each of which is assumed to react as an element of an aerofoil without interference from the neighbouring elements. The relative velocity of the air and the incidence of the aerofoil element are derived from the forward
velocity and angular rotation of the propeller. In this form of
the theory no allowance is made for the interference effects of
the other blades of the propeller, but in recent work the forward
speed of the propeller has been increased by an empirical inflow
velocity depending on the thrust, diameter, and forward speed,
which is assumed to represent this interference effect. No allow-
ance is made for any interference effect in the rotational velocity.

2. The assumption of streamline flow of the air leads to the
conclusion that the inflow velocity is half the final velocity
in the slipstream or tail race. Consider a propeller rotating in a
stream of air moving with velocity $V$. Let $aV$ be the inflow
velocity and $bV$ the slipstream velocity, i.e., the increased velocity
over a plane where the pressure has regained its initial value.
Ignoring all rotational effects, the propeller may be regarded as
a disc of pressure discontinuity $p$. Then if $P_0$ be the undisturbed
pressure and $P$ the pressure just in front of the propeller,

$$
P_0 + \frac{1}{2} \rho V^2 = P + \frac{1}{2} \rho V^2(1 + a)^2
\]

or

$$
P_0 + \frac{1}{2} \rho V^2(1 + b)^2 = P + p + \frac{1}{2} \rho V^2(1 + a)^2
\]

Thus the thrust on an element $A$ of the propeller disc is

$$A \rho V^2 b (1 + \frac{1}{2} b).$$

The mass of air passing the element $A$ in unit time is

$$A \rho V (1 + a),$$

and so the momentum generated is $A \rho V^2 (1 + a) b$.
This is another measure of the thrust, and it follows at once that
$a = \frac{1}{2} b$, i.e., the inflow velocity is half the slipstream velocity.

3. In practice the inflow velocity has been chosen so as to
obtain agreement between the calculated and observed character-
istics of the propeller, and so is of the nature of an empirical factor
which is made to absorb in itself any small errors of the theory
employed in the calculations. It should be noted that two em-
pirical factors are really required to obtain agreement for both
thrust and torque. Direct measurements have also been obtained
of the velocity at a short distance in front of a model propeller,
but care must be taken in using the values of the inflow so obtained
in the theoretical equations. It is a well-known fact that an
aerfoil disturbs the flow of air for some distance in front of its
leading edge, and in consequence the inflow of a propeller
measured experimentally consists of two parts—(1) the distur-
rance of the air in front of the blade element under consideration
due to the blade itself (corresponding to the disturbance mentioned
above in the case of an aerfoil), and (2) the interference effects
of the other blades and of itself (at an angular distance of
$\pm 2\pi$). It is the latter part only which should be used for the inflow
correction in theoretical work, since the former is taken into account
directly in the characteristics of the aerfoil section.
4. The experiments dealt with in this report are based on a new method of visualising the interference effect of the propeller blades on each other. Accepting Drzwiecki's assumption that the blade of a propeller can be divided into a large number of independent aerofoil elements, it follows logically that the interference effect caused by the corresponding elements of the other blades is exactly analogous to the downwash and other interference effects experienced by one plane of an infinite multiplane structure. Consider elements at some definite radial distance along the blades. These elements will all lie relative to the air on a definite helix along which they will be evenly spaced, and if the helix be unrolled, we shall derive a multiplane structure with large negative stagger. This arrangement is represented in Fig. 1A, where AA is the plane of rotation of the propeller.

Let $V =$ forward velocity of the propeller.

$n =$ revolutions per second.

$W =$ resultant velocity of the blade element considered.

$r =$ radial distance of the element.

$\theta =$ blade angle.

$\phi =$ angle of helix.

$\alpha =$ angle of incidence of blade element.

Then if the interference effects are ignored and the propeller is assumed to have four blades—

$$V = W \sin \phi,$$

$$P_1, P_2 = P_1 P_2 = \frac{1}{2} \pi r.$$

The lift and drag of the elements act respectively at right angles to and along the direction of the resultant velocity $W$.

The effect of the interference of the various blade elements will be represented in the subsequent analysis by changes in the direction and magnitude of the resultant velocity $W$ which determines the force on the blade element under consideration. This is an assumption which can only be justified by examining the agreement obtained between theory and practice in a number of cases. It may very possibly be found that no general assumption of this kind is applicable to all aerofoil sections. This assumption is made implicitly in any theory which makes use of an inflow factor.

The characteristics of a propeller depend on the advance per revolution, or on the value of $\phi$, and so may be investigated by setting the axis AA at different angles to the axis of the wind channel. The lift and drag measured on one of the series of aerofoils will represent the forces on the blade of a propeller when all interferences are taken into account, while the difference between these forces and those obtained on a single aerofoil will give a measure of the interference effect. It was considered sufficient to use a series of five aerofoils and to measure the forces on the fourth of these (measuring down wind) to obtain the full
interference effect. The validity of this assumption might well be investigated in carrying this investigation further.

5. The aerofoils used were of 2" chord and 12" span. Their lower surface was flat and their upper surface a circular arc. The maximum thickness was 0.2". They were set at 23 degs. to the axis AA, and were spaced 14.3" apart. In this form the aerofoils correspond fairly closely to a section of the propeller T.7448 (pitch diameter ratio 1.2) at a radial distance of one-third of the diameter, except that the aerofoil section is different. The aerofoils were supported with their span vertical by 1" of spindle, 3" of spindle and 1 foot of spindle. The aerofoil tested was the fourth down wind and the forces were measured in these conditions, the results being given in Table 1. Experiment A is the test of the aerofoil alone, corrected for the interference of the spindle; B is the same aerofoil with the interference of the stand and mounting for the series of aerofoils; C is the test with all the aerofoils in position. The forces required for the aerofoil theory are A + B - C, which will represent the aerofoil with the interference of the series of aerofoils, but without that of the stand and mounting. Here, of course, an assumption has been made which should be investigated before proceeding further with these experiments. The assumption made is that the interference of the aerofoils is not itself affected by the presence of the mounting. For the purpose of the subsequent analysis the ratio of lift to drag is required, and this ratio is given in Fig. 1. All three tests showed curious discontinuities of slope in the neighbourhood of 5 degs. incidence, which may be due to the low value of IV of the tests. In analysing the results, the curves have been smoothed out a little in this region (see Fig. 2). The experiments were carried out in a 4-ft. wind channel at a speed of 60 f.p.s. (IV = 10).

6. The results have been analysed along two independent lines. In the first place an estimate was made of the corrections necessary to the angle of incidence of the element and to the relative velocity to represent the interference effects, and in the second place the translational and rotational velocities in the slipstream were derived on the assumption that these are due to the momentum imparted to the air by the reaction of the aerofoils.

Let $k_L$, $k_D$ be the lift and drag coefficients of the aerofoil alone, and $k'_L$, $k'_D$ those with the interference of the series. Then if the interference effect is equivalent to an inflow angle $i$ and an increase $w$ in the relative velocity, we obtain by resolving at right angles to the corrected velocity direction,

\[
k_L(x - i) \left[ \frac{W + w^2}{W} \right]^2 = k'_L(x) \cos i + k'_D(x) \sin i
\]

\[
k_D(x - i) \left[ \frac{W + w^2}{W} \right]^2 = k'_D(x) \cos i - k'_L(x) \sin i
\]
where $k_L (a)$ represents the value of $k_L$ at incidence $a$. To a sufficient approximation this gives

$$k_D (a - i) = k_D' (a) = k_L' (a) - \tan i$$

which is an equation for determining $i$ by means of the curves of Fig. 4. The correction $w$ to the relative velocity is given by either of the preceding equations. It is useful to express these interference effects as corrections to the forward velocity $V$ and the rotational velocity $2\pi nr$. Now

$$V = W \sin \phi$$

$$2\pi nr = W \cos \phi$$

and so

$$\Delta V = w \sin \phi + W \cos \phi \sin i$$

$$\Delta (2\pi nr) = w \cos \phi - W \sin \phi \sin i$$

or

$$\Delta V \frac{\sin i}{V} = \tan \phi + \frac{w}{W}$$

$$\Delta (2\pi nr) \frac{1}{2\pi nr} = - \tan \phi \sin i + \frac{w}{W}$$

The results of analysis on these lines are given in Table 2 and Figs. 5 and 6. The results are plotted against $V / nD$, and show that the inflow correction to the rotational velocity is never important.

7. To obtain the velocities in the slipstream, it is necessary to use a statistical method of analysis. Consider a column of air of breadth $b$ across the channel and of unit length along the span of the aerofoils (Fig. 4c). The number of aerofoils which meet this column is $\frac{b}{\sin \phi} 14$ and so the equations of momentum along and at right angles to the axis of the channel are

$$b \rho Wu = k_L' \rho \frac{1}{\sin \phi} W^2$$

$$b \rho Wv = k_D' \rho \frac{1}{\sin \phi} W^2$$

or

$$u = k_L' \frac{0.14}{\sin \phi}$$

$$v = k_D' \frac{0.14}{\sin \phi}$$

where $u, v$ are the components of the change in velocity, in a plane where the static pressure has resumed its initial value. If, then, $V_r$ and $V_t$ are the rotational and translational components of the slipstream, we shall have

$$V_r = u \cos \phi - v \sin \phi$$

$$V_t = u \sin \phi + v \cos \phi$$
The results of the analysis on these lines are given in Table 3 and Fig. 7, and it appears that for the blade section considered the rotational component is approximately half the translational component. These values will, however, depend on the aerofoil section used for the propeller blade.

8. To complete the analysis it is necessary to derive values for the thrust and torque of the element considered. For this purpose, the quantities $t$ and $q$ are defined by the equations

$$t = k'L \cos \phi - k'D \sin \phi$$

$$q = k'L \sin \phi + k'D \cos \phi$$

The thrust and torque of the element are proportional to these two quantities, which are given in Table 3 and Fig. 8.

The translational component of the slipstream vanishes with the thrust, and the rotational component with the torque, but the corresponding components of the inflow vanish at a slightly lower value of $V/nD$. The inflow angle appears to vanish approximately with the thrust.

9. The experiment described above is an attempt to deal with the interference problem in propeller theory on new lines. The results obtained are not directly comparable with any propeller that has been tested, but represent in type the results to be expected from this method of experiment. Too much attention should not be paid to the numerical results until a more careful and comprehensive experiment has been carried out; but so far they indicate rather different conclusions than the theory of R. and M. 328 assumes. The interference effects are there represented by an increased translational velocity only, which is a definite proportion (about half) of the translational slipstream velocity. Fig. 5 shows the comparison between the translational inflow obtained by the method of this report and that assumed by R. and M. 328. It will be seen that they differ considerably, are not in a constant ratio, and do not vanish together. Fig. 6 shows the inflow angle deduced by the two methods. That the difference is partly accounted for by the neglect of the rotational inflow can be seen from Fig. 1d. To give the same force on the blade the translational inflow must be higher and the inflow angle higher if the rotational inflow is ignored, since the relative velocity of the blade through the air will be higher and the incidence must therefore be lower.

10. The assumption that the interference of the blades can be represented by a translational inflow only is open to the objection that there is only one parameter available with which to obtain agreement of thrust and torque between calculated and observed values. In R. and M. 328 the inflow has been chosen to obtain agreement of thrust, and it was found that the torque might then differ by as much as \( \pm 5 \) per cent. It is therefore suggested that rotational inflow should be introduced to obtain
FIG 1a

FIG 1b

FIG 1c

FIG 1d

THE DOTTED LINE IS OBTAINED WHEN THE ROTATIONAL INFLOW IS NEGLECTED.
FIG 2

LIFT CURVES

ANGLE OF INCIDENCE $\alpha$

LIFT CORRECTION
CURVES OF DRAG/LIFT

These curves indicate how the efficiency of the propeller is affected by the interference of inflow.

![Diagram showing curves of drag/lift vs. incidence.](#)
INFLOW VELOCITY

PERCENTAGE CHANGE OF EFFECTIVE FORWARD AND ROTATIONAL SPEED DUE TO INTERFERENCE

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**FIG. 5**

![Graph showing percentage change of effective forward and rotational speed due to interference.](image)
INFLOW ANGLES

INFLOW ANGLE

INFLOW ANGLE WHEN ROTATIONAL VELOCITY IS IGNORED

SCALE OF INCIDENCE ASSUMING NO INFLOW
SLIPSTREAM VELOCITY (CALCULATED)

PERCENTAGE INCREASE OF TRANSLATIONAL
AND ROTATIONAL SPEED IN THE SLIPSTREAM

[Graph showing percentage increase of speed vs. v/nd]
THRUST AND TORQUE

VALUES OF THE COEFFICIENTS $t$ AND $q$

OF THE ELEMENT

![Graph showing thrust and torque values for the element]
agreement of torque if the inflow method is to be used; but on the other hand, it seems more reasonable to base propeller interference on a comprehensive series of experiments on the lines of this report rather than upon any inflow assumptions. The inflow theory is purely empirical, and is certainly quite unjustifiable when the blade is working near stalling incidence.

### Table 1.

<table>
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<tr>
<th>A.</th>
<th>B.</th>
<th>C.</th>
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<tr>
<td>$a$</td>
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<td>30</td>
<td>0.415</td>
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### Table 2.

INFLOW VELOCITIES.

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<th>$V$ in D</th>
<th>$a$</th>
<th>$z$</th>
<th>$w$</th>
<th>$\Delta v$</th>
<th>$\Delta (2\pi nr)$</th>
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<tr>
<td>0.44</td>
<td>16.1</td>
<td>4.5</td>
<td>0.046</td>
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<td>0.55</td>
<td>13.3</td>
<td>3.7</td>
<td>0.051</td>
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<tr>
<td>0.66</td>
<td>10.5</td>
<td>2.3</td>
<td>0.027</td>
<td>0.154</td>
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<tr>
<td>0.77</td>
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<td>1.2</td>
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<tr>
<td>0.88</td>
<td>5.3</td>
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<td>0.155</td>
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<td>0.007</td>
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Table 3.

THrust, Torque AND Slipstream Velocities.

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<th>V/nD</th>
<th>Vr/V</th>
<th>Vr/V</th>
<th>q</th>
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<td>0.061</td>
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<tr>
<td>1.21</td>
<td>—0.005</td>
<td>0.009</td>
<td>—</td>
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