APPLICATION OF REACTION RATE THEORY
TO COMBUSTION CHAMBER ANALYSIS

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Paper prepared for the
Combustion and Fuels Sub-Committee of the A.R.C.
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APPLICATION OF REACTION RATE THEORY TO COMBUSTION CHAMBER ANALYSIS.

SUMMARY.

The performance of a combustion chamber at low altitude conditions is probably controlled primarily by mixing rates. Chemical reaction rates may, however, provide a limit at low combustion chamber inlet pressures, and also in the high intensity designs at present being considered.

In the following analysis the rate of chemical reaction between fuel and air is determined as a function of the composition of the burning mixture. The optimum mixtures in a combustion chamber are then calculated, and numerical values of the possible heat release rates in a chamber derived in terms of the inlet pressure and temperature, and the combustion efficiency and reversal flow proportion.

Although drastically simplifying assumptions are made, the results of the analysis appear to be qualitatively and quantitatively correct.
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INTRODUCTION.

In present day combustion chambers, working at comparatively high design air pressures, mixing rates probably control the quantity of fuel that can be burned. As the operating pressure is reduced, proportional mixing rates are little altered, and with duple burners fuel atomization is still satisfactory, but chemical reaction rates may be drastically reduced. Similarly, as the velocity through a chamber is increased at given pressure and temperature, the mixing rates are increased, but the reaction rates are unaltered. Under these circumstances, the reaction rates, rather than mixing, may limit the combustion process. This may also be the case in reheat systems where the pressures may be very low, although fuel distribution is also a problem there.

A simplified analysis based on reaction rate theory has therefore been attempted, to investigate possible limitations, particularly on altitude performance.

REACTION RATE THEORY.

The first simplifying assumption is that the rate of reaction depends only on the frequency of useful collisions between fuel and oxygen molecules. The collision frequency is obtained from the kinetic theory of gases. The proportion of collisions that is useful depends on an activation energy: the value of 40,000 CHU/lb. mol., which agrees with experiments on flame speed, spontaneous ignition, etc., has been taken.

Secondly, since the basis of combustion chamber operation is a turbulent mixing process, heat transfer over appreciable distances by conduction or radiation is assumed to be negligible. It is therefore possible to relate the temperature at a point to the composition there. Since the simplified analysis considers only three species, fuel, air and combustion products, the temperature can be related to the local unburnt fuel, and the local oxygen concentration.

The method of analysis is given in detail in Appendix I.

In fact, of course, the reaction is a complex process involving a very large number of collisions between many different active radicals: but however many such collisions occur the reaction rate will finally depend on the proportions of stable compounds - fuel, air, CO, CO₂, H₂, H₂O etc. - present. Although every reaction involved will have its own activation energy it does seem possible, since the final mixing and reaction must be accomplished by diffusion and conduction, that the assumption of an overall activation energy based on results of laminar experiments would give an answer of the right order. We therefore consider that although the calculations are based on extreme physical simplifications, the results will be qualitatively correct.

CONCLUSIONS FROM THEORY.

The rate of heat release in a given mixture is proportional to the volume and to the square of the absolute pressure. This is because doubling the pressure doubles the number of each variety of molecule present in a given volume and therefore quadruples the collision rate.
2) The rate of heat release per unit volume in a given mixture at a given pressure varies exponentially as the temperature of the air before mixing or burning. This is because higher initial temperatures mean higher mixture temperatures for the same concentrations, and the effect of mixture temperature is exponential on the basis of the activation energy theory.

In reheats systems the effective initial temperature is the nozzle temperature and not the jet pipe temperature, since the latter has been achieved by burning. Similarly, in combustion chamber tests where a preheater is used the initial temperature is that before preheat. The variation of heat release rate for inlet temperatures between 300 and 600°C is given on graph A.

The fuel inlet temperature is assumed constant at 15°C. The effect of preheating the fuel is identical to that of adding the same quantity of heat to the air, leaving the fuel at 15°C.

3) For a given unburnt-fuel:air ratio, the heat release in unit volume is greatest when the mixture temperature is 80% of stoichiometric ($T_m = 0.8 T_p$). Dividing the heat release at other temperatures by that at 0.8 $T_p$ (for the same inlet temperature and unburnt f.a.r.), and plotting against $T_m/T_p$, gives graph B. A single curve covers the ranges for which calculations were done (initial air temperature between 300 and 600°C, a.f.r. between 5 and 30) with a maximum error of 5% of $T_p - T_m$. For $T_m$ greater than 0.8 $T_p$ the reaction rate is reduced because the concentrations of oxygen and fuel are low. For $T_m$ smaller than 0.8 $T_p$ the reaction rate is reduced because few molecules have the requisite activation energy for successful collisions.

4) At a given temperature the heat release in unit volume is greatest when the unburnt f.a.r. ($f$) is double (i.e., twice as rich as) the stoichiometric value ($f^*$). At richer mixtures the extra fuel concentration is offset by the reduction in temperature due to heat capacity of the fuel, while at weaker f.a.r. the reaction rate is reduced by lack of fuel.

Graph C shows the variation of heat release at a given temperature with unburnt f.a.r.

5) When only limited quantities of fuel and products are available it is convenient to base the reaction rate on the unburnt-fuel:products ratio, $j$. Graph D shows the variation of heat release (divided by the value at optimum temperature for the same value of $j$) with $j$ and $T_m$. These curves correspond to that of graph B. However, it will be seen that, whereas curve B is unique for all fuel:air ratios, and so gives a single optimum temperature, the curves of graph D give a range of optimum temperatures for different values of $j$. As $j$ decreases to zero from its optimum value $j^*$ (at $f = 2f^*$, $T_m = 0.80 T_p$) the highest possible rates of heat release per unit
volume are obtained at successively higher temperatures. When hardly any fuel remains the optimum temperature is 0.9 T_p. The corresponding best heat release values are plotted against T/J* on graph E, which corresponds to graph C.

The optimum temperature for final consumption of fuel is 0.9 T_p, which corresponds roughly to mixing with 16% excess air. It is interesting to note that this is similar to the accepted optimum for complete combustion in boiler furnaces.

It must also be remembered that the limiting rate is calculated for a mixture. In laminar flames heat is conducted to raise the temperature of the combustibles to a high value before burning, and so the intensity based on the volume of the reaction zone alone is high.

**THE IDEAL COMBUSTION CHAMBER.**

The ideal chamber is here defined as that occupying the minimum volume for a given rate of fuel consumption, and given air entry temperature and pressure. If fresh air and fuel could be introduced at will into any section of the burning zone, and if no restriction were placed on the amount of fuel that was not burnt, the burning zone volume would correspond to that required by the limiting heat release rate of the previous analysis. It is assumed, however, that the ideal chamber is stabilized by an aerodynamically produced reversal flow, which returns some of the burning mixture to the upstream section where all the fresh flow enters. Space is then wasted transferring the fresh flow to the downstream sections where it can safely be mixed in. It is also assumed that a combustion efficiency approaching 100% is required. The maximum reaction rate, which depends on the presence of a quantity of unburnt fuel, cannot then be everywhere maintained. These factors determine, as will now be shown, that the heat release rate per unit volume, even of the ideal chamber, is significantly less than the maximum rate in an optimum mixture.

Across any section of the chamber there is a steady flow of air, fuel, and the products from combustion upstream of the section. To attain the minimum overall volume, all the products must be mixed with the proportions of air and available fuel that give the maximum combustion rate in the chamber. This is achieved when the composition of the burning mixture is that which gives the maximum reaction rate per unit volume. (It is shown in the appendix that the extra volume made available by mixing in more combustibles does not compensate for the reduction in reaction rate per unit volume due to the lowering of the temperature). No amount is here taken of the practical difficulty of mixing incompletely.

Analyses 3 and 4 of the previous section show that the optimum burning mixture is at 6% stoichiometric temperature with an unburnt-fuel-air ratio of twice the stoichiometric. If fuel and air burn to products, fresh air must be mixed in to maintain the optimum conditions, however, when all the fuel has been added, nearly 50% will still remain unburnt. Therefore fresh air can be mixed in.

P.T.O.
In the ideal chamber there are two distinct burning zones, primary and secondary. In the primary zone burning takes place in a rich mixture, air and fuel being supplied to maintain the highest reaction rate, until all the fuel has been added. In the secondary zone combustion is completed at successively weakening f.a.r. as fresh air is mixed in.

In Appendix II are calculated the ideal primary zone mixture proportions and those of the combustibles to be mixed in. The f.a.r. of the latter must be the overall f.a.r. of the optimum mixture and not the unburnt f.a.r. in order to preserve optimum conditions. At first sight it seems surprising that an unburnt f.a.r. of twice stoichiometric can be preserved by mixing in only slightly over-rich combustibles: the explanation is that an initially over-rich mixture is always tending to richen itself further by combustion.

Result 5 of the previous section shows that the temperature of the secondary zone should be allowed to rise until final consumption of fuel occurs at 90% stoichiometric temperature. The second worked example (page 18 of appendix) gives the optimum mixture proportions when nearly all the fuel has disappeared. It is interesting to note that the air concentration is nearly the same as that of the previous case, the difference in temperature being mainly due to the reduced heat capacity of the unburnt fuel. Combustion is thus completed with an excess of air about 16% greater than the total stoichiometric requirement.

The ideal mixture proportions, referred to 1 lb. fuel, are given in Table I.

Provided the air or products travels unmixed completely round the reversal, the flow is part of the primary zone and wastage no volume. Some space is wasted, however, in transferring the entry air to the section where it can be mixed in. This wasted volume is calculated in the appendix (pages 22-23), and the result plotted on graph E. The primary volume $V_1$ is expressed as a proportion of the volume $V$ necessary to burn all the fuel at the maximum reaction rate.

It is evident that the larger the reversal quantity, the less volume is wasted, but for reversal proportions above 4 the waste is not very large. (If the reversal flow were infinite compared with the inflow, the latter could all be mixed in immediately and no space would be wasted).

In the secondary zone the rate of fuel disappearance depends primarily on the quantity remaining. That is, the disappearance is approximately exponential, and a large volume is required to bring the combustion near completion. The secondary zone volume is calculated in terms of the unburnt fuel remaining, (i.e. combustion inefficiency), (Appendix, page 23). The results are plotted on graph G. It will be seen that attainment of efficiencies above 99% requires a disproportionately large secondary volume.

Combining graphs F and G gives graph H which shows the proportion of the maximum intensity that is attainable with a given reversal flow and efficiency. Used in conjunction with graph A it enables the heat release rate of the ideal chamber to be calculated.
If the required combustion chamber outlet temperature is below 0.9 stoichiometric, then either the combustion products must be diluted after burning is complete, or final burning must take place at a lower temperature. In either case extra volume will be required; outlet temperature and mixing volume requirements decide which arrangement gives the lesser volume. Since the reaction rate falls very quickly with temperature (being half its maximum value at 0.65 stoichiometric temperature or, say, 15000K), the minimum volume will be achieved with a separate dilution zone for all outlet temperatures at present considered practical.

**COMPARISON OF IDEAL AND PRACTICAL CHAMBERS.**

The major points of the preceding theory are that it gives

- a) the optimum mixture proportions in the burning zones,
- b) the qualitative variation of limiting heat release with chamber inlet conditions, and with design efficiency and reversal flow proportion,
- c) quantitative figures for the limiting heat release rates.

In most present day chambers air for burning is supplied through the swirler and secondary holes. The air proportions are not yet known exactly, though this is the subject of an aerodynamic investigation. Assuming, however, that the air entering the first row of secondary holes flows into the reversal, and is thus effectively primary air, while that flowing through the second row of secondary holes is true secondary air, the Table II (page 9) is obtained. Flow quantities are quoted as percentages of the stoichiometric air requirement.

The agreement between theory and practice is encouraging, but suggests that recent chambers are rather too rich. Further comment must await the results of more thorough airflow investigations.

According to the theory, the limiting heat release rate varies as the square of the chamber pressure, and exponentially as the inlet temperature. The heat release required by an engine, however, varies directly with the chamber pressure, and relatively little with inlet temperature. Reaction rate limitations are only likely to be apparent, therefore, at very low temperature and pressure operating conditions.

The lowest pressure information immediately available is from tests of the R.A.2 and R.A.7 chambers on the R.A.E. altitude rig (see NOTE Reports R.65 and R.124). From these reports we have taken the highest recorded heat release rates at engine idling mass flow, pressure and temperature conditions at 40, 50, 60 and 65 thousand feet simulated altitude. Two figures from tests at Burnley on the R.A.14 chamber have been added. The figures are compared in Table III (page 10) with the limiting theoretical heat release, based on the preceding theory, on the assumption that the reversal flow proportion is $T/5$. (It is hoped to check this figure experimentally one day, but the calculation is not critical to it).
The correspondence between actual and theoretical values is surprising, as one would have expected the theoretical value to have been higher. Errors in the assumptions of molecular sizes, activation energies, burning zone volumes etc., could easily make a big difference to the result. Nevertheless, the nearly constant value of the maximum \( Q/Vp^2 \) at various altitudes does suggest that the limiting reaction rate is in fact responsible for the fall off in combustion efficiency at these conditions.

The last line of Table III refers to a typical cruise condition at 50,000 feet. It is evident that engine requirements at such conditions are well below the reaction rate limit. This suggests that the necessity of making possible operation at idling speed at very high altitudes has meant that the burning zone is much larger than is really necessary for the cruise conditions.

**APPLICATION TO REHEAT SYSTEMS.**

An ideal reheat system can be specified on the same basis as the ideal combustion chamber. The optimum mixture proportions are unaltered, but since the jet pipe flow is already vitiated by products from the combustion chamber the volume flows needed to provide the required oxygen quantities are increased. Due to this prior vitiation the quantity of fuel that can be burned under optimum conditions in the reheat primary zone is reduced to 0.65 of the reheat fuel flow and the primary and secondary air requirements are 83% and 40% of stoichiometric. The volume versus efficiency and reversal flow quantity curves are shown on graphs F and G.

Assuming that in a typical reheat system the reversal flow proportion is 2% of the total flow, the theoretical limiting heat release of the ideal reheat system can be calculated. This is compared in Table IV (page 10) with some of the highest heat release rates recorded with an Avon system in flight.

It is again evident that reaction rate theory provides a limit of the right order, and explains the observed rate of fall off in efficiency with altitude. The actual limit appears to be only about 60% of the theoretical, but it must be remembered that even under the most favourable conditions of pressure and temperature the reheat system was only about 80% efficient, due largely to inadequate fuel distribution.

**COMBUSTION INTENSITY.**

The Combustion Intensity of a chamber has in the past been defined as the heat release rate per unit of volume and pressure (CHU/Hr. atmos. cu.ft.). No satisfactory theory has yet been advanced as to why this factor should be uninfluenced by operating pressures or temperatures. As defined, therefore, it is not a satisfactory basis on which to compare different combustion chambers, and its sole advantage is as a specification of engine requirements.

Since the theory of this paper shows that one controlling parameter is the heat release per volume per time per \((\text{atmosphere})^2\) it is suggested that the correct way to compare combustion chambers is on the basis of their curves of efficiency at design f.a.r. against this parameter.
STABILITY.

In a steady mixing process, points on the left hand side of graph B are unstable. If the reaction rate in the mixture momentarily exceeds the rate at which fresh combustibles are mixed in, the temperature will rise: this in turn will further increase the reaction rate, and so on. Conversely, momentarily increasing the mixing rate will quench the flame. Similarly, points on the right hand side of the curve are stable since increasing the mixing rate reduces the temperature which increases the reaction rate to compensate.

Applying this argument to the recirculating zone of a combustion chamber or stabilizer it will be seen that as the velocity and mixing rate are increased the reaction rate will increase until the maximum intensity point is reached. A further increase in velocity makes the system unstable and blow off results.

It is not easy to predict blow off conditions numerically, since the rate into the reversal zone is not known, and the volume of the zone is vague, so that the intensity requirements, even assuming uniform intensity in the zone, are difficult to assess. However, the variation of blow off velocity with temperature has been satisfactorily correlated with experiment, and it is hoped to extend the analysis.

CONCLUSIONS.

For the bimolecular reaction assumed, the rate of heat release in a given mixture is proportional to the volume and to the square of the absolute pressure, and varies exponentially with the initial temperature of the combustibles. The maximum rate of heat release is attained with a mixture of air, fuel and combustion products in the proportions 1:2:84 by weight.

The "ideal" combustion chamber is defined as that giving the maximum heat release rate per unit volume at a given inlet pressure and temperature, and for a design efficiency and reversal flow proportion. It is shown to consist of two zones:

1) Primary zone, in which all the fuel is introduced with about 86% of the air required for complete combustion. The volume required is shown to depend on the reversal flow proportion.

2) Secondary zone, in which the burning mixture is diluted with a further 30% of fresh air, to bring burning near completion. The volume required is shown to depend on the desired combustion efficiency.

Table I gives the optimum mixture proportions in the burning zones. These are compared with those of present day combustion chambers in Table II. By combining the primary and secondary zone volumes necessary for a given efficiency, and a given reversal flow proportion, the intensity of the ideal chamber can be compared with the limiting heat release rate in the optimum mixture (Graph H).

* Note: since this analysis was started, a paper on similar lines correlating blow off velocity with f.a.r., has been published, by Longwell, Frost and Weiss. See "Industrial and Engineering Chemistry" Vol. 45, No. 8, p.1629 (1953).
Since the calculated heat release of the ideal chamber varies as the square of the operating pressure, its intensity is only approached by practical chambers at the lowest pressure conditions. Table III compares the limiting altitude idling performance of the Avon with that of the ideal chamber; there is surprisingly close agreement. At higher operating pressures the Avon chamber is far below the limiting, or ideal, intensity. This shows how the size of a combustion chamber must be determined by the minimum pressure at which operation is required.

Flight results from Avon reheat systems suggest that the same limiting criteria apply there, although the practical limiting heat release values are only some 50% of the ideal. This discrepancy is probably accounted for by fuel maldistribution. (Table IV).

The analysis suggests that the performance of combustion chambers should be compared at the same value of heat release per volume per (atmosphere)$^2$ and not per atmosphere as is conventionally done. The standard of performance of a chamber should be its curve of efficiency versus intensity thus redefined.
### TABLE I. Ideal mixture proportion in burning zone (referred to 1 lb. fuel).

<table>
<thead>
<tr>
<th></th>
<th>Primary</th>
<th>Secondary</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel input</td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>Air input</td>
<td>0.86/f*</td>
<td>0.30/f*</td>
<td>1.16/f*</td>
</tr>
<tr>
<td>F.A.R. of in mixture</td>
<td>1.16 f*</td>
<td>0</td>
<td>0.86 f*</td>
</tr>
<tr>
<td>Burning zone temperature</td>
<td>0.8 T_P</td>
<td>0.8 to 0.9 T_P</td>
<td>-</td>
</tr>
<tr>
<td>air concentration</td>
<td>0.15</td>
<td>0.15 to 0.13</td>
<td>-</td>
</tr>
<tr>
<td>unburnt f.a.r.</td>
<td>2f*</td>
<td>2f* towards zero</td>
<td>-</td>
</tr>
<tr>
<td>Quantity of fuel burnt</td>
<td>0.72</td>
<td>towards 0.28</td>
<td>towards 1.0</td>
</tr>
</tbody>
</table>

(f* is the stoichiometric f.a.r.)

(T_P is the temperature of undiluted combustion products).

### TABLE II. Actual burning zone mixture proportions.

<table>
<thead>
<tr>
<th>Chamber (a.f.r.)</th>
<th>Primary air</th>
<th>Secondary air</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dart (60)</td>
<td>92%</td>
<td>28%</td>
<td>120%</td>
</tr>
<tr>
<td>Avon R.A.7 (65)</td>
<td>82%</td>
<td>32%</td>
<td>114%</td>
</tr>
<tr>
<td>Avon R.A.14 (67)</td>
<td>78%</td>
<td>31%</td>
<td>109%</td>
</tr>
<tr>
<td>Ideal</td>
<td>86%</td>
<td>30%</td>
<td>116%</td>
</tr>
</tbody>
</table>

(Flow quantities are quoted as percentages of the stoichiometric air requirement).

P.T.O.

<table>
<thead>
<tr>
<th>Chamber inlet pressure</th>
<th>Chamber inlet temp.</th>
<th>Airflow</th>
<th>a.f.r.</th>
<th>Combustion Efficiency</th>
<th>Heat release ($\frac{Q}{Vp^2}$) limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.105</td>
<td>266</td>
<td>0.165</td>
<td>37</td>
<td>47</td>
<td>17</td>
</tr>
<tr>
<td>0.132</td>
<td>266</td>
<td>0.21</td>
<td>32</td>
<td>55</td>
<td>19</td>
</tr>
<tr>
<td>0.204</td>
<td>324</td>
<td>0.71</td>
<td>47</td>
<td>65</td>
<td>21</td>
</tr>
<tr>
<td>0.214</td>
<td>266</td>
<td>0.34</td>
<td>35</td>
<td>85</td>
<td>16</td>
</tr>
<tr>
<td>0.300</td>
<td>344</td>
<td>1.01</td>
<td>60</td>
<td>81</td>
<td>14</td>
</tr>
<tr>
<td>0.346</td>
<td>266</td>
<td>0.545</td>
<td>35</td>
<td>95</td>
<td>11</td>
</tr>
<tr>
<td>1.0</td>
<td>450</td>
<td>2.5</td>
<td>60</td>
<td>97</td>
<td>4</td>
</tr>
<tr>
<td>atmos.</td>
<td>0K</td>
<td>1 lb/sec.</td>
<td>-</td>
<td>%</td>
<td>million CHU hr. cu.ft. (atmos)$^2$</td>
</tr>
</tbody>
</table>

### TABLE IV. Limiting Heat Release Rates (Reheat System).

<table>
<thead>
<tr>
<th>Jet Pipe static pressure</th>
<th>Nacelle temp.</th>
<th>Reheat fuel flow</th>
<th>Reheat Efficiency</th>
<th>Heat release ($\frac{Q}{Vp^2}$) limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.277</td>
<td>236</td>
<td>2077</td>
<td>33</td>
<td>3.8</td>
</tr>
<tr>
<td>0.365</td>
<td>218</td>
<td>2164</td>
<td>56</td>
<td>3.9</td>
</tr>
<tr>
<td>0.41</td>
<td>251</td>
<td>2586</td>
<td>56</td>
<td>3.7</td>
</tr>
<tr>
<td>0.454</td>
<td>254</td>
<td>2716</td>
<td>60</td>
<td>3.7</td>
</tr>
<tr>
<td>0.67</td>
<td>239</td>
<td>4100</td>
<td>76</td>
<td>2.9</td>
</tr>
<tr>
<td>atmos.</td>
<td>0K</td>
<td>1 lb/hr.</td>
<td>%</td>
<td>million CHU hr. (atmos)$^2$ cu.ft.</td>
</tr>
</tbody>
</table>
APPENDIX

Symbols.

Most of the symbols used are defined where they occur, or are so commonplace as not to need definition. The following are used throughout:

- \( m \) molecular weight
- \( M \) mass (lb.)
- \( \dot{m} \) mass flow (lb./sec.)
- \( f \) unburnt-fuel:air ratio by weight in a mixture.
- \( f^* \) stoichiometric, or theoretical, fuel:air ratio
- \( j \) unburnt-fuel:products ratio by weight in a mixture.
- \( \Omega = \frac{\dot{Q}}{V_p} \) heat release rate \( \frac{\text{million CHU}}{\text{hr.(atmos)}^2 \text{cu.ft.}} \)

\( \phi = p^2 \Omega^* \) expressed in lb. fuel per sec. per cu.ft.

\( \Omega^* = \text{max. heat release rate in the optimum mixture} \)

Suffix \( p \) refers to products of complete combustion of air and fuel in correct proportions.

\( (T_p \) is the temperature of such products before dilution and is referred to as stoichiometric temperature).

Suffix \( u \) refers to combustibles unmixed with products.

Suffix \( m \) refers to a mixture of products, air and fuel.

P.T.O.
APPENDIX I. REACTION RATE THEORY.

METHOD.

The reaction is idealised to the direct combination of fuel (F) and oxygen (O) molecules in a uniform mixture.

From the Kinetic Theory of Gases, the number of collisions between P and O molecules in unit volume in unit time (Z per cu.ft. per sec.) is given by

\[ Z = N_O \cdot N_F \left( \frac{\sigma_O \cdot \sigma_F}{2} \right)^2 \sqrt{\frac{8 \pi N}{\delta R g T}} \left( \frac{1}{m_O} + \frac{1}{m_P} \right) \]  

where
- \( N_O \), \( N_F \) are the numbers of molecules in unit vol.
- \( \sigma_O \), \( \sigma_F \) are the effective diameters of the molecules
- \( m_O \), \( m_P \) are the molecular weights
- \( R \) is the universal gas constant 2780 ft. lb.wt./\( ^{\circ}\)K lb.mol.
- \( T \) is the absolute temperature, \( ^{\circ}\)K.
- \( g \) is the gravitational constant = 32.2 lb.ft.sec.\(^{-2}\)/lb.wt.

The above units are self consistent.

The number of oxygen molecules in unit volume is

\[ N_O = C_O \cdot \frac{p \cdot m_O}{RT} \cdot \frac{1}{m_O \cdot h} = C_O \cdot \frac{p}{RT h} \]  

where \( C_O \) is the oxygen concentration by volume (mol/mol.aire)
- \( p \) is the absolute pressure lb.wt./sq.ft.
- \( h \) is the mass of an H atom, lb.

If "E" is the activation energy (ft.lb.wt./lb.mol.) corresponding to the oxygen fuel reaction, the number of collisions leading to reaction is

\[ Z^* = Z \cdot e^{-\frac{E}{RT}} \]  

---------- (3)
Disappearance of fuel mass \( M_f \) (lb.) per sec. in a given volume, is then given by
\[
- \frac{1}{M_f} \frac{dM_f}{dt} = \frac{\gamma}{N_f} = C_0 \cdot p \cdot e^{-E/RT} \cdot T^{-\frac{3}{2}} \left[ \frac{(\sigma - \tau p)^2}{h} \sqrt{\frac{2k}{m}} \left( \frac{1}{m_0} + \frac{1}{m_p} \right) \right]
\]

The term in brackets depends on physical constants. Inserting values from Kaye and Laby,
\[
h = \frac{1}{6 \times 10^{25} \times 453} \text{ lb.}
\]
\[
\sigma = \frac{3 \times 10^{-8}}{12 \times 2.54} \text{ ft. approximately}
\]
\[
m_0 = 32 \text{ and } m_p = 28 \text{ (figure for CO)}
\]
Thus
\[
- \frac{1}{M_f} \frac{dM_f}{dt} = 1.15 \times 10^8 \times C_0 \cdot p \cdot e^{-E/RT} \cdot T^{-\frac{3}{2}} \quad (5)
\]
For a given temperature and concentration the proportional rate of reaction varies directly as the pressure.

If \( f^* \) is the stoichiometric f.a.r., then \( f^* \) lb. of fuel at \( T_f \) react with 1 lb. air at \( T_u \) to form \((1 + f^*)\) lb. products at \( T_p \), (the stoichiometric temperature) the heat liberated being
\[
(1 + f^*) (I_p - I_u) + f^* (I_u - I_f)
\]
where \( I_u \) is the enthalpy (CHU/lb.), at \( T_u \), etc. and the suffices \( f \) and \( p \) outside the brackets refer to fuel and products respectively. The second term will be neglected.

Neglecting radiation and conduction of heat, the temperature \( T_m \) of a mixture of \( M_a \) of fresh air, \( f \cdot M_a \) of fuel (\( f \) being the unburnt fuel air ratio by weight) and \( M_p \) of products is given by
\[
M_a (I_m - I_u) + f \cdot M_a (I_m - I_f) + M_p (I_m - I_u) = M_p (I_p - I_u)
\]

P.T.O.
The oxygen concentration in air is 0.21 mol/mol., and so that in the mixture is given by:

\[ C_0 = \frac{0.21 M_a/m_a}{M_a/m_a + M_p/m_p + f M_a/m_f} \]

\[ = \frac{0.21}{1 + \frac{m_b}{M_f} + \frac{m_p}{m_a}} \]

since \( m_p = m_a \), very nearly.

\[ = \frac{0.21 (I_p - I_m)p + f(I_m - I_u)f + (I_m - I_u)a + f \frac{m_a}{m_f} (I_p - I_m)p}{(I_p - I_m)p + f(I_m - I_u)f + (I_m - I_u)a + f \frac{m_a}{m_f} (I_p - I_m)p} \]

Over the range in which we are interested, the denominator can be approximated to by the expression \((I_p - I_u)a + f(I_m - I_u)f\).

We can further approximate in terms of specific heats by assuming \((Kp)_p = 1.1 (Kp)_a\) and \((Kp)_f = 8 (Kp)_a\). In fact, the specific heat of unburnt fuel vapour is not accurately known above 1000°K, but the factor of 8 allows for the known variation of products temperature when extra fuel is added.

Thus

\[ C_0 = \frac{0.21 (T_p - T_m) x 1.1}{(T_p - T_u) + 8f (T_m - T_u)} \]

The following table gives reasonable values for \( T_p \) for aviation kerosine for different values of \( T_u \):

<table>
<thead>
<tr>
<th>( T_u )</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_p - T_u )</td>
<td>1955</td>
<td>1895</td>
<td>1840</td>
<td>1780</td>
<td>1780</td>
</tr>
<tr>
<td>( T_p )</td>
<td>2255</td>
<td>2295</td>
<td>2340</td>
<td>2380</td>
<td>2380</td>
</tr>
</tbody>
</table>

The volume occupied by the mixture (cu. ft.) is related to the oxygen concentration, thus:

\[ p \cdot V = M_a \cdot \frac{21}{C_0} \cdot \frac{R T_m}{m_a} \]
So (5) becomes

\[-\frac{dM_p}{dt} = 1.15 \times 10^8 \times C_0 p e^{-\frac{E}{RT_m}} T_m^{-\frac{3}{2}} f \frac{C_O^* p V m_a}{21 RT_m} = 5.7 \times 10^6 (C_o)^2 \cdot p^2 \cdot e^{-\frac{E}{RT_m}} T_m^{-\frac{3}{2}} f \cdot V \cdot (9)\]

Let \( \Omega \) be the rate of heat release, expressed in million CHU per hour per (atmos.)\(^2\) per cu.ft. Taking the calorific value at \(10^4\) CHU/lb.. fuel

\[\Omega = -\frac{dM_p}{dt} \times 10^4 \times 3600 \times (14.7 \times 144)/10^2 x V F^2\]

Substituting from (9)

\[\Omega = 0.92 \times (100 C_0)^2 \left(\frac{10^5}{e^{\frac{E}{RT_m}}}\right) \left(\frac{10^4}{T_m}\right)^{\frac{3}{2}} \cdot (10)\]

Calculations based on equations (7) and (10) show that \( \Omega \) can be expressed as the product of three independent functions of \( T_u \), \( T_m \) and \( f \) respectively. These are shown on graphs A, B and C, the ranges covered being 300 to 600\(^\circ\)K for \( T_u \), 1.0 to 0.6 for \( T_m \) and 1 to 3 (a.f.r. between 30 and 5) for \( f \).

In an idealisation of a combustion chamber the quantities of fuel and products passing a given section are determinable, and we must arrange to mix in that amount of air that gives the maximum reaction rate for unit volume. It is therefore sometimes convenient to express the reaction rate in terms of the fuel:products ratio \( j \) rather than the fuel:air ratio \( f \). Equation (6) then becomes

\[M_a (I_m - I_u)_a + j M_p (I_m - I_u)_f + M_p (I_p - I_u)_p = M_p (I_p - I_u)_p \cdot (6a)\]
and \[ C_o = \frac{0.21 M_p}{M_a + M_p} \]

\[
= \frac{0.21 [(I_p - I_m)_p - J(I_m - I_u)_f]}{(I_p - I_m)_p - J(I_m - I_u)_f + (1 + \frac{M_p}{M_a})(I_m - I_u)_a}
\]

Applying the same approximations as before.

\[
C_o = \frac{0.21 [1]}{T_p - T_u - 8(T_m - T_u)J} \tag{7a}
\]

The volume occupied is related to the products concentration

\[
C_p \left( 1 + \frac{M_p}{M_a} \right) = 1 - \frac{C_o}{0.21}
\]

\[
p V = C_p \frac{RT}{m_p} \tag{8a}
\]

So from (5)

\[
-\frac{dM_r}{dt} = 1.15 \times 10^8 \times C_0 \ p \ e^{-E/RT_m} - T_m \ J \ C_p \ p \ V \ m_p \tag{9a}
\]

which reduces to

\[
\Omega = 0.193 \ J \ (100 \ C_o)(100 \ C_p) \left( \frac{10^5}{e^{E/RT_m}} \right) \left( \frac{10^6}{T_m} \right)^{3/2} \tag{10a}
\]

Graph D, showing this relation, is plotted similarly to graph B, in that for each value of \( J \), \( \Omega \) is divided by its max. value \( \Omega_j \) at that \( j \). As \( j \) decreases from its value, \( j^* \), (at \( \frac{T_m}{T_p} = 0.8, \frac{c}{c^*} = 2 \)) to zero it pays to let the temperature increase (to \( \frac{T_m}{T_p} = 0.9 \) for \( J = 0 \)).

The approximate relation governing the maximum intensity for values of \( j/j^* \) below \( \frac{2}{3} \) is

\[
\frac{\partial \frac{J}{\Omega^*}}{\partial \frac{J}{j^*}} = 2.7 \ \frac{j}{j^*} \left( 1 - \frac{3}{4} \ \frac{j}{j^*} \right) \tag{11a}
\]
and for values of \( \frac{J}{J^*} \) between 0.4 and 1.0

\[
\frac{\partial f}{\partial J^*} = 0.72 (2.18 - \frac{J}{J^*})(0.18 + \frac{J}{J^*}) --- (11b)
\]

These relations are plotted on graph E, which corresponds to graph C.

**Calculation of typical points.**

**Primary Zone Case:**

Suppose \( T_u = 450^\circ K \)

\[ T_m/T_p = 0.80 \]

\[ f/f^* = 2 \quad (f^* = \frac{1}{15}) \]

Then \( T_p - T_u = 1870^\circ C \)

\[ T_p = 2320^\circ K \]

\[ T_m = 1855^\circ K \]

\[ E/RT_m = 10.8 \]

\[ \frac{10^5}{e^{E/RT_m}} = 2.08 \]

\[ \left(\frac{10^4}{\frac{E}{RT_m}}\right)^{3/2} = 12.5 \]

From equation (7) \( C_o = 0.21 \times 0.15 = 0.032 \)

From equation (10) \( \mathcal{O} = 32.5 \) units,

(million CHU/hr. (atmos)² cu.ft.)

The air concentration in this optimum mixture is 0.15, the combustible concentration 0.17 and the products concentration 0.83.

The corresponding value of \( j \) is \( \frac{0.02}{0.83} = 0.024 \).

P.T.O.
Secondary Zone Case:

In the second case, suppose $T_u = 450 \, ^\circ K$.

\[ \frac{T_m}{T_p} = 0.9 \]

and $r = \frac{1}{200}$

Then $T_m = 2090 \, ^\circ K$

\[ \frac{E}{RT_m} = 9.5 \]

\[ \frac{10^5}{e^{E/RT_m}} = 6.9 \]

\[ \left( \frac{10^4}{e^{E/RT_m}} \right)^{3/2} = 10.5 \]

From equation (7) $C_o = 0.21 \times 0.13 = 0.028$

From equation (10) $\Omega = 2.5$ units.

The air concentration is now 0.13, and the products concentration 0.87. The value of $j$ is .00075.
APPENDIX II. IDEAL BURNING ZONES.

1) Mixture Proportions.

In the primary zone the composition of the burning mixture is: 2% fuel, 15% air, 83% products (which consist of 83 $\frac{r^*}{1+r^*} = 5.2\%$ reacted fuel and 78% reacted air). The overall f.a.r. is thus $\frac{.02 + .052}{.78 + .15} = 1.16 f^*$. This must be the f.a.r. of the fresh combustible mixed in if the proportion is to be maintained. All the fuel is introduced in the primary zone. Thus the quantity of air brought in must be $\frac{1}{1.16}$ x the stoichiometric requirement.

The proportion of fuel that has reacted in the primary zone when the mixing process is complete is $\frac{.052}{.052 + .02} = 72\%$.

In the reheat case the optimum mixture will be the same, but the entering airflow will already have been contaminated by products from the combustion chamber. Suppose the air proportion in the jet pipe flow is .75. Then of the products in the optimum mixture 25% will have come from the chamber. The remaining 58% contain 3.6% reacted reheat fuel. Thus the proportion of reheat fuel that has reacted when mixture is complete is $\frac{.036}{.036 + .02} = 65\%$.

The overall f.a.r. of the in-mixture must be 1.16 as before, but as the air is already associated with products, the unburnt f.a.r. of the inflow is $1.16 - \frac{.25}{.75} f^* = 1.21 f^*$. Thus the air quantity brought in in this case is 33% of the stoichiometric requirement.

At exit from the secondary zone, with fuel nearly completely burnt, the optimum mixture proportions are air 13%, products 87% (of which 5.45% is reacted fuel, and 81.5% reacted air). The overall fuel air ratio is therefore $\frac{.0545}{.0545 + .13} = \frac{f^*}{1.16}$ . Since 86% of the
stoichiometric requirement was introduced in the primary zone, the other 30% must have been introduced in the secondary.

Similarly, in the reheat case the same optimum mixture in the secondary zone would be required. As before 25% of the mixture is products from the combustion chamber proper, leaving 62% reheat products, of which 3.5% is reacted fuel and 58% reacted air. The reheat fuel to reheat air ratio in the mixture is therefore \[ \frac{0.39}{0.58 + 0.13} = \frac{f^*}{1.22} \]. Since 83% of the stoichiometric requirements were supplied to the primary zone, the other 39% must have been introduced into the secondary.

2) **Primary Zone Volume.**

Assume that fresh air, fuel and products are crossing an elementary section of the chamber at the same velocity. All the products are mixed with just sufficient air and fuel to give the highest reaction rate in the total volume available, the rest of the combustibles being untouched. Let the cross sectional area occupied by the unmixed flow, \( W_u \), be \( A_u \) and that by the mixture flow, \( W_m \), be \( A_m \).

Then \[ \frac{W_u}{\rho_u A_u} = \frac{W_m}{\rho_m A_m} \] \hspace{1cm} (12)

Considering two sections \( \delta x \) apart, the difference in fuel flow, \( -\delta W_f \), due to reaction, is given by

\[ -\delta W_f = \left[-\frac{1}{\rho} \frac{dM_f}{dt}\right]_{\text{optimum}} A_m \delta x \]

\[ = \beta A_m \delta x \] \hspace{1cm} (13)

Where \( \beta = \rho^2 \mathcal{S}_{\text{optimum}} \), with due allowance for units.
Now let C be the proportion of air and fuel in the optimum mixture, and \( W_p = (1 - C)W_m \) the products flow. Then, assuming volumes unaltered by mixing

\[
\frac{W_m}{\rho_m} = \frac{CW_u}{\rho_u} + \frac{W_p}{\rho_p} = \left[ \frac{C}{1-C} \frac{\rho_p}{\rho_u} + 1 \right] \frac{W_p}{\rho_p} \tag{14}
\]

The element of volume concerned, \( \delta V = (A_u + A_m) \delta x \)

After manipulation

\[
\delta V = \frac{\delta W_f}{\rho} \left[ \frac{W_u + CW_m}{\rho_u} + \frac{W_p}{\rho_p} \right] \left[ \frac{W_p}{\rho_p} \left( 1 + \frac{C}{1-C} \frac{\rho_p}{\rho_u} \right) \right] \tag{15}
\]

The numerator bracket corresponds to the total volume flow and is independent of the way mixing is done. The second term in the denominator bracket is so small as to be negligible, so the denominator is fixed. Thus to obtain a maximum value of \(-\frac{\delta V}{\delta W_f}\) we require a maximum value of \(\phi\). That is, \(\phi_{\text{optimum}}\) is in fact \(\phi^*\), the maximum heat release rate in the burning mixture.

To maintain this rate we must have

\[
-\delta W_u = \delta W_m = \frac{\delta W_f}{1 - C} = -\delta W_f \frac{1 + \phi^*}{\rho_u} \tag{16}
\]

Equation (15) can then be written

\[
\delta V = -\frac{\delta W_f}{\rho} \left( 1 - \frac{\rho_m}{\rho_u} \right) + \frac{W_u + W_m}{\rho_u} \frac{\rho_m}{\rho_u} \frac{\rho_p}{\rho_p} \frac{\phi^*}{1 + \phi^*} (1 - C) \delta W_m \tag{17}
\]

The total mass flow \( W_u + W_m \) is the same for all sections. At the point where the air first enters \( W_m \) is only the reversal mass flow, \( W_R \). At exit from the primary zone all the flow is mixed, so \( W_m = W_R + W_U \), \( W_U \) being the entry inflow to the primary. The change in fuel flow in the primary is the quantity of fuel burnt, which is \( .72 \) of the total flow \( W_p \) (see page 19) and must also be \( W_U (1 - C) \frac{\phi^*}{1 + \phi^*} \).
Integrating for the whole primary zone

\[ \beta V_1 = 0.72 W_p \left(1 - \frac{\rho_m}{\rho_u}\right) + \frac{\rho_m}{\rho_u} V^* \left(1-C\right) \left(W_u + W_R\right) \ln \frac{W_R + \rho_u}{\rho_u} \]

\[ \beta V_1 \quad \frac{0.72 W_p}{V^*} = 1 + \frac{\rho_m}{\rho_u} \left[(1 + r) \ln \left(1 + \frac{1}{r}\right) - 1\right] \quad \text{----- (18)} \]

The term in brackets gives the extra volume needed to transfer the inlet air downstream to its mixing point. It approaches zero as \( r \) approaches infinity.

Let \( V^* \) be the volume necessary if all the fuel was burnt at the highest rate. Then \( V^* = W_p/\beta \). Taking \( \frac{\rho_m}{\rho_u} = \frac{1}{4} \) as a typical value,

\[ \frac{V_1}{V^*} = 0.72 \left[1 + \frac{1}{4} \left((1 + r) \ln \left(1 + \frac{1}{r}\right) - 1\right)\right] \quad \text{----- (19)} \]

If we also allow for the extra volume needed to pass the secondary airflow through the primary zone, the equations are identical except that the factor in front of the log term is \( W_u + W_m + W_2 \) instead of \( W_u + W_m \). \( W_u \) refers to the primary zone flow only, and \( W_2 \) to the secondary flow. We know (pages 19 and 20)

\[ \frac{W_2}{W_u} = \frac{0.30}{0.86 \left(1 + 1.16 \cdot f^*\right)} = 0.32 \]

Then

\[ \frac{V_1}{V^*} = 0.72 \left[1 + \frac{1}{4} \left((1.32 + r) \ln \left(1 + \frac{1}{r}\right) - 1\right)\right] \quad \text{----- (20)} \]

This increases the extra volume by 40%.

The relations are plotted on graph F.

Similar formulae apply to the reheat case, but the value of \( \frac{\rho_m}{\rho_u} \) is altered. Taking 900°K jet pipe temperature as a typical value, \( \frac{\rho_m}{\rho_u} = \frac{1}{2} \). 0.65 of the reheat fuel is burnt in the primary zone, so
This relation is also illustrated on graph $E$. 

In the reheat case 
\[
\frac{W_2}{W_u} = \frac{0.39}{0.85} (1 + 1.21 f^*) = 0.44
\]

Thus if we allow space in the primary zone for all the secondary air to flow through it, 
\[
\frac{V_1}{V_s} = 0.65 \left\{ 1 + \frac{1}{2} \left[ (1 + r) \ln (1 + \frac{1}{x}) - 1 \right] \right\} \quad (19a)
\]

3) **Ideal Secondary Zone.**

Before leaving the primary zone, all the fuel is mixed in, and at the exit the flow is burning at 80% stoichiometric temperature and twice stoichiometric unburnt f.a.r. Thereafter, fresh air is mixed in to maintain the optimum reaction rate as the f.a.r. weakens.

The volume occupied by the extra fresh air proceeding downstream to its mixing point is negligible here (though not in the primary zone) as this flow is such a small proportion of the volume flow.

The rate of burning may be approximated to by the relations from Appendix I.

\[
- \frac{1}{V} \frac{dM_r}{dt} = 0.72 \phi (2.18 - \frac{1}{\phi})(0.18 + \frac{3}{j_*}) \quad \text{------------------ (11b)}
\]

for $1 \gg \frac{3}{j_*} \gg 0.4$

\[
- \frac{1}{V} \frac{dM_r}{dt} = 2.7 \phi \frac{3}{j_*} (1 - 0.75 \frac{3}{j_*}) \quad \text{------------------ (11b)}
\]

for $0.4 \gg \frac{3}{j_*} \gg 0$

At any section of the secondary zone
\[
W_p = \frac{1 + f^*}{f^*} \left( \frac{r_r}{r_r} - W_f \right) \quad \text{------------------ (21)}
\]

\[
\frac{j}{W_p} = \frac{f^*}{1 + f^*} \frac{r_f}{W_p - W_f} \quad \text{------------------ (22)}
\]

\[
- \delta W_f = \left[ - \frac{1}{V} \frac{dM_r}{dt} \right]_{\text{optimum}} \delta V \quad \text{------------------ (23)}
\]
From these equations an expression can be derived which may be integrated. The process is tedious, and is only summarised here.

For $1 \gg \frac{1}{j} \gg 0.4 \ (0.28 \ W_P \gg W_T \gg 0.13 \ W_P)$

$$0.72 \ \frac{\varphi}{W_P} = \frac{\alpha^*}{2.36 \ (\alpha^* - 0.18)^2} \ \ln \left[ \frac{0.18 + (\alpha^* - 0.18) \ 0.28}{0.18 + (\alpha^* - 0.18) \ \xi} \right]$$

$$+ \frac{\alpha^*}{2.36 \ (\alpha^* + 2.18)^2} \ \ln \left[ \frac{2.18 - (\alpha^* + 2.18) \ \xi}{2.18 - (\alpha^* + 2.18) \ 0.28} \right]$$

$$- \frac{0.28 - \xi}{(\alpha^* - 0.18)(\alpha^* + 2.18)} \quad \text{--------} \quad (24a)$$

Where $\varphi$ has its previous significance ($\varphi = \rho \ \xi^*$)

$\xi$ is the remaining unburnt fuel fraction

and $\alpha^* = \frac{f^*}{j} \ (1 + f^*)$

For $0.4 \gg \frac{1}{j} \gg 0$

$$2.7 \ \frac{\varphi}{W_P} = \frac{1}{\alpha^*} \ \ln \frac{0.13}{\xi} + \frac{\alpha^*}{(\frac{1}{2} + \alpha^*)} \ \ln \left[ \frac{1 - (1 + \frac{1}{2} + \alpha^*) \xi}{1 - (1 + \frac{1}{2} + \alpha^*) \ 0.13} \right]$$

$$- \frac{0.13 - \xi}{\alpha^* \ (1 + \frac{1}{2} + \alpha^*)} \quad \text{--------} \quad (24b)$$

Dividing by $V^* \ (= W_P/\varphi)$ the ideal volume, and taking $\alpha^* = 2.6$ gives the following table. The results are plotted on graph G.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>0.38</th>
<th>0.20</th>
<th>0.10</th>
<th>0.05</th>
<th>0.02</th>
<th>0.01</th>
<th>0.005</th>
<th>0.002</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{V_2}{V^*}$</td>
<td>0</td>
<td>0.09</td>
<td>0.20</td>
<td>0.32</td>
<td>0.45</td>
<td>0.55</td>
<td>0.65</td>
<td>0.78</td>
<td>0.88</td>
</tr>
</tbody>
</table>

$\xi$ is in effect the combustion inefficiency. The relation may be closely approximated to by the equation $\frac{V_2}{V^*} = 0.14 \ \ln \frac{0.13}{\xi}$ for values of $\xi$ below 0.20 (i.e. efficiencies greater than 80%).
The dashed curve on graph G shows the volume that would have been required if the max. heat release rate of the primary zone could be maintained in the secondary.

The secondary zone equations must be modified for the reheat case. $W_p$, the useful products at any section are those produced by reheat burning plus those previously associated with the air in the burning mixture. Then

$$W_p = \frac{1 + f^*}{f^*} (W_P - W_f) + \frac{1}{1-f} (W_P - W_f) + W_p \frac{1}{1-f} C$$

$C$, the proportion of combustible in the ideal mixture, varies slightly with $f$, but taking a mean value gives $W_p = 1.4 \frac{1 + f^*}{f^*} (W_P - W_f)$. All the preceding equations then hold, if $\alpha$ is taken as $\frac{f^*}{1.4 f^* (1 + f)}$, and the lower limit of integration $W_1 = 0.35 W_p$.

The new curve is also plotted on graph G, values being tabulated below.

<table>
<thead>
<tr>
<th>C</th>
<th>V_2/V_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.10</td>
</tr>
<tr>
<td>0.25</td>
<td>0.16</td>
</tr>
<tr>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>0.15</td>
<td>0.32</td>
</tr>
<tr>
<td>0.10</td>
<td>0.47</td>
</tr>
<tr>
<td>0.05</td>
<td>0.65</td>
</tr>
<tr>
<td>0.02</td>
<td>0.79</td>
</tr>
<tr>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

4) **Intensity of Ideal Burning Zone.**

Suppose the reversal flow proportion be $\eta$ and the design efficiency be $\eta_c$. Then from graphs F and G we obtain values of $V_1/V^*$ and $V_2/V^*$. The total volume $V = V_1 + V_2$, so for a fuel flow rate of $W_P$ lb./hr., the "intensity"

$$= \frac{W_P \cdot \eta_c \cdot 10^4}{p^2 V \cdot 10^5} \text{ million CHU/hr.}$$

$$\text{atmos}^2 \text{ cu.ft.}$$

$$\frac{\eta_c \cdot \beta \cdot V^* \cdot 10^4}{p^2 V \cdot 10^5} = \eta_c \left( \frac{V_1}{V^*} + \frac{V_2}{V^*} \right) \tag{25}$$

P.T.O.
On curve H are plotted values of $\frac{\Omega}{\eta^2}$ for different values of $r$ and $\eta_c$. For example, suppose $r = 0.2$ and $\eta_c = 0.97$ and $T_u = T_2 = 450^\circ \text{K}$. Then from graph H, $\frac{\Omega}{\eta^2} = 0.74$ while from graph A, $\Omega^* = 32.5$ units.

$. \therefore \Omega = 24 \text{ million CHU/hr.(atmos)}^2 \text{ cu.ft.}$

72% of the fuel (65% for reheat systems) is assumed to burn in the primary zone. If the combustion efficiency is less than 72%, the whole of the burning is assumed to take place at the maximum intensity, so $\frac{\Omega}{\eta^2} = \frac{0.72}{\sqrt{1/\eta^*}}$
MAXIMUM HEAT RELEASE RATE

\[ \Omega^* \] MAXIMUM HEAT RELEASE IN DRY MIXTURE

UNBURNED FUEL : 2 x STOICHIOMETRIC

MIXTURE - TEMP : 0 x STOICHIOMETRIC

UNITS OF HEAT RELEASE (\( \Omega^* \)):

MILLION C.H.U.

HR - FUEL (MASS)²

APPROX RELATION: \( \Omega^* = 9 \times \frac{T_o}{500} \)

<table>
<thead>
<tr>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
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</tbody>
</table>

INLET TEMPERATURE \( T_o \) \( \text{°K} \)

GRAPH A
CD 1894
HEAT RELEASE RATE

\( \dot{Q} \) = HEAT RELEASE RATE AT TEMP \( T_p \)

\( \dot{Q}_f \) = HEAT RELEASE RATE AT SAME FUEL R
AT STOICHIOMETRIC TEMP (\( T_f \))

MEAN CURVE FOR \( T = 450^\circ \text{K}, \text{STOICH. FUEL} \)

MAX. ERROR IS \( (T_f - T_p)/T_p \) FOR \( 300 < T < 600 \)
AND \( T > 1/10 \)

\( T_r \) HIGH RICH FUEL

\( T_r \) LOW WEAK FUEL

MIXTURE TEMPERATURE

\[ \frac{T_m}{T_p} \]

GRAPH B
CD 1895
HEAT RELEASE RATE

$Q_1 =$ HEAT RELEASE RATE AT F.A.R. $f$
$Q_{2*} =$ HEAT RELEASE RATE AT SAME TEMP. &
AT 2X STOICHIOMETRIC F.A.R. ($f^*$)
MEAN CURVE FOR $T_0 = 450^\circ$ K, $T_h = 0^\circ$ K

Approx. Relation

$Q_{2*} = 0.6 \frac{q}{f^*}/(q_1 + \frac{1}{2})^2$

UNBURNT FUEL : AIR RATIO

GRAPH C
CD 1696
HEAT RELEASE RATE

\[ Q = Q_1 \text{ at fuel/prop ratio } J \]

\[ Q = Q_2 \text{ at same F/P ratio and at optimum temperature for that F/P ratio} \]

\[ J^* \text{ F/P ratio corresponding to twice stoichiometric at } 0.85 \text{ stoichiometric temp} \]

\[ \frac{T}{T_{\text{stoch}}} \]

MIXTURE TEMP.

\[ \frac{T}{T_{\text{stoch}}} \]

GRAPH I

CD 1897
HEAT RELEASE RATE

\( \frac{Q_f}{Q_o} = \text{HEAT RELEASE AT FUEL PROD. RATIO } j \)

\( \frac{Q*}{Q_o} = \text{HEAT RELEASE AT OPTIMUM F.P.R. } j^* \)

Both taken at their respective optimum temps (Graph E)

APPROX. RELATIONS

1) \( \frac{Q_f}{Q_o} = 2.7 \frac{j}{j^*} (1 - \frac{3}{2} \frac{j}{j^*}) \quad 0 < \frac{j}{j^*} < 0.6 \)

2) \( \frac{Q_f}{Q_o} = 0.72 (0.18 + \frac{j}{j^*}) (2.18 - \frac{j}{j^*}) \quad 0.4 < \frac{j}{j^*} < 1.0 \)

UNBURNt FUEL: PRODUCTS RATIO

\[ 0 \leq \frac{j}{j^*} \leq 1.0 \]

GRAPH E

CD 1898
SECONDARY ZONE VOLUME

$V_2$ SECONDARY ZONE VOLUME

$V^*$ VOLUME NEEDED TO BURN ALL FUEL AT MAXIMUM HEAT RELEASE RATE

HEAT SYSTEM (65% FUEL BURNT IN PRIMARY)

COMBUSTION CHAMBER (72% FUEL BURNT IN PRIMARY)

CONSTANT INTENSITY CURVES

COMBUSTION EFFICIENCY

99.9 93 90

INEFFICIENCY $\varepsilon$ (LOG. SCALE)

GRAPH G

CD 1800