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THEORETICAL ANALYSIS OF THE DOWNWASH DISTRIBUTION OVER HELICOPTER ROTORS IN FORWARD FLIGHT

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Prepared by:

E. Seckel

D. C. Hazen

A. A. Nikolsky

Director of Project

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SUMMARY

In order to provide a basis for the experimental study of the downwash distribution over a rotor disc in forward flight, an attempt has been made to analyze the problem theoretically by extending the well-known monoplane vortex theory to cover the more complicated situation presented by the helicopter rotor. The theoretical analysis of a helicopter equipped with a fully articulated rotor is presented in detail. In order to check the theory, a detailed investigation of rigid rotor was made. The lack of logical results from this investigation has been traced to the difficulty of locating the points at which the downwash is measured with respect to the trailing vortices. A possible solution to this problem has been developed and is included in this report.

INTRODUCTION

There is, at present, no satisfactory theoretical method of predicting the downwash distribution over a rotor disc in forward flight; the most frequently used method being to arbitrarily assume that the distribution is either constant, or triangular in shape. This lack of information may seriously affect the ability of the engineer to predict the performance of a helicopter, or the air load distribution over the blades, and hence the structural limitations of the machine.

In order to surmount this difficulty, an attempt has been made to develop a theory which relates the downwash to the vorticity shed from the blades. It is the purpose of this report to develop this theory and to present it in such a form that it may be used by the average practicing engineer without a detailed knowledge of the theory.

To this end, the results of this investigation have been presented in the form of simple numerical equations. That these equations are in error is not an indication that the theory is in error, but rather that it has over simplified a very complex problem.

It should be noted that the suggested modifications to the theory will add complications to the investigation which it was originally hoped could be avoided, but will not, in any way, increase the complexity of the equations which express the final results.

SYMBOLS

W	gross weight of helicopter, pounds
T	rotor thrust, pounds
Q	rotor torque, foot pounds
M	rotor pitching moment, foot pounds
L	rotor rolling moment, foot pounds
V	resultant velocity at a blade element, feet per second
R	blade radius, feet
r	distance of blade element from origin, feet

x ratio of distance of blade element from origin to blade radius, $\frac{r}{R}$

l lift of a blade element, pounds

c_l two dimensional lift coefficient, $\left(\frac{l}{\frac{1}{2}\rho V^2 c}\right)$

ρ mass density of air, slugs per cubic foot

a two dimensional slope of the lift curve, per radian

c blade chord, feet.

α angle of attack, radians

Ω rotational velocity of rotor, radians per second

μ tip speed ratio, $\frac{V \cos\alpha}{\Omega R}$

b number of blades

σ solidity ratio, $\frac{bc}{\pi R}$

c_{d_o} two dimensional drag coefficient, $\frac{dD}{\frac{1}{2}\rho V^2 c}$

δ_o, δ_2 coefficients of quadradic equation for the two dimentional drag coefficient, $c_{d_o} = \delta_o + \delta_2 \alpha^2$

C_T rotor thrust coefficient, $\left(\frac{T}{\rho \Omega^2 \pi R^4}\right)$

C_m rotor pitching moment coefficient, $\left(\frac{M}{\rho \Omega^2 \pi R^5}\right)$

C_L rotor rolling moment coefficient, $\left(\frac{L}{\rho \Omega^2 \pi R^5}\right)$

C_Q rotor torque coefficient, $\left(\frac{Q}{\rho \Omega^2 \pi R^5}\right)$

w downwash, feet per second

Γ circulation

ψ angular displacement of blade about origin, radians

θ_o constant part of blade root incidence, radians

θ_1 total blade twist, radians

θ_c blade incidence caused by cyclic pitch, radians

c_1, c_2 cyclic pitch control coefficients, $\theta_c = c_1 \cos\Psi + c_2 \sin\Psi$

β flapping angle, radians

$\left.\begin{matrix} a_0, a_1, a_2 \\ b_1, b_2 \end{matrix}\right\}$ flapping angle coefficients,
$\beta = a_0 - a_1 \cos\Psi - b_1 \sin\Psi - a_2 \cos 2\Psi - b_2 \sin 2\Psi$

τ automatic pitch change coefficient, $\frac{\partial\theta}{\partial\beta}$

METHOD OF ANALYSIS

Simplified Conception of the Rotor Disc and Vortex Wake

It is assumed that 24 discrete vortices are shed from the rotor disc; one from each of four blade stations, for each of 6 different blade azimuth positions. The downwash at the disc may thus be computed in terms of the 24 vortices shed. The strength of each vortex, on the other hand, is related to the strength of the circulation on the blades at 24 "pivotal" points, and hence depends on the downwash created by all the other trailing vortices.

In more detail, the rotor disc is split up into 6 pie-shaped segments, which are further divided into 4 zones each by concentric circles. (See Fig. 1). The reasons for choosing 6 segments and 4 zones are purely those of compromising ease of computation with accuracy. More accuracy would require a smaller division of the disc, but would involve a tremendous increase in the cost and difficulty of solving the mathematics numerically.

Referring to Fig. 1, it will be seen that the separate zones are bounded by solid radial lines and solid concentric circles. It is assumed that any blade section anywhere within one zone would have a constant value of circulation Γ. The value of that circulation is to be determined at a "pivotal" point within the zone. As shown by Fig. 1, these points are located symmetrically in such a way that all points lie within the trail-

ing vortices and give as nearly average values within the zone as is practical.

Six blade positions are considered, represented by each of the solid radial lines and hence by the radial boundaries of the zones. Consider blade II in Fig. 1. Since the blade lies on the boundary between the zones, it is assumed that at any blade station the circulation strength is the average between the zones before and behind. That is, from point IIA to point IIB, $\Gamma = \frac{\Gamma_{3a} + \Gamma_{2b}}{2}$; and from point IIB to point IIC, $\Gamma = \frac{\Gamma_{3b} + \Gamma_{2c}}{2}$. There is a vortex shed at point IIB, then, of strength

$$\Delta\Gamma = \frac{1}{2}\left\{\Gamma_{3b} + \Gamma_{2b} - \Gamma_{3c} - \Gamma_{2c}\right\} \qquad (1)$$

There are 24 of these vortices shed, one from each intersection of a solid radial line with a circular boundary. The strength of each is determined by the circulation strength in the four neighboring zones. Each of these vortices is a complete horseshoe, extending inward along the blade to the root, and peeled off there. The downwash at each pivotal point due to each of the horseshoe vortices of strength $\Delta\Gamma$ is determined purely by geometry.

Theoretical Justification for the Simplified Description of the Rotor Disc and the Wake

The basic assumption involved in this simplified picture is that the wake vortex system may be treated as though the rotor consisted of a finite number of stationary blades, (in this case 6). This results in a comparatively simple wake vortex pattern.

The downwash over the disc being a function of the geometry of the wake pattern, is therefore determined by this assumption. The blade loading, and hence the strength of the trailing vortices, are, however, determined by the boundary conditions appropriate to rotating blades. This simplified wake pattern would be approached as the forward speed became large compared to the tip speed.

There are other assumptions which are implied:

1.) Small rotor angle of attack, and light disc loading - (so that the trailing vortices may be assumed to lie in the plane of the rotor disc).

2.) Replacement of the actual pulsating flow through the disc by what may be considered an average steady flow. This first implies an infinite number of blades, reduced to 6 to simplify the treatment. In addition, however, the vorticity shed from a blade due to the cyclic variation of the bound vorticity is neglected as implied in the basic assumption of stationary blades for determining the wake.

Notation on the Rotor Disc

Referring to Fig. 1, it will be seen that the azimuth blade positions considered are identified by the Roman numerals I through VI. The radial boundaries of the zones, and hence the radii at which the vortices peel off, are denoted by the capital letters, A. through D. The capital letter N is used as a sub-

script to denote any one of the Roman numerals. The capital letter J is used similarly to denote any of the capital letters, A to D. $\Delta\Gamma_{NJ}$ can thus be used to identify the horseshoe vortices. For instance, if N=II, and J=B, then $\Delta\Gamma_{2B}$ identifies the horseshoe vortex illustrated in Fig. 1.

The azimuth positions of the pivotal points are denoted by the arabic numerals 1 to 6. The radial positions of the pivotal points by the lower case letters a through d. The subscripts n and j are used to denote any values of the arabic numerals and small letters respectively. Thus w_{nj} for n=2 and j=b would be w_{2b}, which represents the downwash at the pivotal point with azimuth position 2 and radius b. The circulation strength is also denoted by nj for the pivotal point in the zone. Thus Γ_{nj} for n=2 and j=b is Γ_{2b}, the circulation in the zone whose pivotal point is 2b.

Choice of Number and Location of Zones and Pivotal Points

It seems desirable to have the zonal division of the disc symmetrical, if not from esthetic considerations, then from the standpoint of symmetry of the calculations. This means that an even number of radial divisions are called for. Further, since the downwash at the pivotal point is assumed to be the average downwash [illegible] in the immediate vicinity of, a trailing vortex where the downwash tends toward infinity, nor can any be located outside of the trailing vortex pattern as this violates the actual physical picture. Hence, the radial divisions from which the vortices

spring must be oriented so that one of the divisions lies on the fore-and-aft diameter of the disc. Purely by intuition it seems that six radial divisions is about the minimum for reasonable accuracy in finding the azimuth variations in Γ and w. Also for finding the radial variation of these terms it seems that four points on a blade is the minimum for reasonable accuracy. This, of course, means 24 points in all. Fortunately, the 24 equations resulting are about the largest number that can be solved efficiently with standard IBM equipment.

One very important consideration in choosing the radii of the zone boundaries is to keep the pivotal points from falling too near a trailing vortex. Some time was spent on trying to establish some analytical method for placing the pivotal points, but it proved far too intricate to be practical. By trial and error the best solution seemed to be locate the points at which the vortices spring from the blades at equal distances along the blades, hence x_A=.25, x_B=.50, x_C=.75 and x_D=1.0. The pivotal points were located so that they fell at points equidistant between the trailing vortices on the lateral diameter of the rotor. These have the following locations x_a=.108, x_b=.325, x_c=.541 and x_d=.758.

Evaluation of Coefficients

The downwash at a pivotal point, nJ, due to one of the horse-shoe vortices, $\Delta\Gamma_{nJ}$, may be expressed as

$$\Delta w_{nJ} = \left(\frac{\Delta\Gamma_{nJ}}{2\pi R}\right) k_{nJNJ} \qquad (2)$$

where the linear dimensions involved in k_{nJNJ} are in per cent radius of the rotor.

The vortex is assumed to have a positive sign if its sense is for more positive Γ inboard, and less positive Γ outboard of the point NJ. The downwash w, is positive downwards, so k is positive if the positive vortex induces downwash at the pivotal point.

Fig. 2 shows a vortex shed from the "upstream" side of the rotor ($\Psi_N < 180°$). The vortex shown is positive as defined above.

The k coefficient is in three parts $k = k_1 + k_2 + k_3$ due, respectively, to the parts of the horseshoe labelled 1, 2 and 3 in the figure.

$$k_1 = \frac{1 + 1/e}{d}$$

$$k_1 = \frac{1 + \cos \Psi_n}{x_j \sin \Psi_n}$$

$$k_2 = \frac{1/e + 1/h}{f}$$

where

$$1/e = \cos (\Psi_N - \Psi_n)$$

$$1 = x_J - x_j \cos (\Psi_N - \Psi_n)$$

$$h = \sqrt{a^2 + c^2}$$

$$a = x_j \cos \Psi_n - x_J \cos \Psi_N$$

$$c = x_J \sin \Psi_N - x_j \sin \Psi_n$$

$$d = \sqrt{x_J^2 + x_j^2 - 2x_J x_j \cos (\Psi_N - \Psi_n)}$$

$$f = x_j \sin (\Psi_N - \Psi_n)$$

$$\therefore k_2 = \frac{\cos(\Psi_N - \Psi_n) + \dfrac{x_c - x_j \cos(\Psi_N - \Psi_n)}{\sqrt{x_c^2 + x_j^2 - 2x_c x_j \cos(\Psi_N - \Psi_n)}}}{x_j \sin(\Psi_N - \Psi_n)} \tag{4}$$

$$k_3 = \frac{1 + a/b}{c}$$

$$k_1 = \frac{\dfrac{x_j \cos \Psi_n - x_J \cos \Psi_N}{1 + \sqrt{x_c^2 + x_j^2 - 2x_c x_j \cos(\Psi_N - \Psi_n)}}}{x_c \sin \Psi_N - x_j \sin \Psi_n} \tag{5}$$

On the downstream side of the rotor ($\Psi_N > 180°$), the vortex [illegible] shown in Fig. 3. Since the direction of the trailing [illegible] of the horseshoe 1 and 2 have changed, the signs of k_1 and k_2 are reversed, ($360° > \Psi_N > 180°$). The sign of part 3 is the same, so k_3 has the same sign as before.

[illegible] the rotor $\Psi_N = 0°$ and $\Psi_N = 180°$. If we consider $\Psi_N = 180°$ applying in the zone $\Psi_N < 180°$ then the signs of equations 2 and 5 are correct. If we consider it part of the zone $\Psi_N > 180°$, k_1 and k_2 must have the opposite signs. Therefore, for $\Psi_N = 0°$ or $180°$, k_1 and k_2 must drop out leaving only k_3. This may be seen to be reasonable by sketching the vortices for $\Psi_N = 0° + \epsilon$ and $\Psi_N = 180° - \epsilon$ and taking the average.

The formula, then, for the constant, is

$$\sin\Psi_c = \frac{\cos(\Psi_N - \Psi_n) + \dfrac{x_c - x_j\cos(\Psi_N - \Psi_n)}{\sqrt{x_c^2 + x_j^2 - 2x_c x_j \cos(\Psi_N - \Psi_n)}}}{x_j \sin(\Psi_N - \Psi_n)}$$

$$+ \left\{\frac{|\sin\Psi_N|}{\sin\Psi_N}\right\} \left[\frac{1 + \cos\Psi_n}{x_j \sin\Psi_n} + \frac{1 + \dfrac{x_j\cos\Psi_n - x_c\cos\Psi_N}{\sqrt{x_c^2 + x_j^2 - 2x_c x_j\cos(\Psi_N - \Psi_n)}}}{(x_c \sin\Psi_N - x_j \sin\Psi_n)} \right] \quad (6)$$

where, for:

$\Psi_N = 0°$ or $180°$, $\left\{\frac{|\sin\Psi_N|}{\sin\Psi_N}\right\} = 0$

$0° < \Psi_N < 180°$, $\left\{\frac{|\sin\Psi_N|}{\sin\Psi_N}\right\} = +1$

$360° > \Psi_N > 180°$, $\left\{\frac{|\sin\Psi_N|}{\sin\Psi_N}\right\} = -1$

Equations for the Circulation Distribution.

The basic equation relating the circulation and the lift in two dimensional flow is

$$l = \rho V \Gamma \quad (7)$$

Also

$$l = \tfrac{1}{2}\rho V^2 c_l c \quad (8)$$

where

$$c_l = a\alpha_{net} = a(\alpha_{geo} - \tfrac{w}{V})$$

Equating equation 7 to equation 8 and solving for Γ, the following result is obtained:

$$\Gamma = \frac{ac}{2} V\alpha - \frac{ac}{2} w \quad (9a)$$

where α is the geometric angle of attack.

At a specific nivotal point, equation 9a is subscripted in the following manner.

$$\Gamma_{\underline{nj}} = \frac{ac}{2} V_{\underline{nj}} \alpha_{\underline{nj}} - \frac{ac}{2} w_{\underline{nj}} \tag{9b}$$

In equation 9b the subscript nj has been underlined to indicate that a specific nivotal point is meant. This is of value when it is necessary to differentiate this nivotal point from all the other points involved in summations.

The expression for the total downwash at a specific nivotal point may be written as

$$w_{\underline{nj}} = \frac{1}{2\pi} \sum k_{\underline{nj}NC} \Delta\Gamma_{NC} \tag{10}$$

Because of the relation between the vortex strength, $\Delta\Gamma$, and the circulation in the neighboring zones, expressed by equation 1, equation 10 may be rewritten as

$$w_{\underline{nj}} = \frac{1}{\pi} \sum L_{\underline{nj}nj} \Gamma_{nj} \tag{11}$$

where

$$L_{\underline{nj}nj} = \frac{1}{2\pi} (k_{\underline{nj}NC} + k_{\underline{nj}(N-1)C} - k_{\underline{nj}(C-1)} - k_{\underline{nj}(N-1,C-1)}) \tag{11a}$$

Note that n=N, and j=C in the expression of the L coefficient. In other words, if the L coefficient at the nivotal point 2c due to the vortex shed at point IIIb is being considered, it would be written as:

$$L_{2c2b} = \frac{1}{2\pi} (k_{2c\,IIIB} + k_{2cIIB} - k_{2cIIIA} - k_{2cIIA})$$

For any nivotal point, nj, then two simultaneous equations in $\Gamma_{\underline{n'}}$ and $w_{\underline{n'}}$ apply:

$$\Gamma_{\underline{n'}} = \frac{ac}{2} V_{\underline{n'}} \alpha_{\underline{n'}} - \frac{ac}{2} w_{\underline{n'}} \tag{9b}$$

$$w_{\underline{n'}} = \frac{1}{R} \sum L_{\underline{n'}nj} \Gamma_{nj} \tag{11}$$

in which the summation is over all the zones nj.

Notice that either Γ or w can be elimated from equations 9b and 11. If the downwash w were desired directly, Γ should be eliminated. However, from the point of view of finding the aerodynamic loads on the blades, it is more convenient to solve for Γ. Therefore, substituting equation 11 into equation 9b and dividing through by $\frac{ac\Omega R}{2}$, the following non-dimensional equation is obtained.

$$\frac{2}{R}\left(\frac{\Gamma_{\underline{n'}}}{c\Omega R}\right) + \frac{c}{R} \sum L_{\underline{n'}nj} \left(\frac{\Gamma_{nj}}{c\Omega R}\right) = \frac{V_{\underline{n'}} \alpha_{\underline{n'}}}{\Omega R} \tag{12}$$

Equation 12 is completely non-dimensional and applies for the particular nivotal point $\underline{n'}$. There are 24 unknowns, Γ_{nj} corresponding to the 24 zones into which the rotor disc is divided. Note that the summation of the second term includes a $\Gamma_{\underline{n'}}$ so that the total coefficient of $\left(\frac{\Gamma_{\underline{n'}}}{c\Omega R}\right)$ is:

$$\left(\frac{2}{R} + \frac{c}{R} L_{\underline{n'n'}}\right)$$

Note that there is an equation 12 for each nivotal point $\underline{n'}$, of which there are 24. Thus, equation 12 represents a s.s.

tem of 24 linear equations in 24 unknowns. Strictly speaking, the coefficients of these equations are known only for the specific helicopter being considered, but experience has shown that if typical values of a and $\frac{c}{R}$ are assumed, say a = 5.6/rad. and $c/R = \frac{1}{15}$, no appreciable error is introduced in dealing with conventional helicopters.

The right hand side of equation 1[illegible] is expressible in terms of the pitching and flapping coefficients, etc., and is treated as if it is known:

$$V_{\underline{n}} = \Omega R\,(x_{\underline{n}} + \mu \sin \Psi_{\underline{n}}) \qquad (13a)$$

$$\frac{V_{\underline{n}}}{\Omega R} = x_{\underline{n}} + \mu \sin \Psi_{\underline{n}} \qquad (13b)$$

or, dropping the subscripts on x and Ψ:

$$\frac{V_{\underline{n}}}{\Omega R} = x + \mu \sin \Psi \qquad (13c)$$

In general, the [illegible] of a blade element may be expressed as:

$$\alpha_{\underline{n}} = \theta_c + \theta_1 x - \frac{\alpha \mu \Omega R}{V} + \tau_1 \rho - \frac{x \rho R}{V} + \theta_c \qquad (14a)$$

$$= \theta_c + \theta_1 x - \frac{\alpha \mu \Omega R}{V}$$

$$+ \tau_1 (\theta_c - a_1 \cos \Psi - b_1 \sin \Psi - a_2 \cos 2\Psi - b_2 \sin 2\Psi)$$

$$- \frac{\alpha \Omega R}{V} (a_1 \sin \Psi - b_1 \cos \Psi + a_2 \sin 2\Psi - b_2 \cos 2\Psi)$$

$$+ [illegible] \Psi + a_2 \sin \Psi$$

$$\alpha_{\underline{1}} = a_0 + a_1 x + a_0 \tau_1 - \frac{\alpha \mu \Omega R}{V} - \left(a_1 \tau_1 - \frac{b_1 x \Omega R}{V} - c_1\right)\cos\Psi$$

$$- \left(b_1 \tau_1 + \frac{a_1 x \Omega R}{V} - c_2\right)\sin\Psi - \left(a_2 \tau_1 - \frac{b_2 x \Omega R}{V}\right)\cos 2\Psi$$

$$- \left(b_2 \tau_1 + \frac{2a_2 x \Omega R}{V}\right)\sin 2\Psi \qquad (14b)$$

In equations 14a and 14b, and hereafter, α on the right hand side of the equation is the rotor angle of attack.

Multiplying equation 13a by equation 14b, the following is obtained:

$$\frac{V \alpha}{\Omega} = x(a_0 + a_1 x + a_0 \tau_1) - \alpha\mu + \left\{-x(a_1 \tau_1 - a_1 - b_1)\right\}\cos\Psi$$

$$+ \left\{-x(b_1 \tau_1 - c_2 + a_1) + \mu(a_1 + a_1 + a_0 \tau_1)\right\}\sin\Psi$$

$$+ \left\{-\mu(a_1 \tau_1 - c_1)\right\}\sin\Psi\cos\Psi + \left\{-\mu(b_1 \tau_1 - c_2)\right\}\sin^2\Psi$$

$$+ \left\{-x(a_2 \tau_1 - 2b_2)\right\}\cos 2\Psi + \left\{-x(b_2 \tau_1 + 2a_2)\right\}\sin 2\Psi$$

$$+ \left\{-\mu a_2 \tau_1\right\}\cos 2\Psi\sin\Psi + \left\{-\mu b_2 \tau_1\right\}\sin 2\Psi\sin\Psi \qquad (15a)$$

[illegible]

$$\sin\Psi\cos\Psi = \frac{1}{2}\sin 2\Psi$$

$$\sin^2\Psi = \frac{1}{2} - \frac{1}{2}\cos 2\Psi$$

$$\cos 2\Psi \quad \sin\Psi = \tfrac{1}{2}\sin 3\Psi - \tfrac{1}{2}\sin\Psi$$

$$\sin 2\Psi \quad \sin\Psi = \tfrac{1}{2}\cos\Psi - \tfrac{1}{2}\cos 3\Psi$$

and neglecting all harmonics higher than the second, equation (15a) may be rewritten in a simpler form

$$\frac{V_{nj}\alpha_{nj}}{\Omega R} = \left\{\frac{\mu}{2}(c_2 - b_1\tau_1 - 2\alpha) + x(e_0 + a_0\tau_1) + x^2(e_1)\right\}$$

$$+\left\{\left(-\frac{\mu b_2\tau_1}{2}\right) + x(c_1 + b_1 - a_1\tau_1)\right\}\cos\Psi$$

$$+\left\{\mu\left(e_0 + a_0\tau_1 + \frac{a_2\tau_1}{2}\right) + x(c_2 - b_1\tau_1 - a_1 + \mu e_1)\right\}\sin\Psi$$

$$+\left\{\frac{\mu}{2}(b_1\tau_1 - c_2) + x(2b_2 - a_2\tau_1)\right\}\cos 2\Psi$$

$$+\left\{\frac{\mu}{2}(c_1 - a_1\tau_1) + x(-b_2\tau_1 - 2a_2)\right\}\sin 2\Psi \qquad (15b)$$

where, of course, x and Ψ refer to the specific pivotal point $\underline{nj}$.

Because of the linearity of the system of equations 12, the complete solution for $\frac{\Gamma_{nj}}{c\Omega R}$, for the right side given by equation 15b may be calculated as the sum of the separate solutions to each of the eleven terms in equation 15b. Further, the separate solutions may be calculated for unit values of the terms above, and later multiplied by the actual values.

Let the solution, $\frac{\Gamma_{nj}}{c\Omega R}$, for the unity value of any of the separate terms in equation 15b be X_{nj}. If, for any zone, j, we plotted X_{nj} vs. Ψ we would obtain a step graph. This is, of

course, consistent with the previous assumptions concerning the distribution of Γ over the disc. For convenience in determining the airload distribution, it is convenient to express X_j as a trignometric series in Ψ. Using the least meaned squared error method of approximation, we obtain the well-known harmonic analysis formulae.

The series may be expressed as:

$$X_j = s_{jo} + s_{j1}\cos\Psi + t_{j1}\sin\Psi + s_{j2}\cos 2\Psi + t_{j2}\sin 2\Psi \tag{16}$$

where

$$s_{jo} = \frac{1}{2\pi}\int_0^{2\pi} X_j\,(\Psi)\,d\Psi$$

$$s_{jn} = \frac{1}{\pi}\int_0^{2\pi} X_j\,(\Psi)\cos n\Psi\,d\Psi$$

$$t_{jn} = \frac{1}{\pi}\int_0^{2\pi} X_j\,(\Psi)\sin n\Psi\,d\Psi$$

Thus the expressions for the series coefficients for $X_j\,(\Psi)$ are:

$$s_{jo} = 1/6\left\{X_{1j} + X_{2j} + X_{3j} + X_{4j} + X_{5j} + X_{6j}\right\}$$

$$s_{j1} = \frac{.866}{\pi}\left\{X_{1j} - X_{2j} - X_{4j} + X_{5j}\right\}$$

$$t_{j1} = \frac{1}{3\pi}\left\{X_{1j} + 2X_{2j} + X_{3j} - X_{4j} - 2X_{5j} - X_{6j}\right\}$$

$$s_{j2} = \frac{.433}{\pi}\left\{X_{1j} - 2X_{2j} + X_{3j} + X_{4j} - 2X_{5j} + X_{6j}\right\}$$

$$t_{j2} = \frac{2}{3\pi}\left\{X_{1j} - X_{3j} + X_{4j} - X_{6j}\right\}$$

Equations for the Air Load Distribution

Knowing the harmonic parts of X_j for the various rings, j, we can evaluate the contribution of the unity value for one of the parts of $\frac{V_{n'} \alpha_{n'}}{\Omega R}$ to the rotor rolling and pitching moments, thrust and torque.

a) Thrust:

The basic thrust equation can be expressed as:

$$\frac{dT}{dr} = b \Gamma \rho V$$

or:

$$\frac{dT}{dx} = bR \Gamma \rho V$$

Hence:

$$T = \int_0^1 bR \Gamma \rho V dx$$

or:

$$\frac{T}{\pi \rho R^2 \Omega^2 c} = \int_0^1 \left(\frac{\Gamma}{\Omega R}\right) \left(\frac{V}{\Omega R}\right) dx$$

Therefore:

$$\frac{C_T}{\sigma} = \int_0^1 X_j \left(\frac{V}{\Omega R}\right) dx \qquad (17a)$$

where.

$$X_j \left(\frac{V}{\Omega R}\right) = (s_{j0} + s_{j1} \cos\psi + t_{j1} \sin\psi + s_{j2} \cos 2\psi + t_{j2} \sin 2\psi)(x + \mu \sin\psi)$$

Retaining only those terms which give constant or average thrust, which are, $xs_{j0} + \frac{1}{2}\mu t_{j1}$, equation 17a may be expressed as:

$$\frac{C_T}{\sigma} = \int_0^1 (x s_{jo} + \tfrac{1}{2}\mu t_{j1})\, dx \tag{17b}$$

or:

$$\frac{C_T}{\sigma} = \sum_{j=a}^{d} \frac{x_j^2 - x_{j-1}^2}{2} s_{jo} + \sum_{j=a}^{d} \frac{x_j - x_{j-1}}{2} \mu t_{j1} \tag{17c}$$

b) Pitching Moment.

Assuming a nose up moment as positive, the basic pitching moment equation may be written as:

$$dM = -r \cos\Psi\, dT$$

$$dM = -xR \cos\Psi\, dT$$

Hence,

$$M = -bR \int_0^R x \cos\Psi \frac{dT}{dr}\, dr$$

$$= -bR^2 \int_0^{1.0} x \cos\Psi \frac{dT}{dr}\, dx$$

$$= -bR^2 \int_0^{1.0} x \cos\Psi\, T \rho V dx$$

$$= -\rho b c \Omega^2 R^4 \int_0^{1.0} x \cos\Psi \left(\frac{T}{\rho b c \Omega R}\right) \left(\frac{V}{\Omega R}\right) dx$$

Therefore:

$$\frac{C_M}{\sigma} = -\int_0^{1.0} x(s + \mu \sin\Psi)\, X_j \cos\Psi\, dx \tag{18a}$$

Again, only those terms which contribute to the constant part of C_M are considered.

$$\frac{C_M}{\sigma} = -\int_0^{1.0} (\tfrac{1}{2} x^2 s_{j1} + \tfrac{1}{2}\mu x t_{j2})\, dx \tag{18b}$$

or:

$$-\frac{C_m}{\sigma} = \sum_{j=A}^{d} \frac{x_j^2 - x_{j-1}^2}{6} s_{j1} + \sum_{j=A}^{d} \frac{x_j^2 - x_{j-1}^2}{8} \mu t_{j2} \quad (18c)$$

c) Rolling Moment

Assuming that a roll to the right is positive, the basic rolling moment equation may be written as:

$$dL = -r\sin\Psi\, dT = -xR\sin\Psi\, dT$$

As above, the equation may be put in the form

$$-\frac{C_r}{\sigma} = \int_0^{10} x(x + \mu\sin\Psi)\left(\frac{T}{c\Omega r}\right)\sin\Psi\, d\Psi$$

or:

$$-\frac{C_r}{\sigma} = \int_0^{10} x(x + \mu\sin\Psi)\, X_j \sin\Psi\, dx \quad (19a)$$

Again, retaining only the constant terms, the equation becomes:

$$-\frac{C_r}{\sigma} = \int_0^{10} \left[\frac{x^2}{5} t_{j1} + \frac{x\mu}{4}(2s_{j0} - s_{j2})\right] dx \quad (19b)$$

or:

$$-\frac{C_r}{\sigma} = \sum_{j=A}^{d} \frac{x_j^2 - x_{j-1}^2}{5} t_{j1} + \sum_{j=A}^{d} \frac{x_j^2 - x_{j-1}^2}{8} \mu(2s_{j0} - s_{j2}) \quad (19c)$$

d) Torque

The total torque is in 2 parts: the induced torque, (due to the tilting of the lift vector), and the

part due to the profile drag of the blades.

1) Profile Torque:

The profile drag of a blade element, dD_o can be written as:

$$dD_o = \tfrac{1}{2}\rho V^2 c_{d_o} c$$

Setting

$$c_{d_o} = \delta_o + \delta_2 \alpha^2$$

and

$$\alpha = \frac{c_l}{a}$$

we get

$$c_{d_o} = \delta_o + \delta_2 \left(\frac{c_l}{a}\right)^2$$

But

$$c_l = \frac{2\Gamma}{Vc}$$

Therefore

$$dD_o = \tfrac{1}{2}\rho V^2 c \left\{ \delta_o + \frac{\delta_2}{a^2}\left(\frac{2\Gamma}{Vc}\right)^2 \right\}$$

$$= \tfrac{1}{2}\rho c \delta_o V^2 + 2\rho \frac{\delta_2}{a^2 c}\Gamma^2$$

Hence

$$\frac{dD_o}{\rho(\Omega R)^2 c} = \frac{\delta_o}{2}\left(\frac{V}{\Omega R}\right)^2 + \frac{2\delta_2}{a^2}\left(\frac{\Gamma}{c\Omega R}\right)^2 \qquad (20)$$

2) The Induced Torque

From Fig. 4 it will be seen that

$$\alpha = \theta + \phi$$

where $\theta = \theta_o + \theta_1 x + \theta\tau_1 + \theta_c$

or $$\alpha = \alpha_0 + a_0\tau_1 + \alpha_1 x + (c_1 - a_1\tau_1)\cos\psi$$

$$+ (c_0 - b_1\tau_1)\sin\psi$$

(neglecting the small terms $a_0\tau_1$ and $b_0\tau_1$)

The basic equation for the induced torque is

$$dD_i = -dT\sin\phi$$

But

$$\Gamma = \frac{aVc}{2}\alpha = \frac{aVc}{2}(\alpha + \phi)$$

and

$$\phi \cong \sin\phi \cong -\alpha + \frac{2\Gamma}{aVc}$$

and

$$dT = V\Gamma\rho$$

Therefore:

$$dD_i = -dT\sin\phi = \Gamma\rho V\left(\alpha - \frac{2\Gamma}{aVc}\right)$$

or

$$\frac{dD_i}{\rho c(\Omega R)^2 R^2} = \left(\frac{\Gamma}{c\Omega R}\right)\left\{\alpha\left(\frac{V}{\Omega R}\right) - \frac{2}{a}\left(\frac{\Gamma}{c\Omega R}\right)\right\} \quad (21)$$

Adding this result to the profile drag, the expression for the total drag becomes:

$$\frac{D}{\rho c(\Omega R)^2 R^2} = \frac{\delta_0}{2}\left(\frac{V}{\Omega R}\right)^2 + \alpha\left(\frac{V}{\Omega R}\right)\left(\frac{\Gamma}{c\Omega R}\right) + \frac{2}{a^2}(\delta_2 - a)\left(\frac{\Gamma}{c\Omega R}\right)^2 \quad (22)$$

By substituting for $\left(\frac{V}{\Omega R}\right)$ and X_i for $\left(\frac{\Gamma}{c\Omega R}\right)$, the following equation is obtained.

$$-\frac{C_T}{\sigma} = \int_0^1 x\left\{\frac{\delta_0}{2}(x^2 + \mu^2)\right.$$

$$+ (\alpha_0 + a_0\tau_1 + \alpha_1 x)\left[x\lambda_{ix} + \mu\lambda_{iz}\right]$$

$$+(c_1 - a_1\tau_1)\left[\tfrac{1}{2}\mu s_{j1} + \tfrac{1}{2}\mu t_{j2}\right]$$

$$+(c_2 - b_1\tau_1)\left[\tfrac{1}{2}\mu t_{j1} + \tfrac{1}{2}\mu s_{j0} - \tfrac{1}{2}\mu s_{j2}\right]$$

$$-\frac{2}{a}(a - \delta_2)\left[s_{j0}^2 + \tfrac{1}{2}t_{j1}^2 + \tfrac{1}{2}s_{j1}^2 + \tfrac{1}{2}t_{j2}^2 + \tfrac{1}{2}s_{j2}^2\right]\Bigg\}\,dx \qquad (23a)$$

Upon integrating and retaining the constant terms, this equation becomes:

$$-\frac{C_0}{\sigma} = \frac{\delta_2}{8}(1+\mu^2)$$

$$+\sum_{j=2}^{j} \frac{x_j^2 - x_{j-1}^2}{8}\Big[\mu t_{j1}(c_1 + a_0\tau_1)$$

$$+\mu t_{j2}(c_1 - a_1\tau_1) + \mu(2s_{j0} - s_{j2})(c_2 - b_1\tau_1)$$

$$-\frac{4}{a^2}(a - \delta_2)(2s_{j0}^2 + t_{j1}^2 + s_{j1}^2 + t_{j2}^2 + s_{j2}^2)\Big]$$

$$+\sum_{j=2}^{j} \frac{x_j^3 - x_{j-1}^3}{6}\Big[2(c_0 + a_0\tau_1)s_{j0} + \mu c_1 t_{j1}$$

$$+(c_1 - a_1\tau_1)s_{j1} + (c_2 - b_1\tau_1)t_{j1}\Big]$$

$$+\sum_{j=2}^{j} \frac{x_j^4 - x_{j-1}^4}{4}(c_1 s_{j0}) \qquad (23b)$$

Equations 17a, 18a, 19a and 23b give the contributions to the thrust, rolling moment, pitching moment

and torque of a unit value for any one of the terms on the right side of equation 15b.

Note that if we call the various terms on the right side of equation 15b, $f_1, f_2 \ldots\ldots f_{11}$, and correspondingly denote the respective solutions from equations 17c, 18c, and 19c as $\left(\frac{C_T}{\sigma}\right)_1$, $\left(\frac{C_T}{\sigma}\right)_2 \ldots$ $\left(\frac{C_T}{\sigma}\right)_{11}$ etc., then because of the linearity of these equations in s's and t's, the total thrust and moments may be written as:

$$\frac{C_T}{\sigma} = \left(\frac{C_T}{\sigma}\right)_1 f_1 + \left(\frac{C_T}{\sigma}\right)_2 f_2 + \ldots\ldots\ldots\ldots \left(\frac{C_T}{\sigma}\right)_{11} f_{11} \quad (24)$$

$$\frac{C_M}{\sigma} = \left(\frac{C_M}{\sigma}\right)_1 f_1 + \left(\frac{C_M}{\sigma}\right)_2 f_2 + \ldots\ldots\ldots\ldots \left(\frac{C_M}{\sigma}\right)_{11} f_{11} \quad (25)$$

$$\frac{C_Y}{\sigma} = \left(\frac{C_Y}{\sigma}\right)_1 f_1 + \left(\frac{C_Y}{\sigma}\right)_2 f_2 + \ldots\ldots\ldots\ldots \left(\frac{C_Y}{\sigma}\right)_{11} f_{11} \quad (26)$$

If we regard $\frac{C_T}{\sigma}$, $\frac{C_M}{\sigma}$, and $\frac{C_Y}{\sigma}$, totals as known from equilibrium conditions, equations 24, 25 and 26 can be regarded as simultaneous equations linear in the variables contained in the f's (α_0, α, α_1, etc.). Any of the rotor forces or moments which involve blade lift (such as the torque and H force) are non-linear in s's and t's (see equation 23b) and the simple summation is not permissable.

PRELIMINARY INVESTIGATION OF THE FIXED ROTOR

In order to check the applicability of the theory and the

assumptions upon which it is based, it was decided to make a detailed analysis of the rigid non-flapping rotor, for in this case the equations take a comparatively simple form.

Equation 12 maintains the same form as in the general case:

$$\frac{\ddot{}}{a}\left(\frac{\Gamma_{ni}}{c\Omega R}\right) + \frac{c}{R}\sum L_{\underline{n}jnj}\left(\frac{\Gamma_{nj}}{c\Omega R}\right) = \frac{V_{ni}\alpha_{ni}}{\Omega R} \qquad (12)$$

but equation, 15b, for the right hand side of this equation simplifies to the form

$$\frac{V_{ni}\alpha_{ni}}{\Omega R} = \left\{\frac{\mu}{2}(c_2 - 2x) + x\theta_0 + x^2(c_1)\right\} + x(c_1)\cos\Psi$$

$$+\mu\theta_0\sin\Psi + x(c_2 + \mu\theta_1)\sin\Psi$$

$$-\frac{\mu}{2}(c_2)\cos 2\Psi + \frac{\mu}{2}(c_1)\sin 2\Psi \qquad (15c)$$

The 24 simultaneous equations represented by equation 12 were set up for unit values of each of the eight c's of terms in equation 15c and submitted to the Mathematical Computing Service of Brooklyn, N. Y. for solution. The results of this computation, along with the harmonic coefficients obtained from them are given in Tables I and II.

Knowing the harmonic coefficients, the equations for the thrust, pitching moment, rolling moment and torque were obtained.

$$\frac{C_T}{\sigma} = \sum_{j=n}^{\infty}\frac{x_j^2 - x_{j-1}^2}{2}\,a_{jo} + \sum_{j=n}^{\infty}\frac{x_j - x_{j-1}}{2}\,\mu\,a_{j1} \qquad (17c)$$

$$-\frac{C_T}{\sigma} = \sum_{j=a}^{d} \frac{x_j^3 - x_{j-1}^3}{6} s_{j1} + \sum_{j=a}^{d} \frac{x_j^2 - x_{j-1}^2}{8} \mu t_{j2} \tag{18c}$$

$$-\frac{C_L}{\sigma} = \sum_{j=a}^{d} \frac{x_j^3 - x_{j-1}^3}{6} t_{j1} + \sum_{j=a}^{d} \frac{x_j^2 - x_{j-1}^2}{8} \mu (2s_{jo} - s_{j2}) \tag{19c}$$

$$-\frac{C_Q}{\sigma} = \frac{\delta_0}{8}(1+\mu^2) + \sum_{j=a}^{d} \frac{x_j^2 - x_{j-1}^2}{8} \Big[2\mu t_{j1}(\theta_o) + \mu t_{j2}(c_1)$$

$$+ \mu(2s_{jo} - s_{j2})c_2 - \frac{4}{a^2}(a - \delta_2)(2s_{jo}^2 + t_{j1}^2 + s_{j1}^2 + t_{j2}^2 + s_{j2}^2) \Big]$$

$$+ \sum_{j=a}^{d} \frac{x_j^3 - x_{j-1}^3}{6} \Big[2(\theta_o)\, s_{jo} + \mu\theta_1\, t_{j1} + (c_2)\, t_{j1} + (c_1)\, s_{j1} \Big]$$

$$+ \sum_{j=a}^{d} \frac{x_j^4 - x_{j-1}^4}{4} (\theta_1\, s_{jo}) \tag{23c}$$

Upon substitution of the values from Table II, equations 17c, 18c and 19c become:

$$a_{11}\theta_o + a_{12}\alpha + a_{13}c_1 + a_{14}c_2 = A_1 \tag{17[illegible]}$$

where:

$$a_{11} = .7171 - .006901\mu - .[illegible]\mu^2$$

$$a_{12} = -1.[illegible]\mu + .003[illegible]\mu^2$$

$$a_{13} = .02615 + .[illegible]\mu + .005537\mu^2$$

$$a_{14} = -.00[illegible] + 1.07176\mu - .001856\mu^2$$

$$A_1 = \frac{C_T}{\sigma} - (.4847 + .001935\mu + .[illegible]61\mu^2)\theta_1$$

$$a_{21}a_o + a_{22}\alpha + a_{23}c_1 + a_{24}c_2 = A_2 \qquad (18d)$$

where:

$$a_{21} = -.01076 - .04442\mu - .05927\mu^2$$

$$a_{22} = +.02100\mu - .0006877\mu^2$$

$$a_{23} = +.2809 - .01418\mu + .1347\mu^2$$

$$a_{24} = -.01653 - .02901\mu - .01349\mu^2$$

$$A_2 = -\frac{C_m}{\sigma} - (.01280 - .01658\mu - .01373\mu^2)a_1$$

$$a_{31}a_o + a_{32}\alpha + a_{33}c_1 + a_{34}c_2 = A_1 \qquad (19d)$$

where:

$$a_{31} = -.0002717 + .6590\mu - .0004[illegible]\mu^2$$

$$a_{32} = +.0008425\mu - .6112\mu^2$$

$$a_{33} = +.01368 + .02339\mu + .008283\mu^2$$

$$a_{34} = +.2336 - .003599\mu + .4356\mu^2$$

$$A_3 = -\frac{C_l}{\sigma} - (.0005604 + .4439\mu - .01529\mu^2)a_1$$

This set of three equations in four unknowns may be solved in terms of a_o, if a_o is treated as a known quantity. Thus the equations may be rewritten as:

$$a_{12}\alpha + a_{13}c_1 + a_{14}c_2 = A_1 - a_{11}a_o$$

$$a_{22}\alpha + a_{23}c_1 + a_{24}c_2 = A_2 - a_{21}a_o$$

$$a_{32}\alpha + a_{33}c_1 + a_{34}c_2 = A_3 - a_{31}a_o$$

Thus, the solution for α becomes:

$$\alpha = \frac{\begin{vmatrix} (A_1 - a_{11}\theta_o) & a_{13} & a_{14} \\ (A_2 - a_{21}\theta_o) & a_{23} & a_{24} \\ (A_3 - a_{31}\theta_o & a_{33} & a_{34} \end{vmatrix}}{\begin{vmatrix} a_{12} & a_{13} & a_{14} \\ a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \end{vmatrix}} \tag{27a}$$

Expanding this determinant, the solution for α becomes:

$$\alpha = \frac{(A_1 - a_{11}\theta_o)\left[a_{23}a_{34} - a_{33}a_{24}\right] - (A_2 - a_{21}\theta_o)\left[a_{13}a_{34} - a_{33}a_{14}\right]}{a_{12}(a_{23}a_{34}-a_{33}a_{24}) - a_{22}(a_{13}a_{34}-a_{33}a_{14}) + a_{32}(a_{13}a_{24}-a_{23}a_{14})}$$

$$+ \frac{(A_3 - a_{31}\theta_o)\left[a_{13}a_{24} - a_{23}a_{14}\right]}{a_{12}(a_{23}a_{34}-a_{33}a_{24}) - a_{22}(a_{13}a_{34}-a_{33}a_{14}) + a_{32}(a_{13}a_{24}-a_{23}a_{14})} \tag{27b}$$

Therefore α may be expressed as:

$$\alpha = K_1 + K_1^1\,\theta_o \tag{27c}$$

where

$$K_1 = \frac{A_1(a_{23}a_{34}-a_{33}a_{24}) - A_2(a_{13}a_{34}-a_{33}a_{14}) + A_3(a_{13}a_{24}-a_{23}a_{14})}{\Delta}$$

$$K_2^1 = \frac{-a_{11}(a_{23}a_{34}-a_{33}a_{24}) + a_{21}(a_{13}a_{34}-a_{33}a_{14}) - a_{31}(a_{13}a_{24}-a_{23}a_{14})}{\Delta}$$

$$\Delta = a_{12}(a_{23}a_{34} - a_{33}a_{24}) - a_{22}(a_{13}a_{34} - a_{33}a_{14})$$
$$+ a_{32}(a_{13}a_{24} - a_{23}a_{14})$$

By an exactly similar process the expressions for c_1 and c_2 in terms of e_o may be obtained

$$c_1 = K_2 + K_2^1 e_o \tag{28}$$

$$c_2 = K_3 + K_3^1 e_o \tag{29}$$

where

$$K_2 = \frac{A_1(a_{32}a_{24} - a_{22}a_{34}) - A_2(a_{32}a_{14} - a_{12}a_{34}) + A_3(a_{22}a_{14} - a_{12}a_{24})}{\Delta}$$

$$K_2^1 = \frac{-a_{11}(a_{32}a_{24} - a_{22}a_{34}) + a_{21}(a_{32}a_{14} - a_{12}a_{34}) - a_{31}(a_{22}a_{14} - a_{12}a_{24}}{\Delta}$$

$$K_3 = \frac{A_1(a_{33}a_{22} - a_{32}a_{23}) - A_2(a_{33}a_{12} - a_{32}a_{13}) + A_3(a_{23}a_{12} - a_{22}a_{13}}{\Delta}$$

$$K_3^1 = \frac{-a_{11}(a_{22}a_{33} - a_{32}a_{23}) + a_{21}(a_{32}a_{13} - a_{33}a_{12}) - a_{31}(a_{23}a_{13} - a_{22}a_{13})}{\Delta}$$

and Δ is given by equation 27c.

Substituting the values from Table II into equations 17c, 18c and 19c, the expressions for the K and K^1 coefficients become:

$$\begin{aligned}\Delta K_1 = {} & (.06304 - .003120\mu + .1535\mu^2 - .005973\mu^3 + .05880\mu^4)\frac{C_T}{\sigma} \\ & + (.005879 - .007540\mu - .01248\mu^2 + .004930\mu^3 + .002427\mu^4)\frac{C_m}{\sigma} \\ & + (-.0002197 + .3024\mu - .01467\mu^2 + .1450\mu^3 - .0001754\mu^4)\frac{C_L}{\sigma} \\ & + (-.03048 + .001265\mu + .03030\mu^2 - .002470\mu^3 - .03555\mu^4 \\ & + .002562\mu^5 - .01744\mu^6)\,\theta_1 \end{aligned} \tag{20}$$

$$\begin{aligned}\Delta K_1^1 = {} & -.04527 + .002280\mu + .0318\mu^2 - .003[illegible]\mu^3 - .0[illegible]\mu^4 \\ & + .003994\mu^5 - .05346\mu^6 \end{aligned} \tag{21}$$

$$\begin{aligned}\Delta K_2 = {} & (-.004730\mu + .01029\mu^2 + .008520\mu^3 + .008544\mu^4)\frac{C_T}{\sigma} \\ & + (.[illegible]\mu - .001614\mu^2 - .1092\mu^3 - .0001866\mu^4)\frac{C_m}{\sigma} \\ & + (.02176\mu + .01369\mu^2 + .01759\mu^3 - .0000422\mu^4)\frac{C_L}{\sigma} \\ & + (.005899\mu - .0004206\mu^2 - .0011[illegible]\mu^3 + .0006643\mu^4 \\ & - .002536\mu^5 - .003980\mu^6)\,\theta_1 \end{aligned} \tag{22}$$

$$\begin{aligned}\Delta K_2^1 = {} & +.0002[illegible]\mu - .006182\mu^2 - .008[illegible]\mu^3 + .001[illegible]\mu^4 - .0[illegible]\mu^5 \\ & - .007[illegible]\mu^6 \end{aligned} \tag{23}$$

$$\Delta K_3 = (.0000509\mu + .1722\mu^2 - .008622\mu^3 + .08024\mu^4)\frac{C_T}{\sigma}$$
$$+ (-.01709\mu - .01331\mu^2 + .009018\mu^3 + .003409\mu^4)\frac{C_m}{\sigma}$$
$$+ (.3525\mu - .01707\mu^2 + .1689\mu^3 - .0004114\mu^4)\frac{C_L}{\sigma}$$
$$+ (-.0000405\mu + .07313\mu^2 - .004031\mu^3 - .04505\mu^4$$
$$+ .003338\mu^5 - .03842\mu^6)\, a_1 \qquad (34)$$

$$\Delta K_3^1 = .0000692\mu + .1094\mu^2 - .006221\mu^3 - .1035\mu^4 + .003991\mu^5$$
$$-.07485\mu^6 \qquad (35)$$

where:

$$\Delta = -.07911\mu + .003874\mu^2 - .007311\mu^3 - .001252\mu^4 + .01488\mu^5$$
$$+ .0000728\mu^6 \qquad (36)$$

A simple form of the torque equation is lacking because it is not a linear equation, but involves squared terms and cross products. By substituting the values of Table II into equation 23c, the torque equation may be expressed as:

$$B_1 a_0^2 + B_2 a_0 + B_3 a_0\alpha + B_4 a_0 c_1 + B_5 a_0 c_2 + B_6\alpha^2 + B_7\alpha$$
$$+ B_8\alpha c_1 + B_9\alpha c_2 + B_{10} c_1^2 + B_{11} c_1 + B_{12} c_1 c_2 + B_{13} c_2^2$$
$$+ B_{14} c_1 + B_{15} = 0 \qquad (37)$$

where:

$$B_1 = (.1134 - .009444\mu + .1050\mu^2) + (.07313 + .0[illegible]\mu^2)\delta_2$$
$$B_2 = (.7887 - .009168\mu + .6234\mu^2)a_1 + \frac{\delta_1}{3}(1 + \mu + \mu^2 + \mu^3)$$
$$B_3 = (-.0[illegible]7\mu + .001377\mu^2)$$
$$B_4 = (.01076 - .03564\mu - .05623\mu^2)$$
$$B_5 = (-.001986 + 1.3585\mu - .01022\mu^2)$$

$$B_6 = (.2010\mu^2\delta_2 - 1.1255\mu^2)$$

$$B_7 = (-.6156\mu + .0008415\mu^2)\,e_1 - (\mu + \mu^3)\,\frac{\delta_0}{\sigma}$$

$$B_8 = (.02100\mu - .0006877\mu^2)$$

$$B_9 = (.0008415\mu + .5140\mu^2) - .2010\mu^2\delta_2$$

$$B_{10} = (.05054 - .01418\mu + .00727\mu^2) + (.04114 + .01017\mu^2)\delta_2$$

$$B_{11} = (.02138 - .01458\mu - .01201\mu^2)\,e_1 + \left(1 + \frac{\mu}{2} + \mu^2 + \frac{\mu^3}{2}\right)\frac{\delta_0}{\sigma}$$

$$B_{12} = (-.002912 - .005618\mu - .005205\mu^2)$$

$$B_{13} = (.07202 - .002599\mu - .03927\mu^2) + (.02707 + .07078\mu^2)\delta_2$$

$$B_{14} = (-.0008474 + .6504\mu - .002595\mu^2 + .05414\mu\delta_2)\,e_1 + (1 + \mu^2)\,\frac{\delta_0}{\sigma}$$

$$B_{15} = \left(\frac{\delta_0}{\sigma} + (1 + \mu + \mu^2 + \mu^3)\frac{e_1\delta_2}{\sigma}\right) + (.09019 - .0008474\mu + .0[illegible]\mu^2)\,e_1^2$$

$$+ (.03824 - .03707\mu^2)\,\delta_2 e_1^2$$

By substituting the values of α, e_1 and e_2 from equations 16c, 18 and 19 into the torque equation 37, it can be seen that a quadratic relation results.

$$Ae_0^2 + Be_0 + C = 0 \qquad (38)$$

where:

$$A = B_1 + (B_2 + B_6K_1^1 + B_8K_2^1 + B_9K_2^1)K_1^1 + (B_4 + B_{10}K_2^1 + B_{12}K_2^1)K_2^1 + (B_5 + B_{13}K_2^1)K_2^1$$

$$B = B_2 + B_7K_1^1 + B_{11}K_2^1 + B_{14}K_2^1 + (B_3 + 2B_6K_1^1 + B_8K_2^1 + B_9K_2^1)K_1$$

$$+ (B_4 + B_8K_1^1 + 2B_{10}K_2^1 + B_{12}K_2^1)K_2 + (B_5 + B_9K_1^1 + B_{13}K_2^1 + B_{12}K_2^1)K_2$$

$$C = B_{15} + (B_3K_1 + B_7 + B_8K_2 + B_9K_2)K_1 + (B_{10}K_2 + B_{11} + B_{12}K_2 + B_{13}K_2 + B_{14})K_2$$

Therefore the solution for a_0 becomes:

$$a_0 = \frac{-B \pm \sqrt{B^2 - 4Ac}}{2A} \tag{39}$$

Having obtained a_0 from equation 39, the values of α, c_1 and c_2 can be obtained by substituting the appropriate value of a_0, (only one root of equation 39 will have physical meaning), into equations 27c, 28 and 29.

Evaluation of the Circulation Distribution

The values of the variables a_0, α, c_1 and c_2 may be substituted into equation 15c, and the values of $\frac{\Gamma}{c\Omega R}$ in Table I which were tabulated for unit values of each of the eight terms may now be corrected.

Term No.	Correction Factor
1	$\mu/2(c_2 - 2\alpha)$
2	a_0
3	a_1
4	c_1
5	μa_0
6	$(c_2 + \mu a_1)$
7	$-\frac{\mu c_2}{2}$
8	$\frac{\mu c_2}{2}$

The final value of $\frac{\Gamma}{c\Omega R}$ at each pivotal point is the sum of the eight corrected contributions.

Evaluation of the Downwash Distribution

Knowing the circulation Γ at each pivotal point n_j, and the values of θ_0, α, c_1 and c_2, it is possible to determine the value of the downwash at each pivotal point by solving equation 9b.

$$\Gamma_{nj} = \frac{ac}{2} V_{nj}\, \alpha_{nj} - \frac{ac}{2} w_{nj} \tag{9b}$$

Dividing by $c\Omega R$ and solving for the downwash, the equation becomes,

$$\frac{w_{nj}}{\Omega R} = \frac{V_{nj}\, \alpha_{nj}}{\Omega R} - \frac{2}{a}\left(\frac{\Gamma_{nj}}{c\Omega R}\right)$$

By substituting the expression for $\frac{V_{nj}\, \alpha_{nj}}{\Omega R}$ from equation 15c, the final expression for the downwash becomes:

$$w_{nj} = \Omega R \Big\{ \frac{\mu}{2}(c_2 - 2\alpha) + x_j\, \theta_0 + x_j^2\, \theta_1$$

$$+ x_j\, c_1 \cos\Psi_n + \left[\mu\theta_0 + x_j(c_2 + \mu\theta_1)\right] \sin\Psi_n$$

$$- \frac{c_2\mu}{2} \cos^2\Psi_n + \frac{c_1\mu}{2} \sin 2\Psi_n - \frac{2}{a}\left(\frac{\Gamma_{nj}}{c\Omega R}\right) \Big\}$$

where $\left(\frac{\Gamma_{nj}}{c\Omega R}\right)$ may be obtained from the corrected values of Table I.

Evaluation of the Air Load Distribution

The lift distribution may be obtained from the basic equation relating lift to circulation

$$l = \rho V \Gamma \tag{7}$$

The value of $\frac{\Gamma}{c\Omega R}$ around each ring of "pivotal" points is expressed by the relation

$$\frac{\Gamma}{c\Omega R} = \sum_{1}^{8} s_{jo} + s_{j1}\cos\Psi + t_{j1}\sin\Psi + s_{j2}\cos 2\Psi + t_{j2}\sin 2\Psi$$

where the symbol $\sum_{1}^{8}$ indicates that the values of s_{jo}, s_{jn} and t_{jn} used in the equation are the summations of the corrected values of s_{jo}, s_{jn} and t_{jn} for each of the 8 terms tabulated in Table I. Note that there are four such equations, one for each ring of pivotal points, a, b, c, and d.

Thus the lift at each row of pivotal points may be expressed as:

$$\Gamma = c\Omega R\left\{(x_j + \mu\sin\Psi)\sum_{1}^{8} s_{jo} + s_{j1}\cos\Psi + t_{j1}\sin\Psi + s_{j2}\cos 2\Psi + t_{j2}\sin 2\Psi\right\}$$

Note that all of the equations presented here apply to the general case of a rigid rotor, only so long as the assumptions of $a = 6.1/\text{rad}$, and $\frac{c}{R} = \frac{1}{15}$ do not introduce appreciable error. In most normal configurations these assumptions cause little error and equations 27 through 39 may be considered as a general solution.

[illegible]

It is the object of this section to demonstrate how equations 27 through 39 may be used, and to check the method by comparing [illegible] obtained by use of these equations with the [illegible] by assuming a constant distribution of [illegible].

The [illegible] characteristics of the helicopter assumed for this example are as follows:

Gross weight W	2700 lbs.
Rotor radius, R	19 ft.
Number of blades, b	2
Ratio of average blade chord to rotor radius, $\frac{c}{R}$	$\frac{1}{15}$
Blade twist, θ_1	-3°
Blade aerodynamic characteristics	
a	5.6/rad.
δ_0	.011
δ_2	.29

The flight condition analyzed is described by the data below:

B.H.P. delivered to the rotor	135 HP
Rotor r.p.m.	220
Air density, ρ, slugs/ft.^{3}	.00238
Tip speed ratio, μ	.35

Evaluation of Constants

$$\sigma = \frac{bc}{\pi R} = .06366$$

$$C_T = \frac{T}{\rho(\Omega R)^2 \pi R^2} = .00522$$

$$Q = \frac{550\ BHP}{\Omega} = -3223 \text{ ft lbs}$$

$$C_Q = \frac{Q}{\rho \pi (\Omega R)^2 R} = -.000328$$

$$\frac{C_T}{\sigma} = .081998$$

$$\frac{C_Q}{\sigma} = .005152$$

As only the rotor is being considered C_L and C_m are assumed to be equal to zero.

Solution With Constant Induced Velocity

From Reference 1, the equations that will be used in this solution are:

Thrust Equation:

$$T = \tfrac{1}{2} bc\rho a\Omega^2 R^3 \Big[\tfrac{1}{2}\lambda_v (B^2 + \tfrac{1}{2}\mu^2) + \theta_0 (1/3 B^3 + \tfrac{1}{2}\mu^2 B) + \theta_1 \left(\frac{B^4}{4} + \frac{\mu^2 B^2}{4} \right) + c_2 \left(\frac{\mu B^2}{2} - \frac{\mu^2}{8} \right) \Big]$$

where $B = 1 - \dfrac{\sqrt{2C_T}}{b}$

Rolling Moment Equation:

$$\left(+ 2c_2 + \frac{4\lambda_v}{\mu} \right) \frac{\mu^2}{8} + \frac{2\theta_0 \mu B}{3} + \frac{c_2 B^2}{4} + \frac{\mu \theta_1 B^2}{4} = 0$$

Pitching Moment Equation:

$$\frac{c_1 \mu^2}{8} - \frac{\beta_b \mu B}{3} + \frac{c_1 B^2}{4} =$$

Torque Equation:

$$Q = \tfrac{1}{2} \rho abc\Omega^2 R^4 \Big\{ \lambda_v^2 \left[\frac{B^2}{2} - \frac{\delta_2}{2a} - \frac{\mu^2}{4} \right] + \lambda_v \Big[-\frac{1}{a}\left(\frac{\delta_1}{3} + \frac{2}{3}\delta_2 \theta_0 + \frac{\delta_1 \theta_1}{2} \right) + \frac{B^3 \theta_0}{3} + \frac{B^4 \theta_1}{4} + \frac{\mu B^2}{4} c_2 \Big] - \frac{1}{a} \left[\frac{\delta_0}{4} + \frac{\delta_1 \theta_0}{4} + \delta_2 \left(\frac{\theta_0^2}{4} + \frac{2}{5}\theta_1 \theta_0 \right) \right] + \frac{\mu B^2}{4} \beta_b^2 - \frac{\mu B^3 \beta_b c_1}{6} \Big\}$$

Upon solving these equations the following results were obtained:

$$\theta_o = 12.85^\circ$$

$$c_1 = 0$$

$$c_2 = -.09480$$

$$\alpha = -13.3^\circ$$

Solution With Variable Induced Velocity

a) Evaluation of coefficients

1. From equations 30 through 36:

$$\Delta = -.01964$$

$$K_0 = -.2764$$

$$K_1' = 2.1950$$

$$K_2 = .005262$$

$$K_2' = .02150$$

$$K_3 = -.03427$$

$$K_3' = -.3229$$

2. From equation 37:

$$B_1 = .1478$$

$$B_2 = -.04120$$

$$B_3 = -.2266$$

$$B_4 = -.001664$$

$$B_5 = .2270$$

$B_6 = -.06544$

$B_7 = .007690$

$B_8 = .005230$

$B_9 = .08745$

$B_{10} = .06522$

$B_{11} = .0003260$

$B_{12} = -.004642$

$B_{13} = .08611$

$B_{14} = -.007394$

$B_{15} = -.004945$

3. From equation 28:

$A = -.7460$

$B = .1554$

$C = -.01641$

Evaluation of θ_0

From equation 30, the equation for θ_0 may be written as:

$$\theta_0 = \frac{-.1554 \pm \sqrt{(.1554)^2 - 4(-.7460)(-.01641)}}{-2(.7460)}$$

$$\therefore \theta_0 = .1050 \pm .05280$$

DISCUSSION OF RESULTS

The imaginary value obtained for the angle θ_c clearly demonstrates that there is some error in the solution. The major contribution to this result seems to come from the large negative value of the coefficient A of equation 38.

Upon examining the results of the sample calculations, an investigation was started to determine where the difficulty was arising. At the onset of the project, difficulty was encountered in locating the pivotal points and the erroneous results obtained caused a re-examination of these points. In Reference 3, similar troubles were encountered and solved by a trial and error method, but this method appears to be unsatisfactory for the present problem due to the expense involved in working each trial solution.

As stated previously, if a pivotal point is placed too close to a vortex line, the value of the circulation or downwash calculated at this point cannot be assumed to represent the average downwash in the zone. By examining Table I, it will be seen that at pivotal points 1b, 1c, 6b and 6c the values of $\frac{\Gamma}{c\Omega R}$ are somewhat erratic. Since the downwash at all the other points is a function of the downwash at these points, it is to be expected that a slight change in all the other values would occur, if the values at 1b, 1c, 6b and 6c were corrected; this change, of course, being more severe at those pivotal points in the immediate vicinity, i.e. 1a, 1d, 6a and 6b.

This change in the other values makes it practically impos-

sible to predict how a correction of the faulty points will affect the final results, without reworking the entire computations. Some estimation may be obtained, however, by plotting the values of $\frac{\Gamma}{c\,\Omega\,R}$ as shown in Figs. 5, 6, 6 and 8, and recalculating the harmonic coefficients. This is admittedly a very crude approximation, but is is about the only method of estimation available. This procedure was followed for terms 2 and 5, yielding the following results:

Term 2

Harmonic Coefficients	Former values	Changed values
s_{b_0}	1.126	.9147
s_{b_1}	.3872	[illegible]
s_{b_2}	.2208	[illegible]
t_{b_1}	.03133	.014976
t_{b_2}	.03515	-.00[illegible]
s_{c_0}	1.215	1.[illegible]
s_{c_1}	-.3705	.0[illegible]
s_{c_2}	-.[illegible]891	.1[illegible]
t_{c_1}	.[illegible]15[illegible]	.[illegible]
t_{c_2}	-.00[illegible]	.0[illegible]

Term 5

Harmonic Coefficients	Former values	Changed values
s_{b_0}	.0364[illegible]	-.[illegible]
s_{b_1}	-.[illegible]	-.4[illegible]
s_{b_2}	.06383	-.[illegible]

Harmonic Coefficients	Former values	Changed values
t_{b_1}	2.330	1.6931
t_{b_2}	.1407	-.7516
s_{c_0}	-.04902	-.05433
s_{c_1}	-.3200	-.3388
s_{c_2}	-.01622	-.02054
t_{c_1}	1.857	1.9141
t_{c_2}	-.7281	-.6467

The above changes in the harmonic coefficient for just two of the eight terms of equation 15c was sufficient to change several of the K and T coefficients in equation 38 in such a manner as to indicate that perhaps the A coefficient in equation could be made to change sign if all B sets were corrected.

On the basis of these calculations, an investigation was started to determine the best method of surmounting the difficulty of obtaining the average downwash in each zone. In zones 2, 3, 4 and 5 the problem is not particularly acute and the values of the downwash computed at the pivotal points is probably reasonably close to the average value in the zone. Near the trailing edge of the rotor, however, due to the increased number of vortices passing through each zone, the situation becomes critical.

A rigorous mathematical treatment of this problem becomes too complex for practical solution, so the possibilities are reduced to some form of an emperical approach. Two possibilities which were investigated were firstly, the solving of several

different rotors with different zonal disivions and averaging the results; and secondly, the computing of several different pivotal points within the same zone and the averaging of the different values of downwash to obtain the average downwash in the zone.

The results of both methods should be very nearly the same, providing the area of the zones being considered is not too great. As the first method would require solving each set of simultaneous equations several times, it was discarded as being impractically expensive.

The second method, however, could be applied without too great an increase in the computational expense. In detail this method would be worked as follows:

Fig. 9 shows a typical zone near the trailing edge of the rotor with several vorcitices passing through it. Instead of considering but one pivotal point, it will be assumed that the downwash will be computed at the points labeled 1, 2 and 3. Equation 2 for the downwash has become:

$$\Delta w_{nj1} = \frac{\Delta \Gamma_{NJ}}{4 \pi R} K_{njNJ_1}$$

$$\Delta w_{nj2} = \frac{\Delta \Gamma_{NJ}}{4 \pi R} K_{njNJ_2}$$

$$\Delta w_{nj3} = \frac{\Delta \Gamma_{NJ}}{4 \pi R} K_{njNJ_3}$$

The average downwash may thus be expressed as:

$$\Delta w_{nj} = \frac{\Delta w_{nj1} + \Delta w_{nj2} + \Delta w_{nj3}}{3}$$

or

$$\Delta w_{nj} = \frac{\Delta \Gamma_{nj}}{2n\pi R} K'_{njNJ}$$

where $K'_{njNJ} = K_{njNJ_1} + K_{njNJ_2} + K_{njNJ_3}$

Upon determining the value of K'_{njNJ} the rest of the method remains unchanged.

Due to the lack of funds and time remaining for the completion of the contract, further investigations were discontinued.

CONCLUSIONS

It must be concluded that the method as investigated in this report has several major faults. The relatively small number of five at points considered makes it extremely difficult to plot curves with any degree of accuracy, while the geometry of the rotor disc makes it difficult to locate the control points so the value of downwash or circulation computed at these points represents the average values for the zone.

Without further investigation, it is impossible to say whether or not these two problems may be satisfactorily resolved, for an increase in the zonal divisions of the rotor disc to obtain more points greatly increases the complexity of the rotor geometry and hence the problem of determining the average value within any zone.

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2. Falkner, V. M.: The Calculation of Aerodynamic Loading on Surfaces of Any Shape. R. & M. No. 1910, 1943

TABLE I

Values of $\left(\frac{\Gamma}{c\Omega R}\right)$ for Unit Values of the

Terms of Equation 15c

Pivotal Point	Term 1	Term 2	Term 3	Term 4
	$\frac{\mu}{2}(c_2 - c_1)$	$x\theta_0$	$x^2\theta_0$	$x(c_1)\cos\psi$
1a	2.6010	.4564	.1369	.1497
1b	2.5339	1.6429	.6973	.8795
1c	2.5341	.8686	.6164	1.2648
1d	2.4070	2.0230	1.8447	.0152
2a	2.6012	.4820	.1496	1.3733
2b	2.5699	.8816	.3619	.1374
2c	2.4821	1.2963	.7279	.1227
2d	2.1305	1.4655	1.0540	.3406
3a	2.6158	.4140	.1127	.1381
3b	2.6420	.9088	.3148	-.6929
3c	2.6651	.4741	.8060	-1.2229
3d	2.6942	2.0728	1.5820	-1.7646
4a	2.6258	.3755	.0908	.2737
4b	2.6552	.9075	.3083	-.8317
4c	2.6812	1.4837	.8092	-1.2495
4d	2.7105	2.0869	1.5904	-1.8666
5a	2.6194	.4241	.1158	.2229
5b	2.5877	.8262	.3239	-.1306
5c	2.4968	1.2758	.7091	-.0394
5d	2.1409	1.4612	1.0429	.1824
6a	2.5922	.4865	.1540	.2472
6b	2.5368	1.5781	.6641	.8054
6c	2.5255	.8903	.6224	1.2549
6d	2.3951	2.0875	1.8610	1.8586

TABLE I (cont'd.)

Values of $\left(\frac{\Gamma}{c\Omega R}\right)$ for Unit Values of the

Terms of Equation 15c

Pivotal Point	Term 5	Term 6	Term 7	Term 8
	$\mu\theta_0 \sin\Psi$	$x(c_2+\mu\theta_1)\sin\Psi$	$-\frac{\mu c_2}{2}\cos 2\Psi$	$\frac{\mu}{2}c_1 \sin 2\Psi$
1a	1.2468	-.1766	2.0606	1.5320
1b	1.3079	.8347	1.6051	2.0316
1c	-.5828	-.1275	1.7718	2.4101
1d	.2152	.7469	1.7064	1.3801
2a	1.7733	.1901	-2.3498	.9268
2b	2.2194	.7770	-2.6926	.4099
2c	2.3550	1.2792	-2.7029	.3028
2d	2.1715	1.5815	-2.6202	.3870
3a	1.7854	.3326	.3847	-2.6158
3b	1.6098	.5383	1.0928	-2.5346
3c	1.5309	.7188	1.2344	-2.4927
3d	1.4815	1.1025	1.3330	-2.4661
4a	.1042	.0462	2.0923	1.5464
4b	.8632	-.2167	1.6989	2.1616
4c	1.0681	-.6253	1.5970	2.2687
4d	1.1813	-.9520	1.5305	2.3353
5a	-2.1043	-.2623	-2.4093	.9573
5b	-2.3009	-.2652	-2.6274	.3344
5c	-2.4128	-1.3918	-2.6786	.1257
5d	-2.2307	-1.6047	-2.6174	.1207
6a	.2052	-.0311	.3731	-2.6392
6b	-1.7544	-1.0728	1.0779	-2.7798
6c	-.1152	-.0918	1.3052	-2.5188
6d	-.4649	-.3067	1.3790	-2.4044

TABLE II

Harmonic Coefficients

Harmonic Coefficients	Term 1	Term 2	Term 3	Term 4
s_{ao}	2.609	.4397	.1266	-.02736
s_{a1}	-.01308	.04227	.02408	.2229
t_{a1}	-.006172	.01980	.01151	.1004
s_{a2}	-.0007288	-.01101	-.005043	.03911
t_{a2}	.004239	-.01640	-.009316	-.05562
s_{bo}	2.588	1.126	.4451	.02784
s_{b1}	-.06246	.3872	.2035	.8847
t_{b1}	-.008225	.03133	.01843	.1192
s_{b2}	.007262	.2208	.08449	.02020
t_{b2}	.002458	.01515	.006378	-.01547
s_{co}	2.564	1.215	.7152	.01643
s_{c1}	-.07905	-.3305	-.1038	1.404
t_{c1}	-.005863	.001534	.004513	.09494
s_{c2}	.06178	-.05851	-.002774	-.04897
t_{c2}	.005875	-.002888	-.0006520	-.02787
s_{do}	2.413	1.865	1.496	.00977
s_{d1}	-.1661	-.01636	.1470	2.021
t_{d1}	-.004[illegible]	-.009551	-.0004938	.06736
s_{d2}	.2294	.3320	.3700	-.1366
t_{d2}	.006720	-.01012	-.001885	-.02847

TABLE II (cont'd.)

Harmonic Coefficients

Harmonic Coefficients	Term 5	Term 6	Term 7	Term 8
s_{a0}	-.01411	-.0001937	.02536	-.04874
s_{a1}	-.8631	-.1341	-.01206	-.01043
t_{a1}	1.1758	.1505	.01565	-.008280
s_{a2}	.2915	.02971	1.989	-.8174
t_{a2}	-.7002	-.07922	.8106	1.989
s_{b0}	.03643	-.0006127	.02529	-.06447
s_{b1}	-.3289	-.1297	-.02942	-.1034
t_{b1}	2.320	.9304	-.03397	.04555
s_{b2}	.06383	-.005389	2.221	-.3570
t_{b2}	.1407	.2039	.2710	2.270
s_{c0}	-.04002	-.006402	0.8781	.05098
s_{c1}	-.3200	-.1136	.06771	.08417
t_{c1}	1.257	1.043	.008798	.04963
s_{c2}	-.01622	-.00005966	2.298	-.1350
t_{c2}	-.7321	-.3533	.1980	2.268
s_{d0}	-.03477	-.005747	.1135	-.1491
s_{d1}	-.2067	-.08500	.06943	-.2463
t_{d1}	1.901	1.605	.01496	.001683
s_{d2}	-.004306	.004844	2.260	-.2310
t_{d2}	-.5211	-.09617	.1335	2.050

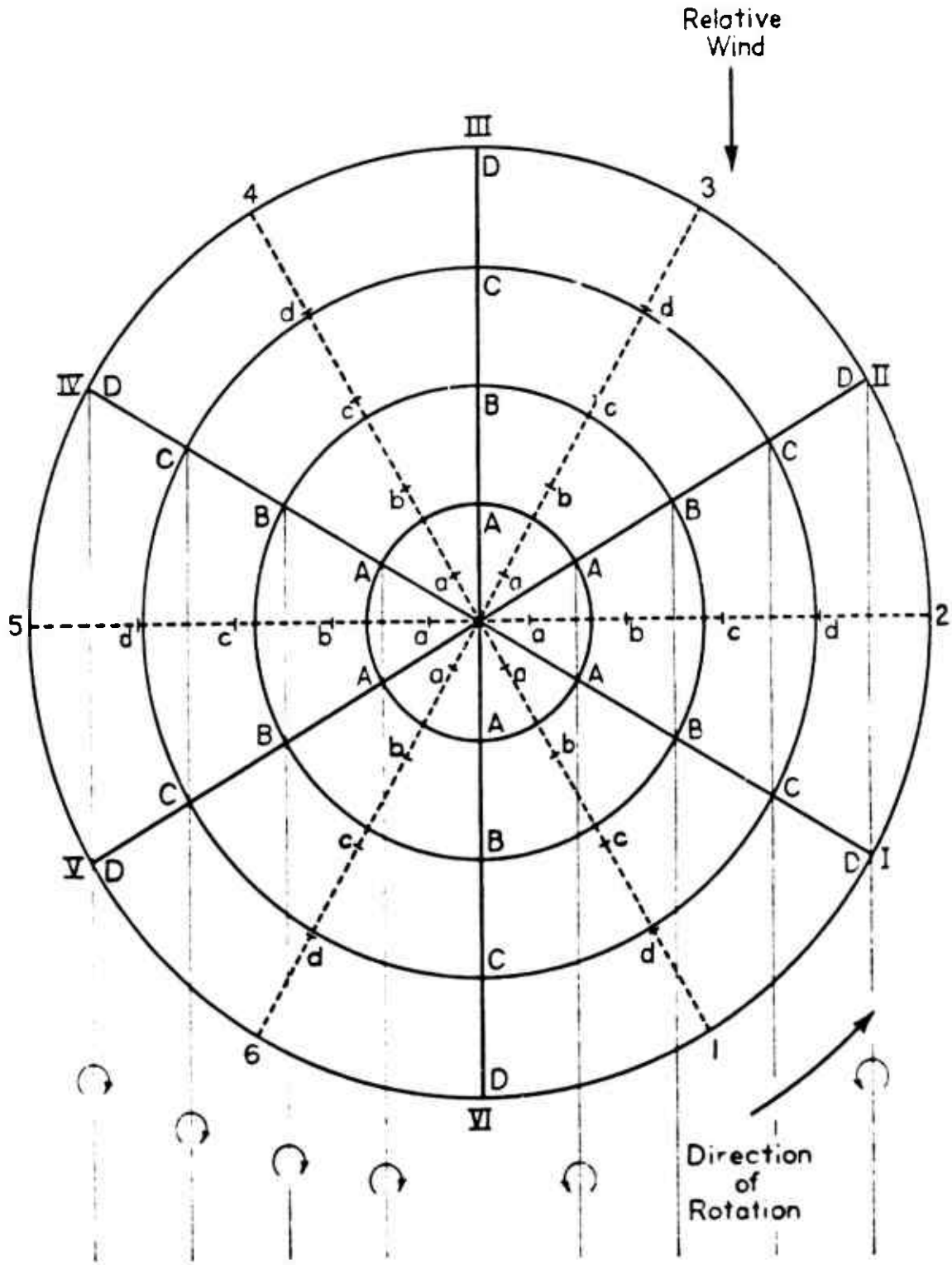

Relative Wind
III
D
4
3
C
d
d
IV
D
D
II
c
B
c
C
C
b
b
B
B
A
A
A
a
a
5
d
c
b
a
a
b
c
d
2
A
a
a
A
A
B
B
b
b
C
C
c
B
c
V
D
D
I
d
C
d
6
1
D
VI
Direction of Rotation

Fig. 1

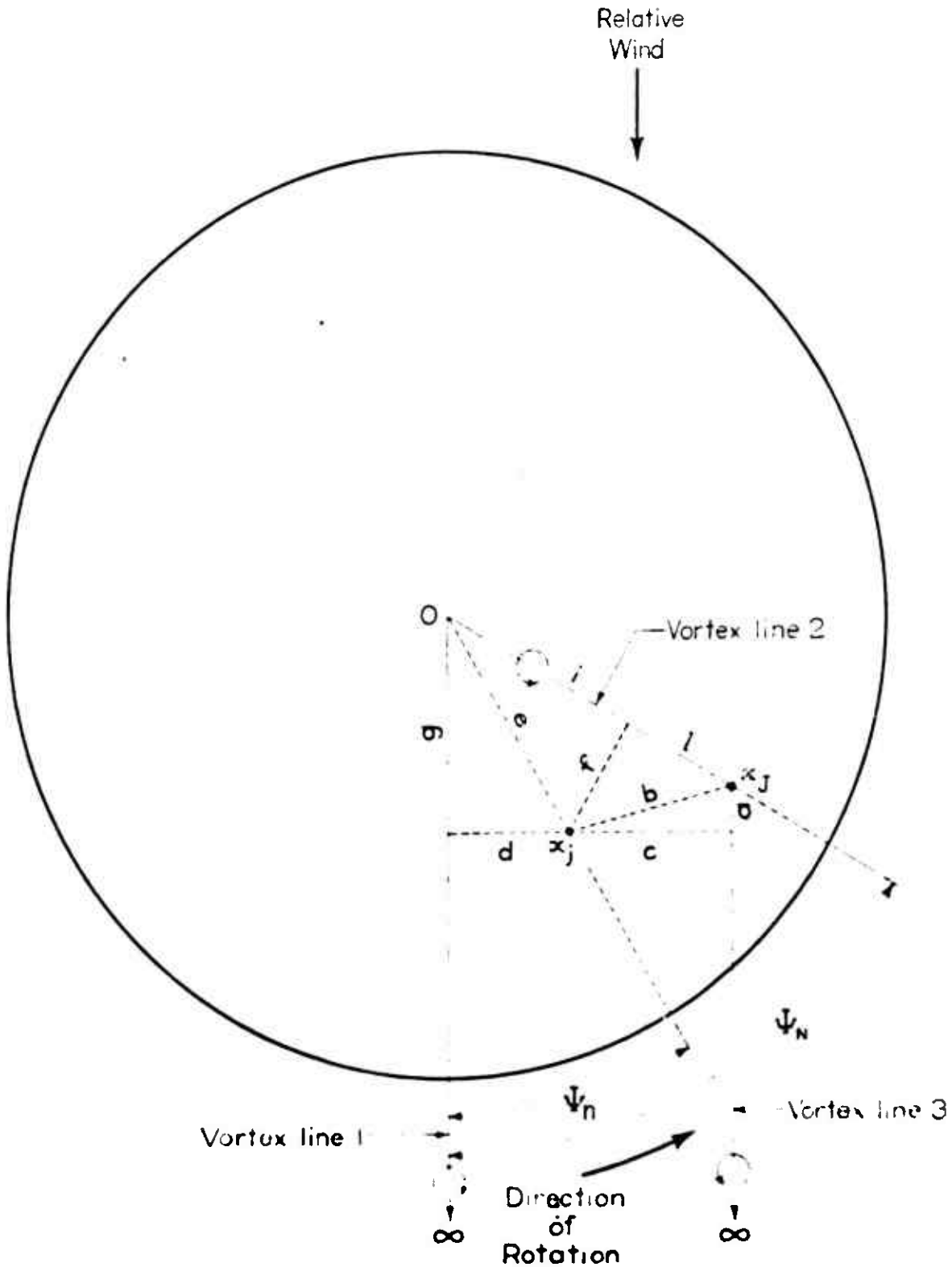

Relative
Wind
O
Vortex line 2
g
e
f
l
b
x_J
d
x_j
c
Ψ_N
Ψ_n
Vortex line 1
Vortex line 3
Direction
of
Rotation
∞
∞

Fig. 2

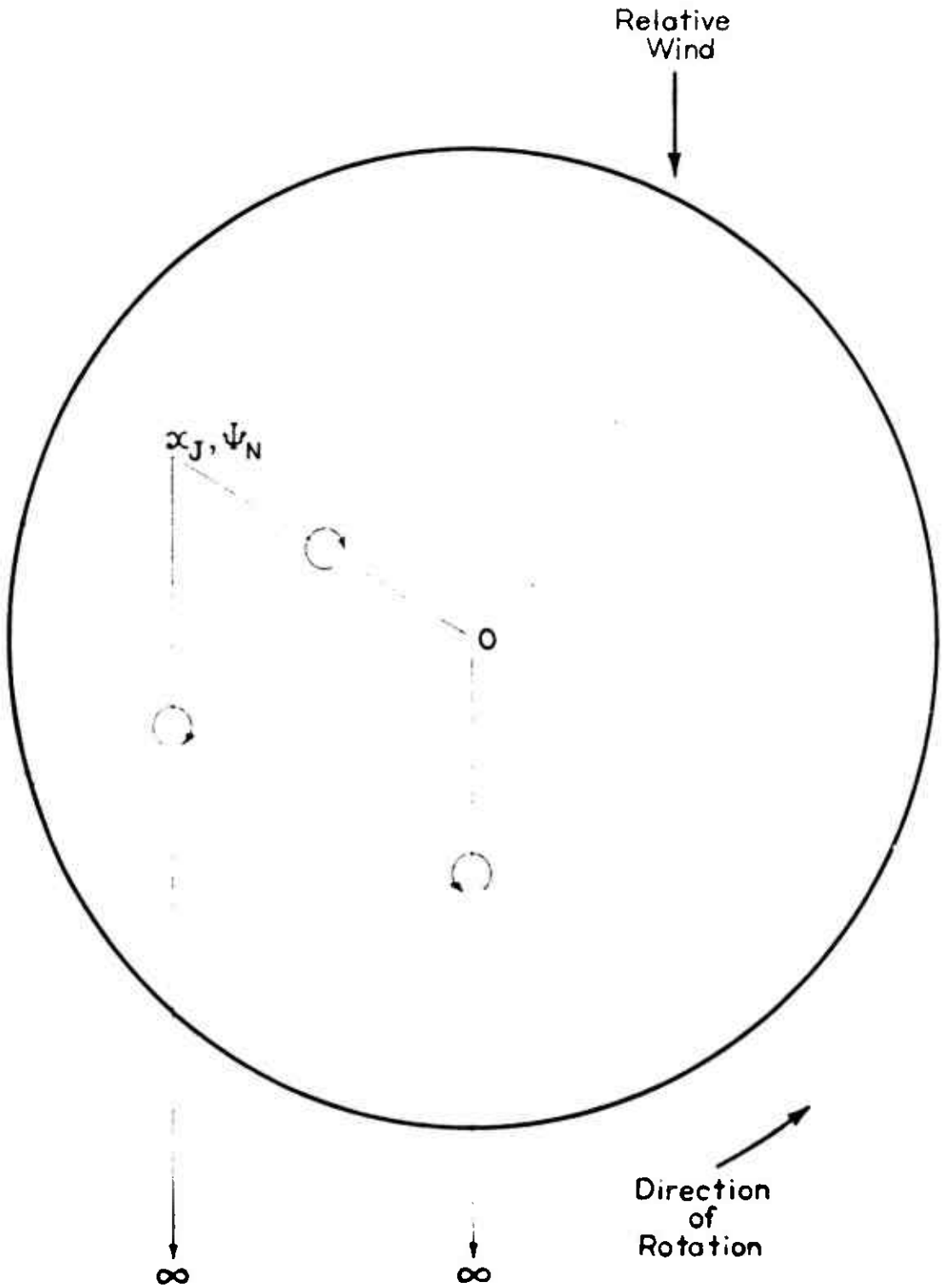


Fig. 3

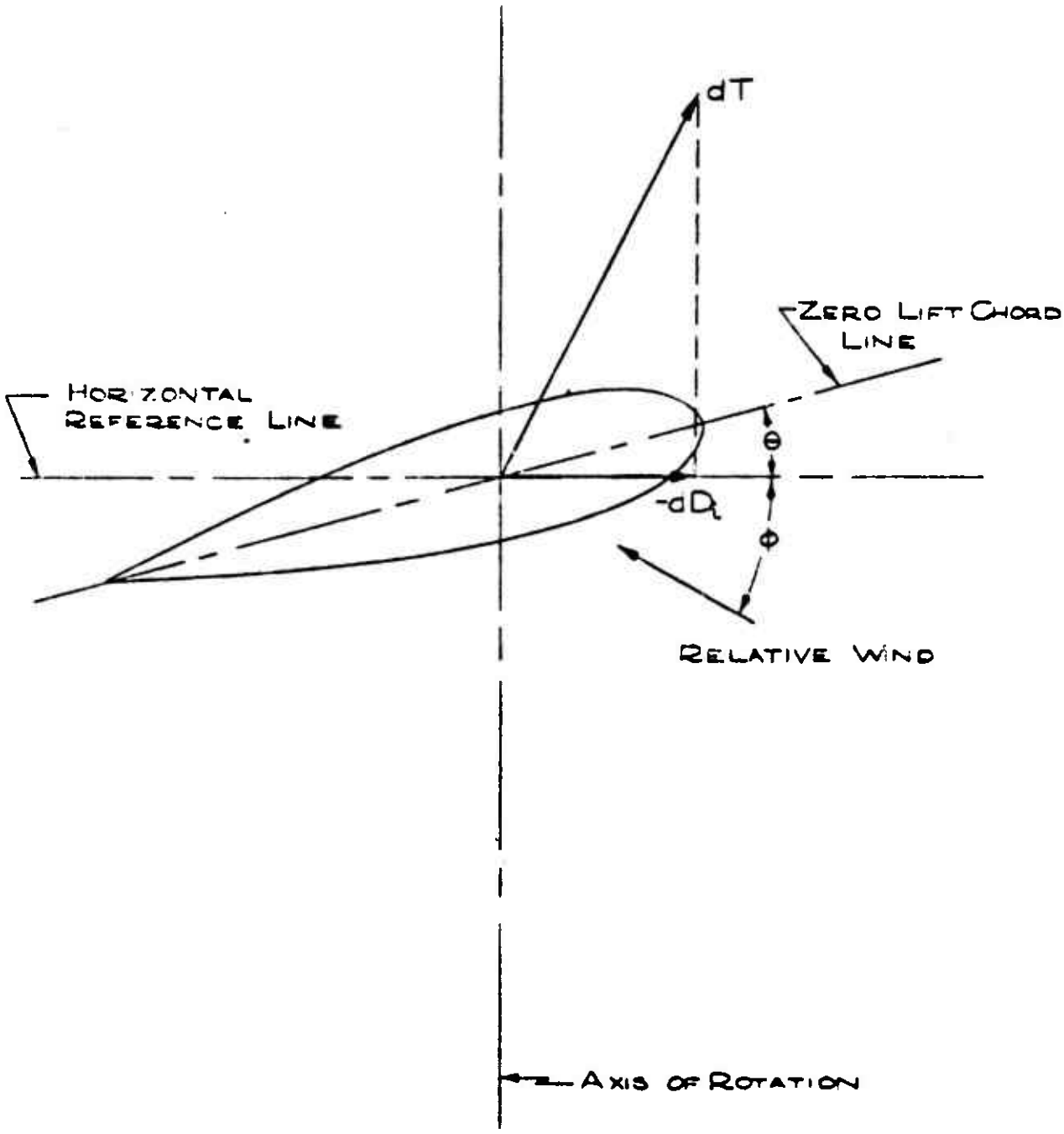
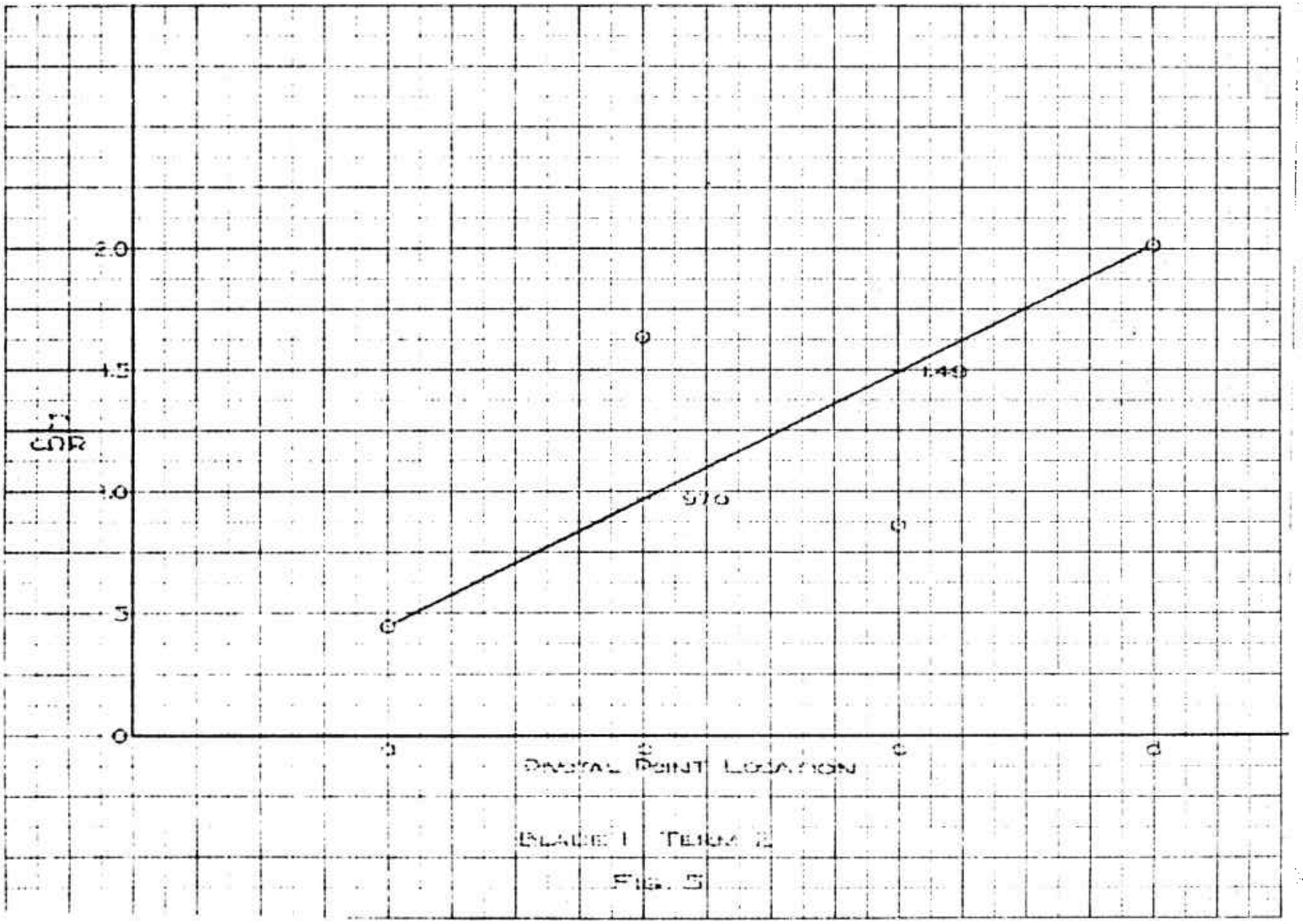
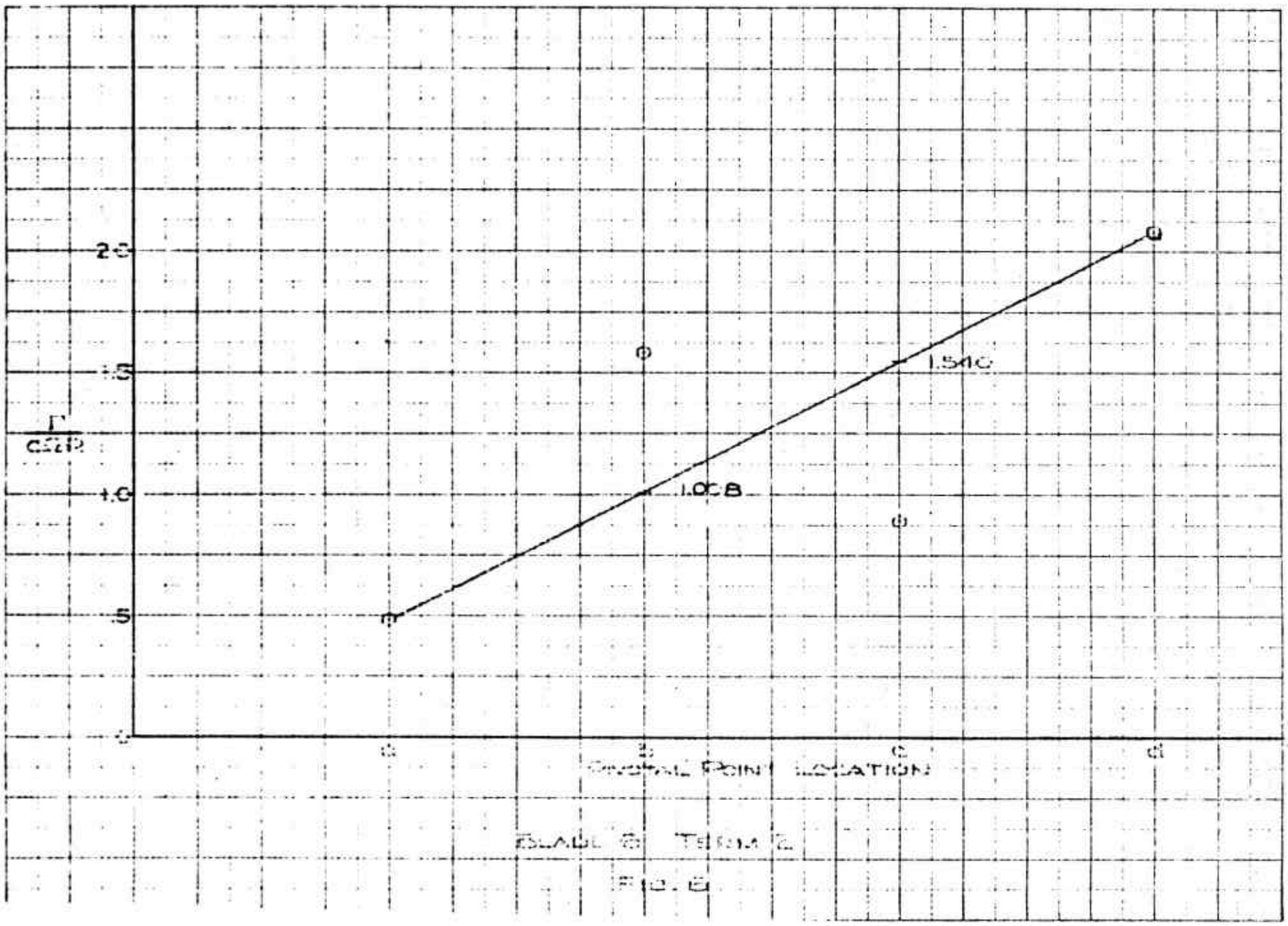

dT
ZERO LIFT CHORD LINE
HORIZONTAL REFERENCE LINE
θ
-dD
φ
RELATIVE WIND
AXIS OF ROTATION

FIG. 4



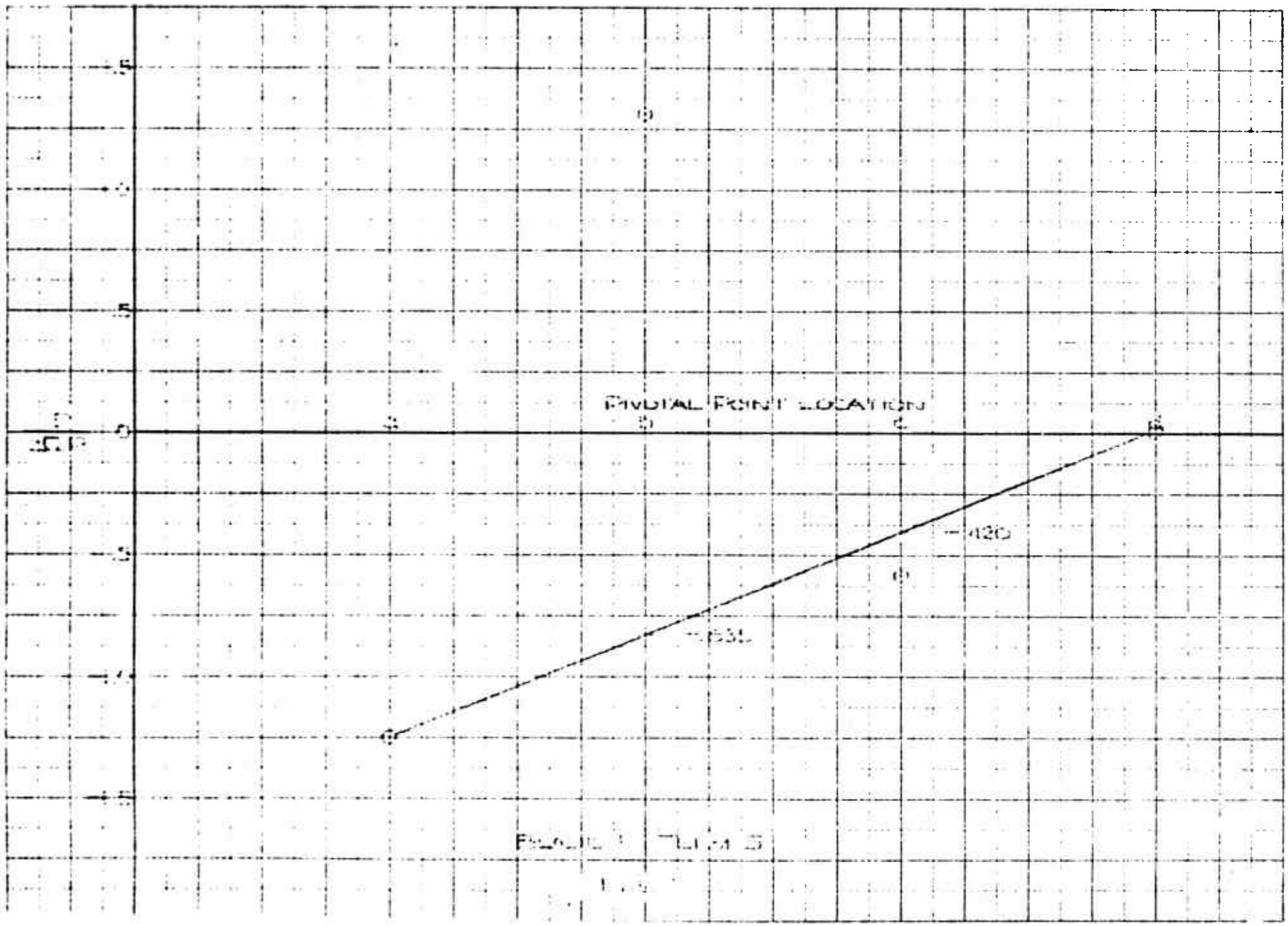
BLADE 1 TERM 2

FIG. 5



SLADE @ TERM 2

FIG. 8


PIVOTAL POINT LOCATION
-420
-531

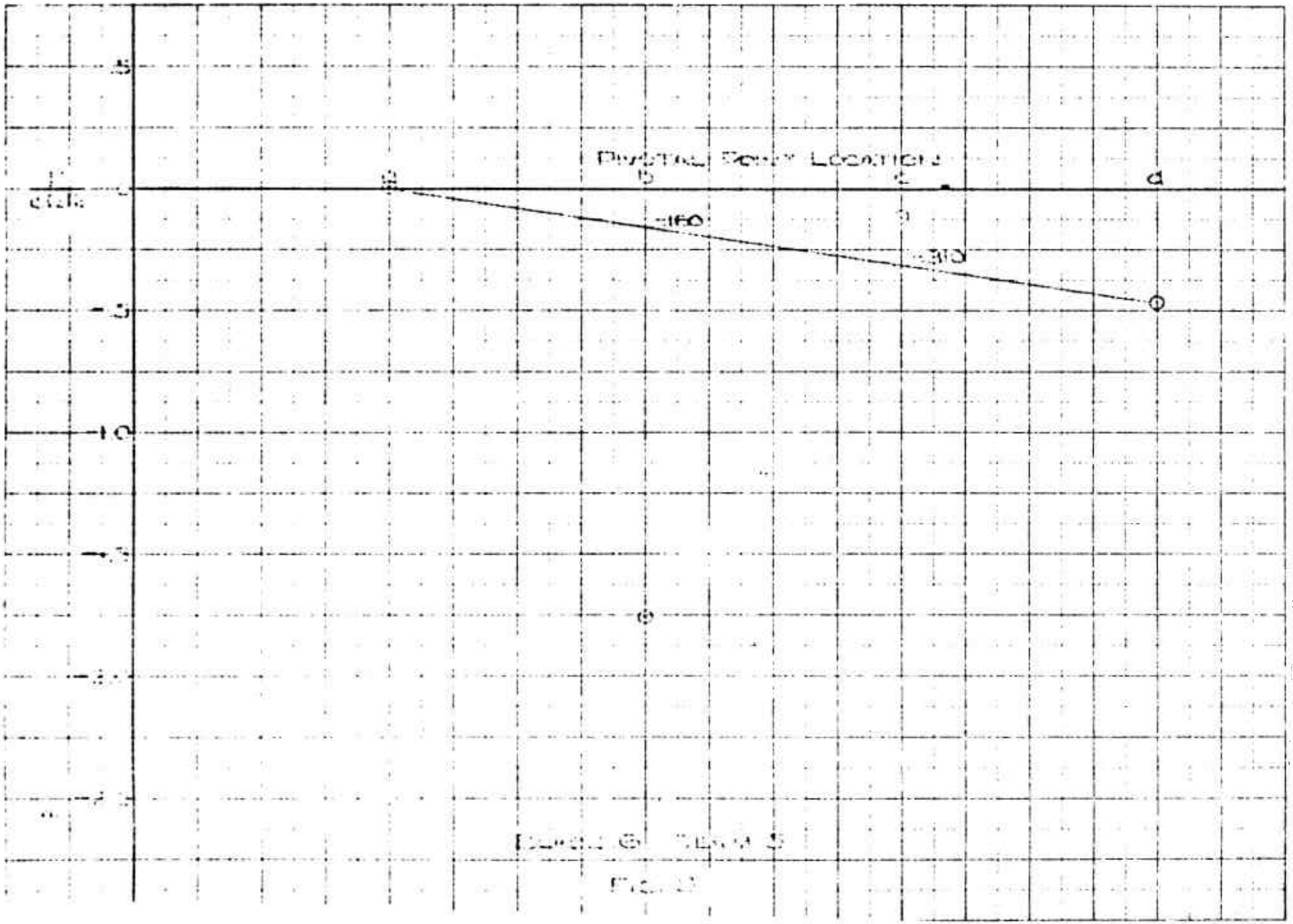


FIG. 3

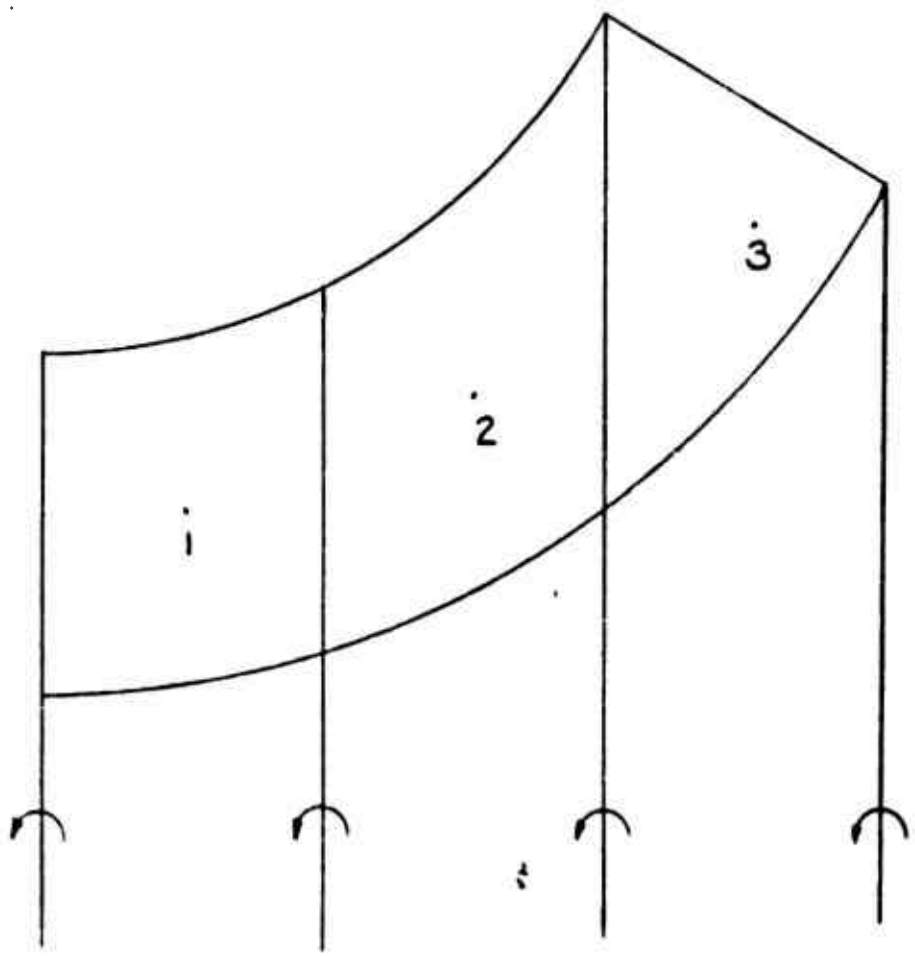


FIG. 9