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ANALYSIS OF A HALF-WAVE RECTIFIER CIRCUIT INVOLVING INDUCTANCE, RESISTANCE, AND CAPACITY

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Report 867 December 26, 1945

ANALYSIS OF A HALF-WAVE RECTIFIER CIRCUIT INVOLVING INDUCTANCE, RESISTANCE, AND CAPACITY

Abstract

A description of an approximate solution of the half-wave rectifier circuit problem discussed in Radiation Laboratory Report T-12 is given. Design curves for choosing optimum values of rectifier constants are given. The exact solution is involved, inasmuch as it contains the circuit constants in complicated expressions; the approximate analysis presented here contains the circuit constants in a simplified form. Several less complete analyses of this circuit have been made, but their results do not agree with experiment. The curves in this report agree closely with results of experiments made at the Radiation Laboratory.

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Title page
Acknowledgment page
29 numbered pages
10 pages of figures

Acknowledgment

Our sincere thanks are extended to Professor R. D. Douglass who supplied the nomographic chart which was used in connection with the solution of equations (16) and (17) and to Mrs. Donald Read who performed all the computations necessary to obtain the curves and tables at the end of the report.

E. E. Bothwell

P. D. Grout

ANALYSIS OF A HALF-WAVE RECTIFIER CIRCUIT INVOLVING

INDUCTANCE, RESISTANCE, AND CAPACITY

The following is a description of an approximate solution of the half-wave rectifier problem discussed in Report T-12 by E. J. White. The circuit is shown in Figure 1.

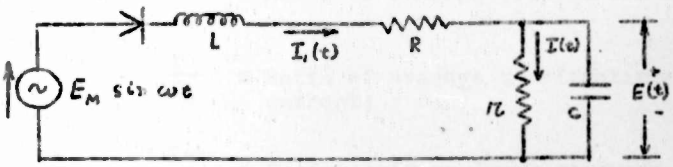


Fig. 1.

Let:

E = Average load voltage;

ΔE = Difference of the maximum and minimum values of the load voltage;

I = Average load current;

E_m = Peak input voltage to rectifier;

I_m = Peak rectifier current;

E_e = Effective value of input voltage $= \frac{E_m}{\sqrt{2}}$;

I_e = Effective value of rectifier current;

E_p = Peak inverse rectifier voltage $= E + E_m$;

E_{po} = No load peak inverse rectifier voltage $= 2E_m$;

The following ratios are used in the graphs at the end of the report:

$\frac{EI}{E_e I_e}$ = Transformer secondary utilization factor (T. U. F.);

$\frac{EI}{E_p I_m}$ = Rectifier utilization factor(R.U.F.);

$\frac{EI}{E_{po} I_m}$ = No load rectifier utilization factor;

$\frac{E_p}{E}$ = Inverse peak voltage ratio of rectifier;

$\frac{I}{I_m}$ = Ratio of average to peak rectifier current;

$\frac{I}{I_e}$ = Ratio of average to effective rectifier current;

$\frac{L\omega}{R}$ = Q of rectifier input circuit;

$\frac{L\omega}{r}$ = Ratio of inductive reactance to load resistance;

$\frac{R}{r}$ = Ratio of rectifier input resistance to load resistance;

$rc\omega \frac{\Delta E}{E}$ = Product of load voltage ripple factor and ratio of load resistance to condenser reactance.

In general, in the set of curves at the end of the report, the ratio, I/I_m, is taken as independent variable and Q = 0, 2, 4, 8, 16, 32, ∞ as parameter. The wave form of the rectifier current is shown in the graphs, and there are listed tables of each of the above ratios.

<u>Analysis of the Circuit</u> The analysis of the circuit of Fig. 1 is involved inasmuch as there are several parameters under consideration. However since the choice of the condenser will be such as to maintain nearly constant load voltage, the following circuit will approximate Fig. 1,

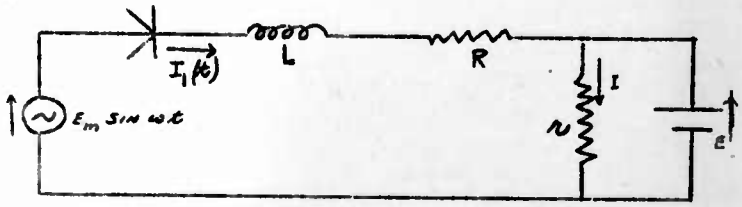


Fig. 2.

providing that it is required that the battery deliver no power to the load.

Since the method of analysis is less involved if the special cases, $Q = 0$ and $Q = \infty$ are considered separately, we postpone the analysis of these two cases until later. For $0 < Q < \infty$, the network equations for Fig. 2 are

$$L\frac{dI_1}{dt} + RI_1 = E_m \sin \omega t - E,$$
$$rI = E. \tag{1}$$

The general solution for $I_1(t)$ is

$$I_1 = De^{-\frac{R}{L}t} + B \sin \omega t - C \cos \omega t - \frac{E}{R}. \tag{2}$$

Substituting (2) into (1), and equating coefficients of sin terms and of cosine terms, we obtain the following equations for B and C:

$$\omega LB = RC,$$
$$RB + \omega LC = E_m.$$

From these two equations, we obtain

$$B = \frac{1}{1+Q^2}\frac{E_m}{R},$$

$$C = \frac{Q}{1+Q^2}\frac{E_m}{R},$$

where

$$Q = \frac{\omega L}{R}, \tag{3}$$

so that

$$I_1(t) = De^{-\frac{1}{Q}\omega t} + \frac{E_m}{R}\frac{1}{1+Q^2}\left[\sin\omega t - Q\cos\omega t - \frac{E}{E_m}(1+Q^2)\right]. \tag{4}$$

In order to determine D, we use the condition that the rectifier starts to conduct when the voltage across its terminals becomes zero, that is when

$$E_m \sin\omega t - E = 0.$$

Let

$$\frac{E}{E_m} = \sin A,$$

A being given in radians. Then conduction begins when

$$\omega t = x = A. \tag{5}$$

Substituting (5) in (4), we obtain

$$0 = De^{-\frac{1}{Q}A} + \frac{E_m}{R}\frac{Q}{1+Q^2}\left[\frac{1}{Q}\sin A - \cos A - \left(\frac{1}{Q}+Q\right)\sin A\right]$$

or

$$D = \frac{E_m}{R}\frac{Q}{1+Q^2}\, e^{\frac{1}{Q}A}(\cos A + Q\sin A).$$

Placing this value of D in (4), we have

$$I_1(t) = \frac{E_m}{R}\frac{1}{1+Q^2}\Big[\sin x - Q\cos x + Q(\cos A + Q\sin A)\, e^{\frac{1}{Q}(A-x)} - (1+Q^2)\sin A\Big],$$

or, using (3),

$$\frac{\omega L}{E_m} I_1(t) = \frac{Q}{1+Q^2} f(x),$$

where

$$f(x) = \sin x - Q\cos x + Q(\cos A + Q\sin A)\times e^{\frac{1}{Q}(A-x)} - (1+Q^2)\sin A. \qquad (6)$$

Now we wish to impose the condition that the battery supply no power to the load. This condition will be obtained if the total current supplied to the load during one cycle of the generator voltage is equal to the total current that flows through the rectifier during the cycle. Since the rectifier stops conducting when its current drops to zero, that is when

$$x = x_1, \qquad (7)$$

the total charge flowing through the rectifier is given by

$$q = \int_{A/\omega}^{x_1/\omega} I_1(t)\,dt = \frac{E_m}{\omega^2 L}\frac{Q}{1+Q^2}\int_A^{x_1} f(x)\,dx \qquad (8)$$

whereas the total charge flowing through the load is

$$q = \frac{2\pi}{\omega}\frac{E}{r}. \qquad (9)$$

Carrying out the integration indicated in (8) and equating the result to (9), we have

$$\frac{E_m}{\omega L}\frac{Q}{1+Q^2}\Big[-\cos x_1 - Q\sin x_1 - Q^2(\cos A + Q\sin A)\, e^{\frac{1}{Q}(A-x_1)} - (1+Q^2)x_1 \sin A + \cos A + Q\sin A + Q^2(\cos A + Q\sin A) + (1+Q^2)A\sin A\Big]$$

$$= 2\pi\frac{E}{r},$$

or

$$\frac{\omega L}{r} = \frac{1}{2\pi}\frac{Q}{1+Q^2}\csc A\Big[(1+Q^2)(A-x_1)\sin A + (1+Q^2)(\cos A\, Q\sin A) - Q^2(\cos A + Q\sin A)\times e^{\frac{1}{Q}(A-x_1)} - (\cos x_1 + Q\sin x_1)\Big]. \qquad (10)$$

Substituting (7) in (6), we have the condition for cut-off,

$$-Q(\cos A + Q\sin A)\, e^{\frac{1}{Q}(A-x_1)} = \sin x_1 - Q\cos x_1 - (1+Q^2)\sin A. \qquad (11)$$

Substituting (11) into (10), we obtain

$$\frac{\omega L}{r} = \frac{Q}{2\pi}\Big[(A-x_1) + \csc A\,(\cos A - \cos x_1)\Big].$$

To find the peak current, we set

$$\frac{dI_1}{dt} = \frac{E_m}{L} \frac{Q}{1+Q^2} \Big[\cos x_2 \; Q \sin x_2 - (\cos A + Q \sin A) \times$$

$$e^{\frac{1}{Q}(A-x_2)}\Big] = 0 \; ,$$

and solve for x_2. Then by equation (6)

$$\frac{\omega L}{E_m} \cdot I_m = \frac{Q}{1+Q^2} \Big[\sin x_2 - Q \cos x_2 + Q(\cos A + Q \sin A) \times$$

$$e^{\frac{1}{Q}(A-x_2)} - (1+Q^2) \sin A\Big]$$

$$= \frac{Q}{1+Q^2} \Big[\sin x_2 - Q \cos x_2 + Q \cos x_2 + Q^2 \sin x_2$$

$$-(1+Q^2) \sin A\Big]$$

$$= Q(\sin x_2 - \sin A).$$

The effective rectifier current is evidently

$$\frac{\omega L}{E_m} I_e = \frac{Q}{1+Q^2} \sqrt{\frac{1}{2\pi} \int_A^{x_1} [f(x)]^2 \, dx} \; .$$

To summarize the results so far, let

$$a = A - x_1 + \csc A \, (\cos A - \cos x_1) \; ,$$

$$b = \sin x_2 - \sin A \; ,$$

$$c = \sqrt{\frac{1}{2\pi} \int_A^{x_1} [f(x)]^2 \, dx} \; . \qquad (12)$$

Then

$$\frac{X}{E_m} = \sin A \; ,$$

$$\frac{E}{I} = r \,,$$

$$\frac{\omega L}{r} = \frac{Q}{2\pi} a \,, \qquad (13)$$

$$\frac{\omega L}{E_m} I_m = Qb \,,$$

$$\frac{\omega L}{E_m} I_e = \frac{Q}{1+Q^2} d \,.$$

From equations (13) we may derive the following equations:

$$\frac{I}{I_e} = \frac{E}{r} \frac{\omega L}{E_m} \frac{1+Q^2}{Q} = \frac{1+Q^2}{2\pi} \frac{a \sin A}{d} \,,$$

$$\frac{I}{I_m} = \frac{E}{r} \frac{\omega L}{E_m} \frac{1}{Qb} = \frac{1}{2\pi} \frac{a \sin A}{b} \,,$$

$$\frac{E_p}{E} = \frac{E+E_m}{E} = 1 + \csc A \,, \qquad (14)$$

$$\frac{EI}{E_e I_e} = \frac{E}{E_m} \frac{E_m}{E_e} \frac{I}{I_e} = \frac{1+Q^2}{\sqrt{2}\,\pi} \frac{a \sin^2 A}{d} \,,$$

$$\frac{EI}{E_p I_m} = \frac{1}{2\pi} \frac{a}{b} \frac{\sin A}{1+\csc A} \,,$$

$$\frac{EI}{E_{pe} I_m} = \frac{1}{4\pi} \frac{a \sin^2 A}{b} \,.$$

Summary of Formulas

Let:

A = Angle at which conduction starts, $0 \leq A \leq \frac{\pi}{2}$

x_1 = Angle at which conduction stops,

x_2 = Angle at which rectifier current reaches peak,

$$Q = \frac{\omega L}{R} , \tag{15}$$

$$a = A - x_1 + \csc A(\cos A - \cos x_1) ,$$

$$b = \sin x_2 - \sin A ,$$

$$f(x) = \sin x - Q \cos x + Q(\cos A + Q \sin A)e^{\frac{1}{Q}(A-x)} - (1 + Q^2) \sin A ,$$

$$d = \sqrt{\frac{1}{2\pi}\int_a^{x_1} \left[f(x)\right]^2 dx} .$$

Then x_1 and x_2 are the roots of the following equations:

$$\sin x_1 - Q \cos x_1 + Q(\cos A + Q \sin A)e^{\frac{1}{Q}(A-x_1)} - (1 + Q^2) \sin A = 0 , \tag{16}$$

$$\cos x_2 + Q \sin x_2 - (\cos A + Q \sin A)e^{\frac{1}{Q}(A-x_2)} = 0 . \tag{17}$$

Having found these roots, $x_1(A,Q)$ and $x_2(A,Q)$ we have

$$\frac{EI}{E_0 I_s} = \frac{1 + Q^2}{\sqrt{2}\,\pi} \; \frac{a(A,Q) \sin^2 A}{d}$$

$$\frac{EI}{E_p I_m} = \frac{1}{2\pi} \; \frac{a(A,Q)}{b(A,Q)} \; \frac{\sin A}{1 + \csc A} ,$$

$$\frac{EI}{E_{po}I_m} = \frac{1}{4\pi}\,\frac{a(A,\theta)\,\sin^2 A}{b(A,Q)}\,,$$

$$\frac{I}{I_e} = \frac{1+Q^2}{2\pi}\,\frac{a(A,\theta)\,\sin A}{d(A,Q)}\,,$$

$$\frac{I}{I_m} = \frac{1}{2\pi}\,\frac{a(A,\theta)\,\sin A}{b(A,Q)}\,, \qquad (18)$$

$$\frac{E}{E_m} = \sin A\,,$$

$$\frac{E_r}{E} = 1+\csc A\,,$$

$$\frac{\omega L}{r} = \frac{Q}{2\pi}\,a(A,Q)\,.$$

Roots of equations (16) and (17). The roots x_1 and x_2 are each functions of A and Q. We are interested in five values of Q; these are 2, 4, 8, 16, and 32, and in a range of A from zero to 90°. A nomographic chart, with Q as a parameter, was constructed from which x_1 and x_2 were obtained correct to three places. However, it was found that in obtaining a(A,Q), b(A,Q), and d(A,Q), as many as four places were lost for values of A greater than seventy or eighty degrees.

Roots accurate to six places were obtained as follows. Ten values of A between zero and 90°, spaced at 10° intervals, were chosen.

For any one pair of values of A and Q, equations (16) and (17) were approximated by second degree polynomials near the roots. Since the roots were known to three places,

a Lagrangean interpolation table could be used to extend the accuracy to one more place. Using the table twice again, the accuracy was extended to six places. By using the Lagrangean error factors, it was shown that the error involved in the use of the tables was less than one part in 10^6. However, in order to have a rigid check on the accuracy of the roots, each root together with values on either side of the root was substituted back into the corresponding formula (16) or (17). In every case the root checked to six places. The fifty values of $d(A,\theta)$ were found by a seven point Simpson's rule integration.

$\underline{Q = \infty,\ A \neq 0}$. In this case

$$R = 0 ,$$

and equation (1) reduces to

$$L \frac{dI_1}{dt} = E_m \sin \omega t - E .$$

Then

$$I_1 = D' - \frac{E_m}{L\omega} \left(\cos \omega t + \frac{E}{E_m} \omega t\right) .$$

Since

$$I_1(A) = 0 ,$$

then

$$D' = \frac{E_m}{L\omega} \left(\cos A + \frac{E}{E_m} A\right),$$

and

$$\frac{\omega L}{E_m} I_1(t) = \cos A - \cos x - (x-A) \sin A.$$

The maximum current occurs when

$$x_2 = \pi - A ,$$

and is given by

$$\frac{\omega L}{E_m} I_m = 2 \cos A - (\pi - 2A) \sin A .$$

The current is again zero when

$$\cos x_1 = \cos A + A \sin A - x_1 \sin A . \qquad (19)$$

The total charge flowing through the rectifier is given by

$$q_1 = \frac{E_m}{\omega^2 L} \int_A^{x_1} \left[(A-x) \sin A + \cos A - \cos x\right] dx$$

$$= \frac{E_m}{\omega^2 L} \left[\sin A - \sin x_1 + (x_1 - A) \cos A - \tfrac{1}{2}(x_1 - A)^2 \sin A\right]$$

$$= \frac{E_m}{\omega^2 L} \left[\sin A - \sin x_1 + \tfrac{1}{2} \csc A(\cos^2 A - \cos x_1)\right]$$

$$= \frac{E_m}{2\omega^2 L} \csc A(2 \sin^2 A - 2 \sin A \sin x_1 \cos^2 A - \cos^2 x_1)$$

$$= \frac{E_m}{2\omega^2 L} \csc A (\sin x_1 - \sin A)^2 , \qquad (20)$$

in which (19) has been used to simplify (20). Now, as in equation (9), the total charge flowing through the load in one cycle is

$$q_1 = \frac{2\pi}{\omega} \cdot \frac{E}{r} .$$

Equating this to (20), we obtain

$$\frac{\omega L}{r} = a_1 ,$$

where

$$a_1 = \frac{1}{4\pi} \left(\frac{\sin x_1 - \sin A}{\sin A}\right)^2 .$$

Evidently

$$\frac{\omega L}{L_n} I_o = \frac{1}{2\pi} \int_A^{x_1} \left[f_1(x)\right]^2 dx ,$$

in which

$$f(x) = \cos A - \cos x - (x-A) \sin A . \qquad (21)$$

Now

$$\int_A^{x_1} \left[f(x)\right]^2 dx = (x_1-A) \cos^2 A + \tfrac{1}{2}(x_1-A) + \tfrac{1}{2} \sin x_1 \cos x_1$$

$$-\tfrac{1}{2} \sin A \cos A + 1/3\ (x_1-A)^3 \sin^2 A$$
$$-2 \cos A \sin x_1 + 2 \cos A \sin A$$
$$-(x_1-A)^2 \sin A \cos A - 2A \sin A \sin x_1$$
$$+2A \sin^2 A + 2 \sin A \cos x_1 - 2 \sin A \cos A$$
$$+2x_1 \sin x_1 \sin A - 2A \sin^2 A$$

$$= 1/3\ (x_1-A)^3 \sin^2 A - (x_1-A)^2 \sin A \cos A$$
$$+(x_1-A)(\cos^2 A + \tfrac{1}{2} - 2 \sin x_1 \sin A)$$
$$+2 \sin A \cos x_1 - 2 \cos A \sin x_1$$
$$+\tfrac{1}{2} \sin x_1 \cos x_1 - \tfrac{1}{2} \sin A \cos A$$

$$= \csc A \Big[1/3 \cos^3 A - \cos^2 A \cos x_1 + \cos A \times$$
$$\cdot \cos^2 x_1 - 1/3 \cos^3 x_1 - \cos^3 A + 2 \cos^2 A \cos x_1$$
$$- \cos A \cos^2 x_1 + \cos^3 A - \cos^2 A \cos x_1 + \tfrac{1}{2}(\cos A - \cos x_1)$$
$$+ 2 \sin A \cos A \sin x_1 - 2 \sin A \cos x_1 \sin x_1$$
$$- 2 \sin A \cos A \sin x_1 + 2 \sin^2 A \cos x_1$$
$$- \tfrac{1}{2} \sin^2 A \cos A + \tfrac{1}{2} \sin A \cos x_1 \sin x_1\Big]$$

$$= \csc A \Big[1/3 \cos^3 A - 1/3 \cos^3 x_1 + \tfrac{1}{2} \cos A - \tfrac{1}{2} \cos x_1$$
$$+ 2 \cos x_1 \sin^2 A - 3/2 \cos x_1 \sin x_1 \sin A$$
$$- \tfrac{1}{2} \sin^2 A \cos x\Big]$$

$$= \Big[1/6\,(5 \cos^3 A - 2 \cos^3 x_1) + 3/2 \cos x_1 \times$$
$$(1 - \sin x_1 \sin A) - 2 \cos x_1 \cos^2 A\Big] \csc A \quad .$$

Thus

$$\frac{\omega L}{E_m} I_0 = d_1 \ ,$$

in which

$$d_1 =$$

$$\sqrt{\frac{1/6(5 \cos^3 A - 2 \cos^3 x_1) + 3/2 \cos x_1 (1 - \sin x_1 \sin A) - 2 \cos x_1 \cos^2 A}{2\pi \sin A}}$$

<u>Summary of formulas for $Q = \infty$, $A \neq 0$</u> .

$$\cos x_1 = \cos A + A \sin A - x_1 \sin A \ ,$$

$$x_2 = \pi - A \ ,$$

$$a_1 = \frac{1}{4\pi} \left(\frac{\sin x_1 - \sin A}{\sin A}\right)^2 \ ,$$

$$b_1 = 2\cos A - (\pi - 2A)\sin A ,$$

$$d_1 = \sqrt{\frac{1/6\,(5\cos^3 A - 2\cos^3 x_1) + 3/2\cos x_1(1-\sin x_1 \sin A) - 2\cos x_1 \cos^2 A}{2\pi \sin A}}$$

$$\frac{I}{I_e} = \frac{a_1}{d_1}\sin A ,$$

$$\frac{I}{I_m} = \frac{a_1}{b_1}\sin A ,$$

$$\frac{E_p}{E} = 1 + \csc A ,$$

$$\frac{EI}{E_e I_e} = \sqrt{2}\,\frac{a_1}{d_1}\sin^2 A , \qquad (22)$$

$$\frac{EI}{E_p I_m} = \frac{a_1}{b_1}\,\frac{\sin A}{1+\csc A} ,$$

$$\frac{EI}{E_{po} I_m} = \frac{1}{2}\,\frac{a_1}{b_1}\sin^2 A ,$$

$$\frac{E}{E_m} = \sin A ,$$

$$\frac{\omega L}{r} = a_1 .$$

<u>$Q = \infty$, $A = 0$</u> Equation (21) becomes

$$f(x) = 1 - \cos x .$$

Thus

$$\frac{2\pi}{\omega}\frac{E}{r} = Q = \frac{E_m}{\omega^2 L}\int_0^{2\pi}(1-\cos x)\,dx = 2\pi\,\frac{E_m}{\omega^2 L} ,$$

or

$$\frac{L\omega}{r} = \frac{E_m}{E} = \csc \theta = \infty .$$

also

$$\frac{\omega L}{E_m} I_e = \sqrt{3/2} .$$

In summary,

$$\begin{aligned}
&A = 0_1 \\
&x_1 = 2\pi , \\
&x_2 = \pi , \\
&\frac{\omega L}{E_m} I(t) = 1 - \cos \omega t , \\
&\frac{I}{I_e} = \sqrt{\frac{2}{3}} , \\
&\frac{I}{I_m} = \tfrac{1}{2} , \\
&\frac{E_p}{E} = \infty , \\
&\frac{E}{E_m} = 0 , \\
&\frac{EI}{E_e I_e} = 0 , \\
&\frac{EI}{E_p I_m} = 0 , \\
&\frac{EI}{E_{po} I_m} = 0 , \\
&\frac{\omega L}{r} = \infty .
\end{aligned} \tag{23}$$

$\underline{Q = 0}$ — For this case

$$L = 0$$

and by inspection of Fig. 2, we have

$$I_1 = \frac{E_m}{R} (\sin x - \sin A) ,$$

$$x_1 = \pi - A ,$$

$$x_2 = \frac{\pi}{2} ,$$

$$\frac{R}{E_m} I_m = (1 - \sin A) .$$

Since

$$\int_A^{\pi - A} \left[f(x)\right]^2 dx = \frac{\pi}{2} - A + \sin A \cos A - 4 \sin A \cos A$$

$$+ (\pi - 2A) \sin^2 A ,$$

$$= \left(\frac{\pi}{2} - A\right) (2 - \cos 2A) - \frac{3}{2} \sin 2A$$

then

$$\frac{R}{E_m} I_e = \sqrt{\frac{1}{2\pi} \left[\left(\frac{\pi}{2} - A\right) (2 - \cos 2A) - \frac{3}{2} \sin 2A\right]} ,$$

also

$$\frac{2\pi}{\omega} \frac{E}{r} = q = \frac{E_m}{\omega R} \left[2 \cos A - (\pi - 2A) \sin A\right] ,$$

or

$$\frac{R}{r} = \frac{1}{2\pi \sin A} \left[2 \cos A - (\pi - 2A) \sin A\right] .$$

In summary,

$$Q = 0 ,$$

$$x_1 = \pi - A ,$$

$$x_2 = \frac{\pi}{2},$$

$$I_1(t) = \frac{E_m}{R}(\sin \omega t - \sin A),$$

$$a_2 = \frac{1}{2\pi}\left[2 \cos A - (\pi - 2A) \sin A\right],$$

$$b_2 = 1 - \sin A,$$

$$d_2 = \sqrt{\frac{1}{2\pi}\left[\left(\frac{\pi}{2} - A\right)(2 - \cos 2A) - 3/2 \sin 2A\right]},$$

$$\frac{I}{I_e} = \frac{a_2}{d_2},$$

$$\frac{I}{I_m} = \frac{a_2}{b_2},$$

$$\frac{E_p}{E} = 1 + \csc A, \qquad (24)$$

$$\frac{EI}{E_e I_e} = \sqrt{2}\,\frac{a_2}{d_2} \sin A,$$

$$\frac{EI}{E_p I_m} = \frac{a_2}{b_2}\,\frac{1}{1 + \csc A},$$

$$\frac{EI}{E_{po} I_m} = \frac{1}{2}\,\frac{a_2}{b_2} \sin A,$$

$$\frac{E}{E_m} = \sin A,$$

$$\frac{R}{r} = \frac{a_2}{\sin A}.$$

Choice of Condenser If the capacitance of the load condenser were infinite, then the varying part of $I_1(t)$ would be completely by-passed by the condenser and the load voltage would be exactly

$$E = \frac{I}{r} ,$$

in which I is the steady component of $I_1(t)$. In fact, this assumption was made in order to obtain an approximate solution of the problem. The effect of the finiteness of the capacity of the condenser may be approximately determined by keeping the assumption that the varying part of $I_1(t)$ is completely by-passed by the condenser and finding the corresponding condenser voltage increase. That is

$$\Delta E(x) = \frac{1}{\omega c} \int_0^x (I_1 - I)\, dx \quad . \tag{25}$$

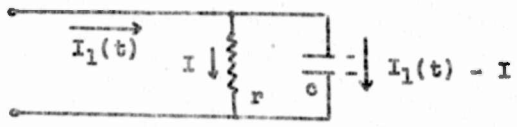


Fig. 3

Correct boundary conditions are assured by the fact that the average value of $I_1 - I$ is zero. Thus

$$\Delta E(0) = \Delta E(2\pi) = 0 .$$

The maximum and minimum values of $\Delta E(x)$ are obtained by setting the derivative of (25) equal to zero. These points occur when

$$I_1(x) = I \, .$$

Now

$$rc\omega \frac{\Delta E(x)}{E} = I\int_0^x \left[I_1(t) - I\right] dx$$

$$= \int_0^x \left[\frac{1}{I} I_1(t) - 1\right] dx$$

$$= -x + \int_A^x \frac{1}{I} I_1(t)\, dx \quad , \qquad (26)$$

since $I_1(t) = 0$ when $x < A$. Since the average value of $I_1 I$ is zero,

$$\int_0^{2\pi} \left[I_1(t) - \frac{E}{r}\right] dx = 0 \, ,$$

and (26) could have been written

$$rc\omega \frac{\Delta E(x)}{E} = 2\pi - x + \int_{x_1}^x \frac{1}{I} I_1(t)\, dx \, . \qquad (27)$$

From the graphs of $I_1(t)$ at the end of the report it was possible to obtain the solution of (26) graphically. Now each of the currents had the form shown in

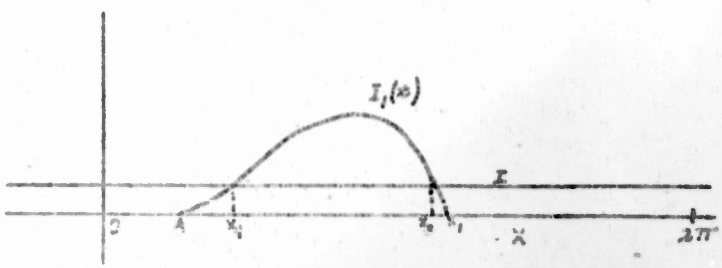


Fig. 4

Fig. 4. The maximum value of $\Delta E(x)$ is obtained by placing $x = x_4$ in (26) and the minimum is obtained by placing $x = x_3$ in (27). Since the portion of $I_1(t)$ included between x_1 and x_3 is nearly linear, (27) is approximately

$$-r\omega\frac{\Delta E(x_3)}{E} = 2\pi - x_3 + \tfrac{1}{2}(x_3 - x_1)$$
$$= 2\pi - \tfrac{1}{2}(x_1 + x_3). \qquad (28)$$

The same procedure applied to (26) yields

$$r\omega\frac{\Delta E(x_4)}{E} = \tfrac{1}{2}(x_4 + A) \quad .$$

The ripple voltage, ΔE, is defined as the difference of the maximum and minimum values of $E(x)$, that is

$$r\omega\frac{\Delta E}{E} = \frac{r\omega}{E}\left[E + \Delta E(x_4) - E - \Delta E(x_3)\right]$$
$$\frac{r\omega}{E}\left[\Delta E(x_4) - \Delta E(x_3)\right]$$
$$= 2\pi + \tfrac{1}{2}(A + x_4 - x_3 - x_1).$$

F. E. Bothwell
P. D. Crout

November 30, 1945

$Q = 0$

$A°$	$x_1°$	$x_2°$	$x_3°$	$\frac{E}{E_m}$	$\frac{E_p}{E}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	180	90	161.5	.0000	∞	.3183	.6366
10	170	90	156.0	.1736	6.759	.2860	.5022
20	160	90	149.7	.3420	3.924	.2524	.5648
30	150	90	142.5	.5000	3.000	.2180	.5270
40	140	90	134.8	.6428	2.556	.1828	.4793
50	130	90	126.8	.7660	2.305	.1469	.4293
60	120	90	118.2	.8660	2.155	.1106	.3722
70	110	90	109.3	.9397	2.064	.07392	.3040
80	100	90	99.85	.9848	2.015	.03701	.2149
90	90	90	90	1.0000	2.000	.00000	.0000

$A°$	$\frac{EI}{E_e I_e}$	$\frac{E_p I}{E_p I_m}$	$\frac{\omega I}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{\omega L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1479	.04231	.02483	1.361	0
20	.2732	.05434	.04317	.4857	0
30	.3725	.07267	.05450	.2180	0
40	.4337	.07151	.05874	.1016	0
50	.4651	.06373	.05527	.04487	0
60	.4558	.05133	.04789	.01711	0
70	.4040	.03581	.03473	.004744	0
80	.2992	.01836	.01822	.0005709	0
90	.0000	.00000	.00000	.0000000	0

$Q = 1$

$A°$	$x_1°$	$x_2°$	$x_3°$	$\frac{E}{E_m}$	$\frac{E_p}{E}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	224.190	130.870	203.5	.0000	∞	.3614	.6893
10	212.212	129.114	196.3	.1736	6.759	.3221	.6444
20	198.667	126.942	186.2	.3420	3.924	.2856	.6073
30	185.018	124.256	169.0	.5000	3.000	.2483	.5659
40	171.101	120.969	165.0	.6428	2.556	.2100	.5196
50	156.756	116.968	153.2	.7660	2.305	.1707	.4679
60	141.784	112.105	139.9	.8660	2.155	.1303	.4090
70	125.941	106.190	124.45	.9397	2.064	.08663	.3371
80	108.853	98.954	108.7	.9848	2.015	.04552	.2421
90	90	90	90	1.0000	2.000	.00000	.0000

$A°$	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{EI}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{\omega L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1582	.04830	.02786	1.116	1.116
20	.2937	.07279	.04885	.3819	.3819
30	.4002	.08276	.06207	.1622	.1622
40	.4724	.08216	.06749	.07013	.07013
50	.5069	.07404	.06538	.02790	.02790
60	.5009	.06049	.05644	.009101	.009101
70	.4480	.04294	.04164	.001948	.001948
80	.3372	.02258	.02241	.0001390	.0001390
90	.0000	.00000	.00000	.0000000	.0000000

$\varrho = 2$

A°	x_1°	x_2°	x_3°	$\frac{E}{E_m}$	$\frac{E}{E_p}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	249.3	146.2	223.0	.0000	∞	.3872	.7113
10	231.012	142.645	211.2	.1736	6.759	.3469	.6684
20	213.648	138.469	199.0	.3420	3.924	.3055	.6291
30	196.688	133.704	188.1	.5000	3.000	.2638	.5832
40	179.834	128.314	173.8	.6428	2.556	.2214	.5338
50	162.870	122.255	158.9	.7660	2.305	.1786	.4792
60	145.606	115.472	143.4	.8660	2.155	.1352	.4167
70	127.857	107.899	127.2	.9397	2.064	.0911	.3150
80	109.407	99.449	109.1	.9848	2.015	.0463	.2481
90	90	90	90	1.0000	2.000	.0000	.0000

A°	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{EI}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{\omega L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1641	.05133	.03012	.8655	1.731
20	.3043	.07786	.05225	.2867	.5734
30	.4124	.08794	.06595	.1177	.2354
40	.4852	.08661	.07114	.04885	.09769
50	.5191	.07746	.06840	.01857	.03713
60	.5104	.06275	.05855	.005740	.01148
70	.4587	.04414	.04280	.001155	.002309
80	.3456	.02295	.02278	.00007630	.0001526
90	.0000	.00000	.00000	.00000000	.0000000

$Q = 4$

$A°$	$x_1°$	$x_2°$	$x_3°$	$\frac{E}{E_m}$	$\frac{E}{E_p}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	272.9	159.0	239.8	.0000	∞	.4216	.7446
10	247.966	153.379	225.5	.1736	6.759	.3704	.6944
20	226.030	147.223	212.0	.3420	3.924	.3226	.6463
30	205.65	140.553	195.3	.5000	3.000	.2737	.5913
40	185.103	133.381	182.7	.6428	2.556	.2298	.5440
50	166.980	125.713	163.1	.7660	2.305	.1839	.4881
60	148.010	117.552	146.0	.8660	2.155	.1383	.4215
70	128.987	108.892	128.1	.9397	2.064	.0931	.3106
80	109.703	99.717	109.5	.9848	2.015	.0464	.2442
90	90	90	90	1.0000	2.000	.0000	.0000

$A°$	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{EI}{E_{po} I_m}$	$\frac{P}{r}$	$\frac{\alpha L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1705	.05481	.03216	.5855	2.342
20	.3126	.08222	.05517	.1881	.7522
30	.4181	.09125	.06844	.07400	.2960
40	.4946	.08988	.07384	.03003	.1201
50	.5288	.07978	.07045	.01102	.04409
60	.5162	.06418	.05988	.003285	.01314
70	.4128	.04512	.04375	.0006380	.002552
80	.3523	.02303	.02285	.00003985	.0001594
90	.0000	.00000	.00000	.00000000	.0000000

Q = 8

A°	x_1°	x_2°	x_3°	$\frac{E}{E_m}$	$\frac{E}{E_p}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	294.4	168.0	252.5	.0000	∞	.4493	.7663
10	260.760	160.637	236.8	.1736	6.579	.3884	.7092
20	234.603	152.928	217.6	.3420	3.924	.3348	.6589
30	211.488	144.855	200.8	.5000	3.000	.2840	.6057
40	189.986	136.449	182.8	.6428	2.556	.2350	.5504
50	169.416	127.734	164.5	.7660	2.305	.1872	.4927
60	149.377	118.724	147.5	.8660	2.155	.1400	.4241
70	129.598	109.430	128.5	.9397	2.064	.0933	.3461
80	109.866	99.856	109.6	.9848	2.015	.0473	.2421
90	90	90	90	1.0000	2.000	.0000	.0000

A°	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{EI}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{\omega L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1742	.05746	.03372	.3531	2.825
20	.3187	.08532	.05725	.1107	.8856
30	.4283	.09467	.07100	.04298	.3438
40	.5003	.09195	.07553	.01690	.1352
50	.5338	.08120	.07170	.006093	.04850
60	.5194	.06497	.06062	.001765	.01412
70	.4600	.04518	.04382	.0003331	.002665
80	.3371	.02346	.02328	.00002081	.0001664
90	.0000	.00000	.00000	.00000000	.0000000

Q = 16

A°	x_1°	x_2°	x_3°	$\frac{E}{E_m}$	$\frac{E}{E_p}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	311.69	173.5	261.3	.0000	∞	.4709	.7871
10	269.133	164.990	242.0	.1736	6.759	.4002	.7228
20	239.818	156.258	222.2	.3420	3.924	.3421	.6562
30	214.886	147.305	203.5	.5000	3.000	.2888	.6108
40	192.175	138.157	186.0	.6428	2.556	.2380	.5537
50	170.750	128.833	169.2	.7660	2.305	.1890	.4932
60	150.107	119.348	148.0	.8660	2.155	.1410	.4258
70	129.919	109.711	129.1	.9397	2.064	.0937	.3468
80	109.946	99.928	109.75	.9848	2.015	.0458	.2440
90	90	90	90	1.0000	2.000	.0000	.0000

A°	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{EI}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{\omega L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1775	.05921	.03475	.1967	3.147
20	.3225	.08719	.05851	.06056	.9699
30	.4319	.09626	.07219	.02319	.3711
40	.5033	.09311	.07648	.009000	.1440
50	.5343	.08198	.07239	.003191	.05105
60	.5215	.06543	.06105	.0009169	.01467
70	.4609	.04501	.04403	.0001709	.002734
80	.3399	.02303	.02305	.00001034	.0001654
90	.0000	.00000	.00000	.00000000	.0000000

Q = 32

A°	x_1°	x_2°	x_3°	$\frac{E}{E_m}$	$\frac{E}{E_p}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	325.05	176.58	265.0	.0000	∞	.4839	.7993
10	274.059	167.40	245.0	.1736	6.759	.4068	.7295
20	242.727	158.072	224.5	.3420	3.924	.3464	.6706
30	216.731	148.619	205.0	.5000	3.000	.2913	.6137
40	193.339	139.060	186.5	.6428	2.556	.2397	.5560
50	171.451	129.408	167.1	.7660	2.305	.1899	.4943
60	150.485	119.670	148.5	.8660	2.155	.1415	.4270
70	130.083	109.854	129.35	.9397	2.064	.0938	.3429
80	109.990	99.964	109.875	.9848	2.015	.0468	.2427
90	90	90	90	1.0000	2.000	.0000	.0000

A°	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{EI}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{\omega L}{r}$
0	.0000	.00000	.00000	∞	∞
10	.1791	.06019	.03532	.1043	3.336
20	.3243	.08827	.05923	.03181	1.018
30	.4340	.09712	.07284	.01208	.3865
40	.5054	.09377	.07702	.004813	.1489
50	.5355	.08239	.07275	.001637	.05237
60	.5229	.06567	.05127	.0004581	.01498
70	.4557	.04542	.04405	.00008566	.002773
80	.3380	.02321	.02304	.000005172	.0001655
90	.0000	.00000	.00000	.000000000	.0000000

$Q = \infty$

A°	$x_1{}^\circ$	$x_2{}^\circ$	$x_3{}^\circ$	$\frac{E}{E_m}$	$\frac{E}{E_p}$	$\frac{I}{I_m}$	$\frac{I}{I_e}$
0	360.000	180	270.0	.0000	∞	.5000	.8165
10	279.647	170	249.0	.1736	6.759	.4150	.7364
20	245.879	160	232.0	.3420	3.924	.3510	.6735
30	218.687	150	207.7	.5000	3.000	.2942	.6180
40	194.557	140	188.0	.6428	2.556	.2413	.5594
50	172.175	130	167.8	.7660	2.305	.1908	.4973
60	150.873	120	148.8	.8660	2.155	.1412	.4270
70	130.250	110	129.4	.9397	2.054	.0942	.3492
80	110.031	100	109.8	.9848	2.015	.0469	.2540
90	90	90	90	1.0000	2.000	.0000	.0000

A°	$\frac{EI}{E_e I_e}$	$\frac{EI}{E_p I_m}$	$\frac{SI}{E_{po} I_m}$	$\frac{R}{r}$	$\frac{mL}{r}$
0	.0000	.00000	.00000	0	∞
10	.1808	.05140	.03603	0	3.548
20	.3258	.08944	.05002	0	1.071
30	.4370	.09805	.07354	0	.4029
40	.5085	.09441	.07754	0	.1540
50	.5388	.08278	.07309	0	.05380
60	.5229	.05555	.06116	0	.01518
70	.4641	.04565	.04428	0	.002806
80	.3537	.02329	.02311	0	.0001684
90	.0000	.00000	.00000	0	.0000000

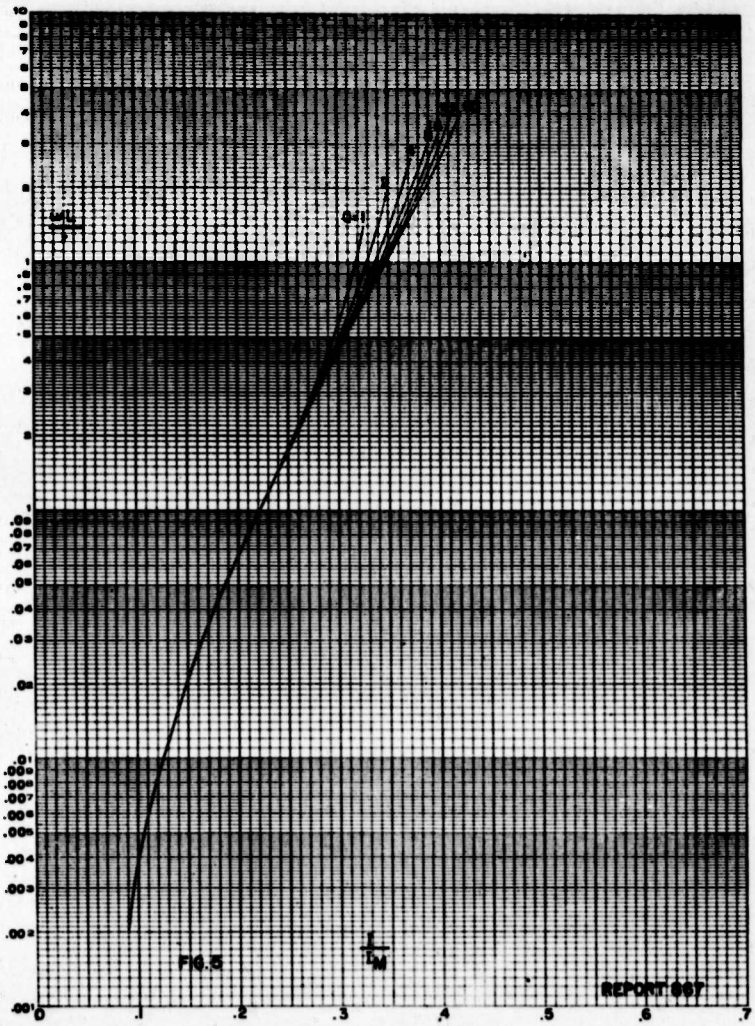


FIG. 5

REPORT 667

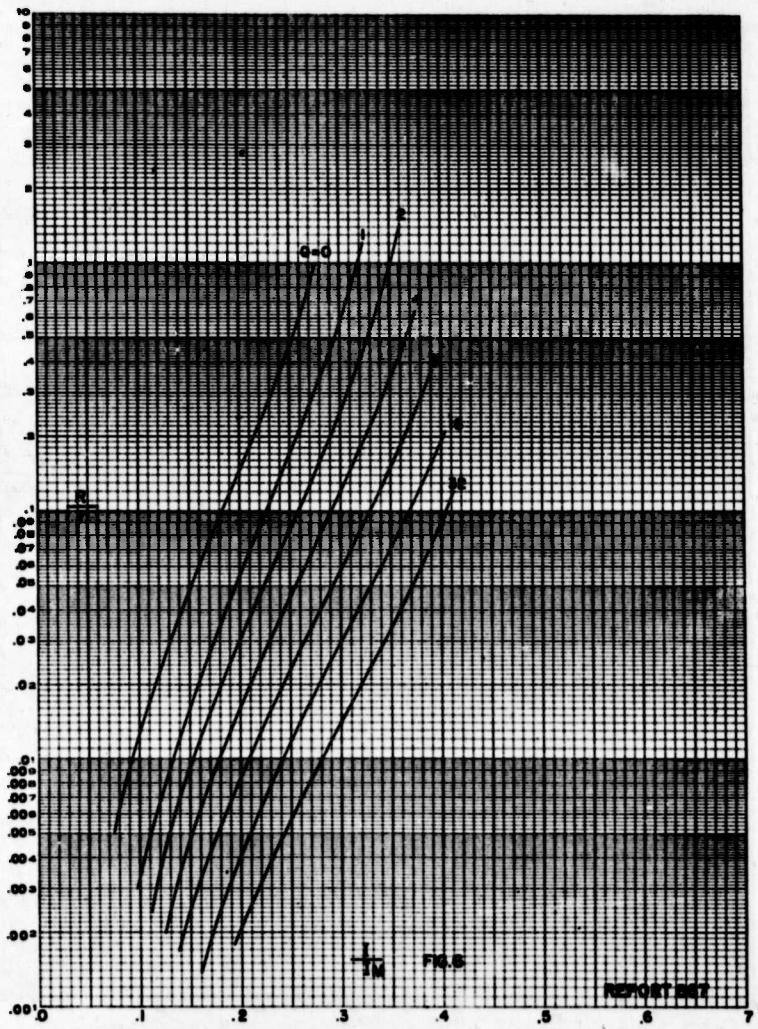


FIG.6

REPORT 867

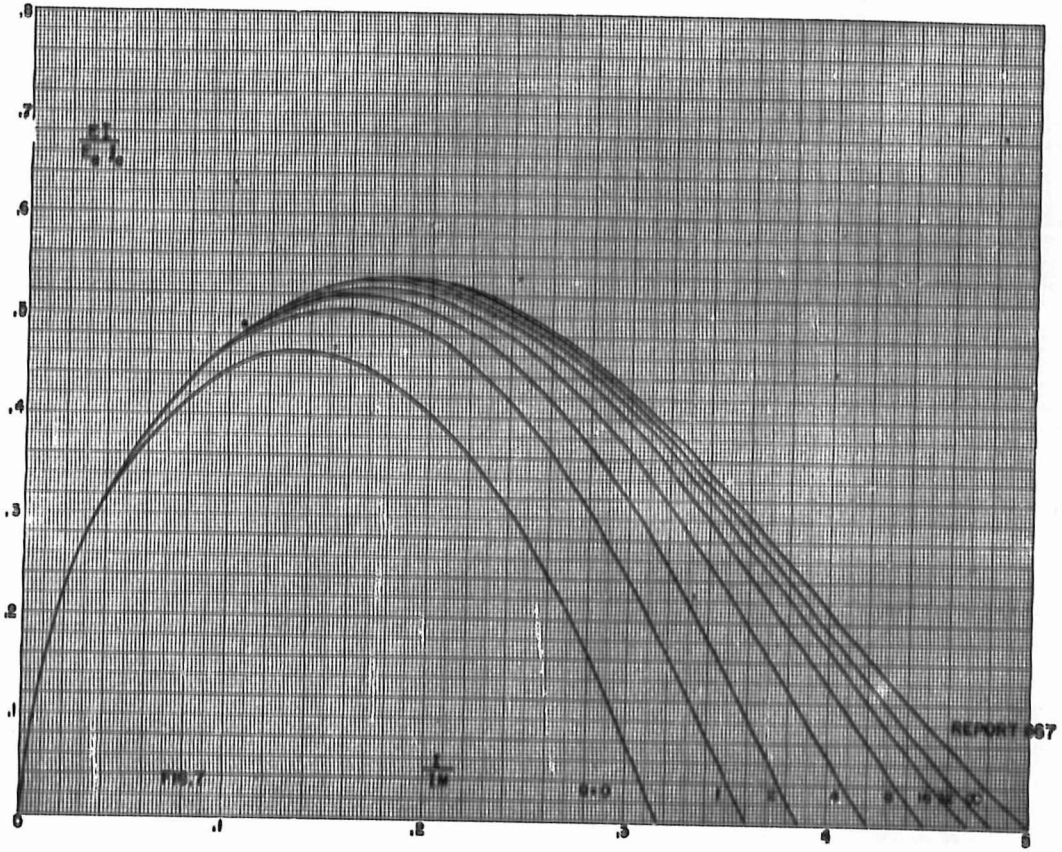
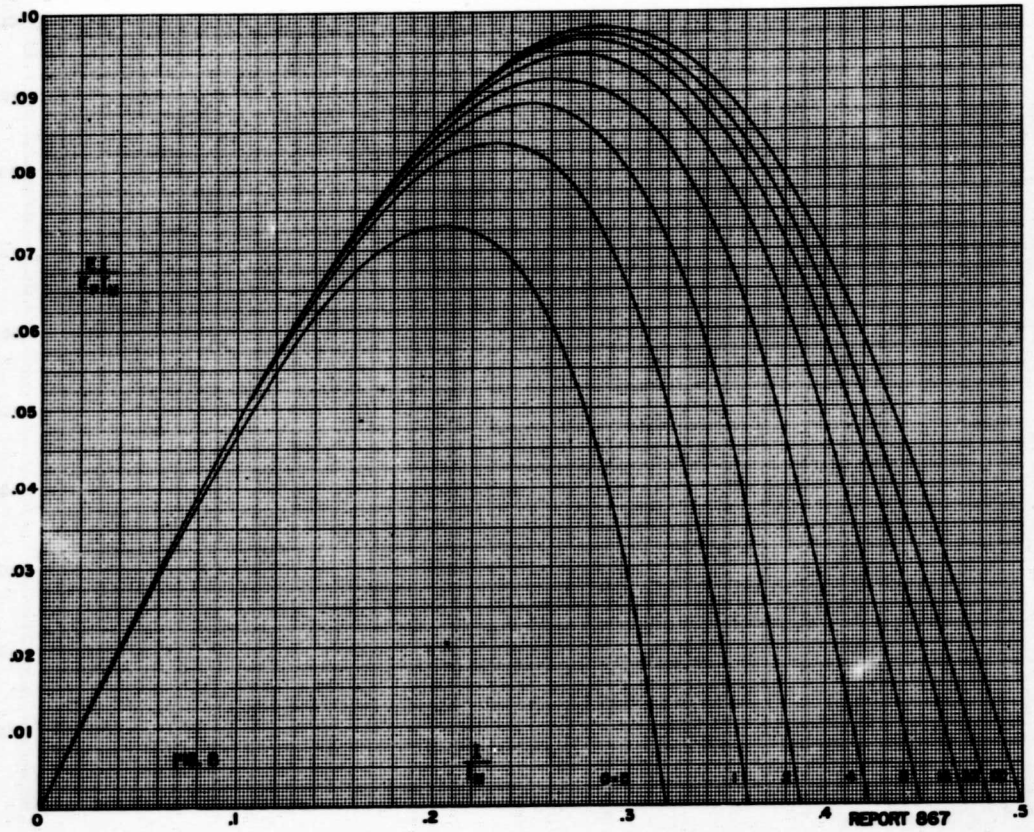


FIG. 7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.4
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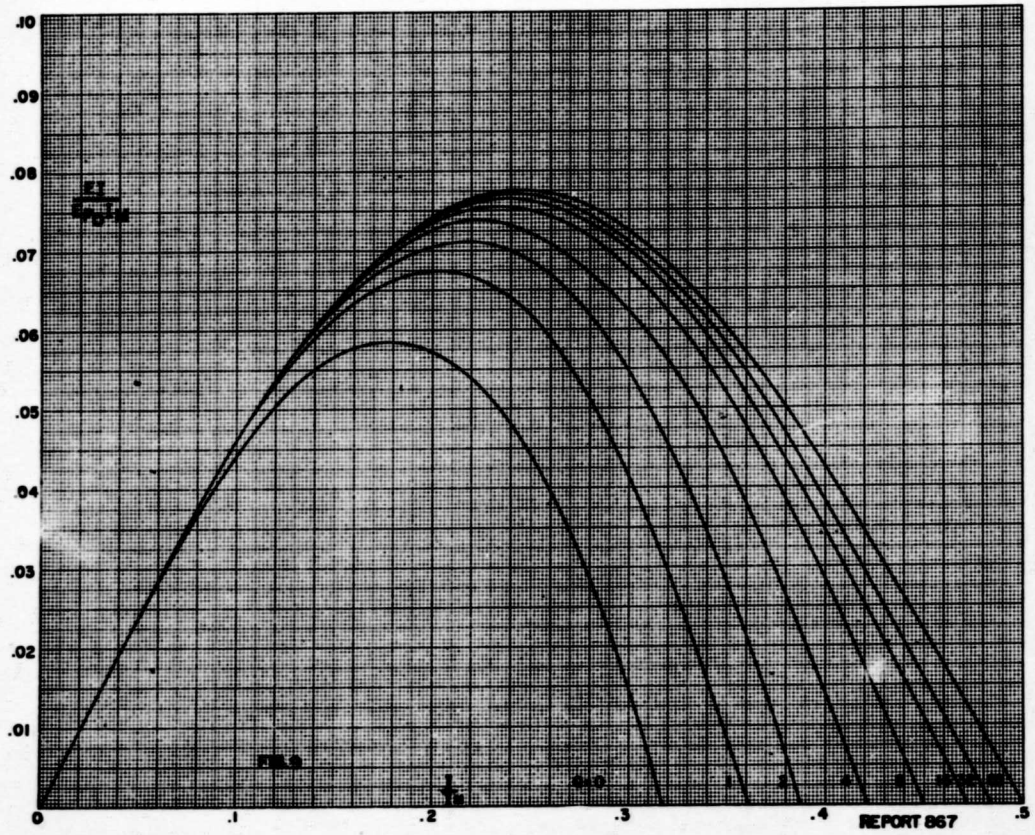
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REPORT 867

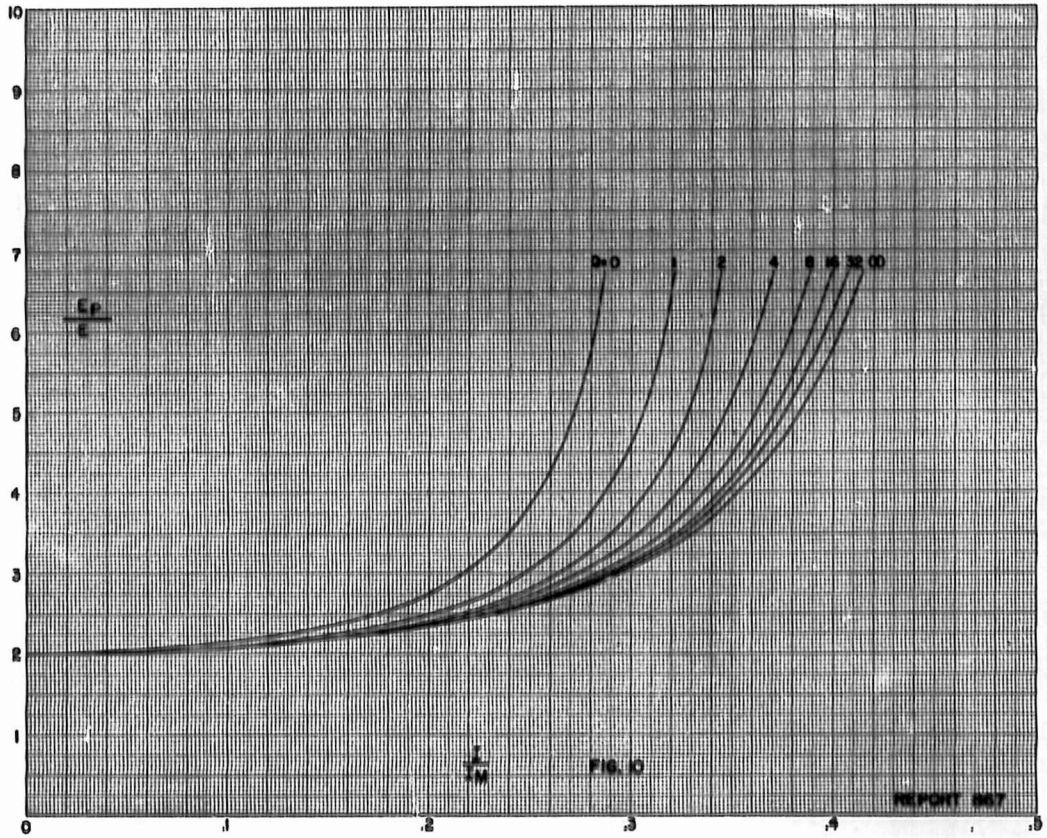
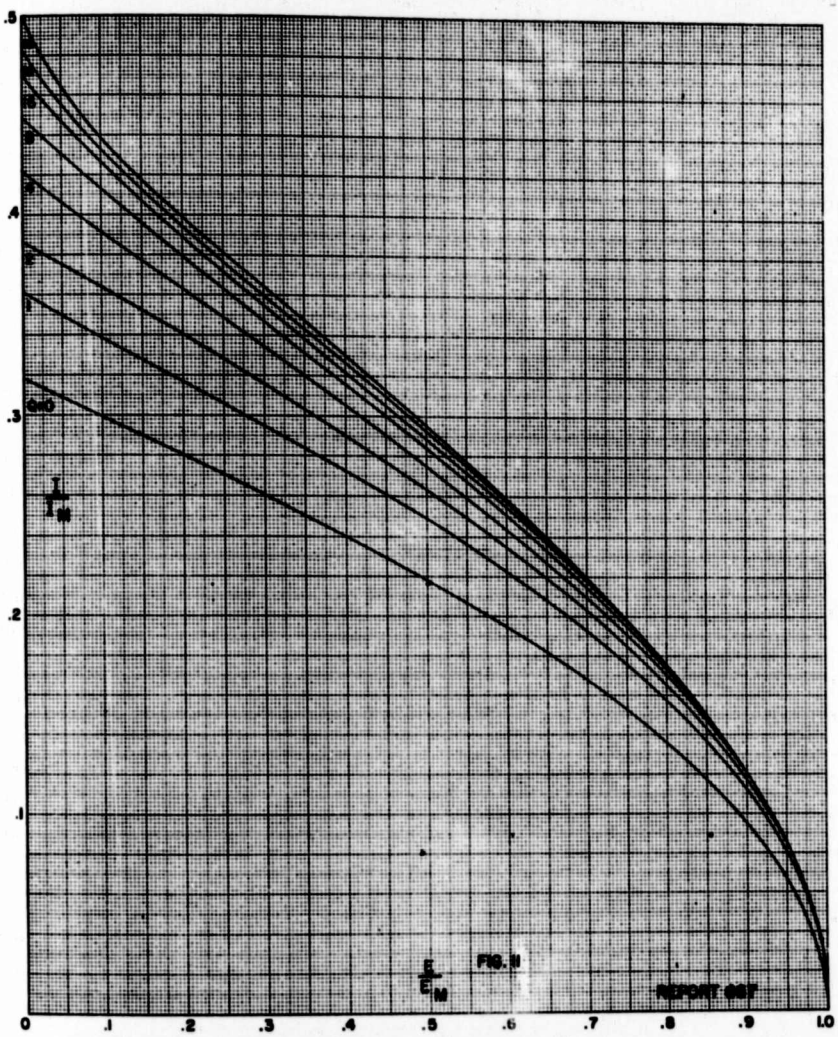


FIG. 10

REPORT 867


.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\frac{I}{I_M}$
$\frac{E}{E_M}$
FIG. 11
REPORT 687

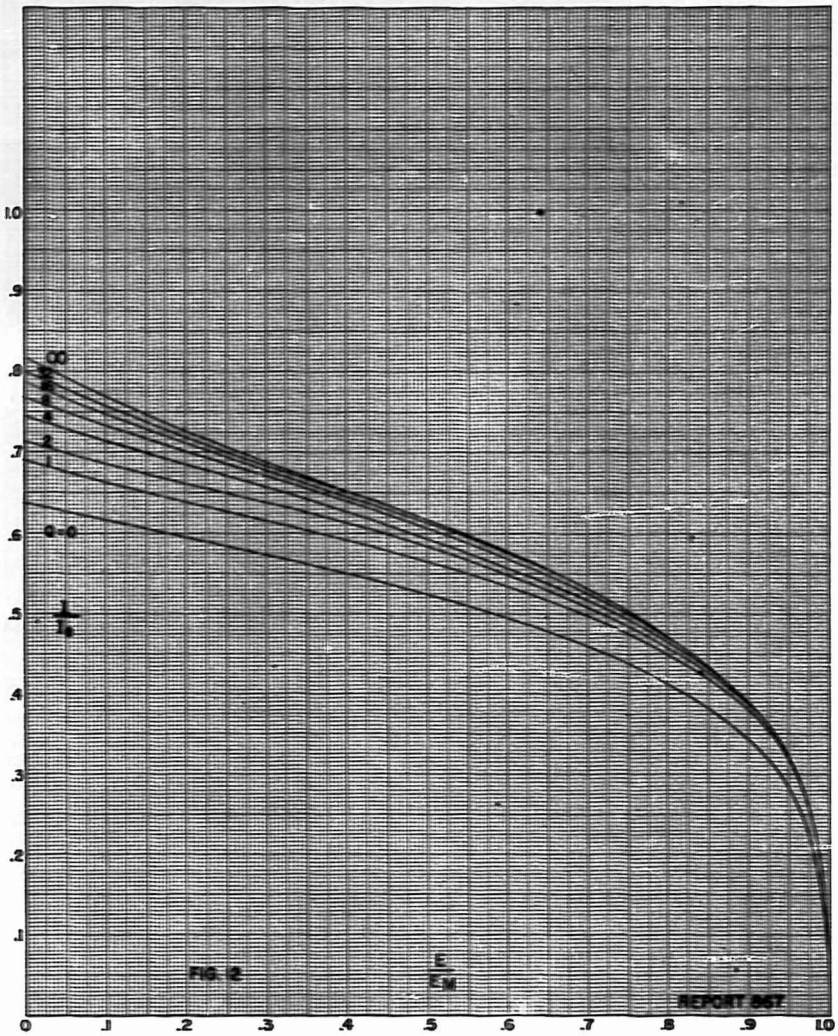


FIG. 2

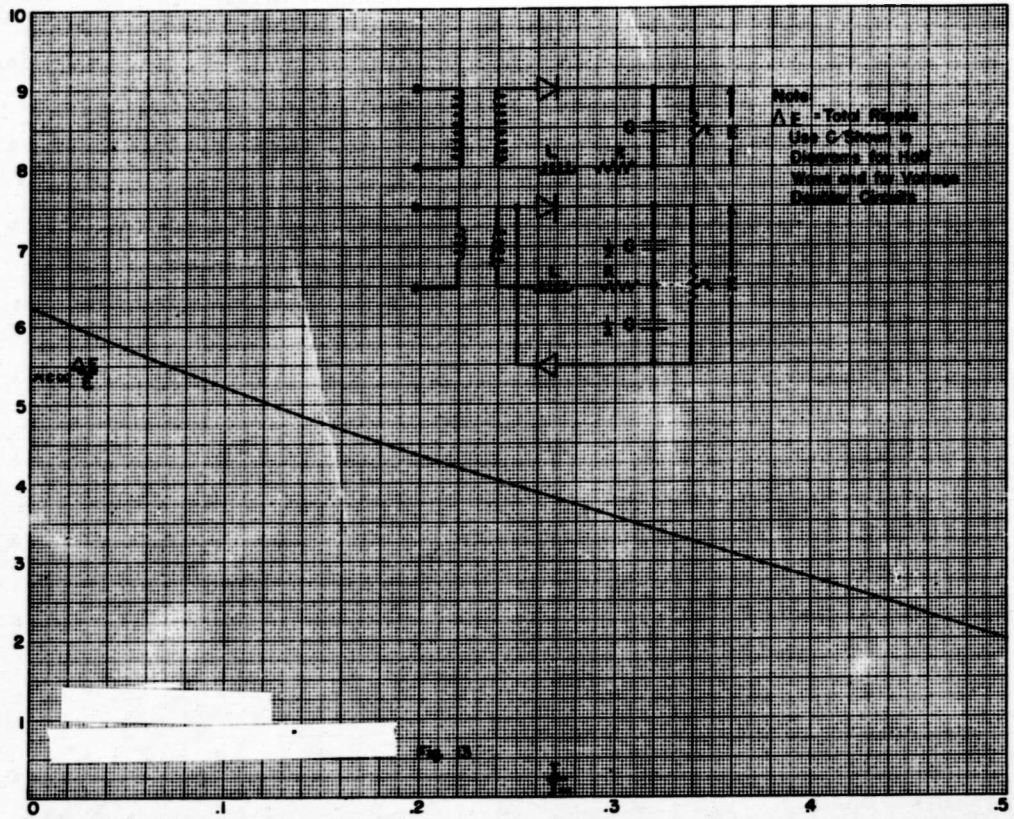

Note:
ΔE = Total Ripple
Use C Shown in
Diagrams for Half
Wave and for Voltage
Doubler Circuits
10
9
8
7
6
5
4
3
2
1
0
.1
.2
.3
.4
.5

Fig. 13

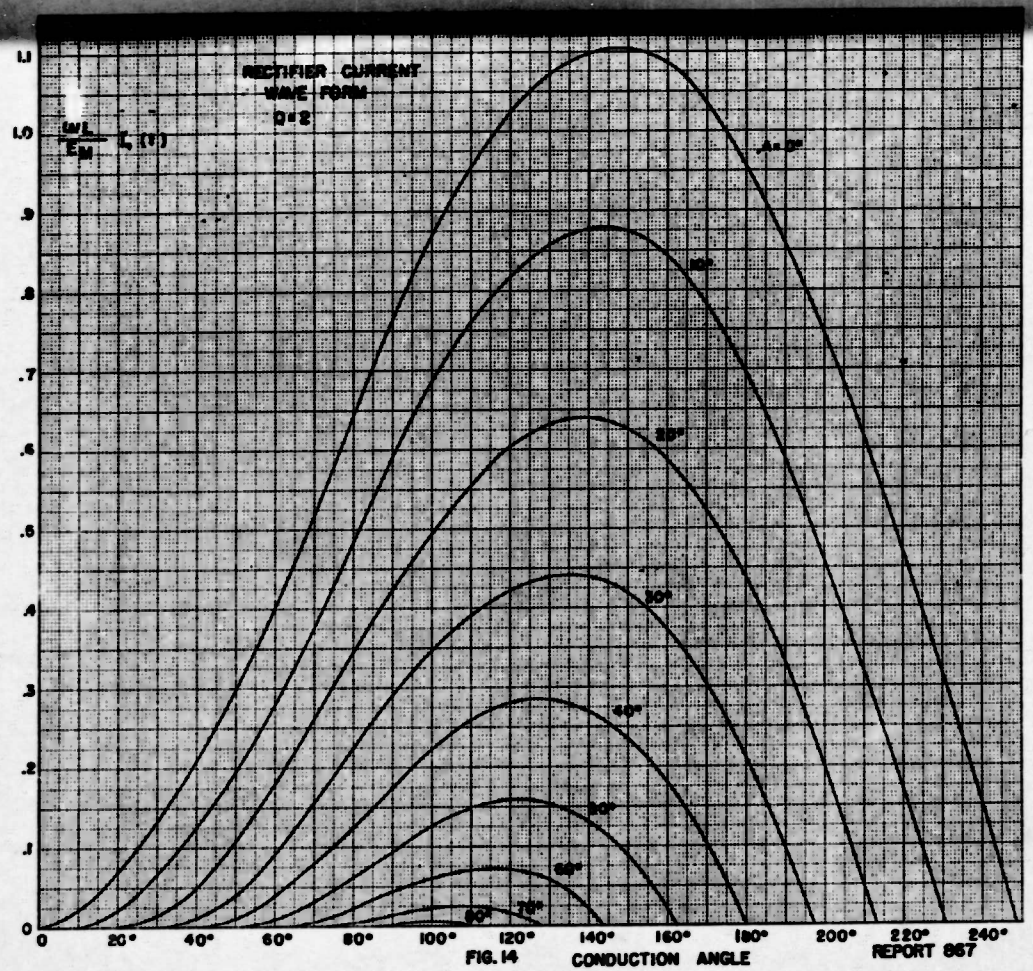

RECTIFIER CURRENT
WAVE FORM
Q=2
$\frac{\omega L}{E_M} I_0(t)$
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
20°
40°
60°
80°
100°
120°
140°
160°
180°
200°
220°
240°
A=0°
10°
25°
30°
40°
50°
60°
70°
80°
CONDUCTION ANGLE

FIG. 14

REEL – C

487

A.T.I.

13803

CADO CONTROL NO:	US CLASSIFICATION:	ATI NO:	OA NO:
-	Unclass.	ATI 13803	R-567

TITLE:
Analysis of a Half-Wave Rectifier Circuit Involving Inductance, Resistance, Capacity

AUTHOR(S):
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ORIGINATING AGENCY:
Mass. Inst. of Technology, Radiation Lab., Cambridge

FOREIGN TITLES:

PUBLISHED BY:	PUBLISHING NO:
OSRD, NDRC, Div 14	None

TRANSLATED BY:	TRANSLATION NO:

PREVIOUSLY CATALOGED AS:

MF-L-19 AUG 49 450M
CADO FORM NO. 5C — LIBRARY CARD

(23) * Half Wave rectifiers; Circuits

ATI-24498 (25)

REVISION (None)

ORIG. AGENCY NO. R-867

PUBLISHING AGENCY NO. (None)

TITLE: Analysis of a Half-Wave Rectifier Circuit Involving Inductance, Resistance, and Capacity

AUTHOR(S): Bothwell, F. E.; Crout, P. D.

ORIGINATING AGENCY: Massachusetts Inst. of Technology, Radiation Lab., Cambridge

PUBLISHED BY: Office of Scientific Research and Development, NDRC, Div 14

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Nov '45	Unclass.	U.S.	Eng.	41	diagrs, graphs

ABSTRACT:

An analytical analysis is presented of an approximate solution of a half-wave rectifier circuit involving inductance, resistance, and capacitance along with design curves for choosing optimum values of rectifier constants. The exact solution is involved, inasmuch as it contains the circuit constants in complicated expressions; the approximate analysis presented contains the circuit constants in a simplified form. The results of this analysis agree closely with those of laboratory experiments.

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DIVISION: Electronics ~~(3)~~ 8

SECTION: Components ~~(10)~~ 2

SUBJECT HEADINGS: Rectifier circuits, Half-wave - Analysis (81240)

ATI SHEET NO.: R-3-10-23

ELECTRONICS (COMPONENTS)