DETERMINATION OF THE DAMPING MOMENT IN YAWING FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS

By Sidney M. Harmon

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SUMMARY

A method for determining the damping moment in yawing for tapered wings with partial-span flaps is presented herein. Charts are given for untwisted wings with taper ratios of 0.25, 0.50, and 1.00, with aspect ratios from 6 to 16, and with center-span flaps extending from 25 to 100 percent of the wing semispan. The results are also applicable to tip-span flaps extending from 0 to 75 percent of the wing semispan. The calculated damping moment in yawing is compared with experimental results for a rectangular wing with a flap having a span 60 percent of the wing span.

INTRODUCTION

The calculation by Wieselsberger of the wing damping moment for an untwisted elliptical wing in yawing is summarized and extended in reference 1 to the case of an untwisted rectangular wing. Reference 2 presents the results of calculations for a wide range of taper ratio for untwisted wings and also for the special angle-of-attack distribution that results from the deflection of partial-span flaps of constant chord ratio when the rest of the span is at zero angle of attack.
The results in reference 2 for the yawing derivative due to induced drag, however, contain inaccuracies because of the omission of an important term in the formula for the yawing moment. Also, as noted in reference 2, the results cannot be applied by simple superposition to the case in which the lift is contributed simultaneously by partial-span flaps and by the plain portions of the wing. This limitation follows from the fact that the damping moment varies as the square of the angle-of-attack distribution; hence, separate components of the lift distribution have interactions that contribute to the resultant value of the yawing derivative.

The present analysis gives the results of calculations for the yawing derivative $\frac{\partial C_n}{\partial \alpha} \left( \frac{rb}{2V_s} \right)$ for untwisted tapered wings with partial-span flaps of constant chord ratio at various angles of attack. The results are presented for the same range of taper ratio as is considered in reference 2 and for center-span flaps extending from 25 to 100 percent of the wing semispan. The results presented for the center-span flaps may be applied to tip-span flaps extending from 0 to 75 percent of the wing semispan. The computations in the present paper do not include the part of the yawing moment contributed by the changes in spanwise profile drag. The effect of this factor on the yawing derivative is discussed in reference 2.
SYMBOLS

\( C_n \) yawing-moment coefficient due to spanwise induced-drag distribution

\( r \) angular velocity in yaw, radians per second

\( b \) wing span

\( V_s \) wind velocity along plane of symmetry of airplane

\( V \) local relative wind velocity at any section

\( y \) coordinate measured along lateral axis of airplane

\( \Gamma \) circulation around any section

\( \alpha_l \) section lift coefficient

\( c \) wing chord at any section

\( \alpha_l \) induced angle of attack at any section, radians

\( y_0 \) coordinate indicating fixed spanwise position

\( w \) normal component of velocity

\( N \) yawing moment due to spanwise induced-drag distribution \( (C_nq_Ssb) \)

\( q_S \) dynamic pressure at plane of symmetry \( \left( \frac{1}{2} \rho V_s^2 \right) \)

\( S \) wing area

\( \rho \) density

\( \theta \) parameter defining spanwise position \( \left( y = \frac{b}{2} \cos \theta \right) \)

\( \theta_0 \) parameter defining fixed spanwise position

\( c_s \) wing chord at plane of symmetry of airplane

\( m_{0s} \) slope of section lift curve at plane of symmetry of airplane, per radian

\( A_1, \ldots, A_n \) coefficients of Fourier series (see reference 3)
\( \mu \) parameter \( \left( \frac{c_{s m_0}}{4b} \right) \)

\( \lambda \) aspect ratio

\( C_{nr} \) yawing derivative \[ \partial C_{n}/\partial (\frac{rb}{2V_s}) \]

\( C_L \) over-all lift coefficient

\( K_1, K_2, \) and \( K_3 \) proportionality constants used to express induced-yawing-moment derivative in terms of its component parts

\( \lambda \) taper ratio, that is, ratio of extended tip chord to chord at plane of symmetry

\( m_0 \) slope of section lift curve, per radian

\( B \) arbitrary constant

\( \Delta C_{Lf} \) increment of over-all lift coefficient due to deflection of flaps

\( b_f \) flap span

\( c_{d_0} \) section profile-drag coefficient

\( \Delta c_{d_0f} \) increment of section profile-drag coefficient due to deflection of flaps

\( \Delta C_{nr} \) increment of yawing derivative due to spanwise changes in profile drag

Subscripts and superscripts:

\( w \) refers to \( c_L \) or \( c_{d_0} \)-distribution resulting from wing angle of attack

\( f \) refers to \( c_L \) or \( c_{d_0} \)-distribution resulting from deflection of flaps

\( f_c \) center-span flaps

\( f_t \) tip-span flaps
METHOD AND ANALYSIS

The method used in the present analysis for calculating the stability derivatives is based on the assumptions outlined in reference 2. Because the angular motions considered in the present analysis are small (rb/2Vs < 0.1), the influence of the curvature of the wing wake is considered to be negligible for practical purposes. Powers of rb/2Vs higher than unity are also neglected.

In yawing motion, the following relationships hold:
\[ V = V_s \left( 1 - \frac{rV}{V_s} \right) \]
where \( V \) is the local relative velocity at a section and \( y \) varies along the span from \( \frac{b}{2} \) to \(-\frac{b}{2}\)

\[ \Gamma = \frac{c_l c V_s}{2} \left( 1 - \frac{rV}{V_s} \right) \quad (1) \]

\[ a_{iYo} = \frac{w}{V} = \frac{1}{2 \pi V} \int_{b/2}^{-b/2} \frac{d\Gamma}{y - Yo} dy \quad (2) \]

where \( a_{iYo} \) is the induced angle of attack at any section \( Yo \) due to the trailing-vortex system.

\[ N = -q_s \int_{b/2}^{-b/2} c_l c a_1 \left( 1 - \frac{2rV}{V_s} \right) y \; dy \quad (3) \]

where \( N \) is the yawing moment due to the spanwise changes in induced drag.
It is convenient to make the substitution
\[ y = \frac{b}{2} \cos \theta \]

Then, by differentiating equation (1) with respect to the new variable \( \theta \), the downwash equation (2) becomes
\[
\alpha_{i_0} = \frac{1}{4mb} \int_{0}^{\pi} \left[ \frac{d(c_1 c)}{d\theta} + \frac{c_1 c}{2V_s} \frac{rb}{\cos \theta - \cos \theta_{o}} \right] d\theta
\]  
(4)

Equation (4) is equivalent to the downwash equation (6) given in reference 2.

If the Lotz method for determining the span load distribution (references 3 and 4) is followed, the circulation
\[ \Gamma = \frac{c_1 c V}{2} = \frac{c_s m_o s V}{2} \sum_{1}^{\infty} A_n \sin n\theta \]  
(5)

Substituting for \( c_1 c \) and \( \frac{d(c_1 c)}{d\theta} \) from equation (5) in equation (4) and integrating gives
\[
\alpha_1 = \frac{c_s m_o s}{4b} \left[ \sum_{1}^{\infty} nA_n \sin n\theta - \frac{rb}{2V_s} \sum_{1}^{\infty} A_n \cos n\theta \right]
\]

The yawing moment \( N \) from equation (3) now becomes
\[
N = -q_s \frac{b}{16} \left( c_s m_o s \right)^2 \int_{0}^{\pi} \left( \sum_{1}^{\infty} A_n \sin n\theta \sum_{1}^{\infty} nA_n \frac{\sin n\theta}{\sin \theta} \right) \left( 1 - \frac{rb}{V_s} \cos \theta \right) \cos \theta \sin \theta d\theta
\]  
(6)
Integration of equation (6) and conversion to the nondimensional coefficient $C_n$ results in

$$C_n = -\frac{\mu^2 n A}{4} \left[ -\sum_{l}^{\infty} (2n + 1)A_n A_{n+1} + \frac{r b}{2V_s} \left[ \sum_{l}^{\infty} 2n A_n^2 - \frac{A_{1}^2}{2} + \sum_{l}^{\infty} (2n + 2)A_n A_{n+2} \right] \right]$$  \hspace{1cm} (7)

where

$$\mu = \frac{c_s m \sigma_s}{4 b}$$

It will be noted that the even-numbered $A$-coefficients in equation (7) are directly proportional to the asymmetric lift produced in yawing. These coefficients are therefore proportional to $rb/2V_s$ and their products or powers greater than unity can be neglected. It follows that, if the even-numbered $A$-coefficients have the values for unit $rb/2V_s$, equation (7) becomes

$$\frac{\partial C_n}{\partial \frac{rb}{2V_s}} = C_{n \rho} = -\frac{\mu^2 n A}{4} \left[ -\sum_{l}^{\infty} (2n + 1)A_n A_{n+1} + \sum_{l}^{\infty} 2n A_n^2 \right. \left. - \frac{A_{1}^2}{2} + \sum_{l}^{\infty} (2n + 2)A_n A_{n+2} \right]$$  \hspace{1cm} (8)

In equation (8), each of the $A$-coefficients is a linear function of the angle-of-attack or $c_l$-distribution. For a given wing, therefore, similarly numbered $A$-coefficients due to any number of $c_l$-distributions may be superposed, and the
sum of these similarly numbered coefficients may be substituted in equation (8) to give the resultant value for $C_{n_r}$ due to the combined lift distributions.

The subsequent analysis considers the case of the $c_l$-distribution caused by the combined distributions of the angle of attack and the deflection of partial-span flaps. The superscripts $w$ and $f$ refer to the $c_l$-distributions due to the angle of attack and to the flap deflection, respectively. By combining these $c_l$-distributions, equation (8) becomes

$$C_{n_r} = -\frac{\mu^2 n a}{4} \left[ \sum_{1}^{\infty} \frac{(2n + 1) (A_n^w + A_n^f) (A_{n+1}^w + A_{n+1}^f)}{2} + \sum_{1}^{\infty} 2n (A_n^w + A_n^f)^2 - \frac{(A_1^w + A_1^f)^2}{2} \right]$$

Equation (9) may be expanded to the form

$$C_{n_r} = -\frac{\mu^2 n a}{4} \left[ \left( \sum_{1}^{\infty} (2n + 1) A_n^w A_{n+1}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} \right) + \sum_{1}^{\infty} (2n + 2) A_n^w A_{n+2}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} \right]$$

$$+ \sum_{1}^{\infty} (2n + 2) A_n^w A_{n+2}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} + \sum_{1}^{\infty} (2n + 2) A_n^f A_{n+2}^f \right]$$

$$= -\frac{\mu^2 n a}{4} \left[ \left( \sum_{1}^{\infty} (2n + 1) A_n^w A_{n+1}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} \right) + \sum_{1}^{\infty} (2n + 2) A_n^w A_{n+2}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} \right]$$

$$+ \sum_{1}^{\infty} (2n + 2) A_n^w A_{n+2}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} + \sum_{1}^{\infty} (2n + 2) A_n^f A_{n+2}^f \right]$$

$$= -\frac{\mu^2 n a}{4} \left[ \left( \sum_{1}^{\infty} (2n + 1) A_n^w A_{n+1}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} \right) + \sum_{1}^{\infty} (2n + 2) A_n^w A_{n+2}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} \right]$$

$$+ \sum_{1}^{\infty} (2n + 2) A_n^w A_{n+2}^w + \sum_{1}^{\infty} 2n (A_n^w)^2 - \frac{(A_1^w)^2}{2} + \sum_{1}^{\infty} (2n + 2) A_n^f A_{n+2}^f \right]$$

(10)
The three groups of terms on the right-hand side of equation (10) are equal to the yawing derivative due to the \( w \)-distribution, to the interaction of the \( w \)- and \( f \)-distributions, and to the \( f \)-distribution. It should be noted that, in equation (10), the even-numbered \( A \)-coefficients have the values appropriate to unit \( rb/2V_s \). In the equation, each \( A \)-coefficient is directly proportional to the over-all lift coefficient \( C_L \) that results from the particular \( C_L \)-distribution to which this \( A \)-coefficient refers. It follows that

\[
C_{n_r} = \frac{C_{n_{rw}}}{C_{lw}} C_L^2 + \frac{C_{n_{wrf}}}{C_{lw} \Delta C_{r_f}} C_L \Delta C_{r_f} + \frac{C_{n_{rf}}}{\Delta C_{r_f}^2} \Delta C_{r_f}^2
\]  

(11)

where \( C_{n_{rw}}, C_{n_{wrf}}, \) and \( C_{n_{rf}} \) are obtained from equation (10), and the superscript \( \text{wf} \) denotes the interaction of the \( w \)- and \( f \)-distributions.

For a given wing, equation (11) may be written

\[
C_{n_r} = K_1 C_L^2 + K_2 C_L \Delta C_{r_f} + K_3 \Delta C_{r_f}^2
\]  

(12)

where \( K_1, K_2, \) and \( K_3 \) are constants referring to the yawing derivative per unit \( C_L^2 \) for the angle-of-attack and flap-deflection distributions and for the interaction of these distributions.
RESULTS AND DISCUSSION

Theoretical Results

Calculations were made in the present analysis to determine the yawing derivative for the lift distribution that results from a combined uniform and symmetrical distribution of angle of attack and uniform deflection of partial-span flaps of constant chord ratio. The computations were made by equation (10). The A-coefficients for the angle-of-attack and flap-deflection distributions were obtained by the method given in references 2 to 4. (See equation (9) of reference 2.) The computations assumed a value of 5.67 per radian for the slope of the section lift curve. The range of the investigation includes three taper ratios: 0.25, 0.50, and 1.00; three aspect ratios: 6, 10, and 16; and flaps extending from the wing center to 25 to 100 percent of the wing semispan. The plan forms of the wings, which have rounded tips, are shown in figure 1 of reference 2. The results of the calculations are given in figures 1 to 3. Figures 1 and 3 are similar to figures 13 and 12, respectively, of reference 2. The results presented in these figures in reference 2 are in error because of the omission of the term \(-\frac{Al^2}{2}\), which appears in equation (8) of the present paper.

The variation of the yawing-derivative factor \(\text{K}_2\) with aspect ratio is shown in figure 1 for taper ratios of 0.25, 0.50, and 1.00, for a uniform and symmetrical angle-of-attack
distribution, and for $\Delta C_{L_p} = 0$. Figure 1 shows that the magnitude of $K_1$ decreases as the aspect ratio decreases and as the taper becomes sharper.

Figure 2 shows the variation with flap span of $K_2$, which gives the yawing derivative due to the interaction of the combined distributions of angle of attack and flap deflection. The actual computations for the taper ratios of 0.25, 0.50, and 1.00 were made for flap spans of 50 and 100 percent of the wing span. In order to obtain the correct fairing for the curves, similar results were computed for the elliptical wing with $A = 10$, for flap spans of 25, 50, 75, 87.5, and 100 percent of the wing span. The data of figure 2 indicate that the magnitude of $K_2$ decreases with aspect ratio and increases with flap span. For flap spans that are greater than approximately 50 percent of the wing span, the magnitude of $K_2$ increases as the wing becomes less tapered; the increase becomes greater as the flap span is increased. For flap spans of approximately 50 percent of the wing span, $K_2$ varies only slightly with wing taper. It is interesting to note in figure 2 that, for flaps of approximately 30 percent of the wing span, the curve for $K_2$ reverses sign; this condition indicates a destabilizing influence for this factor.

The variation of $K_3$ with flap span for uniform deflection of flaps of constant chord ratio and for $C_{L_w} = 0$
is shown in Figure 3. The actual computations and fairing of the curves were obtained in the same manner as those for Figure 2. The values for $K_3$ show variations with wing taper, aspect ratio, and flap span similar to those for $K_2$ in Figure 2.

The total value of the yawing derivative due to induced drag for the combined distributions of angle of attack and flap deflection is obtained from Figures 1 to 3 and Equation (12).

It should be noted from the derivation of Equation (12) that the values for $C_{n_r}$ obtained from this equation and also from Figures 1 to 3 are equally applicable when the angle of attack or the flap deflection is either positive or negative, as long as proper account is taken of the signs. Because the analysis in deriving the formula for $C_{n_r}$ shows that $K_1$, $K_2$, and $K_3$ are negative for either a positive or a negative angle of attack or flap deflection, the application of Equation (12) and Figures 1 to 3 requires only a consideration of the sign of $C_{L_w}$ and $\Delta C_{L_f}$.

Figures 1 to 3, as noted previously, are based on a value of 5.67 per radian for the slope of the section lift curve. In order to apply the figures for other values of $m_o$ - for example, $m_o = 5.67B$, where $B$ is an arbitrary constant - each ordinate is multiplied by $B^2$ or each curve in the figures can be shifted to an equivalent aspect
ratio \( BA \), where \( A \) is the desired aspect ratio. The figures refer specifically to center-span flaps but the data may be applied to tip-span flaps of 0 to 75 percent of the wing span by the following relationship:

\[
C_{n_{w+f_t}} = K_1 \left( C_{L_w} + C_{L_{f_c+t}} \right)^2 - K_2 \left( C_{L_w} + C_{L_{f_c+t}} \right) \Delta C_{L_{f_c}} + K_3 \Delta C_{L_{f_c}}^2 \quad (13)
\]

In the equation \( f_t \) refers to the tip flaps, \( f_c \) refers to center flaps for which the span is \( \left( 1.00 - \frac{b_{f_t}}{b} \right) b \). If \( \frac{b_{f_t}}{b} \) is the ratio of the span of the tip flap to the wing span, and \( f_{t+c} \) indicates full-span flaps. The flap deflection corresponding to \( C_{L_{f_c+t}} \) and \( C_{L_{f_c}} \) is the same as that for the tip flaps. Equation (13), of course, can be used in conjunction with figures 1 to 3 only for the particular case of the combined \( c_l \)-distribution that results from a uniform deflection of flaps of constant chord ratio and from a uniform and symmetrical angle-of-attack distribution.

Comparison of Theoretical with Wind-Tunnel Results

The calculated results presented in figures 1 to 3 are compared in figure 4 with wind-tunnel results obtained from reference 5 for a rectangular wing with partial-span flaps of constant chord ratio. The wing has the following characteristics:

**Tips**
- Aspect ratio, \( A \): Square 6
- Taper ratio, \( \lambda \): 1.00
- \( b_{f_t}/b \): 0.60
- \( \Delta C_{L_{f_c}} \): 0.56
- \( c_{d_{ow}}, \) per unit wing span: 0.024
- \( \Delta c_{d_{of}}, \) per unit flap span: 0.08
and

\[ C_{L_w} = C_L - \Delta C_{L_f} \]

where \( C_L \) is the over-all lift coefficient. From figures 1 to 3, based on rounded tips, \( K_1 = -0.0225 \), \( K_2 = -0.0219 \), and \( K_3 = -0.0125 \); therefore, the yawing derivative due to induced drag

\[ C_{n_p} = -0.0225C_{L_w}^2 - 0.0219C_{L_w} \Delta C_{L_f} - 0.0125 \Delta C_{L_f}^2 \quad (14) \]

The yawing derivative due to profile drag from equation (12) of reference 2 is

\[ \Delta C_{n_p} = -0.33(0.024) - 0.072(0.08) \quad (15) \]

By combining equations (14) and (15), the yawing derivative for the wing in terms of the over-all lift coefficient becomes

\[ C_{n_p} = -0.0225C_L^2 + 0.0129\frac{1}{4}C_L - 0.01779 \]

The comparison of the theoretical values with the wind-tunnel results is shown for a range of lift coefficient from 0 to 1.6. (See Fig. 4.) It will be seen that the agreement generally is good except at high lift coefficients. The greater experimental damping moment at the high lift coefficients is probably due to the localized tip vortex caused by the square tips, which adds an increment of drag.
increasing approximately as the square of the lift coefficient.

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REFERENCES


Figure 1.- Variation with aspect ratio of yawing-derivative factor due to induced drag for uniform and symmetrical angle-of-attack distribution. \( \Delta C_{Df} \), \( C_{nr} = k_1 C_{Lw}^2 \).
Figure 2.— Variation with flap span of yawing-derivative factor due to induced drag for interaction of uniform and symmetrical angle-of-attack distribution and uniform deflection of constant-chord-ratio flaps. $C_n = K_2 C_L \Delta C_y$. 
Figure 3—Variation with flap span of yawing-derivative factor due to induced drag for uniform deflection of constant-chord-ratio flaps. \( C_{\phi} = 0 \).

\[ C_{\phi} = K_y \Delta C_{\phi}^2 \]
Figure 4.- Comparison of theoretical and wind-tunnel results for yawing derivate for partial-span flapped wing.

\[ A = 6; \lambda = 1.00; \frac{b_f}{b} = 0.60; \Delta C_{L_f} = 0.56; \text{ and } C_{L_w} = C_L - 0.56. \]
A method is presented for determining the damping moment in yawing for tapered wings with partial-span flaps. Charts are given for untwisted wings with taper ratios of 0.25, 0.50, and 1.00 with aspect ratios from 6 to 16 and with center-span flaps extending from 25 to 100% of the wing semispan. The results are also applicable to tip-span flaps extending from 0 to 75% of the wing semispan.
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