CHARACTERIZATION OF LANL A1-SERIES MARX FLASH RADIOGRAPHY SOURCE∗

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Abstract
An end to end electrical model has been developed for a flash-radiography diagnostic used at Los Alamos National Laboratory (LANL). The diagnostic consists of a compact, 108 J, A1-series Marx[1] that drives a bremsstrahlung diode radiation source. The modeling was done using a transmission-line code, BERTHA[2], developed at the Naval Research Laboratory (NRL). Model predictions agree well with experimental data. The calculations suggest that, as presently configured, the system efficiency can be increased.

I. INTRODUCTION
The diagnostic consists of a compact, 108 J, A1-series Marx that drives a small, cylindrical bremsstrahlung diode radiation source through a length of 50 n cable that is long compared to the pulse. The modeling was done using a transmission-line code BERTHA that was developed at NRL and recently improved for this work. The most significant addition to the code is a physics-based, e-beam diode model for the cylindrical diode that includes both space-charge-limited and critical-current phases. The diode model was initially benchmarked against data from experiments at NRL that used a similar, lower energy (17 J) Marx to drive the standard LANL e-beam diode. The full, end to end model was then compared with data from the LANL radiographic system. The model predictions and the experimental data were in good agreement. Analysis reveals that only about 31 J is delivered to the load and the endpoint voltage is significantly lower than expected. This shortfall is due to losses in the barium-titanate ceramic capacitors and in switching. The modeling indicates that improvements may result in a 40% increase in peak diode power thus increasing the dose.

II. MODELING
Detailed, lumped-circuit models for the A1 Marx were created at LANL[3] and NSWC[4] and serve as the starting point for this work. An equivalent circuit was formulated in BERTHA, and the code was upgraded to handle specific elements of the Marx. A physics-based, cylindrical diode model was developed and added to the code to complete the model.

A. Equivalent Circuit
The A1 Marx has 25 stages with two, 2.7 nF capacitors per stage charged to a maximum 40 kV. A drawing of a single Marx stage is shown in Fig. 1a. Overlaying the drawing is a circuit diagram which represents the equivalent of each physical element including parasitic reactances. A simple RLC analysis of NSWC data taken with the Marx terminated in a low inductance short circuit indicated that the total inductance is about 950 nH and the total resistance is about 7.3 Ω. The inductance is accounted for in three stages: the switch arc, the rest of the stage (including the capacitor) and the output section (~100 nH). The module inductance then is about 850 nH/25=34 nH. In the end, the best distribution was about 20 nH for each switch and about 12.5 nH for the remaining components in the stage. The translation of the circuit diagram to the BERTHA elements is shown in Figure 1b. The switch inductance was split into two, 10-nH elements for symmetry, with the actual switch junction in between. The same was done for the remaining inductance with the capacitor in between. A series resistor junction was inserted between the two sets of inductances which represented all the resistive contributions from the capacitor ESR, contacts, skin resistance, etc. The switch resistance was accounted for in the switch model running at the junction. The total resistance in each module was targeted at around 1/25th of the total of 7.3 Ω found in the RLC calculation. In the modeling done here, neither a charging resistor chain nor a stage to stage capacitance Css is included due to the complexities of connecting parallel elements in BERTHA. The effect of the charging resistors on the output pulse is a small resistive loss. These losses were accounted for in the stage resistance element. In the A1-series design, Css is made to be significantly less than the stage-to-ground capacitance, so its effects on the Marx
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switching and erection are less deleterious. In the BERTHA model, switching occurs as each stage reaches a specified threshold voltage. This criterion is sufficient to model the circuit behavior of interest here. If detailed modeling of the Marx erection were required, $C_{st}$ would have to be included.

![Image](image)

Figure 1- A1-series Marx: a) single stage equivalent circuit schematic and b) BERTHA module.

**B. BERTHA**

For the A1 Marx, the rise time of the pulse is comparable to the electrical length of the unit so the transmission line code BERTHA is used to ensure that the finite, electrical transit-time effects associated with distributed circuit elements is properly modeled. For this work, the code was improved to handle modules (e.g. Marx stages), a physics-based, cylindrical-pinched-beam diode model was implemented, and a subroutine to handle the capacitive voltage coefficient of the ceramic capacitors was added. The voltage coefficient model adjusts the capacitance of the specified element as a linear function of the voltage. Also added was a closing switch resistance model based on T. L. Martin’s analysis[5]. Inputs to the model are the switch gap, gas pressure and species, and number of channels, as well as an initial channel resistance. A self-consistent, time-dependent switch resistance was calculated.

**C. Cylindrical Diode**

A model for the cylindrical e-beam diode load[6] was built by examining the current-voltage (I-V) characteristics from an intrinsic, diode physics point of view, including effects of geometry and self electric and magnetic fields. The 2½-D particle-in-cell (PIC) code “MAGIC”[7] was used to characterize the charged particle flows. Benchmarking and determining appropriate values for turn-on thresholds and gap closure were investigated experimentally (see Section IIIA). The I-V characteristics for many different geometries were examined for voltages in the range 0.1-2 MV. Charged-particle flow in the diode can be divided into three fundamental regimes: a space-charge-limited (SCL) regime for currents small compared to the critical current, a magnetically-limited (ML) regime for currents limited to the critical current, and an intermediate regime where the beam is weakly pinched. In the SCL regime, the I-V characteristic of the cylindrical diode is described by the Langmuir-Blodgett (L&B) formula. In the ML regime, the diode current is high enough that the self-magnetic field associated with it can significantly alter the electron trajectories in the diode AK gap if ions are present. High current densities (i.e. a pinched beam) can result. This high current operation is described by the critical current formula[8]. At intermediate currents, the beam is weakly pinched and neither formula is accurate. The results of the PIC simulations are summarized by the universal curve of Fig. 2 that describes the diode I-V characteristic over a wide range of conditions. This curve was developed as a result of analyzing many runs with different geometries (Fig. 2 table) and through the intuition gained from the scaling laws provided by the L&B and critical current formulas. It is characterized by a single parameter, $V_T$, which is the voltage where the L&B current equals the critical current. This parameter contains all the detailed information about the diode geometry and the presence of ions. For voltages small compared to $V_T$, the current follows L&B scaling. For voltages large compared to $V_T$, the current follows critical current scaling. For intermediate voltages there is a smooth transition between the two regimes. This universal curve was implemented in BERTHA to accurately model the transition from SCL to ML flow.

**III. BENCHMARKING**

Experiments were conducted in order to validate the cylindrical diode and A1 Marx models. Code predictions compared favorably with experimental results.

**A. Cylindrical Diode**

Experiments were done at NRL to benchmark the diode model and to determine appropriate values for electron and ion emission thresholds, $E_e$ (kV/cm) and $E_r$ (J/cm²); anode and cathode plasma evolution times, $t_A$ and $t_c$ (ns); and anode and cathode closure velocities, $v_A$ and $v_c$ (cm/µs). An A1 Marx was not available so a 17.3 J, 8 stage Marx of similar design was used as the driver for the standard LANL diode. Because a 33 ft. long, 50 Ω cable essentially isolates the Marx from the diode load during the time of the main output pulse,
detailed modeling of the small Marx was not necessary for these tests. An open circuit voltage waveform into the 50 Ω cable was derived from the measured Marx output current. This voltage waveform was then used to drive the BERTHA model of the standard LANL load assembly which included the NRL cylindrical diode model. A comparison with measured data is shown in Fig. 3 for two different anode-cathode (AK) gaps. In both cases the cathode radius was 2.77 mm. The anodes were tungsten rods, 0.8 mm or 2.0 mm in radius. Note that the values of the diode model adjustable parameters were the same, and agreement was attained in both cases.

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\begin{array}{cccccc}
R_C (cm) & R_A (cm) & L (cm) & Ions & V_I (kV) \\
0.37 & 0.15 & 1 & N & 564 & 1.4 \\
0.37 & 0.15 & 1 & Y & 333 & 1.6 \\
0.37 & 0.15 & 2 & N & 229 & 1.3 \\
0.18 & 0.075 & 1 & N & 258 & 1.4 \\
0.74 & 0.3 & 1 & N & 1320 & 1.4 \\
0.37 & 0.15-0.0 & 1 & N & 1340 & 1.4 \\
0.37 & 0.15-0.0 & 1 & Y & 706 & 1.6 \\
0.37 & 0.15-0.0 & 2 & N & 590 & 1.4 \\
0.37-0.22 & 0.15-0.0 & 1 & N & 490 & 1.6 \\
0.37-0.22 & 0.15-0.0 & 2 & Y & 224 & 1.6 \\
\end{array}
\]

Figure 2- Cylindrical diode characterization and modeling.

B. Short Circuit Shot

Experimental data from LANL work is compared with the BERTHA model in Fig. 4. The data was taken with a 30 foot length of 50 Ω cable terminated in a short circuit. In the model, a distributed total of 8.5 pF of stage-to-ground capacitance gave the best comparison with the data. The initial resistance used in the switch model was 4.5 Ω. This low value is probably not a realistic choice but is required to match the falltime features of the initial pulse. The Marx delivers only about 5.5 kA to the 50 Ω load of the cable. This equates to a voltage of about 275 kV at the Marx/cable interface.

C. Full System

Comparison of the complete end to end BERTHA model with LANL data from a flash-radiography test is shown in Fig. 5. The standard e-beam diode is used with a ¼" dia. cathode and a 3 mm dia. tapered tungsten anode and the Marx is charged to 38 kV. Since there are no electrical diagnostics other than a Rogowski loop at the Marx output, diode behavior was examined by studying the reflected pulse from the diode as measured by the loop. The values used for the parameters in the cylindrical diode model are given in the figure. This particular set was chosen so that the overall fit to the reflected pulse was best on average and the width of the radiation pulse (IV product) was comparable with the measurement. If instead we use \( v_C = 2.2 \text{ cm/μs}, \quad v_A = 1.2 \text{ cm/μs}, \quad t_C = 12 \text{ ns}, \quad \text{and} \quad t_A = 5 \text{ ns}, \) the rise time and peak of the reflected pulse match the data exactly but the falltime behavior is less accurate and the radiation pulsewidth is reduced. BERTHA calculations indicate that the peak diode voltage \( V_D \) is about 200 kV and the peak diode current \( I_D \) is about 8 kA.

IV. ANALYSIS AND DISCUSSION

From the calculations and measurements in Fig. 5, 31 J is contained in the initial pulse launched on the 50-Ω cable. The model predicts that only 21 J is delivered to the diode as a result of the diode impedance mismatch. The BaTiO₃ capacitors used in the Marx have a poor voltage coefficient \( C(V) \) (40% of nominal capacitance lost at full 40 kV charge) and poor tolerance (C is +80%/-20%). So, in the worst case, there could be as little as 48 J actually stored at 38 kV charge. In the full,
end-to-end simulation that best fit the data, a linear voltage coefficient of 0.8 %/kV was used and the stage capacitance was increased by 10%. These values indicate that there was initially only 74 J stored in the Marx, compared with 97.5 J for the nominal Marx capacitance at 38-kV charge. Thus, the efficiency for the energy in the cable on first pulse is \( \frac{31}{74} = 42\% \). The switch arcs dissipate a total of 22 J. Series resistance accounts for another 4 J lost. Using the BERTHA model of the A1 Marx only (no cable or diode), with the Marx terminated in a pure resistive load \( R_L \), the power delivered to the load was greatest when \( R_L = 45 \) Ω. Thus the 50 Ω cable is a good match to the Marx and power losses are only ~1% at the interface mismatch. Thus, identifiable losses account for about 80% of the actual stored energy. Use of the actual, non-linear \( C(V) \) and the measured individual capacitances, incorporating a more refined switch model, and inclusion of other losses should result in more accurate energy inventory.

There are two main areas for improvement: better matching of the diode impedance (potential 50% increase in diode energy) and improvements in the Marx. Significant losses are experienced due to the poor ceramic capacitors, both from lower initial energy stored and from increasing capacitance as the Marx discharges. Similar capacitors made with a SrTiO3 ceramic have a much better \( C(V) \). However, for the same voltage rating, the capacitance is lower so that probably three capacitors per stage would be required. Fitting three capacitors into the available space is mechanically feasible. Simulations suggest that small improvements in switching losses can be realized by reducing the switch gaps by half and increasing the pressure by a factor of two to maintain the voltage hold-off. For ideal capacitors and the above changes in the switches, the full end-to-end simulation shows 64 J delivered to the cable (66% efficiency, assuming 97.5 J stored). For this case, with the diode geometry of Fig. 5, \( V_D = 295 \) kV and \( I_D = 11.5 \) kA. This results in an increase of nearly 40% in peak diode power.

Other improvements can be considered but require substantial physical changes. Converting to +/- charging would halve the number of switches but the switch gap would see twice the voltage. Simply increasing the gap by a factor of two would negate any gains since switching loss is proportional to the gap. However, the pressure could be increased and the gap decreased, and perhaps 10 J may be saved overall. More capacitance per stage, if mechanically feasible, would store more energy and lower the characteristic impedance of the generator. Adding more stages at some point is not beneficial because it means also increasing the number of switches and the inductance.

Figure 5- Comparison of full end-to-end BERTHA model with LANL data.

Figure 4- Comparison of BERTHA model with LANL data for short circuit case.