LONG-TERM GOALS

Considering the crucial importance of nonlinear interaction \( S_{nl} \) for the development of third generation wave models the long-term goal of this work is to improve accuracy of calculating nonlinear interaction \( S_{nl} \) in wind wave models, and hence improving wave prediction in general.

OBJECTIVES

The objective of this work is to develop a \textit{computationally cheap yet accurate approximation} for \( S_{nl} \).

APPROACH

The approach is based on the neural networks (NN) technique. It is used to accelerate the calculations and improve the accuracy of the parameterization of nonlinear interaction \( S_{nl} \). The nonlinear interaction source term can be considered as a nonlinear mapping between a source term \( S_{nl} \) and a spectrum \( F \)

\[ S_{nl} = T(F), \quad (1) \]

where \( T \) in is the exact nonlinear operator given by the full Bolzmann interaction integral [1]. This algorithm is a factor \( 10^4 \) too expensive for use in operational wave models. We intend to use the NN technique to produce a cheap and accurate alternative approach.
# Using Neural Network Parametrizations of the Nonlinear Energy Transfer for Application in Wave Models

Considering the crucial importance of nonlinear interaction $S_{nl}$ for the development of third generation wave models, the long-term goal of this work is to improve the accuracy of calculating nonlinear interaction $S_{nl}$ in wind wave models, and hence improving wave prediction in general.

### Subject Terms
- Nonlinear energy transfer
- Neural network parametrizations
- Wave models

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NNs are a generic tool for fast and accurate approximation of continuous mappings and, therefore, can be used to replace the exact algorithm. In order to convert the mapping (1) to a continuous mapping of two finite vectors (independent of the actual spectral discretization), the spectrum $F$ and source function $S_{nl}$ are expanded using systems of two-dimensional functions each of which ($\Phi_i$ and $\Psi_q$) creates a complete and orthogonal two-dimensional basis

$$F = \sum_{i=1}^{n} x_i \Phi_i, \quad S_{nl} = \sum_{q=1}^{m} y_q \Psi_q,$$

(2)

where for $x_i$ and $y_q$ we have

$$x_i = \iint F \Phi_i, \quad y_q = \iint S_{nl} \Psi_q,$$

(3)

where the double integral identifies integration over the spectral space. Because both sets of basis functions $\{ \Phi_i \}_{i=1,\ldots,\infty}$ and $\{ \Psi_q \}_{q=1,\ldots,\infty}$ are complete, increasing $n$ and $m$ in (2) improves the accuracy of approximation, and any spectrum $F$ and source function $S_{nl}$ can be approximated by (2) with a required accuracy. Substituting (2) into Eq. (3) we can get

$$Y = T(X),$$

(4)

which represents a continuous mapping of the finite vectors $X \in \mathbb{R}^n$ and $Y \in \mathbb{R}^m$, and where $T$ still represents the full nonlinear interaction operator. As described in the previous section, this operator can be approximated with a NN with $n$ inputs and $m$ outputs and $k$ neurons in the hidden layer

$$Y \approx T_{NN}(X).$$

(5)

The accuracy of this approximation ($T_{NN}$) is determined by $k$, and can generally be improved by increasing $k$.

To train the NN approximation $T_{NN}$ of $T$, a training set has to be created which consists of pairs of vectors $X$ and $Y$. To create this training set, a representative set of spectra $F_p$ has to be generated with corresponding (exact) interactions $S_{nl,p}$. For each pair $(F, S_{nl,p})$, the corresponding vectors $(X,Y)_p$ are determined using Eq. (3). All pairs of vectors are then used to train the NN to obtain $T_{NN}$.

After $T_{NN}$ has been obtained by training, the resulting NN Interaction Approximation (NNIA) algorithm consists of three steps:

1. Decompose $F$ by applying Eq. (3) to calculate $X$.
2. Estimate $Y$ from $X$ using Eq. (5).
3. Compose $S_{nl}$ from $Y$ using Eq. (2).
WORK COMPLETED

This year we completed a study, which addressed the basic feasibility of the NNIA approach. We (1) select basis functions $\Phi_i$ and $\Psi_q$ and the number of each $(n,m)$; (2) designed a NN topology (number of neurons $k$); (3) constructed a representative training set; and (4) selected training strategies. The first three issues all have a significant impact on both accuracy and economy of a NNIA. Unfortunately, there is no pre-defined way to tackle these issues. It is therefore unavoidable that the development of a NNIA involves many iterations. This year we completed the first iteration. The major requirement of an NNIA to be potentially useful in operational wave modeling, is that the exact interactions $S_{nl}$ are closely reproduced for computational costs comparable to that of the DIA. The feasibility study we have completed showed the potential of this approach with the design of a simple ad-hoc NNIA.

RESULTS

We have considered an NNIA to estimate the nonlinear interactions $S_{nl}(f,\theta)$ as a function of frequency $f$ and direction $\theta$ from the corresponding spectrum $F(f,\theta)$ in deep water only. To train and test this NNIA, we used a set of about 20,000 simulated realistic spectra for $F(f,\theta)$, and the corresponding exact estimates of $S_{nl}(f,\theta)$ [2]. Simulation has been performed using a generator that calculated a spectral function composed of several Pierson-Moskowitz spectra for different peak frequencies oriented randomly in $[0,2\pi]$ interval. Comparison of simulated spectra with spectra simulated by WAVEWATCH model [3, 4] shows that this approach allowed us to simulate reasonably realistic and complicated spectra describing a broad range of wave systems. Spectra with four peaks were used in calculations below. Separate data sets have been generated for training and validation.

As is common in parametric spectral descriptions, we choose separable basis functions where frequency and angular dependence are separated. For $\Phi_i$ this implies

$$\Phi_i(f,\theta) \Rightarrow \Phi_{ij} = \phi_{f,\phi_j}(f)\phi_{\theta,i}(\theta)$$

A similar separation is used for $\Psi_q$. Considering the strongly suppressed behavior of $F$ and $S_{nl}$ for $f \to 0$, and the exponentially decreasing asymptotic for $f \to \infty$, generalized Laguerre’s polynomials are used to define $\phi_f$ and $\psi_f$. Considering that no directional preferences exist in $F$ and $S_{nl}$, a Fourier decomposition is used for $\phi_\theta$ and $\psi_\theta$. The number of base functions is chosen to be $n = 51$ and $m = 64$ to keep the accuracy of approximation for $F$ on average better than 2% and for $S_{nl}$ - better than 5-6%. The number of hidden neurons was taken $k = 30$ which allows a satisfactory approximation (5) for the mapping (4).

Table 1. RMSE statistics for 10,000 $S_{nl}$

<table>
<thead>
<tr>
<th></th>
<th>Mean RMSE</th>
<th>$\sigma_{RMSE}$</th>
<th>Max RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIA</td>
<td>0.0133</td>
<td>0.0111</td>
<td>0.104</td>
</tr>
<tr>
<td>NNIA</td>
<td>0.0068</td>
<td>0.0063</td>
<td>0.065</td>
</tr>
</tbody>
</table>
Table 1 compares three important statistics for source function RMS errors (with respect to exact solution) calculated using DIA and NNIA for 10,000 spectra (independent validation set). NNIA improves accuracy about twice as compared with DIA.

Figure 1 shows mean RMSE as function of the frequency $f$ (left) and the angle $\theta$ (right). Numbers in Table 1 correspond to NNIA with NN with 30 neurons in the hidden layer (51:30:64).

Figure 2 shows 3 pairs (one row in the figure corresponds to one pair) of one dimensional, integrated over $\theta$, source functions $S_{nl}(f)$ (left column) and one dimensional, integrated over $f$, source functions $S_{nl}(\theta)$ (right column) from the validation data set. Thick solid curves correspond to the exact $S_{nl}$. Dashed curves correspond to DIA of $S_{nl}$. Curves with triangles correspond to the NNIA estimate of $S_{nl}$. Numbers inside the panels show DIA and NNIA errors in percents with respect to exact solution.
Figure 2. See explanations in the text above.

The results in Fig. 2 are fairly representative for the validation data set. In general, the NNIA reproduces the exact $S_{nl}$ accurately. Even if clear oscillations are present in the decomposed spectrum (e.g., line in middle panel on left), the NNIA shows no spurious oscillations, and gives reasonable results. Note that many DIA source functions exhibit complicated behavior and spurious oscillations. Major peaks in these functions coexist with more or less random small-scale fluctuations. These fluctuations are probably an artifact produced by a simplified nature of DIA. Exact interactions are the result of averaging over much larger number of resonant sets of wave numbers, and are therefore much smoother than the results of the DIA.

IMPACT/APPLICATIONS

The NNIA under development is intended for implementation in operational wave models such as WAVEWATCH.
TRANSITIONS

The resulting NNIA algorithm will be transitioned to other members of the AWPP-SNL group upon completion.

RELATED PROJECTS

AWPP-SNL group

REFERENCES


PUBLICATIONS


Krasnopolsky V.M., D.V. Chalikov, and H.L. Tolman, 2001: Use of Neural Networks to Improve Computational Efficiency of Environmental Numerical Models, OMB/EMC/NCEP Technical Note, No. 199, pp.37