A key step in the construction of quantum computing technologies is the identification and elimination of unwanted sources of noise and its resulting quantum decoherence. In solid state devices, one important source of noise is the fluctuating electric and magnetic fields that come from the device elements themselves. The ultimate source of these fluctuations is the random electric currents that are strongest in the metallic electrodes and leads. This random motion has both thermal and quantum components, though in most devices presently under investigation, the thermal part dominates. In the past, this important physical effect has generally been modeled by picturing the evanescent wave, Johnson noise, decoherence, metal surface.

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

15. SUBJECT TERMS
evanescent wave, Johnson noise, decoherence, metal surface
Final Report: Theory of Evanescent-wave Johnson Noise in Qubit Devices

ABSTRACT

A key step in the construction of quantum computing technologies is the identification and elimination of unwanted sources of noise and its resulting quantum decoherence. In solid state devices, one important source of noise is the fluctuating electric and magnetic fields that come from the device elements themselves. The ultimate source of these fluctuations is the random electric currents that are strongest in the metallic electrodes and leads. This random motion has both thermal and quantum components, though in most devices presently under investigation, the thermal part dominates. In the past, this important physical effect has generally been modeled by picturing the device using an equivalent RC circuit and applying the Nyquist theorem to this circuit. This approach has limitations, since it depends on global quantities, rather than on the detailed geometry and the local electromagnetic properties of the components. Our aim in this research project was to compute the effect from first principles. We considered the underlying physical law (quantum electrodynamics) governing the devices in question. Given the position- and frequency-dependent dielectric function, the electromagnetic noise spectrum can be calculated, and the decohering effect can be deduced. Semiconducting quantum dot and superconducting flux qubits were investigated.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received | Paper
---|---
05/28/2015 | Amrit Poudel, Maxim G. Vavilov, Canran Xu. Nonadiabatic dynamics of a slowly driven dissipative two-level system, PHYSICAL REVIEW A, (05 2014): 52102. doi:

TOTAL: 4
Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received  Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(e) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received  Paper

TOTAL:

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Patents Submitted

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Amrit Poudel - van Vleck award for graduate students at University of Wisconsin based on research accomplishments.

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### Sub Contractors (DD882)

### Inventions (DD882)

### Scientific Progress

See Attachment

### Technology Transfer
Theory of Evanescent-wave Johnson Noise in Qubit Devices: Final Annual Report 5/22/15

Institution: University of Wisconsin-Madison
Principal Investigators: Robert Joynt, Maxim Vavilov
Postdoctoral Associate: Luke Langsjoen
Graduate Student: Amrit Poudel
Award Number: MSN138117

Introduction

The overall goal of the program was to compute the effect of Evanescent-wave Johnson Noise (EWJN) on qubits. It is expected that this sort of noise will form the “white noise floor” in ultimate qubit designs. Many present-day experiments are limited by decoherence coming from two sources: external noise and materials issues. The first problem is vexatious but reasonably well understood. Its solution requires mainly working on improving filtering and isolation to get rid of noise in external electrical connections. The second problem is long-standing and less well understood. This has to do with charged defects giving rise to 1/f-like noise. Nevertheless, after having made relatively little progress in the last three decades, some remarkable advances have been made in the last few years, particularly in the design and materials for superconducting qubits. There is now hope that this type of noise may be overcome in the near future. The present situation is that superconducting qubits and charged semiconducting qubits still seem to be limited by these materials issues; they will have to face EWJN later. Semiconductor spin qubits are now limited by EWJN, at least at lower magnetic fields.

Work started when funds became available early in 2011. The stated period of the grant was 3 years, and that period concluded in November 2013, at which point there were no more students or postdocs supported on the grant. We were granted a 1-year no-cost extension. With this, we were able to support Luke Langsjoen, a postdoc, for the summer of 2014. At this point, all funds are essentially expended, and we can now summarize all achievements.

Achievements

1. Model Geometries

In the first 18 months, as reported at the 2012 review, we solved some simple geometries exactly. This included the metal half-space, the finite slab, capacitor plates, and metallic quantum wires. We developed numerical techniques for 1- and 2-dimensional systems, finally settling on finite-difference methods with a Yee grid. We used the exact solutions to benchmark
the numerical calculations, finding excellent agreement.

Figure 1. Electric field

Off-Diagonal $D(r,r')$ near a Quantum Wire

![Graph showing electric field noise produced at $r$ by a source at $r'$ in the presence of a quantum wire.]

Numerical results clearly show scattering effects

**Figure 1.** Electric field noise produced at $r$ by a source at $r'$ in the presence of a quantum wire.

2. Non-local Theory

We solved the infamous divergence problem for the noise power in the limit of zero separation from a half space by using a non-local dielectric function $\varepsilon(r,r',\omega)$. The modifications required to the physical picture are fairly substantial very near the surfaces. This is important for certain qubit architectures, as we have recently discovered in the case of NV-centers. The good news is that the inclusion of non-locality renders $T_1$ finite everywhere.

Fortunately for experiment, the characteristic signatures of EWJN are not altered by the non-local corrections, since those signatures have to do mainly the temperature and frequency dependences that come from the fluctuation-dissipation theorem for photons.
3. \( T_1 \) for existing devices

The above calculations were carried out for simple geometries. We went beyond this and computed \( T_1 \) numerically for a laterally-confined double quantum dot of the kind used for singlet-triplet and hybrid qubits. We found that the dipole approximation overestimates the effect of the noise since there is an effective wavelength cutoff in the real system that is not present in the dipole approximation.

Several important experiments can be reasonably approximated by half-space geometries. This is crude compared to what we hope ultimately to do, but nevertheless gives us guidance as to where to focus our future efforts. These estimates are shown in the Table: it should be kept in mind that they are based on very sketchy parameters for the device geometries. To get the range of results, we have estimated the gate conductances as \( 6 \times 10^6 \) - \( 6 \times 10^7 \) S/m, a reasonable range for good to very good conductors.

<table>
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TABLE 1: Estimates of relaxation and decoherence times for some experiments.

There is a strong suggestion from these results that present-day charge qubits are suffering from a decoherence mechanism that is stronger than EWJN that kicks in first. This is perhaps not surprising, since all heterostructures seem to have some charged defects. Spin qubits, on the other hand are probably limited by EWJN. The charge noise does not affect the spins. Furthermore, metallic gates and dielectrics (such as pure Si or SiGe itself) screen electric fields but not magnetic fields, so all currents contribute to spin decoherence. Recent spin qubit designs have the metallic gates even closer than before, so EWJN is very likely to play a strong role in the future.

4. Computations of $T_2$

The theory for $T_2$ is based on the expression for the off-diagonal part of the density matrix:

$$\rho_{01}(t) = \rho_{01}(0)e^{-\Gamma(t)}$$

In the long-time limit the dephasing time $T_\phi$ (or $T_2$) can be extracted from the linear term in $\Gamma$. For a double quantum dot we have that

$$\Gamma(t) = \frac{1}{\hbar^2} \sum_{ij} \int d\omega \int d\vec{r} \int d\vec{r}' M_{\phi}^{i*}(\vec{r}) M_{\phi}^j(\vec{r}') \nu_{ij}(\vec{r}, \vec{r}', \omega) \frac{\sin^2 \omega t}{2} \coth \frac{\hbar \omega}{2k_B T}$$

In this equation, $M_{\phi}$ is the integrand of the matrix element of $A \cdot p$ that determines the energy modulation of the two-level system and $\nu_{ij}$ is proportional to the Green’s function of the electric field.
The general conclusion of our $T_2$ calculations, first of all, is that $T_2$ is the same order of magnitude as $T_1$. This is a consequence of the fact that the frequency dependence of the noise spectrum is essentially linear. For many noise mechanisms (hyperfine coupling and 1/f-type noise, for example) $T_2 \ll T_1$, but this is because the spectrum is concentrated at low frequencies. This overall conclusion is generally good news for quantum engineers: for EWJN they only need to worry about one number.

### 5. Thin Film Results

An important step toward more realistic geometries is the calculation of EWJN from thin films, rather than half-spaces. Of course, all flat sources have finite thickness. We did not expect to find anything dramatic, regarding these calculations more as a necessary preliminary step. Instead we discovered a strong enhancement of the noise over the half-space case, as illustrated in Fig. 2.
Figure 4. The relaxation time $T_1$ for a charge qubit in the presence of a metallic thin film. $d$ is the distance from the near surface and $\lambda_F$ is the Fermi wavelength. Dashed and solid lines indicate local and non-local response, respectively. Red curves are for a half space and blue curves are for a film with thickness $a = 10$ nm.

Typical separation values in a real device would be about $d/\lambda_F = 20-200$, so there is very substantial enhancement of the noise in the physical region – note the log scale! The physical effect responsible for this is an enhanced leakage of the field in the photon mode because of the finite thickness – the wave evanesces much more slowly.
Figure 5. The relaxation time $T_1$ for a spin qubit in the presence of a metallic thin film. $d$ is the distance from the near surface and $\lambda_F$ is the Fermi wavelength. Dashed and solid lines indicate local and non-local response, respectively. Red curves are for a half space and blue curves are for a film with thickness $a = 10 \text{ nm}$.

We have also computed the analogous quantities for magnetic noise. There is a remarkable contrast between Figure 2 and Figure 3. There is no enhancement at all for the magnetic noise – indeed the noise is suppressed for the film as compared to the half-space. The point is that in the case of the electric field there is a competition between screening and thinning out of the source, while for the magnetic field the source simply thins out. Of course as $a \to 0$, there is no source at all.

The dashed lines are included in order to show the physical effects of non-locality; in particular the smoothing of singular behaviors at short distances. Furthermore, they demonstrate that the full machinery that we have developed is necessary in order to obtain credible results: unfortunately, one cannot get away with the much simpler local calculations.
6. Noise from Superconducting Surfaces

One crucial goal of the project is to provide experimental signatures that differentiate EWJN from other noise sources. When the noise source is superconducting, this is a relatively simple matter, since the noise is strongly temperature-dependent near $T_c$, the critical temperature. We have evaluated the effect of EWJN produced by a thick superconducting film below $T_c$ in the Pippard limit where we can use the Mattis-Bardeen expression for the conductivity. Just below $T_c$, the noise due to quasiparticles is enhanced relative to the normal state, but it then begins to drop quickly when $T < T_c/3$. This drop would be a clear and dramatic signature of EWJN. We are working out the quantitative details for arbitrary parameters of superconductor, such as mean free path, transition temperature, Fermi velocity.

![Figure 6. The dissipative part of the electromagnetic response function of a superconductor below the critical temperature $T_c$, normalized to the conductivity of the normal phase, (indicated by an arrow), based on Mattis-Bardeen theory. Note the enhancement when $T > T_c/2$.](image-url)
7. Relaxation time of Nitrogen Vacancy Center Qubits in Diamond

Measurements of EWJN near metallic surfaces requires a small probe interacting with high-frequency electric or magnetic fields that are placed on submicron distances from metal surface. Nitrogen vacancy centers in diamond\textsuperscript{7} can serve as such probe embedded in a diamond crystal close to the diamond interface with a metal. The ground state of an NV center is split due to spin-spin interaction with energy splitting 2.87 GHz between \( m_s=0 \) and \( m_s=\pm 1 \), and a fluctuating magnetic field causes transition between spin states. Devices consisting of a diamond film on a silver substrate were recently studied in experiment\textsuperscript{8}. The relaxation rate of many (>2) centers near the surface was measured as a function of temperature. The relaxation is due to the magnetic field component of EWJN. We evaluated this both within the local conductivity theory and the non-local conductivity theory. At low temperatures, the relaxation rate deviated very strongly from the local expression. Performing calculations with a non-local conductivity, we found excellent agreement with experiments (see Fig. 7.). This obviously has profound implications for all kinds of qubit devices and for other physical phenomena such as the Casimir effect.

![Figure 7. The spin relaxation rate of NV spin in diamond as a function of temperature. The solid curve is obtained from non-local response theory of metal on the surface of a diamond crystal and the dashed curve is a local response result. The temperature dependence of the conductivity was measured directly for the same metallic film on diamond and was used to determine the mean free time of electron as a function of temperature. The largest discrepancy between the curves happens when the mean free path of electrons in metal is comparable to the distance of an NV center from the metal surface. The agreement between the experimental data\textsuperscript{8} and the non-local theory is excellent. Figure adapted from Ref. 8.](image_url)
Summary

We believe all major milestones from the original proposal were accomplished during the grant period, as summarized below. The one major exception was the plan to write software that would be made generally available. This milestone was eliminated with the agreement of the program officers in 2013. Nearly all of the work has been published: the comparison of theory and experiment with NV-centers and superconductors is essentially complete and the papers are in preparation.

Milestone 1 Complete: Full 3-D calculations for dot systems

Major Findings:
- Dipole moment controls the strength of decoherence
- Inclusion of non-local dielectric functions softens divergences ($T_1 \to 0$) at short distances
- Better lateral confinement means more coherence for dot systems
- Spin qubits may be the most susceptible to EWJN because of slower falloff of noise with distance.

Milestone 2 Complete: Thin film calculations systematized by use of reflection coefficients, which enables the use of k-dependent dielectric functions

Major Findings:
- Charge noise enhanced in thin-film geometries
- Magnetic noise suppressed in thin-film geometries
- Unphysical divergence in Casimir force removed by use of nonlocal response functions

Milestone 3,4 More complex gate geometries and extension to other qubits, such as NV-centers and superconductors

Major Findings:
- Enhancement of EWJN just below superconducting transition and suppression of EWJN at temperatures below half of the transition temperature
- Explained temperature dependence of the relaxation rate of NV centers in diamonds due to the non-local conductivity of metallic films
- Evaluated effect of EWJN on current in double quantum dot (work in progress)
References


Talks and Presentations

L. Langsjoen, Amrit Poudel, Maxim Vavilov, Robert Joynt, "Relaxation of quantum dots due to evanescent-wave Johnson noise from a metallic backgate”, APS Meeting, Baltimore, March 21, 2013

L. Langsjoen, Amrit Poudel, Maxim Vavilov, Robert Joynt, "Electromagnetic fluctuations near thin metallic films”, APS Meeting, Denver, March 5, 2014

R. Joynt, “Casimir and the Qubit: Decoherence from Evanescent-wave Johnson Noise”, presented at:

Seminar, Hong Kong University, July, 2012

Colloquium, Univ. of N. Carolina, Chapel Hill, NC, December, 2012

Seminar, Beijing Univ., May, 2013


Talk, KITPC Workshop on Electron Spin Qubits, July, 2014

Seminar, Zhejiang University, October, 2014

Seminar, Institute of Physics, Beijing, November, 2014