AFRL-RQ-WP-TP-2015-0109

ELLIPTIC LENGTH SCALES IN LAMINAR, TWO-DIMENSIONAL SUPERSONIC FLOWS

James H. Miller

Vehicle Technology Branch
High Speed Systems Division

JUNE 2015

AIR FORCE RESEARCH LABORATORY
AEROSPACE SYSTEMS DIRECTORATE
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7541
AIR FORCE MATERIEL COMMAND
UNITED STATES AIR FORCE

NOTICE AND SIGNATURE PAGE

AFRL-RQ-WP-TP-2015-0109 HAS BEEN REVIEWED AND IS APPROVED FOR PUBLICATION IN ACCORDANCE WITH ASSIGNED DISTRIBUTION STATEMENT.

*//Signature//
JAMES H. MILLER
Project Manager
Vehicle Technology Branch
High Speed Systems Division

//Signature//
CLARK L. ALLRED, Chief
Vehicle Technology Branch
High Speed Systems Division
Aerospace Systems Directorate

//Signature//
THOMAS A. JACKSON
Deputy for Science
High Speed Systems Division
Aerospace Systems Directorate

*Disseminated copies will show "//Signature//" stamped or typed above the signature blocks.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)* June 2015

2. REPORT TYPE Technical Paper

3. DATES COVERED *(From - To)* 01 June 2015 – 30 June 2015

4. TITLE AND SUBTITLE
ELLIPTIC LENGTH SCALES IN LAMINAR, TWO-DIMENSIONAL SUPERSONIC FLOWS

5a. CONTRACT NUMBER In-house

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER 62201F

6. AUTHOR(S)
James H. Miller

5d. PROJECT NUMBER 2405

5e. TASK NUMBER N/A

5f. WORK UNIT NUMBER Q1K7

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Vehicle Technology Branch (AFRL/RQHV)
High Speed Systems Division
Air Force Research Laboratory, Aerospace Systems Directorate
Wright-Patterson Air Force Base, OH 45433-7541
Air Force Materiel Command, United States Air Force

8. PERFORMING ORGANIZATION REPORT NUMBER
AFRL-RQ-WP-TP-2015-0109

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Research Laboratory
Aerospace Systems Directorate
Wright-Patterson Air Force Base, OH 45433-7541
Air Force Materiel Command
United States Air Force

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL/RQHV

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-RQ-WP-TP-2015-0109

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
PA Case Number: 88ABW-2014-5736; Clearance Date: 04 Dec 2014.

14. ABSTRACT
A similarity approach to characterizing the influence of shocks and compression ramps on flat plate flows is presented. New correlations for laminar compressive interactions on two-dimensional compression ramps and shock impingements for adiabatic walls have been developed that utilize a universal form which is enabled by the discovery of a new similarity parameter of the Navier-Stokes equations. These new correlations have been developed for two-dimensional, laminar adiabatic wall flows over compression ramps and flows with shock impingements. The new correlations are derived from existing numerical data and incorporate the new similarity parameter for the Navier-Stokes equations that represents the ratio of pressure forces to viscous forces. The new correlations are in excellent agreement with previously published experimental and numerical data. By utilizing the correlations, known freestream conditions, and empirical relations for the undisturbed boundary layer thickness, it is possible to determine a priori estimates for the streamwise length of the region influenced by a shock impinging on a flat plate or a region influenced by a flat plate / compression ramp juncture.

15. SUBJECT TERMS
supersonic, shock-boundary layer interaction, upstream influence, influence regions, similarity parameters

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	b. ABSTRACT	c. THIS PAGE			
Unclassified	Unclassified	Unclassified	SAR	11	James H. Miller **19b. TELEPHONE NUMBER** *(Include Area Code)* N/A

Elliptic Length Scales in Laminar, Two-Dimensional Supersonic Flows

James H. Miller[1]

Air Force Research Laboratory, Wright-Patterson AFB 45433

Nomenclature

C_p = pressure coefficient, $C_p = \frac{p - p_\infty}{1/2\, \rho_\infty V_\infty^2}$

l = length of elliptic region, *m*

L = reference length, *m*

$\frac{l_u}{\delta_L}$ = ratio of upstream influence length to undisturbed (flat plate) boundary layer thickness

$\frac{l}{\delta_L}$ = ratio of elliptic region length to undisturbed (flat plate) boundary layer thickness

l_u = upstream influence length, *m*

l_d = downstream influence length, *m*

M = Mach number, ratio of velocity to speed of sound, $M = \frac{V}{\sqrt{\gamma p/\rho}}$

p = pressure, N/m^2

Re = Reynolds number, ratio of inertia forces to viscous forces, $Re = \frac{\rho_\infty V_\infty L}{\mu_\infty}$

t = time, *s*

u,v,w = Cartesian velocity components in the x,y,z directions respectively, *m/s*

V_∞ = reference or freestream velocity, *m/s*

x,y,z = Cartesian coordinates, *m*

Δp = reference change in pressure (assumed positive) relative to freestream (e.g., $p_c - p_\infty$, where p_c is the inviscid pressure downstream of a shock interaction with a flat surface or compression ramp), N/m^2

δ_L = boundary layer thickness at location L, *m*

γ = ratio of specific heats, 1.4 for air.

θ = compression ramp angle, degrees

---

[1] Lead Senior Aerospace Engineer, Aerospace Systems Directorate, AFRL/RQHV, B18A, Rm A114, 1950 5th Street, Wright-Patterson AFB OH, 45433.

τ = shear stress, N/m^2

μ = viscosity, $kg / (m\ s)$

Superscripts:

~ = nondimensional quantity normalized by freestream conditions or reference change in pressure

Subscripts:

d = downstream boundary of elliptic region

L = x location of compression ramp leading edge or shock impingement point

u = upstream boundary of elliptic region

xx,xy = Cartesian components of shear stress

∞ = freestream conditions

I. Introduction

Several theories [1-3] and correlations [4] exist to describe the length scales associated with the interaction of a pressure disturbance with a boundary layer in a predominantly supersonic flowfield. The theoretical approaches are essential to understanding the detailed structure of the viscous layer as it encounters pressure disturbances that are either compressive or expansive in nature. The interactions in compressive flows are more straightforward for laminar flows but take on different characteristics when applied to turbulent interactions, requiring that theoretical descriptions become more sophisticated. Throughout the literature, the length scales of these interactions are typically described in terms of the undisturbed boundary-layer thickness; and, the correlations developed to describe these length scales take many forms based on dimensional analysis and geometric considerations. In the present article, another approach to these types of theories and correlations based on a similarity perspective is presented. New correlations for laminar compressive interactions for two-dimensional compression ramps and shock impingements for adiabatic walls have been developed. These correlations highlight a more universal form which is based on a new similarity parameter of the Navier-Stokes equations. Results show that compressive interactions can be combined into a single correlation for both upstream influence lengths and total elliptic lengths of interactions.

II. Analysis

Pressure and viscous forces play a dominant role in determining the length scale of an elliptic region caused by a pressure disturbance in viscous supersonic flow. Elliptic regions are defined as regions in which mathematically elliptic methods (time-iterative) must be used to accurately capture the details of the flowfield, while parabolic regions can be solved using space-marching methods where no significant upstream influence affects the flow. The ratio of inertial forces to viscous forces in the form of Reynolds number plays an important role in defining the length scales of the elliptic regions because of the importance of the boundary layer thickness. In this work another similarity parameter is utilized to determine the length of elliptic regions, and this similarity parameter represents the ratio of pressure forces to viscous forces. To the author's knowledge, there are presently no similarity parameters for the Navier-Stokes equations that have been described in the literature for shock boundary layer interactions where the ratio of pressure forces to viscous forces is an important consideration. Although reference to a similar parameter was discussed in Ref. [5] to capture small scale disturbances, the present work is focused on the larger scale of the interaction.

This discussion is restricted to the 2D compressible momentum equation in the Cartesian x direction[6]:

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}\left[\rho u^2 + p - \tau_{xx}\right] + \frac{\partial}{\partial y}\left[\rho u v - \tau_{xy}\right] = 0$$

with

$$\tau_{xx} = \frac{2}{3}\mu\left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right);\ \tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

In the following nondimensionalization approach, nondimensional quantities are annotated with a tilde:

$$\tilde{x},\, \tilde{y} = \frac{x}{L},\frac{y}{L}\ ;\ \tilde{t} = \frac{t}{L/V_\infty}\ ;\ \tilde{\rho} = \frac{\rho}{\rho_\infty}\ ;\ \tilde{u},\tilde{v} = \frac{u}{V_\infty},\frac{v}{V_\infty}\ ;\ \tilde{p} = \frac{p}{\Delta p}$$

There is nothing unusual about this approach to nondimensionalization, except for the pressure. In this approach the pressure is nondimensionalized by a change in pressure that is representative of the interaction causing the formation of the elliptic region. For example, an appropriate value for Δp would be the overall pressure rise associated with a 2D compression ramp in an inviscid flow. For internal flows, the pressure change could be tied to the difference between a nozzle exit pressure and a back pressure. For rather simple 2D external flows, the change in pressure is easily computed *a priori* using analytical shock relations; however, for 3D interactions, the change in pressure would likely need to be computed with more sophisticated computational fluid dynamics (CFD) methods. Additionally, for 3D interactions, the length scales would require determination in spanwise as well as streamwise directions.

Substituting for the dimensional quantities in the momentum equation, and then rearranging terms and simplifying leads to the following equation in nondimensional form:

$$\frac{\partial \tilde{\rho}\tilde{u}}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{x}}[\tilde{\rho}\tilde{u}^2 + \frac{1}{Re}[\left(\frac{Re\ \Delta p}{\gamma\ M_\infty^2 p_\infty}\right)\ \tilde{p}\ -\ \tilde{\tau}_{xx}\ ]]\ +\ \frac{\partial}{\partial \tilde{y}}\left[\tilde{\rho}\tilde{u}\tilde{v}\ -\ \frac{1}{Re}\ \tilde{\tau}_{xy}\right] = 0$$

In preliminary studies of scaling elliptic regions, it is more useful to treat the parameter multiplying the pressure term, $\frac{Re\ \Delta p}{\gamma\ M_\infty^2 p_\infty}$, as independent of Reynolds number when there is an elliptic interaction. This approach is consistent with the assumptions in boundary-layer theory in which the pressure distribution is imposed on the boundary layer. However, the present approach allows for more insight into the relative importance of the pressure term when compared to the viscous terms in the equation. From a mathematical perspective, the eigenvalue analysis of Vigneron et al.[7] resulted in a relation for the mathematical character of the parabolized Navier-Stokes equations that utilized a weighting factor on the streamwise pressure gradient. In that analysis of the local mathematical character, the weighting factor was determined to be a function of the streamwise Mach number only. Discussed herein is how a new similarity parameter can be used to predict length scales of elliptic interactions, but not used to explore details of the local mathematical character of the equations nor for an order-of-magnitude analysis of the terms in the equation.

Before a discussion of the correlation results can be fully appreciated, a description of how the new similarity parameter represents a ratio of pressure forces to viscous forces is warranted:

Considering a fluid element [8] the ratio of inertia forces to viscous forces is:

$$Re = \frac{\rho V L}{\mu}$$

and the ratio of inertia forces to pressure forces is:

$$\frac{V^2}{\Delta p/\rho} = \frac{M^2\ \gamma p/\rho}{\Delta p/\rho} = \frac{M^2\ \gamma p}{\Delta p}$$

Therefore, dividing the Reynolds number by the above ratio gives the ratio of pressure forces to viscous forces, $\frac{Re\ \Delta p}{\gamma\ M_\infty^2 p_\infty}$. It should be noted that this parameter can be written in an incompressible form, namely, $\frac{Re\ C_p}{2}$, where the pressure coefficient is defined in the usual way in terms of the reference change in pressure and the freestream dynamic pressure.

It is appropriate to investigate the data used to develop previous correlations and understand how they may be predicted from correlations based on the new parameter for laminar, compressive interactions.

In Figure 1 the various lengths are shown for a two-dimensional compression ramp.

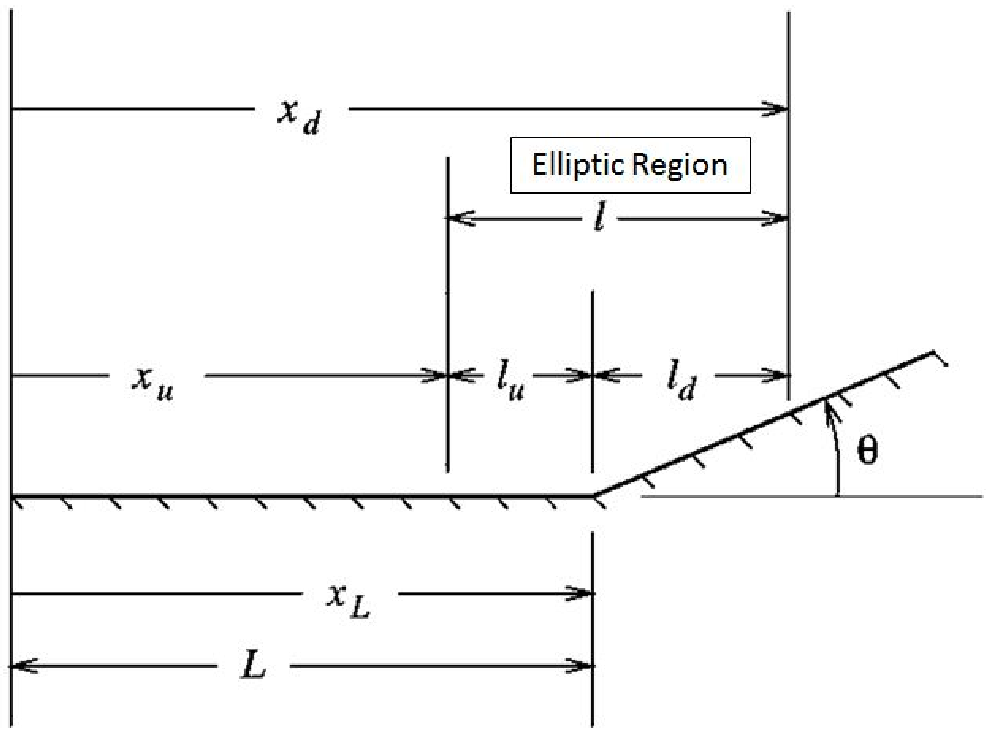

Figure 1: Nomenclature for Elliptic Regions Caused by 2D Compression Ramp

The correlations in the present work predict the ratio of the upstream influence distance, l_u , and elliptic distance (which is the sum of the upstream and downstream distances), l, to the boundary-layer thickness δ_L. The distance *L* is defined as the length measured from the leading edge to the source of the disturbance. In the case of an impinging shock, *L* is the location of the impingement point. Correlations and data for 2D laminar compression ramps and shock impingement studies were published previously [9]. Additionally, data for laminar expansions [10], turbulent compressions [11], and expansions [12] are also available and will be the subject of future work. In those previous studies, the forms of the correlations were developed using a statistical approach combined with cross-plotting techniques that assumed three independent parameters for adiabatic flows: Mach number, *M*, Reynolds number, *Re* and the relative change in pressure in the inviscid flow,$(\Delta p)/p\_\infty$. The present study treats the new similarity parameter as the only parameter in determining streamwise length scales for elliptic interactions that are scaled by the undisturbed boundary layer thickness.

Figure 2 contains the data from previous correlation studies for the upstream influence length, $l_u/_{\delta_L}$, for 2D laminar interactions for both compression ramps and shock impingements[9].

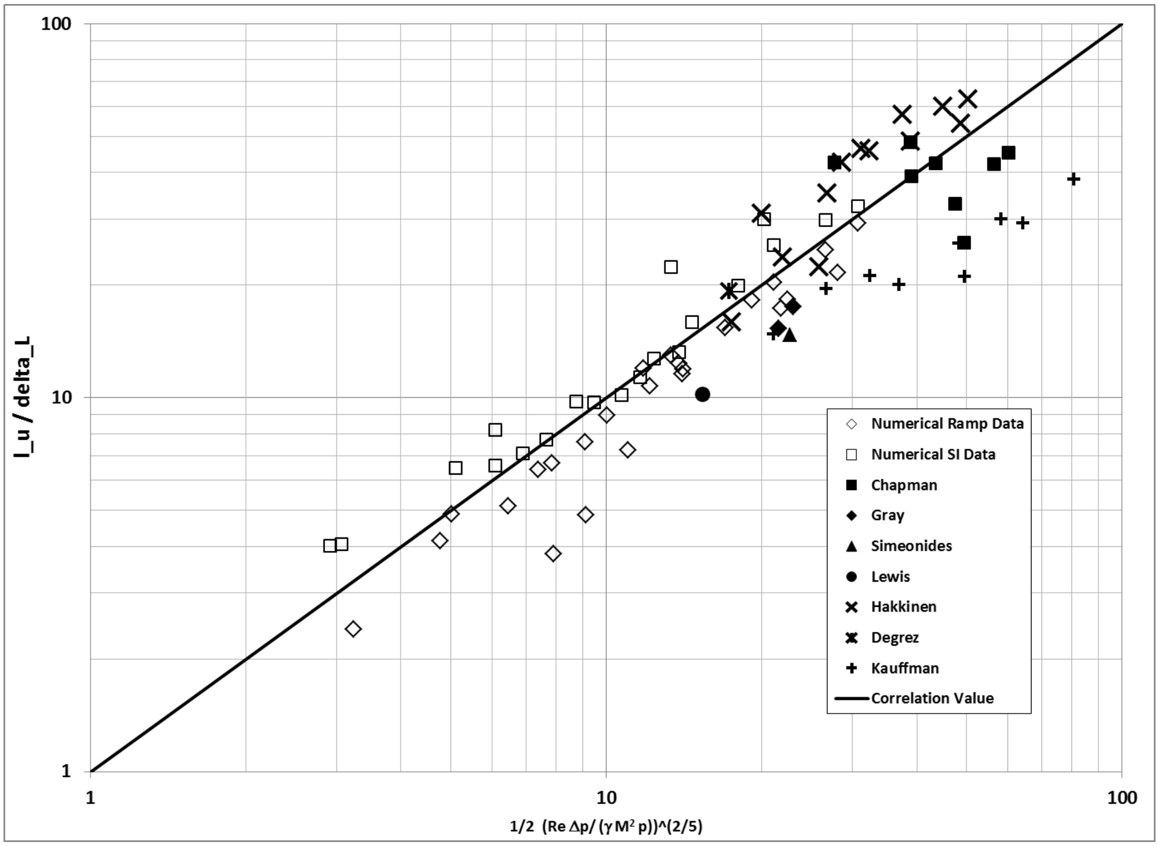

Figure 2: Correlation Results for Upstream Influence due to 2D Compression Ramps and Shock Impingement Flowfields

The numerical data for ramps and shock impingements alone were used for determining the relation for the upstream influence in terms of undisturbed boundary layer thickness, $l_u/\delta_L = \frac{1}{2}\left(\frac{Re\,\Delta p}{\gamma\, M_\infty^2 p_\infty}\right)^{2/5}$. The experimental data for adiabatic wall conditions was taken for ramps [13-16] and shock impingements [17-19] from previous work. The results in Figure 2 show good agreement with the data. In this figure, the correlation value for the upstream influence is defined to be zero when the change in pressure is zero. Previous correlations were developed for ramps and shock impingements separately for Mach number and Reynolds number ranges of 2.0 to 6.06, and 1 x 10^4 to 8 x 10^5, respectively [4]. However the present new correlation combines both ramps and shock impingements into one correlation. Therefore, the previous correlations are slightly more accurate than the present correlation for the upstream influence length. Although the present correlation for the upstream influence length is not as accurate as previously developed, the results for the total length or elliptic length are much improved as shown in Figure 3.

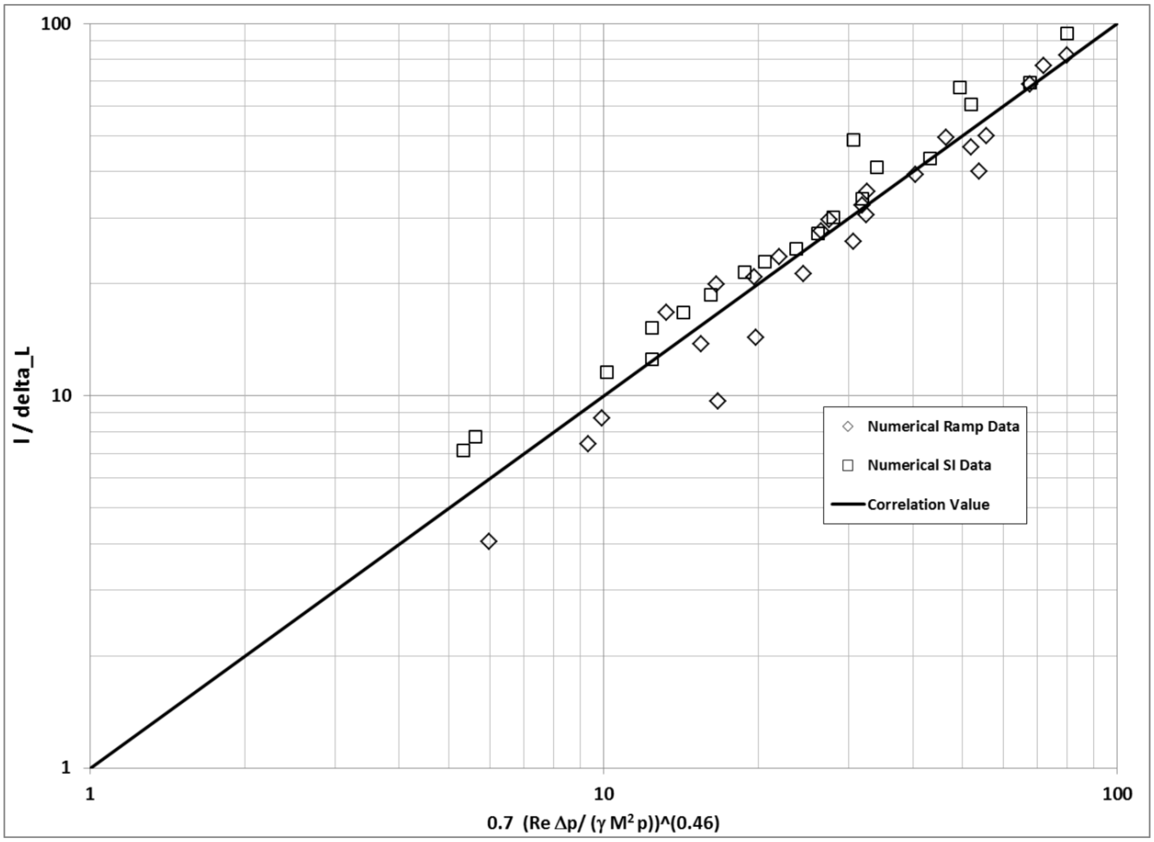

Figure 3: Results for Elliptic Region Length

Figure 3 has the data used to develop previous correlations [9] for the length of the elliptic region, $l/\delta_L = 0.7\left(\frac{Re\,\Delta p}{\gamma\, M_\infty^2 p_\infty}\right)^{0.46}$, plotted as a function of the elliptic parameter. As was done for the results in Figure 2, the correlation developed in Figure 3 was developed from only numerical data for both compression ramps and shock impingements. There were no experimental data from previous references available for the total elliptic length as discussed in Ref. [9]. The present correlation for total elliptic length fits the data very well and enables one to determine *a priori* the total length of an elliptic interaction for 2D, laminar interactions over compression ramps and shock impingement flowfields.

III. Conclusion

New correlations have been developed for 2D, laminar adiabatic wall flows over compression ramps and flows with shock impingements. These correlations are derived from existing numerical data and incorporate the use of a new similarity parameter for the Navier-Stokes equations that represents the ratio of pressure forces to viscous forces. The new correlations are in excellent agreement with previously published data. Development of experimentally-derived correlations for transitional flows over compression ramps is currently underway.

IV. References

[1] Lighthill, M.J., "On Boundary Layers and Upstream Influence II. Supersonic Flows Without Separation," Proceedings of the Royal Society A, Vol. 217, 1953, pp. 478-507.

[2] Stewartson, K. and Williams, P.G., "Self-Induced Separation," Proceedings of the Royal Society A, Vol. 312, 1969, pp. 181-206.

[3] Delery, J., and Marvin, J.G., "Shock-Wave Boundary-Layer Interactions," *AGARDograph* AG-280, Feb. 1986.

[4] Miller, J.H., Tannehill, J.C., and Lawrence, S.L., "Parabolized Navier-Stokes Algorithm for Solving Supersonic Flows with Upstream Influences," *AIAA Journal*, Vol. 38, No. 10, Oct. 2000, pp. 1837-1845.

[5] Miller, J.H., "PNS Algorithm for Solving Supersonic Flows with Upstream Influences," Ph.D. Dissertation, Dept. of Aerospace Engineering and Engineering Mechanics, Iowa State University, Ames, IA, Dec. 1997.

[6] Tannehill, J.C., Anderson, D.A., and Pletcher, R.H., Computational Fluid Mechanics and Heat Transfer, 2nd ed., Taylor and Francis, Washington, DC, 1997, pp. 263-264.

[7] Vigneron, Y.C., Rakich, J.V., and Tannehill, J.C., "Calculation of Supersonic Flow over Delta Wings with Sharp Subsonic Leading Edges," AIAA Paper 78-1137, July 1978.

[8] Karamcheti, K. *Principles of Ideal-Fluid Aerodynamics*, John Wiley and Sons, Inc., New York, 1966. Pp. 16-17.

[9] Miller, J.H., Tannehill, J.C., and Lawrence, S.L., "Computation of Supersonic Flows with Embedded Separated Regions Using an Efficient PNS Algorithm," AIAA 97-1942, June 1997.

[10] Miller, J.H., Tannehill, J.C., and Lawrence, S.L., "PNS Algorithm for Solving Supersonic Flows with Upstream Influences," AIAA Paper 98-0226, Jan. 1998.

[11] Ramesh, M.D., Tannehill, J.C., and Miller, J.H., "Correlations to Predict the Streamwise Influence Regions of Two-Dimensional Turbulent Shock Separated Flows," AIAA Paper 2000-0932, Jan. 2000.

[12] Ramesh, M.D., and Tannehill, J.C., "Correlations to Predict the Streamwise Influence Regions of 2-D Supersonic Turbulent Flows," AIAA Paper 2001-0884, Jan. 2001.

[13] Chapman, D.R., Kuehn, D.M., and Larson, H.K., “Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effect of Transition,” NACA Report 1356, 1957.

[14] Gray, J.D., and Rhudy, R.W., “Effects of Blunting and Cooling on Separation of Laminar Supersonic Flow,” *AIAA Journal*, Vol. 11, No. 9, Sept. 1973, pp. 1296-1301.

[15] Simeonides, G., Haase, W. and Manna, M. “Experimental, Analytical, and Computational Methods Applied to Hypersonic Compression Ramp Flows,” *AIAA Journal*, Vol. 32, No. 2, Feb. 1994, pp. 301-310.

[16] Lewis, J.E., Kubota, T. and Lees, L., “Experimental Investigation of Supersonic Laminar, Two-Dimensional Boundary-Layer Separation in a Compression Corner with and without Cooling,” *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 7-14.

[17] Hakkinen, R.J., Greber, I., Trilling, L., and Abarbanel, S.S., “The Interaction of an Oblique Shock Wave with a Laminar Boundary Layer,” NASA Memo 2-18-59W, March 1959.

[18] Degrez, G., Boccadoro, C.H., and Wendt, J.F., “The Interaction of an Oblique Shock Wave with a Laminar Boundary Layer Revisited. An Experimental and Numerical Study,” *Journal of Fluid Mechanics*, Vol. 177, 1987, pp. 247-263.

[19] Kaufmann II, L.G., and Johnson, C.B., ”Weak Incident Shock Interactions with Mach 8 Laminar Boundary Layers,” NASA TN D-7835, Dec. 1974.