Metamaterials and Transformation Optics

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The report outlines the PI's month of agreed time in the USA (talks at Berkeley, UCSD, and several conferences) and collaborations with US researchers David Smith and Xiang Zhang. The main advance of this year's research is extending the analytical work in transformation optics (relating complex systems to simpler systems with the same spectral properties) to singular 3d objects such as touching spheres, which gives rise to strong enhancement of local fields and thus nonlinear phenomena, opening the possibility of, eg, single-molecule detectors. The analytic solutions of these systems facilitate a quantitative understanding of the systems and qualitative appreciation of how the enhancement comes about. For example, they used this technique to calculate the Van der Waals forces between two nearly-touching nanospheres. Resulting in much more accurate predictions than other approximations. Future steps include calculation of heat flow between particles due to Van der Waals forces. A concerning effect is the nonlocality of the dielectric response, in which charge is slightly spread through a material rather than accumulating purely on the surface; the team arrived at a simple approximation and helps increase the understanding of subnanometre regime in optics.
During the fifth year of activities there were several visits to the USA:

- Gave a talk at UC Berkeley 7-10 February 2013
- Gave an invited lecture and received the McGroddy Prize at the APS March meeting, Baltimore, 17-22 March 2013
- Consultations with Professor David Smith in Seattle and a talk at UC San Diego 15-22 June 2013
- Invited talk at MRS Boston, University Lecture at LSU Baton Rouge, 1-8 December 2013
- Invited talk at Photonics West, San Francisco, seminar at UC Berkeley 1-7 February 2014

This makes up the one month agreed time in the USA.

Once more there have been extensive interactions between my group and Prof. David Smith’s group, as well as with the group of Prof Xiang Zhang at UCB with whom I now have a formal collaboration through the Gordon & Betty Moore Foundation.
Plasmonics at the sub-nanoscale

The main advance of the past year has been to extend our analytical work to singular 3D objects such as touching spheres. As explained in previous reports, the significance of singular plasmonic structures is that they are associated with strong enhancement of local fields thus giving rise to strong non-linear phenomena and the possibility of single molecule detectors. Although finding analytic solutions may at first sight be thought an academic exercise, the solutions have practical importance in that they facilitate not only a quantitative understanding of these systems, which is very difficult to achieve by purely computational techniques, but also a qualitative appreciation of how the enhancement comes about. Our chosen technique is once more transformation optics which via a transformation relates complex systems to simpler systems possessed of the same spectral properties.

One good example is to be found in reference [5] where we use the technique to calculate the Van der Waals forces between two nearly touching nanospheres. These forces are mediated by the quantum fluctuations in electron density at the metal surfaces and are the most long ranged forces between nanoparticles.

![Diagram](image)

**FIG. 1** (color online). Applying an inversion transformation about the point $\mathbf{R}_0$, two separated spheres (a) are mapped into an annulus (b), and vice versa. The annulus geometry comprises an inner solid sphere and an outer hollow one. The geometric inversion is accompanied by a transformation of the system permittivities. The small gap inside the cone indicated by the two dashed lines in (a) is enlarged in the annulus and is located further from the inversion point in (b). The opening angle of the cone in (a) is unchanged under the conformal transformation.

Fig. 1(a) shows the system we wish to study which is very difficult to treat, whilst fig. 1(b) is the transformed system which has more symmetry and therefore is more easily solved.
In fig. 3 we see a graphical representation of the some the modes responsible for the Van der Waals forces plotted both in the physics system to the left of each panel and in the transformed system. Note how the fields in the physical system are strongly concentrated in the gap.

**Fig. 3** (color online). (a) Deviations from the exact prediction of different approximations: expressions Eqs. (8) and (9), the PFA [9], the dipole term corrected results, and the dipole approximations. (b) The retarded vdW energy calculated by replacing the nonretarded dipole contribution with the retarded one, normalized to the exact nonretarded vdW energy. Dashed and solid curves in each panel correspond to the sphere-sphere and the sphere-plane geometry, respectively.

Fig. 3 shows the resulting calculations, $U$, compared to the exact values for the Van der Waals energy, $U_{\text{exact}}$. When the spheres are well separated our calculations have...
to be corrected for retardation effects but the figure shows that we can do this to better than 4% accuracy (red curve in fig 3(a)). Other approximations which have previously been used to attack the problem, such as the Proximity Force Approximation (PFA) are shown to be far less accurate that our new theory.

This work is on going and our next step will be to calculate the heat flow between two nano particles. For large objects separated by vacuum heat is transferred via radiation, but if particles are closer that the characteristic wavelength of the heat a new mechanism kicks in: near field heat transport due to fluctuations Van der Waals forces and this is the dominant mechanism on the nanoscale.

Another effect that has concerned us is the so called “non locality” of the dielectric response. The conventional description of the response of a surface to an external electric field assumes that the polarisation charge all accumulates at the interface. In reality the charge is spread out, admittedly over a tiny length scale of the screening length in the metal, about 0.2nm, but when we are dealing with particles that approach this close, we must worry about this effect. Some of this work has been in collaboration with David’s Smith’s group [8]. The non local corrections are complex and difficult to implement so we have tried to make a more simple representation which gives the same effect. We find that non locality can be mimicked by displacing the metal surface and coating with a thin layer of dielectric. This model system is much simpler to calculate that the full blown non locality and will increase the precision of our understanding of the sub nanometre regime in optics.

By way of illustration I show fig.4 taken from ref. [3]. Here we show the effect of non locality on the fields in the gap between two metal tips separated by 0.5nm, as excited by external radiation. Very large shifts in the resonant frequencies are seen. Also non locality substantially reduces the enhancement seen in the gap and is the limiting factor in obtaining really large enhancements. Note that our effective local theorot does an extremely good job of describing these effects.
FIG. 4 (color online). Numerical treatments of a 3D conical dimer within different models. (a) Absorption spectra for three-dimensional gold conical dimers with geometric parameters as shown in the top right inset of the panel. All the geometric edges are chamfered with a 2 nm rounding radius. The top left inset plots the absorption cross-section for a single conical nanoparticle. (b) Field enhancement spectra evaluated at the gap center for the same structures as panel (a). The insets compare the electric field map at the gap region obtained numerically from the nonlocal hydrodynamic treatment (bottom) and the LAM with $\Delta d = 0.1 \text{ nm}$ (top).
Finally I should like to thank AFOSR for their generous support which has helped me maintain my long standing links with the USA. During the year I was elected a Foreign Associate of the National Academy of Sciences in recognition of my involvement with US science, an award which gave me great pleasure.

JB Pendry
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25 March 2014
List of Publications associated with my group in 2013

1. Title: **Harvesting light with transformation optics**  
Author(s): Luo Yu; Zhao Rongkuo; Fernandez-Dominguez, Antonio, I; et al.  
Source: SCIENCE CHINA-INFORMATION SCIENCES Volume: 56 Issue: 12 Article Number: 120401 DOI: 10.1007/s11432-013-5031-2 Published: DEC 2013

2. Title: **Nonlocal propagation and tunnelling of surface plasmons in metallic hourglass waveguides**  
Author(s): Wiener, Aeneas; Fernandez-Dominguez, Antonio I.; Pendry, J. B.; et al.  
Source: OPTICS EXPRESS Volume: 21 Issue: 22 Pages: 27509-27518 DOI: 10.1364/OE.21.027509 Published: NOV 4 2013

3. Title: **Surface Plasmons and Nonlocality: A Simple Model**  
Author(s): Luo, Yu; Fernandez-Dominguez, A. I.; Wiener, Aeneas; et al.  
Source: PHYSICAL REVIEW LETTERS Volume: 111 Issue: 9 Article Number: 093901 DOI: 10.1103/PhysRevLett.111.093901 Published: AUG 26 2013

4. Title: **Capturing photons with transformation optics**  
Author(s): Pendry, J. B.; Fernandez-Dominguez, A. I.; Luo, Yu; et al.  
Source: NATURE PHYSICS Volume: 9 Issue: 8 Pages: 518-522 DOI: 10.1038/NPHYS2667 Published: AUG 2013

5. Title: **Description of van der Waals Interactions Using Transformation Optics**  
Author(s): Zhao, Rongkuo; Luo, Yu; Fernandez-Dominguez, A. I.; et al.  
Source: PHYSICAL REVIEW LETTERS Volume: 111 Issue: 3 Article Number: 033602 DOI: 10.1103/PhysRevLett.111.033602 Published: JUL 17 2013

6. Title: **Electron-Energy Loss Study of Nonlocal Effects in Connected Plasmonic Nanoprism**s  
Author(s): Wiener, Aeneas; Duan, Huigao; Bosman, Michel; et al.  
Source: ACS NANO Volume: 7 Issue: 7 Pages: 6287-6296 DOI: 10.1021/nn402323t Published: JUL 2013

7. Title: **NEGATIVE REFRACTION Imaging through the looking-glass**  
Author(s): Oulton, Rupert F.; Pendry, John B.  
Source: NATURE PHYSICS Volume: 9 Issue: 6 Pages: 323-324 DOI: 10.1038/nphys2645 Published: JUN 2013

8. Title: **Hydrodynamic Model for Plasmonics: A Macroscopic Approach to a Microscopic Problem**  
Author(s): Ciraci, Cristian; Pendry, John B.; Smith, David R.  
Source: CHEMPHYSCHEM Volume: 14 Issue: 6 Pages: 1109-1116 DOI: 10.1002/cphc.201200992 Published: APR 15 2013

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