
Ayhan Altintasa
Vladimir B. Yurchenko

BILKENT UNIVERSITESİ
EDUCATION AND RESEARCH
ESKİSEHIR YOLU 8. KM
ANKARA 06800 TURKEY

EOARD Grant 12-2073

Report Date: October 2013

Final Report from 1 August 2012 to 30 September 2013

Distribution Statement A: Approved for public release distribution is unlimited.

Air Force Research Laboratory
Air Force Office of Scientific Research
European Office of Aerospace Research and Development
Unit 4515 Box 14, APO AE 09421

- **Contract Number**: FA8655-12-1-2073
- **Grant Number**: Grant 12-2073
- **Program Element Number**: 61102F

**7. Performing Organization Name(s) and Address(es)**
Bilkent Universitesi
Education and Research
Eskisehir Yolu 8. KM
Ankara 06800 Turkey

**9. Sponsoring/Monitoring Agency Name(s) and Address(es)**
EOARD
Unit 4515
APO AE 09421-4515

**12. Distribution/Availability Statement**
Distribution A: Approved for public release; distribution is unlimited.

**14. ABSTRACT**
We proposed new design concepts and solutions for non-paraxial quasi-optical dielectric lenses that exceed in performance more conventional approaches. One of new advances is the use of optimized dual-aspheric surfaces for dielectric lenses which allow one to minimize (and equalize) the refraction angles at the lens opposite surfaces. This reduces the total on-axis thickness of the lens which is important under the conditions of significant non-paraxial operation of the lens. In addition, this also reduces the aberrations of polarization pattern in the image domain. Another advance is the use of split-step double-sided Fresnel profile of the lens along with optimized dual-aspheric shape of lens surfaces between the Fresnel steps. This design solution should further reduce both the lens thickness and possible aberrations as compared to more conventional solutions. For the electromagnetic analysis and simulation of dielectric lenses of complicated shape and design, we have proposed a new kind of asymptotic hybrid approach that combines both the wave-based and ray-tracing approximations in an improved manner which is based on the use of newly proposed extended physical optics (EPO) diffraction integral formulation specifically created for the modeling of quasi-optical components with curvilinear refractive surfaces. We also made focal field measurements of double-sided split-step dual-aspheric mm-wave Fresnel lenses designed for producing well-shaped Gaussian beams in the focal domain. Two lenses, one of Teflon and another of polyamide, were compared in their performance with a double-sided Fresnel lens of different design, with full height of Fresnel steps. We confirm a good focusing ability of all the lenses. They produced the target beam width according to the design. Yet, the lenses of split-step design are thinner, have lower insertion losses, and a greater focal depth as compared to more conventional Fresnel lenses. Experimental results of our investigations are presented in the paper submitted for publication. MM-wave lenses made of lossy materials could serve as a good model for the analysis of lenses for THz applications where losses are always significant and unavoidable.

**15. Subject Terms**
EOARD, Optics, W-band Dual-Aspheric Lenses

<table>
<thead>
<tr>
<th>16. SECURITY CLASSIFICATION OF:</th>
<th>17. LIMITATION OF ABSTRACT</th>
<th>18. NUMBER OF PAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. REPORT UNCLAS</td>
<td>SAR</td>
<td>26</td>
</tr>
<tr>
<td>b. ABSTRACT UNCLAS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. THIS PAGE UNCLAS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**19a. Name of Responsible Person**
John Gonglewski

**19b. Telephone Number** (Include area code)
+44 (0)1895 616007

A. Altintas and V. Yurchenko

EEE Department, Bilkent University
Ankara, Turkey

23 October, 2013
Contents

List of Figures 3

Summary 4

1 Introduction 4

2 New design approaches and simulation methods for quasi-optical lenses 8
   2.1 Dual-aspheric non-paraxial lens profiles 9
   2.2 Double-sided and split-step Fresnel lenses with dual-aspheric surfaces 11

3 A 3D complex-point-source model and extended physical optics method for lenses 13
   3.1 A 3D vector-field complex-point-source model 13
   3.2 An extended physical optics method for dielectric lenses 14

4 Fabrication and testing of mm-wave Fresnel lenses 16
   4.1 Focal field measurements of quasi-optical Fresnel lenses 16
   4.2 Selection of optimal design solutions 18

5 Conclusions 19

References 21

List of Symbols, Abbreviations, and Acronyms 23

Acknowledgement of Sponsorship 24

Disclaimer 24

Disclosure of Inventions 24

Distribution A: Approved for public release; distribution is unlimited.
## List of Figures

1. Non-paraxial quasi-optical system using a dual-aspheric double-sided Fresnel lens .................................................. 6
2. Profiles of (a) conventional aspheric (plano-convex) and (b) optimized dual-aspheric W-band PTFE lenses as needed for a non-paraxial system of Fig. 1 . . 10
3. Profiles of (a) full-step and (b) split-step double-sided dual-aspheric W-band PTFE Fresnel lenses as needed for a non-paraxial system of Fig. 1 . . . . . . . 12
4. A lens of the design of Fig. 3, a, mounted for experimental testing . . . . . . . 12
5. Quasi-optical setup for mm-wave experiments at the NUI Maynooth, Ireland . . 16
6. Focal field power and phase distributions as measured for the split-step dual-aspheric double-side Fresnel lens of new design made of Teflon (lens #1, \( H = 30 \) mm) ................................................................. 18
7. Focal field power and phase distributions as measured for the split-step dual-aspheric double-sided Fresnel lens of new design made of polyamide (lens #2, \( H = 20 \) mm) ................................................................. 19
8. Focal field power and phase distributions as measured for the full-step dual-aspheric double-sided Fresnel lens of old design made of Teflon (lens #3, \( H = 35 \) mm) ................................................................. 20
Summary

The aim of the project is the development of extended physical optics method (EPO) for simulation of refractive quasi-optical components with curvilinear surfaces (e.g., diffractive lenses etc) and its verification in the design, manufacturing and testing of double-sided dual-aspheric Fresnel lenses via comparison of different kinds of design and dielectric materials being used.

When implemented as a hybrid ray-and-wave simulation method, the EPO approach allows one to achieve a better design of complicated quasi-optical refractive elements, e.g., compact short-focus non-paraxial double-sided dual-aspheric Fresnel lenses and similar components.

After design being made, manufacturing and testing of double-sided Fresnel lenses for verification of simulation method will be performed. In this part, the comparison of different kinds of design will be made. Traditional Fresnel and aspheric lenses are one-sided (e.g., planar-convex) because available formulations, for simplicity, assume one surface being planar. With a new method, we shall design and produce double-sided dual-aspheric Fresnel lenses, which are superior for compact short-focus non-paraxial quasi-optical systems. In this case, one can reduce total field distortions in the focal domain and minimize unwanted cross-polarization by optimizing the relative shapes of two surfaces (e.g., by equalizing the incidence angles of the waves on both surfaces).

With advanced design being available, a question of practical importance will be addressed on which of two options is preferable for minimizing aberrations and cross-polarization effects in the focal plane of a short-focus non-paraxial quasi-optical lens. One option is a bulky lens of low dielectric constant material and of large curvature of its surfaces whereas another option is a relatively thin lens of high dielectric constant material and of smaller curvature of surfaces.

In the end, an optimal design of non-paraxial double-sided dual-aspheric Fresnel lens will be proposed for transforming divergent beams of Gaussian horns to well-collimated or convergent high-quality beams when using a compact short-focus quasi-optical imaging system.

1 Introduction

Quasi-optical refractive components such as compact diffractive lenses are in growing demand in modern applications of THz and sub-THz wave bands. High-quality quasi-optical components and efficient methods of their simulations are needed, e.g., in astrophysics (particularly, in deep-space satellite missions), remote sensing, Earth observations, climate and atmospheric
research, space- and aircraft guidance, car-collision avoidance systems and so on [1-4]. Available devices and simulation methods (either ray- or wave-based [5-9]) are not sufficient in this domain. They require further developments based on new ideas and new kinds of design solutions.

A promising simulation method, which is beneficial for components of intermediate scale (the size of 10 to 1000 radiation wavelengths) that have curvilinear surfaces (especially in the difficult case of non-paraxial propagation), is the one proposed recently for 2D problems [10]. The method is based on a new kind of dyadic Green’s function that we propose for curvilinear surfaces, which is an extension of Sommerfeld idea. A new Green’s function is actually a parametric family of functions that deliver improved representations for diffraction integrals over curvilinear surfaces, thus, providing better solutions as compared to conventional formulations based on canonical Sommerfeld integrals defined for planar surfaces [11].

The project proposed for implementing these ideas consists of five basic stages:

1. Development of new design approaches and simulation methods for quasi-optical lenses of non-conventional shape and performance. For improving the performance of quasi-optical dielectric lenses with demanding requirements on their operation in mm-wave, THz, infra-red and visible-light applications (e.g., with improved near-field operation, short focal length and relatively small apertures in terms of wavelength), new kinds of lens design solutions are needed. They require new approaches to the lens design and enhanced simulation methods for the wave field computing. In practice, enhanced asymptotic methods are of primary interest because of their superior efficiency as compared to any exact simulation tools that could be developed.

To address the problem, we propose a new kind of dual-aspheric double-sided Fresnel-step quasi-optical lenses and develop an improved asymptotic simulation technique for computing the wave field of these lenses. The technique is a kind of extended physical optics (EPO) simulation method designed specifically for refractive components with curvilinear surfaces.

The EPO method is a generalization of Sommerfeld diffraction integral for the case of curvilinear refractive surfaces [10] rather than planar apertures treated by canonical formulations [11]. Due to more consistent account for boundary conditions on curvilinear surfaces and a possibility of exclusion of less accurately defined terms in the diffraction integrals, the formulation of this kind is superior to Kirchhoff approximation. Other formulations suggested in recent years for, supposedly, similar purposes [12-13] do not follow the actual idea of Sommerfeld
and, therefore, do not perform so well as one would expect in such cases.

When tested in 2D problems, the EPO method proved to be beneficial in evaluation of diffraction integrals over curvilinear surfaces of quasi-optical lenses [10]. Now, generalization of the method to real-world 3D problems along with relevant software development are necessary for possible practical applications, which is the subject of this part of research project.

Implementation of the EPO method in application to dielectric lenses will be made through the hybrid approach where free-space wave propagation and focusing on rather long distances is represented by diffraction integrals whereas short-path in-lens propagation is simulated in terms of ray tracing and field transfer along the rays. This approach is naturally combined with both the lens design problems, which are usually (and better) treated in terms of rays, and the lens field simulations, which require complex field amplitude computations either in the far zone or in the focal domain of quasi-optical lenses.

2. Design and manufacturing of a few kinds of quasi-optical dual-aspheric Fresnel lenses. Quasi-optical mm-wave applications require compact short-focus non-paraxial imaging systems (Fig. 1). To satisfy the demands, a special design is needed that allows one to reduce the lens thickness, field aberrations, cross-polarization effects, etc. An advanced design is suggested in this proposal that utilizes a possibility of making double-sided dual-aspheric Fresnel lenses optimized for both the given source field and the required image field at the target surface in the given frequency band.

![Fig. 1: Non-paraxial quasi-optical system using a dual-aspheric double-sided Fresnel lens](image)

By making a double-sided Fresnel lens, one may split each step of the Fresnel pattern between two surfaces of a lens. This is a new design solution that allows one to reduce the overall
lens thickness and, also, the curvature of lens surfaces. In addition, by equalizing the angles that both the entrance and exit rays make with lens surfaces at the given non-paraxial source and target beams, one can find optimized aspheric profiles of both lens surfaces, thus, minimizing cross-polarization effects and further reducing lens thickness, aberrations, etc (Figs. 2 – 3).

A few kinds of Fresnel lenses of this design will be manufactured of different materials for experimental testing and comparison with EPO simulations in mm-wave frequency band.

3. Development of 3D complex-point-source horn field representation method followed with experimental testing of horn patterns and software verification. Accurate design of quasi-optical systems requires good knowledge of radiation fields of feed horns. Gaussian horns are often used in these cases where scalar Gaussian beams serve as a source model for rapid design. In more complicated cases as those in the project where polarization is of interest, a more accurate vector field model is needed. Though some vector corrections to scalar Gaussian beams are known [14], they appear quite inconvenient and seems never used in practice.

In this work, we propose a vector form of 3D complex-point-source (CPS) approximation as a simple model for representing vector fields of Gaussian horns. Though scalar CPS model was introduced earlier [15], to our knowledge, it has never been extended to a vector form and never used in practice. In the meantime, a clear advantage of 3D vector CPS model, in addition to its simplicity, is the fact that it supplies an exact solution to Helmholtz equation. Also, it coincides with a scalar Gaussian beam in paraxial domain in far field of Gaussian horns.

In the project, we shall develop 3D vector CPS model, implement it into simulation code, and apply it to quasi-optical lens design. We shall measure Gaussian horn field patterns at various distances from horn aperture and compare them with 3D CPS model predictions. A vector CPS model will also be used for verification of EPO code, since the latter should predict CPS field on arbitrary surface at any distance from the horn aperture and then compute field propagation further to another location so as the results could be compared directly with an exact CPS solution.

4. EPO simulations of quasi-optical lenses and of entire imaging system. Once the EPO code developed and lenses designed, EPO simulations of lenses will be made. In these simulations, the main questions of interest are the prediction of lens power and polarization patterns in the far field and in the focal domain at the given source field model, and verification of expected high-quality lens operation at the given dual-aspheric Fresnel lens design.
5. Measurement of power patterns of quasi-optical Fresnel lenses, comparison with simulation results, and selection of optimal design solutions. Measurement of power patterns will be made for different kinds of quasi-optical Fresnel lenses manufactured of different dielectric materials. The main goal is the comparison of simulation results with experimental data in a variety of situations, kinds of design, and values of parameters for dielectric lenses. In conclusion, an optimal design solution will be selected, particularly, in those cases when we have to choose between the options of either a bulky lens of low refraction index material or a thinner lens made of dielectric of higher refraction index.

2 New design approaches and simulation methods for quasi-optical lenses

Development of enhanced dielectric lenses for non-paraxial quasi-optical systems (e.g., those as in Fig. 1) requires new design solutions and simulation techniques for the entire setup optimization. Modern mm-wave and THz imaging systems could be significantly non-paraxial, of rather short focal length in terms of radiation wavelength, of reduced aberration of power and polarization patterns in both the focal area and the far-field domain, etc. As a typical case, we consider a quasi-optical setup for experimental testing of W-band light-controllable photonic-crystal switch developed in [16].

Because of strong requirements on both the edge taper and the quality of the wave field distribution in the focal domain, the lens in this example should be of sufficiently large diameter (with clear aperture \( D_c = 140 \text{mm} \) at the free-space wavelength \( \lambda = 3 \text{mm} \) when operating at the frequency \( f = 100 \text{GHz} \)) and, also, of relatively short focal length so as the horn-to-lens distance \( L_{S1} \) and the lens-to-target distance \( L_{S2} \) (see Fig. 1) were of typical size \( L_{S1} = 125 \text{mm} \) and \( L_{S2} = 435 \text{mm} \) at the lens thickness \( H = 35 \text{mm} \). With account of the horn aperture radius \( a = 7.14 \text{mm} \) and the target beam waist \( w_0 = 16.8 \text{mm} \), it means that the wavefront curvature radii of incoming and outgoing waves are about \( R_1 = 134 \text{mm} \) and \( R_2 = 671 \text{mm} \), respectively, which is comparable with the given lens diameter \( D_c = 140 \text{mm} \).

Following conventional solutions, non-paraxial lenses of plano-convex type are usually employed in such cases that is explained by relative simplicity of their design and manufacturing. In this case, there is only one surface of special profile that has to be defined when designing and manufacturing the lens for the particular system. The advantage is that the lens profile can...
easily be computed when the system parameters are specified.

Mathematically, the lens is defined by two surfaces $S_1$ and $S_2$, of which $S_1$ is the given planar surface and $S_2$ is the curvilinear surface that has to be found (we describe surfaces in cylindrical coordinates $(r_1, z_1)$ and $(r_2, z_2)$, respectively). In practice, at the design stage, a quasi-optical lens could be analyzed by means of ray-tracing techniques that are capable to provide a basic approximation for the lens shape. Then, at the next stage, the shape could further be optimized by applying more advanced methods, e.g., full-wave approaches, etc.

When considering plano-convex lenses by ray-tracing methods, we arrive at the differential equation that defines $S_2$ surface as a function $z_2 = z_2(r_2)$. The equation could be written as follows

$$
\frac{dy(x)}{dx} = t(x)\{1 + a(x)y(x)\}/\{1 - b(x)t(x)\}
$$

where $y(x) = z_2(r_2(x))$ is the $z$ coordinate of the ray exit point $\{r_2, z_2\}$ on $S_2$ surface when the ray entrance point on $S_1$ is $\{r_1, z_1\}$, $x = r_1$, $r_2 = x + y(x)\tan\theta$, $\theta = \arcsin(x/n\sqrt{R_1^2 + x^2})$, $a(x) = n^2 R_1^2/(n^2 R_1^2 + (n^2 - 1)x^2)^{3/2}$, $b(x) = x^2/(n^2 R_1^2 + (n^2 - 1)x^2)^{1/2}$, $t(x) = (\sin\beta + n\sin\theta)/(\cos\beta - n\cos\theta)$, $\beta = \arctan((x + y\tan\theta)/(R_2 - y))$, $n$ is the refractive index of the lens material, $R_1$ and $R_2$ are the wavefront curvature radii of incoming and outgoing waves at the lens surface $S_1$, respectively ($R_1$ and $R_2$ are the distances of effective phase centers of incoming and outgoing waves from the planar reference surface $S_1$ specified as $z_1 = 0$).

Now, when applying the design solution to the optical system of Fig. 1 with parameters specified above and choosing PTFE (Teflon) as a low-loss lens material ($n = 1.431$), we obtain a bulky and heavy lens as shown by solid curves in Fig. 2, a (thin lines represent the rays). Using the lens of this kind is clearly out of question for the given system. In addition to heavy weight and bulky shape, the lens would have significant aberrations in power and polarization patterns since the incidence angle near the rim at the inner side of $S_2$ surface ($\theta_{i2} = 39^\circ$) is close to the critical angle of total reflection ($\theta_c = 44^\circ$) and exceeds the inner Brewster angle ($\theta_{b2} = 35^\circ$), thus, particularly distorting polarization.

### 2.1 Dual-aspheric non-paraxial lens profiles

To mitigate the problem, we have developed dual-aspheric lenses which have both the $S_1$ and $S_2$ surfaces optimized for the given conditions. In this case, both the surfaces have special profiles defined by a set of two equations of the kind resembling Eq. (1) though having more complicated right-hand parts due to connection between $S_1$ and $S_2$ established by the refraction law when
both the surfaces are curvilinear. In general, one surface could be chosen arbitrary whereas another one is defined via the choice being accepted. Alternatively, an external requirement could be imposed on the choice of surface profiles, thus, establishing an additional link between the profiles and making a possible solution unique. With account of this degree of freedom, one can additionally optimize the lens shape.

In this work, we impose the requirement that the ray entrance and exit angles at the lens surfaces $S_1$ and $S_2$, respectively, to be equal one to another. In this way, we reduce the incidence and transmission angles at the surfaces so as they both differ as much as possible from the critical angle of total reflection $\theta_c$. This allows us to reduce aberrations that occur due to the large angles of incidence in significantly non-paraxial systems as shown in Fig. 2, b.

The results presented in Fig. 2, b, show that the incidence angle near the rim at the inner side of $S_2$ is reduced down to $\theta_{i2} = 30^\circ$ against $\theta_{i2} = 39^\circ$ for the plano-convex lens of a similar size. So, all the angles are now smaller and do not exceed critical values whereas all the rays emerging from the source phase center converge at the equivalent target point, thus, minimizing potential aberrations.

![Fig. 2: Profiles of (a) conventional aspheric (plano-convex) and (b) optimized dual-aspheric W-band PTFE lenses as needed for a non-paraxial system of Fig. 1](image-url)
2.2 Double-sided and split-step Fresnel lenses with dual-aspheric surfaces

A known way of reducing the lens thickness is the use of zone lenses (more generally, lenses with diffractive elements) proposed yet by Fresnel. Traditionally, Fresnel steps (or other diffractive elements) are created on one side of such a lens. In recent time, some design solutions appeared when both the surfaces have the Fresnel steps [17]. To make the lens with achromatic properties in the broad range of THz frequencies, the authors of [17] introduced Fresnel steps of large phase retardation on both sides of the lens whereas smooth rings between the steps are made as spherical surfaces of different curvature radii for different rings. The design of this kind is more advanced as compared to conventional one, though it is not of precisely dual-aspheric type, and Fresnel steps on the opposite sides are not mutually adjusted.

For our application in the mm-wave band [16] where we need significantly non-paraxial lenses, we have already found an optimal dual-aspheric lens profile as described in the preceding section. Once the lenses of this profile are quite heavy and bulky, we also introduced Fresnel steps on both sides of our lenses.

In distinction from other solutions, our lenses of double-sided Fresnel profile with dual-aspheric surfaces have two essential advantages: (a) positions of the Fresnel steps on the opposite surfaces are adjusted one with respect to another so as the wave disruptions arising due to Fresnel steps are localized along the same rays and, therefore, made reasonably minimal (assuming the given positions of both the source and target points) and (b) rings of smooth surfaces between the steps are of dual-aspheric shape computed with account of the actual step heights and positions so as all the parameters of both the smooth surfaces and Fresnel steps are found self-consistently, being adjusted one to another (Fig. 3, a).

The lens of this kind is designed to have a smooth central part rather extended so as to minimize the distortions of the main part of mm-wave beam and, in the same time, to make the edge taper of the entire lens at the level of -40 dB for the incident Gaussian beam as used in the actual application [16]. Notice, the total on-axis thickness of double-sided Fresnel lens is reduced to 35 mm as compared to 55 mm of a smooth-wall design with no Fresnel steps.

To achieve further reduction in the lens thickness, we propose a lens with a split-step double-sided Fresnel profile and dual-aspheric surfaces between the steps (Fig. 3, b). The lens thickness in this case is only 30 mm, though the main advantage of the design is the reduction of wave disruption at the Fresnel steps once the latter are now split in fractions between two surfaces.

As compared to the design of Fig. 3, a, where the phase retardation along the pair of Fresnel
steps of adjusted positions is twice the wave period ($4\pi$), we now obtain the phase retardation at each pair of fractional steps being of just one period ($2\pi$). Therefore, we expect a better performance of the lens as compared to other Fresnel lenses of more conventional design. The only restriction of the design is that the lens is optimized for the given positions of both the source and the image points and for the given shape of the incident Gaussian beam at the given operation frequency (in practice, in a certain band around the main operation frequency).

Fig. 4 shows an example of a Teflon lens of the design presented in Fig. 3, a, being mounted for experimental testing in W frequency band ($f = 75 - 110$ GHz).
3 A 3D complex-point-source model and extended physical optics method for lenses

3.1 A 3D vector-field complex-point-source model

Quasi-optical mm-wave beams are traditionally described as Gaussian beams or various kinds of series expansions in terms of Hermit-Laguerre-Bessel-Gaussian functions [18]. This has a major advantage of providing a simple analytical transformation law for the beam parameters as the beam propagates through the medium.

This approach, however, is insufficient for our purposes. The matter is the traditional approach with various improvements is based on essential assumptions which are not valid or not sufficiently justified in certain cases and, specifically, in our applications.

The first assumption is the paraxial propagation of beams that means small angular divergence and large width of propagating beams. Though the assumption is often valid, it is far from being correct in our problem. The reason is that we deal with the source beams radiated by the Gaussian horns with rather small beam width at the horn aperture $w_a = 4.6$ mm. This corresponds to the beam width $w_0$ (beam radius at $1/e$ field amplitude at the beam neck) being only $w_0 = 4.4 - 4.5$ mm at the radiation wavelength $\lambda = 3 - 4$ mm, respectively. This leads to significant beam divergence of about $\theta_d \approx 20^\circ - 30^\circ$ as measured from the beam axis and defined by the conventional formula $\theta_d = \lambda/\sqrt{\pi w_0}$.

The second assumption is the scalar form of the Gaussian beams whereas real electromagnetic beams are of 3D vector-field character. Though scalar approximation is valid for the paraxial beams, in our case the approximation is insufficient because of large divergence of our beams. There are some formulations proposed earlier on how to account, approximately, for the vector character of electromagnetic beams of typical Gaussian shape [14]. In our case, however, an exact formulation would be preferable.

Finally, the Gaussian beam is not an exact solution to the Helmholtz equation that governs the propagation of electromagnetic waves, though it is a solution to the scalar paraxial form of this equation.

To avoid these limitations when simulating the propagation of beams, we applied an approach based on the concept of the complex-point-source (CPS) model [15]. The model has been proposed a long time ago in both 2D and 3D formulations, though it was presented in a scalar form and, up to our knowledge, it has never been used in 3D vector-field version when
the dyadic Green function has to be derived and applied in numerical simulations.

The CPS model is based on the idea that, once the point source position is specified as a complex number whose real part is the actual source position in the real space and the imaginary part is non-zero, the wave field emitted by the source acquires the shape of the beam in real space so as the beam width (divergence) and directions are controlled by the imaginary part of the source position [15].

In our work, we implemented a CPS model in the vector-field form, with 3D CPS dyadic Green’s function and proper account of polarization, so as we could use the model for representing the beams of quasi-Gaussian corrugated feed horns. This was accomplished by recognizing the fact that the horn aperture field is linearly polarized so as, in terms of polarization, the horn radiates the field as an electric dipole, though with a special beam-like directivity pattern.

Thus, we have used an electric dipole point source with a given dipole orientation in real space according to the orientation of horn aperture and assigned an imaginary part to the source position so as to obtain a radiation beam of required width and divergence ($\theta_d \approx 20^\circ - 30^\circ$ as stated above) that propagates away from the horn in the positive direction along the horn axis. In doing this, we have also used a standard relationship [11, 15] that allows one to find the components of dyadic Green’s function when using a scalar point-source generating function $g(\vec{r}, \vec{r}') = \exp(ikR)/4\pi R$ where $R = |\vec{r} - \vec{r}'|$ is the complex-valued norm (the "length") of the difference vector between the (real-valued) observation point $\vec{r}$ and the (complex-valued) radiation source point $\vec{r}'$.

The validity of this representation of the source beam has been verified by comparing simulated CPS beam patterns with known power patterns of real horns and, in the paraxial domain, with Gaussian beam approximations developed earlier for this kind of horns.

### 3.2 An extended physical optics method for dielectric lenses

Electromagnetic simulations of quasi-optical dielectric lenses has become an important issue in mm-wave and THz technology, yet it is lagging behind of closely related problems such as antenna simulations and wave scattering in terms of availability of advanced software and quality of provided solutions. The reason is a longer history and higher priority of the latter problems due to significant importance of related applications, but also a more complicated character of computational problems involved.

Whereas antenna and scattering problems could, mostly, be treated in either wave-based
or ray-tracing approximation (depending on the size of the object, with possible corrections accounting for alternative effects), the lens problems of typical quasi-optical size inherently require a mixture of both approximations because of two different scales being present simultaneously. On the one hand, the wave propagation between the lens and either the source or the image domain is the far-field transformation. On the other hand, propagation inside the lens is, in fact, a very near field transformation that has to be described in terms alternative to the far-field propagation analysis.

For this reason, a promising approach is the hybrid one which combines one class of methods applied to ”in-lens” wave simulations, with an alternative class of techniques being used for ”out-of-lens” propagation of the wave field. Although certain problems concerning lenses of limited size (particularly, two-dimensional) and of special canonical shape (of the kind of ”smooth-wall” class) could efficiently be solved in a full-wave formulation when using regularization methods [8, 9], hybrid approaches of various kinds would still be more practical for engineering applications.

In this work, we develop an asymptotic hybrid approach where the wave passage from the source domain to the lens and from the lens to the image domain is treated in terms of the wave optics by using the relevant diffraction integral whereas the wave propagation through the lens (including the refraction and reflection at the lens surfaces) is analyzed by ray-tracing methods which are perfectly justified in this domain.

This is an approximate formulation since it does not treat the entire wave field self-consistently, yet the method is well justified for quasi-optical lenses of intermediate scale (for lenses of the size of 10 to 1000 of radiation wavelengths) assuming the proper diffraction integral being used. The latter condition appears to be more crucial for dielectric lenses (refractive elements) than for conventional conductive scatterers typically considered (reflector antennas etc) as one could see later from the examples to be presented.

An essential part of success in this approach is based on the use of extended (generalized) form of the diffraction integral of a Sommerfeld type specifically adjusted for curvilinear surfaces (as we proposed and tested earlier in 2D problems [10]) rather than those canonical forms developed for planar apertures [11]. Thus, the approach represents a kind of extended physical optics (EPO) formulation suited for dielectric refractive elements with curvilinear surfaces. The implementation of the approach into software for asymptotic simulation of 3D quasi-optical lenses is partially completed, though it requires further enhancement and optimization.
4 Fabrication and testing of mm-wave Fresnel lenses

4.1 Focal field measurements of quasi-optical Fresnel lenses

Here we report on fabrication and testing of double-sided split-step mm-wave Fresnel lenses designed for the operation in the frequency range of \( f = 75 - 110 \) GHz (W-band) and specifically optimized for the frequency \( f_0 = 95 \) GHz. The lenses are needed for the development of light-controllable mm-wave photonic-crystal beam switches similar to those presented in [16].

A pair of lenses is used for initially focusing a source beam onto the switch as a device under test (DUT) and then re-focusing the exit beam for matching the pattern of a receiver horn. A view of quasi-optical bench implementing this measurement setup with Teflon lenses of a new split-step design is shown in Fig. 5. The measurements are carried out using a vector network analyzer (VNA) facility operating at the Experimental Physics Department of the NUI Maynooth (Maynooth, Ireland).

![Fig. 5: Quasi-optical setup for mm-wave experiments at the NUI Maynooth, Ireland](image)

We need lenses of clear aperture diameter \( D_C = 140 \) mm for capturing the beam of feed horn at the edge taper of \(-40\) dB when the horn aperture of radius \( a = 7.14 \) mm is at the distance \( L_{S1} = 125 \) mm from the front lens surface \((S_1)\). The exit beam is focused on the DUT of clear aperture \( D_0 = 72 \) mm where the beam width should be \( \omega_0 = 17 - 24 \) mm depending on the DUT structure. The DUT is located at \( L_{S2} = 450 - 435 \) mm from the rear lens surface \((S2)\) that corresponds to the fixed distance \( L_2 = L_{S2} + H = 470 \) mm from the front surface used as a reference \((F\)-number of a lens is about \( F/0.7\)). Here, \( H \) is the lens thickness that varies in

\[ \text{Distribution A: Approved for public release; distribution is unlimited.} \]
the range of $H = 20 - 35$ mm for our Fresnel lenses made of different materials (Teflon and polyamide) but may be up to $H = 55$ mm for non-Fresnel lenses if made of Teflon.

We designed, fabricated, and tested two sets of double-sided split-step dual-aspheric Fresnel lenses optimized for the operation frequency of $f_0 = 95$ GHz. One set of lenses (those shown in Fig. 5) are made of Teflon (lenses #1), having thickness $H = 30$ mm, and designed by assuming Teflon permittivity $\varepsilon = 2.00$. The other lenses (#2) are made of polyamide, being of thickness $H = 20$ mm, and designed assuming $\varepsilon = 2.80$ as found for the given material in our earlier measurements [19]. The lenses of this kind were fabricated by using a CNC technique (MN Group, Ankara, Turkey) following the design described above.

Split-step Fresnel lenses of both materials were designed to provide the same beam on the DUT in the focal plane, with the beam waist $w_0 = 24$ mm and the edge taper of $-20$ dB at the DUT aperture $D_0 = 72$ mm. The values were chosen for reducing the beam convergence and improving the wavefront flatness at the DUT that should improve the functionality of switch structures. Contrary to this, full-step double-sided Fresnel lenses of an old design, also made of Teflon, were fabricated earlier (lenses #3) to provide a smaller beam waist of about $w_0 = 17$ mm on the DUT for reducing the edge taper down to $-40$ dB.

For detailed characterization of these lenses, we carried out measurements of their field distributions in the focal domain (both the field power and the phase) using the VNA scanner facility at the NUI Maynooth (Maynooth, Ireland). The measurements were made in the frequency band of $75 - 110$ GHz when scanning through the frequency with the increment of $5$ GHz. A special care was taken to ensure sufficiently perfect alignment of quasi-optical system, which is a very crucial issue in this experiment.

The results obtained for three kinds of Fresnel lenses are presented in Figs. 6 - 8. The plots show spatial distributions of power (in decibels and in the relative units) and phase (in degrees) of the wave field in the horizontal cut of the focal domain behind the lens (the $XZ$ plane, where $Z$ is the axial coordinate with origin $Z = 0$ placed at the expected focal point and $Z > 0$ pointing away from the source horn). The plots are built for all three lenses at the same operation frequency $f_0 = 95$ GHz (the phase plots are shown with aliasing effects because of too small wavelength for a clear presentation of these plots in a more detailed manner).

Focal field measurements confirmed a good focusing ability of all three kinds of lenses considered. Quantitative analysis shows that the target beam width values $w_0 = 17$ mm and $w_0 = 24$ mm are achieved with relevant lenses as expected.
Fig. 6: Focal field power and phase distributions as measured for the split-step dual-aspheic
double-side Fresnel lens of new design made of Teflon (lens #1, \( H = 30 \) mm)

4.2 Selection of optimal design solutions

The best focal field distributions are obtained with lenses made of Teflon as the material of
choice for mm-wave applications. Yet, a thinner lens of polyamide has also shown a good
performance despite greater losses in this material. The focal field of this lens, though less
structured with troughs and sidelobes outside the main beam, is well shaped in the main area
and, more so, shows a significant focal depth that is a useful quality in many applications.

An increased focal depth seems to be a characteristic feature of split-steps Fresnel lenses (#1
and #2) whereas the full-step lens (#3) has a shorter focal depth (this can be seen by comparing
the green zones in the relative power plots where the green color indicates the half-magnitude
power level). The effect, though, may be a result of special aspheric shape of lens surfaces
between the Fresnel steps, which appears to be rather close to paraboloidal shape.

All the lenses show some standing waves in the focal area, which is unavoidable in any
system. Yet, again, a lens of polyamide shows a bit peculiar property of having the "focal
point" as if split in two points along the beam axis, which are located at about 100 mm one
before and another after the main "focal point" expected at \( Z = 0 \) mm. This is in line with the
fact of extend focal depth obtained with split-step Fresnel lenses.
Fig. 7: Focal field power and phase distributions as measured for the split-step dual-aspheric double-sided Fresnel lens of new design made of polyamide (lens #2, $H = 20$ mm)

5 Conclusions

We proposed new design concepts and solutions for non-paraxial quasi-optical dielectric lenses that exceed in performance more conventional approaches.

One of new advances is the use of optimized dual-aspheric surfaces for dielectric lenses which allow one to minimize (and equalize) the refraction angles at the lens opposite surfaces. This reduces the total on-axis thickness of the lens which is important under the conditions of significant non-paraxial operation of the lens. In addition, this also reduces the aberrations of polarization pattern in the image domain.

Another advance is the use of split-step double-sided Fresnel profile of the lens along with optimized dual-aspheric shape of lens surfaces between the Fresnel steps. This design solution should further reduce both the lens thickness and possible aberrations as compared to more conventional solutions.

For the electromagnetic analysis and simulation of dielectric lenses of complicated shape and design, we have proposed a new kind of asymptotic hybrid approach that combines both the wave-based and ray-tracing approximations in an improved manner which is based on the use of newly proposed extended physical optics (EPO) diffraction integral formulation specifically...
Fig. 8: Focal field power and phase distributions as measured for the full-step dual-aspheric double-sided Fresnel lens of old design made of Teflon (lens #3, $H = 35$ mm) created for the modeling of quasi-optical components with curvilinear refractive surfaces.

We also made focal field measurements of double-sided split-step dual-aspheric mm-wave Fresnel lenses designed for producing well-shaped Gaussian beams in the focal domain. Two lenses, one of Teflon and another of polyamide, were compared in their performance with a double-sided Fresnel lens of different design, with full height of Fresnel steps.

We confirm a good focusing ability of all the lenses. They produced the target beam width according to the design. Yet, the lenses of split-step design are thinner, have lower insertion losses, and a greater focal depth as compared to more conventional Fresnel lenses. Experimental results of our investigations are presented in the paper submitted for publication [20].

MM-wave lenses made of lossy materials could serve as a good model for the analysis of lenses for THz applications where losses are always significant and unavoidable.
References


List of Symbols, Abbreviations, and Acronyms

W-band = frequency band of 75 – 110 GHz
PTFE = Teflon
EPO = Extended Physical Optics
CPS = Complex Point Source
DUT = Device Under Test
VNA = Vector Network Analyzer
CNC machines = Computer Numerical Control machines

\( \lambda \) is the radiation wavelength in free space
\( f \) is the frequency of radiation
\( f_0 \) is the lens optimal operation frequency
\( D_c \) is the lens clear aperture diameter
\( D \) is the lens external diameter
\( D_0 \) is the DUT aperture diameter
\( H \) is on-axis lens thickness
\( S_1, S_2 \) are the 1st (entrance) and the 2nd (exit) lens surfaces
\( L_{S1}, L_{S2} \) are the source-to-\( S_1 \) and \( S_2 \)-to-DUT distances
\( a \) is the feed horn aperture radius
\( w_0 \) is the beam width at the DUT
\( R_1, R_2 \) are the wavefront curvature radii of incoming and outgoing waves at \( S_1 \) and \( S_2 \)
\( r_1, z_1 \) are the cylindrical coordinates of lens \( S_1 \) surface points
\( r_2, z_2 \) are the cylindrical coordinates of lens \( S_2 \) surface points
\( \varepsilon \) is the complex dielectric constant of lens material
\( n = \sqrt{\varepsilon} \) is the refraction index of lens material
\( \theta_{i2} \) is the wave incidence angle at the inner side of lens \( S_1 \) surface
\( \theta_c \) is the critical angle of total internal reflection
\( \theta_b \) is the Brewster angle
\( \theta_d = \lambda/w_0\sqrt{\varepsilon} \) is the beam divergence angle
\( e = 2.7182... \) is the Euler number
\( g(\vec{r}, \vec{r}') = \exp(ikR)/4\pi R \) is the scalar Green’s function
\( R = |\vec{r} - \vec{r}'| \) is the complex-valued norm of the (complex) difference vector
\( X, Y, Z \) are the VNA scanner coordinates
Acknowledgement of Sponsorship: Effort sponsored by the Air Force Office of Scientific Research, Air Force Material Command, USAF, under grant number FA8655-12-1-2073. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purpose notwithstanding any copyright notation thereon.

Disclaimer: The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Office of Scientific Research or the U.S. Government.

Disclosure of Inventions: We certify that there were no subject inventions to declare during the performance of this grant.

Distribution A: Approved for public release; distribution is unlimited.