Geometry Effects on Steady and Acoustically Forced Shear-Coaxial Jet Sprays

Sophonias Teshome, Ivett Leyva, Juan Rodriguez, Doug Talley

This experimental study investigated the mixing behavior and characteristics of dynamic flow structures of cryogenic, non-reactive shear-coaxial jet sprays under varying flow conditions, with and without the presence of pressure perturbations due to acoustic forcing transverse to the flow direction. The role of injector geometry was examined using shear-coaxial injectors with different outer-to-inner jet area ratios and different inner jet post thickness to inner jet diameter ratios. Flow conditions spanning a high pressure spray (reduced pressure of 0.44) ranging up into the supercritical regime (reduced pressure of 1.05), with varying outer-to-inner jet momentum flux ratios (0.5 – 20), and maximum or minimum amplitude in the pressure perturbation at the jet axis location were considered. Nitrogen was used as the test fluid. The inner and outer jet temperatures were independently controlled so that the inner condensed flow was cooled down to below saturation temperature of the liquid, and to below the critical temperature for the supercritical fluid. Back-lighting the coaxial spray/supercritical jet resulted in a silhouette of the dense inner core, which appeared as a dark column. This distinguished it from the outer gaseous flow, and thus, enabled so that the inner condensed flow was cooled down to below saturation temperature of the liquid, and to below the critical temperature for the supercritical fluid. Back-lighting the coaxial spray/supercritical jet resulted in a silhouette of the dense inner flow, and length measurements were used to indicate the extent of mixing under the different flow conditions and injector geometries. As expected, increasing the momentum flux ratio resulted in shorter dark-core lengths. However, the extent of influence of momentum flux ratio was dependent on the injector geometry, whereby, the dark-core length of a large outer-to-inner jet area ratio injector flow was more influenced by increasing momentum flux ratios. A small area ratio injector flow, on the other hand, showed a more gradual decrease in its dark-core length with increasing momentum flux ratios. A basic application of proper orthogonal decomposition on the intensity fluctuation of the high-speed images enabled the extraction of the spatial and temporal characteristics of the dominant flow structures that existed in the flow field during exposure to acoustic forcing. Regardless of injector geometry or pressure regime, lower momentum flux ratio (generally less than 5) flows were found to be responsive to acoustic forcing. With increasing momentum flux ratio, however, and their corresponding acoustically forced flows revealed that, for a large-area ratio injector flow with high momentum flux ratio (generally more than 10), the baseline flow behavior was retained in the forced flow, thereby indicating a flow regime that was less sensitive to external pressure disturbances.
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Motivation

- Feedback cycle between liquid rocket engine (LRE) combustion chamber pressure perturbations and unsteady combustion\textsuperscript{1,2}
- Large amplitude fluctuations in pressure and combustion heat release rates ⇒ combustion instability

Injector head molten after a combustion instability event

Courtesy: U.S. Rocket and Space Center, Huntsville, AL
Objective

- Impose external acoustic perturbations, and examine the response and stability characteristic of shear-coaxial injector flow to pressure perturbation

- Investigate influence of injector geometry on flow response to external pressure perturbation

- Vary the outer-to-inner jet momentum flux ratio, $J$, under subcritical chamber pressure condition, i.e., reduced pressure $Pr = 0.44$

$$J = \frac{\rho_o u_o^2}{\rho_i u_i^2} \quad Pr = \frac{P_{\text{chamber}}}{P_{\text{critical},N_2}} \quad P_{\text{critical},N_2} = 493 \text{ psi} (3.4 \text{ MPa})$$

- Characterize mixing using dark-core length measurements

- Apply proper orthogonal decomposition of high-speed image pixel intensity fluctuations to extract spatial and temporal characteristics of prevalent coherent flow structures
Schematic of Experimental Facility

- Main Pressure Chamber
- Inner Chamber (Test Section)
- GN2 (Ambient T)
- Inner Jet
- Outer Jet
- Chamber Pressurization GN2
- Acoustic Waveguide
- PIEZO-SIREN
- High-Speed Camera
- Exhaust
- Inner Chamber (Test Section)
- Xenon Arc-Lamp
- Main Pressure Chamber
- LN2 In
- LN2 Out
- LX2 In
- LX2 Out
- HEX

Distribution A: Approved for Public Release; Distribution Unlimited
Image of Experimental Facility

- Piezo-Siren
- Waveguide
- High-Speed Camera
- Coaxial Injector
- Differential Pressure Transducers
- Thermocouple Probe
- Inner Chamber

Distribution A: Approved for Public Release; Distribution Unlimited
## Additional Geometric Configurations

### All dimensions in mm

<table>
<thead>
<tr>
<th>Injector</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$t/D_1$</th>
<th>$A_o/A_i$</th>
<th>$l/D_1$</th>
</tr>
</thead>
<tbody>
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<td>LAR-thick</td>
<td>0.51</td>
<td>1.59</td>
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<td>1.05</td>
<td>12.9</td>
<td>0.5</td>
</tr>
<tr>
<td>SAR-thin</td>
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<td>2.44</td>
<td>3.94</td>
<td>0.09</td>
<td>1.6</td>
<td>0.5</td>
</tr>
<tr>
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<td>4.70</td>
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<td>0.84</td>
<td>2.9</td>
<td>-0.1</td>
</tr>
<tr>
<td>LAR-thick</td>
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<td>0.89</td>
<td>2.44</td>
<td>3.94</td>
<td>0.13</td>
<td>10.6</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

SAR, LAR → Small, Large Area Ratio
Thick, Thin → Post thickness

Davis, Rodriguez, Leyva *et al.*

Rodriguez, Graham *et al.* ($l/D_1 = 0$)

Present study (-0.11 $D_1$ recess)

Present study (-0.21 $D_1$ recess)
Acoustic Field Set-Up: Pressure Antinode

- Pressure antinode (PAN) – condition of maximum pressure perturbation in the acoustic field
- Piezo-sirens forced in-phase
- Superposition of quasi-1D acoustic waves traveling in opposite directions ⇒ PAN at the jet location (geometric center of test section)
Dark-Core Length Measurement

- First raw grayscale images were converted to binary images
- A contour was drawn around the “dark-column” in the binary image
- Axial length of the dark-column measured and defined as the Dark-Core Length, \( L \)
Baseline Dark-Core Lengths

\[ L_{B/D_1} \]

\[ J \]

\( \triangle \) LAR Thick

\( \diamond \) SAR Thick

\( \bigcirc \) LAR thin

\( \square \) SAR Thin

Distribution A: Approved for Public Release; Distribution Unlimited
Proper Orthogonal Decomposition

• Proper Orthogonal Decomposition (POD) or Principal Component Analysis (PCA) was used for extracting dominant dynamical processes embedded in high-speed images.

• A time-resolved set of images $A(x,t)$ can be represented as a linear combination of orthonormal basis functions $\phi_k$ (aka proper orthogonal modes)\textsuperscript{1,2,3}:

$$A(x,t) = \sum_{k=1}^{M} a_k(t)\phi_k(x)$$

where $a_k(t)$ are time dependent orthonormal amplitude coefficients and $M$ is the number of modes.

• Main idea: POD modal amplitudes capture the maximum possible “energy” in an average sense\textsuperscript{4}, i.e.,

$$\sum_k \langle a_k(t)a_k(t) \rangle \geq \sum_k \langle b_k(t)b_k(t) \rangle$$

where $b_k(t)$ are the temporal coefficients of a decomposition with respect to an arbitrary orthonormal basis $\psi_k$.

\textsuperscript{1} Chatterjee, A. Current Science, Vol. 78, No. 7 (2000)
Construction of Data Set

• First, form a row vector consisting of all pixel intensity values of each snapshot image (with resolution of $n$ rows by $m$ columns) in order of increasing columns, then increasing rows.

- Pixel Intensity
  - $m$ columns
  - $n$ rows
  - $N$ frames

- Image Frame

• Then, combine all such row vectors for $N$ sequences of image frames resulting in a matrix $A$ consisting of $N$ rows by $P = n \times m$ columns of intensity values.

$$A = \begin{pmatrix}
\end{pmatrix}$$

$P = n \times m$

$N$ time steps

pixel intensities
Orthogonal Decomposition Technique

• Eigenvalue decomposition or singular value decomposition (SVD) can be used

• SVD preferred since
  1. Applicable to non-square matrices (most likely the case)
  2. Decomposition matrices are orthogonal
  3. Subroutine readily available in MATLAB®

• Subtracted temporal mean of \( A \Rightarrow \) matrix of intensity fluctuations \( \tilde{A} \)

• Applied SVD

\[
\begin{align*}
\Rightarrow \tilde{A} &= USV^T = QV^T \\
& \quad \text{Columns of } Q \sim a_k(t) \text{ contain temporal information} \\
& \quad \text{Columns of } V \sim \phi_k(x) \text{ contain spatial information}
\end{align*}
\]

\( N \times N \)

Orthogonal Matrix of Left Singular Column Vectors of \( \tilde{A} \)

\( N \times P \)

Orthogonal Matrix of Right Singular Column Vectors of \( \tilde{A} \) \( \leftrightarrow \) proper orthogonal modes (POM)

\( N \times N \)

Diagonal Matrix of Singular Values
Results – Subcritical Baseline at Low $J$

- LAR, $Pr = 0.44$, $J = 0.5$

Amplitude information contained in singular values

Antisymmetric Structures
Identified with Characteristic Frequencies

Power Spectral Densities (PSD) of Temporal Coefficients of POMs 1 and 2
Results – Subcritical PAN at Low $J$

- LAR, $Pr = 0.44$, $J = 0.5$, forcing Frequency, $f_F = 3.14$ kHz

**Symmetric Structures**
Identified with Characteristic Frequency at $f_F$
Cross-Power Spectral Density (CPSD)

- CPSD yields the FFT of the cross-correlation of the temporal coefficients
  \[
  \text{CPSD} = \sum_{k=0}^{N-1} \frac{\text{cov}(a_x, a_y)}{\sigma_x \sigma_y} e^{-i\omega k}
  \]

- Magnitude and phase plots used to determine existence of propagating structures

LAR, \( Pr = 0.44, J = 0.5 \)

Baseline

PAN

\( f_F = 3.14 \text{ kHz} \)
Sample Animation – PAN ($f_f = 3.14 \text{ kHz}$)

- $\text{LAR } Pr = 0.44, J = 0.5$

Superposition of POMs 1 and 2 Resulted in Downstream Propagating Structures
Results – LAR-thin

• Antisymmetric flow structures indicated helical type flow instabilities for all $J$

$J = 2.1$

Baseline

PAN

$f_F = 3.12 \text{ kHz}$

$J = 20$

Baseline

PAN

$f_F = 3.11 \text{ kHz}$
Previous Works on Jet Instability

• Michalke and Hermann (1982) did linear, inviscid instability analysis of a circular jet with coflow
  • Showed that with increasing coflow velocity, \( U_\infty \)
    -- Helical disturbances more unstable than axisymmetric ones farther downstream of exit
    -- Jet flow becomes less unstable, but spectrum of spatial growth rate becomes broader and the peak shifts to higher frequencies

Dahm et al., Wicker and Eaton (1994) conducted experimental investigation of large-scale vortex structures in the near field of coaxial jets
  • For outer-to-inner jet velocity ratios greater than one, found that coherent structures in the outer shear layer dominate those in the inner shear layer
  • At large axial distances, shear-layer vortices exhibit helical structures

Dahm et al., JFM 1992
Results – SAR-thin

- Helical type flow instabilities became more well-defined with increasing $J$

<table>
<thead>
<tr>
<th>$J = 2.0$</th>
<th>$J = 17$</th>
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<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td><strong>PAN</strong> ($f_F = 2.97$ kHz)</td>
</tr>
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</table>

![Images of flow instabilities and CFSD Magnitude plots for different $J$ values]
Results – SAR-thick

- Helical type flow instabilities became more well-defined with increasing $J$

<table>
<thead>
<tr>
<th>$J = 2.1$</th>
<th>$J = 21$</th>
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<tr>
<td><strong>Baseline</strong></td>
<td><strong>PAN</strong> ($f_F = 3.07$ kHz)</td>
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Graphs showing differences in flow patterns and CPFD magnitude with varying $J$ values.
Summary and Conclusions

- Examined the effect of different exit geometries on the mixing characteristics as well as the behavior of flow disturbance structures with and without transverse acoustic forcing -- **mixing not only depends on momentum flux ratio!**
- Proper orthogonal decomposition of high-speed image intensity fluctuation data revealed key spatial and temporal characteristics of flow structures
- Low $J$, SAR injector flows, had significantly lower $L_B/D_1$ than the large area ratio flows
- Low $J$ LAR-thin injector flow showed a strong response at the PAN forcing frequency while the high $J$ appeared less responsive and retained the baseline flow spectral characteristic
- SAR-thin injector flow showed strong response at the PAN forcing regardless of $J$
- Low $J$ SAR-thick injector flow showed no response at the forcing frequency, while the high $J$ flow did
- Operated at high enough $J$, LAR injector flows less vulnerable to external pressure disturbances
Acknowledgement

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  – Randy Harvey, David Hill, Earl Thomas (ERC)
  – Todd Newkirk (Jacobs Engineering)

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Back-Up
## Data Summary Table

<table>
<thead>
<tr>
<th>Injector</th>
<th>$J$</th>
<th>$R$</th>
<th>$T_{ch}$ K</th>
<th>$\rho_{ch}$ kg/m$^3$</th>
<th>$P_{ch}$ MPa</th>
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<th>$m_o$ mg/s</th>
<th>$\rho_o$ kg/m$^3$</th>
<th>$U_o$ m/s</th>
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<th>$m_i$ mg/s</th>
<th>$\rho_i$ kg/m$^3$</th>
<th>$U_i$ m/s</th>
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