# Non-Reactive Shear-Coaxial Jets with and without Transverse Acoustic Forcing

This research has concentrated on the characterization of mixing for typical liquid rocket injectors. The conditions of interest have encompassed subcritical to supercritical pressures with temperatures below and above the critical temperature, with and without an acoustic field. The project has been largely experimental in-house. A single shear coaxial injector element has been thoroughly studied. This type of injector element is commonly used in many rocket engines (e.g., Space Shuttle Main Engine (SSME), Vulcain, RS-68, etc). It was found that the mixing in these injectors is governed primarily by the presence or absence of an acoustic field, the outer to inner jet area ratio and the inner jet lip thickness to inner jet diameter, the outer to inner momentum flux ratio, the reduced density of the inner jet, and on whether the pressure is subcritical and supercritical.
Non-Reactive Shear-Coaxial Jets with and without Transverse Acoustic Forcing

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Motivation

- Feedback cycle between liquid rocket engine (LRE) combustion chamber pressure perturbations and unsteady combustion\textsuperscript{1,2}
- Large amplitude fluctuations in pressure and combustion heat release rates $\Rightarrow$ combustion instability

\textsuperscript{1}Harrje, D.T., and Reardon, F.H. \textit{Scientific and Technical Information Office}, National Aeronautics and Space Administration, NASA SP-194, 1972.

Objective

- Impose external acoustic perturbations, and examine the response and stability characteristic of shear-coaxial injector flow to pressure perturbation

Acoustic/Pressure Perturbation  →  Shear-Coaxial Injector Flow

- Investigate influence of injector geometry on flow response to external pressure perturbation
- Vary the outer-to-inner jet momentum flux ratio, $J$, under subcritical chamber pressure condition, i.e., reduced pressure $Pr = 0.44, 1.05$

$$J = \frac{\rho_o u_o^2}{\rho_i u_i^2} \quad Pr = \frac{P_{chamber}}{P_{critical,N_2}} \quad P_{critical,N_2} = 3.4 \text{ MPa (493 psi)}$$

- Characterize mixing using dark-core length measurements
- Apply proper orthogonal decomposition of high-speed image pixel intensity fluctuations to extract spatial and temporal characteristics of prevalent coherent flow structures
Schematic of Experimental Facility
Image of Experimental Facility

Piezo-Siren

Waveguide

High-Speed Camera

Coaxial Injector

Differential Pressure Transducers

Thermocouple Probe

Inner Chamber

Distribution A: Approved for Public Release; Distribution Unlimited
Injector Assembly

EC-4 Injector Top Adapter
SwageLock 1/16" Male Connector to 1/16" Male NPT Pipe P/N -160-1-1
SwageLock 1/16" Front Ferrule P/N -163-1
SwageLock 1/16" Back Ferrule P/N -104-1
SwageLock 1/16" Nut P/N -102-1
EC-4 Injector Center/Fuel Adapter
EC-4 Injector Tube Centering Plate
EC-4 Injector Bottom Plate

Injector Assembly Diagram

IJ Inlet Port
PT/TC Port
PT/TC Ports
IJ Plenum
PT/TC Plenum
IJ Injector Tube
OJ Injector Tube
Dowel Pin 416 Stainless Steel Two 1/8" Dia. 1/2" Length

Distribution A: Approved for Public Release; Distribution Unlimited
Injector Configuration

- Two types of outer-to-inner jet cross-sectional area ratios
  - Large Area Ratio, thin post (LAR-thin)
  - Small Area Ratio, thick post (SAR-thick)

All dimensions in mm

<table>
<thead>
<tr>
<th>Injector</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$t/D_1$</th>
<th>$A_o/A_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAR-thin</td>
<td>0.70</td>
<td>0.89</td>
<td>2.44</td>
<td>3.94</td>
<td>0.13</td>
<td>10.6</td>
</tr>
<tr>
<td>SAR-thick</td>
<td>1.47</td>
<td>3.96</td>
<td>4.70</td>
<td>6.35</td>
<td>0.84</td>
<td>2.90</td>
</tr>
</tbody>
</table>
Waveguides

Acoustic Source – fits in circular end of waveguide

Circular end

Rectangular end – which connects to inner chamber

Flow direction

Standing wave (pressure anti-node case)

Waveguides viewed the front – cross section

Inner Chamber

Waveguide viewed from side (going from round to rectangular cross section)
Acoustic Field Set-Up: Pressure Antinode

- Pressure antinode (PAN) – condition of maximum pressure perturbation in the acoustic field
- Piezo-sirens forced in-phase
- Superposition of quasi-1D acoustic waves traveling in opposite directions ⇒ PAN at the jet location (geometric center of test section)

![Diagram of acoustic field setup with PAN, Coaxial Jet, Standing Acoustic Pressure Waves, Piezo-Siren, and time trace graphs of pressure fluctuations.](image)
BASELINE FLOW IMAGES AND DARK-CORE LENGTHS
Baseline Flows: LAR-thin Injector

$P_r = 0.44$

$J = 0.1 \quad 0.5 \quad 2.1 \quad 5.2 \quad 11 \quad 14 \quad 20$

$P_r = 1.05$

$J = 0.1 \quad 0.5 \quad 1.9 \quad 5.0 \quad 8.5 \quad 12$
Baseline Flows: SAR-thick Injector

$P_r = 0.44$

$J = 0.1 \quad 0.5 \quad 2.1 \quad 5.7 \quad 10 \quad 15 \quad 21$

$P_r = 1.05$

$J = 0.1 \quad 0.5 \quad 2.1 \quad 5.2 \quad 9.2 \quad 14 \quad 21$
Dark-Core Length (DCL) Measurement

- First raw grayscale images were converted to binary images
- A contour was drawn around the “dark-column” in the binary image
- Axial length of the dark-column measured and defined as the Dark-Core Length, $L$
Baseline Normalized DCL, $L_B/D_1$
Geometric Configurations

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$t/D_1$</th>
<th>$A_o/A_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAR-thick</td>
<td>0.51</td>
<td>1.59</td>
<td>2.42</td>
<td>3.18</td>
<td>1.05</td>
<td>12.9</td>
</tr>
<tr>
<td>SAR-thin</td>
<td>1.40</td>
<td>1.65</td>
<td>2.44</td>
<td>3.94</td>
<td>0.09</td>
<td>1.6</td>
</tr>
<tr>
<td>SAR-thick</td>
<td>1.47</td>
<td>3.96</td>
<td>4.70</td>
<td>6.35</td>
<td>0.84</td>
<td>2.9</td>
</tr>
<tr>
<td>LAR-thin</td>
<td>0.70</td>
<td>0.89</td>
<td>2.44</td>
<td>3.94</td>
<td>0.13</td>
<td>10.6</td>
</tr>
</tbody>
</table>

SAR, LAR → Small, Large Area Ratio
Thick, Thin → Post lip thickness

Two recesses
Baseline Normalized DCL, $L_B/D_1$ – $P_r = 0.44$

\begin{align*}
L_B/D_1 & \quad J \\
\text{LAR-thin} & \quad \text{SAR-thin (Rodriguez)} \\
\text{SAR-thin (Graham et al.)} & \quad \text{LAR-thick (Rodriguez)} \\
\text{LAR-thick (Leyva et al.)} & \quad \text{SAR-thick}
\end{align*}
Baseline Normalized DCL, $L_B/D_1 \rightarrow P_r = 1.05$
Baseline Normalized DCL, $L_B/D_1$ Collapse

$$\left( \frac{L_B}{D_1} \right) = c_1 J^{c_2} \left( \frac{t}{D_1} \right)^{c_3} \left( \frac{A_o}{A_i} \right)^{c_4}$$

$P_r = 0.44$

$P_r = 1.05$

<table>
<thead>
<tr>
<th>$P_r$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.44</td>
<td>9</td>
<td>-0.34</td>
<td>-0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>1.05</td>
<td>11</td>
<td>-0.43</td>
<td>-0.12</td>
<td>0.15</td>
</tr>
</tbody>
</table>
ACOUSTICALLY FORCED FLOW IMAGES AND DARK-CORE LENGTHS
Acoustically Forced Flows: LAR-thin – $P_r = 0.44$

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.82%
  - PN: 0.09%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.39%
  - PN: 0.13%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.96%
  - PN: 0.25%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.42%
  - PN: 0.23%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.13%
  - PN: 0.25%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.42%
  - PN: 0.23%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.96%
  - PN: 0.25%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.39%
  - PN: 0.13%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.82%
  - PN: 0.09%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.42%
  - PN: 0.23%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.96%
  - PN: 0.25%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.39%
  - PN: 0.13%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.82%
  - PN: 0.09%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.42%
  - PN: 0.23%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.96%
  - PN: 0.25%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.39%
  - PN: 0.13%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.82%
  - PN: 0.09%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 1.42%
  - PN: 0.23%

- **Baseline**
  - $p'_{pk-pk}/P_c = 0.01$
  - PAN: 0.96%
  - PN: 0.25%
Acoustically Forced Flows: LAR-thin – $P_r = 0.44$

Baseline

$P_{pk-pk}/P_c = 0.01$

PAN

1.00%

PN

0.19%

$J = 11$, $f_F = 3.10$ kHz

Baseline

$P_{pk-pk}/P_c = 0.01$

PAN

1.04%

PN

0.30%

$J = 14$, $f_F = 3.11$ kHz

Baseline

$P_{pk-pk}/P_c = 0.01$

PAN

1.03%

PN

0.19%

$J = 20$, $f_F = 3.11$ kHz
### Acoustically Forced Flows: LAR-thin – $P_r = 1.05$

<table>
<thead>
<tr>
<th>Case</th>
<th>Baseline $p'_{pk-pk}/P_c$</th>
<th>PAN $p'_{pk-pk}/P_c$</th>
<th>Baseline $p'_{pk-pk}/P_c$</th>
<th>PAN $p'_{pk-pk}/P_c$</th>
<th>Baseline $p'_{pk-pk}/P_c$</th>
<th>PAN $p'_{pk-pk}/P_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J = 0.1$</td>
<td>0.01%</td>
<td>0.77%</td>
<td>0.00%</td>
<td>0.98%</td>
<td>0.01%</td>
<td>0.75%</td>
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<tr>
<td>$f_F$</td>
<td>3.16 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J = 0.5$</td>
<td>0.01%</td>
<td>0.98%</td>
<td>0.00%</td>
<td>0.98%</td>
<td>0.01%</td>
<td>0.54%</td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.10 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J = 1.9$</td>
<td>0.01%</td>
<td>1.22%</td>
<td>0.01%</td>
<td>0.54%</td>
<td>0.01%</td>
<td>1.02%</td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.10 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J = 5.0$</td>
<td>0.01%</td>
<td>0.98%</td>
<td>0.01%</td>
<td>0.54%</td>
<td>0.01%</td>
<td>0.05%</td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.41 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J = 12$</td>
<td>0.01%</td>
<td>1.22%</td>
<td>0.01%</td>
<td>0.54%</td>
<td>0.01%</td>
<td>1.02%</td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.10 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PAN Forcing: 40 μs Interval Image Sequence

LAR-thin, $P_r = 0.44$, $J = 0.5$
PAN Forcing: 40 μs Interval Image Sequence

LAR-thin, $P_r = 0.44$, $J = 11$
Acoustically Forced Flows: SAR-thick – $P_r = 0.44$

<table>
<thead>
<tr>
<th>$J = 0.1$, $f_F = 3.10$ kHz</th>
<th>$J = 0.5$, $f_F = 3.04$ kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{pk-pk}/P_c = 0.02%$</td>
<td>$P_{pk-pk}/P_c = 0.02%$</td>
</tr>
<tr>
<td>$P_{pk-pk}/P_c = 0.02%$</td>
<td>$P_{pk-pk}/P_c = 0.02%$</td>
</tr>
<tr>
<td>$J = 2.1$, $f_F = 3.07$ kHz</td>
<td>$J = 5.7$, $f_F = 3.11$ kHz</td>
</tr>
</tbody>
</table>
Acoustically Forced Flows: SAR-thick – $P_r = 0.44$

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>PAN</th>
<th>PN</th>
<th></th>
<th>Baseline</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_{pk-pk}/P_c$</td>
<td>0.02%</td>
<td>1.11%</td>
<td>0.23%</td>
<td>$P'_{pk-pk}/P_c$</td>
<td>0.02%</td>
<td>1.48%</td>
<td>0.25%</td>
</tr>
<tr>
<td>$J$</td>
<td>10</td>
<td>$f_F = 3.11$ kHz</td>
<td></td>
<td>$J$</td>
<td>15</td>
<td>$f_F = 3.04$ kHz</td>
<td></td>
</tr>
<tr>
<td>$P_{AN}$</td>
<td>1.11%</td>
<td></td>
<td></td>
<td>$P_{AN}$</td>
<td>1.51%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{PN}$</td>
<td>0.23%</td>
<td></td>
<td></td>
<td>$P_{PN}$</td>
<td>0.45%</td>
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</table>

$J = 21$, $f_F = 3.11$ kHz
Acoustically Forced Flows: SAR-thick – $P_r = 1.05$

Baseline

<table>
<thead>
<tr>
<th>$p_{pk-pk}/P_c$</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01%</td>
<td>0.69%</td>
<td>0.13%</td>
</tr>
</tbody>
</table>

$J = 0.1$, $f_F = 3.12$ kHz

Baseline

<table>
<thead>
<tr>
<th>$p_{pk-pk}/P_c$</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01%</td>
<td>1.17%</td>
<td>0.23%</td>
</tr>
</tbody>
</table>

$J = 2.1$, $f_F = 3.04$ kHz

Baseline

<table>
<thead>
<tr>
<th>$p_{pk-pk}/P_c$</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01%</td>
<td>0.67%</td>
<td>0.13%</td>
</tr>
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</table>

$J = 0.5$, $f_F = 3.00$ kHz

Baseline

<table>
<thead>
<tr>
<th>$p_{pk-pk}/P_c$</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01%</td>
<td>0.93%</td>
<td>0.14%</td>
</tr>
</tbody>
</table>

$J = 5.2$, $f_F = 3.08$ kHz

Distribution A: Approved for Public Release; Distribution Unlimited
Acoustically Forced Flows: SAR-thick – $P_r = 1.05$

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_{pk-pk}/P_c$</td>
<td>0.01%</td>
<td>0.99%</td>
<td>0.19%</td>
</tr>
<tr>
<td>$J$</td>
<td>9.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.11 kHz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$J = 9.2$, $f_F = 3.11$ kHz

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_{pk-pk}/P_c$</td>
<td>0.01%</td>
<td>1.22%</td>
<td>0.25%</td>
</tr>
<tr>
<td>$J$</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.04 kHz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$J = 14$, $f_F = 3.04$ kHz

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>PAN</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P'_{pk-pk}/P_c$</td>
<td>0.01%</td>
<td>1.11%</td>
<td>0.13%</td>
</tr>
<tr>
<td>$J$</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_F$</td>
<td>3.12 kHz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$J = 21$, $f_F = 3.12$ kHz
PAN Forcing: Normalized DCL, $L_{PAN}/L_B$
PN Forcing: Normalized DCL, $L_{PN}/L_B$
APPLICATION OF POD ON IMAGE PIXEL INTENSITIES
Proper Orthogonal Decomposition

• Proper Orthogonal Decomposition (POD) or Principal Component Analysis (PCA) was used for extracting dominant dynamical processes embedded in high-speed images.

• A time-resolved set of images $A(x,t)$ can be represented as a linear combination of orthonormal basis functions $\phi_k$ (aka proper orthogonal modes)\(^1\)\(^2\):

$$A(x,t) = \sum_{k=1}^{M} a_k(t)\phi_k(x)$$

where $a_k(t)$ are time dependent orthonormal amplitude coefficients and $M$ is the number of modes.

• Main idea: POD modal amplitudes capture the maximum possible “energy” in an average sense\(^3\), i.e.,

$$\sum_{k} \langle a_k(t)a_k(t) \rangle \geq \sum_{k} \langle b_k(t)b_k(t) \rangle$$

where $b_k(t)$ are the temporal coefficients of a decomposition with respect to an arbitrary orthonormal basis $\psi_k$.

\(^1\) Chatterjee, A. Current Science, Vol. 78, No. 7 (2000)
Construction of Data Set

• First, form a row vector consisting of all pixel intensity values of each snapshot image (with resolution of $n$ rows by $m$ columns) in order of increasing columns, then increasing rows

- Image Frame
  - Pixel Intensity
  - $n$ rows
  - $m$ columns
  - $N$ frames

• Then, combine all such row vectors for $N$ sequences of image frames resulting in a matrix $A$ consisting of $N$ rows by $P = n \times m$ columns of intensity values.

$$A = \begin{cases} \text{N time steps} \\ P = n \times m \text{ pixel intensities} \end{cases}$$
Orthogonal Decomposition Technique

- Eigenvalue decomposition or singular value decomposition (SVD) can be used
- SVD preferred since
  1. Applicable to non-square matrices (most likely the case)
  2. Decomposition matrices are orthogonal
  3. Subroutine readily available in MATLAB®
- Subtracted temporal mean of $A \Rightarrow$ matrix of intensity fluctuations $\tilde{A}$
- Applied SVD

$$ \Rightarrow \tilde{A} = USV^T = QV^T $$

- Orthogonal Matrix of Left Singular Column Vectors of $\tilde{A}$
- Diagonal Matrix of Singular Values
- Orthogonal Matrix of Right Singular Column Vectors of $\tilde{A} \Leftrightarrow$ proper orthogonal modes (POM)
- Columns of $Q \sim a_k(t)$ contain temporal information
- Columns of $V \sim \phi_k(x)$ contain spatial information
Results – Baseline Low $J$ Flow at $Pr = 0.44$

- LAR-thin, $Pr = 0.44$, $J = 0.5$

Amplitude information contained in singular values

Antisymmetric Structures
Identified with Characteristic Frequencies

Power Spectral Densities (PSD) of Temporal Coefficients of POMs 1 and 2
Results – PAN Low $J$ Flow at $Pr = 0.44$

- LAR-thin, $Pr = 0.44$, $J = 0.5$, forcing Frequency, $f_F = 3.14$ kHz
Results – PN Low $J$ Flow at $P_r = 0.44$

- LAR-thin, $P_r = 0.44$, $J = 0.5$, forcing Frequency, $f_F = 3.14$ kHz
Cross-Power Spectral Density (CPSD)

- CPSD yields the FFT of the cross-correlation of the temporal coefficients
  \[
  \text{CPSD} = \sum_{k=0}^{N-1} \frac{\text{cov}(a_x, a_y)}{\sigma_a \sigma_{\tilde{a}}} e^{-i\omega_k}
  \]

- Magnitude and phase plots used to determine existence of propagating structures
  \[
  \text{LAR, } Pr = 0.44, \quad J = 0.5
  \]
  \[
  \text{PAN \quad (} f_F = 3.14 \text{ kHz})
  \]

Baseline

- Baseline Spectrum Completely Removed in PAN Forced Spectrum

-90° Phase Difference Confirmed Downstream Propagating Flow Structures
Sample Animation – PAN ($f_F = 3.14$ kHz)

- LAR $Pr = 0.44$, $J = 0.5$

Superposition of POMs 1 and 2 Resulted in Downstream Propagating Structures
Baseline POM & CPSD: LAR-thin – $P_r = 0.44$
Baseline POM & CPSD: LAR-thin – $P_r = 1.05$
Previous Works on Jet Instability

- Michalke and Hermann (1982) did linear, inviscid instability analysis of a circular jet with coflow
  - Showed that with increasing coflow velocity, $U_{\infty}$
    - Helical disturbances more unstable than axisymmetric ones farther downstream of exit
    - Jet flow becomes less unstable, but spectrum of spatial growth rate becomes broader and the peak shifts to higher frequencies

- Dahm et al. (1992), Wicker and Eaton (1994) conducted experimental investigation of large-scale vortex structures in the near field of coaxial jets
  - For outer-to-inner jet velocity ratios greater than one, found that coherent structures in the outer shear layer dominate those in the inner shear layer
  - At large axial distances, shear-layer vortices exhibit helical structures

Dahm et al., JFM 1992
Baseline POM & CPSD: SAR-thick – $P_r = 0.44$
Baseline POM & CPSD: SAR-thick – $P_r = 1.05$
Inner Shear-Layer Structure Velocity – $Pr = 0.44$

LAR-thin

SAR-thick
Inner Shear-Layer Structure Velocity – Pr = 1.05

LAR-thin

SAR-thick
Forced POM & CPSD: LAR-thin – $P_r = 0.44$

$J = 0.1$

$J = 0.5$
Forced POM & CPSD: LAR-thin – $P_r = 0.44$

$J = 2.1$

$J = 5.2$
Forced POM & CPSD: LAR-thin – $P_r = 0.44$

$J = 11$

$J = 20$
Forced POM & CPSD: LAR-thin – $P_r = 1.05$

$J = 0.5$

$J = 5.0$
Forced POM & CPSD: LAR-thin – $P_r = 1.05$

$J = 1.9$

$J = 8.5$

$J = 12$
Forced POM & CPSD: SAR-thick – $P_r = 0.44$

$J = 0.1$

$J = 0.5$
Forced POM & CPSD: SAR-thick – $P_r = 0.44$

$J = 2.1$

$J = 5.7$
Forced POM & CPSD: SAR-thick – $P_r = 0.44$

$J = 10$

$J = 21$
Forced POM & CPSD: SAR-thick – $P_r = 1.05$

$J = 0.5$

$J = 2.1$

(a) $J = 2.1$, PAN

(b) $J = 2.1$, PN
Forced POM & CPSD: SAR-thick – $P_r = 1.05$

$J = 5.2$

$J = 9.2$
Forced POM & CPSSD: SAR-thick – $P_r = 1.05$

$J = 14$

$J = 21$
Conclusions

- Baseline flow normalized dark-core lengths, $L_B/D_1$:
  - Power-law curve-fit of $L_B/D_1$ variation with $J$ showed that $P_r = 1.05$ cases had a stronger dependence ($J^{-0.50}$ for the LAR-thin and $J^{0.54}$ for the SAR-thick); $P_r = 0.44$ cases varied as $J^{-0.39}$ for the LAR-thin and $J^{-0.35}$ for the SAR-thick.
  - For a given $J$ and same $P_r$, SAR-thick flows had smaller $L_B/D_1$ than LAR-thin flows.
  - Considering the LAR-thick and SAR-thin injector flow data at both $P_r$, it can be deduced that $L_B/D_1$ increased with $A_o/A_i$ while it decreased with $J$ and $t/D_1$.
    - Injector geometry also plays an important role in addition to $J$.

- Acoustically forced flow normalized dark-core lengths, $L_{PAN}/L_B$ or $L_{PN}/L_B$:
  - LAR-thin injector
    - Both $L_{PAN}/L_B$ and $L_{PN}/L_B$ less than one and approached one with increasing $J$.
    - The flow became less sensitive to forcing with increasing $J$.
  - SAR-thick injector
    - For low $J$ (< 10) flows, $L_{PAN}/L_B$ stayed around one.
    - $L_{PN}/L_B$ approached one with increasing $J$.
    - Flow in recirculation zone only effective in dampening the effect of PAN forcing at lower $J$. 


Conclusions (cont’d)

- Dynamic characteristics of baseline flows:
  - LAR-thin injector:
    - inner shear layer formed immediately downstream of the exit
    - baseline flow inner shear layer characterized by antisymmetric flow structures that indicated the presence of helical instabilities
    - peak frequencies of the magnitude spectra became broader and shifted to higher frequencies with increasing $J$
  - SAR-thick injector:
    - region immediately downstream of the thick inner jet post formed a flow recirculation zone delaying formation of shear layer between the bulk inner and outer jets
    - peak frequencies stayed at low frequencies despite increasing $J$
Conclusions (cont’d)

- Dynamic characteristics of forced flows:
  - LAR-thin injector:
    - PAN forcing produced symmetric flow structures in the inner shear layer region for low $J (< 5)$ flows at both $Pr$
    - Magnitude spectra showed dominant peaks identical to the forcing frequency
    - Higher $J (> 5)$ flows under PAN forcing produced antisymmetric structures that resembled that of baseline
    - Magnitude spectra retained baseline flow spectral characteristics
    - Peak frequencies of the magnitude spectra became broader and shifted to higher frequencies with increasing $J$
    - PN forcing produced sinusoid disturbance
  
  - SAR-thick injector:
    - PAN forcing produced no appreciable response in the inner shear layer of the low $J (< 5)$ flows, at both $Pr$
    - Spectra showed low frequency peaks, and no peaks at forcing frequencies were detected
    - at higher $J (>5)$ and both $Pr$, PAN forcing produced symmetric structures below the recirculation zone with peaks in the spectra at the forcing frequencies
    - PN forcing also produced appreciable response at higher $J$
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Back-Up Slides
Acoustic Field Set-Up: Pressure Node

- Pressure node (PN) – condition of minimum pressure perturbation in the acoustic field
- Piezo-sirens forced out-of-phase
- Superposition of quasi-1D acoustic waves traveling in opposite directions ⇒ PN at the jet location (geometric center of test section)
Baseline Normalized DCL, $L_B/D_1$ – LAR-thin

![Graph showing baseline normalized DCL $L_B/D_1$ for LAR-thin with $P_r = 0.44$ and $P_r = 1.05$.]
Baseline Normalized DCL, $L_B/D_1$ – SAR-thick

- $\triangle$ SAR-thick, $P_r = 0.44$
- $\blacktriangle$ SAR-thick, $P_r = 1.05$