**TITLE**

Investigating the Performance of Some Tracking Filter Schema for the Advanced Shipboard Command and Control Technology (ASCAT) Project

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INVESTIGATING THE PERFORMANCE OF SOME TRACKING FILTER SCHEMA
FOR THE ADVANCED SHIPBOARD COMMAND AND CONTROL TECHNOLOGY
(ASCAct) PROJECT

by
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Approved by / approuvé par

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Date
ABSTRACT

Tracking maneuvering targets is a complex problem which has attracted a great deal of effort over the past several years. It has been now well established that, in terms of tracking accuracy, the Interacting Multiple Model (IMM) algorithm where state estimates are mixed perform significantly better for maneuvering targets than the other types of filters (Adaptive Single Model, Input Estimation, Variable Dimension, etc.). However, the complexity and computation cost of the IMM algorithm can prohibit its use in some applications for which simpler algorithms can provide us with the necessary accuracy at a lower computation cost. This document presents the evaluation of the tracking accuracy of a multiple model track filter using three different constant-velocity models running in parallel (3CVPAR) and a maneuver detector. The output estimate is defined by selecting the model having its likelihood function lower than a Target Maneuver Threshold (TMTH). This approach is recommended for the MultiSensor Data Fusion (MSDF) problem to be tested in the Advanced Shipboard Command and Control Technology (ASCACT) testbed. The tracking performance of the 3CVPAR track filter is compared with: 1) an adaptive single motion model Kalman filter (ASMMKF); 2) an IMM algorithm using the same three CV models than the 3CVPAR filter; 3) an IMM filter using a CV model and a constant acceleration (CA) model producing a CVCA filter; 4) an IMM filter using a CV and two CA models (CA1, CA2) differing only by the level of process noise producing a CV2CA filter. Calculations of the average Root Mean Square Error (RMSE) on 100 Monte Carlo runs permit to evaluate the tracking accuracy of the 3CVPAR track filter compared with simpler (ASMMKF) or more complex (IMMs) algorithms on a challenging multiple-sensor scenario.

RÉSUMÉ

Le pistage de cibles qui manoeuvrent est un problème complexe qui continue d’attirer l’attention de nombreux chercheurs depuis plusieurs années. On a établi que le filtrage de Kalman de type modèles multiples interactifs (MMI) donnait la meilleure précision de pistage par rapport à d’autres approches de filtrages (modèle unique adaptatif, estimation de variable d’entrée, dimension variable, etc.). Cependant, la complexité et le coût en calculs de l’algorithme MMI peut entraver son usage pour certaines applications où des algorithmes plus simples peuvent apporter la précision nécessaire à un coût moindre en calculs. Ce document présente les résultats de l’évaluation de plusieurs méthodes de filtrage afin d’en recommander une pour l’application de fusion de données qui fera l’objet de tests dans le banc d’essai que l’on nomme “ASCACT” (Advanced Shipboard Command and Control Technology). L’approche recommandée pour “ASCACT” utilise trois filtres ayant des modèles différents de vitesses constantes (appelés “3CVPAR”) et un détecteur de manoeuvre. L’estimé est choisi par le filtre dont la fonction de vraisemblance du modèle de vitesse dépasse un seuil de détection de manoeuvre. La performance du filtre “3CVPAR” est comparée à: 1) un modèle unique adaptatif; 2) un algorithme MMI utilisant les trois mêmes modèles de vitesses constantes que le filtre “3CVPAR”; 3) un filtre MMI utilisant un modèle de vitesse constante et un modèle d’accélération constante; et finalement 4) un filtre MMI utilisant un modèle de vitesse constante, mais avec deux modèles d’accélération constante. La précision de ces approches de pistage est évaluée en calculant l’erreur quadratique moyenne de 100 essais Monte Carlo.
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EXECUTIVE SUMMARY

The objective of the Advanced Shipboard Command and Control Technology (ASCACT) project is to improve the shipboard data processing capability of the Command and Control Information Systems (CCISs) on Canadian warships. The ongoing final phase of this project will deliver an advanced development model testbed to the Defence Research Establishment Valcartier (DREV) for conducting naval CCIS research. The ASCACT Testbed will be used to investigate the adaptation of commercial off the shelf (COTS) technology to naval CCIS applications such as Multi-Source Data Fusion (MSDF), Situation Threat Assessment (STA), and Resource Management (RM).

The MSDF application of the ASCACT Testbed will focus on fusing simulated data reports from six sensors of the Canadian Patrol Frigate (CPF) in order to derive the best estimates of the kinematic properties for each perceived entity in the environment, and to infer the identity and key attributes of these entities. DREV has been working for over a decade to develop technologies to enable Canada's warships to dynamically and automatically obtain an image of a tactical situation. The scope of this research is within the time frame of the HALIFAX class mid-life refit and the spin-offs will also be of direct benefit to the IROQUOIS class and its potential replacement. The choice of a method for filtering and prediction is usually the first task facing the designer of the target tracking function of a MSDF system, and many approaches exist. It has been now well established that, in terms of tracking accuracy, the Interacting Multiple Model (IMM) algorithm where state estimates are mixed perform significantly better for maneuvering targets than the other types of filters (Adaptive Single Model, Input Estimation, Variable Dimension, etc.). However, the complexity and computation cost of the IMM algorithm can prohibit its use in some applications for which simpler algorithms can provide us with the necessary accuracy at a lower computation cost.

This report presents the evaluation of the tracking accuracy of a multiple model track filter using three different constant-velocity models running in parallel (3CVPAR) and a maneuver detector. The output estimate is defined by selecting the model having its likelihood function lower than a Target Maneuver Threshold (TMTH). This tracking filter scheme as been proposed for ASCACT. The tracking performance of the 3CVPAR track filter is compared with: 1) an adaptive single motion model Kalman filter (ASMMKF) ; 2) an IMM algorithm using the same three CV models than the 3CVPAR filter; 3) an IMM filter using a CV model and a constant acceleration (CA) model producing a CVCA filter ; 4) an IMM filter using a CV and two CA models (CA1, CA2) differing only by the level of process noise producing a CV2CA filter. Calculations of the average RMSE error on 100 Monte Carlo runs permit to evaluate the tracking accuracy of the 3CVPAR track filter compared with simpler (ASMMKF) or more complex (IMMs) algorithms on a challenging multiple-sensor scenario.
1.0 INTRODUCTION

The objective of the Advanced Shipboard Command and Control Technology (ASCACT) project is to improve the shipboard data processing capability of the Command and Control Information Systems (CCISs) on Canadian warships. The ongoing final phase of this project will deliver an advanced development model testbed to the Defence Research Establishment Valcartier (DREV) for conducting naval CCIS research. The ASCACT Testbed will be used to investigate the adaptation of commercial-off-the-shelf (COTS) technology to naval CCIS applications such as Multi-Source Data Fusion (MSDF), Situation Threat Assessment (STA), and Resource Management (RM).

The MSDF application of the ASCACT Testbed will focus on fusing simulated data reports from six sensors of the Canadian Patrol Frigate (CPF) in order to derive the best estimates of the kinematic properties for each perceived entity in the environment, and to infer the identity and key attributes of these entities. DREV and Lockheed Martin Canada Inc. have been working for over a decade to develop technologies to enable Canada’s warships to dynamically and automatically obtain an image of a tactical situation. The scope of this research is within the time frame of the HALIFAX class mid-life refit and the spin-offs will also be of direct benefit to the IROQUOIS class and its potential replacement. The choice of a method for filtering and prediction is usually the first task facing the designer of the target tracking function of a MSDF system, and many approaches exist. However, approaches based on Kalman filters are the most popular when dealing with the problems presented by missing data, variable measurement noise statistics, and maneuvering targets with variable dynamic capabilities.

Reference [1] presents a practical, quantitative demonstration of Kalman filtering techniques for kinematics data fusion in MSDF systems. The report describes the standard linear Kalman filter, but also a number of non-linear filtering techniques. Since tracking a maneuvering target is a complex but very important issue, the main techniques for handling target maneuvers are discussed. The various target kinematics models most
frequently used for Kalman filtering purposes are also discussed in detail but no evaluation has been conducted to specifically recommend an approach to be tested in the ASCACT real-time testbed. This is the aim of the current report.

Tracking maneuvering targets has attracted a great deal of effort over the past several years, leading to algorithms of an increasing complexity. Recent studies [2-3] clearly establishes the superiority of the tracking performance of the Interacting Multiple Model (IMM) algorithm developed by Blom et al. [4-5] compared with the single motion model track filters such as the input estimation [6] and variable dimension filters [7]. Both rely on certain maneuver detection criteria to update the parameters or to modify the structure of the track filters. Most of these detection criteria are based on threshold rules that need to be satisfied a certain number of consecutive times prior for the filter's being switched to a maneuver mode. The consequence is an unavoidable delay in the maneuver response of the track filter, which may lead to dramatic consequences such as target loss. This problem is solved by the track filters algorithms using a mix of the state estimates of each model (IMM algorithms). Each model represents one of the many different motion regimes of the target during its movement. Other types of multiple model track filters do not combine each model state estimate but rather select one of them as output according to decision rules analogous to those used for the single model track filters. While these track filters perform better than the single model track filter, their maneuver responses are also delayed since the model switching algorithm is based on decision rules applied in the (recent) history of the target. Despite the complexity of such algorithms, their computational loads are smaller than those for the IMM algorithm. Even if their performances are somewhat lower, they may perform adequately for applications involving moderate maneuvers.

This report presents the results of a study evaluating the effect of mixing the state estimates on the tracking performance of a multiple model track filter. For this purpose, the same motion models are used to build: 1) a multiple model track filter using decision rules to select which model will provide the output estimate; 2) an IMM track filter
where state estimate of each model are mixed. In both cases, the motion models used correspond to a constant velocity motion with different process noise covariances. The tracking performance of these filters is compared with that of the known track filters. First, a comparison with an adaptive single motion model track filter confirms the improvement obtained by using multiple models either interacting (IMM) or not (selection of the output state of one model according to specific decision rules). Then, the performance of our two filters is compared with that of: 1) an IMM track filter using a constant-velocity (CV) and a constant-acceleration (CA) models to form a CVCA track filter; 2) an IMM track filter using a constant-velocity (CV) and two constant-acceleration models (CA1 and CA2) to form a CV2CA track filter. The tracking accuracy of these filters is evaluated by computing the average Root Mean Square Error (RMSE) on position and velocity resulting from a Monte-Carlo experiment of 100 independent runs.

The report is organized as follows. Chapter 2 presents some background material. Then, the algorithms that are to be compared are successively presented: 1) the adaptive single motion model track filter (Chapter 3); the multiple model using decisions rules (Chapter 4); the formalism of the IMM algorithm (Chapter 5). The motion models used to build the CVCA and CV2CA track filters are presented in Chapter 6. Simulation results are presented in Chapter 7.

The results contained in this report have been presented unrefereed at the SPIE conference [8] without any recommendation for the ASCACT project. The work was carried out under Lockheed Martin Canada Internal R&D funding and at DREV under Work Unit 1ba14 (Support to the ASCACT Integration Working Group) between January 1998 and February 1999.
2.0 THE KALMAN FILTER EQUATIONS

The discrete time model for a dynamic system can be written as:

\[ x_k = F_{k-1} x_{k-1} + G_{k-1} w_{k-1} \]  
\[ z_k = H_k x_k + v_k \]

where \( w_{k-1} \) is a \( px1 \) process noise vector supposed white and Gaussian \( (w_{k-1} \sim N(0,Q_{k-1}) \), \( v_k \) is a \( nx1 \) measurement error vector also supposed white and Gaussian \( (v_{k-1} \sim N(0,R_k) \), and \( z_k \) is a \( nx1 \) measurement vector. Noting \( x_{k|k-1} \) and \( P_{k|k-1} \) the estimate of the state and covariance of the system prior to assimilating the measurement at time \( t_k \) and described by the prediction equations are written as:

\[ x_{k|k-1} = F_{k-1} x_{k-1|k-1} \]

\[ P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + G_{k-1} Q_{k-1|k-1} G_{k-1}^T \]

The updated (a posteriori) state and covariance estimates using the measurement \( z_k \) to improve the prior estimate \( x_{k|k-1} \) is written as:

\[ x_{k|k} = x_{k|k-1} + K_k (z_k - H_k x_{k|k-1}) \]

\[ P_{k|k} = (I - K_k H_k) P_{k|k-1} \]

with \( K_k \) is the Kalman gain defined as:

\[ K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \]
3.0 ADAPTIVE SINGLE MOTION MODEL TRACK FILTER

The Adaptive Single Motion Model track filter (noted from now on ASMMKF) is based on a constant velocity motion model. The discrete-time model is given by:

\[
x_k = \begin{bmatrix}
x_k^* \\
x_k^* \\
y_k \\
y_k^*
\end{bmatrix}
F_k = \begin{bmatrix}
1 & \Delta t & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & \Delta t \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Equation 9 shows that the process noise covariance \( \sigma \) has the dimension of an acceleration \((m \cdot s^{-2})\). This acceleration is estimated recursively by a linear regression on the last five estimates of the target velocity. This simple algorithm uses the estimated acceleration to increase the process noise and has the advantage of being decision free [9].
4.0 MULTIPLE MODEL TRACK FILTER USING DECISION RULES

A multiple model track filter is designed to track maneuvering targets by applying decision rules on the statistical distance provided by three Kalman filters running in parallel (3CVPAR). Each of these filters uses a constant-velocity model for the system dynamic, which can still be described by Eq. 9. However, each model is characterized by a different level of process noise by the definition of three different values for $\sigma$. For a low value of $\sigma$, the Kalman filter will be more confident in the state estimate than on the measurement and will filter out the measurement noise as well as making the motion model unable to respond adequately to a target maneuver. On the other hand, for a high value of $\sigma$, the Kalman filter will be more confident in the measurement than on the state estimate, which will put the motion model in a better position to respond adequately to a target maneuver but producing a jerky track (RMSE in position higher than for a low value of $\sigma$). The three values of $\sigma$ taken in our experiments were chosen to enter a maneuver magnitude of respectively 0.03, 0.3 and 3 g in the process noise covariance.

Figure 1 shows a block diagram of the 3CVPAR algorithm. For each filter ($i$), the weighted norm of the innovations $|d_k'|^2$ is computed as:

$$
|d_k'|^2 = (z_k - H_k x_{k|k-1})^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} (z_k - H_k x_{k|k-1})
$$

$$
|d_k'|^2 = [v_k']^T [S_k']^{-1} [v_k']^T
$$

(10)

The innovations are supposed white, Gaussian processes $v_k' \sim N(0, S_k')$. Following this assumption, the weighted norm of innovations is chi-square distributed with $M$ degrees of freedom, where $M$ is the measurement dimension (number of independent measures). In our application $M$ will be equal to 2. The value taken by $|d_k'|^2$ is compared with a threshold given by the chi-square distribution. For instance, taking the target maneuver threshold (TMTh) equal to 9 will mean that 1% of the measurements can
exceed this threshold only by random fluctuations (i.e. no maneuver involved). However, if this threshold is exceeded for a certain amount of consecutive time, it is reasonable to think that a maneuver is likely ongoing.

![Diagram](attachment:image.png)

**FIGURE 1** – The multiple model filter using decision rules (3CVPAR)

Source: Ref.[1]

Figure 1 shows that, for each model the weighted norm of innovations $|d_k^i|^2$ is computed and compared with the predetermined TMTh, and the number of times $|d_k^i|^2$ exceeds this value is stored in counters $F^i$. The values in these counters is then compared with some predetermined thresholds $NF^i$ which will determine the state estimate taken as output for the 3CVPAR track filter but also for each mode if a reconfiguration is necessary.
Table I summarizes the set of decision rules that are used. Columns 3-5 show that, depending on the magnitude of the maneuver, the state estimate of certain modes no longer represents the true state of the target. In such cases, the output state estimate of the 3CVPAR filter replaces the state estimate(s) of the deficient mode(s). For instance, the third line shows that a moderate maneuver is ongoing. The state estimate of mode 2 is taken as the 3CVPAR output and replaces the state estimate of mode 1, which no longer represents the dynamic of the target.

**TABLE I:**

Decisions rules if $|\mathbf{d}_k|^2 > \text{TMTh}$

<table>
<thead>
<tr>
<th>$[d_k]^2 &gt; \text{TMTh}$ and</th>
<th>Maneuver Magnitude</th>
<th>Output State Estimate of mode 1</th>
<th>mode 2</th>
<th>mode 3</th>
<th>3CVPAR</th>
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<tr>
<td>$F^1 &lt; NF^1$</td>
<td>No maneuver</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$F^1 &gt; NF^1$</td>
<td>moderate</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$F^1 &lt; NF^2$</td>
<td>wild</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$F^2 &gt; NF^2$</td>
<td>re-initialize</td>
<td>re-initialize</td>
<td>re-initialize</td>
<td>re-initialize</td>
<td>none</td>
</tr>
<tr>
<td>$F^3 &gt; NF^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
5.0 THE IMM ALGORITHM

The IMM algorithm assumes that the dynamic system can be exactly described at a given time of its evolution by one out of M possible models with jumps between these models following a Markov chain with known transition probabilities. The IMM algorithm works according to the following steps:

- A different mix of the state estimates is used for each filter. This respective weighting of each state estimate in the mix depends on both the probability of each model and the model switching probability.

- Then, each filter computes its own a posteriori state estimate and its associated model likelihood. The a posteriori model probability is then computed using the model likelihood function, the a priori model probabilities and the model switching probabilities.

- The output state estimate of the IMM track filter is computed by weighting the a posteriori state estimate of each filter by the corresponding a posteriori model probability. The output state estimate of the IMM filter is no longer used for the estimation.

Figure 2 shows a block diagram of the IMM algorithm for a dynamic system supposed to follow two discrete-time models. Let $x_k^{ji}$ be the state estimate at time $k$ based on the model $j$ at time $k$ and model $i$ at time $k-1$. Let $\Lambda_k$ be the (Mx1) vector of model likelihood at time $k$, and $\mu_k$ the (Mx1) vector of model probabilities once all the likelihood functions have been taken into account. Let $\mu_{k-1}^{ji}$ be the probability associated with the model $j$ at time $k$ and model $i$ at time $k-1$. 
FIGURE 2 – IMM algorithm using two models (Source: Ref.[1])
The above three steps can be expressed mathematically as:

- **Prior State Estimates**

\[
\bar{x}_{k-|k-1}^0 = \sum_{i=1}^{m} x_{k-|k-1}^i \mu_{k-|k-1}^i \mu_{k-|k-1}^i
\]  \hfill (11)

where the prior model probability are given by

\[
\mu_{k-|k-1}^i = \frac{p_{ji}^{i}}{C_j}
\]  \hfill (12)

\[
\hat{C}_j = \sum_{i=1}^{m} p_{ji}^{i} \mu_{k-|k-1}^i
\]  \hfill (13)

with the mixed covariance given by

\[
p_{k-|k-1}^{ij} = \sum_{i=1}^{m} \mu_{k-|k-1}^i [P_{k-|k-1}^i + (x_{k-|k-1}^i - \bar{x}_{k-|k-1}^i)(x_{k-|k-1}^i - \bar{x}_{k-|k-1}^i)^T]
\]  \hfill (14)

- **Model Likelihood**

Each model \(i\) provides its a posteriori state estimate \(x_{k|k-1}^i\) given the measurement \(z_k\). The likelihood function of the model \(i\) is given by:

\[
\Lambda_i^k = \frac{1}{\sqrt{2\pi S_k}} \exp \left[ -\frac{1}{2} (z_k - H_k x_{k|k-1}^i)^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} (z_k - H_k x_{k|k-1}^i) \right]
\]

\[
\Lambda_i^k = \frac{1}{\sqrt{2\pi S_k}} \exp \left[ -\frac{1}{2} v_k^T S_k^{-1} v_k \right]
\]  \hfill (15)
The a posteriori model probabilities are given by:

$$
\mu_k^i = \frac{\tilde{C}_i \Lambda_k^i}{C} \quad (16)
$$

$$
C = \sum_{j=1}^{m} \Lambda_k^i \tilde{C}_j \quad (17)
$$

- Output State Estimate

$$
x_{k|k} = \sum_{i=1}^{m} \mu_k^i x_{k|k}^i \quad (18)
$$

$$
P_{k|k} = \sum_{i=1}^{m} \mu_k^i \left[ P_{k|k}^i + (x_{k|k}^i - x_{k|k}^i)(x_{k|k}^i - x_{k|k}^i)^T \right] \quad (19)
$$
6.0 MOTION MODELS

A three model IMM version of the above 3CVPAR is designed. As described in Chapter 4, the motion models are constant velocity (CV) and differ in the value of $\sigma$ used in the calculation of the covariance of the process noise (resp. 0.03 g, 0.3 g, 3 g). This filter is called, from now-on, the IMM-3CV track filter. The tracking performances of the 3CVPAR and IMM-3CV filters are compared with those of two IMM algorithms: 1) an IMM algorithm using one CV model and one CA model to form a CVCA track filter; 2) an IMM algorithm using one CV model and two CA models (CA1 and CA2) to form a CV2CA track filter. The CV model used in these two IMM algorithms is described in Chapter 3 (with $\sigma = 0.03$ g).

The discrete-time model for the constant acceleration (CA) is given by:

$$x_k = \begin{bmatrix} x_k & x_k & x_k & y_k & y_k & \cdots \end{bmatrix}^T,$$

$$F_k = \begin{bmatrix} 1 & t & t/2 & 0 & 0 & 0 \\ 0 & 1 & t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t & t/2 \\ 0 & 0 & 0 & 0 & 1 & t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G_k Q G_k^T = \sigma^2.$$

$$\begin{bmatrix} \Delta t^4 / 4 & \Delta t^3 / 2 & \Delta t^2 / 2 & 0 & 0 & 0 \\ \Delta t^3 / 2 & \Delta t^2 & \Delta t & 0 & 0 & 0 \\ \Delta t^2 / 2 & \Delta t & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta t^4 / 4 & \Delta t^3 / 2 & \Delta t^2 / 2 \\ 0 & 0 & 0 & \Delta t^3 / 2 & \Delta t^2 & \Delta t \\ 0 & 0 & 0 & \Delta t^2 / 2 & \Delta t & 1 \end{bmatrix}.$$
In this study, the values of the constant \( \sigma \) used in the calculation of the covariance of the process noise of the two constant acceleration filter are \( \sigma = 0.3 \text{ g (CA1)} \) and \( \sigma = 3 \text{ g (CA2)} \).
7.0 RESULTS

The tracking performance of the 3CVPAR track filter is compared with that of its IMM version, as well as to that of the ASMMKF, CVCA and CV2CA track filters.

7.1 Scenario

The scenario consists of a series of maneuvers executed by an air target. Two surveillance radars are used: a short-range radar R1 tracking the target (at a rate of 30 rpm) up to a range of 45 miles, and a medium-range radar R2 tracking the target at a rate of 12 rpm during the entire scenario.

The target executes the following maneuvers:

- segment 2: right turn-of ~ 0.2 g at (x,y)=(26,30),
- segment 2: right turn-of ~ 0.2 g at (x,y)=(23,35),
- segment 4: 3 consecutive left turns of ~ 1 g at (x,y)=(65,37), (66,40), (64,40);
- segment 5: a speed change from 300 to 750 knots at (x,y)=(63,25),
- segment 5: a right turn of 5 g at (x,y)=(63,0).

Segments (1, 3, 6) do not contain maneuvers. Figure 3 represents the ground-truth generated by the scenario simulator (the curvilinear dashed line materializing at the range limit of R1). A measurement noise is added to the ground-truth using realistic values for the nominal uncertainties of each sensor. A Monte Carlo experiment of 100 runs has been performed and all the following results are averaged quantities over those 100 runs.
7.2 Tuning of Parameters

Some of the filters needed prior parameter tuning before any evaluation of their performance could be made. For each of these filters a preliminary Monte-Carlo experiment of 100 runs was conducted to select the parameters that provided an average RMSE in position lower than the RMSE on measurements.

- The pre-defined parameters for the 3CVPAR filter are:

\[ \sigma = [0.03g, 0.3g, 3g]; \text{NF} = [3, 3, 3]; \text{TMTh} = 9.21 \]
• Since two measurement rates were considered (12 and 30 rpm), a Markov transition matrix \( \Pi \) for an arbitrary time of update needed to be defined [10]. It is made possible by defining the constant probability flow matrix \( A \), which relates to \( \Pi \) and to the time of update by the relation:

\[
\Pi_m = e^{A\Delta t} \approx I + A\Delta t + \frac{AA}{2!}\Delta t^2 + \cdots
\]  

(22)

The pre-defined parameters for the IMM filters are summarized in Table II as:

<table>
<thead>
<tr>
<th>TABLE II: IMM filter parameters</th>
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<tbody>
<tr>
<td><strong>IMM filter parameters</strong></td>
</tr>
<tr>
<td>Model probabilities</td>
</tr>
<tr>
<td>( \mu_0 = [0.9 \ 0.1 \ 0.0] )</td>
</tr>
<tr>
<td>( \mu_0 = [0.9 \ 0.1] )</td>
</tr>
<tr>
<td>Probability flow matrix ( A )</td>
</tr>
<tr>
<td>( \Pi = \begin{bmatrix} 0.99 &amp; 0.01 &amp; 0 \ 0.2 &amp; 0.7 &amp; 0.1 \ 0.02 &amp; 0.0 &amp; 0.98 \end{bmatrix} )</td>
</tr>
<tr>
<td>( \Pi = \begin{bmatrix} 0.99 \ 0.01 \ 0.05 \ 0.95 \end{bmatrix} )</td>
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<tr>
<td>( \Pi = \begin{bmatrix} 0.98 &amp; 0.02 &amp; 0 \ 0.01 \ 0.98 &amp; 0.01 \ 0.02 \ 0.0 &amp; 0.98 \end{bmatrix} )</td>
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<tr>
<td>( \sigma )</td>
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<tr>
<td>[0.03 0.3 3.0]g</td>
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<tr>
<td>[0.03 0.3]g</td>
</tr>
<tr>
<td>[0.03 0.3 3.0]g</td>
</tr>
</tbody>
</table>

7.3 Main results

Figures 4-11 show the comparison of the variation of the covariance state estimate (left column: [\( x \)], right column: [\( y \)]) over time for each studied filter. The abrupt drop at the beginning of the scenario (around \( t=56.9 \text{ Ks} \)) is due to the fusion of the radar R1, which has higher accuracy than R2. At around \( t=57.32 \text{ Ks} \), the target flies out of the range of R1, measurements become less accurate and the values of the state covariance reflect it. Then, at \( t=58.07 \text{ Ks} \), the target re-enters the range of R1, the measurements are more accurate and the value of the state covariance decreases.
The time-shift of the peaks clearly shown on Figs. 4-11 demonstrates that the ASMMKF has the slowest response to the maneuvers. Then comes the 3CVPAR algorithm, which is slower than the three IMM algorithms which perform, equally in that respect. These figures confirm that the mix of the state estimates characterizing the IMM algorithm allows the filter to quickly react to maneuvers.

FIGURE 4 - Average covariance (x) - ASMMKF, 3CVPAR
FIGURE 5 - Average covariance (y) - ASMMKF, 3CVPAR

FIGURE 6 - Average covariance (x) - 3CV-IMM, 3CVPAR
FIGURE 7 - Average covariance (y) - 3CV-IMM, 3CVPAR

FIGURE 8 - Average covariance (x) - CVCA, 3CVPAR
FIGURE 9 - Average covariance (y) - CVCA, 3CVPAR

FIGURE 10 - Average covariance (x) - CV2CA, 3CVPAR
Figures 12-19 show the comparison of the average RMSE on position (left column: [x], right column: [y]) over time for each studied filter. Since the ASMMKF reacts too late to the various maneuvers, it has the poorest performance. Among the IMM algorithms, the CVCA produces the best results, followed by the CV2CA and then the IMM-3CV. The 3CVPAR has a performance that puts it between the CV2CA and the IMM-3CV.

Figures 20-27 show the comparison of the average RMSE on velocities (left column: [x], right column: [y]) over time for each studied filter. As expected, the IMM algorithms produce the best results and are very comparable. The 3CVPAR comes close behind, the ASMMKF reacts poorly to wild maneuvers (t=57.7 Ks) due to its too long time of response.
FIGURE 12 - Average RMSE pos. (x) - ASMMKF, 3CVPAR

FIGURE 13 - Average RMSE pos. (y) - ASMMKF, 3CVPAR
FIGURE 14 - Average RMSE pos. (x) 3CV-IMM, 3CVPAR

FIGURE 15 - Average RMSE pos. (y) 3CV-IMM, 3CVPAR
FIGURE 16 - Average RMSE pos. (x) - CVCA, 3CVPAR

FIGURE 17 - Average RMSE pos. (y) - CVCA, 3CVPAR
FIGURE 18 - Average RMSE pos. (x) -CV2CA, 3CVPAR

FIGURE 19 - Average RMSE pos. (y) -CV2CA, 3CV-PAR
FIGURE 20 - Average RMSE vel. (x) - ASMMKF, 3CVPAR

FIGURE 21 - Average RMSE vel. (y) - ASMMKF, 3CVPAR
FIGURE 22 - Average RMSE vel. (x) -3CV-IMM, 3CVPAR

FIGURE 23 - Average RMSE vel. (y) -3CV-IMM, 3CVPAR
FIGURE 24 - Average RMSE vel. (x) - CVCA, 3CVPAR

FIGURE 25 - Average RMSE vel. (y) - CVCA, 3CVPAR
FIGURE 26 Average RMSE vel. (x) - CV2CA, 3CVPAR

FIGURE 27 - Average RMSE vel. (y) - CV2CA, 3CVPAR
Figure 28 shows, for the 3CVPAR algorithm, the index of the model selected for output after applying the decision rules of the maneuver detector. Figures 29-31 show the probabilities of the models for the different IMM algorithms. Some similarities exist between the switching pattern used by the 3CVPAR and the one used by the CVCA. It demonstrates that the peaks of the RMSE on velocity correspond to the various reconfiguration stages of the 3CVPAR filter. Those peaks also correspond to a higher weight put on the CA model for the CVCA filter. The 3CV-IMM and 3CVPAR switch in a similar manner to the high $\sigma$ model (3 g). When the CVCA filter uses the CV model, the CV2CA uses the CA1 model.

FIGURE 28 - Index of model selected for output (3CVPAR)
FIGURE 29 - Model probabilities CVCA

FIGURE 30 - Model probabilities 3CV-IMM
7.4 When 3CVPAR Outperforms The Other Algorithms?

A statistical test [11] is performed to evaluate when the performances of the 3CVPAR are lower, equal or even better those of other filters (especially, the IMM algorithms). The ratio $\frac{\Delta}{\sigma_\Delta}$ is computed for each of the 6 segments (defined by the boundaries $[k,l]$ for simplicity) of the scenario. Denoting $\Delta$ is the average (on the 100 Monte Carlo runs) of the differences:

$$\Delta_{k,l} = \frac{1}{l-k+1} \sum_{m=k}^{l} [(x_{m|m}^\text{filter} - x_{m|m}^\text{ground-truth})^2 - (x_{m|m}^{3\text{CVPAR}} - x_{m|m}^\text{ground-truth})^2]$$

and $\sigma_\Delta$ is the associated standard deviation.
The working hypothesis is that the tracking quality of the 3CVP AR algorithm is better than with the other filters:

\[ H_1: \Delta_k^{\text{filter}} = e_k^{\text{filter}} - e_k^{3\text{CVPAR}} > 0 \]  

(24)

It is shown that \( H_1 \) is accepted if the significance of \( H_0: \Delta_k^{\text{filter}} \leq 0 \) is less than 5% which implies \( \Delta_k^{\text{filter}} / \sigma_{e_k} > 1.65 \).

Table III summarizes the results obtained by computing the ratio \( \bar{\Delta}/\sigma_{\Delta} \) for the RMSE on position and velocities (separated by a / ) averaged on the 100 runs of our Monte-Carlo experiment. Each column corresponds to the section of the scenario identified on the Figure 3.

**TABLE III:**
Comparison of 3CVPAR to The Other Filters (100 runs)

<table>
<thead>
<tr>
<th>Filter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASMMKF</td>
<td>2.84 / 3.24</td>
<td>-12.8 / -12.1</td>
<td>1.78 / 1.73</td>
<td>26.4 / 29.7</td>
<td>-3.21 / -1.36</td>
<td>3.85 / 4.11</td>
</tr>
<tr>
<td>IMM-3CV</td>
<td>2.67 / 3.30</td>
<td>-12.5 / -12.6</td>
<td>0.15 / 0.65</td>
<td>7.25 / 8.76</td>
<td>-2.85 / 0.87</td>
<td>-0.75 / -0.83</td>
</tr>
<tr>
<td>CVCA</td>
<td>4.72 / 5.81</td>
<td>-17.4 / -15.9</td>
<td>1.81 / 2.30</td>
<td>-13.3 / -9.38</td>
<td>-10.61 / -11.54</td>
<td>4.56 / 1.45</td>
</tr>
<tr>
<td>CV2CA</td>
<td>10.4 / 10.40</td>
<td>-17.46 / -15.24</td>
<td>11.9 / 9.6</td>
<td>-13.2 / -9.48</td>
<td>-8.64 / -7.44</td>
<td>-1.07 / 0.60</td>
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</tbody>
</table>

As described in Section 7.1, segments 1, 3, and 6 correspond to quiescent portions of the target trajectory. The test shows that, during those periods, our working hypothesis is satisfied, which means that the tracking accuracy of the 3CVPAR algorithm is higher than that of the other filters.
Segment 2 contains two moderate maneuvers (0.2 g) spaced in time. The test shows that, for this type of maneuver, the other filters outperform the 3CVPAR algorithm. The difference in magnitude of the two first peaks on Figs. 12-19 explain this effect. However, this is not critical at this point, the performance of the 3CVPAR for such maneuvers stays acceptable.

Segment 4 contains two moderate maneuvers (1 g) close in time. It is interesting to see that, this time, the ASMMKF and the 3CV-IMM have real difficulties to track adequately the target. The 3CVPAR algorithm performs better than those filters while being largely under the level of performance attained by the CVCA and CV2CA algorithms for this type of maneuver.

Segment 5 contains a linear acceleration and a wild maneuver (5 g). The reading of the results is, in that case, quite different from that of segment 2 for which all the filters were performing more or less adequately. This time the performances of the ASMMKF, 3CV-IMM and 3CVPAR algorithms are not good enough to guarantee that they will not lose the target.
8.0 CONCLUSION

A parallel filter using three constant-velocity models is presented. This filter is called “3CVPAR”. Its tracking accuracy has been compared with simpler algorithms (Adaptive Single Motion Model Kalman Filter), with its IMM version as well as with more classical IMM algorithms such as the CVCA and CV2CA track filters. The respective tracking accuracy of each algorithm has been evaluated by performing a Monte-Carlo study of 100 runs on a challenging multiple-sensor scenario including various types of maneuvers. This study shows that the 3CVPAR filter will perform reasonably well for applications involving moderate maneuvers. Its simplicity and its low computational load compared with the classical IMM algorithms can make it the filter of choice. This is recommended for the baseline MSDF application of the ASCACT testbed. When maneuvers of higher magnitude are expected (evasive), IMM algorithms that include a maneuver modeling are preferable. This is recommended for future advanced MSDF application of ASCACT.
9.0 REFERENCES


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Tracking maneuvering targets is a complex problem which has attracted a great deal of effort over the past several years. It has been now well established that, in terms of tracking accuracy, the Interacting Multiple Model (IMM) algorithm where state estimates are mixed perform significantly better for maneuvering targets than the other types of filters (Adaptive Single Model, Input Estimation, Variable Dimension, etc.). However, the complexity and computation cost of the IMM algorithm can prohibit its use in some applications for which simpler algorithms can provide us with the necessary accuracy at a lower computation cost.

This document presents the evaluation of the tracking accuracy of a multiple model track filter using three different constant-velocity models running in parallel (3CVPAR) and a maneuver detector. The output estimate is defined by selecting the model having its likelihood function lower than a Target Maneuver Threshold (TMTH). This approach is recommended for the Multi-Sensor Data Fusion (MSDF) problem to be tested in the Advanced Shipboard Command and Control Technology (ASCACT) testbed. The tracking performance of the 3CVPAR track filter is compared with: 1) an adaptive single motion model Kalman filter (ASMMKF); 2) an IMM algorithm using the same three CV models than the 3CVPAR filter; 3) an IMM filter using a CV model and a constant acceleration (CA) model producing a CVCA filter; 4) an IMM filter using a CV and two CA models (CA1, CA2) differing only by the level of process noise producing a CV2CA filter. Calculations of the average Root Mean Square Error (RMSE) on 100 Monte Carlo runs permit to evaluate the tracking accuracy of the 3CVPAR track filter compared with simpler (ASMMKF) or more complex (IMMs) algorithms on a challenging multiple-sensor scenario.

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