Transverse jets play an important role in many propulsion-related applications including gas turbine burner dilution, exhaust from V/STOL aircraft, and fluidic thrust vectoring. Although this flow has received extensive research attention over several decades, a lack of universality exists regarding scaling laws available in literature. Using data from existing literature, a foundational scaling law framework has been proposed for the jet trajectory and mixture uniformity. A newly derived parameter demonstrates an improved collapse of trajectory data in literature. This parameter was derived using theoretical arguments that both entrainment and aerodynamic drag should be considered as relevant mechanisms of momentum transport between the jet and cross flow. An experimental study was conducted and the results indicate the utility of the new scaling law parameter for defining flow regimes and correlating mixing performance. Future work will extend this scaling law framework for multiple transverse jet configurations.
Trajectory and Mixing Scaling Laws for Confined and Unconfined Transverse Jets

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Outline

• Objectives
• Background on scaling laws
  — Unconfined transverse jet trajectories
  — Confined transverse jet mixing
• New scaling law variable
• Experimental facility
• Mixing results
• Conclusions and future work
Objectives

- Transverse jets present in environment and engineering
  - Smoke stacks
  - Thrust vectoring
  - Combustion chamber mixing
  - Flow reactors

Lack of universal scaling laws and parameters that span wide domains of the operating space
Scaling Laws

• Jet trajectories
  
  Keffer and Baines (1963)
  \[ \frac{y - y_o}{r^2 D} = f\left(\frac{x - x_o}{r^2 D}\right) \]

  Kamotani and Greber (1972)
  \[ \frac{y}{D} = 0.89 J^{0.47} \left(\frac{x}{D}\right) \]

  Hasselbrink and Mungal (2001)
  \[ \frac{y}{rD} = f\left(\frac{x}{rD}\right) \]

• Mixing optimization

  From Fric and Roshko (1994)
  \[ r = \frac{U_j}{U_o} \quad \text{and} \quad J = \frac{\rho_i U_j^2}{\rho_o U_o^2} \]

  One jet:
  Maruyama et al. (1983)
  \[ r = 1.49 \left(\frac{D_j}{D_o}\right)^{-0.415} \]

  8-20 jets:
  Holdeman (1993)
  \[ \frac{S}{H} J^{1/2} = 2.5 \quad \Rightarrow \quad J = 1.27 \frac{n^2}{D_o^2} \]

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Trajectory Scaling

• Traditional \( r_d \) and \( r^2d \) scaling laws

“entrainment mechanism”

“drag mechanism”

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Momentum Transport to the Jet

- **Entrainment**

\[
\frac{\dot{m}(x)}{m_{jet}} = C_e \left( \frac{\rho_{jet}}{\rho_\infty} \right)^{1/2} \left( \frac{x}{d} \right)
\]

Cross flow fluid mass entrained over a streamwise distance of 1 jet diameter

\[
m_{\text{entrained}} = m_{jet} C_e \frac{1}{S^{1/2}}
\]

\[
m_{jet} = \pi \frac{d^2}{4} \rho_j U_j
\]

Rate of momentum addition to the jet in the cross flow direction

\[
m_{\text{entrained}} U_o = m_{jet} C_e \frac{1}{S^{1/2}} U_o
\]

Ratio of new to original momentum rates

\[
\frac{m_{\text{entrained}} U_o}{m_{jet} U_j} = \frac{m_{jet} C_e \frac{1}{S^{1/2}} U_o}{m_{jet} U_j} = \frac{C_e}{J^{1/2}}
\]

Ricou and Spalding (1961)

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Momentum Transport to the Jet

• Drag

Consider a 1 diameter length element of the jet near the injection location:

Rate of momentum transport to the jet due pressure = drag force

\[ F = \frac{1}{2} \rho_o U_o^2 AC_d \]

Considering a streamwise distance of 1 jet diameter

\[ A = d^2 \]

Rate of transport of momentum from the jet orifice

\[ \dot{m}_{jet} U_j = \pi \frac{d^2}{4} \rho_j U_j^2 \]

Ratio of new to original momentum rates

\[ \frac{F}{\dot{m}_{jet} U_j} = \frac{1}{2} \rho_o U_o^2 d^2 C_d = \frac{2C_d}{\pi J} \]
Momentum Transport to the Jet

• Momentum and drag

Combine the total momentum transport ratios:

\[ \frac{C_{ef}}{J^{1/2}} + \frac{2C_d}{\pi J} \]

Invert this ratio and define as a new parameter B:

\[ B = \frac{J}{\frac{2C_d}{\pi} + 0.32J^{1/2}} \]

\( B_d \) represents the streamwise distance at which the magnitude of the total new momentum is equal to the jet momentum. The trajectory should scale with \( B_d \), within the limitations of the assumptions of the analysis.
Trajectory Scaling

- New scaling parameter—entrainment and drag

\[ B = \frac{J}{2C_d} + 0.32J^{1/2} \]

\[ C_d \approx 1.7 \]

Mashayek, Jafari, and Ashgriz (2008)
Optimum Mixing: Single Jet

Single jet optimum mixing correlations

\[ M = \frac{\sigma_c}{\sqrt{c(1-c)}} \]

- Centered jet
- Moderate impaction
- Strong impaction

Locally optimum mixing

Maruyama, one jet
Maruyama, two jets
Cozewith and Busko, one jet
Forney, Nafia, and Vo, one jet

\( d/D = 1/0.69^3 \)

\( B_d/D = 1 \)

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Multiple Jet Optimum Mixing

NASA trade study: 8-20 jets

\[ C_{opt} = \frac{S}{R_o} \sqrt{J} = 2.5 \]

- **No jet diameter dependence**
- Mass flow ratios 0.5 to 2.5
- Purely empirical (i.e. no physical basis)

Planar example:

\[ C = \frac{S}{R_o} \sqrt{J} > 5 \quad \text{Overpenetration} \]

\[ C_{opt} = \frac{S}{R_o} \sqrt{J} = 2.5 \quad \text{Optimum mixing} \]

\[ C = \frac{S}{R_o} \sqrt{J} < 1.25 \quad \text{Underpenetration} \]


Bain, Smith, Holdeman (1995): Jets should penetrate \( \frac{1}{4} \) of channel height

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Optimum Mixing Scaling Law

\[ B = \frac{J}{2C_d} + 0.32J^{1/2} \]

\[ \frac{BD_j}{D_o} = 1 \]

\[ \frac{D_j}{D_o} = 0.05 \]

\[ \frac{D_j}{D_o} = 0.1 \]

\[ \frac{D_j}{D_o} = 0.2 \]

gap in data

Holdeman scaling

\[ n_{opt} = \frac{\pi \sqrt{2}}{2.5} r \]

valid at \( x/D_o > 3?? \)

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Limitations on Entrainment

- Single jet (Tee mixer)

\[ \dot{m}_{\text{entrained}} = \dot{m}_o \]

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Limitations on Entrainment

• Multiple jets

\[ \frac{\dot{m}_j}{\dot{m}_o} = 0.5 \]

\[ \frac{\dot{m}_j}{\dot{m}_o} = 1 \]

\[ \frac{\dot{m}_j}{\dot{m}_o} = 2 \]

\( \dot{m}_{\text{entrained}} = \dot{m}_o \)

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Scalar field measurements using planar laser induced fluorescence (PLIF)

Maruyama (1983)

$C = \frac{S}{R_o} \sqrt{J} = 2.5$

$n_{opt} = \frac{\pi \sqrt{2}}{2.5} \cdot r$

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Water Experiment

- Experimental conditions
  - $\text{Re}_j > 6000$
  - $\text{Re}_o > 6000$
  - $1.3 < r < 7$
  - $1.8 < J < 50$
  - $D_j/D_o = 0.12, 0.165, 0.21$
  - $x/D_o = 3.0$
  - $0.25 < BD_j/D_o < 1.75$
- Turbine and rotameter flow meters

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Fluorescence

Sodium fluorescein

From Karasso and Mungal (1997)

Use calibration pictures with known homogenous mixtures

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PLIF Images

• Single Jet PLIF samples

Instantaneous

1.67 second average

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Mean Mixture Fraction Distributions

\[ \frac{BD_j}{D_o} = 0.25 \]

\[ \frac{BD_j}{D_o} = 0.75 \]

\[ \frac{BD_j}{D_o} = 1.5 \]

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Unmixedness

- **Single Jet**

\[ U_z = \frac{\sigma_c}{\sqrt{c(1-c)}} \]

<table>
<thead>
<tr>
<th>( D_j/D_o )</th>
<th>( J_{opt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>14.1</td>
</tr>
<tr>
<td>0.165</td>
<td>9</td>
</tr>
<tr>
<td>0.21</td>
<td>6.9</td>
</tr>
</tbody>
</table>

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Optimum Mixing Scaling Law

\[ \frac{D_j}{D_o} = 0.75 \]

Maruyama, one jet
Maruyama, two jets
Cozewith and Busko, one jet
Forney, Nafia, and Vo, one jet

For current study, \( \frac{D_j}{D_o} = 1 \)

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Conclusions

• Definition of a new scaling law for jet trajectory
  — Entrainment and drag mechanisms
  — Improved universality for unconfined single transverse jets
  — $BD_j/D_o = C$ predictive for optimum mixing scaling law for Tee mixer

• New experimental data on single confined transverse jets
  — $BD_j/D_o$ value indicates flow regime for different size jets
  — Local optimum point at $BD_j/D_o \sim 0.75$
Optimum Mixing Scaling Law

Possible sources of discrepancy:

- Cozewith data based on chemical reaction—microscale mixing
- Forney data based on RANS
- Current data limited to only $x/D = 3.0$