

Joint Design and Separation Principle for Opportunistic Spectrum Access in the Presence of Sensing Errors

Yunxia Chen, *Student Member, IEEE*, Qing Zhao, *Member, IEEE*, and Ananthram Swami, *Fellow, IEEE*

Abstract—Opportunistic spectrum access (OSA) that allows secondary users to independently search for and exploit instantaneous spectrum availability is considered. The design objective is to maximize the throughput of a secondary user while limiting the probability of colliding with primary users. Integrated in the joint design are three basic components: a spectrum sensor that identifies spectrum opportunities, a sensing strategy that determines which channels in the spectrum to sense, and an access strategy that decides whether to access based on potentially erroneous sensing outcomes. This joint design is formulated as a constrained partially observable Markov decision process (POMDP), and a separation principle is established. The separation principle reveals the optimality of myopic policies for the design of the spectrum sensor and the access strategy, leading to closed-form optimal solutions. Furthermore, it decouples the design of the sensing strategy from that of the spectrum sensor and the access strategy, and reduces the constrained POMDP to an unconstrained one. Numerical examples are provided to study the tradeoff between sensing time and transmission time, the interaction between the physical layer spectrum sensor and the MAC layer sensing and access strategies, and the robustness of the ensuing design to model mismatch.

Index Terms—Cognitive radio, opportunistic spectrum access, partially observable Markov decision process (POMDP).

I. INTRODUCTION

OPPORTUNISTIC spectrum access (OSA), first envisioned by Mitola [1] under the term “spectrum pooling” and then investigated by the DARPA XG program [2], has recently received increasing attention due to its potential for improving spectrum efficiency. The basic idea of OSA is to

Manuscript received February 27, 2007; revised January 17, 2008. This work was supported in part by the Army Research Laboratory CTA on Communication and Networks under Grant DAAD19-01-2-0011 and by the National Science Foundation under Grants CNS-0627090 and ECS-0622200. The material in this paper was presented in part at the IEEE Asilomar Conference on Signal, Systems, and Computers, Asilomar, CA, October/November 2006 and the IEEE Workshop on Signal Processing Advances in Wireless Communications, Helsinki, Finland, June 2007.

Y. Chen was with the Department of Electrical and Computer Engineering, University of California, Davis, CA 95616 USA. She is now with Cisco Systems, Inc., San Jose, CA USA (e-mail: yxchen@ece.ucdavis.edu).

Q. Zhao is with the Department of Electrical and Computer Engineering, University of California, Davis, CA 95616 USA (e-mail: qzhao@ece.ucdavis.edu).

A. Swami is with the Army Research Laboratory, Adelphi, MD 20783 USA (e-mail: aswami@arl.army.mil).

Communicated by A. Høst-Madsen, Associate Editor for Detection and Estimation.

Color versions of Figures 1–4 and 6–10 in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TIT.2008.920248

allow secondary users to search for, identify, and exploit instantaneous spectrum opportunities while limiting the interference perceived by primary users (or licensees).

In this paper, we address the design of OSA strategies for secondary users overlaying a slotted primary network. Integrated in the design are three basic components: 1) a spectrum sensor at the physical (PHY) layer that identifies instantaneous spectrum opportunities; 2) a spectrum sensing strategy at the medium access control (MAC) layer that specifies which channels in the spectrum to sense in each slot; and 3) a spectrum access strategy, also at the MAC layer, that determines whether to access the chosen channels based on imperfect sensing outcomes. The design objective is to maximize the throughput of a secondary user under the constraint that the probability of collision perceived by any primary user is below a predetermined threshold.

A. Fundamental Design Tradeoffs

We provide first an intuitive understanding of the fundamental tradeoffs in the joint design of the three basic components.

Spectrum Sensor: False Alarm Versus Miss-Detection: The spectrum sensor of a secondary user identifies spectrum opportunities by detecting the presence of primary signals, i.e., by performing a binary hypothesis test. With noise and fading, sensing errors are inevitable: false alarms occur when idle channels are detected as busy, and miss-detections occur when busy channels are detected as idle. In the event of a false alarm, a spectrum opportunity is overlooked by the sensor, and eventually wasted if the access strategy trusts the sensing outcome. On the other hand, miss-detections may lead to collisions with primary users. The tradeoff between false alarm and miss-detection is captured by the receiver operating characteristic (ROC) of the spectrum sensor, which relates the probability of detection (PD) and the probability of false alarm (PFA) (see an example in Fig. 1, where we consider an energy detector). The design of the spectrum sensor and the choice of the sensor operating point are thus important issues and should be addressed by considering the impact of sensing errors on the MAC layer performance in terms of throughput and collision probability. In particular, we are interested in the following fundamental question: which criterion should be adopted in the design of the spectrum sensor, the Bayes or the Neyman–Pearson (NP)? If the former, how do we choose the risks? If the latter, how should we set the constraint on the PFA?

Sensing Strategy: Gaining Immediate Access Versus Gaining Information for Future Use: Due to hardware limitations and the energy cost of spectrum monitoring, a secondary user may

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE JAN 2008		2. REPORT TYPE		3. DATES COVERED 00-00-2008 to 00-00-2008	
4. TITLE AND SUBTITLE Joint Design and Separation Principle for Opportunistic Spectrum Access in the Presence of Sensing Errors				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of California, Department of Electrical and Computer Engineering, Davis, CA, 95616				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT Opportunistic spectrum access (OSA) that allows secondary users to independently search for and exploit instantaneous spectrum availability is considered. The design objective is to maximize the throughput of a secondary user while limiting the probability of colliding with primary users. Integrated in the joint design are three basic components: a spectrum sensor that identifies spectrum opportunities, a sensing strategy that determines which channels in the spectrum to sense, and an access strategy that decides whether to access based on potentially erroneous sensing outcomes. This joint design is formulated as a constrained partially observable Markov decision process (POMDP), and a separation principle is established. The separation principle reveals the optimality of myopic policies for the design of the spectrum sensor and the access strategy, leading to closed-form optimal solutions. Furthermore, it decouples the design of the sensing strategy from that of the spectrum sensor and the access strategy, and reduces the constrained POMDP to an unconstrained one. Numerical examples are provided to study the tradeoff between sensing time and transmission time, the interaction between the physical layer spectrum sensor and the MAC layer sensing and access strategies, and the robustness of the ensuing design to model mismatch.					
15. SUBJECT TERMS Cognitive radio, opportunistic spectrum access, partially observable Markov decision process (POMDP)					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

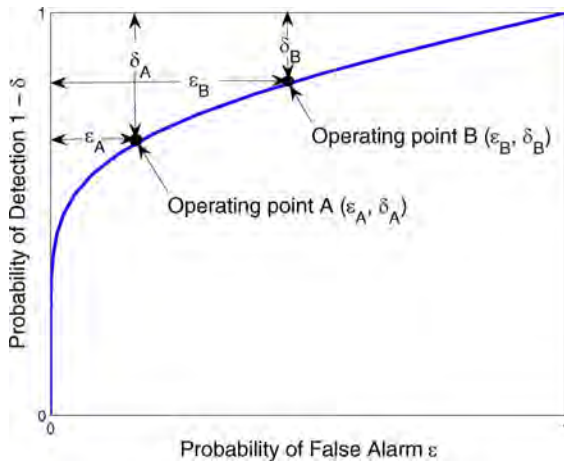


Fig. 1. The ROC of an energy detector. Each point on the ROC curve corresponds to a sensor operating characteristic resulting from different detection threshold of the energy detector. (ϵ : probability of false alarm; δ : probability of miss-detection.).

not be able to sense all the channels in the spectrum simultaneously. A sensing strategy is thus needed for intelligent channel selection to track the rapidly varying spectrum opportunities. The purpose of a sensing strategy is twofold: to find idle channels for immediate access and to gain statistical information on the spectrum occupancy for better opportunity tracking in the future. The optimal sensing strategy should thus strike a balance between these two often conflicting objectives.

Access Strategy: Aggressive Versus Conservative: Based on the imperfect sensing outcomes given by the spectrum sensor, the secondary user needs to decide whether to access. An aggressive access strategy may lead to excessive collisions with primary users while a conservative one may result in throughput degradation due to overlooked opportunities. Whether to adopt an aggressive or a conservative access strategy depends on the operating characteristic of the spectrum sensor and the collision constraint at the MAC layer. Hence, a joint design of the PHY layer spectrum sensor and the MAC layer access strategy is necessary for optimality.

B. Main Results

By modeling primary users' spectrum occupancy as a Markov chain, we establish a decision-theoretic framework for the optimal joint design of OSA based on the theory of partially observable Markov decision processes (POMDPs). This framework captures the fundamental design tradeoffs discussed above. Within this framework, the optimal OSA strategy is given by the optimal policy of a constrained POMDP.

While powerful in problem modeling, POMDP suffers from the curse of dimensionality and does not easily lend itself to tractable solutions. Constraints on a POMDP further complicates the problem, often demanding *randomized* policies to achieve optimality. Our goal is to develop structural results that lead to simple yet optimal solutions and shed light on the interaction between the PHY and the MAC layers of OSA networks.

Single-Channel Sensing: We focus first on the case where the secondary user can sense and access one channel in each slot

(e.g., in the case of single-carrier communications). We establish a separation principle for the optimal joint design of OSA. We show that the joint design can be carried out in two steps without losing optimality: first to choose a spectrum sensor and an access strategy that maximize the instantaneous throughput (i.e., the expected number of bits that can be delivered in the current slot) under the collision constraint, and then to choose a sensing strategy to optimize the overall throughput. As stated below, the significance of this separation principle is twofold.

- The separation principle reveals the optimality of myopic policies for the design of the spectrum sensor and the access strategy. Myopic policies aim solely at maximizing the immediate reward and ignore the impact of the current action on the future reward. Hence, obtaining myopic policies becomes a static optimization problem instead of a sequential decision-making problem. While myopic policies are rarely optimal for a general POMDP, we show that the rich structure of the problem at hand renders an exception. As a consequence, we are able to obtain an explicit design of the optimum spectrum sensor and a closed-form optimal access strategy. Moreover, this closed-form optimal design allows us to characterize quantitatively the interaction between the PHY layer spectrum sensor and the MAC layer access strategy.
- The separation principle decouples the design of the sensing strategy from that of the spectrum sensor and the access strategy. More importantly, the design of the sensing strategy is reduced to an *unconstrained* POMDP, which admits *deterministic* optimal policies. Unconstrained POMDPs have been well studied, and existing algorithms can be readily applied [3]–[6].

We also provide numerical examples to study design tradeoffs. We will see that miss-detections are more harmful to the throughput of the secondary user than false alarms. The tradeoff study between the spectrum sensing time and the data transmission time indicates that the spectrum sensor should take fewer channel measurements as the maximum allowable probability of collision increases. In other words, when the collision constraint is less restrictive, the secondary user can spend less time in sensing, leaving more time in a slot for data transmission. Robustness studies show that the throughput loss due to inaccuracies in the assumed Markovian model parameters is small, and more importantly, the probability of collision perceived by the primary network is not affected by model mismatch.

Multichannel Sensing: We then consider the scenario where the secondary user can sense and access multiple channels simultaneously in each slot. We show that the separation principle still holds if the spectrum sensor and the access strategy are designed independently across channels. We note that such independent design is suboptimal since it ignores the potential correlation among channel occupancies. We thus propose two heuristic approaches to exploit channel correlation, one at the PHY layer and the other at the MAC layer. Simulation results show that exploiting channel correlation at the PHY layer is more effective than at the MAC layer.

We also find that the performance of the PHY layer spectrum sensor can improve over time by incorporating the MAC layer sensing and access decisions. Such MAC layer decisions pro-

vide information on the evolution of the primary users' spectrum occupancy, from which the *a priori* probabilities of the hypotheses employed by the spectrum sensor can be learned. This finding, along with the quantitative characterization of the impact of the spectrum sensor on the access strategy, illustrates the two-way interaction between the PHY and the MAC layers: the necessity of incorporating the sensor operating characteristics into the MAC design and the benefit of exploiting the MAC layer information in the PHY design.

C. Related Work

Two types of spectrum opportunities have been considered in the literature: spatial and temporal. A majority of existing work on OSA focuses on exploiting spatial spectrum opportunities that are static or slowly varying in time (see [7]–[9] and references therein). A typical example application is the reuse of locally unused TV broadcast bands. In this context, due to the slow temporal variation of spectrum occupancy, real-time opportunity identification is not as critical a component as in applications that exploit temporal spectrum opportunities, and existing work often assumes perfect knowledge of spectrum opportunities in the whole spectrum at any time and location.

The exploitation of temporal spectrum opportunities resulting from the bursty traffic of primary users is addressed in [10]–[13] under the assumption of perfect sensing. In [10], MAC protocols are proposed for an *ad hoc* secondary network overlaying a Global System for Mobile Communications (GSM) cellular network. It is assumed that the secondary transmitter and receiver exchange information on which channel to use through a commonly agreed control channel. Different from [10], optimal distributed MAC protocols developed in [11] can synchronize the hopping patterns of the secondary transmitter and receiver without the aid of additional control channels. More recently, the design of optimal spectrum sensing and access strategies in a fading environment has been addressed under an energy constraint in [12]. In [13], access strategies for a slotted secondary user exploiting opportunities in an unslotted primary network are considered, where a round-robin single-channel sensing scheme is used. Modeling of spectrum occupancy has been addressed in [14]. Measurements obtained from spectrum monitoring testbeds demonstrate the Markovian transition between busy and idle channel states in wireless local-area network (LAN).

Although the issue of spectrum sensing errors has been investigated at the PHY layer [15]–[19], cognitive MAC design in the presence of sensing errors has received little attention. To the best of our knowledge, [20] is the first work that integrates the operating characteristic of the spectrum sensor at the PHY layer with the MAC design. A heuristic approach to the joint PHY-MAC design of OSA is proposed in [20]. In this paper, we establish a decision-theoretic framework within which the optimal joint design of OSA in the presence of sensing errors can be systematically addressed and the interaction between the PHY and the MAC layers can be quantitatively characterized. Interestingly, the separation principle developed in this paper reveals that the heuristic approach proposed in [20] is optimal.

For an overview on challenges and recent developments in OSA, readers are referred to [21].

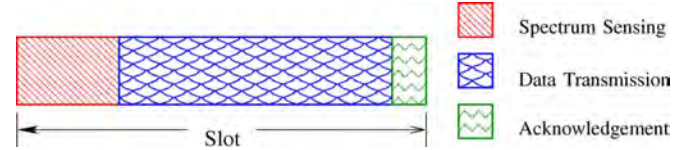


Fig. 2. The slot structure.

D. Organization and Notation

This paper is organized as follows. Section II describes the network model and the basic operations performed by a secondary user to exploit spectrum opportunities. In Section III, we introduce the three basic components of OSA and formulate their joint design as a constrained POMDP. In Section IV, we establish the separation principle for the optimal joint design of OSA with single-channel sensing. Section V extends the separation principle to multichannel sensing scenarios. Section VI concludes this paper.

Random variables and their realizations are denoted by capital and lower case letters, respectively. Vectors are denoted by boldfaced letters.

II. NETWORK MODEL

Consider a spectrum that consists of N channels (e.g., different frequency bands or tones in an orthogonal frequency-division modulation (OFDM) system), each with bandwidth B_n ($n = 1, \dots, N$). These N channels are licensed to a slotted primary network. We model the spectrum occupancy as a discrete-time homogenous Markov chain with 2^N states. Specifically, let $S_n(t) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$ denote the occupancy of channel n in slot t . The spectrum occupancy state (SOS), denoted as $\mathbf{S}(t) \triangleq [S_1(t), \dots, S_N(t)]$, follows a Markov chain with state space $\mathcal{S} \triangleq \{0, 1\}^N$. The transition probabilities of the SOS are denoted as $P(\mathbf{s}'|\mathbf{s}) \triangleq \Pr\{\mathbf{S}(t+1) = \mathbf{s}' | \mathbf{S}(t) = \mathbf{s}\}$. Note that the transition probabilities are determined by the dynamics of the primary traffic. We assume that they are known and remain unchanged in T slots.

We consider a secondary *ad hoc* network whose users independently and selfishly exploit instantaneous spectrum opportunities in these N channels. At the beginning of each slot,¹ a secondary user with data to transmit chooses a set of channels to sense. A spectrum sensor is used to detect the states of the chosen channels. Based on the sensing outcomes, the secondary user decides which sensed channels to access. Due to hardware and energy constraints, we assume that a secondary user can sense and access at most L ($1 \leq L \leq N$) channels in a slot. At the end of the slot, the receiver acknowledges each successful transmission. The basic slot structure is illustrated in Fig. 2.

Our goal is to develop an optimal OSA strategy for the secondary user, which sequentially determines which channels in the spectrum to sense, how to design the spectrum sensor, and whether to access based on the imperfect sensing outcomes. The design objective is to maximize the throughput of the secondary

¹With the knowledge of the slot length and through sensing the transmissions of primary users, secondary users can synchronize to the slot structure. Furthermore, the primary network may broadcast periodic beacon signals to keep its own users synchronized. These beacon signals can be exploited by secondary users for synchronization.

user during a desired period of T slots under the constraint that the probability of collision $P_n(t)$ perceived by the primary network in any channel n and slot t is capped below a predetermined threshold ζ , i.e.,

$$P_n(t) \triangleq \Pr\{\Phi_n(t) = 1 | S_n(t) = 0\} \leq \zeta, \quad \forall n, t \quad (1)$$

where $\Phi_n(t) \in \{0$ (no access), 1 (access) $\}$ denotes the access decision of the secondary user.

Remarks:

- 1) We assume that the transition probabilities of the SOS are known or have been learned. We take the viewpoint that such statistical models of a particular spectrum region should be obtained through measurements before the deployment of secondary networks. This is for the purpose of evaluating the potential gain or profit of secondary market in that spectrum region. Such statistical models can then be made available to secondary users to facilitate the design. We are, however, aware that in some scenarios, secondary users may have imperfect knowledge of the underlying Markovian model. In Section IV-F, we study the robustness of the optimal OSA design to a mismatched Markovian model. For the case where the Markovian model is unknown, formulations and algorithms for POMDP with an unknown model exist in the literature [22] and can be applied to this problem.
- 2) We use the *conditional* probability of collision $P_n(t)$ in the design constraint and impose the collision constraint on every channel n and slot t . This ensures that a primary user experiences collisions no more than ζ fraction of its *transmission time* regardless of where and when it transmits. Note that if the unconditional probability of collision $\Pr\{\Phi_n(t) = 1, S_n(t) = 0\}$ is adopted, the constraint depends on the traffic load of primary users in channels chosen by the secondary users; primary users who have light traffic load may not be as well protected as those with heavy traffic load.
- 3) We assume that secondary users exploit spectrum opportunities independently and selfishly. That is, secondary users do not exchange their information on the SOS and each one aims to maximize its own throughput without taking into consideration the interactions among secondary users. This assumption is suitable for secondary *ad hoc* networks where there is no central coordinator or dedicated control/communication channel. The secondary network can adopt a carrier sensing mechanism to avoid collisions among competing secondary users as detailed in [11], [20]. We point out that such selfish decisions may not be optimal in terms of network-level throughput. Nevertheless, this formulation allows us to focus on the basic components of OSA and highlight the interactions among them.

III. CONSTRAINED POMDP FORMULATION

In this section, we develop a decision-theoretic framework for the optimal joint design of the three basic OSA components based on the theory of POMDP. We focus first on single-channel sensing ($L = 1$). Extensions to multichannel sensing scenarios are detailed in Section V.

A. Spectrum Sensor

Suppose that channel n is chosen in slot t . The spectrum sensor detects the presence of primary users in this channel by performing a binary hypothesis test

$$\begin{aligned} \mathcal{H}_0 : S_n(t) &= 1 \text{ (idle)} \\ \text{vs. } \mathcal{H}_1 : S_n(t) &= 0 \text{ (busy)}. \end{aligned} \quad (2)$$

Let $\Theta_n(t) \in \{0$ (busy), 1 (idle) $\}$ denote the sensing outcome (i.e., the result of the binary hypothesis test). The performance of the spectrum sensor is characterized by the PFA $\epsilon_n(t)$ and the probability of miss detection (PM) $\delta_n(t)$

$$\begin{aligned} \epsilon_n(t) &\triangleq \Pr\{\text{decide } \mathcal{H}_1 | \mathcal{H}_0 \text{ is true}\} \\ &= \Pr\{\Theta_n(t) = 0 | S_n(t) = 1\} \end{aligned} \quad (3a)$$

$$\begin{aligned} \delta_n(t) &\triangleq \Pr\{\text{decide } \mathcal{H}_0 | \mathcal{H}_1 \text{ is true}\} \\ &= \Pr\{\Theta_n(t) = 1 | S_n(t) = 0\}. \end{aligned} \quad (3b)$$

Subject to the constraint that the PFA is no larger than $\epsilon_n(t)$, the largest achievable PD, denoted as $P_{D,\max}^{(n)}(\epsilon_n(t))$, can be attained by the optimal NP detector or an optimal Bayesian detector with a suitable set of risks [23, Sec. 2.2.1]. All operating points (ϵ, δ) above the best ROC curve $P_{D,\max}^{(n)}$ are thus infeasible.

Let

$$\mathbb{A}_\delta(n) \triangleq \{(\epsilon, \delta) : 0 \leq \epsilon \leq 1 - \delta \leq P_{D,\max}^{(n)}(\epsilon)\}$$

denote all feasible operating points of the spectrum sensor.² As illustrated in Fig. 3, the best ROC curve $P_{D,\max}^{(n)}$ achieved by the optimal NP detector forms the upper boundary of the feasible set $\mathbb{A}_\delta(n)$. We also note that every sensor operating point (ϵ_n, δ_n) below the best ROC curve lies on a line that connects two boundary points and hence can be achieved by randomizing between two optimal NP detectors with properly chosen constraints on the PFA [23, Sec. 2.2.2]. For example, the operating point (ϵ_n, δ_n) as shown in Fig. 3 can be achieved by applying the optimal NP detector under the constraint of PFA $\leq \epsilon_n^{(1)}$ with probability $p = \frac{\epsilon_n - \epsilon_n^{(2)}}{\epsilon_n^{(1)} - \epsilon_n^{(2)}}$ and the optimal NP detector under the constraint of PFA $\leq \epsilon_n^{(2)}$ with probability $1 - p$. Therefore, the design of spectrum sensor is reduced to the choice of a desired sensor operating point in $\mathbb{A}_\delta(n)$.

The design of the optimal NP detector is a well-studied problem, which is not the focus of this paper. Our objective is to define the criterion and the constraint under which the spectrum sensor should be designed, equivalently, to find the optimal sensor operating point $(\epsilon_n^*(t), \delta_n^*(t)) \in \mathbb{A}_\delta(n)$ to achieve the best tradeoff between false alarm and miss-detection. Note that the optimal sensor operating point may vary with time (see Section V-D for an example.)

B. Sensing and Access Strategies

In each slot, a sensing strategy decides which channel in the spectrum to sense, and an access strategy determines whether

²Since the two hypotheses in (2) play a symmetric role, we have assumed, without loss of generality, that the PD is no smaller than the PFA, i.e., $1 - \delta \geq \epsilon$.

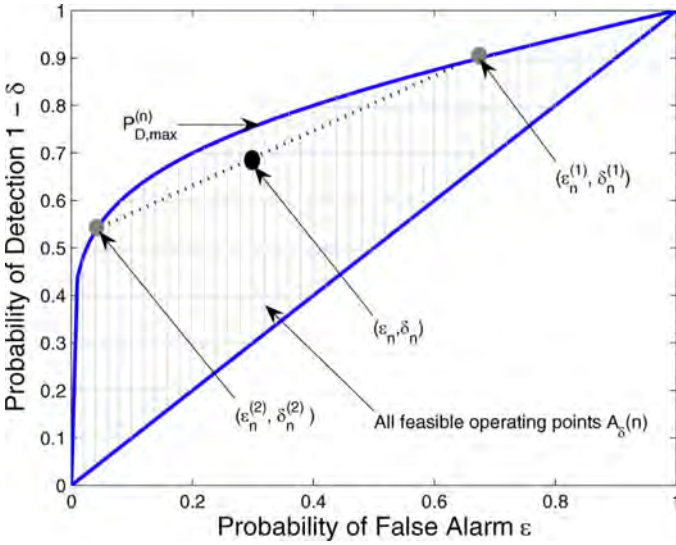


Fig. 3. Illustration of the set $\mathbb{A}_\delta(n)$ of all feasible sensor operating points (ϵ_n, δ_n) . $(\delta_n^{(i)} = 1 - P_{D,\max}^{(n)}(\epsilon_n^{(i)}), i = 1, 2)$.

to access given the sensing outcome.³ Below we illustrate the sequence of operations in each slot.

At the beginning of slot t , the SOS transits to $\mathcal{S}(t) = [S_1(t), \dots, S_N(t)]$ according to the transition probabilities of the underlying Markov chain. The secondary user first chooses a channel $a(t) \in \mathbb{A}_s \triangleq \{1, \dots, N\}$ to sense and a feasible sensor operating point $(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t))$. It then determines whether to access $\Phi_a(t) \in \{0 \text{ (no access), } 1 \text{ (access)}\}$ by taking into account the sensing outcome $\Theta_a(t) \in \{0 \text{ (busy), } 1 \text{ (idle)}\}$ provided by the spectrum sensor that is designed according to the chosen operating point $(\epsilon_a(t), \delta_a(t))$. A collision with primary users happens when the secondary user accesses a busy channel. At the end of this slot, the receiver acknowledges a successful transmission $K_a(t) \in \{0 \text{ (no ACK), } 1 \text{ (ACK)}\}$. We assume that the ACKs are received without errors.⁴

C. Constrained POMDP Formulation

The sequential decision-making process described above can be modeled as a POMDP with constraint given in (1). The underlying system of this POMDP is the SOS with state space $\mathbb{S} = \{0, 1\}^N$ and transition probabilities $P(\mathbf{s}'|\mathbf{s})$. We describe below the actions, observations, and reward structure of the resulting POMDP.

Action Space: The action in the POMDP formulation consists of three parts: a sensing decision $a(t) \in \mathbb{A}_s$, a spectrum sensor design $(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t))$, and an access decision $\Phi_a(t) \in \{0, 1\}$.

Observation Space: As will become clear later, optimal channel selection for opportunity tracking relies on the exploitation of the statistical information on the SOS provided

³An alternative formulation of the joint design is to combine the spectrum sensor with the access strategy. In this case, the access decision is made directly based on the channel measurements. It can be readily shown that this formulation is equivalent to the one adopted here.

⁴Note that the ACK is sent after the successful reception of data. Hence, the channel over which the ACK is transmitted is ensured to be idle in this slot.

by the observation history of the secondary users. To ensure synchronous hopping in the spectrum without introducing extra control message exchange, the secondary user and its desired receiver must have the same history of observations so that they make the same channel selection decisions. Since sensing errors may cause different sensing outcomes at the transmitter and the receiver, the acknowledgment $K_a(t) \in \{0, 1\}$ should be used as the common observation in each slot.

Reward: A natural definition of the reward is the number of bits that can be delivered by the secondary user, which is assumed to be proportional to the channel bandwidth. Given sensing action $a(t)$ and access action $\Phi_a(t)$, the immediate reward $R(t)$ can be defined as

$$R(t) = K_a(t)B_a = S_a(t)\Phi_a(t)B_a. \quad (4)$$

Hence, the expected total reward of the POMDP represents the overall throughput, i.e., the expected total number of bits that can be delivered by the secondary user in T slots.

Belief Vector: Due to partial spectrum monitoring and sensing errors, a secondary user cannot directly observe the true SOS. It can, however, infer the SOS from its decision and observation history. As shown in [3], the statistical information on the SOS provided by the entire decision and observation history can be encapsulated in a belief vector $\mathbf{\Lambda}(t) \triangleq \{\lambda_{\mathbf{s}}(t)\}_{\mathbf{s} \in \mathbb{S}}$, where $\lambda_{\mathbf{s}}(t) \in [0, 1]$ denotes the conditional probability (given the decision and observation history) that the SOS is \mathbf{s} in slot t

$$\lambda_{\mathbf{s}}(t) \triangleq \Pr\{\mathcal{S}(t) = \mathbf{s} | \mathbf{\Lambda}(1), \{a(\tau), \Theta_a(\tau)\}_{\tau=1}^{t-1}\} \quad (5)$$

where $\mathbf{\Lambda}(1)$ is the initial belief vector, i.e., the *a priori* distribution of the SOS at time $t = 1$, which can be set to the stationary distribution of the underlying Markov chain if no information on the initial SOS is available.

Policy: A joint design of OSA is given by policies of the above POMDP. Specifically, a sensing policy π_s specifies a sequence of functions $\pi_s = [\mu_s(1), \dots, \mu_s(T)]$, where $\mu_s(t)$ maps a belief vector $\mathbf{\Lambda}(t)$ to a channel $a(t) \in \mathbb{A}_s$ to be sensed in this slot. Since the optimal policy for a finite-horizon POMDP is generally nonstationary, functions $\{\mu_s(t)\}_{t=1}^T$ are not identical. A sensor operating policy π_δ specifies, in each slot t , a spectrum sensor design $(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t))$ based on the current belief vector $\mathbf{\Lambda}(t)$ and the chosen channel $a(t)$. An access policy π_c specifies an access decision $\Phi_a(t) \in \{0, 1\}$ in each slot t based on the current belief vector $\mathbf{\Lambda}(t)$ and the sensing outcome $\Theta_a(t) \in \{0, 1\}$ at the chosen channel $a(t)$.

The above defined policies are deterministic. For unconstrained POMDPs, there always exist deterministic optimal policies. For constrained POMDPs, however, we may need to resort to randomized policies to achieve optimality. A randomized sensing policy π_s defines a sequence of functions, each mapping a belief vector $\mathbf{\Lambda}(t)$ to a probability mass function (pmf) on the set \mathbb{A}_s of channels, and a randomized sensor operating policy π_δ defines the mapping from $\mathbf{\Lambda}(t)$ to a probability density function (pdf) on the set $\mathbb{A}_\delta(a(t))$ of feasible sensor operating points. A randomized access policy π_c maps $\mathbf{\Lambda}(t)$ and sensing outcome $\Theta_a(t)$ to a transmission probability. In other words, the actions chosen in a randomized policy are probability distributions. Due to the uncountable space

of probability distributions, randomized policies are usually computationally prohibitive.

Objective and Constraint: We aim to develop the optimal joint design of OSA $\{\pi_\delta^*, \pi_s^*, \pi_c^*\}$ that maximizes the expected total number of bits that can be delivered by the secondary user (i.e., the expected total reward of the POMDP) in T slots under the collision constraint given in (1)

$$\begin{aligned} \{\pi_\delta^*, \pi_s^*, \pi_c^*\} &= \arg \max_{\pi_\delta, \pi_s, \pi_c} \mathbb{E}_{\{\pi_\delta, \pi_s, \pi_c\}} \left[\sum_{t=1}^T R(t) \middle| \mathbf{\Lambda}(1) \right] \\ \text{s.t. } P_a(t) &= \Pr\{\Phi_a(t) = 1 \mid S_a(t) = 0\} \leq \zeta, \quad \forall a, t \end{aligned} \quad (6)$$

where $\mathbb{E}_{\{\pi_\delta, \pi_s, \pi_c\}}$ represents the expectation given that policies $\{\pi_\delta, \pi_s, \pi_c\}$ are employed, and $P_a(t)$ is the probability of collision perceived by the primary network in channel $a(t)$ and slot t .

We consider in (6) the nontrivial case where the conditional collision probability $P_a(t)$ is well defined, i.e., $\Pr\{S_a(t) = 0\} > 0$. Note that $\Pr\{S_a(t) = 0\} = 0$ (or 1) implies that the system state $S_a(t)$ is known based on the current belief vector $\mathbf{\Lambda}(t)$. In this case, the optimal access decision is straightforward, and the design of the spectrum sensor becomes unnecessary since the channel state is already known.

IV. SEPARATION PRINCIPLE FOR OPTIMAL OSA

In this section, we solve the constrained POMDP given in (6) to obtain the optimal joint design of OSA. Specifically, we establish a separation principle that reveals the optimality of deterministic policies and leads to closed-form optimal design of the spectrum sensor and the access strategy. It also allows us to characterize quantitatively the interaction between the PHY layer sensor operating characteristics and the MAC layer access strategy.

A. Optimality Equation

The first step to solving (6) is to express the objective and the constraint explicitly as functions of the actions. We establish first the optimality of deterministic sensing and sensor operating policies, which significantly simplifies the action space.

Optimality of Deterministic Policies: In Proposition 1, we show that it is sufficient to consider deterministic sensing and sensor operating policies in the optimal joint design of OSA.

Proposition 1: For the optimal joint design of OSA given by (6), there exist deterministic optimal sensing and sensor operating policies.

Proof: The proof is based on the concavity of the best ROC curve and the fact that the collision constraint is imposed on every channel. See details in Appendix A. $\square\square\square$

As a result of Proposition 1, the secondary user needs to choose, in each slot, a channel $a(t) \in \mathbb{A}_s$ to sense, a feasible sensor operating point $(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t))$, and a pair of transmission probabilities $(f_a(0, t), f_a(1, t))$, where

$$f_a(\theta, t) \triangleq \Pr\{\Phi_a(t) = 1 \mid \Theta_a(t) = \theta\} \in [0, 1]$$

is the probability of accessing channel $a(t)$ given sensing outcome $\Theta_a(t) \in \{0, 1\}$ in the current slot. The composite action space is then given by

$$\begin{aligned} \mathbb{A} &\triangleq \{(a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))) \\ &: a \in \mathbb{A}_s, (\epsilon_a, \delta_a) \in \mathbb{A}_\delta(a), (f_a(0), f_a(1)) \in [0, 1]^2\}. \end{aligned} \quad (7)$$

Objective Function: Let $V_t(\mathbf{\Lambda}(t))$ be the value function, which represents the maximum expected reward that can be obtained starting from slot t ($1 \leq t \leq T$) given belief vector $\mathbf{\Lambda}(t)$. Given that the secondary user takes action $A(t) = A \in \mathbb{A}$ and observes acknowledgment $K_a(t) = k \in \{0, 1\}$, the reward that can be accumulated starting from slot t consists of two parts: the immediate reward $R(t) = kB_a$ and the maximum expected future reward $V_{t+1}(\mathbf{\Lambda}(t+1))$, where

$$\mathbf{\Lambda}(t+1) \triangleq \{\lambda_{\mathbf{s}}(t+1)\}_{\mathbf{s} \in \mathbb{S}} = \mathcal{T}(\mathbf{\Lambda}(t) \mid A, k)$$

represents the updated knowledge of the SOS after incorporating the action $A(t) = A$ and the acknowledgment $K_a(t) = k$ in slot t . Averaging over all possible states $\mathbf{s} \in \mathbb{S}$ and acknowledgment $k \in \{0, 1\}$ and then maximizing over all actions $A \in \mathbb{A}$, we arrive at the following optimality shown in (8a)–(8b) at the bottom of the page, where

$$A \triangleq \{a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))\} \in \mathbb{A}$$

denotes a composite action taken in the current slot t and

$$U_A(k \mid \mathbf{s}) \triangleq \Pr\{K_a(t) = k \mid \mathbf{S}(t) = \mathbf{s}\}$$

is the conditional pmf of the acknowledgment $K_a(t)$ given current state $\mathbf{S}(t) = \mathbf{s}$ and action $A(t)$.

Noting that the acknowledgment can be written as $K_a(t) = S_a(t)\Phi_a(t)$, we obtain its conditional pmf $U_A(k \mid \mathbf{s})$ as

$$\begin{aligned} U_A(1 \mid \mathbf{s}) &= \Pr\{S_a(t) = 1 \mid \mathbf{S}(t) = \mathbf{s}\} \Pr\{\Phi_a(t) = 1 \mid \mathbf{S}(t) = \mathbf{s}, S_a(t) = 1\} \\ &= 1_{[s_a=1]} \sum_{\theta=0}^1 \Pr\{\Theta_a(t) = \theta \mid \mathbf{S}(t) = \mathbf{s}\} f_a(\theta) \\ &= s_a[\epsilon_a f_a(0) + (1 - \epsilon_a) f_a(1)] \end{aligned} \quad (9a)$$

$$\begin{aligned} U_A(0 \mid \mathbf{s}) &= 1 - U_A(1 \mid \mathbf{s}) \end{aligned} \quad (9b)$$

where $1_{[x]}$ is the indicator function and

$$\Pr\{S_a(t) = 1 \mid \mathbf{S}(t) = \mathbf{s}\} = 1_{[s_a=1]}$$

$$V_t(\mathbf{\Lambda}(t)) = \max_{A \in \mathbb{A}} \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) \sum_{k=0}^1 U_A(k \mid \mathbf{s}) [kB_a + V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) \mid A, k))], \quad 1 \leq t < T \quad (8a)$$

$$V_T(\mathbf{\Lambda}(T)) = \max_{A \in \mathbb{A}} \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(T) U_A(1 \mid \mathbf{s}) B_a \quad (8b)$$

is given by the occupancy state s_a of channel a . Applying Bayes' rule, we obtain the updated belief vector $\mathbf{\Lambda}(t+1) = \mathcal{T}(\mathbf{\Lambda}(t) | A, k)$ as

$$\lambda_{\mathbf{s}}(t+1) = \frac{\sum_{\mathbf{s}' \in \mathcal{S}} \lambda_{\mathbf{s}'}(t) P(\mathbf{s} | \mathbf{s}') U_A(k | \mathbf{s}')}{\sum_{\mathbf{s}' \in \mathcal{S}} \lambda_{\mathbf{s}'}(t) U_A(k | \mathbf{s}')}, \quad \mathbf{s} \in \mathcal{S}. \quad (10)$$

We see from (10) that by adopting the acknowledgment $K_a(t)$ as their observation, the transmitter and the receiver will have the same updated belief vector $\mathbf{\Lambda}(t+1)$, which ensures that they tune to the same channel in the next slot.

Note from (8) that the action

$$A = \{a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))\}$$

taken by the secondary user affects the expected total reward in two ways: it acquires an immediate reward $R(t) = kB_a$ and transforms the current belief vector $\mathbf{\Lambda}(t)$ to a new one $\mathbf{\Lambda}(t+1) = \mathcal{T}(\mathbf{\Lambda}(t) | A, k)$ which determines the future reward $V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, k))$. Hence, the function of the secondary user's action is twofold: to exploit immediate spectrum opportunities and to gain information on the SOS (characterized by belief vector $\mathbf{\Lambda}(t+1)$) so that more rewarding decisions can be made in the future. As a consequence, the optimal joint design of OSA should achieve the tradeoff between these two often conflicting objectives. Myopic policies that aim solely at maximizing the instantaneous throughput (i.e., the expected immediate reward) without considering future consequences are generally suboptimal.

Collision Constraint: The collision probability $P_a(t)$ is determined by the sensor operating point (ϵ_a, δ_a) and the transmission probabilities $(f_a(0), f_a(1))$: see (11) at the bottom of the page. In principle, by solving (8) recursively (starting from the last slot T using (8b)) under the constraint of (11), we can obtain the maximum overall throughput $V_1(\mathbf{\Lambda}(1))$ of the secondary user and the corresponding policies $\{\pi_\delta^*, \pi_s^*, \pi_c^*\}$. We, however, note that (8) is generally intractable due to the uncountable action space \mathbb{A} .

B. The Separation Principle

Theorem 1: The Separation Principle for OSA with Single-Channel Sensing

The joint design of OSA given in (8) can be carried out in two steps without losing optimality.

- Step 1: Choose the sensor operating policy π_δ and the access policy π_c to maximize the instantaneous throughput subject to the collision constraint. Specifically, for any

chosen channel a in any slot t , the optimal sensor operating point $(\epsilon_a^*, \delta_a^*)$ and transmission probabilities $(f_a^*(0), f_a^*(1))$ are given by

$$\begin{aligned} & \{(\epsilon_a^*, \delta_a^*), (f_a^*(0), f_a^*(1))\} \\ &= \arg \max_{\substack{(\epsilon_a, \delta_a) \in \mathbb{A}_\delta(a) \\ (f_a(0), f_a(1)) \in [0,1]^2}} \mathbb{E}[R(t) | \mathbf{\Lambda}(t)] \\ &= \arg \max_{\substack{(\epsilon_a, \delta_a) \in \mathbb{A}_\delta(a) \\ (f_a(0), f_a(1)) \in [0,1]^2}} \epsilon_a f_a(0) + (1 - \epsilon_a) f_a(1) \quad (12a) \\ &\text{s.t. } P_a(t) = (1 - \delta_a) f_a(0) + \delta_a f_a(1) \leq \zeta. \quad (12b) \end{aligned}$$

- Step 2: Using the optimal sensor operating and access policies $\{\pi_\delta^*, \pi_c^*\}$ given by (12), choose sensing policy to maximize the overall throughput. Specifically, the optimal sensing policy π_s^* is given by

$$\pi_s^* = \arg \max_{\pi_s} \mathbb{E}_{\pi_s} \left[\sum_{t=1}^T R(t) \middle| \mathbf{\Lambda}(1) \right]. \quad (13)$$

Proof: The proof is based on the convexity of the value function $V_t(\mathbf{\Lambda}(t))$ with respect to the belief vector $\mathbf{\Lambda}(t)$ and the structure of the conditional observation distributions $U_A(k | \mathbf{s})$. See Appendix B for details. $\square \square \square$

The separation principle simplifies the optimal joint design of OSA in two ways. First, it reveals that myopic policies, rarely optimal for a general POMDP, are optimal for the design of the spectrum sensor and the access strategy. We can thus obtain the optimal spectrum sensor $(\epsilon_a^*, \delta_a^*) \in \mathbb{A}_\delta(a)$ and the optimal transmission probabilities $(f_a^*(0), f_a^*(1)) \in [0,1]^2$ by solving the static optimization problem given in (12). This allows us to characterize quantitatively the interaction between the spectrum sensor and the access strategy as given in Proposition 2 and to obtain the optimal joint design in closed form as given in Theorem 2. While the proof is lengthy, there is an intuitive explanation for this apparently surprising result. We note that upon receiving the ACK $K_a(t) = 1$, the secondary user knows exactly that the chosen channel is idle. However, when $K_a(t) = 0$ (no packet is received), the secondary receiver cannot tell whether the chosen channel is busy or not accessed. Hence, $K_a(t) = 1$ provides the secondary user with more information on the current SOS. We also note that accessing the chosen channel maximizes not only the instantaneous throughput but also the chance of receiving more informative observation $K_a(t) = 1$. Hence, getting immediate reward and gaining information for more rewarding future decisions are no longer conflicting here.

Second, the separation principle decouples the design of the sensing strategy from that of the spectrum sensor and the

$$\begin{aligned} P_a(t) &\triangleq \Pr\{\Phi_a(t) = 1 | S_a(t) = 0\} \\ &= \sum_{\theta=0}^1 \Pr\{\Theta_a(t) = \theta | S_a(t) = 0\} \Pr\{\Phi_a(t) = 1 | \Theta_a(t) = \theta, S_a(t) = 0\} \\ &= (1 - \delta_a) f_a(0) + \delta_a f_a(1) \leq \zeta. \end{aligned} \quad (11)$$

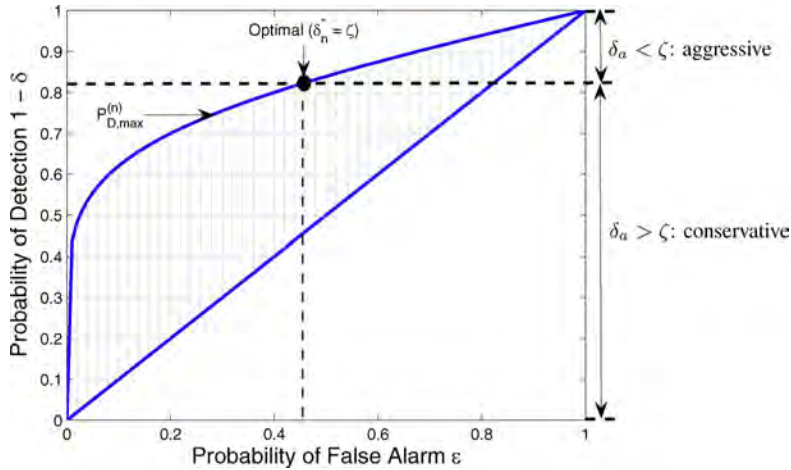


Fig. 4. Illustration of conservative and aggressive regions.

access strategy, and reduces the sensing strategy from a constrained POMDP (6) to an unconstrained one with finite action space (13). This is because the sensor operating points and the transmission probabilities determined by (12) have ensured the collision constraint regardless of channel selections. The optimal sensing policy is thus obtained by maximizing the overall throughput without any constraint.

C. Interaction Between the PHY and the MAC Layers

Before solving for the optimal sensor operating and access policies, we study the interaction between the PHY layer spectrum sensor and the MAC layer access strategy.

We note that when the spectrum sensor at the PHY layer is given, the separation principle still holds for the design of the sensing and access strategies. The optimal access strategy for a given spectrum sensor can thus be obtained.

Proposition 2: Given a chosen channel a and a feasible sensor operating point (ϵ_a, δ_a) , the optimal transmission probabilities $(f_a^*(0), f_a^*(1))$ are given by

$$(f_a^*(0), f_a^*(1)) = \begin{cases} (\frac{\zeta - \delta_a}{1 - \delta_a}, 1), & \delta_a < \zeta \\ (0, 1), & \delta_a = \zeta \\ (0, \frac{\zeta}{\delta_a}), & \delta_a > \zeta. \end{cases} \quad (14)$$

Proof: The proof is based on the separation principle (12) and the fact that all feasible operating points lie above the line $1 - \delta_a = \epsilon_a$. See details in Appendix C. $\square\square\square$

As seen from Proposition 2, randomized access policies are necessary to achieve optimality when $\delta_a \neq \zeta$. Moreover, Proposition 2 quantitatively characterizes the impact of the sensor performance δ_a on the optimal access strategy $(f_a^*(0), f_a^*(1))$. As illustrated in Fig. 4, the set $\mathbb{A}_\delta(a)$ of feasible sensor operating points can be partitioned into two regions: the “conservative” region ($\delta_a > \zeta$) and the “aggressive” region ($\delta_a < \zeta$). When $\delta_a > \zeta$, with high probability, the spectrum sensor detects a busy channel as idle (i.e., a miss-detection occurs). Hence, the access policy should be conservative to ensure that the collision probability is capped below ζ . Specifically, even when the sensing outcome $\Theta_a(t) = 1$ indicates an idle channel, the secondary user should only transmit with probability $\frac{\zeta}{\delta_a} < 1$. When the

channel is sensed as busy $\Theta_a(t) = 0$, the user should always refrain from transmission. On the other hand, when $\delta_a < \zeta$, the probability of false alarm is high; the spectrum sensor is likely to overlook an opportunity. Hence, the secondary user should adopt an aggressive access policy: always transmit when the channel is sensed as idle and transmit with probability $\frac{\zeta - \delta_a}{1 - \delta_a} > 0$ even when the sensing outcome indicates a busy channel. When $\delta_a = \zeta$, the access policy is to simply trust the sensing outcome, i.e., access if and only if the channel is sensed to be available $\Phi_a(t) = \Theta_a(t)$. We will show in Section IV-D that the splitting point $\delta_a = \zeta$ on the best ROC curve $P_{D,\max}^{(a)}$ is the optimal sensor operating point.

Similar to Proposition 2, we can quantitatively study the impact of the access strategy on the spectrum sensor design by solving (12) for the optimal sensor operating points when the transmission probabilities are given. This result is omitted to avoid unnecessary repetition. Details can be found in [25].

D. Optimal Joint Design of Spectrum Sensor and Access Policy

Optimizing (14) over all feasible sensor operating points, we obtain an explicit optimal design for the spectrum sensor and a closed-form deterministic optimal access policy in Theorem 2.

Theorem 2: For any chosen channel a in any slot, the optimal sensor should adopt the optimal NP detector with constraint $\delta_a^* = \zeta$ on the PM. Correspondingly, the optimal access policy is to trust the sensing outcome given by the spectrum sensor, i.e., $f_a^*(0) = 0$ and $f_a^*(1) = 1$.

Proof: The proof of Theorem 2 exploits the convexity of the set \mathbb{A}_δ of feasible sensor operating points, which follows directly from the concavity of the best ROC curve [23]. See Appendix D for details. $\square\square\square$

We find that the optimal sensor operating point coincides with the splitting point $\delta_a^* = \zeta$ of the “conservative” region and the “aggressive” region on the best ROC curve (see Fig. 4). This indicates that at $\delta_a^* = \zeta$, the best tradeoff between false alarm and miss-detection is achieved and the access policy does not need to be conservative or aggressive. We thus have a simple and deterministic optimal access policy: trust the sensing outcome. Summarized below are the properties of the optimal sensor operating and access policies given in Theorem 2.

Properties 1: The optimal spectrum sensor design and the optimal access policy are as follows.

P1.1 time-invariant and belief-independent.

P1.2 model-independent.

As a result of P1.1, the spectrum sensor can be configured off-line, and there is no need to calculate and store the optimal transmission probabilities, leading to significant reduction in both implementation complexity and memory requirement. The second property is that the optimal design of the spectrum sensor and the access strategy does not require the knowledge of the transition probabilities of the underlying Markov process. Since the probability of collision (11) is solely determined by the sensor operating and access policies, P1.2 indicates that the collision constraint on the joint OSA design can be ensured regardless of the accuracy of the Markovian model used by the secondary user. In other words, the primary network is not affected by the inaccurate model adopted by the secondary user. Model mismatch only affects the performance of the secondary user (see Fig. 8 for a simulation example).

E. Optimal Sensing Policy

As revealed by the separation principle, the optimal sensing policy can be obtained by solving an unconstrained POMDP with finite action space \mathbb{A}_s . Specifically, by applying the optimal spectrum sensor design and the optimal access policy given in Theorem 2 to (8), we simplify the optimality equation as shown in (15a)–(15b) at the bottom of the page. By applying $f_a^*(0) = 0$ and $f_a^*(1) = 1$ to (9), we obtain the conditional observation probability $U_a(k|\mathbf{s})$ as

$$U_a(1|\mathbf{s}) = s_a(1 - \epsilon_a^*), \quad U_a(0|\mathbf{s}) = 1 - U_a(1|\mathbf{s}) \quad (16)$$

where ϵ_a^* is the PFA associated with the PD $1 - \delta^* = 1 - \zeta$ on the best ROC curve $P_{D,\max}^{(a)}$. The updated belief vector $\mathcal{T}(\mathbf{\Lambda}(t) | a, k)$ can be obtained by using (10) with $U_a(k|\mathbf{s})$ replaced by $U_a(k|\mathbf{s})$ in (16).

It is shown in [3] that the value function of an unconstrained POMDP with finite action space is piece-wise linear and can be solved via linear programming. We can thus use the existing computationally efficient algorithms [4]–[6] to solve (8) for the optimal sensing policy.

Although myopic sensor operating and access policies are shown to be optimal for the joint design of OSA (see the separation principle), myopic sensing policy is suboptimal in general. Interestingly, it has been shown in [24] that, when the SOS evolves independently and identically across channels, the myopic sensing policy is optimal and has a simple and robust structure that obviates the need for knowing the transition probabili-

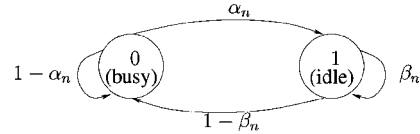


Fig. 5. The Markov channel model.

ties. When the channel occupancy states are correlated, the myopic approach can serve as a suboptimal solution with reduced complexity.

F. Numerical Examples

Here we provide numerical examples to study different factors that affect the optimal joint design of OSA. We consider $N = 3$ channels, each with bandwidth $B_n = 1$. While the separation principle applies to arbitrarily correlated SOS, we consider here the case where the SOS evolves independently but not identically across these three channels for simplicity. In this case, the SOS dynamics can be characterized by the transition probabilities $\boldsymbol{\alpha} \triangleq [\alpha_1, \alpha_2, \alpha_3]$ and $\boldsymbol{\beta} \triangleq [\beta_1, \beta_2, \beta_3]$, where α_n denotes the probability that channel n transits from state 0 (busy) to state 1 (idle), and β_n denotes the probability that channel n stays in state 1 (see Fig. 5). In all examples, the transition probabilities are given by $\boldsymbol{\alpha} = [0.2, 0.4, 0.6]$ and $\boldsymbol{\beta} = [0.8, 0.6, 0.4]$. The horizon length is $T = 10$ slots, and the maximum allowable probability of collision is $\zeta = 0.05$. We use the normalized overall throughput $V_1(\mathbf{\Lambda}(1))/T$, where $\mathbf{\Lambda}(1)$ is the stationary distribution of the SOS, to evaluate the performance of the optimal OSA design.

To illustrate the interaction between the PHY layer spectrum sensor and the MAC layer access policy, we consider a simple spectrum sensing scenario where the background noise and the primary signal are modeled as white Gaussian processes. Let $\sigma_{n,0}^2$ and $\sigma_{n,1}^2$ denote, respectively, the noise and the primary signal power in channel n . At the beginning of each slot, the spectrum sensor takes M independent measurements $\mathbf{Y}_n \triangleq [Y_{n,1}, \dots, Y_{n,M}]$ from chosen channel n and performs the following binary hypothesis test:

$$\begin{aligned} \mathcal{H}_0(S_n = 1) : \mathbf{Y}_n &\sim \mathcal{N}(\mathbf{0}_M, \sigma_{n,0}^2 \mathbf{I}_M) \\ \text{vs. } \mathcal{H}_1(S_n = 0) : \mathbf{Y}_n &\sim \mathcal{N}(\mathbf{0}_M, (\sigma_{n,1}^2 + \sigma_{n,0}^2) \mathbf{I}_M) \end{aligned} \quad (17)$$

where $\mathcal{N}(\mathbf{0}_M, \sigma^2 \mathbf{I}_M)$ denotes the M -dimensional Gaussian distribution with identical mean 0 and variance σ^2 in each dimension. An energy detector is optimal under the NP criterion [23, Sec. 2.6.2]

$$\|\mathbf{Y}_n\|_2 = \sum_{i=1}^M Y_{n,i}^2 \stackrel{\mathcal{H}_1}{\geq} \stackrel{\mathcal{H}_0}{\leq} \eta_m. \quad (18)$$

$$V_t(\mathbf{\Lambda}(t)) = \max_{a \in \mathbb{A}_s} \sum_{\mathbf{s} \in \mathcal{S}} \lambda_{\mathbf{s}}(t) \sum_{k=0}^1 U_a(k|\mathbf{s}) [kB_a + V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | a, k))], \quad 1 \leq t < T \quad (15a)$$

$$V_T(\mathbf{\Lambda}(T)) = \max_{a \in \mathbb{A}_s} \sum_{\mathbf{s} \in \mathcal{S}} \lambda_{\mathbf{s}}(T) U_a(1|\mathbf{s}) B_a. \quad (15b)$$

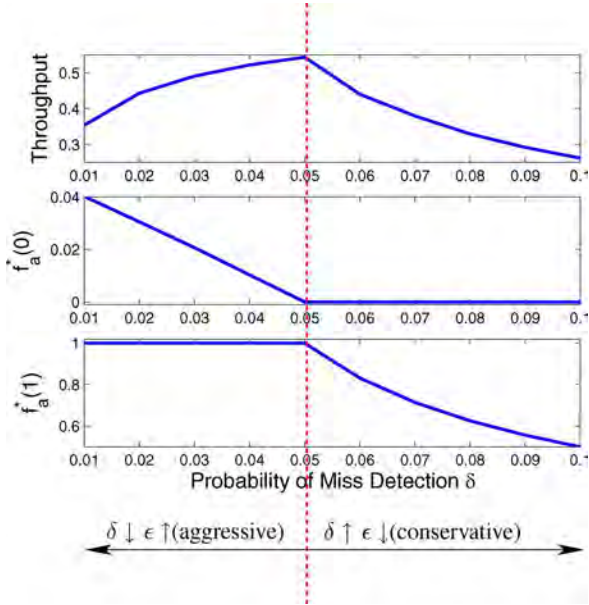


Fig. 6. The impact of sensor operating characteristics on the performance of the optimal OSA design.

The PFA and the PM of the energy detector are given by [23, Sec. 2.6.2]

$$\delta_n = \gamma\left(\frac{M}{2}, \frac{\eta_n}{2(\sigma_{n,0}^2 + \sigma_{n,1}^2)}\right), \quad \epsilon_n = 1 - \gamma\left(\frac{M}{2}, \frac{\eta_n}{2\sigma_{n,0}^2}\right) \quad (19)$$

where

$$\gamma(m, a) = \frac{1}{\Gamma(m)} \int_0^a t^{m-1} e^{-t} dt$$

is the incomplete gamma function. The optimal decision threshold η_n^* of the energy detector is chosen so that $\delta_n^* = \zeta$. Unless otherwise mentioned, we assume that $M = 10$, $\sigma_{n,0}^2 = \sigma_0^2 = 0$ dB, and $\sigma_{n,1}^2 = \sigma_1^2 = 5$ dB for all channels $n = 1, \dots, N$.

Impact of Sensor Operating Characteristics: Fig. 6 shows the impact of sensor operating characteristics on the secondary user's throughput and the optimal access policy. The upper graph plots the maximum normalized throughput $V_1(\mathbf{\Lambda}(1))/T$ versus the PM δ . The optimal transmission probabilities ($f_a^*(0), f_a^*(1)$) are shown in the middle and the lower graph, respectively. We can see that the maximum throughput is achieved at $\delta^* = \zeta = 0.05$ and the transmission probabilities change with δ as given by Theorem 2. Interestingly, the throughput curve is concave with respect to δ in the "aggressive" region ($\delta < \zeta$) and convex in the "conservative" region ($\delta > \zeta$). The performance thus decays at a faster rate when the sensor operating point drifts toward the "conservative" region. This suggests that miss-detections are more harmful to the OSA design than false alarms.

Impact of the Number of Channel Measurements: In this example, we study the tradeoff between the spectrum sensing time, which is determined by the number M of channel measurements taken by the spectrum sensor, and the transmission

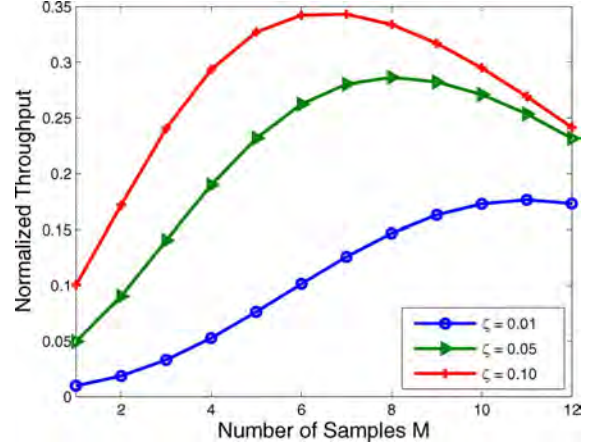


Fig. 7. The impact of the number of channel measurements on the performance of the optimal OSA design.

time. Taking more channel measurements can improve the fidelity of the sensing outcome but will reduce the data transmission time and hence the number of transmitted bits. We are thus motivated to study the throughput of the secondary user as a function of M for different maximum allowable probabilities of collision ζ . We assume that each channel measurement takes $c = 5\%$ of a slot time. The transmission time is thus given by $1 - Mc = 1 - 0.05M$. Assuming that the number of bits that can be transmitted by the secondary user is proportional to both the channel bandwidth and the transmission time, we modify the immediate reward (4) of the POMDP to $R(t) = (1 - Mc)K_a(t)B_a$.

Fig. 7 shows that the throughput of the secondary user increases and then decreases with the number M of channel measurements. Note that the PM is a function of the number M of channel measurements and the detection threshold η_a^* of the energy detector (as seen from (19)). When the PM is fixed to be $\delta_a^* = \zeta$ according to the separation principle, the detection threshold η_a^* increases with M , and hence the PFA ϵ_a^* decreases with M . As a consequence, when M is small, the throughput of the secondary user is limited by the large PFA. On the other hand, when M is large, the PFA is reduced at the expense of less transmission time in each slot, which also leads to low throughput. We observe that the optimal number M^* of channel measurements at which the throughput is maximized decreases with the maximum allowable collision probability ζ . The reason behind this observation is that the PM δ_a^* increases with ζ and hence fewer measurements are required to achieve the same PFA (as seen from (19)).

Impact of Mismatched Markov Model: In this example, we study the impact of mismatched Markovian models on the OSA performance. We assume that the true transition probabilities are given by α and β . The secondary user employs the optimal OSA design based on inaccurate transition probabilities α' and β' . In the upper half of Fig. 8, we plot the relative throughput loss as a function of the relative estimation error Ψ in transition probabilities, where $\Psi = \frac{\alpha'_n - \alpha_n}{\alpha_n} = \frac{\beta'_n - \beta_n}{\beta_n}$ ($n = 1, 2, 3$). Note that when $\Psi = 0$, the secondary user has perfect knowledge of the transition probabilities and hence achieves the maximum throughput. Inaccurate knowledge can cause performance loss.

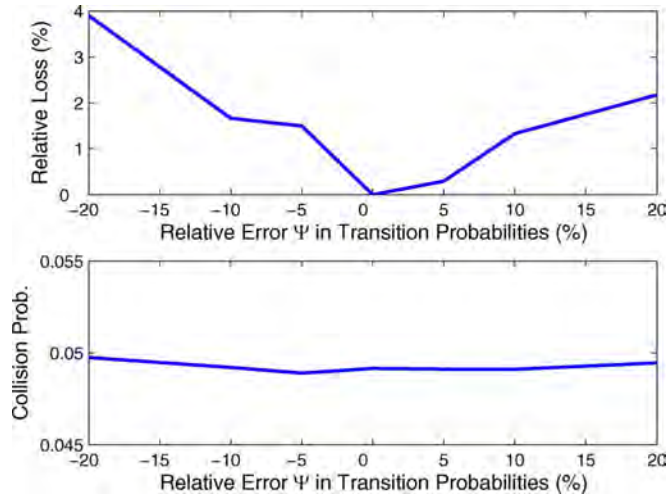


Fig. 8. The impact of mismatched Markov model on the performance of the optimal OSA strategy.

We observe that the relative throughput loss is below 4% even when the relative error is up to 20%. In the lower graph, we examine the probability of collision perceived by the primary network. We see that the probability of collision is not affected by inaccurate transition probabilities, which confirms P1.2.

V. OSA WITH MULTICHANNEL SENSING

In this section, we address the joint design of OSA in the case where multiple channels can be sensed and accessed simultaneously in each slot ($L > 1$). We focus on the extension of the separation principle developed in Section IV.

A. Optimal Joint Design

Within the POMDP framework presented in Section III, we first describe the three basic components of OSA with multichannel sensing and then derive the optimality equation.

Spectrum Sensor: Suppose that a set $\mathcal{A}(t) \subset \{1, \dots, N\}$ of channels is chosen in slot t , where $|\mathcal{A}(t)| = L \geq 1$. The spectrum sensor performs a 2^L -ary hypothesis test

$$\begin{aligned} \mathcal{H}_0 : \mathbf{S}_{\mathcal{A}(t)} &= [1, 1, \dots, 1], \\ \mathcal{H}_1 : \mathbf{S}_{\mathcal{A}(t)} &= [0, 1, \dots, 1], \\ &\vdots \\ \mathcal{H}_{2^L-1} : \mathbf{S}_{\mathcal{A}(t)} &= [0, 0, \dots, 0], \end{aligned} \quad (20)$$

where $\mathbf{S}_{\mathcal{A}(t)} \triangleq \{S_n(t)\}_{n \in \mathcal{A}(t)} \in \{0, 1\}^L$ denotes the occupancy states of the chosen channels $\mathcal{A}(t)$ in the current slot. The *a priori* probabilities of these hypotheses can be learned from the observation and decision history, which is characterized by the belief vector. For example, given current belief vector $\mathbf{\Lambda}(t)$ and chosen channels $\mathcal{A}(t)$, the *a priori* probability of \mathcal{H}_0 in this slot is given by

$$\Pr\{\mathcal{H}_0\} = \sum_{\mathbf{s} \in \mathcal{S}} \lambda_{\mathbf{s}}(t) \prod_{n \in \mathcal{A}(t)} 1_{[s_n=1]}. \quad (21)$$

This indicates that how sensor and access information at the MAC layer (captured by the belief $\mathbf{\Lambda}(t)$) can be used in the design of the spectrum sensor at the PHY layer.

Let $\Theta_{\mathcal{A}(t)} \triangleq \{\Theta_n(t)\}_{n \in \mathcal{A}(t)} \in \{0, 1\}^L$ denote the sensing outcomes. Sensing errors occur if the spectrum sensor mistakes one hypothesis for another, i.e., $\Theta_{\mathcal{A}(t)} \neq \mathbf{S}_{\mathcal{A}(t)}$. Since there are a total of 2^L hypotheses, the performance of the spectrum sensor can be specified by a set $\mathcal{E}(t)$ of $2^L(2^L - 1)$ error probabilities

$$\mathcal{E}(t) \triangleq \{\Pr\{\text{detect } \mathcal{H}_i \mid \mathcal{H}_j \text{ is true}\} : 0 \leq i, j \leq 2^L - 1, i \neq j\}. \quad (22)$$

The optimal design of the spectrum sensor should achieve a tradeoff among these $2^L(2^L - 1)$ error probabilities. Let $\mathbb{A}_{\delta}^{(L)}(\mathcal{A})$ include all sets of achievable error probabilities. A sensor operating policy specifies, in each slot t , a feasible sensor operating point (i.e., a set of achievable error probabilities) $\mathcal{E}(t) \in \mathbb{A}_{\delta}^{(L)}(\mathcal{A}(t))$ based on the current belief vector $\mathbf{\Lambda}(t)$ and the chosen channels $\mathcal{A}(t)$.

Sensing and Access Policies: At the beginning of each slot t , a sensing policy specifies a set

$$\mathcal{A}(t) \in \mathbb{A}_s^{(L)} \triangleq \{\mathcal{A} \subset \{1, \dots, N\}, |\mathcal{A}| = L\}$$

of channels to be sensed based on the current belief vector $\mathbf{\Lambda}(t)$. Based on $\mathbf{\Lambda}(t)$ and the imperfect sensing outcomes $\Theta_{\mathcal{A}(t)}$ given by the spectrum sensor, an access policy decides whether to access $\Phi_{\mathcal{A}(t)} \triangleq \{\Phi_n(t)\}_{n \in \mathcal{A}(t)} \in \{0, 1\}^L$. At the end of slot t , the receiver acknowledges every successful transmission. The acknowledgments are denoted by

$$\mathbf{K}_{\mathcal{A}(t)} \triangleq \{K_n(t)\}_{n \in \mathcal{A}(t)} \in \{0, 1\}^L$$

where $K_n(t) = S_n(t)\Phi_n(t)$. The immediate reward $R(t)$ is given by

$$R(t) = \sum_{n \in \mathcal{A}(t)} K_n(t) B_n. \quad (23)$$

Optimality Equation: Similar to Section III, we can formulate the optimal design of OSA with multichannel sensing as a constrained POMDP. We can also show that Proposition 1 holds, i.e., it is sufficient to consider deterministic sensor operating and sensing policies for the optimal design of OSA with multichannel sensing. Therefore, in each slot, the secondary user needs to make the following decisions: which set $\mathcal{A}(t) \in \mathbb{A}_s^{(L)}$ of channels to sense, which sensor operating point $\mathcal{E}(t) \in \mathbb{A}_{\delta}^{(L)}(\mathcal{A}(t))$ to choose, and which set $\mathcal{F}(t) \triangleq \{f_n(\boldsymbol{\theta}, t)\}$ of transmission probabilities to use, where

$$f_n(\boldsymbol{\theta}, t) = \Pr\{\Phi_n(t) = 1 \mid \Theta_{\mathcal{A}(t)} = \boldsymbol{\theta}\} \in [0, 1], \quad n \in \mathcal{A}(t), \boldsymbol{\theta} \in \{0, 1\}^L$$

is the probability of accessing chosen channel n given belief vector and sensing outcome $\Theta_{\mathcal{A}(t)} = \boldsymbol{\theta}$. The composite action space is denoted by

$$\mathbb{A}^{(L)} = \{\{\mathcal{A}, \mathcal{E}, \mathcal{F}\} : \mathcal{A} \in \mathbb{A}_s^{(L)}, \mathcal{E} \in \mathbb{A}_{\delta}^{(L)}(\mathcal{A}), \mathcal{F} \in [0, 1]^{L2^L}\}.$$

We can obtain the optimality equation and the design constraint as

$$\begin{aligned} &V_t(\mathbf{\Lambda}(t)) \\ &= \max_{\mathcal{A} \in \{\mathcal{A}, \mathcal{E}, \mathcal{F}\} \in \mathbb{A}^{(L)}} \sum_{\mathbf{s} \in \mathcal{S}} \lambda_{\mathbf{s}}(t) \sum_{\mathbf{k}_{\mathcal{A}} \in \{0, 1\}^L} U_A^{(L)}(\mathbf{k}_{\mathcal{A}} | \mathbf{s}) \\ &\quad \times [R(t) + V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | \mathcal{A}, \mathbf{k}_{\mathcal{A}}))], \quad 1 \leq t < T \end{aligned} \quad (24a)$$

$$\begin{aligned}
V_T(\Lambda(T)) &= \max_{A=\{\mathcal{A}, \mathcal{E}, \mathcal{F}\} \in \mathbb{A}(L)} \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) \sum_{\mathbf{k}_{\mathcal{A}} \in \{0,1\}^L} U_A^{(L)}(\mathbf{k}_{\mathcal{A}}|\mathbf{s}) R(t) \quad (24b) \\
\text{s.t. } P_n(t) &= \sum_{\theta_{\mathcal{A}}, \mathbf{s}_{\mathcal{A}} \in \{0,1\}^L} h_{\mathbf{s}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}|0) l_{\Theta_{\mathcal{A}}|\mathbf{S}_{\mathcal{A}}}(\theta_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}}) f_n(\theta_{\mathcal{A}}) \leq \zeta, \forall n, t \quad (24c)
\end{aligned}$$

where $\mathcal{F} \triangleq \{f_n(\theta)\}$ is a set of chosen transmission probabilities,

$$h_{\mathbf{s}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}|i) \triangleq \Pr\{\mathbf{S}_{\mathcal{A}}(t) = \mathbf{s}_{\mathcal{A}} | S_n(t) = i\}$$

is the conditional distribution of channel occupancy states $\mathbf{S}_{\mathcal{A}}(t)$ given current belief vector $\Lambda(t)$,

$$l_{\Theta_{\mathcal{A}}|\mathbf{S}_{\mathcal{A}}}(\theta_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}}) \triangleq \Pr\{\Theta_{\mathcal{A}}(t) = \theta_{\mathcal{A}} | \mathbf{S}_{\mathcal{A}}(t) = \mathbf{s}_{\mathcal{A}}\} \in \mathcal{E}$$

is the error probability determined by the current sensor operating point, and the conditional distribution $U_A^{(L)}(\mathbf{k}_{\mathcal{A}}|\mathbf{s})$ of observations $\mathbf{K}_{\mathcal{A}}(t)$ can be calculated as shown in (25) at the bottom of the page. The updated belief vector $\mathcal{T}(\Lambda(t) | A, \mathbf{k}_{\mathcal{A}})$ can be obtained by using (25) and (10).

In principle, the optimal decisions $\{\mathcal{A}^*, \mathcal{E}^*, \mathcal{F}^*\}$ in each slot can be obtained by solving (24) recursively. However, without any structural results on this constrained POMDP, (24) is computationally prohibitive. A natural question here is whether there exists a separation principle similar to Theorem 1 that can be used to simplify the optimal design of OSA with multichannel sensing.

B. Separation Principle

A general separation principle does not exist for the joint design of OSA with multichannel sensing. We show that under certain conditions, the separation principle established for the single-channel sensing case can be applied in the multichannel sensing scenarios.

Theorem 3: When the spectrum sensor and the access policy are designed independently across channels, the separation principle developed in Theorem 1 is valid for optimal OSA design with multichannel sensing. In this case, the optimal spectrum sensor adopts the optimal NP detector with PM equal to ζ and detects the occupancy of a chosen channel by using the measurements from this channel, and the optimal access decision on a chosen channel is to trust the sensing outcome from this channel. The optimal sensing policy can be obtained by solving an unconstrained POMDP.

Proof: The proof is built upon that of Theorem 1. See Appendix E. $\square\square\square$

We emphasize that the extension of the separation principle to multichannel sensing scenarios is based on the condition that the spectrum sensor and the access policy are designed independently across channels. Specifically, we assume that the occupancy of a channel is detected independently of the measurements taken from other channels and the access decision on a channel is made independently of the sensing outcomes from other channels. Intuitively, in this case, the design of spectrum sensor and access policy for the multichannel $L > 1$ sensing case can be treated as L independent design problems, one for each chosen channel. Hence, the optimal design for the single-channel case can be extended to $L > 1$.

Theorem 3 provides sufficient conditions under which the design given by the separation principle (referred to as the SP approach for simplicity) is optimal. In Proposition 3, we show that the SP approach is locally optimal (i.e., maximizes the instantaneous throughput) under certain relaxed conditions.

Proposition 3: Suppose that the spectrum sensor is designed independently across channels while the access policy jointly exploits the sensing outcomes from all channels. The SP approach is locally optimal when channels evolve independently.

Proof: See Appendix F. $\square\square\square$

It may seem plausible that the SP approach is (globally) optimal when channels evolve independently since in this case the sensing outcomes are independent across channels and independent access decisions seem to suffice. Interestingly, counter examples can be constructed to show that introducing correlation among access decisions across channels can improve the overall throughput. The rationale behind this is that the joint access design enables the secondary user to trade the immediate access to “bad” channels (e.g., channels with small bandwidth) for information on the occupancy states of “good” channels, leading to potentially more rewarding future decisions. Specifically, as noted in Section IV-B, the secondary user cannot distinguish a busy channel $S_n(t) = 0$ from the decision of no access $\Phi_n(t) = 0$ when observing $K_n(t) = 0$. However, if the access decision $\Phi_m(t)$ on channel $m \neq n$ is correlated with $\Phi_n(t)$, then we can infer the occupancy state of channel n from both $K_m(t)$ and $K_n(t)$. That is, by sacrificing the reward that can be obtained in channel m with small bandwidth, we can obtain more information on the occupancy state of channel n .

C. Heuristic Approaches to Exploiting Channel Correlation

While simplifying the design of OSA with multichannel sensing, the condition that the spectrum sensor and the access policy are designed independently across channels can cause throughput degradation since the correlation among channel occupancies is ignored. We propose two heuristic approaches

$$\begin{aligned}
U_A^{(L)}(\mathbf{k}_{\mathcal{A}}|\mathbf{s}) &\triangleq \Pr\{\mathbf{K}_{\mathcal{A}}(t) = \mathbf{k}_{\mathcal{A}} | \mathbf{S}(t) = \mathbf{s}\} \\
&= \sum_{\theta_{\mathcal{A}} \in \{0,1\}^L} l_{\Theta_{\mathcal{A}}|\mathbf{S}_{\mathcal{A}}}(\theta_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}}) \prod_{n \in \mathcal{A}} \Pr\{K_n(t) = k_n | \Theta_{\mathcal{A}}(t) = \theta_{\mathcal{A}}, \mathbf{S}_{\mathcal{A}}(t) = \mathbf{s}_{\mathcal{A}}\} \\
&= \sum_{\theta_{\mathcal{A}} \in \{0,1\}^L} l_{\Theta_{\mathcal{A}}|\mathbf{S}_{\mathcal{A}}}(\theta_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}}) \prod_{n \in \mathcal{A}} [k_n s_n f_n(\theta_{\mathcal{A}}) + (1 - k_n)(1 - s_n f_n(\theta_{\mathcal{A}}))]. \quad (25)
\end{aligned}$$

to exploit the channel correlation: one at the PHY layer and the other at the MAC layer.

1) *Exploiting Channel Correlation at the PHY Layer:* When the occupancy states are correlated across channels, we have correlated channel measurements at the PHY layer. Hence, the measurements at all chosen channels should be jointly exploited in spectrum opportunity identification. With this in mind, we propose a heuristic design of the spectrum sensor: it performs L binary hypothesis tests, one for each chosen channel, by using all channel measurements and adopting the optimal NP detector with PM equal to ζ . We point out that, differently from the SP sensor, the proposed spectrum sensor performs L composite hypothesis tests since it uses all channel measurements and the occupancy states of other channels are unknown in each hypothesis test. Hence, the structure of the optimal NP detector adopted by this heuristic sensor relies on the joint distribution of the channel occupancy states, which is given by the belief vector (see Section V-D for an example). That is, the spectrum sensor design is affected by the observation and decision history and thus varies with time. As illustrated in Fig. 9, the performance of this spectrum sensor improves over time, resulting from more informative distribution of the SOS obtained from accumulating observations. Note that the design of this spectrum sensor is much simpler than the 2^L -ary hypothesis test given in (20).

Based on the sensing outcomes given by this sensor that exploits measurements from all chosen channels, access decisions are made independently across channels, i.e., access if and only if a channel is sensed as idle. We refer this approach as the PHY layer approach.

Proposition 4: Suppose that the access policy is designed independently across channels while the spectrum sensor jointly exploits the measurements taken from all chosen channels. The PHY layer approach is locally optimal. When channels evolve independently, the PHY layer approach reduces to the SP approach.

Proof: See Appendix G. $\square\square\square$

Note that the PHY layer approach is locally optimal even when channels are correlated.

2) *Exploiting Channel Correlation at the MAC Layer:* When channel occupancies are correlated, so are the sensing outcomes given by the spectrum sensor. Hence, the channel correlation can also be exploited at the MAC layer by making access decisions jointly across channels. A heuristic MAC layer

approach is to adopt the spectrum sensor of the SP approach, i.e., to detect the occupancy state of a channel by using only the measurements of this channel, and then choose the access policy that exploits sensing outcomes from all chosen channels to maximize the instantaneous throughput. Specifically, for given chosen channels $\mathcal{A} \in \mathbb{A}_s$ and belief vector $\mathbf{\Lambda}(t)$ in slot t , we choose transmission probabilities $\hat{\mathcal{F}} = \{f_n(\boldsymbol{\theta}_{\mathcal{A}})\} \in [0, 1]^{L2^L}$ as shown in (26a)–(26c) at the bottom of the page, where the conditional probability $h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}|i)$ ($i = 0, 1$) of the current channel occupancies $\mathcal{S}_{\mathcal{A}}(t)$ and the sensing error probability $l_{\boldsymbol{\theta}_{\mathcal{A}}|\mathcal{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}})$ are defined below (24).

The access policy given in (26) can be obtained via linear programming. Proposition 5 shows that this MAC layer approach is equivalent to the SP approach when the SOS evolves independently across channels. This agrees with our intuition that when channels are independent, so are the sensing outcomes from the chosen channels. Hence, independent access decisions perform as well as the joint one in terms of instantaneous throughput.

Proposition 5: Suppose that the spectrum sensor is designed independently across channels while the access policy jointly exploits the sensing outcomes from all chosen channels. When channels evolve independently, the MAC layer approach reduces to the SP approach and hence is locally optimal.

Proof: See Appendix F. $\square\square\square$

D. Numerical Examples

Next, we study the performance of the SP, the PHY layer, and the MAC layer approaches. Note that these three approaches differ in the spectrum sensor and the access policy. We can employ any sensing policy to compare their performance. For simplicity, we consider a myopic sensing policy that chooses the set $\mathcal{A}(t)$ of channels to maximize the expected immediate reward that can be obtained in the absence of sensing errors, i.e., for given belief vector $\mathbf{\Lambda}(t)$ in slot t

$$\mathcal{A}(t) = \arg \max_{\mathcal{A} \in \mathbb{A}_s^{(L)}} \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\}. \quad (27)$$

We adopt the model of Gaussian noise and Gaussian primary signal described in Section IV-F. In this case, the spectrum sensor of the SP approach employs an energy detector given in (18). The detection threshold η_n of the energy detector is chosen so that the PM is fixed at ζ .

$$\hat{\mathcal{F}} = \arg \max_{\mathcal{F} \in [0,1]^{L2^L}} \mathbb{E}[R(t) | \mathbf{\Lambda}(t)] \quad (26a)$$

$$= \arg \max_{\mathcal{F} \in [0,1]^{L2^L}} \sum_{n \in \mathcal{A}} B_n \Pr\{\Phi_n(t) S_n(t) = 1\}$$

$$= \arg \max_{\mathcal{F} \in [0,1]^{L2^L}} \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\} \sum_{\boldsymbol{\theta}_{\mathcal{A}}, \mathbf{s}_{\mathcal{A}} \in \{0,1\}^L} h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}|1) l_{\boldsymbol{\theta}_{\mathcal{A}}|\mathcal{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}}) f_n(\boldsymbol{\theta}_{\mathcal{A}}) \quad (26b)$$

$$\text{s.t. } P_n(t) = \sum_{\boldsymbol{\theta}_{\mathcal{A}}, \mathbf{s}_{\mathcal{A}} \in \{0,1\}^L} h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}|0) l_{\boldsymbol{\theta}_{\mathcal{A}}|\mathcal{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}}|\mathbf{s}_{\mathcal{A}}) f_n(\boldsymbol{\theta}_{\mathcal{A}}) \leq \zeta, \quad \forall n \in \mathcal{A} \quad (26c)$$

Using the measurements $\{\mathbf{Y}_n\}_{n \in \mathcal{A}}$ from all chosen channels, the sensor employed by the PHY layer approach performs a composite hypothesis test for each chosen channel n

$$\begin{aligned} \mathcal{H}_0(S_n(t)=1) : \\ \mathbf{Y}_n &\sim \mathcal{N}(\mathbf{0}_M, \sigma_{n,0}^2 \mathbf{I}_M), \\ \mathbf{Y}_m &\sim \mathcal{N}(\mathbf{0}_M, (\sigma_{m,0}^2 + 1_{[S_m(t)=0]} \sigma_{m,1}^2) \mathbf{I}_M), \forall m \in \mathcal{A}(t) \setminus \{n\} \\ \mathcal{H}_1(S_n(t)=0) : \\ \mathbf{Y}_n &\sim \mathcal{N}(\mathbf{0}_M, (\sigma_{n,1}^2 + \sigma_{n,0}^2) \mathbf{I}_M), \\ \mathbf{Y}_m &\sim \mathcal{N}(\mathbf{0}_M, (\sigma_{m,0}^2 + 1_{[S_m(t)=0]} \sigma_{m,1}^2) \mathbf{I}_M), \forall m \in \mathcal{A}(t) \setminus \{n\}. \end{aligned} \quad (28)$$

Note that the distribution of the measurements under each hypothesis depends on the distribution of the current channel occupancy states $\mathcal{S}_{\mathcal{A}}(t) = \{S_n(t)\}_{n \in \mathcal{A}}$, which is given by $h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | i)$ (defined below (24)) and can be calculated from the current belief vector $\Lambda(t)$. In this case, the optimal NP detector for (28) is given by a likelihood ratio test [23, Sec. 2.5]

$$\frac{\sum_{\mathbf{s}_{\mathcal{A}} \in \{0,1\}^L} h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | 0) \prod_{m \in \mathcal{A}} p(\mathbf{Y}_m | S_m = s_m)}{\sum_{\mathbf{s}_{\mathcal{A}} \in \{0,1\}^L} h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | 1) \prod_{m \in \mathcal{A}} p(\mathbf{Y}_m | S_m = s_m)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau_n \quad (29)$$

where $h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | 0) = 0$ when $s_n \neq 0$ and $p(\mathbf{Y}_n | S_n = s_n)$ is the pdf of independent Gaussian channel measurements \mathbf{Y}_n

$$\begin{aligned} p(\mathbf{Y}_n | S_n = s_n) \\ = \prod_{i=1}^M \frac{1}{\sqrt{2\pi(\sigma_{n,0}^2 + 1_{[s_n=0]} \sigma_{n,1}^2)}} e^{-\frac{Y_{n,i}^2}{2(\sigma_{n,0}^2 + 1_{[s_n=0]} \sigma_{n,1}^2)}}. \end{aligned} \quad (30)$$

Note that when channel occupancies are independent, the above sensor employed by the PHY layer approach is equivalent to that of the SP approach, which demonstrates Proposition 4. The PFA and the PM of this sensor can be evaluated via simulation. In each slot, the detection threshold τ_n is chosen according to the belief vector so that the resulting PM is fixed at ζ , i.e., the design of the spectrum sensor varies with time.

As proven in Propositions 3–5, the PHY layer and the MAC layer approaches are equivalent to the SP approach when channels evolve independently. We thus compare below the performance of these three approaches in correlated channels. Specifically, we consider $N = 4$ correlated channels, each with bandwidth $B_n = 1$. The transition probabilities of the SOS are given by

$$\begin{aligned} P([0111] | [0000]) &= 0.6 \\ P([0000] | [0000]) &= 0.4 \\ P([0000] | [0111]) &= P([0000] | [1011]) \\ &= P([0000] | [1101]) \\ &= P([0000] | [1110]) = 0.2 \end{aligned}$$

and

$$\begin{aligned} P([1011] | [0111]) &= P([1101] | [1011]) \\ &= P([1110] | [1101]) \\ &= P([0111] | [1110]) = 0.8. \end{aligned}$$

The maximum allowable probability of collision is assumed to be $\zeta = 0.05$. In each slot, $L = 3$ channels are chosen. The

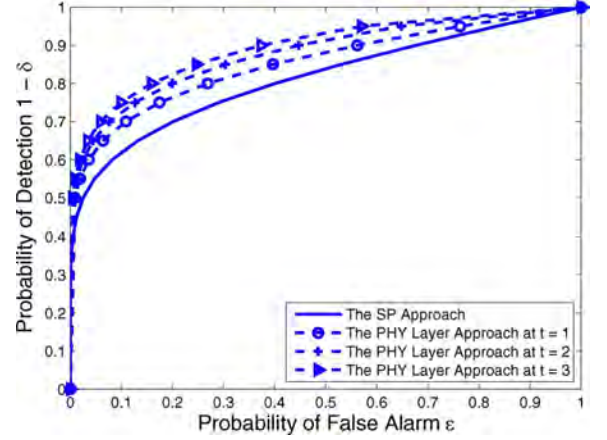


Fig. 9. Comparison of ROC curves.

spectrum sensor takes $M = 1$ measurement at each chosen channel, and the noise and the primary signal powers are given by $\sigma_{n,0}^2 = 0$ dB and $\sigma_{n,1}^2 = 10$ dB for all n .

Comparison of Sensor Performance: In Fig. 9, we plot the ROC curves ($1 - \delta_n$ versus ϵ_n) of the SP sensor and the sensor employed by the PHY layer approach. Note that the sensor employed by the MAC layer approach is the same as the SP sensor. We see that the sensor of the PHY approach outperforms that of the SP sensor. Specifically, for a fixed PM, the PFA of the sensor employed by the PHY approach is smaller than that of the SP sensor. This is because the sensor of the PHY approach exploits the correlation among channel measurements in detection while the SP sensor uses measurements from a single channel. We also observe that the ROC curve of the sensor of the PHY approach improves over time while that of the SP sensor remains the same. This observation can be explained by comparing the optimal detectors (18) and (29). Clearly, the energy detector (18) used by the SP sensor is static and so is its performance. As seen from (29), the decision variable of the sensor of the PHY approach depends on the conditional distribution $h_{\mathcal{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | i)$ of the channel occupancies, which varies with time according to the belief vector. As time t increases, the belief vector provides more information on the SOS due to the accumulating observations, leading to improved sensor performance. Fig. 9 demonstrates that the performance of the spectrum sensor can be improved by incorporating the sensing and access decisions at the MAC layer, which are encoded in the belief vector.

Comparison of Throughput Performance: In Fig. 10, we compare the throughput of these three approaches. As expected, the SP approach, which ignores the channel correlation, performs the worst. By jointly exploiting the sensing outcomes in access decision-making, the MAC layer approach can improve throughput performance. A much larger performance gain is achieved by the PHY layer approach which jointly exploits the channel measurements in spectrum opportunity identification. We can thus see that exploiting channel correlation at the PHY layer is more effective than that at the MAC layer. In other words, independent opportunity identification at the PHY layer hurts the throughput more than independent access decision-making at the MAC layer. This agrees with our intuition because independent opportunity identification makes hard decisions on whether the channel is idle. The correlation among

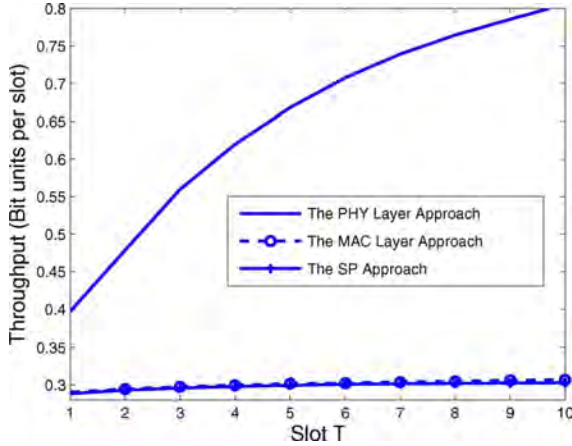


Fig. 10. Comparison of normalized throughput (bit units per slot).

the resulting sensing outcomes is less informative than that in the original channel measurements, leading to throughput degradation.

VI. CONCLUSION

Unique challenges in the design of OSA networks arise from the tension between the secondary users’ desire for performance and the primary users’ need for protection. Such tension dictates the interaction between opportunity identification at the physical layer and opportunity exploitation at the MAC layer, and a cross-layer approach is necessary to achieve optimality.

In this paper, we have developed a POMDP framework that captures basic components and design tradeoffs in OSA. We have shown that, surprisingly, there exists a separation principle in the optimal joint design of OSA that circumvents the curse of dimensionality in general POMDPs. Being able to obtain the optimal joint design in closed form allows us to characterize quantitatively the interaction between the physical and MAC layers. In particular, we have demonstrated how sensing errors at the PHY layer affect MAC design and how incorporating MAC layer information into physical layer leads to a *cognitive* spectrum sensor whose performance improves over time by learning from accumulating observations.

We have not taken into account the interactions among secondary users. The design of multiuser sensing strategies is addressed in [26], where perfect sensing is assumed. The POMDP framework has also been extended in [27] to address the joint design of OSA in unslotted primary networks.

APPENDIX A PROOF OF PROPOSITION 1

We first prove the existence of a deterministic optimal sensor operating policy. Suppose that channel n is chosen in the current slot. Let $\omega : \mathbb{A}_\delta(n) \rightarrow [0, 1]$ be an arbitrary pdf on the set $\mathbb{A}_\delta(n)$ of feasible sensor operating points, i.e.,

$\int_{(\epsilon, \delta) \in \mathbb{A}_\delta(n)} \omega(\epsilon, \delta) d\epsilon d\delta = 1$. We can compute the resulting PFA ϵ_n and the PD $1 - \delta_n$ as

$$\epsilon_n = \mathbb{E}[\epsilon] = \int_{(\epsilon, \delta) \in \mathbb{A}_\delta(n)} \epsilon \omega(\epsilon, \delta) d\epsilon d\delta, \tag{31a}$$

$$1 - \delta_n = \mathbb{E}[1 - \delta] = \int_{(\epsilon, \delta) \in \mathbb{A}_\delta(n)} (1 - \delta) \omega(\epsilon, \delta) d\epsilon d\delta. \tag{31b}$$

Since $0 \leq \epsilon \leq 1 - \delta \leq P_{D, \max}^{(n)}(\epsilon)$ for every sensor operating point in $\mathbb{A}_\delta(n)$, we have

$$0 \leq \epsilon_n \leq 1 - \delta_n \leq \int_{(\epsilon, \delta) \in \mathbb{A}_\delta(n)} P_{D, \max}^{(n)}(\epsilon) \omega(\epsilon, \delta) d\epsilon d\delta. \tag{32}$$

Since the best ROC curve $P_{D, \max}^{(n)}$ is concave, we have

$$\mathbb{E}[P_{D, \max}^{(n)}(\epsilon)] \leq P_{D, \max}^{(n)}(\mathbb{E}[\epsilon])$$

and hence

$$0 \leq \epsilon_n \leq 1 - \delta_n \leq P_{D, \max}^{(n)}(\epsilon_n).$$

That is, the resulting PFA and PM (ϵ_n, δ_n) of any randomized sensor operating policy ω belongs to the set $\mathbb{A}_\delta(n)$. Therefore, it is sufficient to consider deterministic sensor operating policies.

The spectrum sensor and the access policy should ensure that the collision constraint is satisfied no matter which channel is chosen. Let v_n denote the maximum expected remaining reward when channel n is chosen in the current slot. Then, the deterministic sensing policy that chooses channel $n^* = \arg \max_{n \in \mathbb{A}_s} v_n$ in this slot is optimal since the maximum expected remaining reward that can be achieved by a randomized sensing policy is $\sum_{n \in \mathbb{A}_s} v_n \mu(n) \leq v_{n^*}$, where $\mu : \mathbb{A}_s \rightarrow [0, 1]$ is a pmf on the set \mathbb{A}_s .

APPENDIX B PROOF OF THEOREM 1

The proof of the separation principle is built upon the following three lemmas. For ease of presentation, we define $Q_t(\mathbf{\Lambda}(t) | A)$ as the maximum expected remaining reward that can be obtained starting from slot t given that the current belief vector is $\mathbf{\Lambda}(t)$ and action $A = \{a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))\} \in \mathbb{A}$ is taken in this slot, i.e.,

$$Q_t(\mathbf{\Lambda}(t) | A) = \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) \sum_{k=0}^1 U_A(k | \mathbf{s}) [k B_a + V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, k))]. \tag{33}$$

Let

$$A \triangleq \{a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))\} \in \mathbb{A}$$

and

$$A' \triangleq \{a, (\epsilon'_a, \delta'_a), (f'_a(0), f'_a(1))\} \in \mathbb{A}$$

be two actions with the same channel selection but different sensor operating points and transmission probabilities.

Lemma 1: The value function given in (8) is convex in the belief vector. Specifically, at any time t , the value functions $V_t(\mathbf{\Lambda}_1(t))$ and $V_t(\mathbf{\Lambda}_2(t))$ of any two belief vectors $\mathbf{\Lambda}_1(t)$ and $\mathbf{\Lambda}_2(t)$ satisfy

$$V_t(\tau\mathbf{\Lambda}_1(t) + (1-\tau)\mathbf{\Lambda}_2(t)) \leq \tau V_t(\mathbf{\Lambda}_1(t)) + (1-\tau)V_t(\mathbf{\Lambda}_2(t)),$$

where $0 \leq \tau \leq 1$. (34)

Proof: We use mathematical induction. From the value function given in (8b), we can see that $V_T(\mathbf{\Lambda}(t))$ in the last slot $t = T$ is linear and hence convex in the belief vector $\mathbf{\Lambda}(t)$. Suppose that $V_t(\mathbf{\Lambda}(t))$ is convex for every slot $t > t_0$. By the definition of convex functions, we can show that the maximum remaining reward $Q_t(\mathbf{\Lambda}(t) | A)$ under an action $A \in \mathbb{A}$ is convex. Since the maximum of a set of convex functions is convex, the value function $V_{t_0}(\mathbf{\Lambda}(t))$ in slot $t = t_0$ is convex and Lemma 1 follows. $\square\square\square$

Lemma 2: If acknowledgment $K_a(t) = 1$ is observed in a slot t , then the expected future reward, given by the value function $V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, 1))$, is independent of the sensor operating point (ϵ_a, δ_a) and the transmission probabilities $(f_a(0), f_a(1))$ employed in the current slot. That is

$$V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, 1)) = V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A', 1)). \quad (35)$$

Proof: Applying the conditional observation probability $U_A(1|\mathbf{s})$ given in (9) to (10), we obtain the updated belief vector $\mathbf{\Lambda}^1(t+1) \triangleq \mathcal{T}(\mathbf{\Lambda}(t) | A, 1)$ whose element $\lambda_{\mathbf{s}}^1(t+1)$ is given by

$$\lambda_{\mathbf{s}}^1(t+1) = \frac{\sum_{\mathbf{s}' \in \mathbb{S}} \lambda_{\mathbf{s}'}(t) P(\mathbf{s}|\mathbf{s}') s'_a}{\sum_{\mathbf{s}' \in \mathbb{S}} \lambda_{\mathbf{s}'}(t) s'_a} \quad (36)$$

which is independent of the sensor operating point (ϵ_a, δ_a) and the transmission probabilities $(f_a(0), f_a(1))$. $\square\square\square$

Lemma 3: In any slot t , the future rewards $V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, k))$ and $V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A', k))$ satisfy the following inequality:

$$V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, 0)) \leq \tau V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, 1)) + (1-\tau)V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A', 0)) \quad (37)$$

where τ is given by

$$\tau = \frac{\sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) [U_A(0|\mathbf{s}) - U_{A'}(0|\mathbf{s})]}{\sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) U_A(0|\mathbf{s})}. \quad (38)$$

Proof: Applying the conditional observation probability $U_A(k|\mathbf{s})$ given in (9) to (10), we can obtain the updated belief vectors $\mathcal{T}(\mathbf{\Lambda}(t) | A, k)$ and $\mathcal{T}(\mathbf{\Lambda}(t) | A', k)$. After some algebras, we reach the following equality:

$$\mathcal{T}(\mathbf{\Lambda}(t) | A, 0) = \tau \mathcal{T}(\mathbf{\Lambda}(t) | A, 1) + (1-\tau) \mathcal{T}(\mathbf{\Lambda}(t) | A', 0) \quad (39)$$

where τ is given by (38). Lemma 3 follows from the convexity of the value function proven in Lemma 1. $\square\square\square$

With the above three lemmas, we now prove the separation principle. First notice that the expected immediate reward $\mathbb{E}[R(t) | \mathbf{\Lambda}(t)]$ can be obtained as

$$\begin{aligned} \mathbb{E}[R(t) | \mathbf{\Lambda}(t)] &= B_a \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) U_A(1|\mathbf{s}) \\ &= [\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)] B_a \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) s_a. \end{aligned} \quad (40)$$

Since $B_a \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) s_a$ is a constant for given belief vector $\mathbf{\Lambda}(t)$ and sensing action a , the expected immediate reward $\mathbb{E}[R(t) | \mathbf{\Lambda}(t)]$ increases with quantity $\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)$.

Second, we note that the sensor operating point (ϵ_a, δ_a) and the transmission probabilities $(f_a(0), f_a(1))$ only affect the expected remaining reward $Q_t(\mathbf{\Lambda}(t) | A)$ defined in (33) through the observation probability $U_A(1|\mathbf{s}) = s_a [\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)]$. Therefore, if we can show that $Q_t(\mathbf{\Lambda}(t) | A)$ increases with the quantity $\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)$, then this will prove the separation principle.

To this end, we consider two actions A and A' such that $\epsilon'_a f'_a(0) + (1-\epsilon'_a) f'_a(1) \geq \epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)$ in slot t . Comparing the resulting maximum expected remaining rewards $Q_t(\mathbf{\Lambda}(t) | A')$ and $Q_t(\mathbf{\Lambda}(t) | A)$, we have

$$\begin{aligned} &Q_t(\mathbf{\Lambda}(t) | A') - Q_t(\mathbf{\Lambda}(t) | A) \\ &= \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) \{B_a [U_{A'}(1|\mathbf{s}) - U_A(1|\mathbf{s})] \\ &\quad + \sum_{k=0}^1 [U_{A'}(k|\mathbf{s}) V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A', k)) \\ &\quad - U_A(k|\mathbf{s}) V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, k))]\} \\ &\geq \sum_{\mathbf{s} \in \mathbb{S}} \lambda_{\mathbf{s}}(t) \sum_{k=0}^1 [U_{A'}(k|\mathbf{s}) V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A', k)) \\ &\quad - U_A(k|\mathbf{s}) V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) | A, k))]. \end{aligned} \quad (41)$$

Applying Lemmas 2 and 3, we obtain after some algebra

$$Q_t(\mathbf{\Lambda}(t) | A') - Q_t(\mathbf{\Lambda}(t) | A) \geq 0 \quad (42)$$

which proves the monotonicity of the expected remaining reward $Q_t(\mathbf{\Lambda}(t) | A)$ with $\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)$ and hence completes the proof of the separation principle.

APPENDIX C PROOF OF PROPOSITION 2

When $\delta_a = 1$, we have $\epsilon_a = 0$ and the objective function $\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1)$ given in (12a) is maximized when $f_a^*(1) = 1$. When $\delta_a \in [0, 1)$, the constraint given in (12) can be written as

$$0 \leq f_a(0) \leq \frac{\zeta - \delta_a f_a(1)}{1 - \delta_a}. \quad (43)$$

Applying (43) to the objective function in (12a), we obtain that

$$\epsilon_a f_a(0) + (1-\epsilon_a) f_a(1) \leq f_a(1) \left[1 - \frac{\epsilon_a}{1 - \delta_a} \right] + \frac{\epsilon_a \zeta}{1 - \delta_a} \quad (44)$$

where the equality holds when $f_a(0) = \frac{\zeta - \delta_a f_a(1)}{1 - \delta_a}$. Since $1 - \delta_a \geq \epsilon_a$ (see footnote 2), the right-hand side of (44) increases with $f_a(1)$. Hence, to maximize the objective function $\epsilon_a f_a(0) + (1 - \epsilon_a) f_a(1)$, we should choose the largest $f_a(1)$ such that $f_a(0) = \frac{\zeta - \delta_a f_a(1)}{1 - \delta_a} \geq 0$ (see (43)). Therefore, when $\delta_a \leq \zeta$, $f_a^*(1) = 1$ and, correspondingly, $f_a^*(0) = \frac{\zeta - \delta_a}{1 - \delta_a}$. When $\delta_a \geq \zeta$, $f_a^*(1) = \frac{\zeta}{\delta_a}$ and, correspondingly, $f_a^*(0) = 0$.

APPENDIX D PROOF OF THEOREM 2

Applying the optimal transmission probabilities $(f_a^*(0), f_a^*(1))$ given in Proposition 2 to the objective function (12a), we obtain that

$$\epsilon_a f_a(0) + (1 - \epsilon_a) f_a(1) = \begin{cases} 1 - \frac{\epsilon_a}{1 - \delta_a} (1 - \zeta), & \delta_a \leq \zeta \\ \frac{1 - \epsilon_a}{\delta_a} \zeta, & \delta_a \geq \zeta. \end{cases} \quad (45)$$

Since the best ROC curve is concave [23, Sec. 2.2], both $\frac{\epsilon_a}{1 - \delta_a}$ and $\frac{1 - \epsilon_a}{\delta_a}$ increase with ϵ_a and hence decrease with δ_a . From (45), we can see that the objective function $\epsilon_a f_a(0) + (1 - \epsilon_a) f_a(1)$ increases with δ_a when $\delta_a \leq \zeta$, but decreases when $\delta_a \geq \zeta$. Hence, the maximum is achieved when $\delta_a^* = \zeta$. Correspondingly, the optimal transmission probabilities $(f_a^*(0), f_a^*(1))$ are given by $(0, 1)$.

APPENDIX E PROOF OF THEOREM 3

Let $A^{(L)} \triangleq \{\mathcal{A}, \{(\epsilon_n, \delta_n)\}_{n \in \mathcal{A}}, \{(f_n(0), f_n(1))\}_{n \in \mathcal{A}}\} \in \mathbb{A}^{(L)}$ denote a joint composite action taken in a slot t and $A_n \triangleq \{n, (\epsilon_n, \delta_n), (f_n(0), f_n(1))\} \in \mathbb{A}$ denote the corresponding actions taken on each individual chosen channel $n \in \mathcal{A}$. When the spectrum sensor is designed independently across channels, we can write

$$\begin{aligned} l_{\Theta_{\mathcal{A}} | \mathcal{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}} | \mathbf{s}_{\mathcal{A}}) &= \Pr\{\Theta_{\mathcal{A}}(t) = \boldsymbol{\theta}_{\mathcal{A}} | \mathcal{S}_{\mathcal{A}}(t) = \mathbf{s}_{\mathcal{A}}\} \\ &= \prod_{n \in \mathcal{A}} \Pr\{\Theta_n(t) = \theta_n | S_n(t) = s_n\} \end{aligned}$$

in a product form since the occupancy of a channel is detected independently of the measurements at other chosen channels. When the access policy is designed independently across channels, we have $f_n(\boldsymbol{\theta}_{\mathcal{A}}) = f_n(\theta_n)$ for all sensing outcomes $\boldsymbol{\theta}_{\mathcal{A}} \in$

$\{0, 1\}^L$. Therefore, we can write the conditional observation probability $U_{\mathcal{A}}^{(L)}(\mathbf{k}_{\mathcal{A}} | \mathbf{s})$ as (46), shown at the bottom of the page. Similarly, after some algebras, the design constraint in (24c) can be written as (47), also shown at the bottom of the page.

Applying (46) to (24), we can see that the sensor operating point (ϵ_n, δ_n) and transmission probabilities $(f_n(0), f_n(1))$ of a chosen channel $n \in \mathcal{A}$ affect the maximum remaining reward only through $U_{A_n}(1 | \mathbf{s}) = s_n[\epsilon_n f_n(0) + (1 - \epsilon_n) f_n(1)]$, which is independent of the actions $\{A_m\}_{m \in \mathcal{A} \setminus \{n\}}$ taken on the other channels. Moreover, the simplified constraint (47) reveals that the collision probability of a channel n is also independent of the actions $\{A_m\}_{m \in \mathcal{A} \setminus \{n\}}$ taken at other channels. Therefore, the design of the sensor operating and access policies can be decoupled across channels. Following the same proof as given in Appendix B, we can show that the expected remaining reward increases with $\epsilon_n f_n(0) + (1 - \epsilon_n) f_n(1)$ of every chosen channel $n \in \mathcal{A}$.

On the other hand, the expected immediate reward $\mathbb{E}[R(t) | \boldsymbol{\Lambda}(t)]$ is given by

$$\begin{aligned} \mathbb{E}[R(t) | \boldsymbol{\Lambda}(t)] &= \sum_{n \in \mathcal{A}} B_n \Pr\{K_n(t) = 1\} \\ &= \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\} [\epsilon_n f_n(0) + (1 - \epsilon_n) f_n(1)] \quad (48) \end{aligned}$$

which also increases with $\epsilon_n f_n(0) + (1 - \epsilon_n) f_n(1)$. Therefore, the separation principle developed in Theorem 1 holds for $L > 1$.

APPENDIX F PROOF OF PROPOSITIONS 3 AND 5

Let $\mathcal{A} \in \mathbb{A}_{\mathcal{S}}^{(L)}$ denote a set of chosen channels and $\mathcal{A}_n^- = \mathcal{A} \setminus \{n\}$ be all the set of chosen channels excluding n . Since channels evolve independently, we have $h_{\mathcal{S}_{\mathcal{A}_n^-} | S_n}(\mathbf{s}^- | 0) = h_{\mathcal{S}_{\mathcal{A}_n^-} | S_n}(\mathbf{s}^- | 1)$, where $\mathbf{s}^- \triangleq \{s_m\}_{m \in \mathcal{A}_n^-}$ and

$$h_{\mathcal{S}_{\mathcal{A}_n^-} | S_n}(\mathbf{s}^- | i) = \Pr\{\mathcal{S}_{\mathcal{A}_n^-}(t) = \mathbf{s}^- | S_n(t) = i\}.$$

Hence, given belief vector $\boldsymbol{\Lambda}(t)$ and chosen channels \mathcal{A} in slot t , the myopic (i.e., locally optimal) sensor operating point $(\hat{\epsilon}_n, \hat{\delta}_n)$ and transmission probabilities $\hat{\mathcal{F}} = \{\hat{f}_n(\boldsymbol{\theta}_{\mathcal{A}})\}$ are given by

$$\begin{aligned} U_{\mathcal{A}}^{(L)}(\mathbf{k}_{\mathcal{A}} | \mathbf{s}) &= \sum_{\boldsymbol{\theta}_{\mathcal{A}} \in \{0, 1\}^L} \prod_{n \in \mathcal{A}} \Pr\{\Theta_n(t) = \theta_n | S_n(t) = s_n\} [k_n s_n f_n(\theta_n) + (1 - k_n)(1 - s_n f_n(\theta_n))] \\ &= \prod_{n \in \mathcal{A}} \sum_{\theta_n=0}^1 \Pr\{\Theta_n(t) = \theta_n | S_n = s_n\} [k_n s_n f_n(\theta_n) + (1 - k_n)(1 - s_n f_n(\theta_n))] \\ &= \prod_{n \in \mathcal{A}} U_{A_n}(k_n | \mathbf{s}). \quad (46) \end{aligned}$$

$$P_n(t) = \sum_{\theta_n=0}^1 \Pr\{\Theta_n(t) = \theta_n | S_n(t) = 0\} f_n(\theta_n) = (1 - \delta_n) f_n(0) + \delta_n f_n(1) \leq \zeta, \quad \forall n \in \mathcal{A}. \quad (47)$$

$$\begin{aligned}
\{(\hat{\epsilon}_n, \hat{\delta}_n), \hat{\mathcal{F}}\} &= \arg \max_{\substack{(\epsilon_n, \delta_n) \in \mathbb{A}_\delta(n) \\ \mathcal{F} \in [0,1]^{L2L}}} \mathbb{E}[R(t) | \mathbf{\Lambda}(t)] \\
&= \arg \max_{\substack{(\epsilon_n, \delta_n) \in \mathbb{A}_\delta(n) \\ \mathcal{F} \in [0,1]^{L2L}}} \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\} \sum_{\theta_n=0}^1 \Pr\{\Theta_n(t) = \theta_n | S_n(t) = 1\} g_{\theta_n}^n \\
&= \arg \max_{\substack{(\epsilon_n, \delta_n) \in \mathbb{A}_\delta(n) \\ \mathcal{F} \in [0,1]^{L2L}}} \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\} [\epsilon_n g_0^n + (1 - \epsilon_n) g_1^n]
\end{aligned} \tag{49a}$$

$$\begin{aligned}
\text{s.t. } P_n(t) &= \sum_{\theta_n=0}^1 \Pr\{\Theta_n(t) = \theta_n | S_n(t) = 0\} g_{\theta_n}^n \\
&= (1 - \delta_n) g_0^n + \delta_n g_1^n \leq \zeta, \quad \forall n \in \mathcal{A}
\end{aligned} \tag{49b}$$

$$g_n(\theta_n) \triangleq \sum_{\boldsymbol{\theta}^- \in \{0,1\}^{L-1}} f_n(\boldsymbol{\theta}^-, \theta_n) \sum_{\mathbf{s}^- \in \{0,1\}^{L-1}} \Pr\{\mathcal{S}_{\mathcal{A}_n^-} = \mathbf{s}^-\} \prod_{m \in \mathcal{A}_n^-} \Pr\{\Theta_m(t) = \theta_m | S_m(t) = s_m\}, \tag{50}$$

$$\{\hat{\mathcal{E}}, \{(\hat{f}_n(0), \hat{f}_n(1))\}_{n \in \mathcal{A}}\} \tag{51a}$$

$$\begin{aligned}
&= \arg \max_{\substack{\mathcal{E} \in \mathbb{A}_\delta^{(L)} \\ f_n(0), f_n(1) \in [0,1]}} \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\} [\Pr\{\Theta_n(t) = 1 | S_n(t) = 1\} f_n(1) \\
&\quad + \Pr\{\Theta_n(t) = 0 | S_n(t) = 1\} f_n(0)]
\end{aligned} \tag{51b}$$

$$\text{s.t. } P_n(t) = \Pr\{\Theta_n(t) = 1 | S_n(t) = 0\} f_n(1) + \Pr\{\Theta_n(t) = 0 | S_n(t) = 0\} f_n(0) \leq \zeta, \quad \forall n \in \mathcal{A}, \tag{51c}$$

$$\begin{aligned}
&\Pr\{\Theta_n(t) = \theta_n | S_n(t) = s_n\} \\
&= \sum_{\boldsymbol{\theta}^-, \mathbf{s}^- \in \{0,1\}^{L-1}} \Pr\{\boldsymbol{\Theta}_{\mathcal{A}_n^-}(t) = \boldsymbol{\theta}^-, \Theta_n(t) = \theta_n | \mathcal{S}_{\mathcal{A}_n^-}(t) = \mathbf{s}^-, S_n(t) = s_n\} \Pr\{\mathcal{S}_{\mathcal{A}_n^-}(t) = \mathbf{s}^- | S_n(t) = s_n\}
\end{aligned} \tag{52}$$

(26) as shown in (49a)–(49b) at the top of the page, where $g(\theta_n) \in [0, 1]$ is defined as in (50) also at the top of the page, where $\boldsymbol{\theta}^- \triangleq \{\theta_m\}_{m \in \mathcal{A}_n^-}$. We see from (49) that the myopic approach should maximize $\epsilon_n g_n(0) + (1 - \epsilon_n) g_n(1)$ under the constraint $(1 - \delta_n) g_n(0) + \delta_n g_n(1) \leq \zeta$ for every chosen channel $n \in \mathcal{A}$, leading to the same optimization problem as (12). By Theorem 2, $\hat{\delta}_n = \zeta$ and $(\hat{g}_n(0), \hat{g}_n(1)) = (0, 1)$ are the solution to (49). That is, the SP sensor is locally optimal. Furthermore, since $(\hat{g}_n(0), \hat{g}_n(1)) = (0, 1)$ is achieved by choosing $\hat{f}_n(\boldsymbol{\theta}^-, \theta_n) = 1_{[\theta_n=1]}$ in (50), transmission probabilities $\hat{f}_n(\boldsymbol{\theta}_{\mathcal{A}}) = \theta_n$ are locally optimal, which completes the proof of Proposition 3.

Proposition 5 follows directly from the fact that the MAC layer approach employs the myopic access policy and the SP sensor, which has been proven to be locally optimal.

APPENDIX G

PROOF OF PROPOSITION 4

When the access policy is designed independently across channels, we have $f_n(\boldsymbol{\theta}_{\mathcal{A}}) = f_n(\theta_n)$ for any sensing outcome $\boldsymbol{\Theta}_{\mathcal{A}}(t) = \boldsymbol{\theta}_{\mathcal{A}}$ from chosen channels \mathcal{A} . Hence, given belief vector $\mathbf{\Lambda}(t)$ and chosen channels \mathcal{A} in slot t , the myopic spectrum sensor $\hat{\mathcal{E}}$ and access decisions $\{(\hat{f}_n(0), \hat{f}_n(1))\}_{n \in \mathcal{A}}$ are

given by (51a)–(51c) at the top of the page, where (52), also at the top of the page, is determined by the sensor operating point $\mathcal{E} \in \mathbb{A}_\delta^{(L)}$ and the current belief vector $\mathbf{\Lambda}(t)$ (see Appendix F for notation definitions). Since (51) has the same form as (12), the PHY layer approach is locally optimal.

Furthermore, when the SOS evolves independently across channels, the measurements from different channels are independent. Hence, the sensor employed by the PHY layer approach is equivalent to the SP sensor.

REFERENCES

- [1] J. Mitola, "Cognitive radio for flexible mobile multimedia communications," in *Proc. IEEE Int. Workshop on Mobile Multimedia Communications (MoMuC)*, San Diego, CA, Nov. 1999, pp. 3–10.
- [2] *The Next Generation (XG) Program*, Std., DARPA [Online]. Available: <http://www.darpa.mil/sto/smallunitops/xg.html>
- [3] R. D. Smallwood and E. J. Sondik, "The optimal control of partially observable Markov processes over a finite horizon," *Oper. Res.*, vol. 21, pp. 1071–1088, 1973.
- [4] E. J. Sondik, "The Optimal Control of Partially Observable Markov Processes," Ph.D. dissertation, Stanford Univ., Stanford, CA, 1971.
- [5] G. E. Monahan, "A survey of partially observable Markov decision processes: Theory, models, and algorithms," *Manag. Sci.*, vol. 28, no. 1, pp. 1–16, Jan. 1982.
- [6] H.-T. Cheng, "Algorithms for Partially Observable Markov Decision Processes," Ph.D. dissertation, Univ. British Columbia, Vancouver, BC, Canada, 1988.

- [7] H. Zheng and C. Peng, "Collaboration and fairness in opportunistic spectrum access," in *Proc. IEEE Int. Conf. Communications*, Seoul, Korea, May 2005, vol. 5, pp. 3132–3136.
- [8] W. Wang and X. Liu, "List-coloring based channel allocation for open-spectrum wireless networks," in *Proc. of IEEE 62nd Vehicular Technology Conf. (VTC-Fall)*, Dallas, TX, Sep. 2005, vol. 1, pp. 690–694.
- [9] M. E. Steenstrup, "Opportunistic use of radio-frequency spectrum: A network perspective," in *Proc. 1st IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, MD, Nov. 2005, pp. 638–641.
- [10] P. Papadimitratos, S. Sankaranarayanan, and A. Mishra, "A bandwidth sharing approach to improve licensed spectrum utilization," *IEEE Commun. Mag.*, vol. 43, pp. 10–14, Dec. 2005.
- [11] Q. Zhao, L. Tong, and A. Swami, "Decentralized cognitive MAC for dynamic spectrum access," in *Proc. 1st IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, MD, Nov. 2005, pp. 224–232.
- [12] Y. Chen, Q. Zhao, and A. Swami, "Distributed cognitive MAC for energy-constrained opportunistic spectrum access," in *Proc. MILCOM*, Washington, DC, Oct. 2006, pp. 1–7.
- [13] Q. C. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Optimal dynamic spectrum access via periodic channel sensing," in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, Hong Kong, Mar. 2007, pp. 33–37.
- [14] S. Geirhofer, L. Tong, and B. M. Sadler, "Dynamic spectrum access in WLAN channels: Empirical model and its stochastic analysis," in *Proc. 1st Int. Workshop on Technology and Policy in Accessing Spectrum (TAPAS)*, Boston, MA, Aug. 2006.
- [15] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Proc. Allerton Conf. Communication, Control, and Computing*, Monticello, IL, Oct. 2004.
- [16] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. 38th Asilomar Conf. Signals, Systems, and Computers*, Asilomar, CA, 2004, pp. 772–776.
- [17] K. Challapali, S. Mangold, and Z. Zhong, "Spectrum agile radio: Detecting spectrum opportunities," in *Proc. Int. Symp. Advanced Radio Technologies*, Boulder, CO, Mar. 2004.
- [18] B. Wild and K. Ramchandran, "Detecting primary receivers for cognitive radio applications," in *Proc. 1st IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Nov. 2005, pp. 124–130.
- [19] A. Ghasemi and E. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. 1st IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Nov. 2005, pp. 131–136.
- [20] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 589–600, Apr. 2007.
- [21] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, pp. 79–89, May 2007.
- [22] D. Aberdeen, "A Survey of Approximate Methods for Solving Partially Observable Markov Decision Processes," Tech. Rep. National ICT Australia, 2003 [Online]. Available: <http://users.rsise.anu.edu.au/~daa/papers.html>
- [23] H. L. V. Trees, *Detection, Estimation, and Modulation Theory, Part I*. New York: Wiley-Interscience, 2001.
- [24] Q. Zhao and B. Krishnamachari, "Structure and optimality of myopic sensing for opportunistic spectrum access," in *Proc. IEEE Workshop on Towards Cognition in Wireless Networks (CogNet)*, Glasgow, U.K., Jun. 2007.
- [25] Y. Chen, Q. Zhao, and A. Swami, "Joint PHY-MAC Layer Design for Opportunistic Spectrum Access in the Presence of Sensing Errors," technical report Univ. California, Davis, 2007, Tech. Rep. TR-07-02.
- [26] K. Liu, Q. Zhao, and Y. Chen, "Distributed sensing and access in cognitive radio networks," in *Proc. 10th Int. Symp. Spread Spectrum Techniques and Applications*, Bologna, Italy, Aug. 2008.
- [27] Q. Zhao and K. Liu, "Detecting, tracking, and exploiting spectrum opportunities in unslotted primary systems," in *Proc. IEEE Radio and Wireless Symp. (RWS)*, Orlando, FL, Jan. 2008.