Wireless Connectivity of Swarms in Presence of Obstacles

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Prescribed by ANSI Std Z39-18
Motivation

\[ \text{EDGE} = \text{Range} + \text{Line of Sight} \]
Problem Statement

Given:
- N mobile holonomic robots
- Workspace, $W$
- Initial positions, $q^{\text{init}}$
- Final Positions, $q^{\text{final}}$
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\[
\mathbf{\dot{q}_i = u_i}
\]
Obvious Infeasibility

Start and goal in different connected components of W

Cycles in different homoptic equivalence classes
## Related work

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_{Desai, Kumar, Fierro}_

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_{Spanos, Murray; Zavlanos Pappas, Bullo, Cortes, Notarstefano}_

_{Reynolds, Reif, Bishop, Tanner, Pappas, Moore, Jadabaie, Passiano, Olfati-Saber, Murray}_
Approach: Potential Functions

1. Range:
   \[ \phi (\text{cm}^2/\text{sec}) \]
   distance to other robot (cm)

2. Line of Sight:

3. Go To Goal:
   Navigation function
   [Rimon & Kodischek]
Addition of Potentials is Dangerous!

Range

Line of Sight

Go-to goal

Parallel Comp Algorithm
IF exists $\nabla \phi_i$ such that
$(\nabla \phi_i \times \nabla \phi_j) > 0 \lor (\nabla \phi_i \times \nabla \phi_j) < 0, \forall j \neq i$

Then select $\vec{V}$ so that
$- \left[ \frac{\partial \phi_j}{\partial x}, \frac{\partial \phi_j}{\partial y} \right] \cdot [v_x, v_y] \leq 0$

ELSE
Infeasible!
Discard some $\nabla \phi_i$

Low Level Control: Final Velocity
Parallel Composition
controller: concept

Goal Potential

[Esposito Kumar 2002]
Parallel Composition
controller: concept
Parallel Composition controller: concept
Parallel Composition controller: concept

Efficient: Computing directions is $O(P^2)$ (all pairs of cross products)

Complete: Generates solution if feasible. If infeasible, algorithm is conclusive.

Stability: Common Lyapunov function.
Validation
Completeness: Is the composition always feasible?
A Necessary Condition

Neighbors must select paths in same (straight line) homotopy class!

- A connected swarm cannot “split” an obstacle
- **No distributed, global solution !!!**
Conjecture: Feasible, iff initial conditions are not “split” by saddle stable manifolds

1. Any feasible path is a loop homotopic to trivial loop
2. must cross stable manifold an even number of times,
3. requires increasing potential function
Conjecture: Feasible, iff initial conditions are not "split" by saddle stable manifolds

1. Potential peaks in dimension along edge (range violated)
2. Sign of derivative transverse to edge changes >=2 times (LOS violated)
3. Turns out there is no local condition for a stable manifold? Future work....
Swarm Wireless Connectivity w/ Obstacles

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frame 1

frame 2

frame 3

frame 4

goal