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Erosional equivalences of levees: Steady and intermittent wave overtopping

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ABSTRACT

Present criteria for acceptable grass covered levee overtopping are based on average overtopping values but do not include the effect of overtopping duration. This paper applies experimental steady state results for acceptable overtopping to the case of intermittent wave overtopping. Laboratory results consisting of velocities and durations for acceptable land side levee erosion due to steady flows are examined to determine the physical basis for the erosion. Three bases are examined: (1) velocity above a threshold value, (2) shear stress above a threshold value, and (3) work above a threshold value. The work basis provides the best agreement with the data and a threshold work value and a work index representing the summation of the product of work above the threshold and time are developed. The governing equations for flow down the land side of a levee establish that the flows near the land side levee toe will be supercritical. Wave runup is considered to be Rayleigh distributed with the runup above the levee crest serving as a surrogate for overtopping. Two examples illustrating application of the methodology are presented. Example 1 considers three qualities of grass cover: good, average, and poor. The required levee elevations for these three covers differ by 1.8 m. The results for Example 1 are compared with the empirical criteria of 0.1 liters per second per meter (l/s per m), 1.0 l/s per m, and 10.0 l/s per m. It is found that the required crest elevation by the methodology recommended herein for the "poor" cover is only slightly lower than for the criterion for average overtopping of $q = 10.0$ l/s per m but significantly lower than for the overtopping criterion of 1.0 and 0.1 m/s per m. Example 2 considers two durations of the peak surge with the result that the longer duration peak surge requires a levee that is higher by approximately 0.8 m.

1. Introduction

Experience resulting from Hurricane Katrina has shown that land side levee erosion due to wave overtopping can significantly limit levee performance and survival (USACE, 2008a). The options to ensuring levee integrity due to wave overtopping include: (1) a sufficiently high crest elevation such that overtopping does not occur, (2) armoring the levee land side such that the levee can withstand large amounts of overtopping, and (3) establishing a levee elevation that will allow an overtopping quantity that is within the capability of the levee to withstand the associated limited erosion. Erosion is a time dependent process such that a levee can withstand various overtopping magnitudes for different durations. Present guidance for allowable overtopping considers only the average overtopping magnitude and does not recognize the role of duration. Although the specific interest may be in designing the levee for survival during a particular storm (say a 100 year event), there is also interest in the erosional potential during storms that will cause greater overtopping. As computer capabilities progress in representing hurricane induced storm surges, there is a need to improve understanding of the overtopping erosion potential and to provide associated guidance for more rational design.

This paper develops and recommends preliminary equivalences of cumulative combinations of various overtopping quantities and the associated durations that represent the same level of levee erosion hazard. Methodologies are based on experimental results of steady flows on the land side of a levee and are limited to cases in which the mean water level is below the levee crest.

2. Brief literature review

Due to the consequences of wave-induced levee overtopping, considerable research has been conducted encompassing field, laboratory, and numerical approaches. These have led to the

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14. ABSTRACT
Present criteriaforacceptablegrasscoveredleveeovertoppingarebasedonaverageovertoppingvalues but donotincludetheeffectofovertoppingduration.Thispaperappliesexperimentalsteadystate results foracceptableovertoppingtothecaseofintermittentwaveovertopping.Laboratoryresults consistingofvelocitiesanddurationsforacceptablelandsideleveeerosionduetosteadyflowsare examinedtodeterminethephysicalbasisfortheerosion.Threebasesareexamined:(1)velocityabovea threshold value,(2)shearstressaboveathresholdvalue,and(3)workaboveathresholdvalue.The work basisprovidesthebestagreementwiththedataandathresholdworkvalueandaworkindex representingthesummationoftheproductofworkabovethethresholdandtimearedeveloped.The governingequationsforflowdownthelandsideofaleveeestablishthattheflowsnearthelandside levee toewillbesupercritical.WaverunupisconsideredtobeRayleighdistributedwiththerunup abovetheleveecrestservingasasurrogateforovertopping.Twoexamplesillustratingapplicationof the methodologyarepresented.Example1considersthreequalitiesofgrasscover:good,average,and poor.Therequiredleveeelevationsforthesethreecoversdifferby1.8m.TheresultsforExample1are comparedwiththeempiricalcriteriaof0.1literspersecondpermeter(l/sperm),1.0l/sperm,and 10.0l/sperm.Itisfoundthattherequiredcrestelevationbythemethodologyrecommendedhereinfor the ??poor??coverisonlyslightlylowerthanforthecriterionforaverageovertoppingof q ? 10.0l/sper m butsignificantlylowerthanfortheovertoppingcriterionof1.0and0.1m/sperm.Example2 considerstwodurationsofthepeaksurgewiththeresultthatthelongerdurationpeaksurgerequiresa levee thatishigherbyapproximately0.8m.

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development of guidelines represented by Van der Meer (2002, hereafter referred to as the TAW Manual) and the more recent EurOtop Manual authored by Pullen et al. (2007). The Coastal Engineering Manual by the US Army Corps of Engineers (2008b) provides similar guidance. Review of the many relevant individual overtopping contributions is beyond the scope of this paper. In addition to the three manuals noted above, numerical and experimental contributions of Van Gent (2006), Schüttrumpf et al. (2002), Schüttrumpf (2003), Schüttrumpf and Oumeraci (2005), and Van der Meer et al. (2006) have led to a substantially improved understanding of overtopping mechanics. More recently, two approaches have advanced design capabilities of land side grassed slopes to resist various levels of wave overtopping. These include the full scale controlled tests of overtopping to evaluate the erosional resistance of actual levee surfaces, Van der Meer et al. (2006), Hoffmans et al. (2008), Van der Meer (2008), and Akkerman et al. (2007). Second, Young (2005) has investigated the detailed mechanics of grasses within the soil and the characteristics which contribute to soil strength. With particular reference to the main objective of the present paper, Schüttrumpf (2003) has presented an example illustrating the need to consider wave overtopping duration.

3. Governing hydraulic equation

Although steady state considerations are clearly not fully representative of intermittent overtopping by waves, they should provide an approximate measure of the behavior of the overtopped volume on the land side of the levee. Referring to Fig. 1, the steady state momentum equation governing flow down the landward side of a levee is

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial z} + g\sin\alpha \tag{1}$$

in which ρ is the mass density of water, u velocity, g the gravitational constant, τ shear stress and the remainder of the terms are defined in Fig. 1 in which $W \equiv \rho gh\sin\alpha$. Integrating over depth, neglecting surface stresses and considering hydrostatic conditions, Eq. (1) can be reduced to

$$\frac{\partial h}{\partial x} = \frac{\sin\alpha - fu^2/8gh}{1 - Fr^2} \tag{2}$$

in which f is the Weisbach Darcy friction factor with the shear stress definition $\tau = \frac{1}{8}\rho f u^2$ and Fr is the Froude number defined as

$$Fr = \frac{u}{\sqrt{gh}} \tag{3}$$

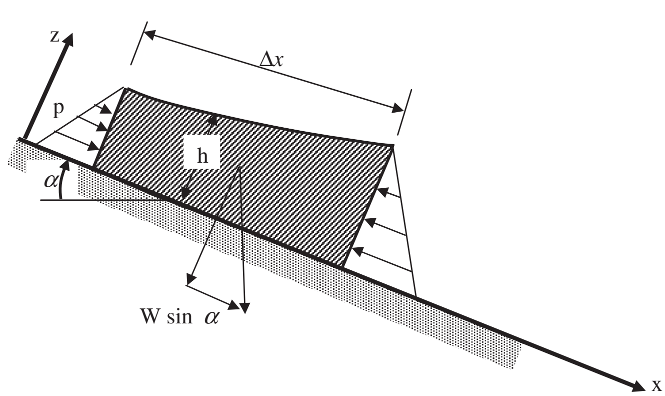

Fig. 1. Definition sketch.

Eq. (2) is the so-called backwater curve used in hydraulics to represent gradually varied flow and will be applied here to determine whether or not the water velocities will approximately approach their terminal or normal velocities while on the land side levee slope. The terminal or normal velocity, u_∞, is from Eq. (2)

$$u_\infty = \sqrt{\frac{8gh\sin\alpha}{f}} \tag{4}$$

or

$$u_\infty = \left(\frac{8gq\sin\alpha}{f}\right)^{1/3} \tag{5}$$

where $q = uh$ is the discharge per unit width and for steady flow is independent of the distance, x. It is noted that Eq. (4) is the same as developed by Schüttrumpf (2003) although the friction factor applied here is defined as four times Schüttrumpf's friction factor. Critical flow conditions are given by (e.g., Chow, 1988)

$$u_o = \sqrt{gh_o}$$
$$q = u_o h_o$$
$$h_o = \frac{q^{2/3}}{g^{1/3}} \tag{6}$$

The critical slope, α_c is given by

$$\alpha_c = \sin^{-1}\left(\frac{f}{8}\right) \tag{7}$$

For a Weisbach Darcy coefficient, $f = 0.08$, the critical slope, $\tan\alpha_c = 0.01$. Thus since levee slopes are more on the order 0.1–0.2, all terminal velocities down the land side slope will be supercritical although it is possible that the overtopping velocities at the levee crest will exceed terminal velocities in which case the velocities will decrease with distance down the landside slope.

4. Velocity transition distances on land side of levee

Eq. (2) was applied to determine the transition distances down the land side levee face for the following three discharges: $q = 50$ l/s per m, 100 l/s per m, and 200 l/s per m. These unit discharges are much greater than the average overtopping of $q = 0.1$ to 10.0 l/s per m considered as possible thresholds for acceptable erosion (discussed later); however with irregular waves, it is the more extreme overtopping that will cause the erosion. Additional variables included in these calculations were: $\tan\alpha = 1{:}6$, $f = 0.08$ and initial depth at the break in slope at the landward side of the levee crest of 0.9 times the critical depth.

Results are presented in Fig. 2 in which the variation of velocity with distance from the landward break in slope is shown along with the terminal velocity asymptotes. It is seen for these three discharges that the terminal velocities are nearly reached within approximate distances of typical land side levee dimensions. Thus, in later applications, it is suitable (somewhat conservative) to base erosion criteria on the terminal velocity as given by Eq. (5) rather than calculate the velocity at the landward toe of the levee.

5. Indices of potential land side levee erosion

Several indices of potential land side levee erosion could be defined, including: (1) velocity, (2) shear stress, and (3) work done on the land side of the levee. For purposes here, we examine these three indices to determine the most effective index in

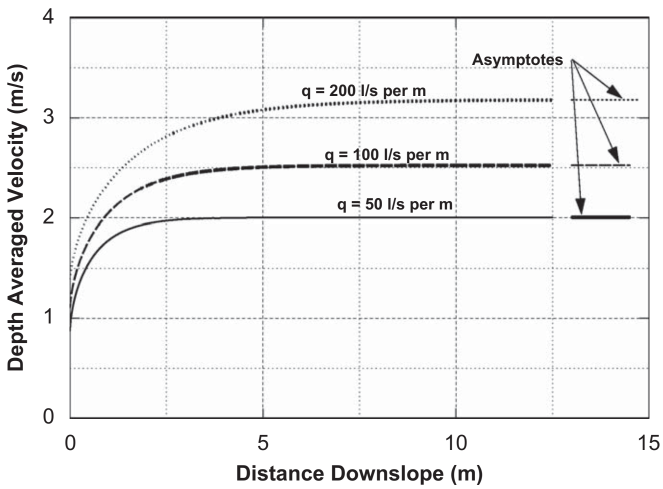

Fig. 2. Variation of depth averaged velocity vs. distance for three unit discharge values. Note these three cases commenced with 90% of critical depth at the landward side of the levee crest. Land side slope = 1:6.

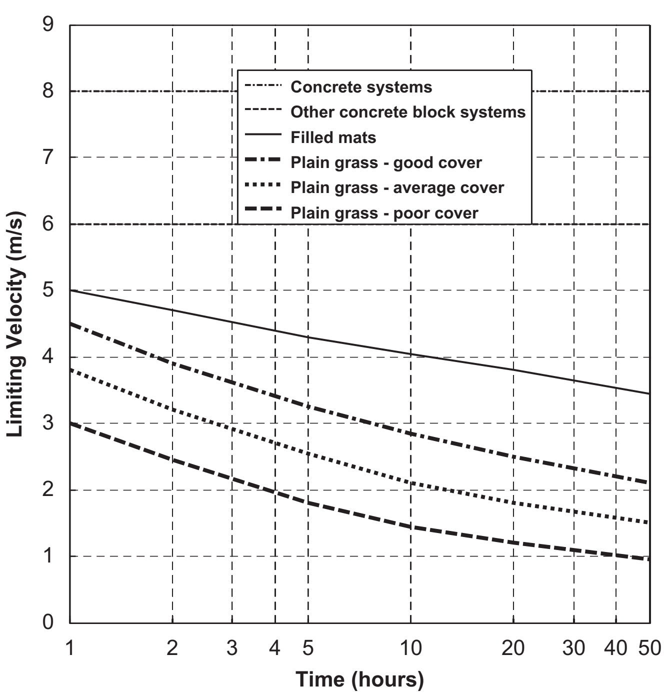

Fig. 3. Combinations of (velocities, durations) for acceptable erosion of various coverings due to steady overtopping (Hewlett et al., 1987) as reported in Hughes (2007).

representing land side levee erosion using steady state results from a CIRIA publication by Hewlett et al. (1987) as reported in Hughes (2007) and shown in Fig. 3. Hughes also notes that an earlier version of Fig. 3 was published by Whitehead et al. (1976). Fig. 3 is based on full scale cases. Van der Meer et al. (2006) state "CIRIA report 116 (Hewlett et al. 1987) has been reworked to wave overtopping in The Netherlands, but without validation." Thus, it appears that the Hewlett et al. results may play some role in the present European guidelines as presented in the TAW and EurOtop manuals. Schüttrumpf (2003) presents Fig. 3 with attribution to Seijffert and Verheij (1998). It is noted that Fig. 3 relates overtopping velocities and durations for various types of land side armoring and grass cover quality. For purposes here, we focus on the following three qualities of grass cover: "Poor", "Average", and "Good". The most appropriate index determined will be applied to develop a method for acceptable combinations of wave overtopping magnitudes and durations. For descriptions of the characters of these three grass covers and their definition of "acceptable erosion", see Hewlett et al. (1987).

Table 1
Pairs of velocities and durations to be used in evaluation of erosion indices (from Fig. 3).

Velocity (m/s)	Duration (h)		
	Plain Grass-Good Cover	Plain Grass-Average Cover	Plain Grass-Poor Cover
4.5	1	NA	NA
4.0	1.8	NA	NA
3.5	4	1.5	NA
3.0	8.5	2.5	1
2.5	20	5	1.9
2.0	50	14	3.8
1.5	NA	50	10
1.0	NA	NA	40

In summary, the three indices of potential erosion to be evaluated are: (1) velocity index, (2) shear stress index, and (3) work index. The data to be applied in this evaluation are represented by the curves "Plain Grass-Good Cover", Plain Grass-Average Cover", and "Plain Grass-Poor Cover" in Fig. 3; the associated combinations of velocity and duration are summarized in Table 1. In applying Hewlett's results, it is recognized that the effects of levee slope and sediment characteristics will affect erosion; however, for purposes here, these are considered as secondary factors.

5.1. Evaluation of effectiveness of erosional indices in representing acceptable land side levee erosion

As noted, three measures of erosion are considered: (1) velocity greater than a threshold velocity, u_c, (2) shear stress above a threshold shear stress, τ_c, and (3) work above a threshold work, W_c. Each of these is discussed below.

5.1.1. Erosion due to excess velocity

For this case, the acceptable erosion E_u due to an excess velocity over the critical value, $u_{c,u}$, is given by

$$E_u = K_u(u_{m,i} - u_{c,u})t_i, \quad u_{m,i} > u_{c,u} \tag{8}$$

where $u_{m,i}$ and t_i are the measured velocities and times in Table 1 and E_u, K_u are unknown coefficients and the critical velocity, $u_{c,u}$ is also an unknown . Eq. (8) can be written as

$$\frac{E_u}{K_u} = (u_{m,i} - u_{c,u})t_i, \quad u_{m,i} > u_{c,u} \tag{9}$$

where the two unknowns are now E_u/K_u and the critical velocity, $u_{c,u}$.

The general approach for all three erosion indices will be illustrated by detailed discussion of this velocity measure of erosion due to an excess velocity. If this measure were perfect, the error, ε_i, associated with each velocity value in Table 1 would be zero where the error is defined as

$$\varepsilon_i = \frac{E_u}{K_u} - (u_{m,i} - u_{c,u})t_i \tag{10}$$

which represents a rectangular hyperbola relationship between $(u_m - u_{c,u})$ and t_i. Applying the method of least squares for the case of "Plain Grass-Good Cover", we determine the two unknown variables E_u/K_u and $u_{c,u}$ such that the sum of squares of the errors is a minimum. Applying this procedure, we find $u_c = 1.93$ m/s and $E_u/K_u = 22.1 \times 10^3$ m.

In this case, the predicted velocity, $u_{p,I}$ for each data point is, from Eq. (10)

$$u_{u,p,i} = \left(\frac{E_u}{K_u t_i} + u_{c,u}\right) \tag{11}$$

and the standard error in the predicted velocities, $\sigma_{u,u}$, is defined as

$$\sigma_{u,u} = \sqrt{\frac{1}{(I-1)}\sum_{i=1}^{I}(u_{m,i} - u_{u,p,i})^2} \tag{12}$$

and I is the number of data points, in this case $I = 5$–6, see Table 1.

5.1.2. Erosion due to excess shear stress

For this case, the erosion, E_τ is given by

$$E_\tau = K_\tau(\tau_i - \tau_c)t_i = K_\tau \alpha_\tau (u_{m,i}^2 - u_{c,\tau}^2)t_i \tag{13}$$

where K_τ is a shear stress erosional coefficient, α_τ a grouping of terms that includes the mass density of water and a shear stress coefficient, and $u_{c,\tau}$ the threshold velocity associated with the shear stress. In this case, the predicted velocity, $u_{\tau,p,i}$ for each data point is given by

$$u_{\tau,p,i} = \left(\frac{E_\tau}{K_\tau \alpha_\tau t_i} + u_{c,\tau}^2\right)^{1/2} \tag{14}$$

and the standard error in the predicted velocities, $\sigma_{u,\tau}$, is defined by Eq. (12) as before

$$\sigma_{u,\tau} = \sqrt{\frac{1}{(I-1)}\sum_{i=1}^{I}(u_{m,i} - u_{\tau,p,i})^2} \tag{15}$$

in which $\sigma_{u,\tau}$ represents the mean square error in the velocities and $u_{m,i}$ and $u_{\tau,p,i}$ represent the measured (Table 1) and predicted velocities on the basis of the shear stress criterion for the ith value, respectively.

5.1.3. Erosion due to excess work

For this case, the erosion, E_W is given by

$$E_W = K_W(W_i - W_c)t_i = K_W \beta_W (u_{m,i}^3 - u_{c,W}^3)t_i \tag{16}$$

where K_W is a work erosional coefficient, β_W a grouping of terms that includes the mass density of water and a shear stress coefficient and $u_{c,W}$ the threshold velocity associated with the work threshold below which no erosion occurs. In this case, the predicted velocity, $u_{W,p,i}$ for each data point is given by

$$u_{W,p,i} = \left(\frac{E_W}{K_W \beta_W t_i} + u_{c,W}^3\right)^{1/3} \tag{17}$$

and the standard error in the predictions, $\sigma_{u,W}$, is defined by Eq. (12) as before. In which $\sigma_{u,W}$ represents the mean square error in the velocities and $u_{m,i}$ and $u_{W,p,i}$ represent the measured (Table 1) and predicted velocities on the basis of the work criterion for the ith value, respectively.

5.2. Quantification of erosion indices

The methodology described in Section 5.1 was applied to determine the limiting erosion quantities, for example $E_\tau/K_\tau\alpha_\tau$ and $u_{c,\tau}$ for the case of an excess shear stress governing the rates of erosion. These quantities were determined for the velocity index, shear stress index, and work index and for levee conditions of "Plain Grass-Good Cover", "Plain Grass-Average Cover", and "Plain Grass-Poor Cover" and the results are presented in Tables 2–4, respectively.

The standard errors in velocities for each of these three grass covers are presented in the last column of each of the tables representing the three grass cover qualities. Of interest is that the work index consistently has the lowest standard error by a significant amount, thereby indicating the superiority of this index. Fig. 4 presents a comparison of the predicted and measured velocities associated with the three indices for Hewlett's measurements for the case of Grass Cover-Good Quality. It is seen that all three indices appear to provide equally good fits for the lower velocities; however, the work index provides a substantially better fit for the higher velocities.

The utility of the method developed is that it provides a basis for accounting for the cumulative effects of variable erosion events. Based on the finding that the cumulative work is more appropriate than the two other indices considered, it appears (perhaps somewhat intuitively), that the cumulative work within acceptable limits can be accomplished by any velocity above the critical value for the particular grass cover. This is the working hypothesis of the method developed and applied herein. For

Table 2
Threshold velocities, erosion limits, and velocity errors for three indices considered Plain Grass-Good Cover.

Index	Threshold velocity (m/s)	Erosion limit	Standard error in velocities (m/s)
Velocity basis	$u_{c,u} = 1.93$	$E_u/K_u = 22.10 \times 10^3$ m	1.70
Shear stress basis	$u_{c,\tau} = 1.88$	$E_\tau/K_\tau\alpha_\tau = 0.118 \times 10^6$ m^2/s	0.76
Work basis	$u_{c,W} = 1.80$	$E_W/K_W\beta_W = 0.492 \times 10^6$ m^3/s^2	0.38

Table 3
Threshold velocities, erosion limits, and velocity errors for three indices considered Plain Grass-Average Cover.

Index	Threshold velocity (m/s)	Erosion limit	Standard error in velocities (m/s)
Velocity basis	$u_{c,u} = 1.43$	$E_u/K_u = 15.55 \times 10^3$ m	0.98
Shear stress basis	$u_{c,\tau} = 1.39$	$E_\tau/K_\tau\alpha_\tau = 0.067 \times 10^6$ m^2/s	0.38
Work basis	$u_{c,W} = 1.30$	$E_W/K_W\beta_W = 0.229 \times 10^6$ m^2/s^2	0.12

Table 4
Threshold velocities, erosion limits, and velocity errors for three indices considered Plain Grass-Poor Cover.

Index	Threshold velocity (m/s)	Erosion limit	Standard error in velocities (m/s)
Velocity basis	$u_{c,u} = 0.94$	$E_u/K_u = 13.01 \times 10^3$ m	0.81
Shear stress basis	$u_{c,\tau} = 0.89$	$E_\tau/K_\tau\alpha_\tau = 0.0408 \times 10^6$ m^2/s	0.26
Work basis	$u_{c,W} = 0.76$	$E_W/K_W\beta_W = 0.103 \times 10^6$ m^2/s^2	0.04

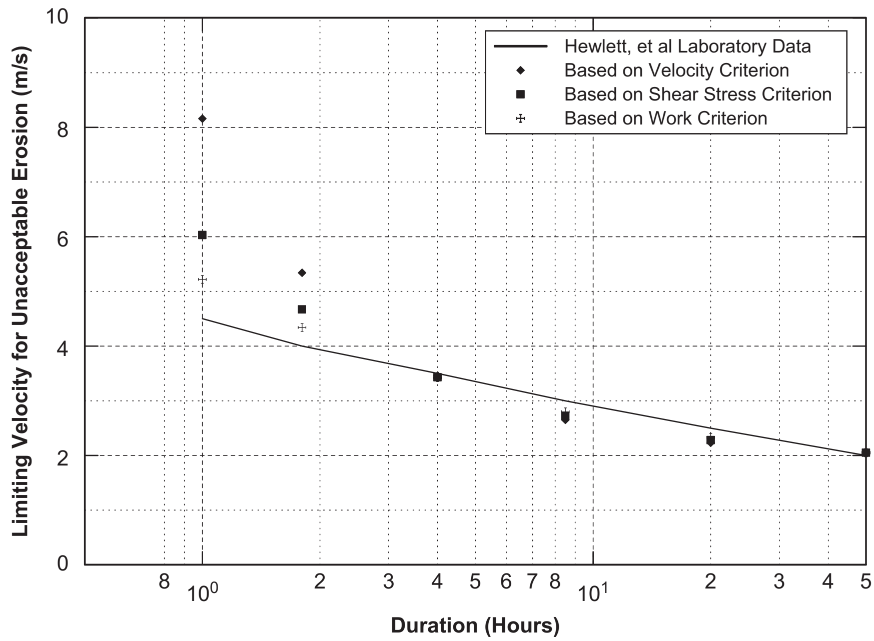

Fig. 4. Comparison of fits of three erosion indices results to Hewlett's measurements for "Plain Grass-Good Cover".

example, if N overtopping events, each of velocity $u_n > u_{c,W}$ and duration, Δt_n occurred, the criterion for acceptable erosion for Plain Grass-Good Cover would be

$$\sum_{n=1}^{N}(u_n^3 - u_{c,W}^3)\Delta t_n \le E_W/K_W\beta_W \le 0.492\times10^6\ \mathrm{m^3/s^2} \tag{18}$$

and $u_{c,W} = 1.80\ \mathrm{m/s}$.

6. Application of critical work index to wave-induced land side levee erosion considerations

Results presented in the previous section support an index based on work as the most appropriate for erosion considerations. Section 2 has shown that for typical levee conditions it is appropriate (somewhat conservative) to consider the so-called terminal velocity, u_∞, as the reference velocity given by Eq. (5) and repeated below

$$u_\infty = \left(\frac{8gq\sin\theta}{f}\right)^{1/3} \tag{19}$$

It is seen that this velocity is proportional to the overtopping discharge to the one-third power which is somewhat surprising and serendipitous since the best erosion index has been shown in the previous section to be proportional to the cube of the velocity or, as shown by Eq. (19), to the discharge to the first power. Thus, Eq. (18) can be rewritten as

$$\sum_{n=1}^{N}\left(\frac{8g\sin\theta q_n}{f} - u_{c,W}^3\right)\Delta t_n \le E_W/K_W\beta_W, \quad q_n > \left(\frac{f u_{c,W}^3}{8g\sin\theta}\right) \tag{20}$$

for erosion to be within the acceptable range.

6.1. Levee overtopping estimates

Because it is necessary to account for only those overtopping events when the overtopping work done on the levee exceeds the threshold, the values of the individual overtopping rates are required, i.e., the rates based on the durations that the individual overtopping events occur. This section develops a relationship between individual overtopping events and individual runup events.

6.1.1. Wave runup

Wave runup has been shown to be approximately Rayleigh distributed (e.g., Walton, 1999; Hedges and Mase, 2004). The wave runup can be expressed in terms of the significant wave height at the toe of the slope, H_{mo}, the Iribarren number, ξ_o, and various coefficients relating to the roughness and other levee characteristics (TAW, 2002). The potential runup is based on the planar levee surface extended upwards without limit, see Fig. 5. The usual runup for design purposes is the 2% runup, i.e., exceeded by only 2% of all runup values. With the 2% runup available and with the consideration that runup is Rayleigh distributed, the complete potential runup distribution is established.

De Waal and Van der Meer (1992) have developed an empirical relationship between the 2% excess runup and average overtopping. However, there is no direct conversion from individual runup to individual wave overtopping as required herein. For purposes here, we will develop a correlation between cumulative excess runup and overtopping which allows this conversion.[1] The runup values are first determined for those runup events that exceed the levee crest elevation. To develop the correlation, we require this cumulative excess runup to be proportional to the total average overtopping as determined by the empirical TAW overtopping methodology (later expressed as a coefficient, K_R). This is a critical step as it ensures that the average overtopping as predicted by the method proposed herein equals the average overtopping as determined by the empirical TAW methodology. In the sections below, the runup distribution will first be examined followed by development of the coefficient relating excess runup to average overtopping.

[1] It is noted that other approaches to determining individual overtopping events can be developed and/or applied such as an empirical overtopping probability distribution presented in TAW (2002).

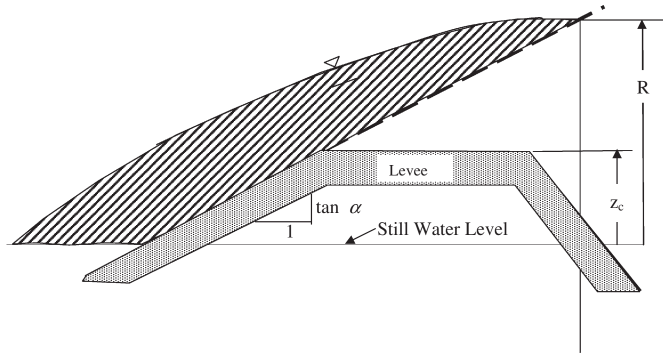

Fig. 5. Definition sketch for potential wave runup.

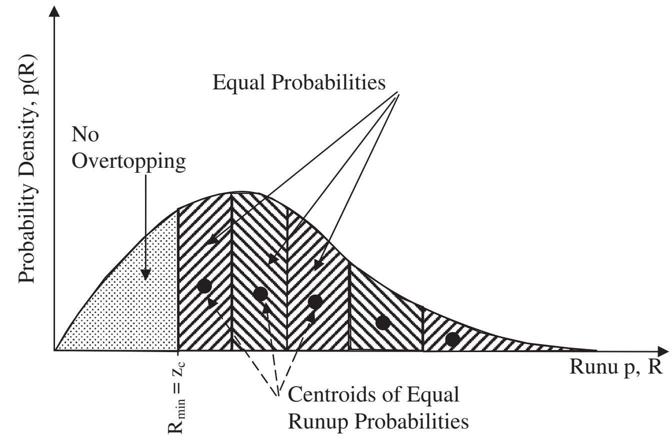

Fig. 6. Schematization of runup probability distribution showing the portion of the distribution which contributes to overtopping.

6.1.2. *Application of methodology considering Rayleigh distribution for wave runup*

The usual relationship for wave runup is in terms of the 2% runup, $R_{2\%}$ and the significant wave height, H_{mo} at the toe of the structure. According to TAW (2002), this relationship is

$$R_{2\%} = H_{mo}\left\{\begin{array}{ll} 1.75\gamma_b\gamma_f\gamma_\beta\xi_o, & 0.5 \le \gamma_b\xi_o < 1.8 \\ \gamma_f\gamma_\beta\left(4.3 - \dfrac{1.6}{\sqrt{\xi_o}}\right), & 1.8 \le \gamma_b\xi_o \end{array}\right\} \tag{21}$$

where H_{mo} is the spectral significant wave height at the toe of the levee, γ_b the reduction factor for influence of levee berm, γ_f the reduction factor for influence of levee roughness, γ_β the reduction factor for influence of oblique wave attack, ξ_o the Iribarren number $= \tan\alpha/\sqrt{H_{mo}/L_o}$, L_o the deep water wave length $= gT^2/2\pi$, and T the wave period.

For demonstration purposes here, we consider normal wave incidence ($\gamma_\beta = 1.0$), that no berm is present ($\gamma_b = 1.0$) and a completely smooth slope such that all of the γ factors are unity.

Because both the waves and runup values are Rayleigh distributed, we can develop a relationship between the runup due to individual waves which is required for application of the methodology. The cumulative Rayleigh probability distribution is expressed for wave runup R as

$$P(R < \hat{R}) = 1 - e^{-(\hat{R}^2/R_{rms}^2)} \tag{22}$$

in which $\hat{R}$ is any specified value of runup and R_{rms} is the root-mean-square of the runup. The runup probability density is determined from the above as

$$p(R) = \frac{dP}{dR} = \frac{2R}{R_{rms}^2} e^{-(R^2/R_{rms}^2)} \tag{23}$$

which can be shown to yield the following:

$$\frac{R_{2\%}}{R_{rms}} = 1.978 \tag{24}$$

With the quantification of the root-mean-square runup, R_{rms}, the runup associated with any probability can be determined from the Rayleigh distribution.

The probability, P_L, that no overtopping will occur by an individual wave is determined from the cumulative probability distribution as

$$P_L = 1 - e^{-(z_c^2/R_{rms}^2)} \tag{25}$$

where z_c is the levee crest height above the instantaneous mean water level. For further application, as illustrated in Fig. 6, the portion of the probability distribution with runup values greater than z_c is divided into N equal portions and the centroids of these elements determined from the Rayleigh distribution.

For each time step increment, Δt_j, for which the storm surge and waves can be considered reasonably constant, the runup levels are determined as described above. The total average overtopping is determined by the TAW empirical method given by

Breaking waves, $\xi_o < 1.8$

$$q_{TAW} = \left\{\begin{array}{l} 0.067\gamma_b\sqrt{\dfrac{gH_{mo}^3 \tan\alpha}{H_{mo}/L_o}}\; e^{-4.3F'} \\ F' = \dfrac{z_c\sqrt{H_{mo}/L_o}}{H_{mo}\gamma_b\gamma_f\gamma_\beta \tan\alpha} \end{array}\right\} \tag{26}$$

Non-breaking waves, $\xi_o > 1.8$

$$q_{TAW} = \left\{\begin{array}{l} 0.2\sqrt{gH_{mo}^3}\; e^{-2.3F'} \\ F' = \dfrac{z_c}{H_{mo}\gamma_f\gamma_\beta} \end{array}\right\} \tag{27}$$

in which all variables have been defined in the preceding runup discussion.

For our purposes the erosion mechanics depend on the average overtopping rate, $q_{j,n}$, during each individual overtopping event (see Eq. (20)) rather than the average over the overall time period considered as provided by the TAW method. For example, it is possible that the velocities based on the average overtopping rate are less than the critical required to cause erosion, but the higher individual overtopping rates may exceed this threshold.

The overtopping time, $\Delta t_{j,n}$ for each of the runup values, $R_{j,n}$ is equal to the representative wave period reduced by two factors. The first reduction factor is the same for all runup values in this jth time interval based on the proportion of the time that runup exceeds the threshold value and the second depends on the proportion of time that the individual runup exceeds the levee crest elevation, $z_{c,j}$. The first factor, $T_{r1,j}$ is

$$T_{r1,j} = P_{L,j} = e^{-(z_{c,j}/Rrms,j)^2} \tag{28}$$

where $P_{L,j} = P(R > z_{c,j})$ as given by Eq. (25). The second factor, $T_{r2,j,n}$, is given by

$$T_{r2,j,n} = \frac{1}{\pi}\cos^{-1}(z_{c,j}/R_{j,n}) \tag{29}$$

which is based on consideration of a sinusoidal runup form and represents the proportion of the individual wave period for which the runup exceeds the levee crest. We require that the average overtopping rate as determined by the TAW method be proportional to the sum of the runup values above the levee crest as shown in Fig. 6. Thus, the overtopping proportionality constant, K_R, is given by

$$K_{R,j} = \frac{q_{TAW,j}}{(1/I)\sum_{n=1}^{N_j}(R_{j,n} - z_{c,j})} \tag{30}$$

which considers the overtopping rate to be proportional to the excess runup although other relationships could be considered (see Footnote 1).

Thus, the average discharge rate, $q_{j,n}$ for the jth time interval and the nth overtopping event of magnitude $R_{j,n}$ is given by

$$q_{j,n} = \frac{K_{R,j}(R_{j,n} - z_{c,j})}{T_{r1,j}T_{r2,j,n}} \tag{31}$$

thereby providing a basis for determining the quantity q_n in Eq. (20).

In application under time-varying storm surge and waves, the times for which the runup would exceed the levee crest are determined as shown in Fig. 6 and the segments during which the storm surge and significant waves considered reasonably constant over time interval Δt_j are determined. During this time interval, the portion of the Rayleigh distribution that would contribute to the overtopping, $(R > R_{\min})$ is divided into N equal probabilities and their respective centroids determined. The overtopping characteristics including the discharge and duration are determined for each of these N probability elements according to Eq. (31). Summation of these results then represents the contribution to levee erosion, $\Delta(E_j/(K_W/\beta_W))$ for the jth time increment of the hydrograph

$$\Delta(E_j/(K_W/\beta_W)) = \sum_{n=1}^{N_j}\left(\frac{8g\sin\theta q_{j,n}}{f} - u_{c,W}^3\right)\Delta t_{j,n} \tag{32}$$

and

$$\Delta t_j = \sum_{n=1}^{N}\Delta t_{j,n} \tag{33}$$

with the overtopping and associated erosion contribution due to storm surge over the jth time interval, Δt_j, the erosional contribution of the (j+1)th segment of the storm surge hydrograph is determined in the same manner as described for the jth segment and added to earlier erosional contributions, etc.

7. Examples illustrating application of the methodology

Two examples are presented illustrating the application of the methodology developed herein. The number of storm surge levels, J, and the number of probabilities to consider for each surge level, N, is a matter of judgment: for each of the two examples presented here, the number of intervals for which the tide was considered constant was 40 and the number of runup probability intervals is 40, i.e., $J = 40$ and $N = 40$.

7.1. Example 1

Example 1 illustrates application of the methodology for the levee cross-section and the time-varying storm surge at the toe of the levee as presented in Figs. 7 and 8, respectively. For purposes of this example, the significant wave height at the toe of the structure is taken as 0.4 times the water depth and the wave period is considered constant at 12 s.

A computer program was developed to carry out the overtopping and erosion calculations described in the previous sections of this paper. The erosion resistance for this example will be illustrated for the three cases of grass cover quality represented in Hewlett's data (Fig. 3).

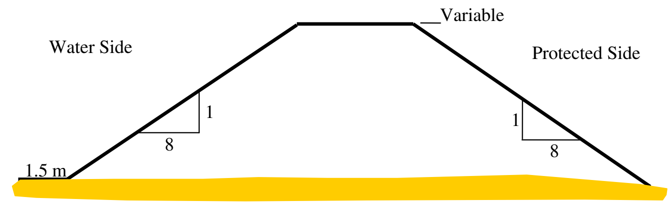

Fig. 7. Example levee cross-section.

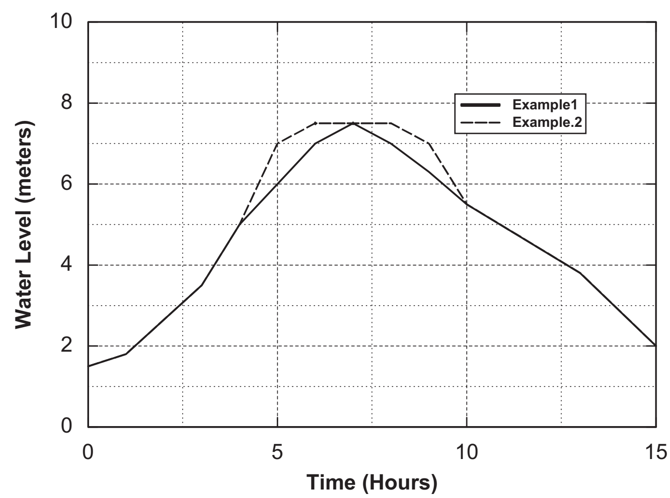

Fig. 8. Time histories of total storm surges for Examples 1 and 2.

Fig. 9 presents the time variation of the storm surge and the calculated cumulative erosion work units along with the criteria for "good", "average", and "poor" quality grass covers as determined by Hewlett (Fig. 3 herein). The levee crest elevation for the case in Fig. 9 is 10 m. It is seen that after the complete storm surge has occurred, the levee with good cover is well below the acceptable erosion whereas the average and poor grass covers have exceeded their allowable values for acceptable erosion.

Fig. 10 presents for Example 1, a comparison of the Cumulative Erosional Work Units (CEWU) for the three grass covers for a range of levee crest elevations. Additionally, the allowable values of the CEWU for acceptable erosion are shown.

First it is seen that because of the different critical velocities associated with the three grass cover qualities, the three grass covers have different CEWUs for the same levee crest elevation. For example, the number of CEWUs for "Good Cover" is less than for "Average Cover" and "Poor Cover" for the same crest elevation. Second, the allowable value of the CEWU is greater for the higher quality grass covers than for the lower quality covers as shown in Tables 2–4 and Fig. 3. As a result, for the conditions of this example, acceptable elevations for the three grass covers are: 9.8, 10.5, and 11.6 m for the "Good", "Average", and "Poor" quality grass covers, respectively.

It is interesting to compare for this example, the acceptable crest elevations by this method with the crest elevations based on current guidance. Fig. 11 presents, for a range of levee crest elevations and the storm surge and wave conditions of this example, the peak average overtopping per unit crest length based on the TAW method. It is seen that these overtopping values for the range of crest elevations considered are much greater than the

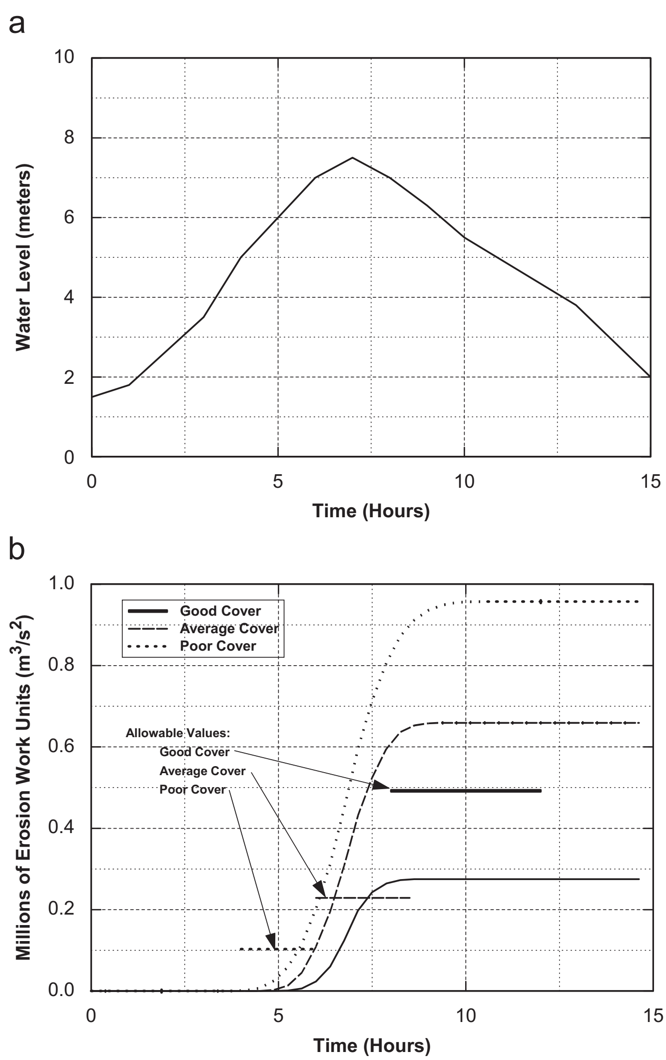

Fig. 9. Time variation of erosion work units for storm surge of Example 1 and three grass cover qualities. Levee crest elevation = 10 m for this case. (a) Time variation of total storm surge for Example 1 and (b) time variation of cumulative erosional work units for Example 1.

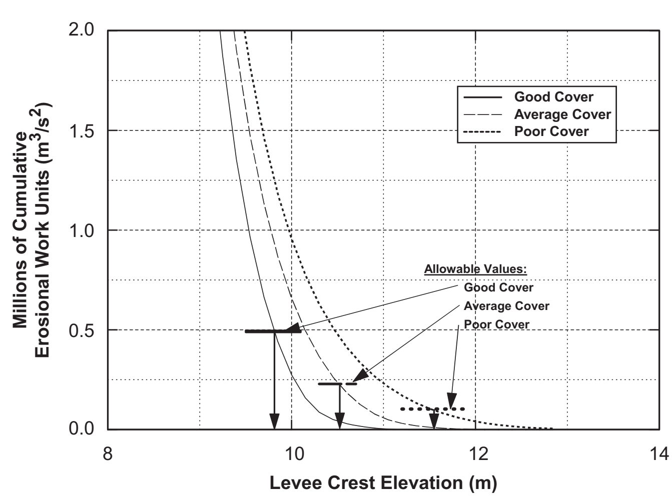

Fig. 10. Example 1: variation of erosional work units vs. levee crest elevation.

present guidance which ranges from 0.1 to 10.0 l/s per m. Table 5 presents a comparison of the required crest elevations by the methodology of this report for Example 1 and for the TAW

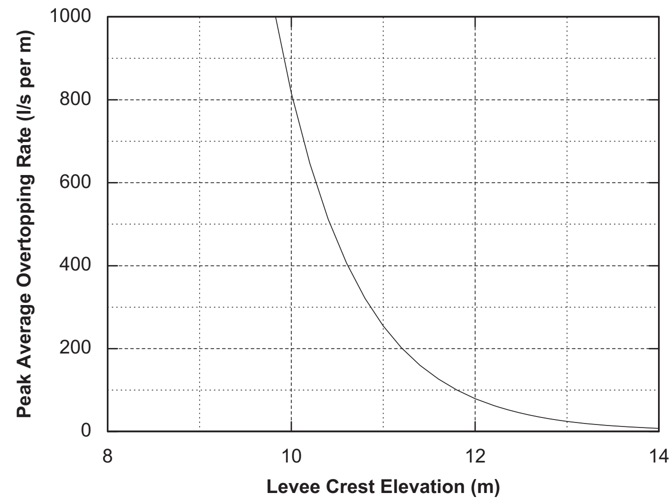

Fig. 11. Peak average overtopping rate (by TAW) vs. levee crest elevation for storm surges shown in Fig. 8.

Table 5
Required levee crest elevations for Example 1 by method of this paper and present guidance.

Grass cover quality	Required crest elevation (m)			
	Method of this paper	Present guidance (TAW, 2002)		
		$q = 0.1$ l/s per m	$q = 1.0$ l/s per m	$q = 10.0$ l/s per m
Good	9.8	15.7	13.8	11.8
Average	10.5	15.7	13.8	11.8
Poor	11.6	15.7	13.8	11.8

method for overtopping values of 0.1, 1.0, and 10.0 l/s per m of levee crest.

It is interesting that for Example 1, the required crest elevation by the methodology presented herein for the "poor" cover is slightly less than that for the largest TAW criterion, $q = 10.0$ l/s per m. For the "good" grass cover, the elevation determined by the methodology here (9.8 m) is lower than those based on the maximum overtopping criteria with the differences ranging from 2.0 m (for $q = 10.0$ l/s per m) to 5.9 m (for 0.1 l/s per m).

7.2. Example 2

The only differences here from Example 1 are that the peak storm surge elevation of 7.5 m and associated waves will be maintained for a considerably longer duration thereby illustrating the significance of surge and wave duration on land side impact. The storm surge hydrograph for Example 2 is presented in Fig. 8 and Fig. 12 presents the variation of CEWUs vs. levee crest elevation for the three grass cover qualities. As expected, the effect of longer peak storm surge duration results in higher required levee elevations for acceptable erosion. The Example 2 crest elevations exceed those for Example 1 by 0.8, 0.9, and 0.8 m, respectively for "Good", "Average", and "Poor" grass covers, respectively.

Fig. 13 presents a comparison of the effect of the storm surge hydrographs of Examples 1 and 2 for Good Grass Cover Conditions. As noted, it is seen that the required levee crest

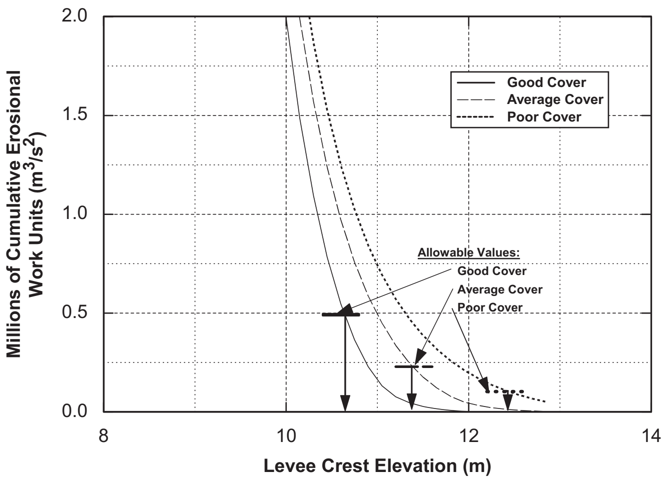

Fig. 12. Example 2: variation of cumulative erosional work units vs. levee crest elevation.

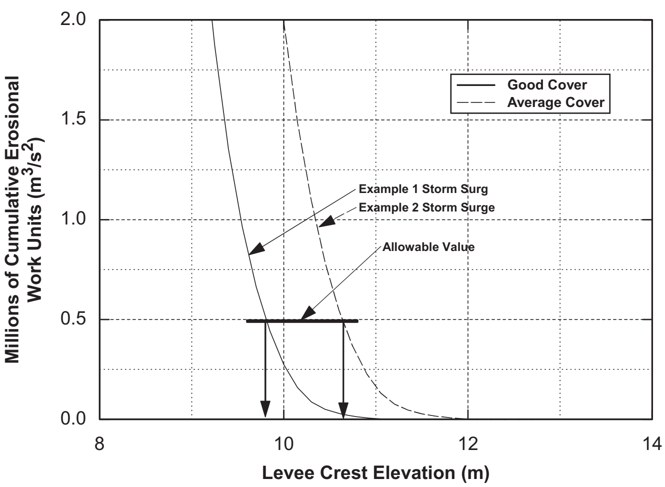

Fig. 13. Effect of storm surge hydrographs: comparison of cumulative erosional work unit results for Examples 1 and 2 and associated levee crest elevations. Good quality grass cover.

elevation increase for acceptable erosion with the greater storm surge and wave durations is approximately 0.8 m as noted above.

8. Summary and conclusions

A method has been developed and illustrated by example applications for transforming the available steady flow levee overtopping erosion relationships of Hewlett et al. (1987) to the case of intermittent wave overtopping. The basic contribution of this methodology is that it provides a capability to account for the time-varying characteristics of the intermittent wave overtopping in contributing to erosion and could also be applied to account for the effects of successive storms prior to the repair of levee damage incurred in one or more preceding storms. Although not demonstrated herein, the method is readily adaptable to stochastic modeling thereby allowing the confidence limits associated with uncertainties in storm surge and wave height and period to be taken into consideration.

The equations for non-uniform steady flows indicate that for realistic conditions, velocities at the toe on the land side of a levee will approximately reach their uniform (supercritical) values which are proportional to the one-third power of the unit discharge. Three erosion criteria were considered with respect to the Hewlett et al. (1987) steady state data relating velocities and durations for acceptable land side levee erosion: a cumulative velocity erosional index, a cumulative shear stress erosional index, and a cumulative work erosional index each with their respective threshold values below which acceptable erosion would occur. The cumulative work index criterion was found to be superior to the other two criteria and was adopted for purposes here. The TAW (2002) relationships for runup have been applied along with the consideration that runup is Rayleigh distributed. For each time increment over which storm surge elevation and associated waves can be considered reasonably constant, the portion of the runup distribution that would contribute to overtopping is considered as N equal probabilities and the associated runup values are calibrated to the average runup for average wave overtopping according to the TAW (2002) relationships. Finally, a method is developed which establishes a work threshold below which no erosion would occur and a cumulative work erosional index above which unacceptable levee erosion occurs. The time-varying characteristics of the storm surge, wave height, and wave period are taken into consideration.

Two examples have been presented illustrating application of the methodology. The examples account for the time-varying surge and wave height and differ only by the durations of the peak surges and wave heights. The grass quality (the Hewlett et al. grass quality ranged from "Poor" to "Good") is shown to affect the levee elevations by approximately 1.8 m. The sensitivity of the required levee crest elevations to the variations in peak storm tide duration was examined. It was found that acceptable levee crest elevations vary by approximately 0.8–0.9 m for the two storm surge hydrographs considered herein. Levee crest elevations based on the method developed were compared with those for the overtopping criterion of 0.1 l/s per m (which does not account for overtopping duration). For good quality grass cover, this difference (with the levee crest elevation higher for the 0.1 l/s per m criterion) is 5.9 m for Example 1 and 5.1 m for Example 2. If an overtopping criterion of 10.0 l/s per m is adopted, the differences are 2.0 m for Example 1 and 1.2 m for Example 2. For poor quality grass cover and an overtopping criterion of 0.1 l/s per m, the levee crest elevation differences are 4.1 m for Example 1 and 3.3 m for Example 2. These results underscore the value of maintaining the grass cover quality.

It is recommended that the methodology developed herein be applied in a provisional manner as follows. The method should be calibrated/verified with large scale data obtained either under laboratory or natural conditions. Specifically, the hydrodynamic characteristics during Hurricane Katrina and erosional characteristics of those levees that failed due to overtopping could be employed to calibrate/verify this methodology as could the full scale tests underway in The Netherlands. The method should not be applied for design prior to calibration/verification but could be used to provide comparisons of the effects of variable grass cover quality and time-varying storm surge and wave characteristics. Hopefully, the results of this paper will contribute to a design method that allows rational consideration of overtopping duration as well as magnitude.

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